Assignment - 2

ME-639 Intro to Robotics

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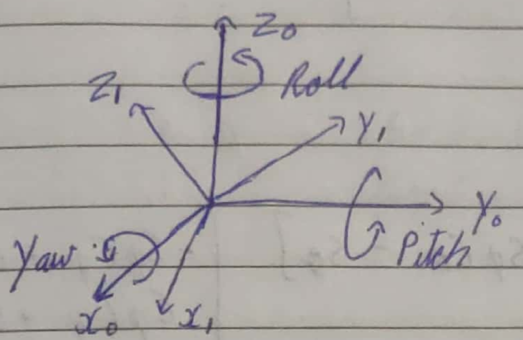

Thus using the yaw - pitch - Roll representation i.e yaw about x_0 through an angle ψ, then pitch about y_0 by an angle θ and finally roll about z_0 by angle ϕ gives the rotation matrix as

$$R_0^1 = R_{z,\phi} R_{y,\theta} R_{x,\psi}$$

$$= \begin{bmatrix} C_\phi & -S_\phi & 0 \\ S_\phi & C_\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_\theta & 0 & S_\theta \\ 0 & 1 & 0 \\ -S_\theta & 0 & C_\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_\psi & -S_\psi \\ 0 & S_\psi & C_\psi \end{bmatrix}$$

$$= \begin{bmatrix} C_\phi C_\theta & -S_\phi C_\psi + C_\phi S_\theta S_\psi & S_\phi S_\psi + C_\phi S_\theta C_\psi \\ S_\phi C_\theta & C_\phi C_\psi + S_\phi S_\theta S_\psi & -C_\phi S_\psi + S_\phi S_\theta C_\psi \\ -S_\theta & C_\theta S_\psi & C_\theta C_\psi \end{bmatrix}$$

Now we have column vectors of the rotation matrix as

$$C_1 = \begin{bmatrix} C_\phi C_\theta \\ S_\phi C_\theta \\ -S_\theta \end{bmatrix} \qquad C_2 = \begin{bmatrix} -S_\phi C_\psi + C_\phi S_\theta S_\psi \\ C_\phi C_\psi + S_\phi S_\theta S_\psi \\ C_\theta S_\psi \end{bmatrix}$$

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$$C_3 = \begin{bmatrix} S_\phi S_\psi + C_\phi S_\theta C_\psi \\ -C_\phi S_\psi + S_\phi S_\theta C_\psi \\ C_\theta C_\psi \end{bmatrix}$$

Now, we have

$$C_1^T C_2 = \begin{bmatrix} C_\phi C_\theta & S_\phi C_\theta & -S_\theta \end{bmatrix} \begin{bmatrix} -S_\phi C_\psi + C_\phi S_\theta S_\psi \\ C_\phi C_\psi + S_\phi S_\theta S_\psi \\ C_\theta S_\psi \end{bmatrix}$$

$$= -\cancel{C_\phi C_\theta S_\phi C_\psi} + C_\phi^2 C_\theta S_\theta S_\psi + \cancel{C_\theta C_\phi C_\psi S_\phi} + S_\phi^2 C_\theta S_\theta S_\psi - S_\theta C_\theta S_\psi$$

$$= C_\theta S_\theta S_\psi (C^2\phi + S^2\phi) - S_\theta C_\theta S_\psi$$
$$= 0$$

$$C_1^T C_3 = \begin{bmatrix} C_\phi C_\theta & S_\phi C_\theta & -S_\theta \end{bmatrix} \begin{bmatrix} S_\phi S_\psi + C_\phi S_\theta C_\psi \\ -C_\phi S_\psi + S_\phi S_\theta C_\psi \\ C_\theta C_\psi \end{bmatrix}$$

$$= \cancel{C_\phi C_\theta S_\phi S_\psi} + C^2\phi C_\theta S_\theta C_\psi - \cancel{S_\phi C_\theta C_\phi S_\psi} + S^2\phi C_\theta S_\theta C_\psi - S_\theta C_\theta C_\psi$$

$$= C_\theta S_\theta C_\psi (C_\phi^2 + S^2\phi) - S_\theta C_\theta C_\psi$$
$$= 0$$

$$C_2^T C_3 = \begin{bmatrix} -S_\phi C_\psi + C_\phi S_\theta S_\psi & C_\phi C_\psi + S_\phi S_\theta S_\psi & C_\theta S_\psi \end{bmatrix} \begin{bmatrix} S_\phi S_\psi + C_\phi S_\theta C_\psi \\ -C_\phi S_\psi + S_\phi S_\theta C_\psi \\ C_\theta C_\psi \end{bmatrix}$$

