

UNIT-II

Search Algorithms in Artificial Intelligence

Search Algorithms in Artificial Intelligence

Search algorithms are one of the most important areas of Artificial Intelligence.

Search in AI is the process of navigating from a **starting state** to a **goal state** by transitioning through **intermediate states**.

Almost any AI problem can be defined in these terms.

- **State** — A potential outcome of a problem
- **Transition** — The act of moving between states.
- **Starting State** — Where to start searching from.
- **Intermediate State** - The states between the starting state and the goal state that we need to transition to.
- **Goal State** — The state to stop searching.
- **Search Space** — A collection of states.

Terminologies:

Search: Searching is a step by step procedure to solve a search-problem in a given search space. A search problem can have three main factors:

Search Space: Search space represents a set of possible solutions, which a system may have.

Start State: It is a state from where agent begins the search.

Goal test: It is a function which observe the current state and returns whether the goal state is achieved or not.

Search tree: A tree representation of search problem is called Search tree. The root of the search tree is the root node which is corresponding to the initial state.

Actions: It gives the description of all the available actions to the agent.

Transition model: A description of what each action do, can be represented as a transition model.

Path Cost: It is a function which assigns a numeric cost to each path.

Solution: It is an action sequence which leads from the start node to the goal node.

Optimal Solution: If a solution has the lowest cost among all solutions.

Properties of Search Algorithms:

Following are the four essential properties of search algorithms to compare the efficiency of these algorithms:

Completeness: A search algorithm is said to be complete if it guarantees to return a solution if at least any solution exists for any random input.

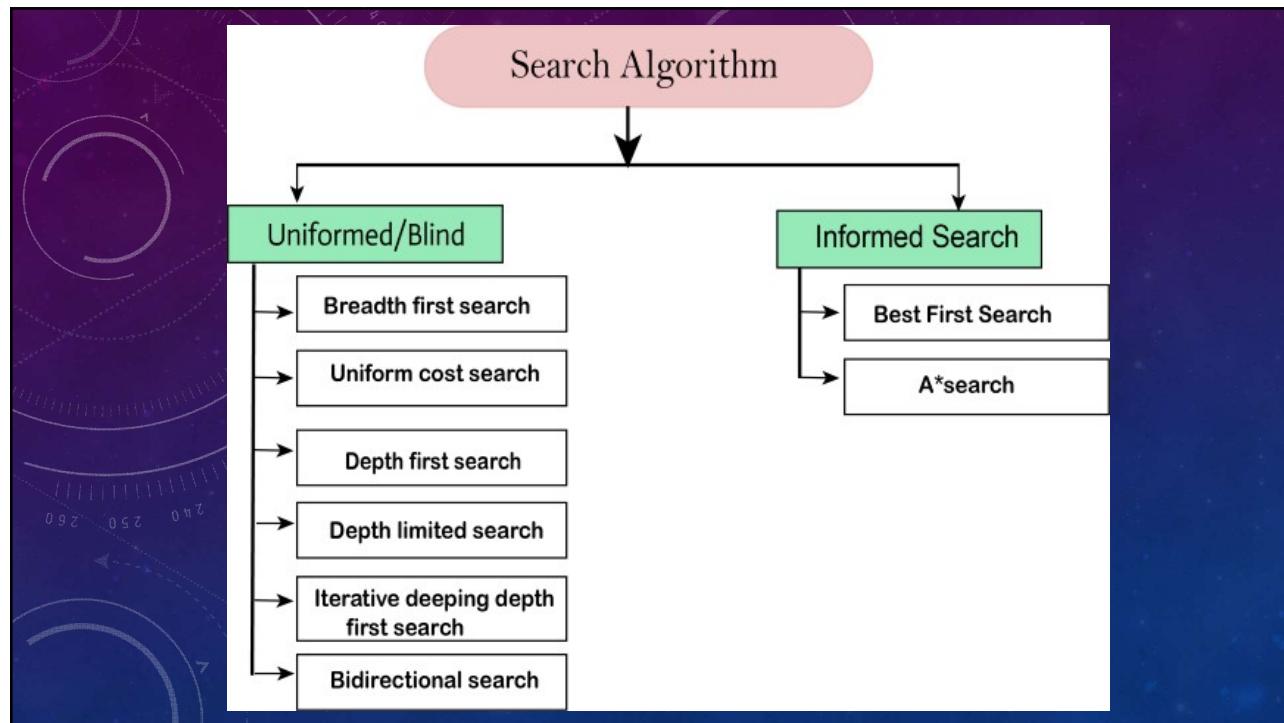
Optimality: If a solution found for an algorithm is guaranteed to be the best solution (lowest path cost) among all other solutions, then such a solution for is said to be an optimal solution.

Time Complexity: Time complexity is a measure of time for an algorithm to complete its task.

Space Complexity: It is the maximum storage space required at any point during the search, as the complexity of the problem.

Types of search algorithms

Based on the search problems we can classify the search algorithms into uninformed (Blind search) search and informed search (Heuristic search) algorithms.



Uninformed/Blind Search:

The uninformed search does not contain any domain knowledge such as closeness, the location of the goal. It operates in a brute-force way as it only includes information about how to traverse the tree and how to identify leaf and goal nodes. Uninformed search applies a way in which search tree is searched without any information about the search space like initial state operators and test for the goal, so it is also called blind search. It examines each node of the tree until it achieves the goal node.

It can be divided into five main types:

- Breadth-first search
- Depth-first search
- Uniform cost search
- Iterative deepening depth-first search
- Bidirectional Search

1. Breadth-first Search:

It is a simple search strategy where the root node is expanded first, then covering all other successors of the root node, further move to expand the next level nodes and the search continues until the goal node is not found. BFS expands the shallowest node first using FIFO order. Thus, new nodes remain in the queue and old unexpanded node which are shallower than the new nodes, get expanded first.

In BFS, goal test (a test to check whether the current state is a goal state or not) is applied to each node at the time of its generation rather when it is selected for expansion.

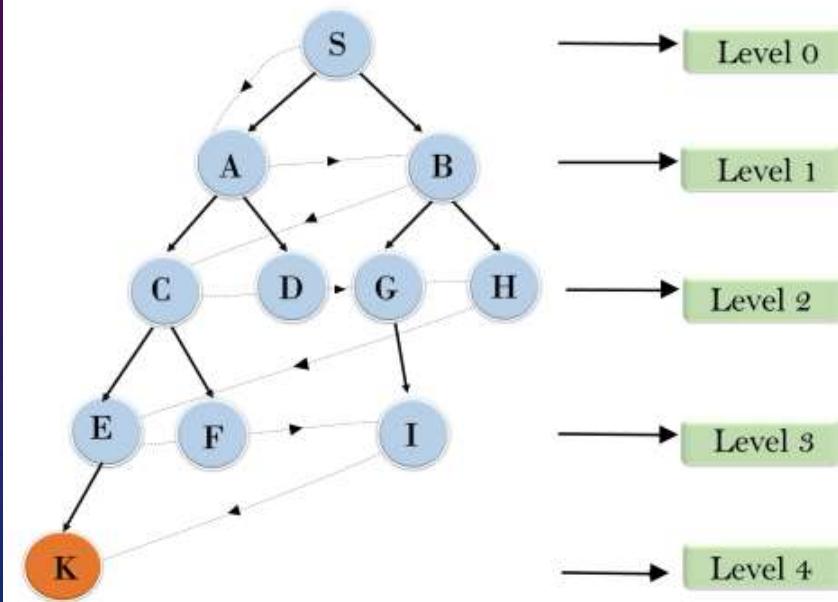
Advantages:

- BFS will provide a solution if any solution exists.
- If there are more than one solutions for a given problem, then BFS will provide the minimal solution which requires the least number of steps.

Example:

In the below tree structure, we have shown the traversing of the tree using BFS algorithm from the root node S to goal node K. BFS search algorithm traverse in layers, so it will follow the path which is shown by the dotted arrow, and the traversed path will be:

S--->A--->B--->C--->D--->G--->H--->E--->F--->I--->K

Breadth First Search

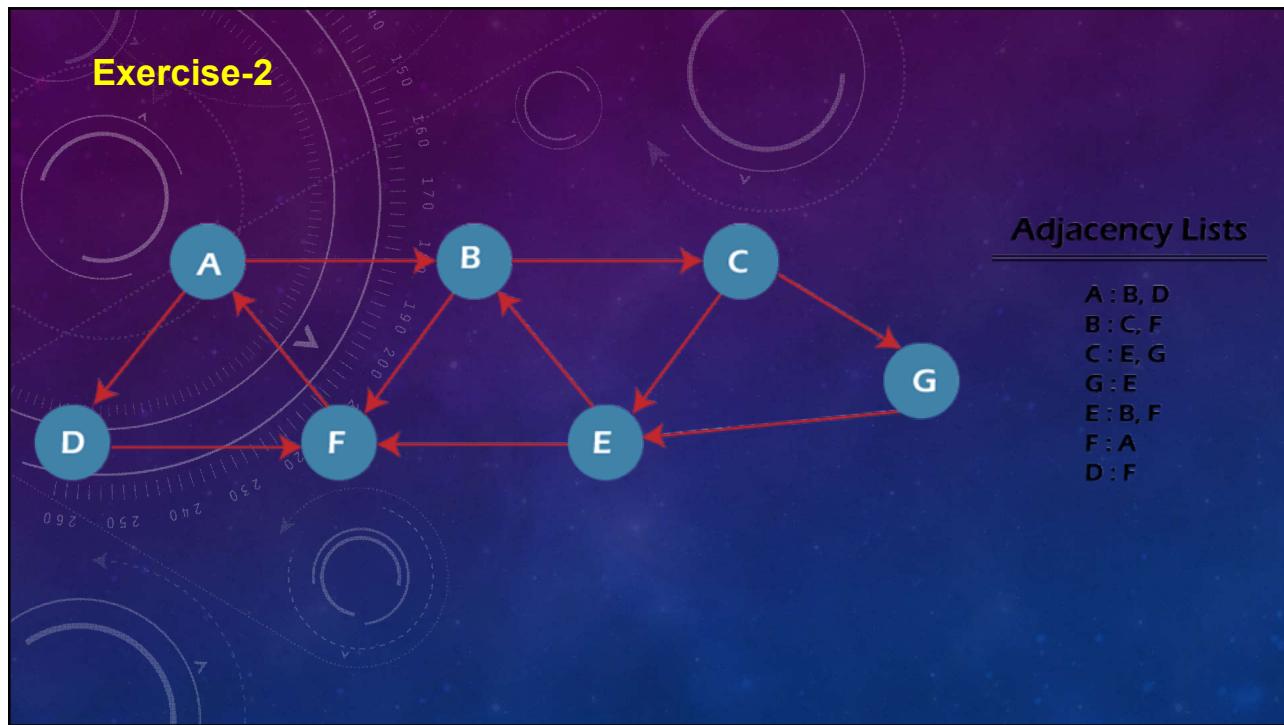
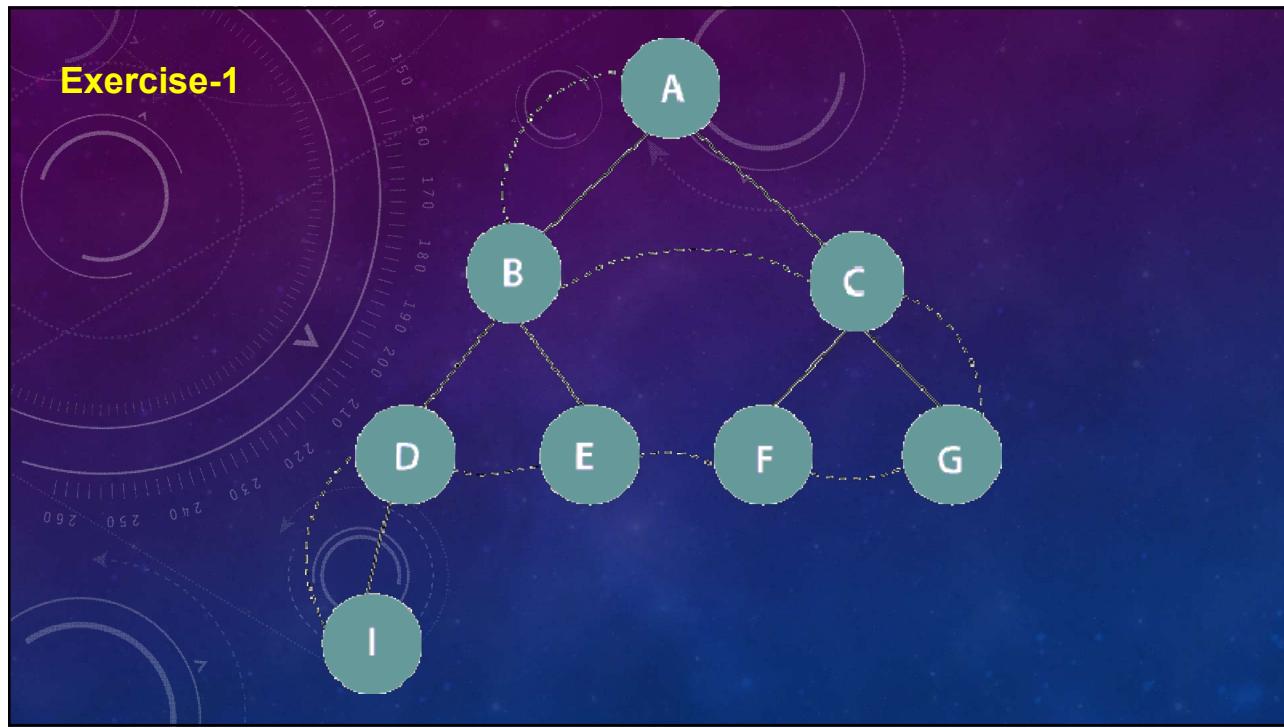
The performance measure of BFS is as follows:

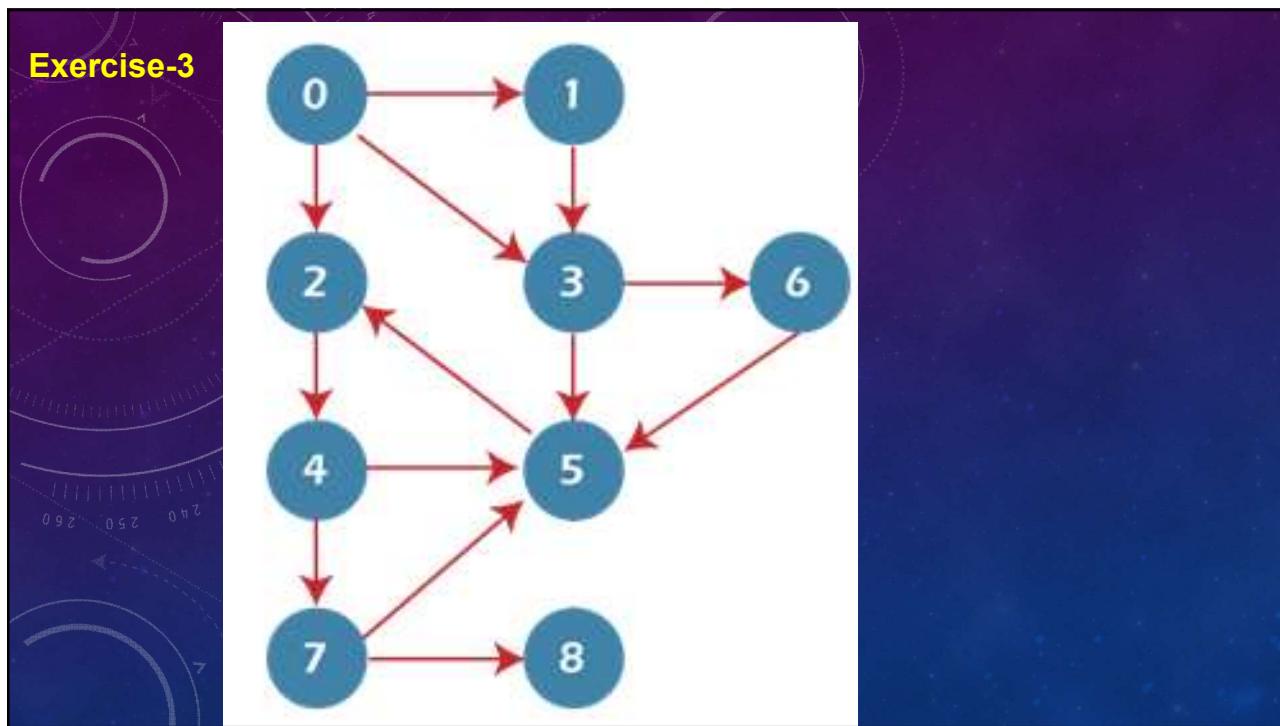
- **Completeness:** It is a complete strategy as it definitely finds the goal state.
- **Optimality:** It gives an optimal solution if the cost of each node is same.
- **Space Complexity:** The space complexity of BFS is $O(b^d)$, i.e., it requires a huge amount of memory. Here, b is the branching factor and d denotes the depth/level of the tree
- **Time Complexity:** BFS consumes much time to reach the goal node for large instances.

Disadvantages of BFS

- The biggest disadvantage of BFS is that it requires a lot of memory space, therefore it is a memory bounded strategy.
- BFS is time taking search strategy because it expands the nodes breadthwise.

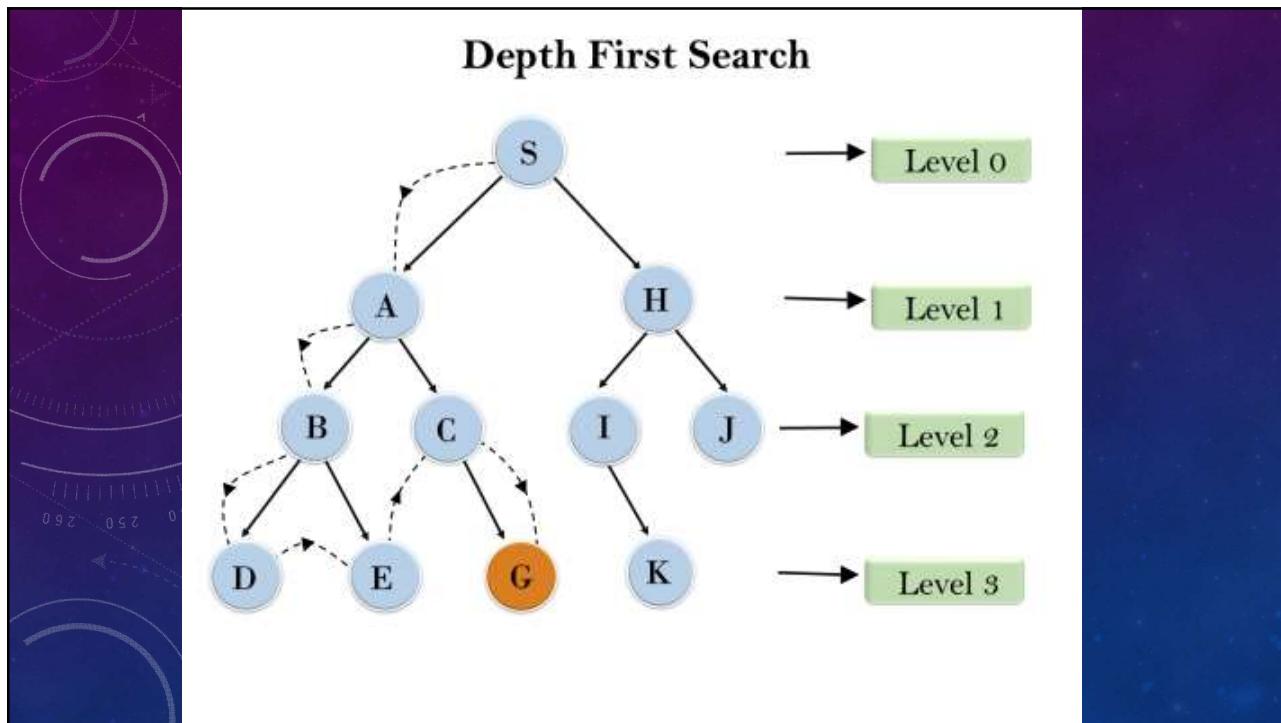
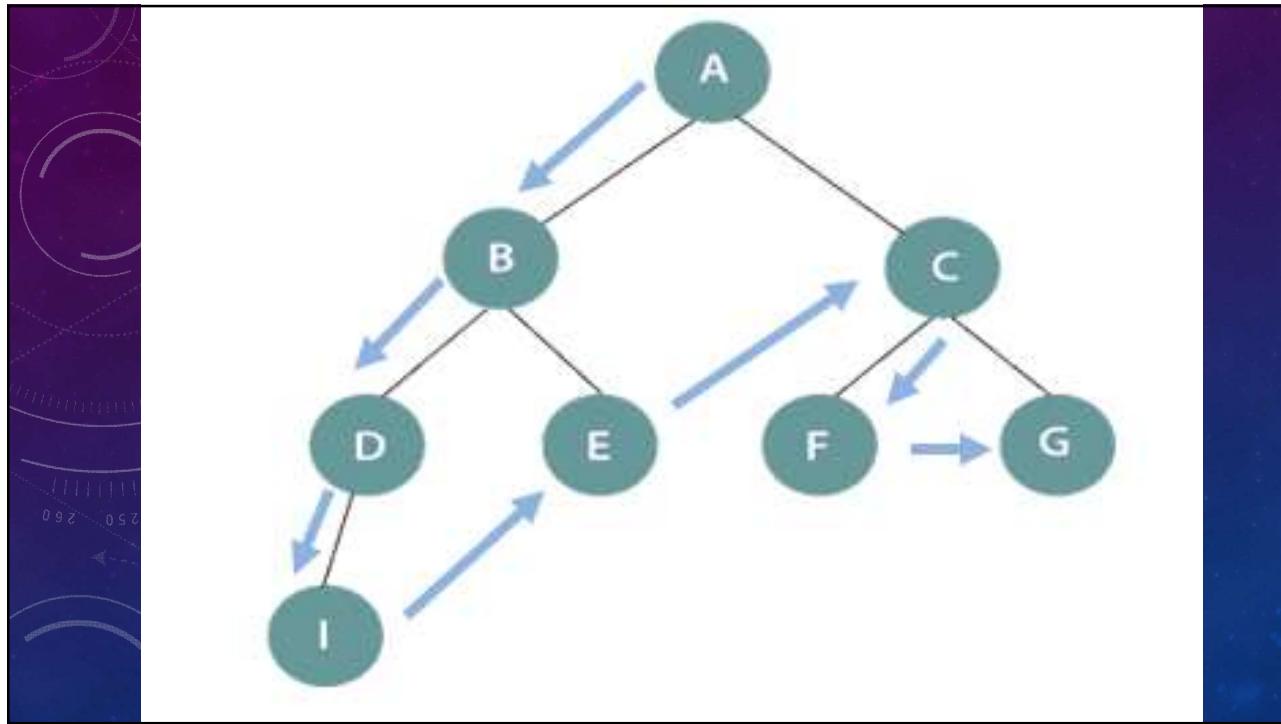
Note: BFS expands the nodes level by level, i.e., breadthwise, therefore it is also known as a Level search technique.





Depth-first search

This search strategy explores the deepest node first, then backtracks to explore other nodes. It uses LIFO (Last in First Out) order, which is based on the **stack**, in order to expand the unexpanded nodes in the search tree. The search proceeds to the deepest level of the tree where it has no successors. This search expands nodes till infinity, i.e., the depth of the tree.



The performance measure of DFS

Completeness: DFS does not guarantee to reach the goal state.

Optimality: It does not give an optimal solution as it expands nodes in one direction deeply.

Space complexity: It needs to store only a single path from the root node to the leaf node. Therefore, DFS has $O(bm)$ space complexity where b is the branching factor(i.e., total no. of child nodes, a parent node have) and m is the maximum length of any path.

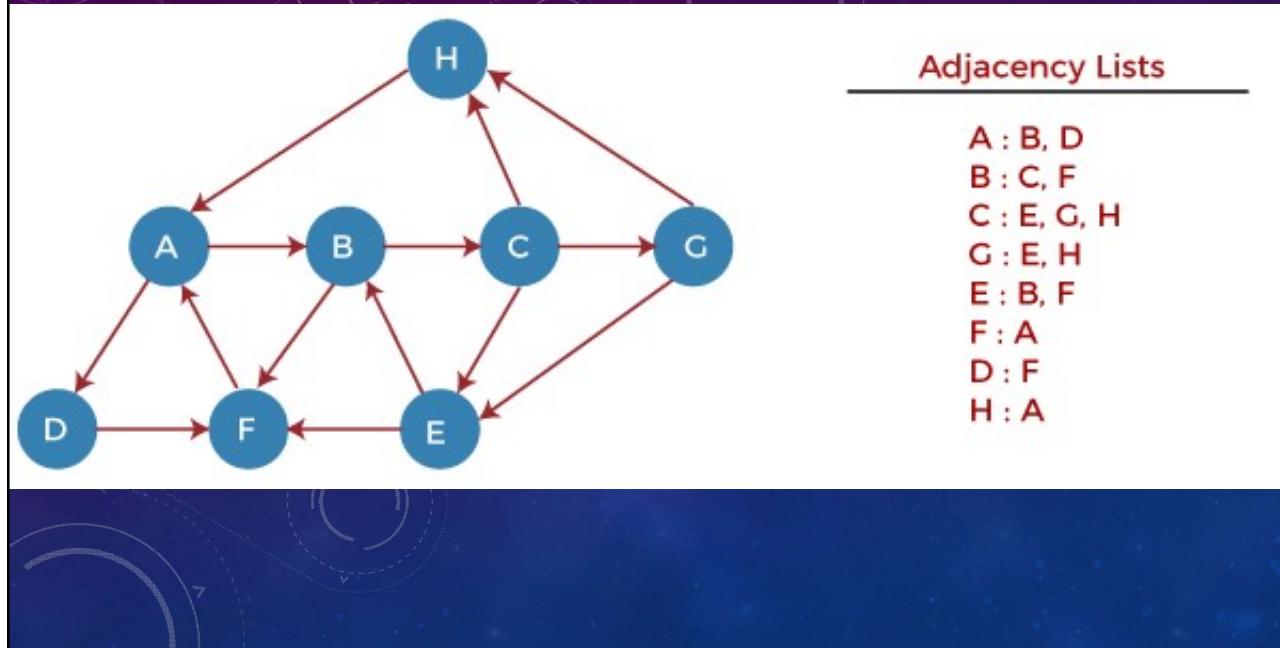
Time complexity: DFS has $O(b^m)$ time complexity.

