Negation of Quantifiers:

$$\sim [\forall x \, P(x)] \equiv \exists x \, \sim P(x)$$

$$\sim [\exists x \, P(x)] \equiv \forall x \, \sim P(x)$$

Example: Let $P(x)$: x is Intelligent , $x = \{\text{all students}\}$

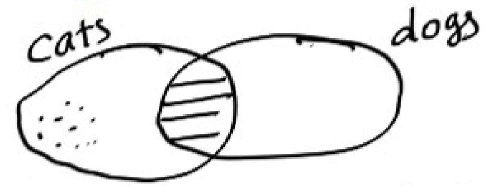
$\forall x \, P(x)$: All students are Intelligents

Negation will be

$\sim [\forall x \, P(x)] =$ Not all Students are intelligents

$\equiv$ Some students are intelligents

$\equiv$ Some students are not intelligents $= \exists x \sim P(x)$



Some cats are dogs

Quantifier	when it is TRUE	when It is FALSE
$\forall x\, P(x)$	$P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge \cdots = \text{TRUE}$	$P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge \cdots = \text{FALSE}$
$\exists x\, P(x)$	$P(x_1) \vee P(x_2) \vee P(x_3) \vee \cdots = \text{TRUE}$	$P(x_1) \vee P(x_2) \vee P(x_3) \vee \cdots = \text{FALSE}$

Q. Consider the predicate p(x) indicates $\boxed{\text{"x is happy"}}$ where universe of discourse is set of {all students in a class}

a) $\exists x\ p(x)$

b) $\forall \sim p(x)$

c) $\exists x \sim p(x)$

d) $\sim[\forall x \sim p(x)]$

@ $\exists x\ P(x)$ = Their exists some x, such that $P(x)$ is TRUE

= Some students are happy ✓

= Some students are not happy ✓

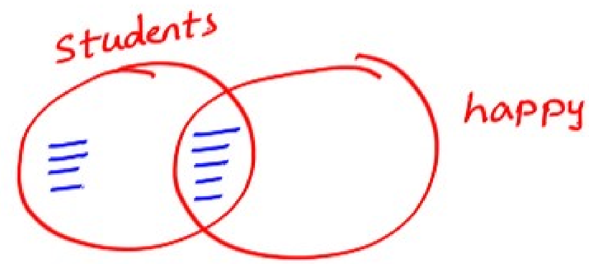
(b) $\forall x \sim P(x)$: All students are not happy

= Each student is not happy

= No student is happy

(c) $\exists x \sim P(x)$ = Some students are not happy ✓

$\Rightarrow$ Some students are happy ✓



(d) $\sim [\forall x \sim P(x)] = \exists x \; P(x)$

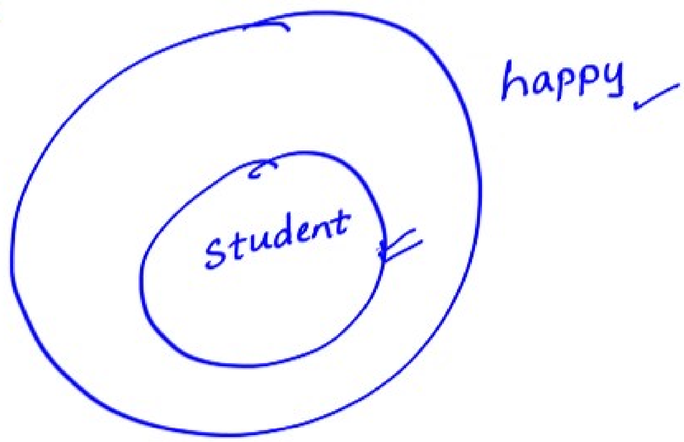
= Some students are happy

$\Rightarrow$ Some students are not happy

$\forall x \, P(x)$: All students are happy

$\Rightarrow$ Some students are happy

$\nRightarrow$



Q. Consider the following predicates

G(x) : x is gold ornament

S(x) : x is silver ornament

P(x) : x is precious

Q. Express the following into appropriate predicate logic "Gold and Silver ornaments are precious".

100

10 gold & 15 silver = 25

2

a) $\forall x[P(x) \rightarrow (G(x) \wedge S(x))]$

b) $\forall x[(G(x) \wedge S(x)) \rightarrow P(x)]$

c) $\forall x[(G(x) \vee S(x)) \rightarrow P(x)]$

d) $\exists x[(G(x) \vee S(x)) \rightarrow P(x)]$

(a) $\forall x \{ P(x) \longrightarrow [G(x) \land S(x)] \}$

For every x, if x is precious then x is will be gold and silver.

