

Eg: Which of the following is NOT a Tautology

a)
$$\sim (p \rightarrow q) \rightarrow p$$

a)
$$\sim (p \to q) \to p$$
 c) $[\sim p \land (p \lor q)] \to q$

$$b) \sim (p \to q) \to \sim q \qquad \qquad d) \; (p \to q) \wedge q \to p$$

d)
$$(p \rightarrow q) \land q \rightarrow p$$



True
$$\longrightarrow$$
 False = False
a) $\sim (P \rightarrow 9) \rightarrow P$
 $\sim (False \rightarrow 9) \rightarrow False$
 $\sim (T) \rightarrow F$
 $F \rightarrow F = T = NO False$
True Tautology
b) $\sim (P \rightarrow T) \rightarrow F$
 $\sim (P \rightarrow T) \rightarrow F$
 $\sim (T) \rightarrow F$
 $F \rightarrow F$
 $T \rightarrow F$

example

prove: X -> y is NOT a Tautology

Sol If X = True and Y = False then $X \rightarrow Y = False$ = NOT aTautology

consider X is TRUE and then Try to Show Approach-I:

y is False

consider Y is False and then Try to show Approad-II:

X is TRUE



Example

prove: X -> Y is a Tautology (always True)

Since 1995

Sol Idea: No case Exists like when X=TRUE and Y=False

Approach - I: consider X = TRUE and try to show

Y = TRUE

Approach-II: consider Y = False and try to show

X = False



Eg: Which of the following is NOT a tautology

a)
$$(p \land q) \rightarrow (p \lor q)$$
 $c) \sim p \rightarrow (p \rightarrow q)$

$$(c) \sim p \rightarrow (p \rightarrow q)$$

b)
$$(p \land q) \rightarrow (p \rightarrow q)$$
 d) $p \rightarrow (p \land q)$

d)
$$p \rightarrow (p \land q)$$

a)
$$(PAQ) \rightarrow (PVQ)$$
 $(TAT) \rightarrow (TVT)$
 $T \rightarrow T = True$

$$Tautologg$$
b) $(PAQ) \rightarrow (P \rightarrow Q)$
 $T \rightarrow F$

$$(TAF) \rightarrow (T \rightarrow F)$$

$$F \rightarrow F = True$$

$$Tautologg$$

c)
$$P \rightarrow (P \rightarrow q)$$
 $T \rightarrow (F \rightarrow q)$
 $T \rightarrow T = True$
 $T \rightarrow (P \rightarrow q)$
 $T \rightarrow T \rightarrow T$
 $T \rightarrow (P \rightarrow q)$
 $T \rightarrow T \rightarrow T$
 $T \rightarrow T \rightarrow T$

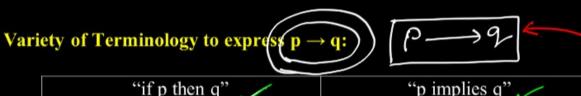
Conditional Statements:





The statement $p \to q$ is called a conditional statement, because $p \to q$ asserts that 'q' is True on the condition that 'p' holds.

The conditional statement $p \rightarrow q$ is the proposition "If p, then q". The conditional statement $p \rightarrow q$ is false when 'p' is true and 'q' is false, otherwise True.







"if p then q"	"p implies q"		
"if p, q"	"p only if q"		
"p is sufficient for q"	"a sufficient condition for q is p"		
$X \rightarrow Y$ ("q if p") Y if X	(q when ever p)		
"q when p"	"q is necessary for p"		
"a necessary condition for p is q"	"q follows from p"		
"q unless ∼ p"	"q provided that p"		

$$\vec{a}$$
 only if \vec{b}

$$\equiv (a \longrightarrow b)$$

 $\overline{X} \longleftrightarrow \overline{A}$

Converse, Inverse and contra positive:

Let Implication: $P \longrightarrow Q$: $a \longrightarrow b$: $x \longrightarrow y$