Disadvantages of DFS

- It may get trapped in an infinite loop.
- It is also possible that it may not reach the goal state.
- DFS does not give an optimal solution.

Note: DFS uses the concept of backtracking to explore each node in a search tree.

Exercise-1



Uniform-cost Search Algorithm:

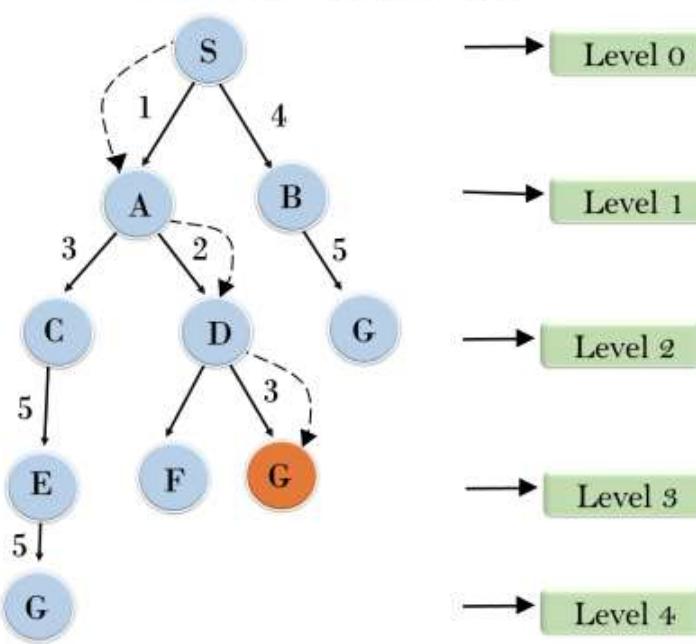
Uniform-cost search is a searching algorithm used for traversing a weighted tree or graph. This algorithm comes into play when a different cost is available for each edge. The primary goal of the uniform-cost search is to find a path to the goal node which has the lowest cumulative cost. Uniform-cost search expands nodes according to their path costs from the root node. It can be used to solve any graph/tree where the optimal cost is in demand. A uniform-cost search algorithm is implemented by the priority queue. It gives maximum priority to the lowest cumulative cost. Uniform cost search is equivalent to BFS algorithm if the path cost of all edges is the same.

Advantages:

Uniform cost search is optimal because at every state the path with the least cost is chosen.

Disadvantages:

It does not care about the number of steps involved in searching and only concerned about path cost. Due to which this algorithm may be stuck in an infinite loop.

Uniform Cost Search

Iterative deepening depth-first Search:

The iterative deepening algorithm is a combination of DFS and BFS algorithms. This search algorithm finds out the best depth limit and does it by gradually increasing the limit until a goal is found.

This algorithm performs depth-first search up to a certain "depth limit", and it keeps increasing the depth limit after each iteration until the goal node is found. This Search algorithm combines the benefits of Breadth-first search's fast search and depth-first search's memory efficiency.

The iterative search algorithm is useful uninformed search when search space is large, and depth of goal node is unknown.

Advantages:

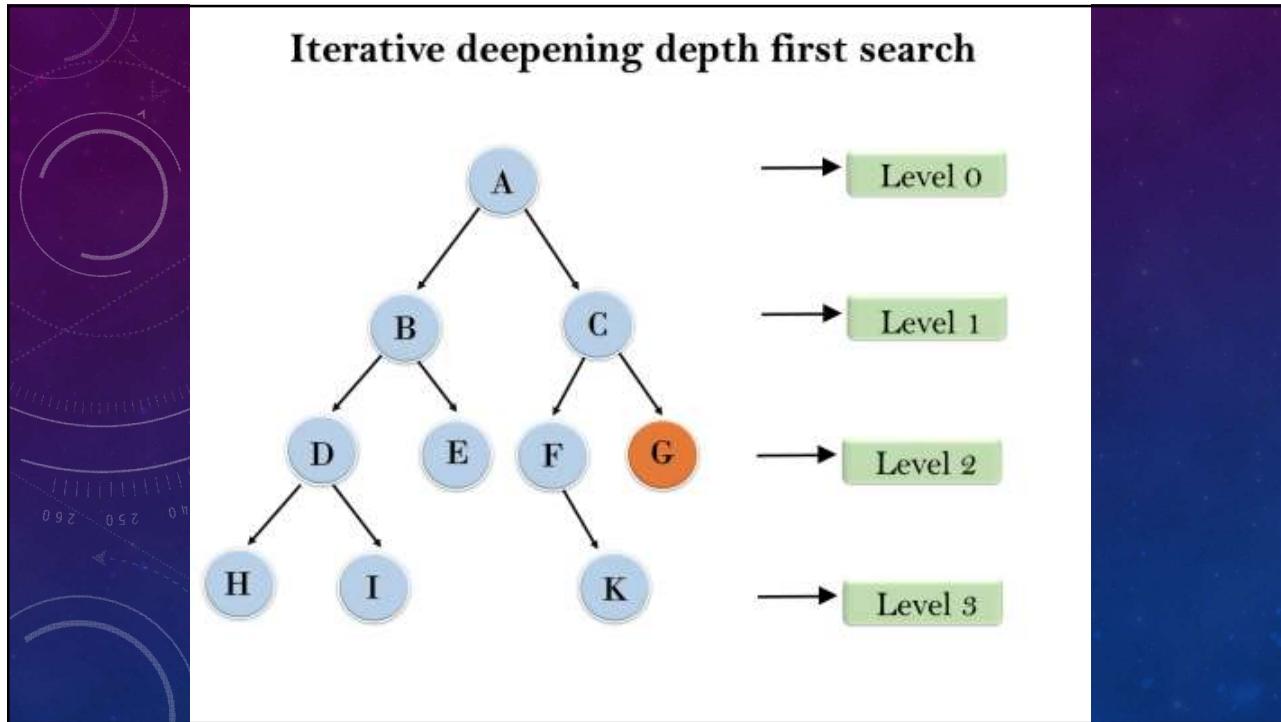
It combines the benefits of BFS and DFS search algorithm in terms of fast search and memory efficiency.

Disadvantages:

The main drawback of IDDFS is that it repeats all the work of the previous phase.

Example:

Following tree structure is showing the iterative deepening depth-first search. IDDFS algorithm performs various iterations until it does not find the goal node. The iteration performed by the algorithm is given as:



1'st Iteration----> A

2'nd Iteration----> A, B, C

3'rd Iteration---->A, B, D, E, C, F, G

4'th Iteration---->A, B, D, H, I, E, C, F, K, G

In the fourth iteration, the algorithm will find the goal node.

Completeness: This algorithm is complete if the branching factor is finite.

Time Complexity:

Let's suppose b is the branching factor and depth is d then the worst-case time complexity is $O(b^d)$.

Space Complexity: The space complexity of IDDFS will be $O(bd)$.

Optimal: IDDFS algorithm is optimal if path cost is a non-decreasing function of the depth of the node.

Bidirectional Search Algorithm:

Bidirectional search algorithm runs two simultaneous searches, one from initial state called as forward-search and other from goal node called as backward-search, to find the goal node. Bidirectional search replaces one single search graph with two small subgraphs in which one starts the search from an initial vertex and other starts from goal vertex. The search stops when these two graphs intersect each other.

Bidirectional search can use search techniques such as BFS, DFS, DLS, etc.

Advantages:

Bidirectional search is fast.

Bidirectional search requires less memory

Disadvantages:

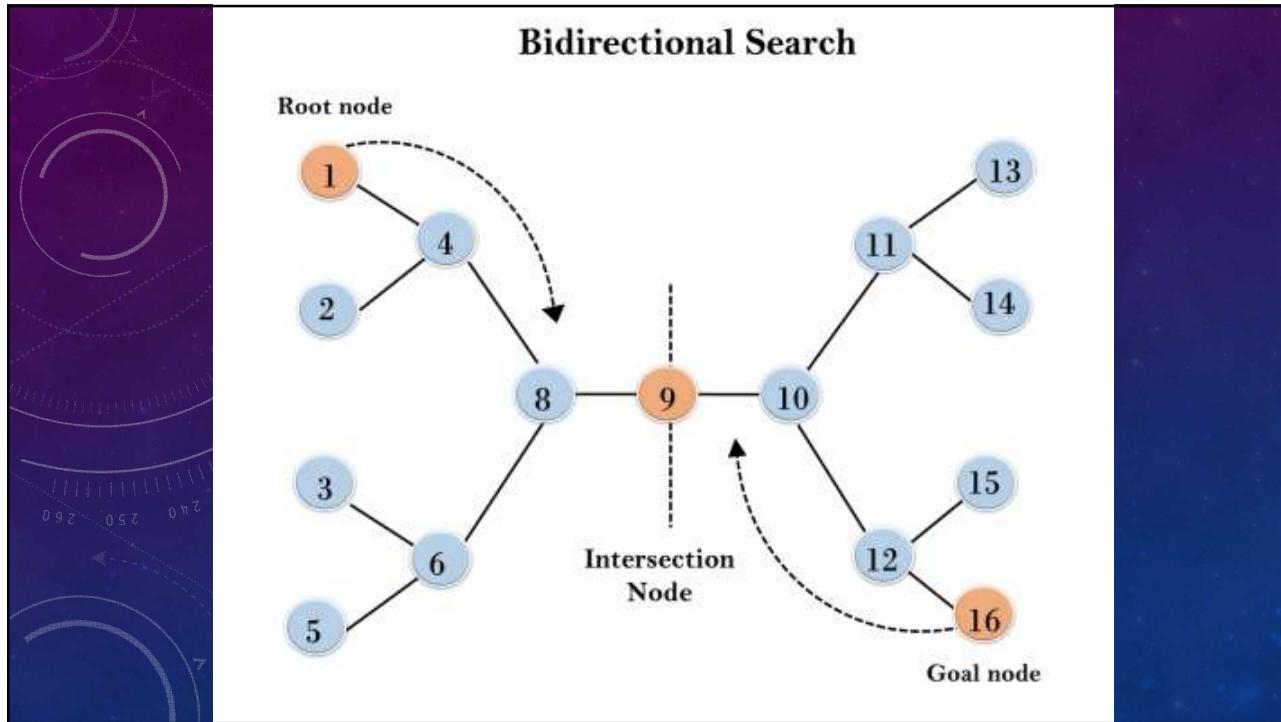
Implementation of the bidirectional search tree is difficult.

In bidirectional search, one should know the goal state in advance.

Example:

In the below search tree, bidirectional search algorithm is applied. This algorithm divides one graph/tree into two sub-graphs. It starts traversing from node 1 in the forward direction and starts from goal node 16 in the backward direction.

The algorithm terminates at node 9 where two searches meet.



Completeness: Bidirectional Search is complete if we use BFS in both searches.

Time Complexity: Time complexity of bidirectional search using BFS is $O(b^d)$.

Space Complexity: Space complexity of bidirectional search is $O(bd)$.

Optimal: Bidirectional search is Optimal.

Informed Search

Informed search algorithms use domain knowledge. In an informed search, problem information is available which can guide the search. Informed search strategies can find a solution more efficiently than an uninformed search strategy. Informed search is also called a Heuristic search.

A **heuristic** is a way which might not always be guaranteed for best solutions but guaranteed to find a good solution in reasonable time.

Informed search can solve much complex problem which could not be solved in another way.

An example of informed search algorithms is a traveling salesman problem.

