American University of Sharjah
MTH 102
Final Exam Review

1. Find the derivative of the following functions.

   (a) $f(x) = \left(5x^4 - 14x^2\right)^2 \left(x^2 - 5\right)$

   (b) $f(x) = \sqrt[5]{8x^2 - 9x + 10}$

   (c) $y(x) = \dfrac{3}{x^2} - \ln\left(5x^2 + 1\right) - e^3 + 10$

   (d) $y(x) = \dfrac{e^{\left(3x^2+1\right)} - 4}{x^3 + x}$

   (e) $y(x) = 5^{\left(x^2+7x\right)}$

   (f) $f(x) = 5x^3 \log_5\left(9x^2 + 7x\right)$

   (g) $g(x) = \ln \sqrt[3]{x^3 + 7x}$

2. Use implicit differentiation to find the y' for $x^3 e^{5y^2+1} + 3x = 8x^2$

3. Let $p = 75 - 0.03x$ and $C(x) = 4x + 4500,\ 0 \leq x \leq 2500$ be the price-demand equation and the cost function, respectively.

   (a) Express the revenue function in terms of x and then find the average revenue at $x = 200$.

   (b) Find the exact revenue of producing the 301st item.

   (c) Use marginal revenue to approximate the revenue of producing the 301st item.

4. Find the equation of the tangent line for $y = \dfrac{3x^2 + 4}{x^2 + 2}$ at $(0, 2)$.

5. Let $f(x) = x^4 - 4x^3$.

   (a) Find the domain for $f(x)$.

   (b) Find the x and y intercepts for $f(x)$.

   (c) Find all critical values of $f(x)$.

   (d) Find all increasing and decreasing intervals for $f(x)$.

   (e) Find all relative (local) maximum and relative (local) minimum for $f(x)$.

   (f) Find all concavity intervals for $f(x)$.

   (g) Find all inflection points for $f(x)$.

6. Find the absolute maximum and minimum for $f(t) = t\left(4 - t^2\right)^3$ on $[0, 3]$.

7. A firm has a total revenue given by $R(x) = 2800x - 8x^2 - x^3$ dollars for x units of a product. Find the maximum revenue from sales of that product.

8. A car rental agency rents 200 cars per day at a rate of \$30 per day. For each \$1 reduction, 10 more cars are rented. At what rate should the cars be rented to produce the maximum revenue? What is the maximum revenue?

9. A company handles an apartment building with 70 units. Experience has shown that if the rent for each of the units is \$1080 per month, all the units will be filled, but 1 unit will become vacant for each \$20 increase in the monthly rate. What rent should be charged to maximize the total revenue from the building if the upper limit on the rent is \$1300 per month?

10. If the total cost function for a product is $C(x) = 250 + 6x + 0.1x^2$ dollars. How many units produced will minimize the average cost? Find the minimum average cost.

11. Suppose that the total cost in dollars of producing x units of a product is given by $C(x) = 10,000 + 20xe^{x/600}$. Find the marginal cost function.

12. Given the price-demand equation for a certain product to be

$$0.04q + p = 30.$$

    (a) Express the demand q as a function of the price p.
    (b) Express the revenue $R(p)$ as a function of the price p.
    (c) Find the elasticity of demand.
    (d) What is the elasticity of demand when the price $p = \$10$? If the price is decreased by 10%, what is the approximate change in demand?
    (e) For which value of p is the demand elastic?
    (f) If $p = \$25$, and the price is decreasing, will the revenue increase or decrease?

13. Find the equation of the tangent line for $x \ln y = y^2 e^x - 1$ at $(0, 1)$.

14. Let $q = f(p) = \sqrt{2500 - 2p^2}$ be the price demand equation. Find the values of p for which demand is elastic.

15. Let $q = f(p) = 2400 - 6p^2$ be the price demand equation. Find the values of p for which demand is elastic, inelastic.

16. The demand for a certain product is given by $p = \frac{900}{(q+2)^2}$, where p is the price per unit in dollars and q is demand in units of the product. Find the elasticity of demand with respect to price when $q = 14$.

17. Evaluate each of the following integrals

(a) $\int_0^1 x^3\sqrt{x^4+8}dx$

(b) $\int x^2 e^{x^3-4}dx$

(c) $\int \left(5\sqrt[3]{x^2} - 4x^{-1} + 2e^2\right) dx$

(d) $\int_0^2 \frac{4x+1}{(4x^2+2x+3)}dx$

18. The marginal price for a weekly demand of x bottles of baby shampoo in a drugstore is given by

$$p^{'}(x) = \frac{-600}{(3x+50)^2}$$

Find the price demand equation if the weekly demand is 150 when the price of a bottle of shampoo is \$4.

