01. Logic

Practice Questions

01. Consider the following conditional statement:
p: If the flood destroys my house or the fire destroys my house, then my insurance company will pay me.
Which of the following is equivalent to p?
(a) If my insurance company pays me, then the flood destroys my house or the fire destroys my house.
(b) If my insurance company pays me, then the flood destroys my house and the fire destroys my house.
(c) If my insurance company does not pay me, then the flood does not destroy my house or the fire does not destroy my house.
(d) If my insurance company does not pay me, then the flood does not destroy my house and the fire does not destroy my house.

02. Consider the following arguments:
S_1: $\{r \to (q \to p), \sim p\} \Rightarrow (\sim r \vee \sim q)$
S_2: $\{(p \to q) \wedge (q \to r), (\sim q \wedge r)\} \Rightarrow p$
Which of the following is true?
(a) Only S_1 is valid
(b) Only S_2 is valid
(c) Both S_1 and S_2 are valid
(d) Both S_1 and S_2 are invalid

03. The statement formula
$$\{(a \vee b) \wedge (\sim a \vee c) \wedge \sim(b \vee c)\}$$
is
(a) a tautology (b) a contradiction
(c) a contingency (d) none of these

04. Consider the following statements:
S_1: $((a \vee b) \to c) \Rightarrow (a \wedge b) \to c$
S_2: $((a \wedge b) \to c) \Rightarrow (a \vee b) \to c$
Which of the following is true?
(a) Only S_1 is valid
(b) Only S_2 is valid
(c) Both S_1 and S_2 are valid
(d) Both S_1 and S_2 are invalid

05. Consider the following statement formulae.
S_1: $\{(\sim p \to (q \to \sim w)) \wedge (\sim s \to q) \wedge \sim t \wedge (\sim p \vee t)\} \to (w \to s)$
S_2: $\{(q \to t) \wedge (s \to r) \wedge (\sim q \to s)\} \to (\sim t \to r)$
Which of the following is true?
(a) only S_1 is valid
(b) only S_2 is valid
(c) Both S_1 and S_2 are valid
(d) Both S_1 and S_2 are invalid

06. Which of the following is valid?
(a) $\{\sim p, p \to q, q \to r\} \Rightarrow \sim r$
(b) $\{p \to q, q \to r, r\} \Rightarrow p$
(c) $\{p \to (q \to r), (p \wedge q)\} \Rightarrow r$
(d) $\{\sim(p \wedge q), \sim p\} \Rightarrow q$

07. Consider the following statements:

S_1: The contra-positive of

$\{(\sim r) \vee (\sim s)\} \rightarrow q$ is $\{q \vee (r \wedge s)\}$

S_2: The converse of

$\{(\sim r) \vee (\sim s)\} \rightarrow q$ is $q \rightarrow \sim(r \wedge s)$

S_3: The inverse of

$\{(\sim r) \vee (\sim s)\} \rightarrow q$ is $(r \wedge s) \rightarrow \sim q$

S_4: The negation of

$\{(\sim r) \vee (\sim s)\} \rightarrow q$ is $\sim(r \wedge s) \wedge \sim q$

Which of the following is true?

(a) Only S_1, S_2 and S_3

(b) Only S_1, S_3 and S_4

(c) Only S_2, S_3 and S_4

(d) S_1, S_2, S_3 and S_4

08. A binary relation $*$ is defined by the following truth table

p	**q**	**p * q**
F	F	F
F	T	F
T	F	T
T	T	F

Then $p \rightarrow q$ is equivalent to

(a) $(\sim p * q)$ (b) $\sim(p * q)$

(c) $\sim(p * \sim q)$ (d) $(p * \sim q)$

09. The formula

$\{(\sim p \wedge q) \vee (p \wedge \sim q) \vee (p \wedge q)\}$

is equivalent to

(a) $p \rightarrow q$ (b) $p \wedge q$

(c) $p \leftrightarrow q$ (d) $p \vee q$

10. The statement formula

$\{p \wedge (\sim p \vee \sim q) \wedge (\sim p \vee q \vee r) \wedge \sim r\}$ is

(a) a tautology

(b) a contingency

(c) not satisfiable

(d) none of these

11. Consider the following compound propositions:

$S_1 = ((P \rightarrow Q) \rightarrow P) \rightarrow Q$

$S_2 = P \rightarrow (Q \rightarrow (P \rightarrow Q))$

Which of the following is true?

(a) $S_1 \Rightarrow S_2$ (b) $S_2 \Rightarrow S_1$

(c) $S_1 \Leftrightarrow S_2$ (d) None of these

12. Consider the following arguments:

S_1: $\{(a \vee b) \rightarrow c, c \rightarrow (d \wedge e)\} \rightarrow (a \rightarrow d)$

S_2: $\{(p \rightarrow q), \sim(p \wedge q)\} \rightarrow \sim p$

Which of the following is true?

