Solution to 12.13.3.16

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Question: A bag contains 4 white and 5 black balls. Another bag contains 9 white and 7 black balls. A ball is transferred from the first bag to the second and then a ball is drawn at random from the second bag. Find the probability that the ball drawn is white.

Solution: Let X be the random variables that denotes which ball is picked and transferred from the first bag to the second.

$$X = \begin{cases} 0, & \text{White} \\ 1, & \text{Black} \end{cases} \tag{1}$$

The pmf of X is:

$$p_X(k) = \begin{cases} \frac{4}{9}, & k = 0 \\ \frac{5}{9}, & k = 1 \end{cases} \tag{2}$$

Let Y be a random variable that denotes which ball is drawn from the second bag after a ball from first bag is transferred to it.

$$Y = \begin{cases} 0, & \text{White} \\ 1, & \text{Black} \end{cases} \tag{3}$$

$$p_Y(0) = \Pr(Y = 0, X = 0) + \Pr(Y = 0, X = 1) \tag{4}$$

$$= \Pr(Y = 0 | X = 0) \Pr(X = 0) + \Pr(Y = 0 | X = 1) \Pr(X = 1) \tag{5}$$

$$= \left(\frac{10}{17}\right)\left(\frac{4}{9}\right) + \left(\frac{9}{17}\right)\left(\frac{5}{9}\right) \tag{6}$$

$$= \frac{5}{9} \tag{7}$$

parameter	value	description
X	$\{0, 1\}$	Denotes which ball is drawn from the first bag
Y	$\{0, 1\}$	Denotes which ball is drawn from the second bag

TABLE 0
Random Variables