Section - A

Question numbers 1 **to** 6 **carry 1 mark each.**

1. Write the number of vectors of unit length perpendicular to both the vectors $\overrightarrow{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\overrightarrow{b} = \hat{j} + \hat{k}$

2. Write the number of all possible matrices of order 2×2 with each entry $1, 2$ or 3.

3. If $x \in N$ and $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$, then find the value of x.

4. Write the position vector of the point which divides the join of points with position vectors $3\overrightarrow{a} - 2\overrightarrow{b}$ and $2\overrightarrow{a} + 3\overrightarrow{b}$ in the ratio $2 : 1$.

5. Find the vector equation of the plane with intercepts $3, -4$ and 2 on x, y and z axis respectively.

6. Use elementary column operation $C_2 \to C_2 + 2C_1$ in the following matrix equation:
$\begin{pmatrix} 2 & 1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$

Section - B

Question numbers 7 **to** 19 **carry** 4 **marks each.**

7. The equation of tangent at $(2, 3)$ on the curve $y^2 = ax^3 + b$ is $4x - 5$. Find the values of a and b.

8. Find the co-ordinates of the point where the line through the points A $(3, 4, 1)$ and B $(5, 1, 6)$ crosses the XZ plane. Also find the angle which this line makes with the XZ plane.

9. Find : $\int (3x + 1)\ \sqrt{4 - 3x - 2x^2}dx$

10. The two adjacent sides of a parallelogram are $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $2\hat{i} + 2\hat{j} + 3\hat{k}$. Find the two unit vectors parallel to its diagnols. Using the diagnol vectors, find the area of the parallelogram.

11. Form the differential equation of the family of circles in the second quadrant and touching the co-ordinate axes.

12. In a game, a man wins ₹5 for getting a number greater than 4 and loses ₹1 otherwise, when a fair die is thrown. The man decided to throw a die thrice but to quit as when he gets a number greater than 4. Find the expected value of the amount he wins/loses.
    OR
    A bag contains 4 balls. Two balls are drawn at random (without replacement) and are found to be white. What is the probability that all balls in the bag are white?

13. A trust invested some money in two types of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received ₹2800 as interest. However if trust had interchanged money in bonds, they would have got ₹100 less as interest. Using matrix method, find the amount invested by the trust. Interest received on this amount will be given to Helpage India as donation. Which value is reflected in this question?

14. Differentiate $x^{\sin x} + (\sin x)^{\cos x}$ with respect to x.
    OR
    If $y = 2\cos(\log x) + 3\sin(\log x)$, prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$.

15. Solve the equation for x : $\sin^{-1} x + \sin^{-1}(1-x) = \cos^{-1} x$.

OR

If $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \alpha$, prove that $\frac{x^2}{a^2} - 2\frac{xy}{ab}\cos\alpha + \frac{y^2}{b^2} = \sin^2\alpha$.

16. If $x = a\sin 2t\,(1+\cos 2t)$ and $y = b\cos 2t\,(1-\cos 2t)$, find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

17. Solve the differential equation: $y + x\frac{dy}{dx} = x - y\frac{dy}{dx}$

18. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$

OR

Evaluate: $\int_0^{\frac{3}{2}} |x\cos\pi x| dx$

19. Find: $\int \frac{x^2}{x^4 + x^2 - 2}$

Section - C

Question numbers 20 **to** 26 **carry** 6 **marks each.**

20. Using properties of determinants, show that $\triangle ABC$ is isosceles if:

$$\begin{vmatrix} 1 & 1 & 1 \\ 1+\cos A & 1+\cos B & 1+\cos C \\ \cos^2 A + \cos A & \cos^2 B + \cos B & \cos^2 C + \cos C \end{vmatrix} = 0$$

OR

A shopkeeper has 3 varieties of pens ‘A’, ‘B’ and ‘C’. Meenu purchased 1 pen of each variety for a total of ₹21. Jeevan purchased 4 pens of ‘A’ variety, 3 pens of ‘B’ variety and 2 pens of ‘C’ variety for ₹60. While Shikha purchased 6 pens of ‘A’ variety, 2 pens of ‘B’ variety and 3 pens of ‘C’ variety for ₹70. Using matrix method, find cost of each variety of pen.

21. There are two types of fertilizers ‘A’ and ‘B’. ‘A’ consists of 12% nitrogen and 5% phosphoric acid whereas ‘B’ consists of 4% nitrogen and 5% phosphoric acid. After testing the soil conditions, farmer finds that he needs at least 12 kg of nitrogen and 12 kg of phosphoric acid for his crops. If ‘A’ costs ₹10 per kg and ‘B’ cost ₹8 per kg, then graphically determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost.

22. Prove that the least perimeter of an isoceles triangle in which a circle of radius r can be inscribed is $6\sqrt{3}r$.

OR

If the sum of lengths of hypotenuse and a side of a right angled triangle is given, show that area of triangle is maximum, when the angle between them is $\frac{\pi}{3}$.

23. Five bad oranges are accidentally mixed with twenty good ones. If four oranges are drawn one by one successively with replacement, then find the probability distribution of number of bad oranges drawn. Hence find the mean and variance of the distribution.

24. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of square bounded by $x = 0, x = 4, y = 4$ and $y = 0$ into three equal parts.

25. Show that the binary operation * on $A = \mathbf{R} - \{-1\}$ defined as $a * b = a + b + ab$ for all $a, b \in A$ is commutative and associative on A. Also find the identity element of * in A and prove that every element of A is invertible.

26. Find the position vector of the foot of perpendicular and the perpendicular distance from the point P with position vector $2\hat{i} + 3\hat{j} + 4\hat{k}$ to the plane $\vec{r}.\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) - 26 = 0$. Also find the image of P in the plane.