19. Suppose that the marginal revenue for a product is given by $\overline{MR} = \frac{-30}{(2x+1)^2} + 30$, where x is the number of units and revenue is in dollars. Find the total revenue.

20. Suppose the sales (in thousands) of a certain product will increase at the monthly rate of $S'(t) = 10 - 10e^{-0.1t}$ with $S(0) = 0$. Find the total sales during the first 7 months (from $t = 0$ to $t = 7$).

21. The marginal cost of a company that manufactures x items is given by $C'(x) = 600 - \frac{x}{4}$ where $0 \leq x \leq 2000$. Compute the increase in cost if production is increased from 200 to 500 items.

22. $\int 2x \ln x \ dx =$

(a) $x \ln x - 1/2 + C$

(b) $x^2 (\ln x - 1/2) + C$

(c) $x (x \ln x + 1/2) + C$

(d) $x (x \ln x - 1) + C$

23. $\int 5te^{2t}dt =$

(a) $\frac{5}{4}(2t-1)e^{2t} + C$

(b) $\frac{5}{4}(2t+1)e^{2t} + C$

(c) $\frac{5}{2}(2t-1)e^{2t} + C$

(d) $\frac{5}{2}(2t+1)e^{2t} + C$

24. $\int_{1}^{2} x^3 \ln x dx =$

(a) $\frac{-15}{16} + 4\ln 2$
(b) $\frac{15}{16} + 4\ln 2$
(c) $\frac{15}{16} - 4\ln 2$
(d) $\frac{-15}{16} - 4\ln 2$

25. Find the consumers' surplus (round your answer) at a price level of $p =$ \$150 for the price demand equation

$$p = D(x) = 400 - 0.05x.$$

(a) 100
(b) 250
(c) 625000
(d) 600

26. Find the producers' surplus (round your answer) at a price level of $p = 67$ for the price-supply equation

$$p = S(x) = 10 + 0.1x + 0.0003x^2.$$

(a) 1000
(b) 5000
(c) 6800
(d) 9900

27. Find (round your answer) the consumers' surplus at the equilibrium price level for the price-demand equation $p = D(x) = 80e^{-0.001x}$ and the price-supply equation $p = S(x) = 30e^{0.001x}$.

(a) 6000
(b) 6500
(c) 10000
(d) 6980

28. Find $\left(\frac{\partial^2 f(x,y)}{\partial x \partial y} = f_{xy}(x,y)\right)$ for $f(x,y) = -4x^3y^5 + 9x^3y^{-2} + y^3$

(a) $-60x^2y^4 + 54x^2y^{-3}$
(b) $60x^2y^4 - 54x^2y^{-3}$
(c) $-60x^2y^4 - 54x^2y^{-3}$

(d) $60x^2y^4 + 54x^2y^{-3}$

29. Find $\frac{\partial^2 f(x,y)}{\partial x \partial y}\Big|_{(2,3)} = f_{xy}(2,3)$ for $f(x,y) = \dfrac{\ln x}{y}$

(a) $\frac{-1}{9}$
(b) $\frac{1}{9}$
(c) $\frac{-1}{18}$
(d) $\frac{1}{18}$

Q30-31: A firm produces two types of computers each month, x of type A and y of type B. The weekly revenue and cost functions (in dollars) are

$$\begin{aligned} R(x,y) &= 80x + 90y + 0.04xy - 0.05x^2 - 0.05y^2 \\ C(x,y) &= 8x + 6y + 20000 \end{aligned}$$

30. Find the marginal profit with respect to x.

(a) $\frac{\partial P(x,y)}{\partial x} = P_x(x,y) = 72 + 0.04y - 0.1x$
(b) $\frac{\partial P(x,y)}{\partial x} = P_x(x,y) = 72 + 0.04y + 0.1x$
(c) $\frac{\partial P(x,y)}{\partial x} = P_x(x,y) = 72 - 0.04y - 0.1x$
(d) $\frac{\partial P(x,y)}{\partial x} = P_x(x,y) = 72 + 0.04x - 0.1x$

31. Find the marginal profit with respect to y.

(a) $\frac{\partial P(x,y)}{\partial y} = P_y(x,y) = 84 + 0.04x + 0.1y$
(b) $\frac{\partial P(x,y)}{\partial y} = P_y(x,y) = 84 - 0.04x - 0.1y$
(c) $\frac{\partial P(x,y)}{\partial y} = P_y(x,y) = -84 + 0.04x - 0.1y$
(d) $\frac{\partial P(x,y)}{\partial y} = P_y(x,y) = 84 + 0.04x - 0.1y$

