Wavelet transform as a potential tool for ECG analysis and compression

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ABSTRACT

The recently introduced wavelet transform is a member of the class of time–frequency representations which include the Gabor short-time Fourier transform and Wigner–Ville distribution. Such techniques are of significance because of their ability to display the spectral content of a signal as time elapses. The value of the wavelet transform as a signal analysis tool has been demonstrated by its successful application to the study of turbulence and processing of speech and music. Since, in common with these subjects, both the time and frequency content of physiological signals are often of interest (the ECG being an obvious example), the wavelet transform represents a particularly relevant means of analysis. Following a brief introduction to the wavelet transform and its implementation, this paper describes a preliminary investigation into its application to the study of both ECG and heart rate variability data. In addition, the wavelet transform can be used to perform multiresolution signal decomposition. Since this process can be considered as a sub-band coding technique, it offers the opportunity for data compression, which can be implemented using efficient pyramidal algorithms. Results of the compression and reconstruction of ECG data are given which suggest that the wavelet transform is well suited to this task.

Keywords: ECG analysis, wavelet transform, signal analysis

INTRODUCTION

Wavelet transform

The wavelet transform (WT) is a recently introduced time-scale representation that has already found applications in a variety of fields[1,2]. In essence, a wavelet decomposition of a signal amounts to its description in terms of shifted and dilated versions of some basis wavelet. The shift operation enables signals to be localized in time whilst the dilation operation allows the scale of the original signal to be determined. Hence, a signal can be localized in both time and scale (which can be considered as similar to frequency) dimensions thus leading to the term 'time-scale' representation.

The basis wavelet chosen can be tailored to suit the application, as long as it meets certain mathematical criteria[3]. A commonly used wavelet is the complex modulated gaussian which has the form $g(t) = e^{i\omega t}\exp(-t^2/2)$. Like all wavelets it is time-limited (due to the exponential envelope), which is why shifted versions of the wavelet are required for reconstruction of the original signal, and the WT can localize events in time. In addition, the nature of $g(t)$ illustrates that wavelets are oscillatory about a zero mean.

Upon dilation (which is essentially a 'stretching' in the time domain), the wavelet's 'frequency', ω, will clearly decrease thus allowing analysis at a different scale. Note that this dilation results in a decreased ability to localize events in time (although the frequency resolution correspondingly increases) because the wavelet itself becomes less localized.

The WT can be considered to belong to the class of time–frequency transforms which include the Wigner–Ville and Gabor short-time Fourier transform (STFT)[3]. Advantages to be gained through its use include the ability to localize singularities more accurately in the time domain; near perfect reconstruction from the transform coefficients without the requirement for oversampling[2]; efficient implementation by pyramidal algorithms[4,5]; and that the scaling operation (via dilation of the basis wavelet) makes it a suitable method for the investigation of fractals[6].

The first two of these advantages are directly related to the sampling patterns of the discrete transforms in phase-space[3,4]. Whereas the STFT utilizes identical resolution cells throughout, the shape of the resolution cells of the wavelet transform vary with scale. The consequence of this is that the WT has better time resolution at higher scales, there being an inverse relationship throughout phase-space between the time and scale resolutions as referred to above. It is this that leads to the constant-Q filtering (i.e. the ratio of the filter's bandwidth to its centre frequency remains constant) performed by the WT[3,7].

Previous applications of the wavelet transform

Areas in which the WT has already been successfully used include firstly, and of particular relevance to the field of biomedical engineering, its use for speech (and music) processing which has been linked to the

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constant-Q filtering that mimics the processing performed by the ear[1,7]. Also significant here, considering the structure of these type of signals, is the ability to localize events in time.

A second application has been the analysis of turbulence and multifractal structures. The WT is of value in these fields because of its scaling properties, via dilation, and the self-similar nature of fractals[6].

Whilst the above two uses are in signal processing, another application of the WT has been for image compression[4]. It is suited to this task because it has a stable reconstruction; there is no need for oversampling of the phase-space and it can be implemented using computationally efficient pyramidal (tree) algorithms. It is therefore not surprising that extremely efficient image compression appears attainable[8].