Converse: $9 \rightarrow P$: $b \rightarrow a$: $Y \rightarrow X$

Inverse: $\sim P \rightarrow \sim 9$: $\sim a \rightarrow \sim b$: $\sim \times \rightarrow \sim \gamma$

Contra positive: ~9->~P: ~b->~a: ~Y->~X

 $P \rightarrow 9 \cong \sim 9 \rightarrow \sim P$ Contra positive law



conditional:

Eg: In Triangle ABC, If $\overrightarrow{AB} = \overrightarrow{AC}$ then $\angle B = \angle C$



Converse: 9→P: In Triangle ABC, If LB=LC then AB=AC

Inverse: ~P->~9: In Triangle ABC, If AB ≠ AC then LB ≠ LC.

contrapositive: ~9 -> ~P: In Triangle ABC, If LB # LC then AB # AC



ACE

Conditional: $P \rightarrow 9 \equiv If It$ is then the home team wins

Converse: 9-1= If the home team wins then It is plaining

Inverse: ~P->~9 = If It is not raining then the home team does not win

Contra positive: $\sim 9 \rightarrow \sim P = If$ the home team does not win then It is not raining.

ACE

GATE:

What is the converse of the following assertion?

"I stay only if you go"

- a) I stay if you go
- √b) If I stay then you go
- x c) If you do not go then I do not stay
 - d) If I do not stay then you go



Laws (or) Properties (or) Logical Equivalences:

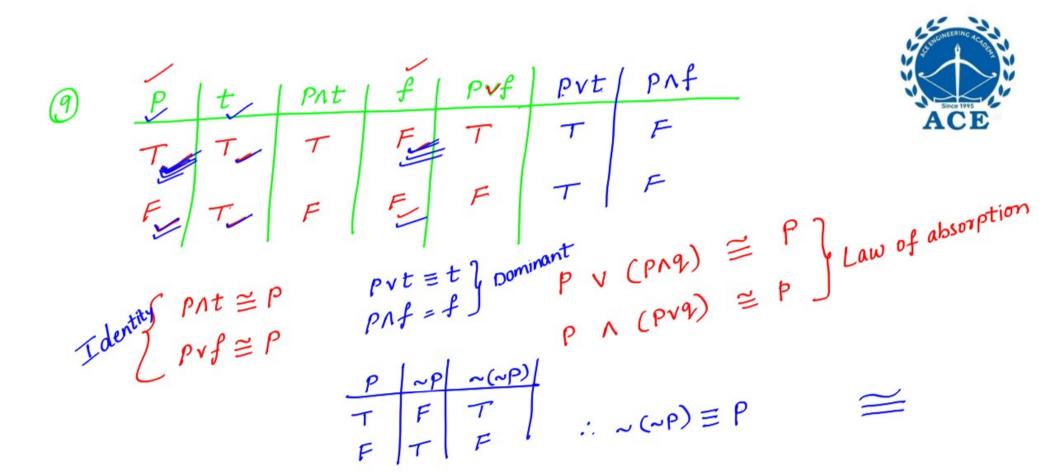
	Rules	Name	ACE
1	$p \lor p \cong p $	Idempotent law	
	$p \wedge p \cong p $	ruempotent iaw	PIP PVP
2	$p \lor q \cong q \lor p \checkmark \qquad 2+3=3+2$	Commutative law	TTT
	$p \wedge q \equiv q \wedge p \nearrow$	Commutative law	FFF
3	$p \vee (q \vee r) \equiv (p \vee q) \vee r$	Associative law	PVP = P
	$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$	1 issociative law	
4	$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$	Distributive Law	
	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive Law	
5	$\sim (p \lor q) \equiv \sim p \land \sim q$	De Morgan's Law	
	(De Moigan's Law	

