ADAPTIVE LEAST SQUARES CORRELATION: A POWERFUL IMAGE MATCHING TECHNIQUE

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ABSTRACT

The Adaptive Least Squares Correlation is a very potent and flexible technique for all kinds of data matching problems. Here its application to image matching is outlined. It allows for simultaneous radiometric corrections and local geometrical image shaping, whereby the system parameters are automatically assessed, corrected, and thus optimized during the least squares iterations. The various tools of least squares estimation can be favourably utilized for the assessment of the correlation quality. Furthermore, the system allows for stabilization and improvement of the correlation procedure through the simultaneous consideration of geometrical constraints, e.g. the collinearity condition. Some exciting new perspectives are emphasized, as for example multiphoto correlation, multitemporal and multisensor correlation, multipoint correlation, and simultaneous correlation/triangulation.

1. INTRODUCTION

Image matching is a key component in almost any image analysis process. Image matching is crucial to a wide range of applications, e.g. to navigation, guidance, automatic surveillance, robot vision, and to the mapping sciences. Stereo image matching in particular is of fundamental importance to photogrammetry. Any automated system for three-dimensional point positioning must include a potent procedure for stereo image or dual image matching.

Cross-correlation and related techniques have dominated the field since the early fifties. The shortcomings of this class of image matching methods have caused a slow-down in the development of operational automated correlation systems. This problem was addressed at a panel session at the annual ASP Convention in Denver, March 1982. Five short articles in *Photogrammetric Engineering and Remote Sensing* **49**(4) reflect a part of this discussion.

Cross-correlation cannot respond appropriately to a number of facts that are inseparably related to stereo images of three-dimensional and sometimes even two-dimensional objects. The conjugate images created under the laws of perspective projection might differ considerably from each other. Terrain slope, height differences, and positional and attitude differences of the sensors give cause to geometrical distortions. Illumination and reflectance conditions might distort the images radiometrically. Under certain conditions this could even trigger a geometrical displacement. Noise from the electrical components and the sampling rate (pixel size) could also influence both the geometrical and the radiometric correspondence of the images.

Cross-correlation works fast and well if the patches to be matched contain enough signal without too much high frequency content and if geometrical and radiometric distortions are kept at a minimum. Both conditions are often not encountered in standard aerial images. Therefore a great effort has been made in recent years to design matching techniques that are more efficient than cross-correlation (e.g. Baker & Binford 1982, Ackermann 1984, Foerstner 1984, Rosenfeld 1984). Concepts which were suggested by the artificial intelligence community are first- and second-order derivative matching, relaxation methods, segmentation and graph structure matching, transform ('Hough transform') matching, feature (edge) matching, etc. Thus, a tendency to switch from area-based to edge-based analysis can be observed. Rosenfeld (1984) remarks in a critical review that these new methods do not solve the problems addressed before.

A method that combines the merits of area-based and edge-based matching was applied to photogrammetry by Foerstner (1982), Ackermann (1984), and Pertl (1984). The author has investigated this technique since 1982. Tests have been performed with synthetic and real images. The method was found to be of great potential for a variety of image and template matching problems. It is of particular value for some essential photogrammetric tasks such as automated detection and measurement of fiducial marks, reseau crosses, stereomodel (Gruber) points, control points, the transfer and measurement of tie points, and the generation of digital terrain models. Multispectral, multitemporal, and multisensor images can be processed, and photo/map registration tasks can be performed. Besides, it can be used as pattern recognizer, feature extractor, and line follower.

The basic equations of this matching technique are set up in the context of a statistical estimation model. The estimation is performed as least squares estimation. The familiar apparatus of the least squares approach with respect to parameter estimation and hypothesis testing can be favourably utilized. Precision and reliability measures are readily available and allow an assessment of the quality of the match in a better way than is feasible with other matching techniques. Algorithmic, computational, and numerical aspects can also be studied in a well-known environment. The method is tagged as 'adaptive' because it can be executed in a self-tuning *modus*, i.e. the parameter set to be estimated can be automatically corrected in order to obtain a most appropriate estimation model set-up with respect to the specific signal content of the patches to be matched.