- Best First Search (Greedy Search)
- A* Search

Best-first Search Algorithm (Greedy Search):

Greedy best-first search algorithm always selects the path which appears best at that moment. It is the combination of depth-first search and breadth-first search algorithms. It uses the heuristic function and search. Best-first search allows us to take the advantages of both algorithms. With the help of best-first search, at each step, we can choose the most promising node. In the best first search algorithm, we expand the node which is closest to the goal node and the closest cost is estimated by heuristic function, i.e.

$$f(n) = g(n).$$

Where, $h(n)$ = estimated cost from node n to the goal.

The greedy best first algorithm is implemented by the priority queue.

Best first search algorithm:

Step 1: Place the starting node into the OPEN list.

Step 2: If the OPEN list is empty, Stop and return failure.

Step 3: Remove the node n_i from the OPEN list which has the lowest value of $h(n)$, and places it in the CLOSED list.

Step 4: Expand the node n_i , and generate the successors of node n_i .

Step 5: Check each successor of node n_i , and find whether any node is a goal node or not. If any successor node is goal node, then return success and terminate the search, else proceed to Step 6.

Step 6: For each successor node, algorithm checks for evaluation function $f(n)$, and then check if the node has been in either OPEN or CLOSED list. If the node has not been in both list, then add it to the OPEN list.

Step 7: Return to Step 2.

Advantages:

Best first search can switch between BFS and DFS by gaining the advantages of both the algorithms.

This algorithm is more efficient than BFS and DFS algorithms.

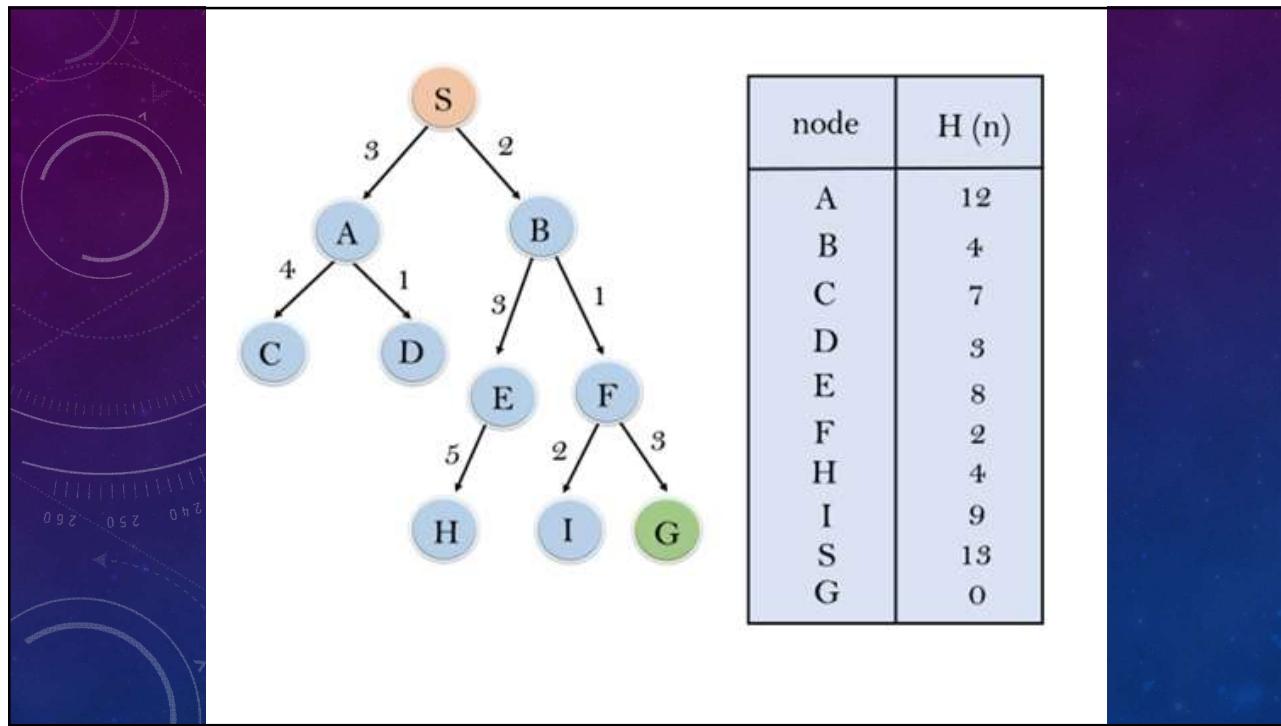
Disadvantages:

It can behave as an unguided depth-first search in the worst case scenario.

It can get stuck in a loop as DFS.

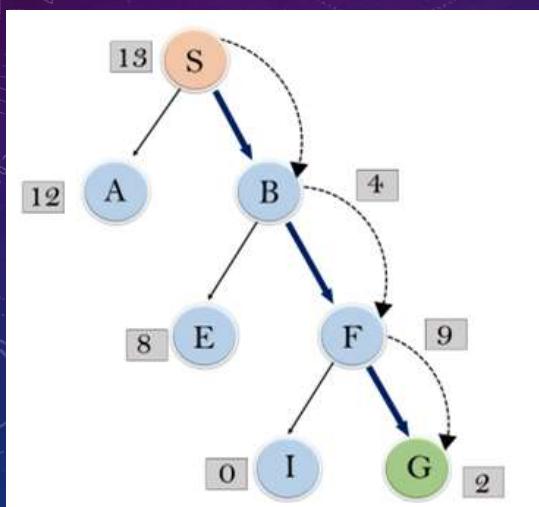
This algorithm is not optimal.

Example: Consider the below search problem, and we will traverse it using greedy best-first search. At each iteration, each node is expanded using evaluation function $f(n)=h(n)$, which is given in the below table.



In this search example, we are using two lists which are OPEN and CLOSED Lists.

Following are the iteration for traversing the above example.



Expand the nodes of S and put in the CLOSED list

Initialization: Open [A, B], Closed [S]

Iteration 1: Open [A], Closed [S, B]

Iteration 2: Open [E, F, A], Closed [S, B]
: Open [E, A], Closed [S, B, F]

Iteration 3: Open [I, G, E, A], Closed [S, B, F]
: Open [I, E, A], Closed [S, B, F, G]

Hence the final solution path will be: S----> B---->F----> G

Time Complexity: The worst case time complexity of Greedy best first search is $O(b^m)$.

Space Complexity: The worst case space complexity of Greedy best first search is $O(b^m)$. Where, m is the maximum depth of the search space.

Complete: Greedy best-first search is also incomplete, even if the given state space is finite.

Optimal: Greedy best first search algorithm is not optimal.

A* Search Algorithm:

A* search is the most commonly known form of best-first search. It uses heuristic function $h(n)$, and cost to reach the node n from the start state $g(n)$. It has combined features of UCS and greedy best-first search, by which it solves the problem efficiently. A* search algorithm finds the shortest path through the search space using the heuristic function. This search algorithm expands less search tree and provides optimal result faster. A* algorithm is similar to UCS except that it uses $g(n)+h(n)$ instead of $g(n)$.

In A* search algorithm, we use search heuristic as well as the cost to reach the node. Hence we can combine both costs as following, and this sum is called as a fitness number.

$$f(n) = g(n) + h(n)$$

Estimated cost
of the cheapest
solution.

Cost to reach
node n from
start state.

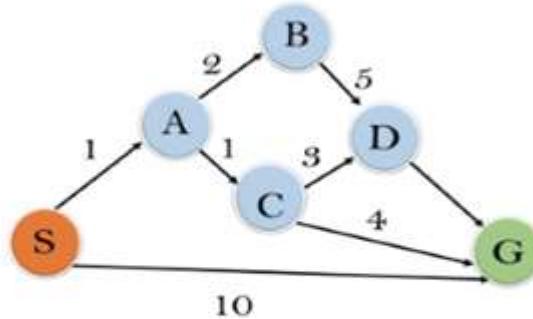
Cost to reach
from node n to
goal node

Advantages:

- A* search algorithm is the best algorithm than other search algorithms.
- A* search algorithm is optimal and complete.
- This algorithm can solve very complex problems.

Disadvantages:

- It does not always produce the shortest path as it mostly based on heuristics and approximation.
- A* search algorithm has some complexity issues.
- The main drawback of A* is memory requirement as it keeps all generated nodes in the memory, so it is not practical for various large-scale problems.

Example:

State	$h(n)$
S	5
A	3
B	4
C	2
D	6
G	0

Points to remember:

- A* algorithm returns the path which occurred first, and it does not search for all remaining paths.
- The efficiency of A* algorithm depends on the quality of heuristic.
- A* algorithm expands all nodes which satisfy the condition $f(n)$

Complete: A* algorithm is complete as long as:

- Branching factor is finite.
- Cost at every action is fixed.

Optimal: A* search algorithm is optimal if it follows below two conditions:

Admissible: the first condition requires for optimality is that $h(n)$ should be an admissible heuristic for A* tree search. An admissible heuristic is optimistic in nature.

Consistency: Second required condition is consistency for only A* graph-search.

If the heuristic function is admissible, then A* tree search will always find the least cost path.

Time Complexity: The time complexity of A* search algorithm depends on heuristic function, and the number of nodes expanded is exponential to the depth of solution d . So the time complexity is $O(b^d)$, where b is the branching factor.

Space Complexity: The space complexity of A* search algorithm is $O(b^d)$

Working on a Local Search Algorithm

Local search algorithms are a type of optimization algorithm that iteratively improves the solution to a problem by making small, local changes to it. Here are the general steps of a local search algorithm:

Initialization:

The algorithm starts with an initial solution to the problem. This solution can be generated randomly or using a heuristic.

Evaluation:

The quality of the initial solution is evaluated using an objective function. The objective function measures how good the solution is, based on the problem constraints and requirements.

Neighborhood search:

The algorithm generates neighboring solutions by making small modifications to the current solution. These modifications can be random or guided by heuristics.

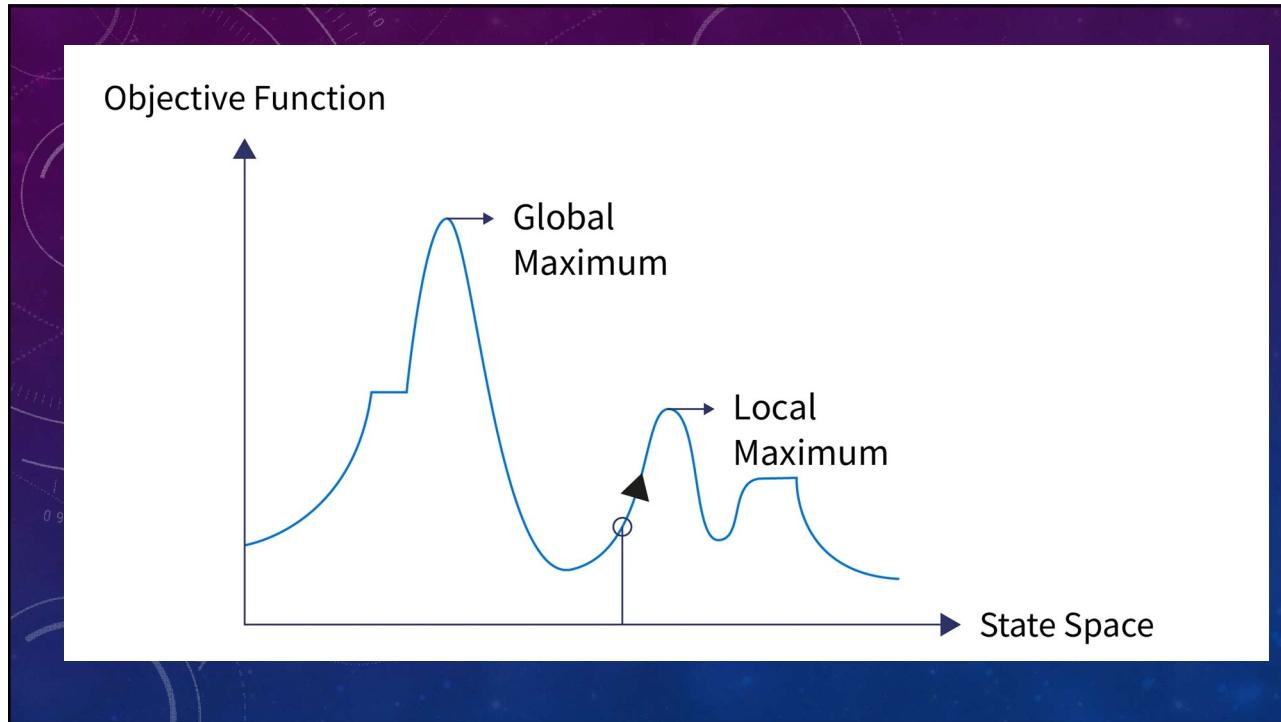
Selection:

The neighboring solutions are evaluated using the objective function, and the best solution is selected as the new current solution.

Termination:

The algorithm terminates when a stopping criterion is met. This criterion can be a maximum number of iterations, a threshold value for the objective function, or a time limit.

