(4) Find Orbital Period of the inner planet, using
(a) Kepler's Third law ⇒

(a) a (semimajor axis)$_{\text{inner planet}}$ = 0.8 A.U.

mass of star = $1\ M_\odot$

$$T^2 = \frac{a^3}{M}$$

$$T^2 = \frac{(0.8)^3}{1}$$

$$T^2 = 0.512$$

$$T \approx 0.715 \text{ years} \approx 261 \text{ days}$$

Hence, the required orbital period of the inner planet is 261 days or 0.7155 years.

(b) Calculate the semi-major axis and orbital period of the outer planet based on resonance ratio.

(b) Given ⇒ 3 : 2 mean-motion resonance, which means that inner planet completes 3 orbits in the time, outer planet completes 2 orbits.

Now,

$$\frac{T_{outer}}{T_{inner}} = \frac{3}{2}$$

$$T_{outer} = \frac{3}{2} T_{inner}$$

$$= \frac{3}{2} \times 0.715 \text{ years}$$

$$\boxed{T_{outer} \approx 1.075 \text{ years}}$$

Now,

$$T_{outer}^2 = \frac{a_{outer}^3}{M_{\odot}} \quad \text{(By Kepler's 3rd law)}$$

$$(1.075)^2 = \frac{a_{outer}^3}{1}$$

$$\boxed{a_{outer} \approx 1.048 \text{ A.U.}}$$

Hence, the required semi-major axis & orbital period of outer planet are :- (i) 1.048 A.U.
(ii) 1.075 years

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(C) Impact of Orbital Resonance on System's Long Term Stability and Habitability ⇒

(a) Long - Term Stability ⇒

(i) Positive Impacts ⇒

① Predictable Orbits ⇒ Synchronized periods between planets, resulting in a repetitive and predictable gravitational influence.

This can help prevent chaotic variations in their orbits over millions of years, contributing to system's overall stability.

② Energy & Angular Momentum Exchange ⇒ Resonance allow the planets to exchange angular momentum in a controlled manner, keeping the system in a balanced configuration.

(ii) Negative Impacts ⇒

① Orbital Eccentricity Growth ⇒ Resonances can lead to gradual increase in orbital eccentricity. For example, repeated gravitational interactions might push one or both planets into more elliptical orbits. Overtime may destabalize the system.

② Chaotic Behaviour ⇒ If there are additional planets in the system or if external pertubations. (e.g. from nearby passing object) occur, resonance might amplify instead of dampening the effect, thus creating the system unstable.

(b) Habitability ⇒

(i) Positive Impacts ⇒

① Orbital Stability ⇒

In systems with well-defined resonances, the planets can remain in a habitable zone for billions of years. This is important for developement of life.

② Geological Activity and Potential Habitability ⇒

Resonances can induce tidal heating, which is a game changer for icy planets / moons. Moderate tidal heating can drive tectonic activity, recycling carbon, which could help stabalize the planet's climate.

(ii) Negative Impacts ⇒

① Tidal Heating and Overheating ⇒

In strong resonance, tidal force may become excessive, leading to extreme internal heating. This can result in :-

1) Intense volcanic activity as seen on "Io".

2) Destruction of atmosphere

② Climate instability ⇒ Changes in eccentricity due to resonance, can cause planet to oscillate in and out of the habitable zone