(a) S_1 is valid and S_2 is not valid

(b) S_2 is valid and S_1 is not valid

(c) Both S_1 and S_2 are valid

(d) Neither S_1 nor S_2 is valid

13. The statement formula $\{P \vee (P \leftrightarrow Q) \vee Q\}$ is equivalent to

(a) P (b) Q

(c) a tautology (d) $(P \wedge Q)$

14. Which of the following is ***not*** a tautology?

(a) $((p \rightarrow q) \rightarrow r) \rightarrow (p \rightarrow (q \rightarrow r))$

(b) $((p \rightarrow (r \vee q)) \rightarrow ((p \rightarrow r) \vee (p \rightarrow q))$

(c) $(p \rightarrow (r \wedge q)) \rightarrow ((p \rightarrow r) \vee (p \rightarrow q))$

(d) $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow r)$

15. Consider the following statement formulae.

S1: $\sim(p \vee q) \rightarrow (p \rightarrow q)$

S2: $\sim(q \rightarrow \sim p) \rightarrow (p \rightarrow \sim q)$

S3: $\sim(p \rightarrow q) \rightarrow (p \vee q)$

S4: $(p \wedge \sim q) \rightarrow (p \leftrightarrow q)$

Which of the following is true?

(a) S1 and S2 are tautologies

(b) S2 and S3 are tautologies

(c) S1 and S3 are tautologies

(d) S3 and S4 are tautologies

16. The number of non equivalent propositional functions possible with three propositional variables p_1, p_2 and p_3 is

(a) 8 (b) 256

(c) 9 (d) 512

17. The statement formula

$\{(a \rightarrow c) \wedge (b \rightarrow d) \wedge (c \rightarrow \sim d)\} \rightarrow (\sim a \vee \sim b)$

is

(a) satisfiable but not valid

(b) valid

(c) not satisfiable

(d) none of these

18. If $(p \rightarrow q)$ is false, which of the following has truth value true?

(a) $((\sim p) \wedge q) \leftrightarrow (p \vee q)$

(b) $(p \leftrightarrow q)$

(c) $(p \vee q) \vee r$

(d) $(p \wedge q) \vee r$

19. Consider the following statements:

S_1: $(a \leftrightarrow b) \rightarrow (a \wedge b)$

S_2: $(a \leftrightarrow b) \leftrightarrow ((a \wedge b) \vee (\sim a \wedge \sim b))$

Which of the following is true?

(a) S_1 is valid and S_2 is not valid

(b) S_1 is not valid and S_2 is valid

(c) Both S_1 and S_2 are valid

(d) Both S_1 and S_2 are not valid

20. Consider the following statements:

S_1: $((a \vee b) \rightarrow c) \Rightarrow ((a \wedge b) \rightarrow c)$

S_2: $((a \wedge b) \rightarrow c) \Rightarrow ((a \vee b) \rightarrow c)$

Which of the following is true?

(a) S_1 is valid and S_2 is not valid

(b) S_1 is not valid and S_2 is valid

(c) Both S_1 and S_2 are valid

(d) Both S_1 and S_2 are not valid

21. Let p, q, r denote primitive statements.

Consider the following statements:

S_1: $p \rightarrow (q \wedge r) \Leftrightarrow (p \rightarrow q) \wedge (p \rightarrow r)$

S_2: $[(p \vee q) \rightarrow r] \Leftrightarrow [(p \rightarrow r) \wedge (q \rightarrow r)]$

Which of the following is true?

(a) S_1 is valid and S_2 is not valid

(b) S_1 is not valid and S_2 is valid

(c) Both S_1 and S_2 are valid

(d) Both S_1 and S_2 are not valid

22. Let p, q, r denote primitive statements.

The statement formula

$[p \rightarrow (q \vee r)] \leftrightarrow [(p \wedge \sim q) \rightarrow r]$ is

(a) valid (b) not satisfiable

(c) satisfiable but not valid

(d) none of these

23. Let p, q and r be primitive statements.

Consider the following statements.

S_1: The dual of $p \rightarrow (q \wedge r)$ is $\sim p \wedge (q \vee r)$

S_2: The dual of $p \leftrightarrow q$ is $(\sim p \vee \sim q) \wedge (q \vee p)$

Which of the following is true?

(a) S_1 is true and S_2 is false

(b) S_1 is false and S_2 is true

(c) Both S_1 and S_2 are true

(d) Both S_1 and S_2 are false

24. Let p, q, r denote primitive statements.

The statement formula

$((a \wedge b) \rightarrow c) \leftrightarrow ((a \rightarrow c) \vee (b \rightarrow c))$ is

(a) valid

(b) not satisfiable

(c) satisfiable but not valid

(d) none of these

25. Consider the following arguments.

Argument1:

Premises: If there was a ball game, then travelling was difficult. If they arrived on time, then travelling was not difficult. They arrived on time.

Conclusion: There was no ball game.

Argument2:

Premises:

1. If jack misses many class through illness, then he fails high school.
2. If jack fails high school, then he is uneducated.
3. If jack reads a lot of books, then he is not uneducated.
4. Jack misses many classes through illness and reads a lot of books.

Conclusion: Jack is smart.

Which of the above arguments is/are valid in propositional logic?