Solution: The final solution is the best solution found during the search process.



Global Minimum: If the elevation corresponds to the cost, then the task is to find the lowest valley, which is known as Global Minimum.

Global Maxima: If the elevation corresponds to an objective function, then it finds the highest peak which is called as Global Maxima. It is the highest point in the valley.

Local Search

The local search algorithm in artificial intelligence is a family of optimization algorithms used to find the best possible solution to a problem by iteratively making small changes to an initial solution.

These algorithms are used to solve optimization problems in which an exact solution is either impossible to find or computationally expensive.

Hill climbing, simulated annealing, tabu search, and genetic algorithms are a few examples of different kinds of local search algorithms. Each of these algorithms operates a little bit differently, but they all follow the same fundamental procedure of iteratively creating new solutions and comparing them to the existing solution to determine whether they are superior.

The local search algorithm in artificial intelligence is a crucial tool in the field of artificial intelligence and is frequently employed to address a range of optimization issues.

Applications for local search algorithms include scheduling, routing, and resource allocation. They are particularly helpful for issues where the search space is very large and can be used to solve both discrete and continuous optimization problems.

Local Search

The nodes are expanded systematically by informed and uninformed searches in different ways:

- storing various routes in memory and
- choosing the most appropriate route,

Hence, a solution state is needed to get to the goal node. Beyond these "classical search algorithms," however, there are some "local search algorithms" that only consider the solution state required to reach the target node and disregard path cost.

In contrast to multiple paths, a local search algorithm completes its task by traversing a single current node and generally following that node's neighbors.

To solve challenging optimization issues, local search algorithms are frequently combined with other optimization methods like constraint satisfaction or linear programming. Since they can quickly converge on a solution that is close to the optimal solution, even if they do not find the exact optimal solution, they are especially helpful for problems where the search space is very large.

Hill Climbing Algorithm in Artificial Intelligence

- o Hill climbing algorithm is a local search algorithm which continuously moves in the direction of increasing elevation/value to find the peak of the mountain or best solution to the problem. It terminates when it reaches a peak value where no neighbor has a higher value.
- o Hill climbing algorithm is a technique which is used for optimizing the mathematical problems. One of the widely discussed examples of Hill climbing algorithm is Traveling-salesman Problem in which we need to minimize the distance traveled by the salesman.
- o It is also called greedy local search as it only looks to its good immediate neighbor state and not beyond that.

- o A node of hill climbing algorithm has two components which are state and value.
- o Hill Climbing is mostly used when a good heuristic is available.
- o In this algorithm, we don't need to maintain and handle the search tree or graph as it only keeps a single current state.

Features of Hill Climbing:

Following are some main features of Hill Climbing Algorithm:

Generate and Test variant: Hill Climbing is the variant of Generate and Test method. The Generate and Test method produce feedback which helps to decide which direction to move in the search space.

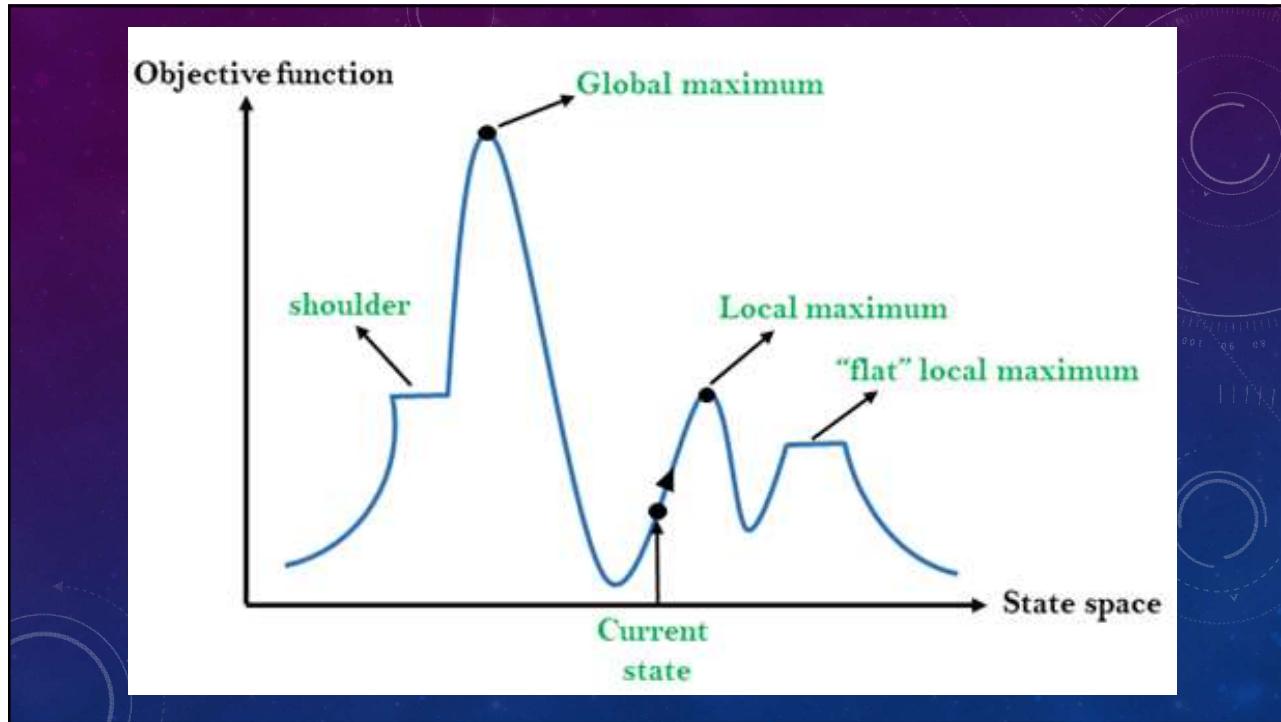
Greedy approach: Hill-climbing algorithm search moves in the direction which optimizes the cost.

No backtracking: It does not backtrack the search space, as it does not remember the previous states.

State-space Diagram for Hill Climbing:

The state-space landscape is a graphical representation of the hill-climbing algorithm which is showing a graph between various states of algorithm and Objective function/Cost.

On Y-axis we have taken the function which can be an objective function or cost function, and state-space on the x-axis. If the function on Y-axis is cost then, the goal of search is to find the global minimum and local minimum. If the function of Y-axis is Objective function, then the goal of the search is to find the global maximum and local maximum.



Different regions in the state space landscape:

Local Maximum: Local maximum is a state which is better than its neighbor states, but there is also another state which is higher than it.

Global Maximum: Global maximum is the best possible state of state space landscape. It has the highest value of objective function.

Current state: It is a state in a landscape diagram where an agent is currently present.

Flat local maximum: It is a flat space in the landscape where all the neighbor states of current states have the same value.

Shoulder: It is a plateau region which has an uphill edge.

Types of Hill Climbing Algorithm:

- Simple hill Climbing:
- Steepest-Ascent hill-climbing:
- Stochastic hill Climbing:

Simple Hill Climbing:

Simple hill climbing is the simplest way to implement a hill climbing algorithm. It only evaluates the neighbor node state at a time and selects the first one which optimizes current cost and set it as a current state. It only checks it's one successor state, and if it finds better than the current state, then move else be in the same state. This algorithm has the following features:

Less time consuming

Less optimal solution and the solution is not guaranteed

Algorithm for Simple Hill Climbing:

Step 1: Evaluate the initial state, if it is goal state then return success and Stop.

Step 2: Loop Until a solution is found or there is no new operator left to apply.

Step 3: Select and apply an operator to the current state.

Step 4: Check new state:

- If it is goal state, then return success and quit.
- Else if it is better than the current state then assign new state as a current state.
- Else if not better than the current state, then return to step2.

Step 5: Exit

Less time consuming

Less optimal solution and the solution is not guaranteed

Algorithm for Simple Hill Climbing:

Step 1: Evaluate the initial state, if it is goal state then return success and Stop.

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- o If it is goal state, then return success and quit.
- o Else if it is better than the current state then assign new state as a current state.
- o Else if not better than the current state, then return to step2..

2. Steepest-Ascent hill climbing:

The steepest-Ascent algorithm is a variation of simple hill climbing algorithm. This algorithm examines all the neighboring nodes of the current state and selects one neighbor node which is closest to the goal state. This algorithm consumes more time as it searches for multiple neighbors

3. Stochastic hill climbing:

Stochastic hill climbing does not examine for all its neighbor before moving.

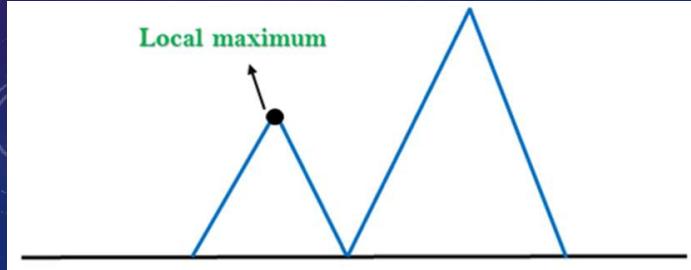
Rather, this search algorithm selects one neighbor node at random and decides whether to choose it as a current state or examine another state.

Problems in Hill Climbing Algorithm:

1. Local Maximum: A local maximum is a peak state in the landscape which is better than each of its neighboring states, but there is another state also present which is higher than the local maximum.

Solution: Backtracking technique can be a solution of the local maximum in state space landscape. Create a list of the promising path so that the algorithm can backtrack the search space and explore other paths as well.

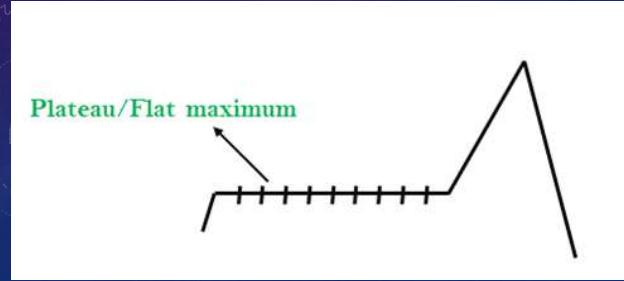
Local maximum



2. Plateau: A plateau is the flat area of the search space in which all the neighbor states of the current state contains the same value, because of this algorithm does not find any best direction to move. A hill-climbing search might be lost in the plateau area.

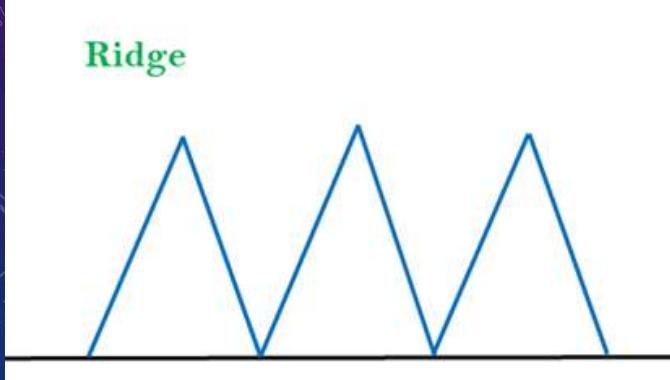
Solution: The solution for the plateau is to take big steps or very little steps while searching, to solve the problem. Randomly select a state which is far away from the current state so it is possible that the algorithm could find non-plateau region.

Plateau/Flat maximum



3. Ridges: A ridge is a special form of the local maximum. It has an area which is higher than its surrounding areas, but itself has a slope, and cannot be reached in a single move.

Solution: With the use of bidirectional search, or by moving in different directions, we can improve this problem.



Simulated Annealing:

A hill-climbing algorithm which never makes a move towards a lower value guaranteed to be incomplete because it can get stuck on a local maximum. And if algorithm applies a random walk, by moving a successor, then it may complete but not efficient.

Simulated Annealing is an algorithm which yields both efficiency and completeness.

In mechanical term Annealing is a process of hardening a metal or glass to a high temperature then cooling gradually, so this allows the metal to reach a low-energy crystalline state. The same process is used in simulated annealing in which the algorithm picks a random move, instead of picking the best move. If the random move improves the state, then it follows the same path. Otherwise, the algorithm follows the path which has a probability of less than 1 or it moves downhill and chooses another path.

Adversarial Search

Adversarial search is a search, where we examine the problem which arises when we try to plan ahead of the world and other agents are planning against us.

- o We have already studied the search strategies which are only associated with a single agent that aims to find the solution which often expressed in the form of a sequence of actions.
- o But, there might be some situations where more than one agent is searching for the solution in the same search space, and this situation usually occurs in game playing.

- o The environment with more than one agent is termed as multi-agent environment, in which each agent is an opponent of other agent and playing against each other. Each agent needs to consider the action of other agent and effect of that action on their performance.

- o So, Searches in which two or more players with conflicting goals are trying to explore the same search space for the solution, are called adversarial searches, often known as Games.