(b) $\forall x \{ [\overset{T}{G(x)} \land \overset{T}{S(x)}] \longrightarrow P(x) \}$

For every x, if x is gold and also x is silver then x will be precious

(c) $\forall x \{ [G(x) \lor S(x)] \longrightarrow P(x) \}$

For every x, if Either x is gold (or) x is silver then x will be precious

(d) $\exists x \{ [G(x) \lor S(x)] \longrightarrow P(x) \}$

For Some x, if Either x is gold (or) x is silver then x will be precious

* Parents and Friends are very important
(2) (8)

* Mother and wife are valuable persons
(1) (1)

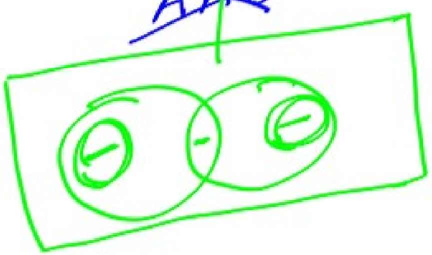
1+1

$1 \cup 1$

$A \cup B$ ✓

~~$A \cap B$~~

$1 \cap 1$



* Gold (or) silver are precious

* fear about Tigers and Lions

Tigers (or) Lions

Q. What is the logical translation of the following statement?

"None of my friends are perfect"

a) $\exists x[F(x) \wedge \sim P(x)]$

b) $\exists x[\sim F(x) \wedge \sim P(x)]$

c) $\exists x[\sim F(x) \wedge P(x)]$

d) $\sim \exists x[F(x) \wedge P(x)]$ ✓

None of my friends are perfect

= No one of my friends is perfect

= All my friends are not perfect

$= \forall x\,[F(x) \longrightarrow \sim P(x)]$

$= \forall x\,[\sim F(x) \vee \sim P(x)]$

$= \sim \exists x\,[F(x) \wedge P(x)]$

$\boxed{a \longrightarrow b \equiv \sim a \vee b}$

ACE

Q. Which of the following is NOT logically equivalent to

$\sim \exists x[\forall y(\alpha) \wedge \forall z(\beta)]$?

a) $\forall x[\exists z(\sim \beta) \rightarrow \forall y(\alpha)]$

b) $\forall x[\forall z(\beta) \rightarrow \exists y(\sim \alpha)]$

c) $\forall x[\forall y(\alpha) \rightarrow \exists z(\sim \beta)]$

d) $\forall x[\exists y(\sim \alpha) \rightarrow \exists z(\sim \beta)]$

a & d

$\sim \exists x[\forall y\, \alpha \wedge \forall z\, \beta]$

$\equiv \forall x[\sim(\forall y\, \alpha) \vee \sim(\forall z\, \beta)]$

$\sim a \vee b \equiv a \rightarrow b$

$\equiv \forall x[\underbrace{\exists y \sim \alpha}_{\sim a} \vee \underbrace{\exists z \sim \beta}_{b}]$

$\equiv \forall x[\sim(\exists y \sim \alpha) \longrightarrow \exists z \sim \beta]$

$\equiv \forall x[\forall y\, \alpha \longrightarrow \exists z \sim \beta]$ = option (c)

$\equiv \forall x[\forall z\, \beta \longrightarrow \exists y \sim \alpha]$ = option (b)

$$a \to b \equiv \sim a \vee b$$

$$a \to b$$

$$\boxed{a \to b \equiv \sim b \to \sim a}$$

$$\sim \exists x \left[\forall y\, \alpha \wedge \forall z\, \beta \right]$$

$$\equiv \forall x \left[\overbrace{\sim (\forall y\, \alpha)}^{\sim a} \vee \overbrace{\sim (\forall z\, \beta)}^{b} \right]$$

$$\equiv \forall x \left[\underbrace{\forall y\, \alpha}_{a} \to \underbrace{\sim (\forall z\, \beta)}_{b} \right]$$

$$\equiv \forall x \left[\forall y\, \alpha \to \exists z \sim \beta \right] \equiv (C)$$

Multiple Quantifiers:

A different ordering of the quantifiers may yield a different statement.