	$p \lor p \cong p$	Idempotent law	
	$p \wedge p \cong p$	ruempotent iaw	
2	$p \lor q \cong q \lor p$	Commutative law	
	$p \wedge q \equiv q \wedge p$	Commutative law	
3/	$p \lor (q \lor r) \equiv (p \lor q) \lor r -$	Associative law	
	$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$. ^
4/	$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$	$xv(yn_3) = (xvy) \wedge (xv_3) / \sqrt{2}$ Distributive Law	•
	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	no (yv3) =(noy)v (noy) // Du	ral
5	$\sim (p \vee q) \equiv \sim p \wedge \sim q / \qquad \neg$		
	$\sim (p \land q) \equiv \sim p \lor \sim q$	De Morgan's Law	
6/*	$p \rightarrow q \equiv \sim p \vee q / a \rightarrow b \equiv \sim a \vee b$	Implication law	



Laws (or) Properties (or) Logical Equivalences:

		Rules	Name	ACE
7		$p \leftrightarrow q \equiv \underbrace{(p \to q)} \land \underbrace{(q \to p)} \mathrel{5}$	Bi-implication law $P \leftrightarrow 9 \equiv (\sim P \land \sim 9) \land (P \leftrightarrow 9) \equiv (\sim P \lor 9) \land (P \leftrightarrow 9) \equiv (\sim P \lor 9) \land (P \leftrightarrow 9) \Rightarrow (P \leftrightarrow 9) \Rightarrow$	(PA9) ~ (~9VP)
8	Dual C	$p \lor (p \land q) \equiv p$ $p \land (p \lor q) \equiv p$	Absorption law $\frac{P}{T}$ $\frac{Q}{T}$ $\frac{P}{T}$ $\frac{Q}{T}$	PY (PA9)
9		$ \begin{array}{l} p \land t \equiv p \\ \hline p \lor f \equiv p \end{array} $ $ \begin{array}{l} l + 0 = 1 \\ 2 + 0 = 2 \end{array} $	Identify law	P = P.
10		~(~ p) ≡ p	Double Negation (or) Involutary law	
11		$p \lor \sim p \equiv t$ $p \land \sim p \equiv f$	Negation law (or) Complements (or)	
		1	Inverse law	



7	$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$	Bi-implication law
8	$p \lor (p \land q) \equiv p$ $p \land (p \lor q) \equiv p$	Absorption law
9	$p \land t \equiv p$ $p \lor f \equiv p$	Identify law /
10	~(~ p) ≡ p	Double Negation (or) Involutary law
11 (Negation law (or) Complements (or) Inverse law $a \vee \sim a = t$ $a \wedge \sim a = f$
12	$p) \lor t \equiv t$ $p \land f \equiv f$	Domination law $P \rightarrow P \cong \neg P \rightarrow \neg P$





Eg:

rg:

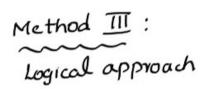
Q. The propositional function $p \lor (q \lor \sim p)$ is

	· II · I T	
a) tautology	Civen pv Cqv~p)
b) contradiction	= pv C~p	v9) (:: commutative)
	= (pv~p)	var C.: Associative)
c) contingency	= (t) vq	(:: complement's law)
d) $p \wedge q$: Dominant)

Method II: Truth table

~~ b	19	~ P	9 v~p	pv(gv~p)
	T		T	T
T	F	F	F_	T
F	T	T	Tu	T
F	F	T	アン	T

: pr(qv-p) = t





$$pv(qv\sim p)$$
i) $Tv() = True$
ii) $Fv(qvT) = True$





Q. The propositional function $(p \rightarrow q) \leftrightarrow (p \land \sim q)$ is

- c) contingency
- d) None

$$(p\rightarrow q) \longleftrightarrow (p \land \neg q)$$

a) tautology
$$(p \rightarrow q) \longleftrightarrow (p \land \neg q)$$

b) contradiction $\equiv (\neg p \lor q) \longleftrightarrow (p \land \neg q)$ $(\because a \rightarrow b \equiv \neg a \lor b)$