

Solution to Gaussian 9.3.10

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Question A bag consists of 10 balls each marked with one of the digits 0 to 9. If 4 balls are drawn successively with replacement from the bag, what is the probability that none is marked with the digit 0?

Solution:

Parameter	Values	Description
n	4	Number of balls drawn
p	0.1	Probability that the ball drawn is marked zero
$\mu = np$	0.4	Mean of distribution
$\sigma^2 = np(1 - p)$	0.36	Variance of distribution
Y	0,1,2,3,4	Number of balls drawn which are zero

$$Y \sim \mathcal{N}(\mu, \sigma^2) \quad (1)$$

The gaussian distribution function is defined as:

$$p_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (2)$$

The probability that none of the balls drawn is marked with zero is given by:

$$p_Y(0) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(-\mu)^2}{2\sigma^2}} \quad (3)$$

$$= 0.532 \quad (4)$$

Using Binomial: Let X be a random variable which denotes the number of balls drawn that are marked with zero,

$$p_X(k) = {}^nC_k p^k (1 - p)^{n-k} \quad (5)$$

$$p_X(0) = {}^4C_0 (0.1)^0 (0.9)^4 \quad (6)$$

$$= 0.6561 \quad (7)$$

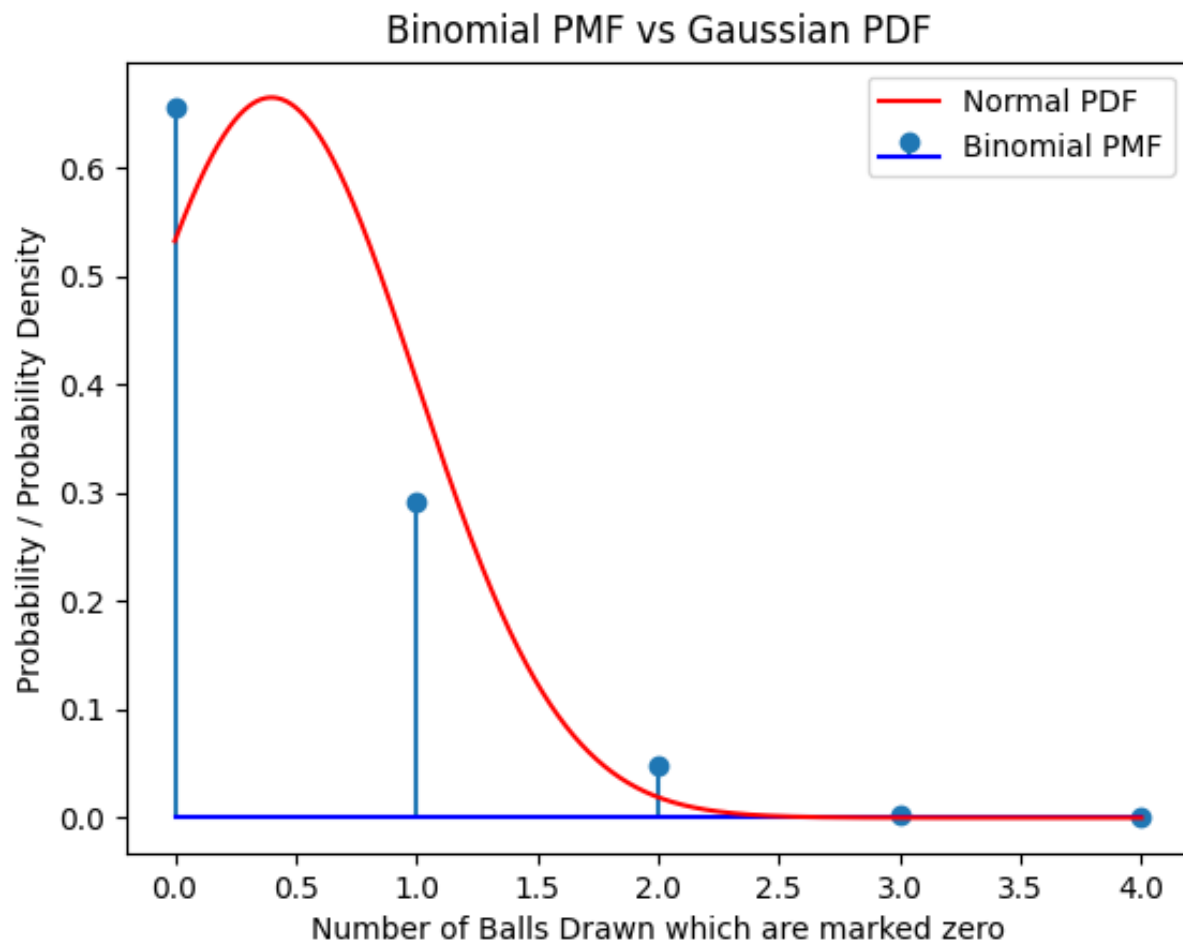


Fig. 0. Binomial PMF vs Gaussian PDF