This paper presents the basic estimation model of the adaptive least squares correlation, discusses the special characteristics of this technique, and outlines some extensions which demonstrate the great flexibility and potential of the method for data matching and for image matching tasks in particular.

2. THE ESTIMATION MODEL-BASIC APPROACH

Although this technique can perform image matching functions in a variety of different applications, we refer in the sequel to the correlation of two digital images only.

Assume two image regions are given as discrete two-dimensional functions f(x,y), and g(x,y), which might have been derived from continuous functions (analogue photographs). f(x,y) and g(x,y) can be defined as conjugate regions of a stereopair in the 'left' and the 'right' photograph respectively. f(x,y) is interpreted in the following as the 'template', g(x,y) as the 'picture'. Correlation is (ideally) established if

$$f(x,y) = g(x,y)\,. \tag{1}$$

Because of random effects (noise) in both photographs (or in one set only if the template is assumed to be free of noise), equation (1) is not consistent. Therefore a noise vector e(x,y) is added, resulting in

$$f(x,y) - e(x,y) = g(x,y)\,. \tag{2}$$

e(x,y) is a true error vector.

The location of the function values g(x,y) must be determined in order to provide for the match point. This is achieved by minimizing a goal function which measures the distances between the grey levels in template and picture. The goal function to be minimized in this approach is the L_2-norm of the residuals of least squares estimation.

In the least squares context equation (2) can be considered as a nonlinear observation equation (standard problem II) which models the vector of observations f(x,y) with a function g(x,y), whose location in the 'right' photograph needs to be estimated. The location is described by shift parameters Δx, Δy, which are counted with respect to an initial position of g(x,y), the approximation of the conjugate picture region $g^0(x,y)$.

In order to account for a variety of systematic image deformations and to obtain a better match, image shaping parameters and radiometric corrections are introduced besides the shift parameters.

If the grey values are given over a grid, the image shaping is achieved by resampling of $g^0(x,y)$ over the transformed grid points. The geometrical transformation is modelled by a bivariate polynomial

$$x = t_y^T\,\bar{A}\,t_x\ , \tag{3a}$$

$$y = t_y^T\,\bar{B}\,t_x\ , \tag{3b}$$

with $$t_x^T = \{1\ x_0\ x_0^2 \ldots x_0^{m-1}\}\ , \tag{4a}$$

$$t_y^T = \{1\ y_0\ y_0^2 \ldots y_0^{m-1}\}\ , \tag{4b}$$

and the parameter matrices $\bar{A}, \bar{B}$

$$\bar{A} = \begin{bmatrix} a_{11} & a_{12} & . & . & a_{1m} \\ . & & & & . \\ . & & & & . \\ . & & & & . \\ a_{m1} & a_{m2} & . & . & a_{mm} \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} b_{11} & b_{12} & . & . & b_{1m} \\ . & & & & . \\ . & & & & . \\ . & & & & . \\ b_{m1} & b_{m2} & . & . & b_{mm} \end{bmatrix} \tag{5}$$

x_0, y_0 are the grid locations of the data points of $g^0(x,y)$.

The transformation parameters $a_{11}, \ldots, a_{mm}, b_{11}, \ldots, b_{mm}$ need to be estimated from (2).

In order to be able to operate with the conventional least squares approach the function g(x,y) in (2) must be linearized.