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$$= -S^2_\phi C_\psi S_\psi - \cancel{C^2_\psi S_\phi C_\phi S_\theta} + S^2_\psi S_\phi C_\phi S_\theta + C^2_\phi S^2_\theta S_\psi C_\psi$$
$$- C^2_\phi C_\psi S_\psi + \cancel{C^2_\psi S_\phi C_\phi S_\theta} - S^2_\psi S_\phi C_\phi S_\theta + S^2_\phi S^2_\theta S_\psi C_\psi$$
$$+ C^2_\theta C_\psi S_\psi$$

$$= -C_\psi S_\psi + S^2_\theta S_\psi C_\psi + C^2_\theta C_\psi S_\psi$$
$$= -C_\psi S_\psi + C_\psi S_\psi$$
$$= 0$$

Thus as $C_1^T C_2 = 0 = C_2^T C_1$

$C_1^T C_3 = 0 = C_3^T C_1$

$C_2^T C_3 = 0 = C_3^T C_2$

The column vectors of Rotation matrix are orthogonal

2 We have

$$R_0^1 = \begin{bmatrix} C\phi C\theta & -S\phi C\psi + C\phi S\theta S\psi & S\phi S\psi + C\phi S\theta C\psi \\ S\phi C\theta & C\phi C\psi + S\phi S\theta S\psi & -C\phi S\psi + S\phi S\theta C\psi \\ -S\theta & C\theta S\psi & C\theta C\psi \end{bmatrix}$$

$$|R_0^1| = C\phi C\theta [C^2\psi C\theta C\phi + C\theta C\psi S\theta S\phi S\psi + S^2\psi C\theta C\phi - C\theta C\psi S\theta S\phi S\psi]$$
$$+ (S\phi C\psi - C\phi S\theta S\psi)[C^2\theta C\psi S\phi - C\phi S\theta S\psi + S^2\theta C\psi S\phi]$$
$$+ (S\phi S\psi + C\phi S\theta C\psi)[C^2\theta S\psi S\phi + C\phi C\psi S\theta + S^2\theta S\phi S\psi]$$

$$= C^2\phi C^2\theta + C^2\psi S^2\phi - \cancel{C\phi C\psi S\theta S\phi S\psi} - \cancel{C\phi C\psi S\theta S\phi S\psi}$$
$$+ C^2\phi S^2\theta S^2\psi + S^2\phi S^2\psi + \cancel{C\phi C\psi S\theta S\phi S\psi} + \cancel{C\phi C\psi S\theta S\phi S\psi}$$
$$+ C^2\phi C^2\psi S^2\theta$$

$$= C^2\phi C^2\theta + S^2\phi + C^2\phi S^2\theta$$
$$= C^2\phi + S^2\phi$$
$$= 1$$

Hence $|R_0^1| = 1$

5 To Prove : $R\,S(a)\,R^T = S(Ra)$

~~the base~~ Let

$$R = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$S(a) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \quad \& \quad a = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

Now

$$R\,S(a)\,R^T = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} a_{12}a_z - a_{13}a_y & -a_{11}a_z + a_{13}a_x & a_{11}a_y - a_{12}a_x \\ a_{22}a_z - a_{23}a_y & -a_{21}a_z + a_{23}a_x & a_{21}a_y - a_{22}a_x \\ a_{32}a_z - a_{33}a_y & -a_{31}a_z + a_{33}a_x & a_{31}a_y - a_{32}a_x \end{bmatrix} \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11}a_{12}a_z - a_{11}a_{13}a_y & -a_{11}a_{12}a_z + a_{12}a_{13}a_x & +a_{11}a_{13}a_y - a_{12}a_{13}a_x \\ a_{11}a_{22}a_z - a_{11}a_{23}a_y & -a_{21}a_{12}a_z + a_{12}a_{23}a_x & +a_{21}a_{13}a_y - a_{22}a_{13}a_x \\ a_{11}a_{32}a_z - a_{11}a_{33}a_y & -a_{31}a_{12}a_z + a_{12}a_{33}a_x & +a_{31}a_{13}a_y - a_{32}a_{13}a_x \end{bmatrix}$$

