

Prob. 1. Avg. Weight = 1 kg =  $\mu_0$   
Std. Dev. = 0.1 =  $\sigma$   
Sample = 100 bags  
Mean = 1.03 ;  $\alpha = 5\%$

ATQ,  $\sum_{i=1}^{100} \frac{W_i}{100} = 1.03$  — (1)

We state the hypothesis as:

$$H_0: \mu = 1 \text{ kg}$$

$$H_1: \mu \neq 1 \text{ kg}$$

Formula for calculating t-stat.

$$t = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{1.03 - 1.00}{0.1 / \sqrt{100}} = \frac{(0.03)(10)}{(0.1)} = 3$$

The degrees of freedom =  $df = 99$

The two-tailed critical value for t-stat at  $df = 99$ ,  $\alpha = 5\%$  is found as 1.660

$$\boxed{\text{Observed } t = 3 > \text{Critical } t = 1.660}$$

So, we fail to accept null hypothesis at 5% level of significance.

Hence, we conclude that process mean is not still 1 kg.

Ques 2. Mean Response Time of a species of pigs to a stimulus is  $\mu_0 = 0.8 \text{ sec}$   
 No of pigs = 28 and given 2oz of alcohol  
 Avg response time = 1sec  
 Std. Dev. = 0.3 sec  
 $\alpha = 5\%$

Let the mean (population mean) response time of a species of pigs to a stimulus is  $\mu$

Now, we have to test (population mean) response time is changed or not after giving 2 oz of alcohol to the pigs.

We state the hypothesis as:

$$H_0: \mu = 0.80 \text{ against } H_1: \mu \neq 0.80$$

The test is two-tailed.

→ Formula for calculating t-stat is

$$t = \frac{\bar{x} - \mu_0}{\left( \frac{s}{\sqrt{n}} \right)}$$

$$= \frac{1 - 0.8}{\frac{0.3}{\sqrt{28}}} = 3.5277$$

The degrees of freedom is  $df = (n-1) = 27$

The two-tailed critical value for t-stat at  $df = 27$  and  $\alpha = 5\%$  is found (from table of t-dist.) as 1.703

$$\text{Observed } t = 3.5277 > \text{Critical } t = 1.703$$

Therefore, we fail to accept the null hypothesis  $H_0: \mu = 0.8$  at 5% level of significance.

Hence, we reject the null hypothesis and we conclude that alcohol affects the mean response time.



Ques 3. Claim: Whether the mean weight of this year's crop is less than 7.6 pounds or not.

1. Construction of Null & Alternative hypothesis

$$H_0: \mu \geq 7.6$$

$$H_1: \mu < 7.6$$

2. Computation of std. error of sample mean weight.

$$\text{No of fish considered} = n = 16$$

$$\text{Sample mean weight} = \bar{x} = 7.2$$

$$\text{Population Std. Dev.} = \sigma = 1.2$$

Now, we determine std. error of the sample mean weights of fishes is  $\frac{\sigma}{\sqrt{n}} = \frac{1.2}{\sqrt{16}} = 0.3$

Level of significance  $\alpha = 0.05$ .

From std. normal table, right tailed  $z_{\frac{\alpha}{2}} = 1.645$ .

This is a z-test because the problem provides the "population" standard deviation

$$z\text{-stat} = \frac{7.2 - 7.6}{1.2/\sqrt{16}} = (-1.33)$$

$$P\text{-Value} = P(z > -1.33) = 0.908$$

this strong enough evidence to reject the hatchery's claim. at the

(a) 5 percent: No, because  $P\text{-value} > 0.05$

(b) 1 percent: No, because  $P\text{-value} > 0.01$

(c)  $P\text{-value} = 0.908$

Ans 4.

$n = 456$  students

avg score = 60 =  $\bar{x}$

$\sigma = s = 5.6$

National avg = 56.5 =  $\mu_0$

$$H_0: \mu \leq 56.5$$

$$H_1: \mu > 56.5$$

$$t = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{(60 - 56.5)}{5.6} \sqrt{456}$$

$$= \left( \frac{3.5}{5.6} \right) \sqrt{456} = 13.34$$

Observed  $t >$  Critical  $T$

hence we reject  $H_0$ .

Ans 5.  $H_0: \mu \leq 100$

$$H_1: \mu > 100$$

$$n = 20$$

$$\sigma = 5$$

$$\bar{X} = 105$$

~~$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$~~

$$z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{105 - 100}{5/\sqrt{20}} = 4.47$$

The result is significant at  $p < 0.05$



Ans 6.