* The statement $\exists x \, \forall y \, \boxed{p(x, y)}$ and $\forall y \, \exists x \, \boxed{p(x, y)}$ are not logically equivalent.

There are 8 ways to apply the two quantifiers.

Example:

p(x, y) : x likes y

x = { Boys }   y = { Girls }

$P(x,y)$: x likes y

1) $\forall x \forall y\ P(x,y)$: Every boy likes Every girl.

2) $\boxed{\forall x \exists y\ P(x,y)}$ * : Every boy likes Some girls

3) $\exists x \forall y\ P(x,y)$: Some boys likes all girls

4) $\exists x \exists y\ P(x,y)$: Some boys likes Some girls

5) $\forall y \forall x\ P(x,y)$: Every girl is liked by Every boy

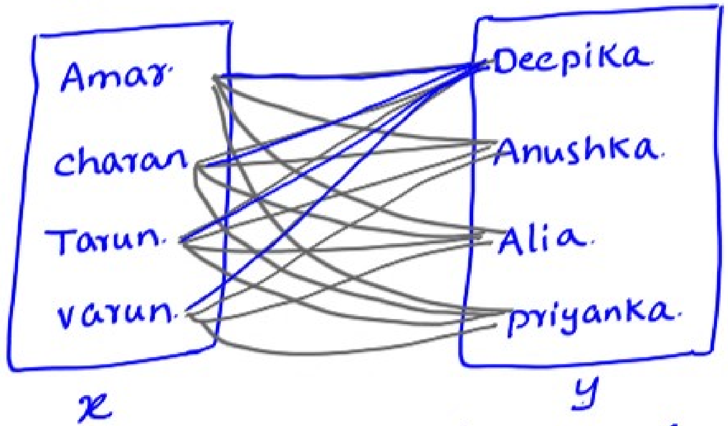
6) $\forall y \exists x\ P(x,y)$: Every girl is liked by Some boys

7) $\boxed{\exists y \forall x\ P(x,y)}$ * : Some girls are liked by all boys.

8) $\exists y \exists x\ P(x,y)$: Some girls are liked by Some boys.

① $\forall x \forall y \; P(x,y)$: Every boy likes Every girl

DIS-I



$4 \times 4 = 16$ mappings

Conclusion : $\overset{\text{True}}{a} \longrightarrow b$ can be True

(i) $\forall x \forall y \; P(x,y) \longrightarrow \forall y \forall x \; P(x,y)$

$b \longrightarrow a$

(ii) $\forall y \forall x \; P(x,y) \longrightarrow \forall x \forall y \; P(x,y)$

(iii) $\forall x \forall y \; P(x,y) \longleftrightarrow \forall y \forall x \; P(x,y)$

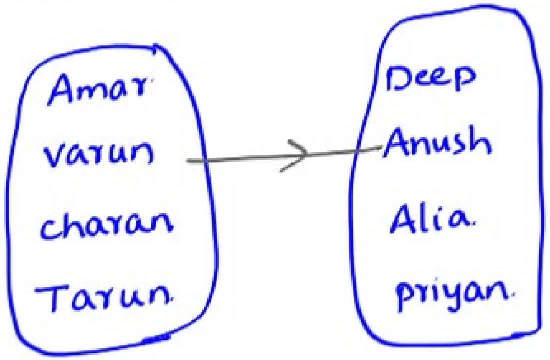
$a \longleftrightarrow b$

From the above mapping, we can conclude that
"Every girl is liked by Every boy"

$= \forall y \forall x \; P(x,y)$

(4) $\exists x \, \exists y \; P(x,y)$: Some boys likes some girls

DIS-II



Conclusion :

$$\exists x \, \exists y \; P(x,y) \longleftrightarrow \exists y \, \exists x \; P(x,y)$$