$a_{12}a_{21}a_z - a_{13}a_{21}a_y \quad -a_{11}a_{22}a_z + a_{13}a_{22}a_x \quad + a_{11}a_{23}a_y - a_{12}a_{23}a_x$

$a_{22}a_{21}a_z - a_{23}a_{21}a_y \quad - a_{21}a_{22}a_z + a_{23}a_{22}a_x \quad + a_{21}a_{23}a_y - a_{22}a_{23}a_x$

$a_{32}a_{21}a_z - a_{33}a_{21}a_y \quad - a_{31}a_{22}a_z + a_{33}a_{22}a_x \quad + a_{31}a_{23}a_y - a_{32}a_{23}a_x$

$$\begin{bmatrix} a_{12}a_{31}a_z - a_{13}a_{31}a_y - a_{11}a_{32}a_z + a_{13}a_{32}a_x + a_{11}a_{33}a_y - a_{12}a_{33}a_x \\ a_{22}a_{31}a_z - a_{23}a_{31}a_y - a_{21}a_{32}a_z + a_{23}a_{32}a_x + a_{21}a_{33}a_y - a_{22}a_{33}a_x \\ a_{32}a_{31}a_z - a_{33}a_{31}a_y - a_{31}a_{32}a_z + a_{33}a_{32}a_x + a_{31}a_{33}a_y - a_{32}a_{33}a_x \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ (a_{11}a_{22} - a_{21}a_{12})a_z + (a_{21}a_{13} - a_{11}a_{23})a_y + (a_{12}a_{23} - a_{22}a_{13})a_x \\ (a_{11}a_{32} - a_{31}a_{12})a_z + (a_{31}a_{13} - a_{11}a_{33})a_y + (a_{12}a_{33} - a_{32}a_{13})a_x \end{bmatrix}$$

$$\begin{bmatrix} -[(a_{11}a_{22} - a_{21}a_{12})a_z + (a_{21}a_{13} - a_{11}a_{23})a_y + (a_{12}a_{23} - a_{22}a_{13})a_x] \\ 0 \\ (a_{32}a_{21} - a_{31}a_{22})a_z + (a_{31}a_{23} - a_{33}a_{21})a_y + (a_{33}a_{22} - a_{32}a_{23})a_x \end{bmatrix}$$

$$\begin{bmatrix} -[(a_{11}a_{32} - a_{31}a_{12})a_z + (a_{31}a_{13} - a_{11}a_{33})a_y + (a_{12}a_{33} - a_{32}a_{13})a_x] \\ -[(a_{32}a_{21} - a_{31}a_{22})a_z + (a_{31}a_{23} - a_{33}a_{21})a_y + (a_{33}a_{22} - a_{32}a_{23})a_x] \\ 0 \end{bmatrix}$$

$= S(B)$; where

$$B = \begin{bmatrix} (a_{32}a_{21} - a_{31}a_{22})a_z + (a_{31}a_{23} - a_{33}a_{21})a_y + (a_{33}a_{22} - a_{32}a_{23})a_x \\ (a_{31}a_{12} - a_{11}a_{32})a_z + (a_{11}a_{33} - a_{31}a_{13})a_y + (a_{32}a_{13} - a_{12}a_{33})a_x \\ (a_{11}a_{22} - a_{21}a_{12})a_z + (a_{21}a_{13} - a_{11}a_{23})a_y + (a_{12}a_{23} - a_{22}a_{13})a_x \end{bmatrix}$$

We have R as a rotation matrix

$$\begin{aligned} B : \quad & a_{32}a_{21} - a_{31}a_{22} \\ &= C_\theta S_\psi (S_\phi C_\theta) - (-S_\theta)(C_\phi C_\psi + S_\phi S_\theta S_\psi) \\ &= C^2_\theta S_\phi S_\psi + S_\theta C_\phi C_\psi - S^2_\theta S_\phi S_\psi \\ &= S_\phi S_\psi + S_\theta C_\phi C_\psi \\ &= a_{13} \end{aligned}$$

$a_{31} a_{23} - a_{33} a_{21}$

$= -S_\theta(-C_\phi S_\psi + S_\phi S_\theta C_\psi) - C_\theta C_\psi (S_\phi C_\theta)$

