Algebra

1. Solve for x : $\tan^{-1}\left(\dfrac{2-x}{2+x}\right) = \dfrac{1}{2}\tan^{-1}\dfrac{x}{2}, x > 0.$

2. Prove that $2\sin^{-1}\left(\dfrac{3}{5}\right) - \tan^{-1}\left(\dfrac{17}{31}\right) = \dfrac{\pi}{4}.$

3. Find the equation of the tangent line to the curve $y = \sqrt{5x-3} - 5$, which is parallel to the line $4x - 2y + 5 = 0$.

Vector Algebra

1. If $\left|\overrightarrow{a}\right| = 4, \left|\overrightarrow{b}\right| = 3$ and $\overrightarrow{a}.\overrightarrow{b} = 6\sqrt{3}$, then find the the value of $\left|\overrightarrow{a} \times \overrightarrow{b}\right|$.

2. Write the coordinates of the point which is the reflection of the point (α, β, γ) in the XZ-plane.

3. Find the position vector of the point which divides the join of points with position vectors $\overrightarrow{a} + 3\overrightarrow{b}$ and $\overrightarrow{a} - \overrightarrow{b}$ internally in the ratio $1 : 3$.

4. Show that the lines $\dfrac{x-1}{3} = \dfrac{y-1}{-1} = \dfrac{Z+1}{0}$ and $\dfrac{x-4}{2} = \dfrac{y}{0} = \dfrac{z+1}{3}$ intersect. Find their point of intersection.

5. Find the angle between the vectors $\overrightarrow{a} + \overrightarrow{b}$ and $\overrightarrow{a} - \overrightarrow{b}$ if $\overrightarrow{a} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\overrightarrow{b} = 3\hat{i} + \hat{j} - 2\hat{k}$, and hence find a vector perpendicular to both $\overrightarrow{a} + \overrightarrow{b}$ and $\overrightarrow{a} - \overrightarrow{b}$.

6. Find the coordinates of the foot of perpendicular and perpendicular distance from the point $P(4, 3, 2)$ to the plane $x + 2y + 3z = 2$. Also find the image of P in the plane.

7. Show that the relation R defined by $(a, b)\mathrm{R}(c, d) \Rightarrow a + d = b + c$ on the $A \times A$, where $A = \{1, 2, 3, \ldots, 10\}$ is an equivalence relation. Hence write the equivalence class $[(3, 4)]$; $a, b, c, d \in A$.

Matrices

1. Write the number of all possible matrices of order 2×3 with each entry 1 or 2.

2. If $A = \begin{pmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{pmatrix}$ and $BA = \left(b_{ij}\right)$, find $b_{21} + b32$.

3. Write the value of $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$.

4. On her birthday Seema decided to donate some money to children of an orphanage home. If there were 8 children less, every one would have got ₹10 more. However, if there were 16 children more, every one would have got ₹10 less. Using the matrix method, find the number of children and the amount distributed by Seema. What values are reflected by Seema's decision?

5. Solve for x : $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$, using properties of determinants.

6. Using elementary row operations find the inverse of matrix $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$ and hence solve the following system of equations $3x - 3y + 4z = 21$, $2x - 3y + 4z = 20$, $-y + z = 5$.

Functions and Relations

1. Show that the function f given by:

$$f(x) = \begin{cases} \dfrac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}, & x \neq 0 \\ -1, & x = 0 \end{cases}$$

is discontinuous at $x = 0$.

2. Verify Mean Value theorem for the function $f(x) = 2\sin x + \sin 2x$ on $[0, \pi]$.

3. Find the intervals in which the function $f(x) = \dfrac{4\sin x}{2+\cos x} - x; 0 \le x \le 2\pi$ is strictly increasing or strictly decreasing.

Integration

1. Evaluate: $\int_1^5 |x-1| + |x-2| + |x-3| dx$

2. Evaluate: $\int_0^\pi \dfrac{x\sin x}{1+3\cos^2 x} dx$

3. Find: $\int (3x+5)\sqrt{5+4x-2x^2} dx$

4. Find: $\int \dfrac{2x+1}{(x^2+1)(x^2+4)} dx$

5. Using integration, find the area of the triangle formed by the negative x-axis and tangent and normal to the circle $x^2+y^2=9$ at $\left(-1, 2\sqrt{2}\right)$.

Differentiation

1. Solve the differential equation:

$$\left(x^2+3xy+y^2\right)dx - x^2 dy = 0$$

given that $y=0$, when $x=1$.

2. If $x = e^{\cos 2t}$ and $y = e^{\sin 2t}$, prove that $\dfrac{dy}{dx} = -\dfrac{y\log x}{x\log y}$.

3. Solve the differential equation:

$$x\frac{dy}{dx} + y - x + xy\cot x = 0; x \neq 0.$$

Probability

1. A committee of 4 students is selected at random from a group consisting of 7 boys and 4 girls. Find the probability that there are exactly 2 boys in the committee, given that at least one girl must be there in the committee.

2. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6
$P(X)$	C	$2C$	$2C$	$3C$	C^2	$2C^2$	$7C^2 + C$

   Find the value of C and also calculate mean of the distribution.

3. A, B and C throw a pair of dice in that order alternately till one of them gets a total of 9 and wins the game. Find their respective probabilities of winning, if A starts first.

Optimization

1. A company manufactures two types of cardigans: type A and type B. It costs ₹360 to make a type A cardigan and ₹120 to make a type B cadigan. The company can make at most 300 cardigans and spend at most ₹72000 a day. The number of cardigans of type B cannot exceed the number of cardigans of type A by more than 200. The company makes a profit of ₹100 for each cardigan of type A and ₹50 for each cardigan of type B. Formulate this problem as a linear programming problem to maximize the profit to the company. Solve it graphically and find maximum profit.

2. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi-vertical angle α is one third that of the cone and the greatest volume of the cyclinder is $\frac{4}{27}\pi h^3 \tan^2 \alpha$.