# "Problem – 5"

① To derive escape velocity formula using the conservation of energy

Theory :- Escape velocity is an occurrence when an object's total mechanical energy (Kinetic + Gravitational) is zero at $\infty$ distance from the said massive body

$E_{initial} = KE_{initial} + GE_{initial}$

$= \frac{1}{2} m v_{esc}^2 + \left(-\frac{GMm}{R}\right)$

$v_{esc}$ → Vescape

G → gravitional constant

M → mass of massive body

m → mass of smaller body

R → radius of massive body

$E_{final} = 0$ (By theory)

Now, by conservation of Energy ⇒

$E_{intial} = E_{final}$

$\frac{1}{2} m v_{esc}^2 - \frac{GMm}{R} = 0$

$$\boxed{V_{esc} = \sqrt{\frac{2GM}{R}}}$$

② Calculate the escape velocity and express it as a fraction of speed of light 'c'.

Using the formula derived in part -1 ⇒

$$V_{escape} = \sqrt{\frac{2GM}{R}}$$

$G = 6.674 \times 10^{-11} \ m^3 \ kg^{-1} \ s^{-2}$

$M = 1.4 \ M_\odot \approx (1.4) \times (1.989 \times 10^{30}) \ kg$

$R = 0.008 \ R_\odot \approx (0.008) \times (6.96 \times 10^{8}) \ m$

$$\boxed{\begin{aligned} V_{escape} &= 8.17 \times 10^{6} \ m/s \\ \text{or} \quad & 8170 \ km/s \end{aligned}}$$

Now,

$c \Rightarrow$ speed of light

$$V_{escape} = \frac{8.17 \times 10^{6}}{c} \times c$$

$$= 2.72 \times 10^{-2} \times c$$

$$= 0.0272 \ c$$

or $2.72\%$ of speed of light

③

The high escape velocity near a white dwarf like ~~87~~ 8170 km/s in this question, profoundly impacts the physics of matter near it. The effects are but not limited to ⇒

(a) Electron degeneracy pressure
(b) Acceleration Disks
(c) Gravitational Redshift
(d) Nuclear burning and Novae

As asked in the question, we will focus on electron degeneracy pressure and ~~accer~~ accretion disks.

(a) <u>ELECTRON DEGENERACY PRESSURE</u> ⇒

Due to high density of white dwarfs, it creates a strong gravitational collapse is provided by electron degeneracy pressure, a quantum mechanical model, which arises from the pauli Exclusion principle. According to this, electrons are forced into higher-energy states as their wave functions cannot overlap, creating pressure independent of temperature. At higher masses (by higher, we mean those approaching chandrasekhar limit of 1.4 $M_{\odot}$ (solar mass), reduces electron degeneracy effect by pushing electrons towards relativistic speeds.

Hence, it prevents white dwarf from collapsing further under it own gravity.

(b) Accoretion Disks ⇒

The process of accretion involves material from a companion star falling and spiralling inwards due to the intense gravitational attraction of the white dwarf, heating up along the way. This process can lead to X-ray emission and other forms of EMR.

As the material approaches the white dwarf, it gains ~~extreme~~ escape velocity, the intense gravity prevents it leaving & continues spiral motion and the angular momentum in disk is transfered, which allows heating disk to high temperatures often producing bright X-rays.

In some cases, ~~especi~~ especially in binary systems systems, accretion disks can give rise to relativistic jets. These jets are result of the high energy interactions near the event horizon and intense magnetic field, which are often generated by the rapid rotation of the white dwarf.