$= C_\phi S_\theta S_\psi - S^2_\theta C_\psi S_\phi - C^2_\theta C_\psi S_\phi$

$= -C_\psi S_\phi + C_\phi S_\theta S_\psi$

$= a_{12}$

$a_{33} a_{22} - a_{32} a_{23}$

$= C_\theta C_\psi (C_\phi C_\psi + S_\phi S_\theta S_\psi) - C_\theta S_\psi(-C_\phi S_\psi + S_\phi S_\theta C_\psi)$

$= C^2_\psi C_\theta C_\phi + C_\theta C_\psi S_\phi S_\theta S_\psi + S^2_\psi C_\theta C_\phi - C_\theta C_\psi S_\theta S_\phi S_\psi$

$= C_\theta C_\phi$

$= a_{11}$

$a_{31} a_{12} - a_{11} a_{32}$

$= -S_\theta(-S_\phi C_\psi + C_\phi S_\theta S_\psi) - C_\phi C_\theta (C_\theta S_\psi)$

$= S_\theta S_\phi C_\psi - S^2_\theta C_\phi S_\psi - C^2_\theta C_\phi S_\psi$

$= -C_\phi S_\psi + C_\psi S_\theta S_\phi$

$= a_{23}$

$a_{11} a_{33} - a_{31} a_{13}$

$= C_\phi C_\theta (C_\theta C_\psi) - (-S_\theta)(S_\phi S_\psi + C_\phi S_\theta C_\psi)$

$= C^2_\theta C_\phi C_\psi + S_\theta S_\phi S_\psi + S^2_\theta C_\phi C_\psi$

$= C_\phi C_\psi + S_\theta S_\phi S_\psi$

$= a_{22}$

$a_{32} a_{13} - a_{12} a_{33}$

$= C_\theta S_\psi (S_\phi S_\psi + C_\phi S_\theta C_\psi) - (-S_\phi C_\psi + C_\phi S_\theta S_\psi)(C_\theta C_\psi)$

$= S^2_\psi C_\theta S_\phi + C_\theta C_\phi \cancel{C_\psi} S_\theta S_\psi + C^2_\psi S_\theta S_\phi - C_\theta C_\phi C_\psi S_\theta S_\psi$

$= C_\theta S_\phi$

$= a_{21}$

$a_{11} a_{22} - a_{21} a_{12}$

$= C\phi C\theta (C\phi C\psi + S\phi S\theta S\psi) - S\phi C\theta (-S\phi C\psi + C\phi S\theta S\psi)$

$= C^2\phi C\theta C\psi + C\phi C\theta S\theta S\phi S\psi + S^2\phi C\theta C\psi - C\theta C\phi S\theta S\phi S\psi$

$= C\theta C\psi$

$= a_{33}$

$a_{21} a_{13} - a_{11} a_{23}$

$= S\phi C\theta (S\phi S\psi + C\phi S\theta C\psi) - C\phi C\theta (-C\phi S\psi + S\phi S\theta C\psi)$

$= S^2\phi C\theta S\psi + C\theta C\phi C\psi S\theta S\phi + C^2\phi C\theta S\psi - C\theta C\phi C\psi S\theta S\phi$

$= C\theta S\psi$

$= a_{32}$

$a_{12} a_{23} - a_{22} a_{13}$

$= (-S\phi C\psi + C\phi S\theta S\psi)(-C\phi S\psi + S\phi S\theta C\psi)$

$\quad - (C\phi C\psi + S\phi S\theta S\psi)(S\phi S\psi + C\phi S\theta C\psi)$

$= \cancel{C\phi C\psi S\phi S\psi} - C^2\psi S^2\phi S\theta - S^2\psi C^2\phi S\theta - \cancel{C\phi C\psi S\phi S\psi}$

$+ S^2\theta \cancel{C\phi C\psi S\phi S\psi} - C^2\phi C^2\psi S\theta - S^2\phi S^2\psi S\theta - S^2\theta \cancel{C\phi C\psi S\phi S\psi}$

$= -S^2\phi S\theta - C^2\phi S\theta$

$= -S\theta$

$= a_{31}$

$$\therefore \quad B = \begin{bmatrix} a_{11} a_x + a_{12} a_y + a_{13} a_z \\ a_{21} a_x + a_{22} a_y + a_{23} a_z \\ a_{31} a_x + a_{32} a_y + a_{33} a_z \end{bmatrix}$$