This yields

$$f(x,y) - e(x,y) = g^0(x,y) + \frac{\partial g^0(x,y)}{\partial x} dx + \frac{\partial g^0(x,y)}{\partial x} dy \quad . \tag{6}$$

$$dx = \frac{\partial x}{\partial p_i} dp_i \quad ,$$

p_i = ith transformation parameter in (3)

$$dy = \frac{\partial x}{\partial p_i} dp_i \quad .$$

For reasons, which are explained later in this section, we specify $\bar{A}, \bar{B}$ to

$$\bar{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & 0 \end{bmatrix} , \tag{7a}$$

$$\bar{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & 0 \end{bmatrix} ,$$

and obtain thus the transformation

$$\begin{aligned} x &= a_{11} + a_{12} x_0 + a_{21} y_0 \ , \\ y &= b_{11} + b_{12} x_0 + b_{21} y_0 \ , \end{aligned} \tag{7b}$$

which allows for a fully affine image shaping of the picture function.

Equation (7b) also include the shift parameters Δx, Δy, which are denoted here by a_{11}, b_{11} .

Differentiation of (7b) gives

$$\begin{aligned} dx &= da_{11} + x_0 da_{12} + y_0 da_{21} \ , \\ dy &= db_{11} + x_0 db_{12} + y_0 db_{21} \ . \end{aligned} \tag{8}$$

Using the simplified notations

$$g_x = \frac{\partial g^0(x,y)}{\partial x} \ , \quad g_y = \frac{\partial g^0(x,y)}{\partial y}$$

and adding a radiometric shift parameter r_s to system equation (6) results with (8) in

$$\begin{aligned} f(x,y) - e(x,y) = g^0(x,y) + g_x da_{11} + g_x x_0 da_{12} + g_x y_0 da_{21} \\ g_y db_{11} + g_y x_0 db_{12} + g_y y_0 db_{21} + r_s \ . \end{aligned} \tag{9}$$

Combining the parameters in (9) in the parameter vector x

$$x^T = \{da_{11}, da_{12}, da_{21}, db_{11}, db_{12}, db_{21}, r_s\} ,$$

their coefficients in the design matrix A, and the vector difference $f(x,y) - g^0(x,y)$ in ℓ, the observation equations are obtained in classical notation (with $e = e(x,y)$) as

$$\ell - e = Ax \ . \tag{10a}$$

With the statistical expectation operator E and the assumptions

$$E(e) = 0 , \ E(ee^T) = \sigma_0^2 P^{-1} \tag{10b}$$

the system (10) is a Gauss-Markov estimation model.

The least squares estimation in model (10a), (10b) leads to the unbiased, minimum variance estimators

$$\hat{x} = (A^T P A)^{-1} A_T P \ell \quad \text{, solution vector} \tag{11a}$$

$$\hat{\sigma}_0^2 = \frac{1}{r} v^T P v \quad \text{, variance factor} \tag{11b}$$

$$v = A\hat{x} - \ell \quad \text{, residual vector} \tag{11c}$$

$r = n - u$

r . . . redundancy

u . . . number of transformation parameters

n . . . number of observations

The residuals v_i can be interpreted as the differences in grey levels between the estimated picture patch (region surrounding the matchpoint) and the template patch.

$$v_i = \hat{g}(x,y)_i - f(x,y)_i \quad , \quad i = 1, \ldots, n \tag{12}$$

^ . . . stands for least squares estimator

Hence $\hat{\sigma}_0$ is an *a posteriori* estimator for the difference of the local template noise and picture noise.

Since the function values g(x,y) in (2) are actually stochastic quantities, we face here the peculiar case that the elements of the design matrix A are not fixed, but stochastic quantities. A strict estimation procedure would have to consider this fact. In order to avoid unnecessary complications the stochastic properties of A are simply ignored. This allows the application of the computationally efficient standard algorithm for least squares estimation as outlined to (10a), (10b). It is expected that the omissions are minor and do not disturb the results significantly.

Because of the nonlinearity of the original setup (2), the final solution is obtained iteratively. With the first approximations

$$a_{11}^0 = b_{11}^0 = a_{21}^0 = b_{12}^0 = 0, \quad a_{12}^0 = b_{21}^0 = 1 \tag{13}$$

(r_s is linear *a priori*)

results the set of coordinates

$$x_i = x_{0i} \; , \; y_i = y_{0i} \; , \; i = 1, \ldots, n$$

for the first iteration step (n = number of grid points (grey levels) in template or picture).