$\Rightarrow$ Some girls are liked by Some boys

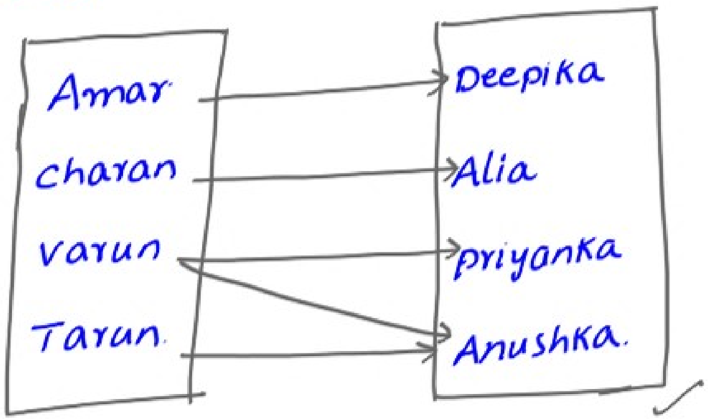
Anushk varun

$\Rightarrow \; \exists y \, \exists x \; P(x,y)$

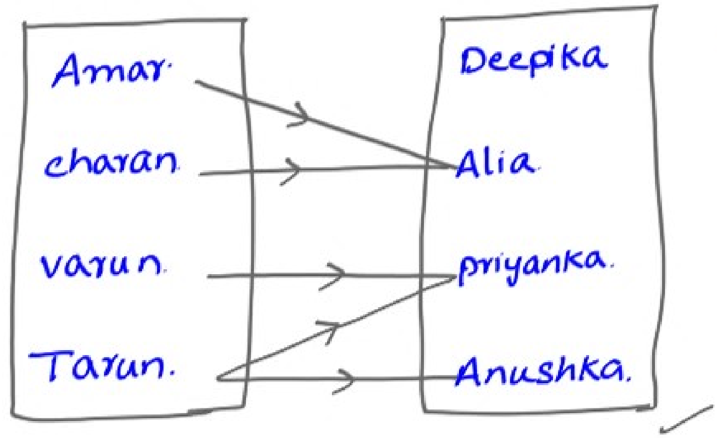
Discussion - III

(2) $\forall x\ \exists y\ P(x,y)$: Every boy likes some girls

(i)

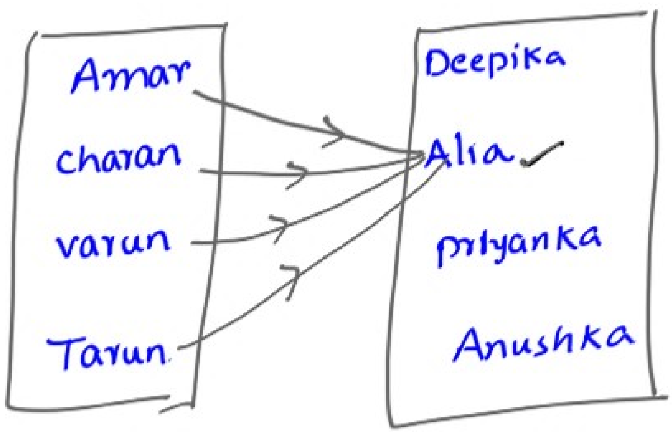


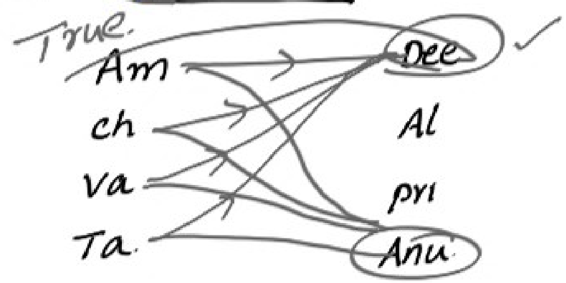
(ii)



(7) $\exists y \, \forall x \; P(x,y)$: There exists some y such that for every x
$P(x,y)$ is TRUE

= Some girls are liked by all boys





Am $\rightarrow$ D, A
ch $\rightarrow$ D, A
V $\rightarrow$ D, A
T $\rightarrow$ D, A

(2) True

True

$$\forall x \, \exists y \; P(x,y) \not\rightarrow \exists y \, \forall x \; P(x,y)$$

(2) NOT implied (7)