Games are modeled as a Search problem and heuristic evaluation function, and these are the two main factors which help to model and solve games in AI.

Types of Games in AI:

- **Perfect information:** A game with the perfect information is that in which agents can look into the complete board. Agents have all the information about the game, and they can see each other moves also. Examples are Chess, Checkers, Go, etc.
- **Imperfect information:** If in a game agents do not have all information about the game and not aware with what's going on, such type of games are called the game with imperfect information, such as tic-tac-toe, Battleship, blind, Bridge, etc.

○ **Deterministic games:** Deterministic games are those games which follow a strict pattern and set of rules for the games, and there is no randomness associated with them. Examples are chess, Checkers, Go, tic-tac-toe, etc.

○ **Non-deterministic games:** Non-deterministic are those games which have various unpredictable events and has a factor of chance or luck. This factor of chance or luck is introduced by either dice or cards. These are random, and each action response is not fixed. Such games are also called as stochastic games.

Example: Backgammon, Monopoly, Poker, etc.

Zero-Sum Game

- Zero-sum games are adversarial search which involves pure competition.
- In Zero-sum game each agent's gain or loss of utility is exactly balanced by the losses or gains of utility of another agent.
- One player of the game try to maximize one single value, while other player tries to minimize it.
- Each move by one player in the game is called as ply.
- Chess and tic-tac-toe are examples of a Zero-sum game.

Zero-sum game: Embedded thinking

The Zero-sum game involved embedded thinking in which one agent or player is trying to figure out:

- What to do.
- How to decide the move
- Needs to think about his opponent as well
- The opponent also thinks what to do
- Each of the players is trying to find out the response of his opponent to their actions. This requires embedded thinking or backward reasoning to solve the game problems in AI.

Formalization of the problem:

A game can be defined as a type of search in AI which can be formalized of the following elements:

- **Initial state:** It specifies how the game is set up at the start.
- **Player(s):** It specifies which player has moved in the state space.
- **Action(s):** It returns the set of legal moves in state space.
- **Result(s, a):** It is the transition model, which specifies the result of moves in the state space.
- **Terminal-Test(s):** Terminal test is true if the game is over, else it is false at any case. The state where the game ends is called terminal states.

- **Utility(s, p):** A utility function gives the final numeric value for a game that ends in terminal states s for player p. It is also called payoff function. For Chess, the outcomes are a win, loss, or draw and its payoff values are +1, 0, $\frac{1}{2}$. And for tic-tac-toe, utility values are +1, -1, and 0.

Game tree:

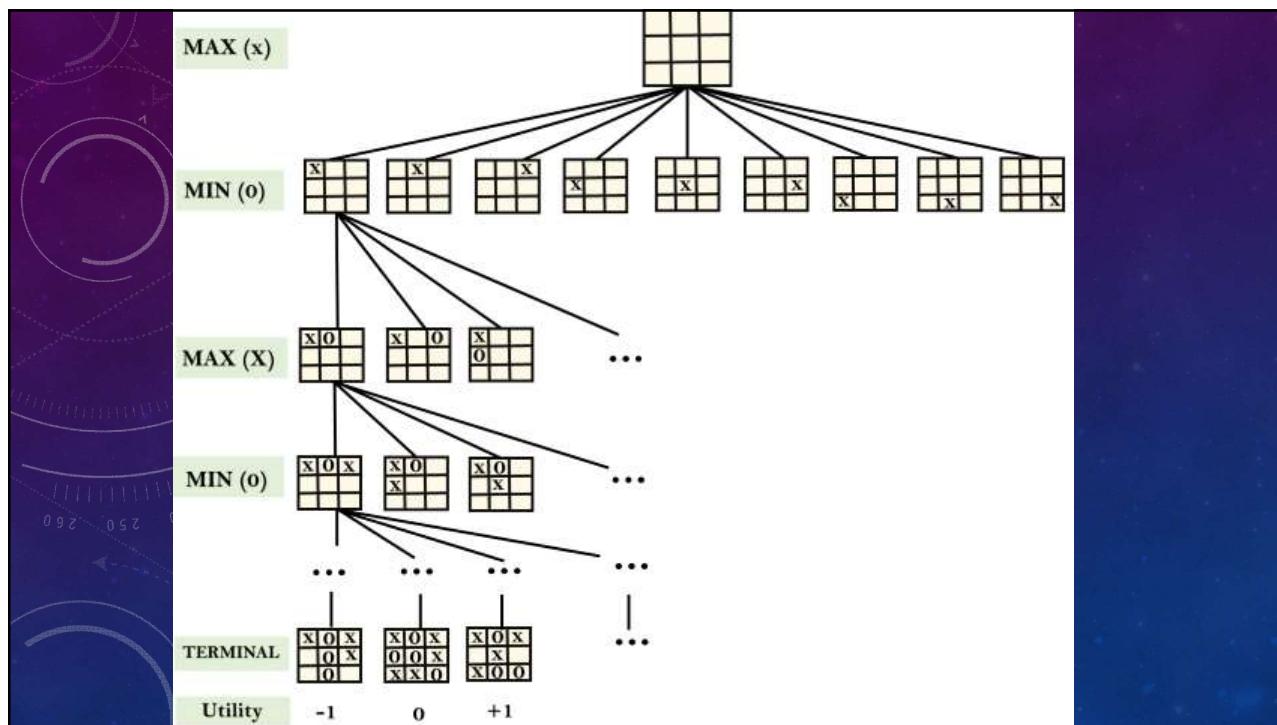
A game tree is a tree where nodes of the tree are the game states and Edges of the tree are the moves by players. Game tree involves initial state, actions function, and result Function.

Example: Tic-Tac-Toe game tree:

The following figure is showing part of the game-tree for tic-tac-toe game.

Following are some key points of the game:

- There are two players MAX and MIN.
- Players have an alternate turn and start with MAX.
- MAX maximizes the result of the game tree
- MIN minimizes the result.



Example Explanation:

- From the initial state, MAX has 9 possible moves as he starts first. MAX place x and MIN place o, and both player plays alternatively until we reach a leaf node where one player has three in a row or all squares are filled.
- Both players will compute each node, minimax, the minimax value which is the best achievable utility against an optimal adversary.
- Suppose both the players are well aware of the tic-tac-toe and playing the best play. Each player is doing his best to prevent another one from winning. MIN is acting against Max in the game.

- So in the game tree, we have a layer of Max, a layer of MIN, and each layer is called as Ply. Max place x, then MIN puts o to prevent Max from winning, and this game continues until the terminal node.
- In this either MIN wins, MAX wins, or it's a draw. This game-tree is the whole search space of possibilities that MIN and MAX are playing tic-tac-toe and taking turns alternately.

Hence adversarial Search for the minimax procedure works as follows:

- It aims to find the optimal strategy for MAX to win the game.
- It follows the approach of Depth-first search.
- In the game tree, optimal leaf node could appear at any depth of the tree.
- Propagate the minimax values up to the tree until the terminal node discovered.

In a given game tree, the optimal strategy can be determined from the minimax value of each node, which can be written as $\text{MINIMAX}(n)$. MAX prefer to move to a state of maximum value and MIN prefer to move to a state of minimum value then:

For a state s $\text{MINIMAX}(s) =$

$$\begin{cases} \text{UTILITY}(s) & \text{If } \text{TERMINAL-TEST}(s) \\ \max_{a \in \text{Actions}(s)} \text{MINIMAX}(\text{RESULT}(s, a)) & \text{If } \text{PLAYER}(s) = \text{MAX} \\ \min_{a \in \text{Actions}(s)} \text{MINIMAX}(\text{RESULT}(s, a)) & \text{If } \text{PLAYER}(s) = \text{MIN.} \end{cases}$$

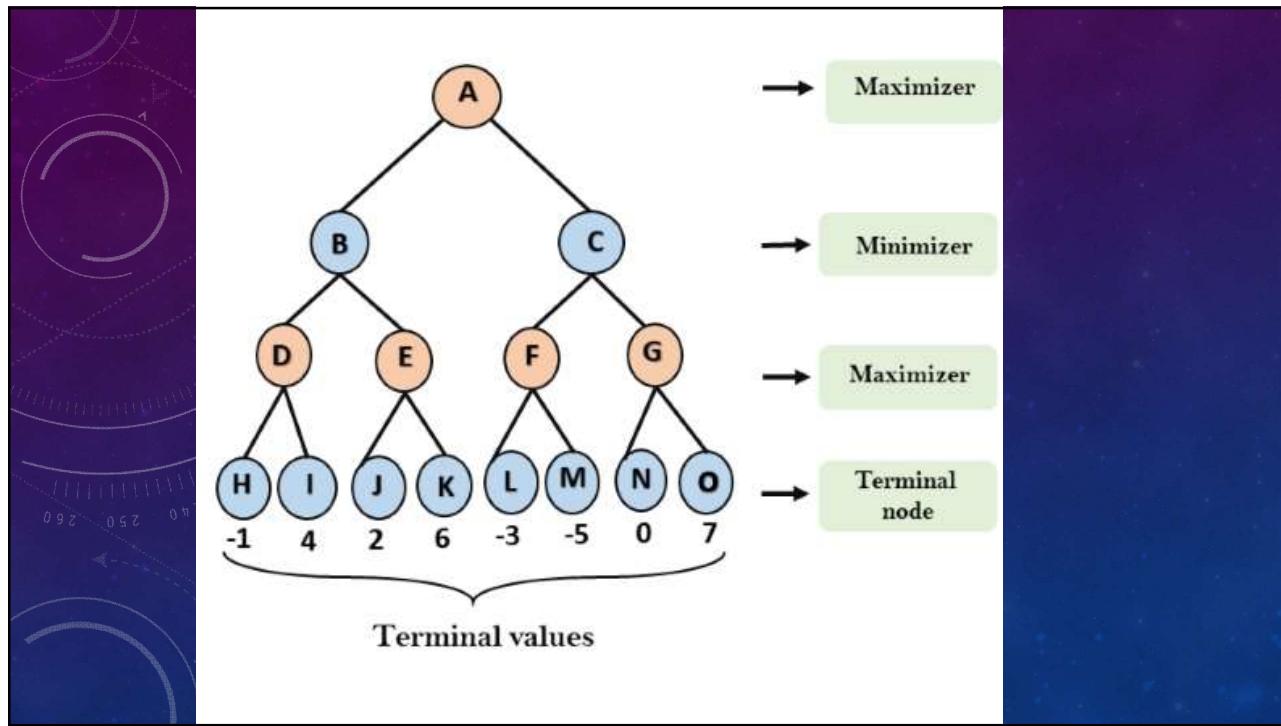
Mini-Max Algorithm in Artificial Intelligence

- o Mini-max algorithm is a recursive or backtracking algorithm which is used in decision-making and game theory. It provides an optimal move for the player assuming that opponent is also playing optimally.
- o Mini-Max algorithm uses recursion to search through the game-tree.
- o Min-Max algorithm is mostly used for game playing in AI. Such as Chess, Checkers, tic-tac-toe, go, and various tow-players game. This Algorithm computes the minimax decision for the current state.
- o In this algorithm two players play the game, one is called MAX and other is called MIN.

- Both the players fight it as the opponent player gets the minimum benefit while they get the maximum benefit.
- Both Players of the game are opponent of each other, where MAX will select the maximized value and MIN will select the minimized value.
- The minimax algorithm performs a depth-first search algorithm for the exploration of the complete game tree.
- The minimax algorithm proceeds all the way down to the terminal node of the tree, then backtrack the tree as the recursion.

Working of Min-Max Algorithm:

- The working of the minimax algorithm can be easily described using an example. Below we have taken an example of game-tree which is representing the two-player game.
- In this example, there are two players one is called Maximizer and other is called Minimizer.
- Maximizer will try to get the Maximum possible score, and Minimizer will try to get the minimum possible score.
- This algorithm applies DFS, so in this game-tree, we have to go all the way through the leaves to reach the terminal nodes.
- At the terminal node, the terminal values are given so we will compare those value and backtrack the tree until the initial state occurs. Following are the main steps involved in solving the two-player game tree:



Step-1: In the first step, the algorithm generates the entire game-tree and apply the utility function to get the utility values for the terminal states. In the below tree diagram, let's take A is the initial state of the tree. Suppose maximizer takes first turn which has worst-case initial value =- infinity, and minimizer will take next turn which has worst-case initial value = +infinity.