$\therefore \quad B = Ra$

Hence $\quad R S(a) R^T = S(Ra)$. Proved

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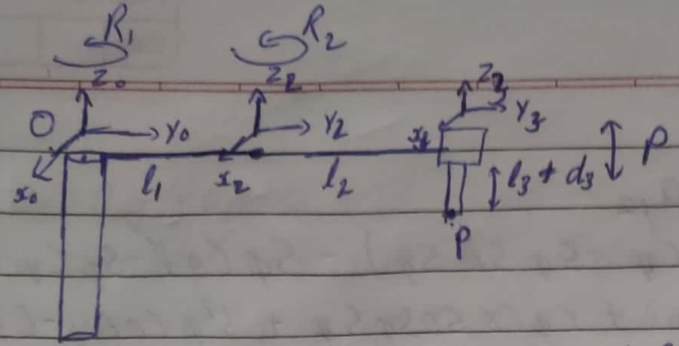
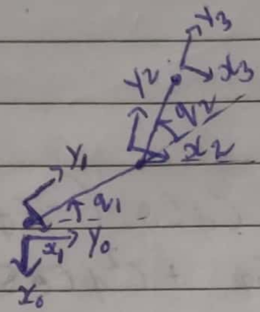


O - Origin frame

~~Front~~ Side view of RRP SCARA



Top View

$\therefore$ We have

$$R_0^1 = \begin{bmatrix} Cq_1 & -Sq_1 & 0 \\ Sq_1 & Cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} Cq_2 & -Sq_2 & 0 \\ Sq_2 & Cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} 0 \\ l_1 \\ 0 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_2^3 = \begin{bmatrix} 0 \\ l_2 \\ 0 \end{bmatrix}$$

$$\therefore \cancel{P_{o e}} \quad P = \begin{bmatrix} 0 \\ 0 \\ -(l_3+d_3) \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 \, H_1^2 \, H_2^3 \begin{bmatrix} P \\ 1 \end{bmatrix}$$

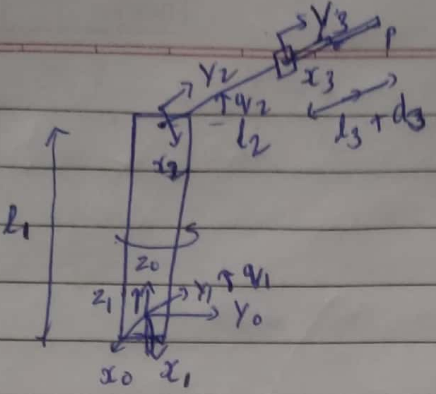
$$= \begin{bmatrix} Cq_1 & -Sq_1 & 0 & 0 \\ Sq_1 & Cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Cq_2 & -Sq_2 & 0 & 0 \\ Sq_2 & Cq_2 & 0 & l_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -(l_3+d_3) \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_{q_1+q_2} & -S_{q_1+q_2} & 0 & -l_1 S_{q_1} \\ S_{q_1+q_2} & C_{q_1+q_2} & 0 & l_1 C_{q_1} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ l_2 \\ -(l_3+d_3) \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} -l_2 S_{q_1+q_2} - l_1 S_{q_1} \\ l_2 C_{q_1+q_2} + l_1 C_{q_1} \\ -(l_3+d_3) \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} -l_2 S_{q_1+q_2} - l_1 S_{q_1} \\ l_2 C_{q_1+q_2} + l_1 C_{q_1} \\ -(l_3+d_3) \end{bmatrix}$$



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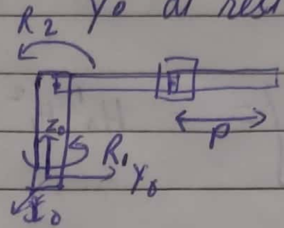
RRP - Stanford Type

Links l_2 & l_3 are along y_0 at rest



We have

$$R_0^1 = \begin{bmatrix} Cq_1 & -Sq_1 & 0 \\ Sq_1 & Cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Rotation along y_1

$$R_1^2 = \begin{bmatrix} C\pi/2 & 0 & S\pi/2 \\ 0 & 1 & 0 \\ -S\pi/2 & 0 & C\pi/2 \end{bmatrix} \begin{bmatrix} Cq_2 & -Sq_2 & 0 \\ Sq_2 & Cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ Sq_2 & Cq_2 & 0 \\ -Cq_2 & Sq_2 & 0 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_2^3 = \begin{bmatrix} 0 \\ l_2 \\ 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 \\ l_3 + d_3 \\ 0 \end{bmatrix}$$