Finally, two cases of direct application of the WT to ECG processing have been in the detection of (simulated) ventricular late potentials (VLP)[9] and ventricular tachycardia (VT)[10]. In the former, it is the coefficients determined at a particular scale that enable the VLP to be detected, whilst patients prone to VT can be distinguished via combined time-scale plots of the WT using a three lead system.

Base wavelet

In contrast to most other transforms the basis function (i.e. wavelet) can be chosen to suit a particular application. However, in practice a small number of wavelets have been found to be particularly suited to certain applications, and it is ones such as these that have been used to produce the results presented here.

It should be noted that whereas certain wavelets are approximations to the ideal, others, derived for instance via the solution to dilation equations, are exact. It is the latter type that allow perfect reconstruction without recourse to oversampling, and which are therefore suited to use in data compression.

Wavelet transform and ECG data

Recently it has been suggested that HRV data (i.e. the tachogram) exhibits chaotic and fractal properties[11], and we felt that the WT may be one of several tools which could be used to investigate this possibility.

However, whilst applying the technique to this problem it was discovered that it offers an extremely efficient means of compressing raw ECG data. Consequently, the results presented here include the WTs of HRV and of ECG data, plus demonstrations of the compression and reconstruction of raw ECG data.

METHODS

All computation during this initial investigation was performed using a Mesh 21 MHz (Landmark) '286 PC' plus Cyrix 'FasMath' co-processor. Programs were written firstly to obtain and display the WTs of the ECG and HRV data, and secondly to perform the compression, reconstruction and comparison of ECG data.

WAVELET TRANSFORM OF HRV AND ECG DATA

Implementation

The WTs were obtained directly by convolving the dilated (sampled) wavelets with the data using the discrete version of the wavelet transform (i.e. without recourse to any of the more efficient algorithms that have been developed)[12,13]. Although this obviously increased the time of computation this was not a problem for the limited amount of data studied to date. The wavelet used to study the ECG data was the complex modulated gaussian, which has been widely used[7,14] whilst for the HRV data the real gaussian wavelet was employed since this has been shown to be of value in analysing fractal signals[6].

Test transforms

To demonstrate the nature of the WT produced, two test signals were used. *Figure 1* shows the time-scale plot of the wavelet transform of a sinusoid switched on approximately half way through the data sequence. The complex modulated gaussian was used in this case and so two time-scale plots are produced corresponding to the amplitude and phase. The ability of the transform to localize in both time and scale is clearly evident.

Figure 2 shows the WT of the Cantor Set (a fractal pattern) produced using the real gaussian wavelet. The repetitive (fractal) nature of the transform can be

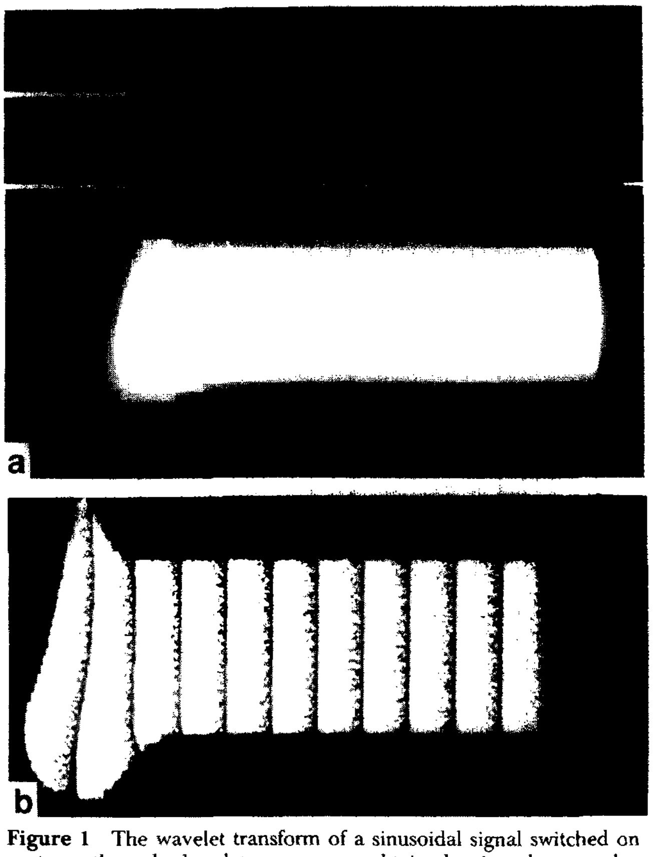

Figure 1 The wavelet transform of a sinusoidal signal switched on partway through the data sequence, obtained using the complex modulated gaussian wavelet, with **a**, amplitude and **b**, phase data displayed separately. Localization of the signal in both time (horizontal axis) and scale (vertical axis) dimensions is clearly visible in the amplitude representation. The period of the signal is readily distinguishable from the phase information