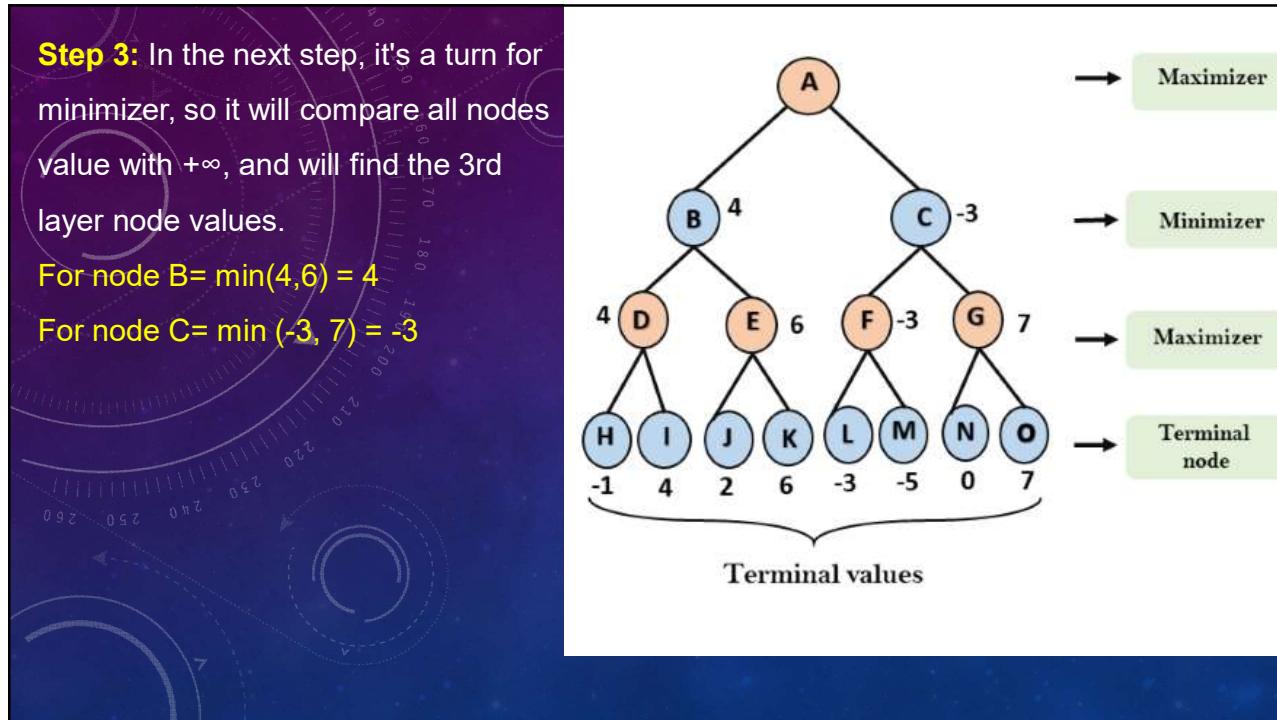
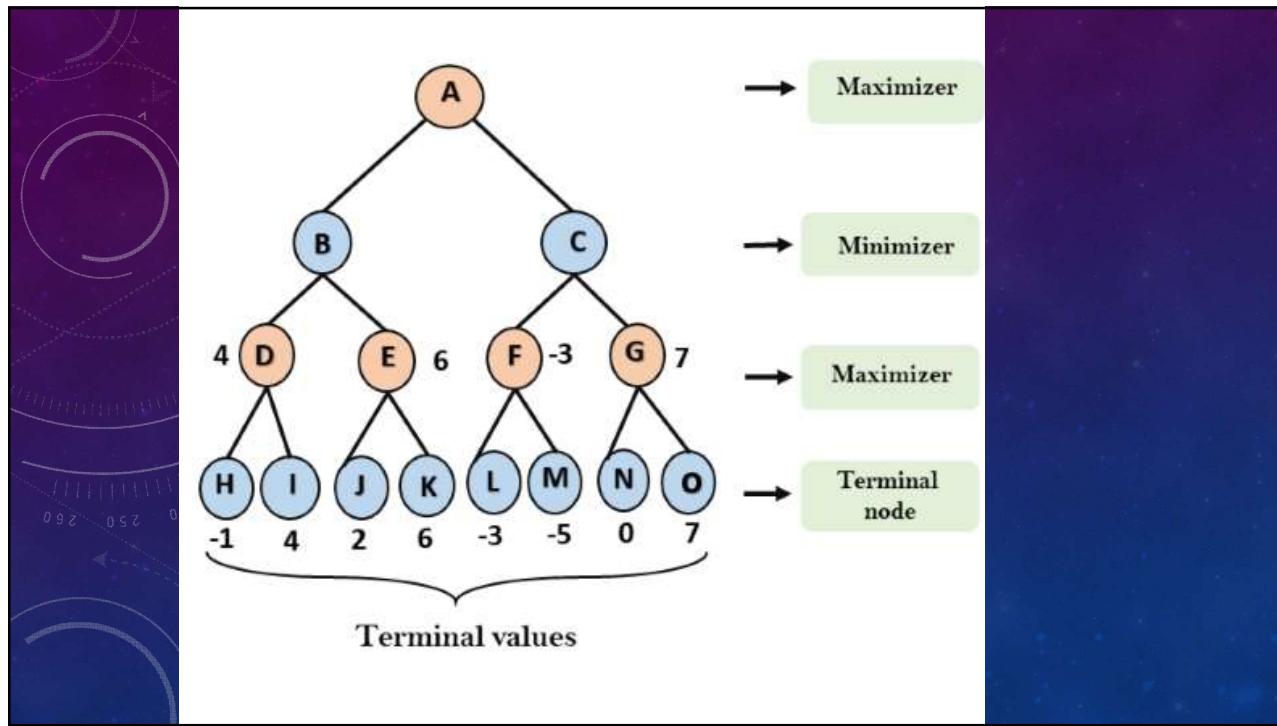
Step 2: Now, first we find the utilities value for the Maximizer, its initial value is $-\infty$, so we will compare each value in terminal state with initial value of Maximizer and determines the higher nodes values. It will find the maximum among the all.

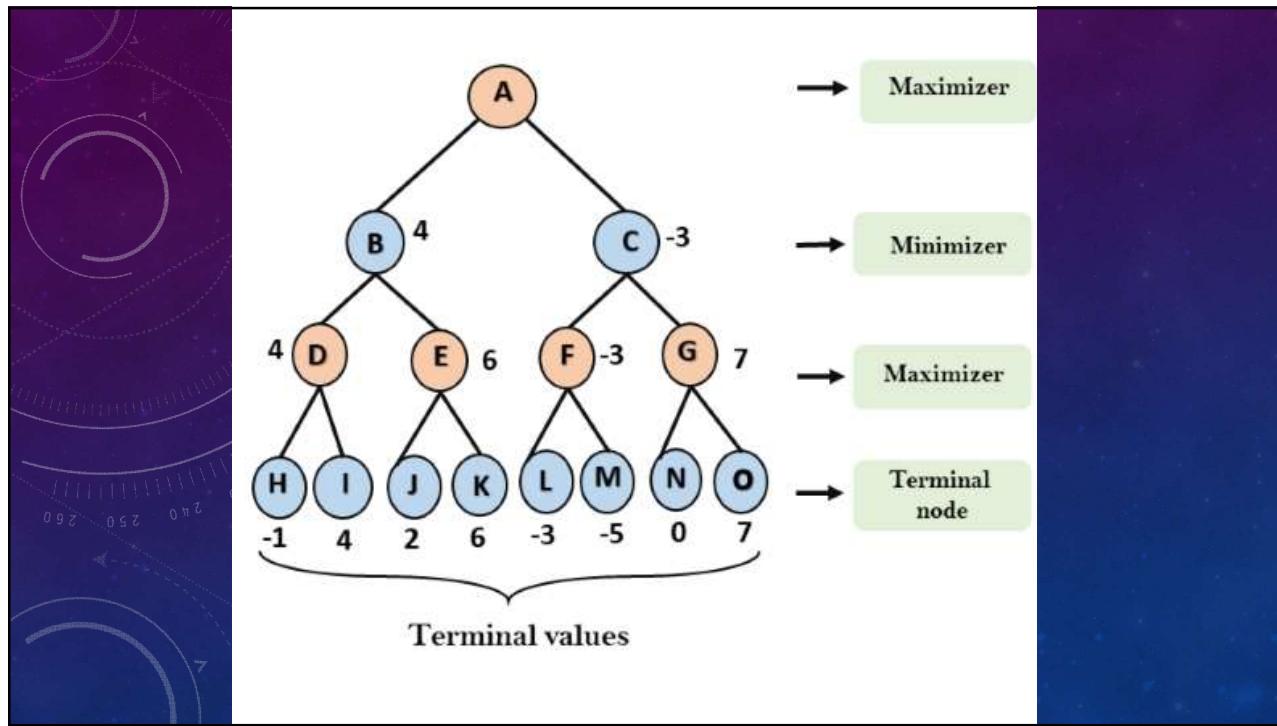
$$\text{For node D} \quad \max(-1, -\infty) \Rightarrow \max(-1, 4) = 4$$

$$\text{For Node E} \quad \max(2, -\infty) \Rightarrow \max(2, 6) = 6$$

$$\text{For Node F} \quad \max(-3, -\infty) \Rightarrow \max(-3, -5) = -3$$

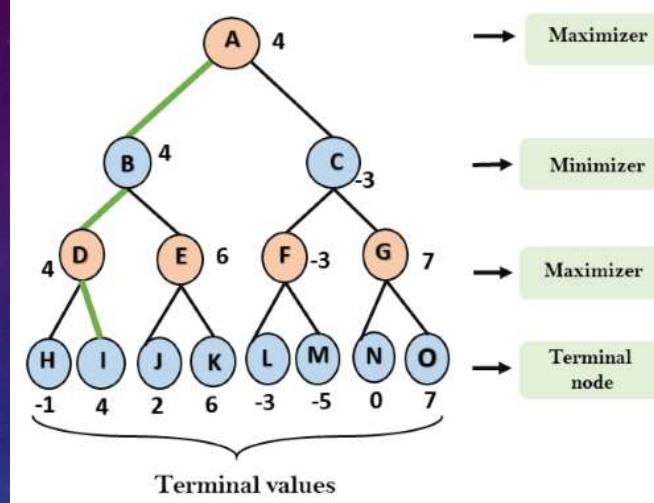
$$\text{For node G} \quad \max(0, -\infty) = \max(0, 7) = 7$$





Step 4: Now it's a turn for Maximizer, and it will again choose the maximum of all nodes value and find the maximum value for the root node. In this game tree, there are only 4 layers, hence we reach immediately to the root node, but in real games, there will be more than 4 layers.

For node A $\max(4, -3) = 4$



Properties of Mini-Max algorithm:

- **Complete-** Min-Max algorithm is Complete. It will definitely find a solution (if exist), in the finite search tree.
- **Optimal-** Min-Max algorithm is optimal if both opponents are playing optimally.
- **Time complexity-** As it performs DFS for the game-tree, so the time complexity of Min-Max algorithm is $O(b^m)$, where b is branching factor of the game-tree, and m is the maximum depth of the tree.
- **Space Complexity-** Space complexity of Mini-max algorithm is also similar to DFS which is $O(bm)$.

Limitation of the minimax Algorithm:

The main drawback of the minimax algorithm is that it gets really slow for complex games such as Chess, go, etc. This type of games has a huge branching factor, and the player has lots of choices to decide. This limitation of the minimax algorithm can be improved from alpha-beta pruning which we have discussed in the next topic.

In chess game on an average 35 choices are possible and moves for both player approximate 100

$$\text{Time complexity } O(b^d) = (35)^{100}$$

Alpha-Beta Pruning

- Alpha-beta pruning is a modified version of the minimax algorithm. It is an optimization technique for the minimax algorithm.
- As we have seen in the minimax search algorithm that the number of game states it has to examine are exponential in depth of the tree. Since we cannot eliminate the exponent, but we can cut it to half. Hence there is a technique by which without checking each node of the game tree we can compute the correct minimax decision, and this technique is called pruning. This involves two threshold parameter Alpha and beta for future expansion, so it is called alpha-beta pruning. It is also called as Alpha-Beta Algorithm.

- Alpha-beta pruning can be applied at any depth of a tree, and sometimes it not only prune the tree leaves but also entire sub-tree.

- The two-parameter can be defined as:
 - Alpha: The best (highest-value) choice we have found so far at any point along the path of Maximizer. The initial value of alpha is $-\infty$.
 - Beta: The best (lowest-value) choice we have found so far at any point along the path of Minimizer. The initial value of beta is $+\infty$.

The Alpha-beta pruning to a standard minimax algorithm returns the same move as the standard algorithm does, but it removes all the nodes which are not really affecting the final decision but making algorithm slow. Hence by pruning these nodes, it makes the algorithm fast.