After the solution vector (11a) is obtained the transformation (7b) is applied and $g^0(x,y)$ is resampled over the new set of coordinates, and the design matrix A is re-evaluated. The iteration stops if each element of the alteration vector $\hat{x}$ in (11a) goes below a certain limit (c_1 , … , c_6)

$$\begin{aligned} |da_{11}| < c_1 \; , \; |db_{11}| < c_2 \; , \\ |da_{12}| < c_3 \; , \; |db_{12}| < c_4 \; , \\ |da_{21}| < c_5 \; , \; |db_{21}| < c_6 \; . \end{aligned} \tag{14}$$

The functional model consists essentially of two parts. One set of parameters is designed to model the geometrical distortions (‘shaping function’), the other set corrects the radiometric anomalies. Two different considerations of diametrical kind lead to the selection of a particular parameter set. The estimation model should accommodate enough parameters in order to be able to model the distortions as comprehensively as possible. This, however, induces the danger of over-parameterization. Therefore the quest for many modelling parameters is balanced by the need to restrict the type and number of parameters to those which are safely determinable. Nondeterminable parameters do have an impairing effect on the estimation model and deteriorate the quality of the match. Hence it is of crucial importance for least squares correlation to include an appropriate test procedure for parameters and to be capable of excluding those individual parameters which are nondeterminable. The designation 'adaptive' indicates such a capability. This is a 'self-tuning' approach, the tuning being performed simultaneously with the parameter estimation. The nonlinearity of the original estimation model requires iterations of the linearized model anyway, so that the tuning process can be advantageously incorporated into the sequence of the iteration steps. The iteration starts with a full parameter set. These parameters might even be considered as stochastic variables. The associated weights must then relate to the size of the distortions to be expected. Even small weights keep the system from becoming singular. In the course of iterations the nondeterminable parameters are excluded. For details on a possible rejection strategy see Gruen (1984).

Since the determinability of individual parameters depends on the structure of the match signal, the aforementioned goal could be achieved by *a priori* analysis of the signal and by rule-based selection of a fixed set of parameters prior to adjustment. This analysis could be performed by the least squares correlation method as well. Hence both options are essentially equivalent, and the use of one or the other depends mainly on external considerations.

A question remains as to what should be considered a "full" parameter set. For the selection of the geometrical shaping function the following considerations might be helpful. Let the terrain surface be approximated by a polyhedron, consisting of plane surface facets. Assume that the images of these facets are used as the patches for the match. If an object facet models the local terrain surface correctly, i.e. if the local terrain surface patch is a plane in sufficient approximation, then the corresponding image can be strictly described by a projective transformation between object and image. Since it can usually be assumed that the facet image is very small with respect to the full image format, i.e. the facet image being formed by a very narrow bundle of rays, this projective transformation (8-parameter transformation) can be approximated by an affine transformation (6-parameter transformation). Therefore the affine transformation model is considered to be sufficient and used as a full parameter set. If the true surface above the object facet deviates significantly from a plane, e.g. in the case of buildings, trees, or other high rise structures, the projective transformation must be replaced by a perspective transformation. This in turn requires knowledge about the spatial structure of the object itself and the orientation parameters of the image. It should be an interesting exercise to utilize such knowledge towards a refinement of this matching procedure. A sub-optimal substitute solution to this problem is to consider any deviations from the projective (affine) transformation model as systematic errors or blunders and to exclude them or compensate them in the estimation procedure. These problems are of concern for future studies.

3. ANALYSIS OF RESULTS AND COMPUTATIONAL ASPECTS

The analysis of results is a key element in this matching procedure, because it is expected that this approach allows for a better assessment of the quality of results than the conventional correlation methods. Baarda's concept for the assessment of the quality of observations and least squares adjustment results is widely accepted in geodesy and photogrammetry and can be applied to this procedure as well.