Conclusion :

i) $\forall x \, \exists y \; P(x,y) \not\longrightarrow \exists y \, \forall x \; P(x,y)$

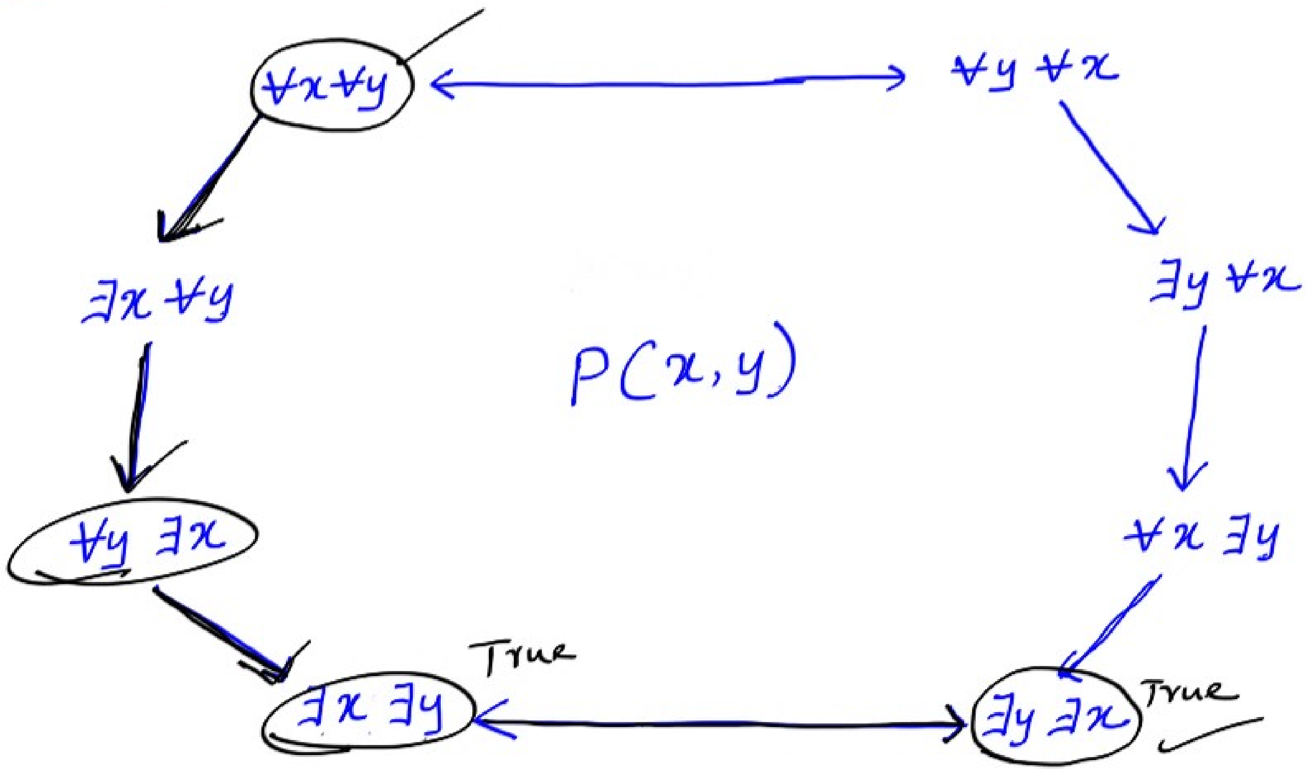
NOT logically implied

= if $\forall x \, \exists y \; P(x,y)$ is TRUE Then $\exists y \, \forall x \; P(x,y)$ may be TRUE or may not be TRUE (No gurantee)

(ii) $\exists y \, \forall x \; P(x,y) \longrightarrow \forall x \, \exists y \; P(x,y)$

= if $\exists y \, \forall x \; P(x,y)$ is Then $\forall x \, \exists y \; P(x,y)$ will be TRUE (always)

Logical Relationship diagram



$\underline{\forall x \forall y}\ P(x,y) \longrightarrow \underline{\exists y \exists x}\ P(x,y)$ ✓

$\forall a \forall b$

$\exists a \forall b$

$\downarrow \quad \uparrow \times$

$\forall b \exists a$