

## Exercise 2.1

#2)

Loop invariant:- At the start of each iteration of the loop, the variable 'sum' is the sum of all the elements in the sub-array  $A[1:i-1]$

Initialisation: Before the first iteration, of  $i = 1$  the for loop,  $i = 1$  so and  $\rightarrow$  is then  $\text{sum} = 0$ . Since the sub-array  $A[1:0]$   $= A[1:0]$  <sup>which</sup> has no elements, and so the sum of its elements is 0 and the loop invariant is true.

Maintenance: Suppose the loop invariant is true before an iteration. That means for some integer  $k < n$ , the subarray 'sum' is the sum of the elements in the <sup>sub</sup>array  $A[1:k]$ .

Then after an iteration

Then ~~after~~ during the iteration

Then <sup>during</sup> after the iteration,  $A[i]$  is added to the 'sum', which was already the sum of  $A[1:i-1]$ , and so by definition of sum, it is now the sum of the elements of  $A[1:i]$ . Since  $i = i + 1$  before the next iteration, the variable sum is hence the sum of the elements of the subarray  $A[1:i-1]$  at the start of the next iteration and hence the invariant is still true.



Termination:- Once the loop variable terminates, 'i' exceeds n, i.e.  $i = n+1$ , the loop terminates. Letting  $i = n+1$  in the wording of the loop invariant yields that the variable 'sum' is the sum of the elements of the subarray  $A[1:(n+1)-1] = A[1:n]$  which is equal to the entire array. Hence, the algorithm is correct.

3) Insertion sort (A, n) // Decreasing

~~for i = 2 to n~~  
~~key =~~

~~for i = n down to 2~~  
~~key = A[i]~~  
~~j =~~

3)

Insertion sort ( $A, n$ ) // Decreasing order

```
for i = 2 to n
    key = A[i]
    j = i - 1
    while j > 0 and A[j] < key
        A[j+1] = A[j]
        j = j - 1
    A[j+1] = key
```

4)

Linear Search ( $A, x$ )

```
n = A.length
for j = 1 to A.length
    if A[j] == x
        return j
return NIL
```

Loop invariant :- Before each iteration of the loop with index  $j$ , there is no element in the sub array  $A[1:j-1]$  such that equals  $x$

Initialisation :- At the start of the first iteration  $j = 1$ , and so there ~~is no~~ invariant is true because there are no elements in the sub array  $A[1:0]$  in the first place



During Maintenance :- ~~If the~~ After each any iteration, either the loop is exited because an index  $i$  is found such that  $A[i] == x$  (and thus the algorithm is correct) or the index  $A[j] == x$  turns out to be false, and so ~~At~~ the subarray  $A[1:j]$  does not contain the variable  $x$ . Before the next iteration,  $j$  is incremented by 1, so the subarray  $A[1:j-1]$  does not contain the element  $x$ , and so the invariant stays true.

Termination :- The loop terminates either when an index  $i$  is found such that  $A[i] == x$  ~~and~~ and then it is returned (thus the algorithm is correct) or when  $j = n+1$ , ~~in Subst~~, which means there is no  $x$  in the subarray  $A[1:n+1-1] = A[1:n]$  (which is simply the entire array) and so NIL is returned and the algorithm is correct.

5/

5) AD

ADD-BINARY-INTEGERS (A, B, n)

C = new integer[n+1]

carry = 0

for i = 0 to n-1

sum = carry + A[i] + B[i]

if sum == 0

C[i] = 0

else - if sum == 1

C[i] = 1

carry = 0

else - if sum == 2

C[i] = 0

carry = 1

else - if sum == 3

C[i] = 1

C[n] = carry

return C