Condition for Alpha-beta pruning:

The main condition which required for alpha-beta pruning is:

$$\alpha \geq \beta$$

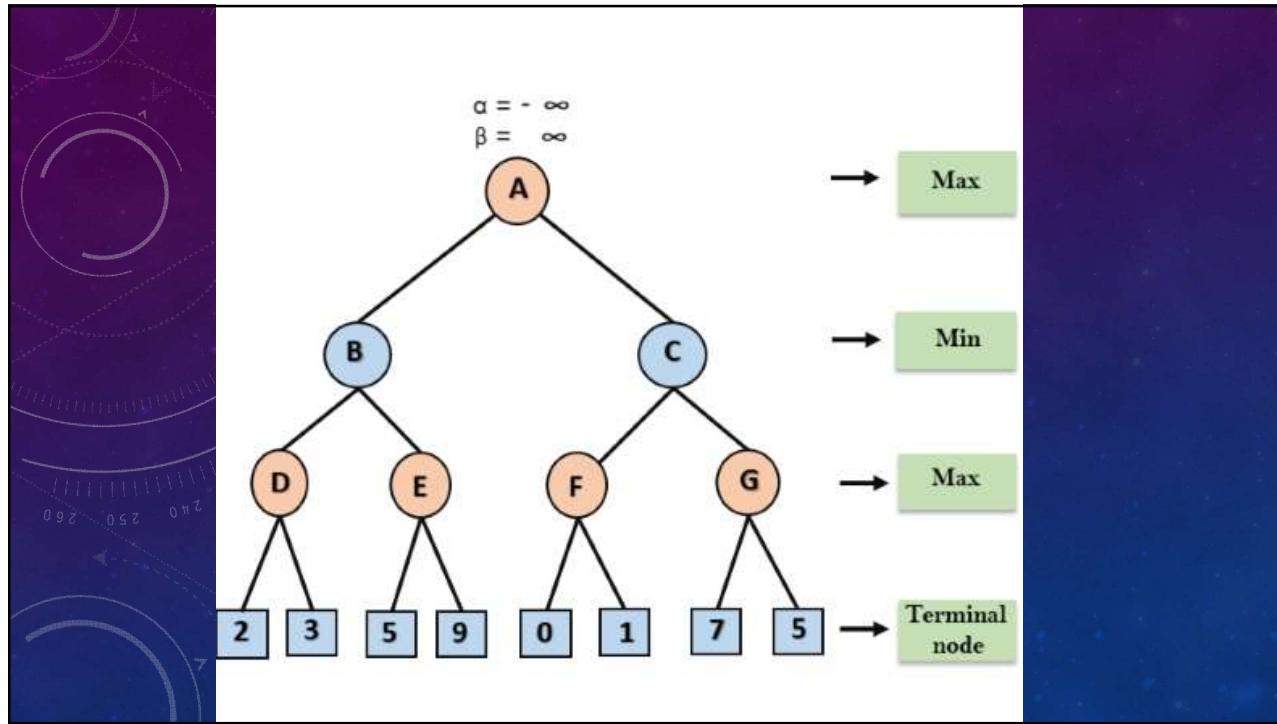
Key points about alpha-beta pruning:

- The Max player will only update the value of alpha.
- The Min player will only update the value of beta.
- While backtracking the tree, the node values will be passed to upper nodes instead of values of alpha and beta.
- We will only pass the alpha, beta values to the child nodes.

Working of Alpha-Beta Pruning:

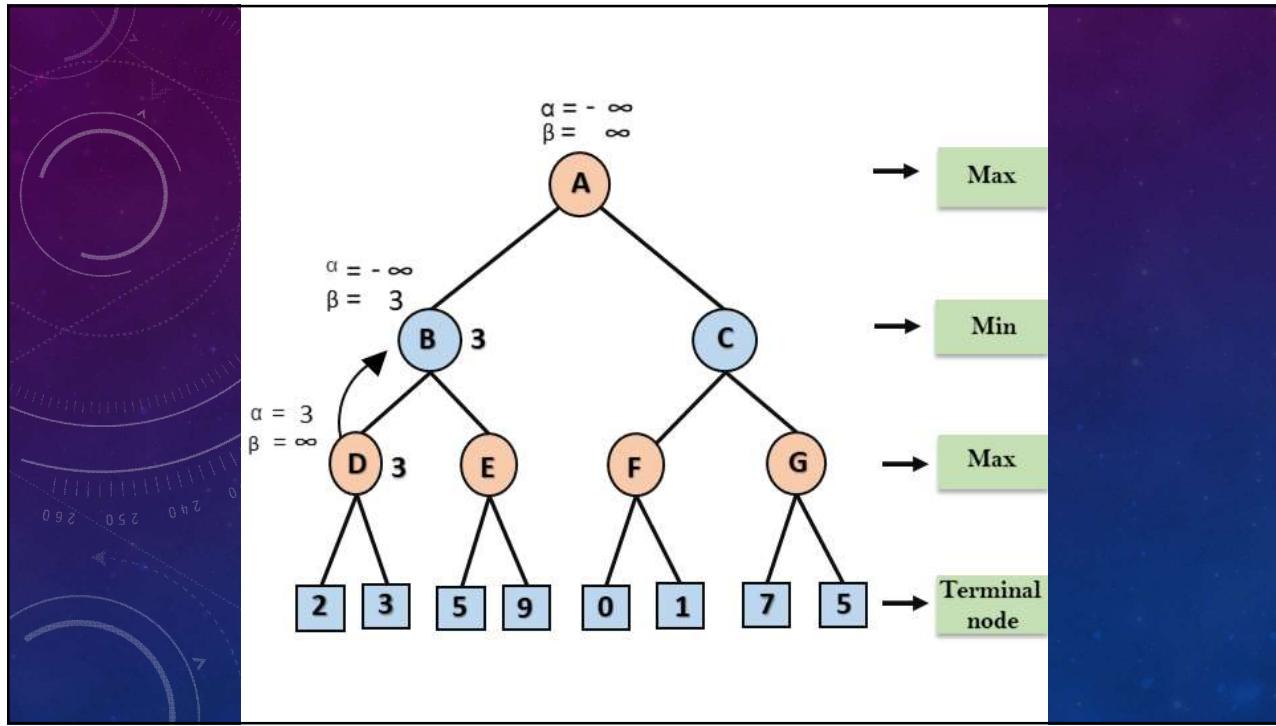
Let's take an example of two-player search tree to understand the working of Alpha-beta pruning

Step 1: At the first step the, Max player will start first move from node A where $\alpha = -\infty$ and $\beta = +\infty$, these value of alpha and beta passed down to node B where again $\alpha = -\infty$ and $\beta = +\infty$, and Node B passes the same value to its child D.



Step 2: At Node D, the value of α will be calculated as its turn for Max. The value of α is compared with firstly 2 and then 3, and the max (2, 3) = 3 will be the value of α at node D and node value will also 3.

Step 3: Now algorithm backtrack to node B, where the value of β will change as this is a turn of Min, Now $\beta = +\infty$, will compare with the available subsequent nodes value, i.e. $\min (\infty, 3) = 3$, hence at node B now $\alpha = -\infty$, and $\beta = 3$.



In the next step, algorithm traverse the next successor of Node B which is node E, and the values of $\alpha = -\infty$, and $\beta = 3$ will also be passed.

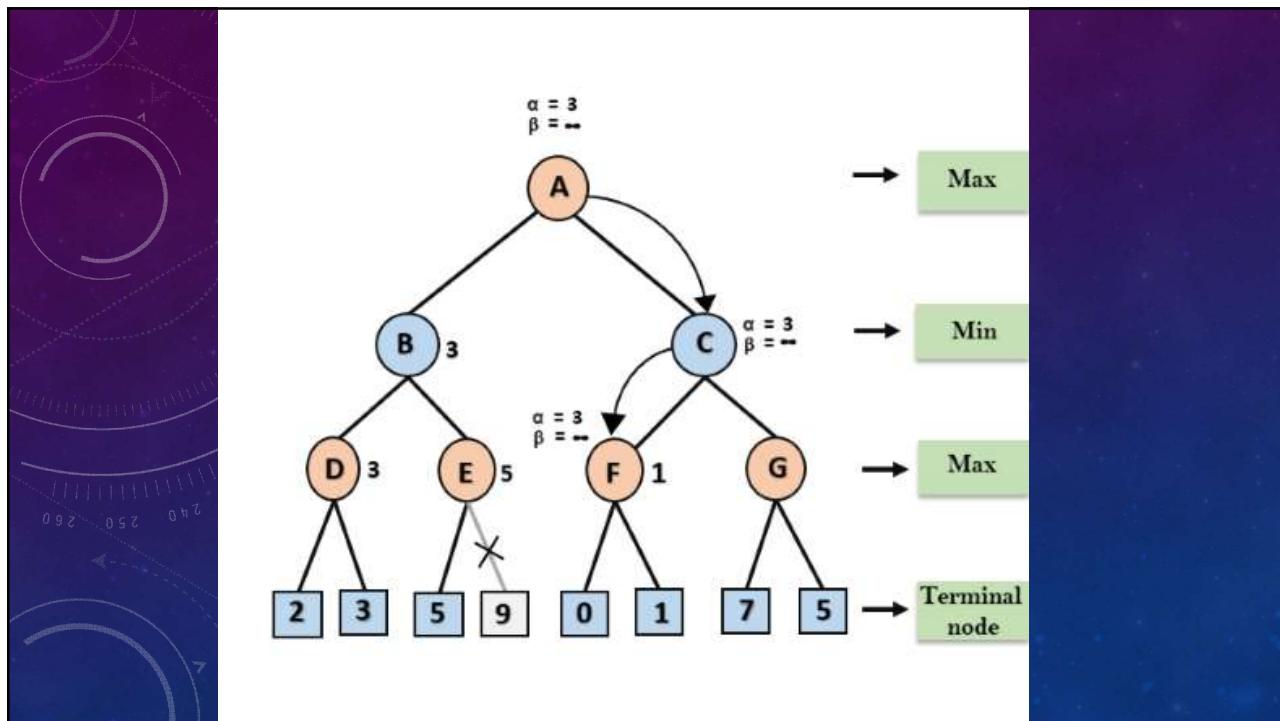
Step 4: At node E, Max will take its turn, and the value of alpha will change. The current value of alpha will be compared with 5, so $\max(-\infty, 5) = 5$, hence at node E $\alpha = 5$ and $\beta = 3$, where $\alpha >= \beta$, so the right successor of E will be pruned, and algorithm will not traverse it, and the value at node E will be 5.

Step 5: At next step, algorithm again backtrack the tree, from node B to node A.

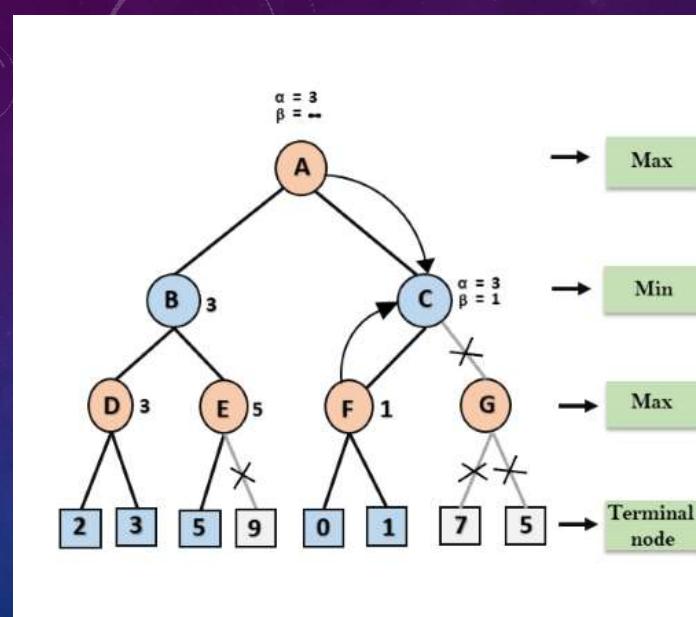
At node A, the value of alpha will be changed the maximum available value is 3 as $\max(-\infty, 3) = 3$, and $\beta = +\infty$, these two values now passes to right successor of A which is Node C.

At node C, $\alpha = 3$ and $\beta = +\infty$, and the same values will be passed on to node F.

Step 6: At node F, again the value of α will be compared with left child which is 0, and $\max(3, 0) = 3$, and then compared with right child which is 1, and $\max(3, 1) = 3$ still α remains 3, but the node value of F will become 1.



Step 7: Node F returns the node value 1 to node C, at C $\alpha = 3$ and $\beta = +\infty$, here the value of beta will be changed, it will compare with 1 so $\min(\infty, 1) = 1$. Now at C, $\alpha = 3$ and $\beta = 1$, and again it satisfies the condition $\alpha \geq \beta$, so the next child of C which is G will be pruned, and the algorithm will not compute the entire sub-tree G.



Step 8: C now returns the value of 1 to A here the best value for A is $\max(3, 1) = 3$. Following is the final game tree which is showing the nodes which are computed and nodes which have never been computed. Hence the optimal value for the maximizer is 3 for this example.