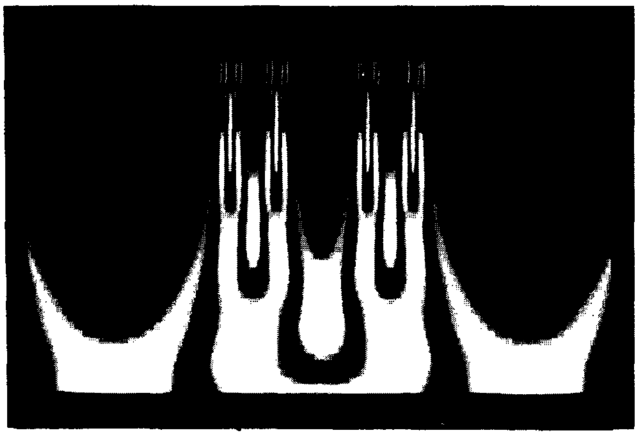

Figure 2 The wavelet transform of the Cantor set, illustrating the application of this method to the study of fractal data

seen, which illustrates the use of the technique in this context. In this example, no phase information is produced since the wavelet is real.

Results

The transforms of raw ECG data, sampled at 500 Hz and 250 Hz respectively, are shown in *Figures 3* and *4*. The complex modulated gaussian wavelet was used. Localization of the QRS complex in the time domain is clearly shown whilst structure in the transform due to the P and T complexes is also clearly visible.

Figures 5a, b show the WTs of HRV data obtained using the real gaussian wavelet. *Figure 5a* is of data from a healthy male subject whilst *Figure 5b* was produced using data from the MIT/BIH database, (patient no. 115, exhibiting normal sinus rhythm with sinus arrythmia present).

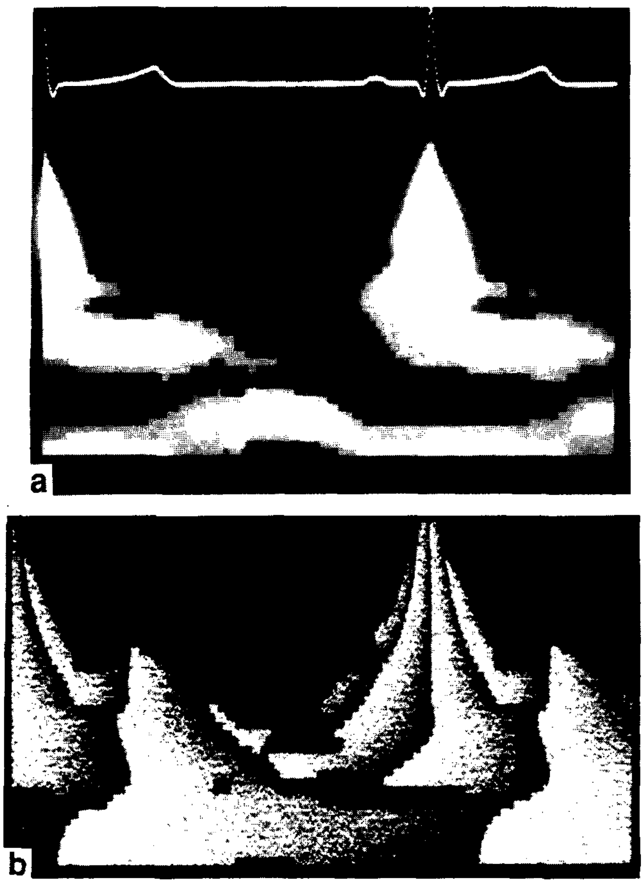

Figure 3 The wavelet transform of ECG data sampled at 500 Hz, with **a**, amplitude and **b**, phase information displayed separately

