Solution to Gaussian 9.3.10

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Question: A bag consists of 10 balls each marked with one of the digits 0 to 9. If 4 balls are drawn successively with replacement from the bag, what is the probability that none is marked with the digit 0?
Solution: Let X be a random variable defined as

Random Variable	Values	Description
X	$0 \le X \le 4$	Number of balls drawn which are not zero

X has a binomial distribution with parameters

$$n = 4 \tag{1}$$
$$p = 0.9 \tag{2}$$

The variance and mean is

$$\mu = np \tag{3}$$
$$= 3.6 \tag{4}$$
$$\sigma^2 = np\,(1-p) \tag{5}$$
$$= 0.36 \tag{6}$$

The pmf of X is given by

$$p_X(k) = {}^nC_k\,(p)^k\,(1-p)^{n-k} \tag{7}$$
$$= {}^4C_k\,(0.9)^k\,(0.1)^{4-k} \tag{8}$$

Now, the probability that none of the balls drawn is marked with zero

$$p_X(4) = {}^4C_4\,(0.9)^4\,(0.1)^0 \tag{9}$$
$$= 0.6561 \tag{10}$$

Let Y be a gaussian variable,

$$Y = \frac{X-\mu}{\sigma} \tag{11}$$

The pdf is given by

$$f_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{12}$$

When $X = 4$

$$Y = \frac{4-3.6}{0.6} \tag{13}$$
$$= \frac{2}{3} \tag{14}$$

$$p_Y\left(\frac{2}{3}\right) = 0.532 \tag{15}$$

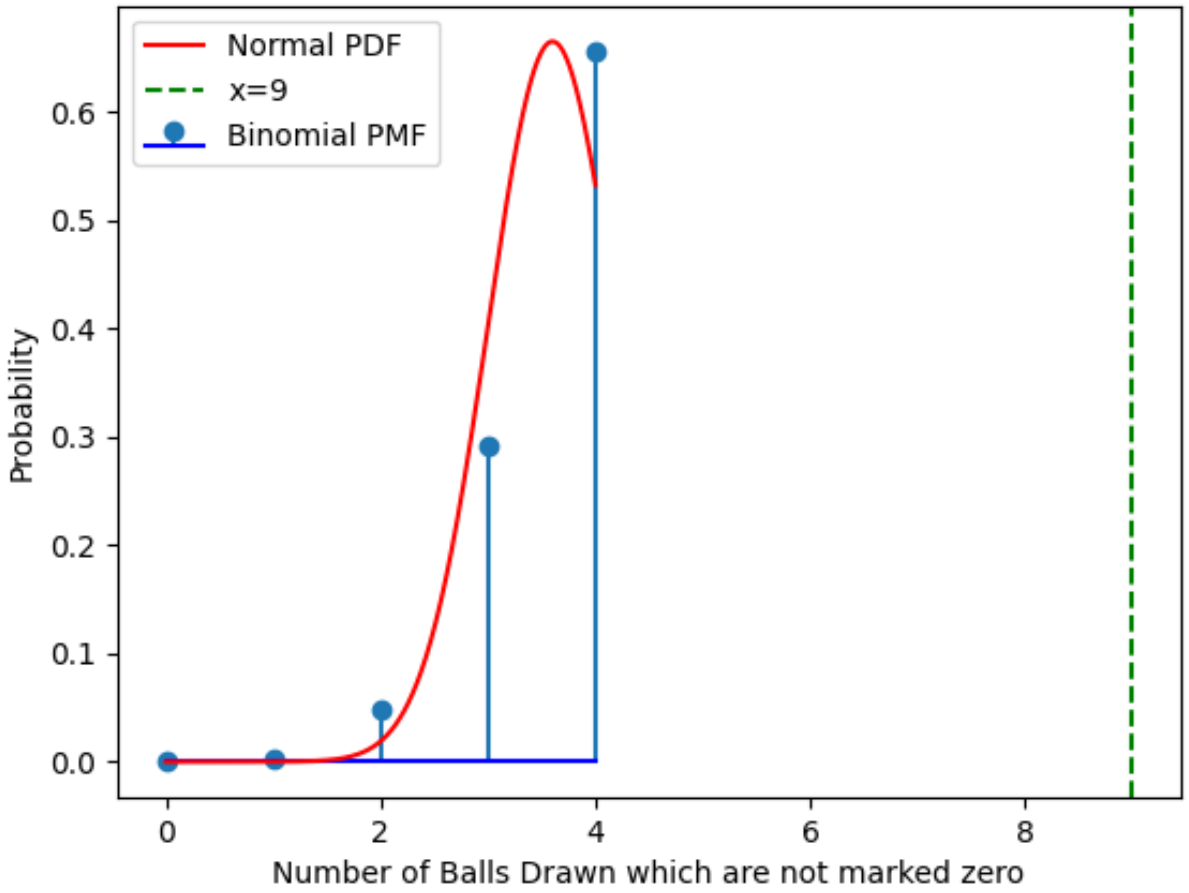

Fig. 0. Binomial pmf vs Gaussian pdf