32. The productivity of an automobile manufacturing company is given approximately by

$$f(x,y) = 50x^{0.8}y^{0.2}$$

with the utilizing of x units of labor and y units of capital. Find the marginal productivity with respect to y

(a) $f_y(x,y) = \frac{10x^{0.8}}{y^{0.8}}$
(b) $f_y(x,y) = \frac{x^{0.8}}{y^{0.8}}$
(c) $f_y(x,y) = \frac{10x^{0.2}}{y^{0.8}}$
(d) $f_y(x,y) = \frac{10x^{0.8}}{y^{0.2}}$

33. Test for relative extrema for $f(x,y) = 2x^2 - xy + y^2 - x - 5y + 8$

(a) Relative maximum at $(1,3,0)$
(b) Relative minimum at $(1,3,0)$
(c) Saddle point at $(1,3,0)$
(d) Relative maximum at $(1,2,0)$

Q34-35: A firm produces two types of calculators per year: x thousand of type A and y thousand of type B. If the revenue and cost equations for the year in millions of dollars are

$$\begin{aligned} R(x,y) &= 2x + 3y \\ C(x,y) &= x^2 - 2xy + 2y^2 + 6x - 9y + 5 \end{aligned}$$

34. How many of each type of calculators should be produced per year to maximize the profit.

(a) 2 thousands of type A and 4 thousands of type B
(b) 2 thousands of type A and 2 thousands of type B
(c) 4 thousands of type A and 4 thousands of type B
(d) 4 thousands of type A and 2 thousands of type B

35. What is the maximum profit?

(a) 10 million dollars
(b) 15 million dollars
(c) 20 million dollars
(d) 12 million dollars

36. Let $z = f(x,y) = \sqrt{2x^2 - 5xy^2}$. Then $f_x(x,y) =$

(a) $\frac{4x-5y^2}{\sqrt{2x^2-5xy^2}}$
(b) $\frac{4x-5y^2}{2\sqrt{2x^2-5xy^2}}$
(c) $\frac{4x-5xy^2}{2\sqrt{2x^2-5xy^2}}$
(d) $\frac{4x+5y^2}{2\sqrt{2x^2-5xy^2}}$

37. Let $z = f(x,y) = \sqrt{2x^2 - 5xy^2}$. Then $f_y(x,y) =$

(a) $\frac{-5xy}{2\sqrt{2x^2-5xy^2}}$
(b) $\frac{-5xy}{\sqrt{2x^2-5xy^2}}$

(c) $\frac{5xy}{\sqrt{2x^2-5xy^2}}$

(d) $\frac{-10xy}{\sqrt{2x^2-5xy^2}}$

38. Let $Q(s,t) = \frac{2s-3t}{s^2+t^2}$. Then $Q_s(s,t) =$

(a) $\frac{2\left(t^2+3st-s^2\right)}{(s^2+t^2)}$

(b) $\frac{\left(t^2+3st-s^2\right)}{(s^2+t^2)^2}$

(c) $\frac{2\left(t^2+3st-s^2\right)}{(s^2+t^2)^2}$

(d) $\frac{2\left(t^2+3st+s^2\right)}{(s^2+t^2)^2}$

39. Let $Q(s,t) = \frac{2s-3t}{s^2+t^2}$. Then $Q_t(s,t) =$

(a) $\frac{\left(3t^2+3s^2-4st\right)}{(s^2+t^2)^2}$

(b) $\frac{\left(3t^2-3s^2+4st\right)}{(s^2+t^2)^2}$

(c) $\frac{\left(3t^2-3s^2-4st\right)}{(s^2+t^2)^2}$

(d) $\frac{\left(3t^2-3s^2-4st\right)}{(s^2+t^2)}$

40. Let $z = xe^{xy}$. Then $z_{xx} =$

(a) $e^{xy}(2+xy)$

(b) $ye^{xy}(2+y)$

(c) $ye^{xy}(2+x)$

(d) $ye^{xy}(2+xy)$

41. Let $z = xe^{xy}$. Then $z_{xy} =$

(a) $e^{xy}(2+xy)$

(b) $ye^{xy}(2+xy)$

(c) $xe^{xy}(2+x)$

(d) $xe^{xy}(2+xy)$

42. Let $z = xe^{xy}$. Then $z_{yy} =$

(a) x^3e^{xy}

(b) $x^3e^{xy}(2+xy)$

(c) xe^{xy}

(d) x^2e^{xy}