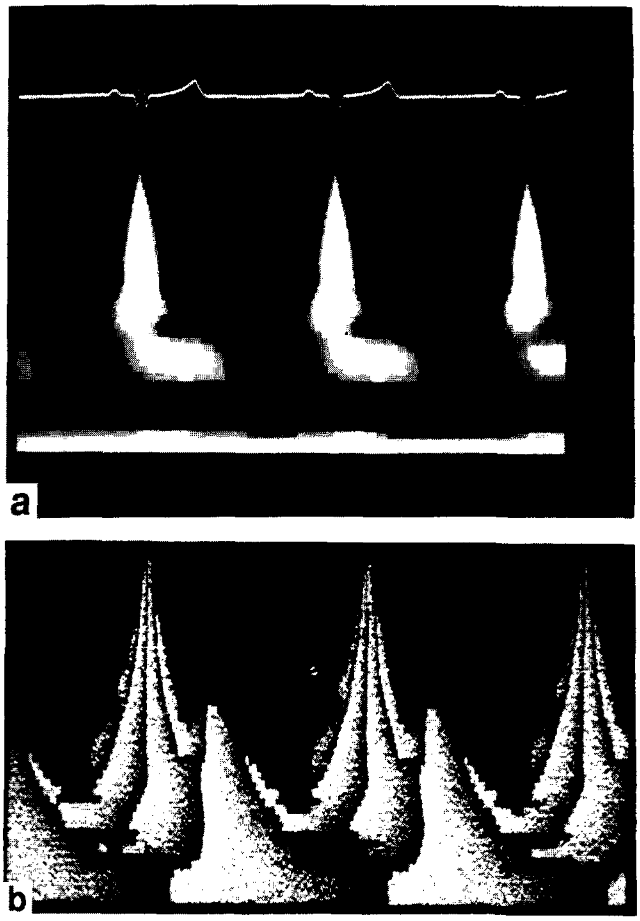

Figure 4 As for *Figure 3* but with a sampling rate of 250 Hz

COMPRESSION OF ECG DATA

Implementation

The wavelet used during ECG data compression is one of those derived by I. Daubechies from her extensive work on the mathematical construction of wavelets[15]. In order to obtain the coefficients of a transform utilizing this wavelet, filter matrices are produced which are then used to process the data and so generate the required coefficients[15], rather than using the convolution process referred to above.

Two such filter matrices are needed (that use the same coefficients but in changed order and with changed signs in places) which effectively perform low and high pass filtering operations on the data. Four coefficients were needed for the chosen wavelet with values: $(1+\sqrt{3})/4$, $(3+\sqrt{3})/4$, $(3-\sqrt{3})/4$ and $(1-\sqrt{3})/4$.

The results of high pass filtering contain information concerning the 'fine' detail of the data and are the actual wavelet coefficients. The results of low pass filtering produce a 'blurred', smoothed version of the data which is subsequently reprocessed by both filters. Further (high passed) wavelet coefficients are then generated together with an even smoother version of the original data which is yet again refiltered.