3.1 PRECISION

The precision of the estimated parameters is expressed by the covariance matrix

$$K_{xx} = \hat{\sigma}_0^2 Q_{xx} = \hat{\sigma}_0^2 (A^T PA)^{-1} \quad . \tag{17}$$

Of particular interest are the standard deviations

$$\begin{aligned} \hat{\sigma}_{\Delta x} &= \hat{\sigma}_{a11} = \hat{\sigma}_0 (q_{a11})^{1/2} \\ \hat{\sigma}_{\Delta y} &= \hat{\sigma}_{b11} = \hat{\sigma}_0 (q_{b11})^{1/2} \end{aligned} \tag{18}$$

of the shift parameters a_{11} , b_{11} , because they describe the precision of the matchpoint location in the picture.

The size of the standard deviations of the shaping parameters and the correlations between themselves and with respect to the shift parameters may give useful information concerning the stability of the system.

3.2 RELIABLITY

The precision measures provide for appropriate indicators of the accuracy of the correlation if the estimation model can be considered correct. Undetected blunders, systematic errors (false modelling), and weight errors require the consideration of the reliability of the results. Baarda's concept of internal, external reliability and data-snooping can be favourably applied here. Local radiometric and geometric distortions between template and picture result in blunders and systematic errors, which can be detected by the data-snooping technique. Because of the regular arrangement of the data and the high redundancy the internal reliability values are very large and homogenous.

The external reliability indicates how the correlation is affected by nondetectable model errors.

3.3 CONVERGENCE OF SOLUTION VECTOR

As opposed to standard least squares adjustments in geodesy and photogrammetry, where the function g(x,y) is unique and convergence problems are rarely encountered, we face here occasionally the problem that the solution vector converges only very slowly, that it oscillates, or that it even converges to a false solution. These cases indicate severe correlation problems and require further attention.

Weak or impossible correlation

Cases of weak or impossible correlation show up in a very slow convergence of the solution vector. Here the problem stems from that the data sets do not have sufficient signal content to allow for a reasonably accurate correlation.

Although such a situation is signalized by an inflation of the standard deviations of one or both shift values, the size of those standard deviations do not always represent a suitable quantitative measure for the amount of insecurity of the correlation. It is crucial that reliable criteria and quality measures be developed for these cases.

Figure 1 shows examples for fast convergence *(a)* and slow convergence *(b)*.

False correlation

False correlation threatens if the data of the picture window allows for multiple solutions and if the first approximations for the least squares process are not close enough. The least squares estimator might then converge to a side-minimum. Internal evidence measures that could signal such cases need to be developed. Investigations should be directed towards the use of external constraints, such as epipolar plane conditions or object point intersection constraints for conjugate rays, which could help to prevent such problems.

