

## Exercise 2.1

Q2)

Loop invariant:- At the start of each iteration of the loop, the variable 'sum' is the sum of all the elements in the sub-array  $\& A[0: i-1]$

Initialisation: Before the first iteration, of  $i=$  the for loop,  $i=1$ , so and is then  $\text{sum} = 0$ . Since The sub-array  $A[1: 0: 1-1] = A[1: 0]$  which has no elements, and so the sum of its elements is 0 and the loop invariant is true.

Maintenance: Suppose the loop invariant is true before #an iteration. That means for some integer  $k$   $k \leq n$ , the subarray 'sum' is the sum of the elements in the <sup>sub</sup>array  $A[1 : k]$ .

Then after an iteration

Then after during the iteration

Then <sup>during</sup> after the iteration,  $\& A[i]$  is added to the 'sum', which was already the sum of  $A[1 : i-1]$ , and so by definition of sum, it is now the sum of the elements of  $A[1 : i]$ . Since  $i = i+1$  before the next iteration, the variable sum is hence the sum of the elements of the subarray  $A[1 : i-1]$  at the start of the next iteration and hence the invariant is still true.

Termination:- Once the loop variable terminates, 'i' exceeds  $n$ , i.e.  $i = n+1$ , the loop terminates. Letting  $i = n+1$  in the wording of the loop invariant yields that the variable 'sum' is the sum of the elements of the subarray  $A[1:(n+1)-1] = A[1:n]$  which is equal to the entire array. Hence, the algorithm is correct.

3) Insertion sort (A, n) //Decreasing

~~for for  $i=2$  to  $n$   
key =~~

~~for  $i=n$  down to 2  
key =  $A[i]$   
 $j=0$~~

3)

Insertion sort ( $A, n$ ) // Decreasing order

for  $i = 2$  to  $n$

key =  $A[i]$

$j = j - 1$

while  $j > 0$  and  $A[j] < \text{key}$

$A[j + 1] = A[j]$

$j = j - 1$

$A[j + 1] = \text{key}$

4)

Linear Search ( $A, n$ )

$n = A.length$

for  $j = 1$  to  $A.length - n$

if  $A[j] == n$

return  $j$

return NIL

Loop invariant :- Before each iteration of the loop with index  $j$ , there is no element in the sub array  $A[1:j-1]$  such that equals  $n$

Initialisation:- At the start of the first iteration  $j = 1$ , and so ~~there is no invariant is true because there are no elements in the sub array  $A[1:0]$  in the first place~~

During

Maintenance :- If the loop is exited because an index  $i$  is found such that  $A[i] == n$  (and thus the algorithm is correct) or the index  $A[j] == n$  turns out to be false, and so ~~the~~ the subarray  $A[1:j]$  does not contain the variable  $n$ . Before the next iteration,  $j$  is incremented by 1, so the subarray  $A[1:j-1]$  does not contain the element  $n$ , and so the invariant stays true.

Termination :- The loop terminates either when an index  $i$  is found such that  $A[i] == n$  (and then it is returned (thus the algorithm is correct)) or when  $j=n+1$  in ~~inSubst~~, which means there is no  $n$  in the subarray  $A[1:n+1-1] = A[1:n]$  (which is simply the entire array) and so NIL is returned and the algorithm is correct.

5)

5) AD

ADD-BINARY-INTEGERS (A, B, n)

C = new integer [n+1]

Carry = 0

for i = 0 to n-1

    sum = carry + A[i] + B[i]

    if sum == 0

        C[i] = 0

    else - if sum == 1

        C[i] = 1

        carry = 0

    else - if sum == 2

        C[i] = 0

        carry = 1

    else - if sum == 3

        C[i] = 1

    C[n] = carry

return C