After each low pass filtering process the data has

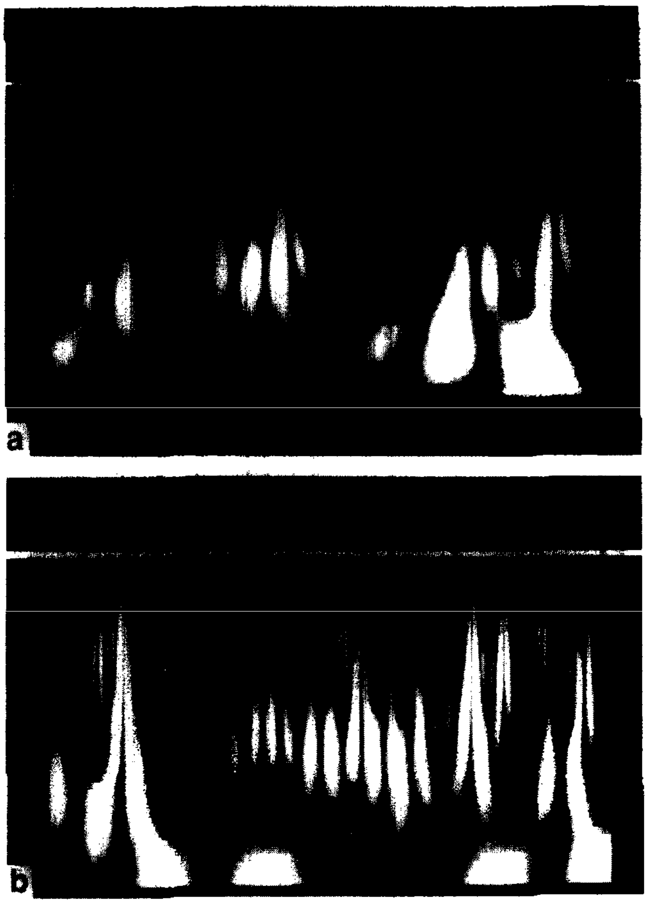

Figure 5 The wavelet transforms of HRV data obtained using the real gaussian wavelet for **a**, a healthy adult male and **b**, showing a section of data from recording E115 in MIT/BIH database

lost fine detail (stored as the wavelet coefficients) and can thus be represented by half the number of data points; this is intrinsically linked to the change in scale of the analysing wavelet at each stage via the dilation process. Therefore the number of wavelet coefficients produced at each stage is halved (related to the aforementioned link between time and scale resolution and sampling of phase-space).

By considering the production of the wavelet coefficients in this manner, the use of a tree algorithm to achieve the data compression becomes clear[5]. Furthermore, reconstruction of the original signal is also performed with a tree algorithm. Here the wavelet coefficients (i.e. fine detail) are fed back at each stage of the tree structure (beginning with the final low pass filtered version of the data) to produce successively higher resolution reconstructions of the data until the original data is reproduced.

Data sets of 1026 points of sampled ECG data were used which, because of the need for zero padding to overcome edge effects, generated 1046 transform coefficients. This corresponds to a 10-stage filtering process. (Larger or smaller blocks of data could have been used.) As described above, filtering simply required the use of the appropriate matrices using four coefficient values. 16-bit integer arithmetic was used throughout.

In order to investigate data compression, after generation of the transform coefficients a threshold was chosen below which all coefficients were set to zero. Reconstruction was then performed with a note being made of the number of non-zero coefficients used.

Only the first 640 of the 1026 data points are shown in the figures due to the limits of the graphics display. The figures show the original data, the reconstructed signal, the magnified (with a factor of 2) difference between the original and reconstructed signals and a graphical display of the wavelet coefficients used for reconstruction.

Regarding this graphical display, each WT coefficient is represented by a point which is highlighted if it is used in the reconstruction. Each row represents a stage in the tree algorithm, with 10 stages in all plus the results of the final low pass filtering operation. The lowest line corresponds to the initial stage (and therefore has the most coefficients), the line above the next stage (with approximately half the number of coefficients) and so on.

Results of ECG compression and reconstruction

Figures 6a–c illustrate the compression and reconstruction of ECG data sampled at 500 Hz using different numbers of non-zero transform coefficients. These correspond to compression ratios (expressed simply as the ratio of number of data points compressed to the number of wavelet coefficients used for reconstruction) of 4.1, 9.9 and 27.0 to 1.

Figure 6c illustrates the construction of the QRS complex even at large compression ratios, although the P and T complexes are significantly distorted. It also shows the noticeable appearance, as expected, of the shape of the base wavelet in the reconstructed signal when a limited number of coefficients are used.

The display of the coefficients used demonstrates that they are concentrated around the two QRS complexes contained within the original data.

DISCUSSION

Wavelet transform of ECG data

As expected the 'time-scale' transform of ECG data consists of a characteristic pattern, with the QRS complex clearly localized in both time and scale. The question that must now be addressed is whether any abnormalities in the ECG may be clearly identified by analysis via the WT. Early results concerning the detection of VLP[9] and VT[10] suggest that this is the case, and that further work is needed to investigate areas such as the optimum means of data display and which lead systems to employ.