(a) *(b)*

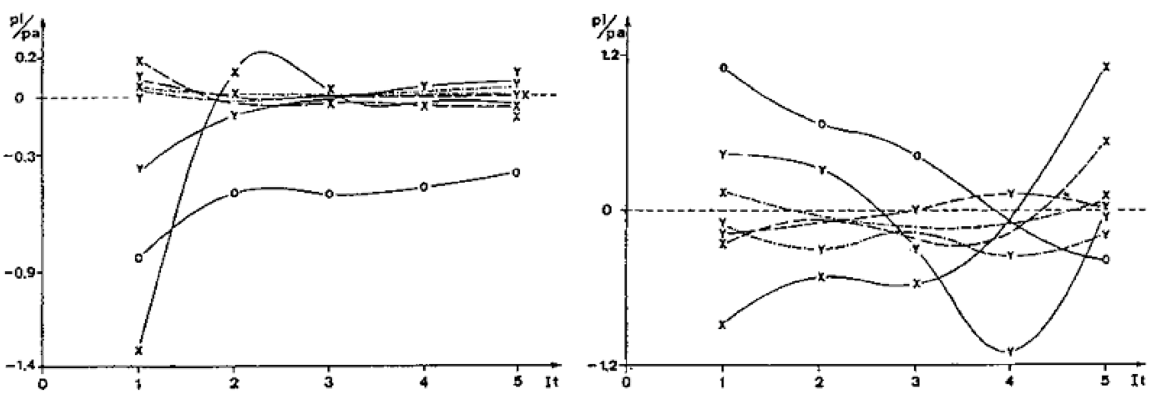

Fig.1. Convergence of the individual components of the solution vector.
(a) fast convergence
(b) slow convergence, partial divergence (low signal content).
pi . . . pixel, pa . . . parameter value

O——O——O radiometric correction(shift)
X——X——X x-shift(Δx)
Y——Y——Y y-shift(Δy)
X----X----X x-scale
Y----Y----Y y-scale
X-·-·-X-·-·-X x-sheering
Y-·-·-Y-·-·-Y y-sheering

3.4 COMPUTATIONAL ASPECTS

Computational aspects are particularly crucial for mass-point correlations. Computational problems that need to be addressed are:

(a) computation of derivatives g_x , g_y from discrete picture function values;
(b) resampling (interpolation) of picture function values $g^0(x,y)$, including an option for filtering;
(c) treatment of the design matrix A in the iteration process (constant A or recomputation);

(d) treatment of the covariance matrix K_{xx} ; and
(e) computation of reliability indicators and variables involved in the data-snooping test criteria.

Test results are available for all problems (a) – (e); they will be subject to publication elsewhere.

4. EXTENSION OF THE TECHNIQUE

Since this technique is set up in the least squares context a variety of measures can be taken and procedures added that may lead to a substantial stabilization of the image match in the cases of low signal content.

Transformation parameters as stochastic variables

An instability of the matching procedure is often indicated by large and unexpected alterations of the transformation parameters during the iteration process. This can be avoided by introducing the transformation parameters as stochastic variables into the estimation model. A *priori* knowledge about the transformation parameters might be available from a previously executed coarse matching step. Besides, experience shows that the size of transformation parameters is restricted. The assignment of *a priori* standart deviations for the parameters prevents the picture function from being deformed in unacceptable manner and from drifting away from the correct match-point. In addition, the reliability of the parameters can be easily monitored. Unreliable parameters are subject to rejection.

Automatic variation of the template/picture

This is particularly important if the predefined patches do not include sufficient signals.

The variations can be done either in terms of the selection of a totally new patch for correlation or as extension of the patch size with the aim to grab a signal that is close but was originally not included. The patch size can be increased stepwise until sufficient signal is found. This process can be controlled by sequentially updating and analysing the K_{xx} matrix, as the additional observations are integrated into the system.

Multiphoto least squares correlation

If images on more than two photographs are to be correlated, multiphoto correlation is performed. Multiphoto correlation adds strength to the solution and allows control of the correlation independently.

Multiphoto correlation is possible either sequentially in pairs, or in form of a simultaneous solution. Further, the simultaneous solution can be supported by external geometrical constraints.

One class of useful constraints is generated by the object point intersection conditions. Assume one template and k pictures, i.e. a total of $k+1$ photographs.

The k sets of grey level matching equations can be formulated as

$$\begin{aligned} -e_1 &= A_1 x_1 - \ell_1 \ ; \ \ P_1 \\ -e_2 &= A_2 x_2 - \ell_2 \ ; \ \ P_2 \\ &\vdots \\ -e_k &= A_k x_k - \ell_k \ ; \ \ P_k \end{aligned} \tag{19a}$$

or in summarized form

$$-e = Ax - \ell \ ; \ P \tag{19b}$$

Since the parameter vectors x_1 , …, x_k do not have any joint components, the sub-system (19a) are orthogonal to each other and they can be solved independently.