$$\therefore \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} Cq_1 & -Sq_1 & 0 & 0 \\ Sq_1 & Cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ Sq_2 & Cq_2 & 0 & 0 \\ -Cq_2 & Sq_2 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ l_3+d_3 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -Sq_1 Sq_2 & -Sq_1 Cq_2 & Cq_1 & 0 \\ Cq_1 Sq_2 & Cq_1 Cq_2 & Sq_1 & 0 \\ -Cq_2 & Sq_2 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ l_3+d_3+l_2 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} -(l_3+l_2+d_3)\, Sq_1 Cq_2 \\ (l_3+l_2+d_3)\, Cq_1 Cq_2 \\ (l_3+l_2+d_3)\, Sq_2 + l_1 \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} -(l_2+l_3+d_3)\, Sq_1 Cq_2 \\ (l_2+l_3+d_3)\, Cq_1 Cq_2 \\ (l_2+l_3+d_3)\, Sq_2 + l_1 \end{bmatrix}$$

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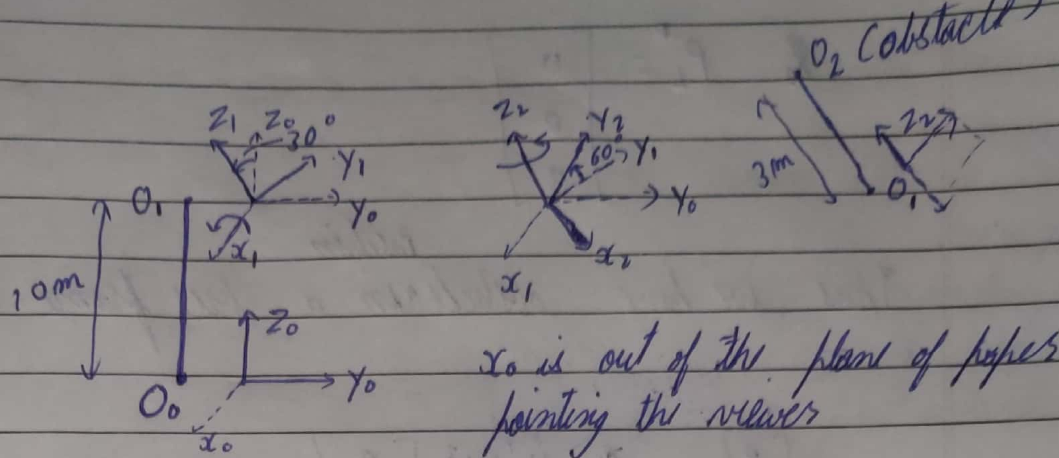

x_0 is out of the plane of paper pointing the viewer

~~He~~ From the above diagram we can notice that we have a base at O_0. The drone moves 10 m in Z_0 direction then rotates along x_0 axis by 30° giving a new frame $O_1 X_1 Y_1 Z_1$. Again the drone rotates along axis Z_1 by 60° to give $O_1 X_2 Y_2 Z_2$. From there it locates an obstacle straight up 3 m along Z_2 direction.

Thus we have

$$\cancel{R_0^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$R_0^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30 & -\sin 30 \\ 0 & \sin 30 & \cos 30 \end{bmatrix} \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} \cos 60 & -\sin 60 & 0 \\ \sin 60 & \cos 60 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$P_2 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$

Thus we have obstacle position in a base frame as

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 \begin{bmatrix} P_2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ \sqrt{3}/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ 3/4 & \sqrt{3}/4 & -1/2 & 0 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -3/2 \\ 3\sqrt{3}/2 + 10 \\ 1 \end{bmatrix}$$

Thus Obstacle position in base frame is $\left(0, -3/2, \frac{3\sqrt{3}}{2}+10\right)$

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10 Few different Types of Gearboxes

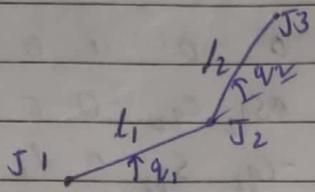
→ Planetary Gearheads :- It has good power density, can achieve high gear ratios in very compact shapes, has a good efficiencies. It has a limitation to reduce backlash & sometimes leads to heavy & bulky solutions. It is broadly used in power trains and were broadly used in the first industrial robots.