Wavelet transform of HRV data

The WT of HRV data presented are representative of typical results obtained. We now intend to use this technique in conjunction with more formal methods (e.g. dimensional analysis) to investigate whether the suggested chaotic and/or fractal nature of the tachogram does indeed exist.

ECG compression using the wavelet transform

The results of ECG data compression using the WT show that efficient compression, in terms of computa-

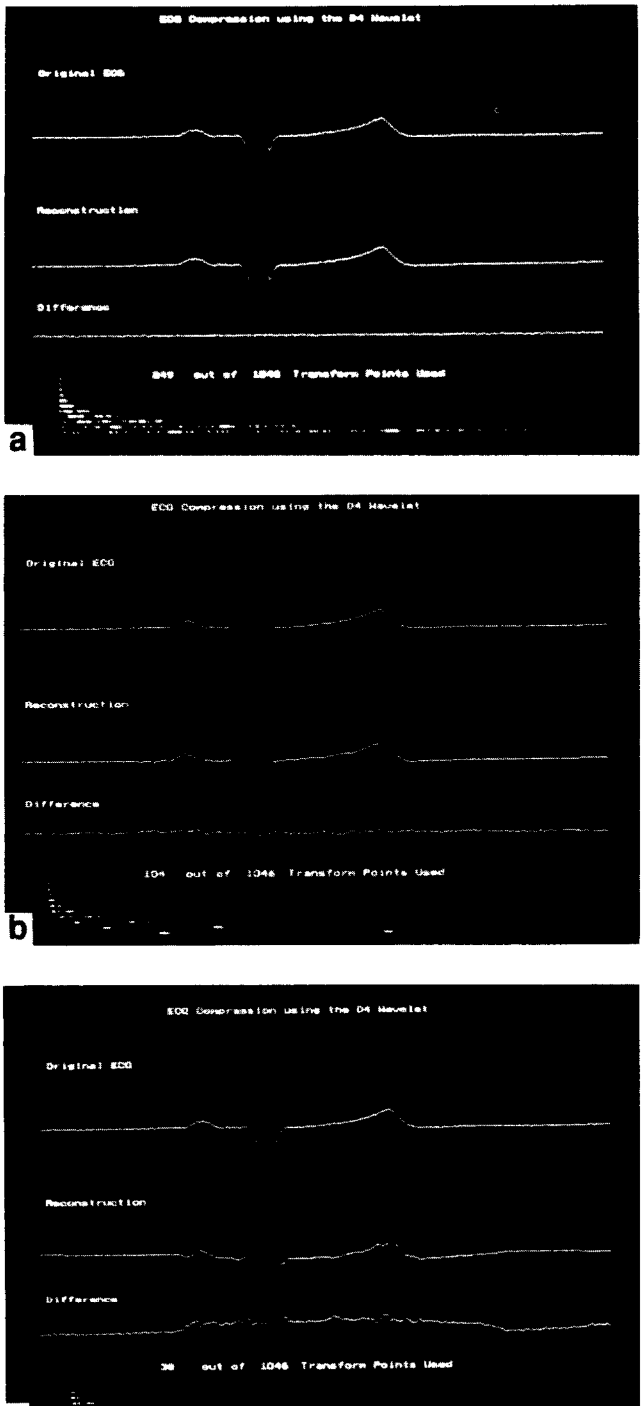

Figure 6 The results of the compression and reconstruction of ECG data sampled at 500 Hz. The original and reconstructed signals are shown together with the difference between them (magnified by a factor of 2). Also shown is a graphical display of the coefficients used for reconstruction. Note that only 640 of the original 1026 data points which were compressed are shown. These results represent compression ratios of **a**, 4.1, **b**, 9.9 and **c**, 27.0 to 1 respectively

tional effort and data storage, can be achieved. Its suitability for this task is probably linked to the nature of both the ECG (the signal being concentrated in time) and the WT (which relies upon reconstructing a signal from dilated and shifted versions of the base wavelet). In addition, the wavelet chosen for this application appears well suited to the task.

CONCLUSION

The preliminary results presented in this paper show that the wavelet transform is worthy of further investigation as a signal analysis tool which can be applied to the ECG. They also clearly demonstrate its potential for the efficient compression of ECG data.

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