If the image forming process followed the law of perspective projection, a set of $k+1$ collinearity conditions can be formulated, in linearized form, as

$$\begin{aligned} B_0 y_0 + t_0 &= 0 \ , \\ B_1 y_1 + t_1 &= 0 \ , \\ &\vdots \\ B_k y_k + t_k &= 0 \end{aligned} \tag{20a}$$

and summarized as

$$By + t = 0 \ . \tag{20b}$$

The index (0) refers to the template.

The connection between (19a) and (20a) is established via the shift parameters $\Delta x_i , \Delta y_i \ (i = 1, \dots, k)$ that pertain in pairs to the parameter vectors x_i ($i = 1, \dots , k$) of (19a) and to the parameter vectors y_j ($j = 1, \dots , k$) of (20a).

Although this is a standard photogrammetric exercise, for the sake of completeness the matrices B_j and the vectors t_j are derived in the following.

With $\vec{X}_p = \begin{bmatrix} X_p \\ Y_p \\ Z_p \end{bmatrix}$, $\vec{X}_p$ … vector of object point coordinates of point P

$\vec{X}_{oj} = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix}_j$, $\vec{X}_{oj}$ … vector of perspective centre coordinates of photograph (j)

$R_j = \{r_1 , r_2 , r_3\}_j$, R_j … rotation matrix of photograph (j)

$j = 0, \dots, k$

the vector form of the collinearity condition is

$$\vec{x}_j = \frac{1}{\lambda_j} R_j^T \left(\vec{X}_p - \vec{X}_{oj} \right) , \tag{21}$$

with $\vec{x}_j = \begin{bmatrix} x \\ y \\ -c \end{bmatrix}_j$, $\vec{x}_j$ … vector of image coordinates of image point P^1 in photograph (j)

c_j …camera constant of photograph (j)

The x, y-components of (21) result in

$$x_j = -c_j \frac{r_{1j}^T\left(\vec{X}_p - \vec{X}_{oj}\right)}{r_{3j}^T\left(\vec{X}_p - \vec{X}_{oj}\right)} \hat{=} -F_j^x \quad , \tag{22a}$$

$$y_j = -c_j \frac{r_{2j}^T\left(\vec{X}_p - \vec{X}_{oj}\right)}{r_{3j}^T\left(\vec{X}_p - \vec{X}_{oj}\right)} \hat{=} -F_j^y \quad , \tag{22b}$$

whereby it is assumed that

$$x_j = x_j^0 + \Delta x_j \;\; , \;\; y_j = y_j^0 + \Delta y_j \quad . \tag{23}$$

For j = 1, … , k the values Δx_j , Δy_j are identical with the shift parameters from the matching equations (19a).

For the template holds

$$\Delta x_0 = \Delta y_0 = 0 \quad . \tag{24}$$

Finally the two component form of the non-linear collinearity conditions assumes

$$\Delta x_j + F_j^x + x_j^0 = 0 \quad , \tag{25a}$$

$$\Delta y_j + F_j^y + y_j^0 = 0 \quad . \tag{25b}$$

If the interior and exterior orientation parameters of the photographs are known, the linearization of (25a) , (25b) with respect to the remaining unknown parameters, the object point coordinates of P, result in

$$\Delta x_j + \frac{\partial F_j^x}{\partial X_p} dX_p + \frac{\partial F_j^x}{\partial Y_p} dY_p + \frac{\partial F_j^x}{\partial Z_p} dZ_p + F_j^{x(0)} + x_j^0 = 0 \quad , \tag{26a}$$

$$\Delta y_j + \frac{\partial F_j^y}{\partial X_p} dX_p + \frac{\partial F_j^y}{\partial Y_p} dY_p + \frac{\partial F_j^y}{\partial Z_p} dZ_p + F_j^{y(0)} + y_j^0 = 0 \quad . \tag{26b}$$

the least squares solution of the joint system

$$-e = Ax - \ell \;\; ; \;\; P \tag{27a}$$

$$Bx + t = 0 \tag{27b}$$

is well known.