From the sample we can compute the following statistics:

$$\bar{X} = 26.4 \quad \& \quad S = 3.502$$

From the question, we want to test the null hypothesis

$$H_0: \mu \geq 30 \quad \text{versus} \quad H_1: \mu < 30.$$

Since,  $n < 30$  and  $\sigma$  is unknown, we use the student t-distribution with  $n-1$  degree of freedom. With  $T_{n-1} = T_9$  being t-random variable with  $n-1=9$  degrees of freedom, we have the following p-value.

$$P \left\{ T_9 < \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \right\} = P \{ T_9 < -3.25 \} \approx 0.005$$

Hence, the hypothesis that  $\mu \geq 30$  is rejected at any reasonable significance level  $\alpha$  larger than 0.005.

Ans 7.

$H_0$ : Mean Value = 210

$H_1$ : Mean Value > 210

Test Statistic:  $t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$

(a)  $n = 25$

$$t = \frac{200 - 210}{35/\sqrt{25}} = -1.4286$$

p-value = 0.083 at 24 df and one-tailed

Since, the p-value is more than 5%, there is insufficient evidence to reject the null hypothesis.

(b)  $n = 64$

$$t = \frac{200 - 210}{35/\sqrt{64}} = -2.28576$$

p-value = 0.01285 at 63 df

Since, the p-value is less than 5%, there is:

(The result is significant at  $p < 0.05$ )

Ans 8. Mean = 9

Bottles Sold = 9, 10, 8, 12, 13, 10, 9

Significance = 0.01

$H_0: \mu = 9$

$H_1: \mu \neq 9$

It's two-tailed test

Population mean = 10.14

Sample Std = 1.64

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{10.14 - 9}{1.64/\sqrt{7}} = 1.839$$

$$df = n - 1 = 7 - 1 = 6$$

$$\alpha = 0.01$$

So, p-value = 0.1155

Hence the result is not significant  $p < 0.01$



Ans 9  $H_0$ : Range of rockets is 2500 kms  
 $H_1$ : Range of rockets is less than 2500 kms

Sample Mean:

$$= (2490 + 2510 + 2360 + 2410 + 2300 + 2440) / 6$$
$$= 2418.33$$

Sample Standard Deviation = 79.352

Sample Size  $n = 6$

Since std of population is unknown we will do a t-test.

And, it's theorized that the range will be reduced after the rockets are in storage for some time.

$$t = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$$

$$= \frac{(2418.33 - 2500) \sqrt{6}}{79.352}$$

$$= \frac{(-81.67) \sqrt{6}}{79.352} = (-1.029) \sqrt{6}$$

$$= \boxed{-2.52}$$

$$df = 6 - 1 = 5$$

At 1% significance  $p$ -value = 0.02. Thus the result is not significant at  $p < 0.01$ .

Ans 10

In such situation:

Alternate Hypothesis is what requires a proof

$$H_0: \sigma \geq 0.10$$

$$H_1: \sigma < 0.10$$

$$\text{So } \chi^2 \text{ calculation: } = \frac{(n-1)s^2}{\sigma^2} = \frac{(49)(0.0064)}{0.01} \\ = 31.36$$

So, if the true standard deviation were 0.1 (the minimum unsafe level), then observing a sample standard deviation of 0.08 or smaller has probability  $1 - 0.976 \approx 0.0235$

ie, the probability that  $\chi^2$  variable with 49 deg. of freedom would have a 97.6513% chance of being above 31.36 :  $P(\chi^2_{49} > 31.36) = 0.976513$ .

That is, the observed sample standard deviation is not really consistent with the hypothesis that the true standard deviation is 0.1 or larger.

So, we have evidence that the true deviation is  $< 0.1$  and that the device is safe.

Ans 11 Given

Population Standard Deviation ( $\sigma$ ) = 5

Sample Size ( $n$ ) = 20

Sample Std. Dev. ( $s$ ) = 8

Population Variance = 25 = ( $\sigma^2$ )

Sample Variance = 64 = ( $s^2$ )

$$H_0: \sigma \leq 5$$

$$H_1: \sigma > 5$$

$$\alpha = 0.05$$

From standard normal table,

Since our test is right-tailed

Reject  $H_0$  if  $(\chi)^2 > 30.144$

We use test statistic

$$(\chi)^2 = \frac{(n-1)s^2}{\sigma^2}$$

$$= 48.64$$

The value of  $(\chi)^2_\alpha$  at  $\log 0.05$  and  $df = 19$  is 30.144

Hence,  $(\chi)^2_{\text{calc.}} > (\chi)^2_{\text{alpha}}$

So, we reject  $H_0$   $(\chi)^2$   $p\text{-value} = 0.0002$