Solution to Gaussian 9.3.10

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Question A bag consists of 10 balls each marked with one of the digits 0 to 9. If 4 balls are drawn successively with replacement from the bag, what is the probability that none is marked with the digit 0?
Solution:

(a) Gaussian PDF

Parameter	Values	Description
n	4	Number of balls drawn
p	0.1	Probability that the ball drawn is marked zero
$\mu = np$	0.4	Mean of distribution
$\sigma^2 = np(1-p)$	0.36	Variance of distribution
Y	0,1,2,3,4	Number of balls drawn which are zero

$$Y \sim \mathcal{N}\left(\mu, \sigma^2\right) \tag{1}$$

The gaussian distribution function is defined as:

$$p_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{2}$$

The probability that none of the balls drawn is marked with zero is given by:

$$p_Y(0) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(-\mu)^2}{2\sigma^2}} \tag{3}$$

$$= 0.532 \tag{4}$$

(b) CDF Approximation

$$\Pr(Y \le 0) = F_Y(0) \tag{5}$$

CDF of Y is:

$$F_Y(x) = \Pr(Y \le x) \tag{6}$$

$$= \Pr(Y - \mu \le x - \mu) \tag{7}$$

$$= \Pr\left(\frac{Y-\mu}{\sigma} \le \frac{x-\mu}{\sigma}\right) \tag{8}$$

Since,

$$\frac{Y-\mu}{\sigma} \sim \mathcal{N}(0,1) \tag{9}$$

Q function is defined

$$Q(x) = \Pr(Y > x) \,\forall x \in Y \sim \mathcal{N}(0,1) \tag{10}$$

$$F_Y(x) = 1 - \Pr\left(\frac{Y-\mu}{\sigma} > \frac{x-\mu}{\sigma}\right) \tag{11}$$

$$= \begin{cases} 1 - Q\left(\frac{x-\mu}{\sigma}\right), & x > \mu \\ Q\left(\frac{\mu-x}{\sigma}\right), & x < \mu \end{cases} \tag{12}$$

$$F_Y(0) = Q\left(\frac{0.4-0}{0.6}\right) \tag{13}$$
$$= Q\left(\frac{2}{3}\right) \tag{14}$$
$$= 0.252 \tag{15}$$

(c) Binomial PMF Let X be a random variable which denotes the number of balls drawn that are marked with zero,

$$p_X(k) = {}^{n}C_k p^k (1-p)^{n-k} \tag{16}$$
$$p_X(0) = {}^{4}C_0 (0.1)^0 (0.9)^4 \tag{17}$$
$$= 0.6561 \tag{18}$$

Y	Gaussian PDF	CDF Approximation	Binomial PMF
0	0.532	0.252	0.6561

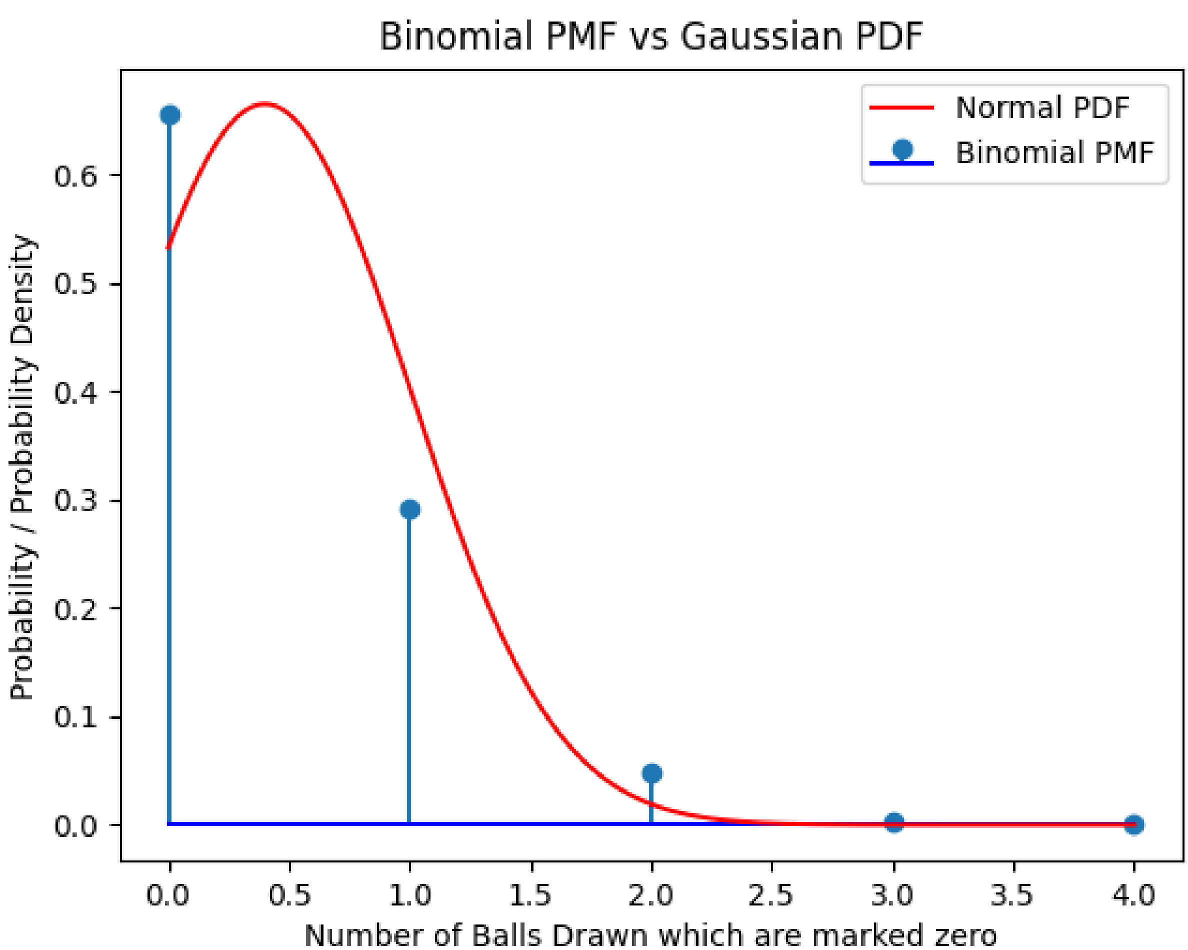

Fig. 3. Binomial PMF vs Gaussian PDF