(y of (20b) has been replaced here by x in order to establish a functional relation to (27a)). System (27a), (27b) is of great sparsity. This should be considered when the computational algorithm is set up for the solution. Despite this fact that the evaluation of the solution equations is fairly expensive. The expense can be reduced to a minimum if the equations (27b) are not introduced as functional constraints, but as a set of observation equations. This can be interpreted as if the approximate values x_j^0 , y_j^0 in (26a) , (26b) are treated as stochastic variables. Instead of (27b) observation equations of the form

$$-e_t = Bx + t \;\; ; \;\; P_t \tag{27c}$$

are used with e_t being distributed as

$$e_t \sim N\left(0, \sigma_0^2 Q_{tt}\right) \; ; \; Q_{tt} = P_t^{-1} \tag{28}$$

The least squares solution for the joint system (27a) , (27c) results in

$$\hat{x} = \left(A^T P A + B^T P_t B\right)^{-1} \left(A^T P \ell - B^T P_t t\right) \tag{29}$$

The computational load becomes particularly small, if the individual components of e_t can be considered uncorrelated and of equal precision.

With $e_t \sim N\left(0, \sigma_0^2 cI\right)\ ;\ c > 0$ (30)

the solution vector assumes

$$\hat{x} = \left(A^T PA + B^T B / c\right)^{-1} \left(A^T P\ell - B^T t / c\right) \ . \tag{31}$$

The vector $\hat{x}$ includes the parameters of the matching equations and the object point coordinates of P. Therefore this is a simultaneous solution to both the image matching and point positioning problems. This solution is currently undergoing extensive testing, using synthetic and real data. First results are encouraging.

5. CONCLUSIONS AND PROSPECTS

The adaptive least squares correlation can be applied to a great variety of data matching problems. This paper focused on its utilization for image matching. The technique shows a number of attractive features, such as

- High matching accuracy
- Monitoring of quality; precision and reliability measures are readily available; meaningful figures-of-merit can be established
- Simultaneous geometrical image shaping and radiometric adjustment; rectification prior to matching unnecessary
- Combination of area-based and edge-based analysis
- Usable in a hierarchical mode ('coarse-to-fine')
- Usable as derivative-operator based matching procedure; i.e. first-order derivatives may generate the vector of observations ('slope observables'), and the coefficients of the design matrix are then functions of the second-order derivatives
- Rule-based correlation; grid-based mass-point correlation may be replaced by a selective procedure, where only those patches are used for correlation which contain enough signal in order provide a reliable solution
- Data patch does not need to be complete; areas of low signal content within the patch can be neglected
- Since the computational algorithm is based on the Gauss/Cholesky factorization it can be implemented in a parallel mode on computers.

In addition, a variety of geometrical constraints can be incorporated to support the grey level matching. Besides the collinearity condition in multi-photo correlation, which was briefly addressed in this paper, the well-known epipolar line-condition can also be used. Virtually every sampling mode for digital terrain data can be modelled by formulating the corresponding geometrical constraints, e.g. profiling, contouring, raster sampling, etc.

An exciting prospect is that these procedures can simultaneously be applied to more than two photographs, thus extending their possibilities beyond the capabilities of a human operator.

The fusion of point positioning with grey level matching increases the precision and reliability of the matching procedure. Blunders can be better controlled. Combined with a suitable algorithm for on-line triangulation, a fully automatic real-time triangulation system is conceivable.

Multi-patch correlation becomes feasible, utilizing neighbourhood conditions, with the adjacent patches controlling and stabilizing each other.

Furthermore, this technique can be applied to multi-spectral and multi-temporal images, to feature extraction, change detection, and line following. It may even be utilized to match and analyse non-sensor data sets, such as digital height models, digital planimetric models and line map information. The incorporation of image understanding techniques will further enhance the performance of the adaptive least squares correlation.

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