→ Harmonic Drives :- These are light weight, with zero backlash gearboxes. The multiple tooth engagement allows for large torque resistance. Peak efficiences are lower, and maximal input speed is another limitation. They ~~found~~ are used in joints closer to end effectors as well as space robots.

→ Cycloid Drives :- They are known for high robustness and ~~torque~~ ~~torsion~~ torsional stiffness. Peak effeciencies are larger and are known to withstand larger loads. They are heavier, ~~clos~~ costly and limited to low input speeds. These are mainly used in boats, cranes & CNC machines.

In drones the gearbox might be used along with the motors to either increase or decrease the propeller speed range if it doesn't matches with the available motor speed range. Other than this various kinds of drones are used for different applications like product delivery where pick up and ~~down~~ ~~drop~~ drop out might require a gripper to be attached. So different kind of gearbox might to used in the gripper ~~drop~~ drop down considering the payload, speed and motion.

11 In SCARA Manipulator we have joints 1, 2 as revolute and joint 3 is prismatic.



J3 is prismatic into the plane

Thus the Jacobian for the given manipulator is given as

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}$$

$$= \begin{bmatrix} z_0 \times (O_3 - O_0) & z_1 \times (O_3 - O_1) & z_2 \, \cancel{\times (O_3 - O_2)} \\ z_0 & z_1 & 0 \; \cancel{z_2} \end{bmatrix}$$

$$\therefore \; O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad O_1 = \begin{bmatrix} l_1 c_{q_1} \\ l_1 s_{q_1} \\ 0 \end{bmatrix}$$

$$O_2 = \begin{bmatrix} l_1 c_{q_1} + l_2 c_{q_1+q_2} \\ l_1 s_{q_1} + l_2 s_{q_1+q_2} \\ 0 \end{bmatrix}$$

$$O_3 = \begin{bmatrix} l_1 c_{q_1} + l_2 c_{q_1+q_2} \\ l_1 s_{q_1} + l_2 s_{q_1+q_2} \\ -(l_3 + d_3) \end{bmatrix}$$

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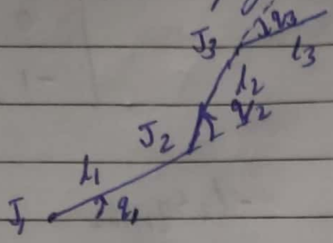
$$Z_0 = Z_1 = \cancel{Bq} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad Z_2 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$\therefore \quad J = \begin{bmatrix} -l_1 S_{q_1} - l_2 S_{q_1+q_2} & -l_2 S_{q_1+q_2} & 0 \\ l_1 C_{q_1} + l_2 C_{q_1+q_2} & l_2 C_{q_1+q_2} & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

13 For RRR configuration, we have all joints 1, 2, 3 as revolute



The Jacobian for the given manipulator is given as

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}$$

$$= \begin{bmatrix} Z_0 \times (O_3 - O_0) & Z_1 (O_3 - O_1) & Z_2 (O_3 - O_2) \\ Z_0 & Z_1 & Z_2 \end{bmatrix}$$

where $O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $\quad O_1 = \begin{bmatrix} l_1 Cq_1 \\ l_1 Sq_1 \\ 0 \end{bmatrix}$

$$O_2 = \begin{bmatrix} l_1 Cq_1 + l_2 Cq_1 + q_2 \\ l_1 Sq_1 + l_2 Sq_1 + q_2 \\ 0 \end{bmatrix} \qquad O_3 = \begin{bmatrix} l_1 Cq_1 + l_2 Cq_1 + q_2 + l_3 Cq_1 + q_2 + q_3 \\ l_1 Sq_1 + l_2 Sq_1 + q_2 + l_3 Sq_1 + q_2 + q_3 \\ 0 \end{bmatrix}$$

$$Z_0 = Z_1 = Z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore J = \begin{bmatrix} -l_1 S_{q_1} - l_2 S_{q_1+q_2} - l_3 S_{q_1+q_2+q_3} & -l_2 S_{q_1+q_2} - l_3 S_{q_1+q_2+q_3} & -l_3 S_{q_1+q_2+q_3} \\ l_1 C_{q_1} + l_2 C_{q_1+q_2} + l_3 C_{q_1+q_2+q_3} & l_2 C_{q_1+q_2} + l_3 C_{q_1+q_2+q_3} & l_3 C_{q_1+q_2+q_3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$