(a) Only argument1 is valid

(b) Only argument2 is valid

(c) Both arguments are valid

(d) Neither argument1 nor argument2 is valid

26. The Simplest form of

$$(p \wedge (\sim r \vee q \vee \sim q)) \vee ((r \vee t \vee \sim r) \wedge \sim q)$$

is

(a) $p \wedge \sim q$ (b) $p \vee \sim q$

(c) t (d) $(p \rightarrow \sim q)$

27. The Simplest form of

$$(p \vee (p \wedge q) \vee (p \wedge q \wedge \sim r)) \wedge ((p \wedge r \wedge t) \vee t)$$

is

(a) $p \wedge t$ (b) $q \wedge t$

(c) $p \wedge r$ (d) $p \wedge q$

28. The statement formula

$$(p \wedge (p \rightarrow q)\ (s \vee r) \wedge (r \rightarrow \sim q)) \rightarrow (s \vee t)$$

is

(a) valid

(b) not satisfiable

(c) satisfiable but not valid

(d) none of these

29. The statement formula

$$\{((\sim p \vee q) \rightarrow r) \wedge (r \rightarrow (s \vee t)) \wedge (\sim s \wedge \sim u) \wedge (\sim u \rightarrow \sim t)\} \rightarrow p$$ is

(a) valid

(b) not satisfiable

(c) satisfiable but not valid

(d) none of these

30. The statement formula

$$((p \rightarrow q) \wedge (\neg r \vee s) \wedge (p \vee r)) \rightarrow (\neg q \rightarrow s)$$

is

(a) valid

(b) not satisfiable

(c) satisfiable but not valid

(d) none of these

31. The statement formula

$$\{(\sim p \leftrightarrow q) \wedge (q \rightarrow r) \wedge (\sim r)\} \rightarrow p$$ is

(a) valid

(b) not satisfiable

(c) satisfiable but not valid

(d) none of these

32. Consider the following statements:

S_1: The conjunctive normal form of

$(a \wedge b) \vee c$ is

$(a \vee b \vee c) \wedge (a \vee \sim b \vee c) \wedge (\sim a \vee b \vee c)$

S_2: The disjunctive normal form of

$a \wedge (b \leftrightarrow c)$ is

$(a \wedge b \wedge c) \vee (a \wedge \sim b \wedge \sim c)$

Which of the following is true?

(a) Only S_1

(b) Only S_2

(c) Both S_1 and S_2

(d) Neither S_1 nor S_2

33. Consider the following arguments:

(I). It is not raining or Rita has her umbrella.
Rita does not have her umbrella or she does not get wet.
It is raining or Rita does not get wet.
$\therefore$ Rita does not get wet.

(II). If superman were able and willing to prevent evil, he would do so.
If superman were unable to prevent evil, he would be impotent.
If he were unwilling to prevent evil, he would be malevolent.
Superman does not prevent evil.
If superman exists, he is neither impotent nor malevolent.
$\therefore$ Superman does not exist.

Which of the following is true?

(a) (I) is valid and (II) is not valid
(b) (II) is valid and (I) is not valid
(c) Both (I) and (II) are valid
(d) neither (I) nor (II) is valid

First Order Logic

34. Let Universe of Discourse be Set of all real numbers.

S_1: $(x^2 + 1) < 0$

S_2: x is odd

Which of the above statements are predicates?

(a) Only S_1
(b) Only S_2
(c) Both S_1 and S_2
(d) Neither S_1 nor S_2

35. Let P(x, y) be a predicate defined as

$P(x, y) : (x \vee y) \rightarrow z$.

The negation of $\forall x\, \exists y\, P(x, y)$ is

(a) $\exists x\ \forall y\ ((x \vee y) \wedge z)$
(b) $\exists x\ \forall y\ ((x \vee y) \wedge \sim z)$
(c) $\exists x\ \forall y\ (\sim(x \vee y) \vee z)$
(d) $\forall x\ \exists y\ ((x \vee y) \wedge \sim z)$

36. Let D_x and D_y denote the domains of x and y, respectively.

Consider the predicate formula ϕ:

ϕ: $\forall x\, \exists y\, [x + y = 17]$

Consider the following domains:

1. $D_x = D_y$ = the set of integers.
2. $D_x = D_y$ = the set of positive integers.
3. D_x = the set of integers and D_y = the set of positive integers.
4. D_x = the set of positive integers and D_y = the set of integers.

In which of the above cases, the quantified predicate ϕ has truth value ***true*** ?

(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1 and 4

37. Consider the following mathematical statement in number theory:

"For every integer *n* bigger than 1, there is a prime strictly between n and 2n":

If the Universe of discourse is set of all integers and p(x) denotes "x is a prime number", then which of the following first order logic sentences correctly represents the above statement?

(a) $\forall n\ [(n > 1) \to \exists x\ \{p(x) \wedge (n < x < 2n)\}]$

(b) $\forall n\ [(n > 1) \wedge \exists x\ \{p(x) \to (n < x < 2n)\}]$

(c) $\exists n\ [(n > 1) \to \forall x\ \{p(x) \wedge (n < x < 2n)\}]$

(d) $\exists n\ [(n > 1) \wedge \forall x\ \{p(x) \wedge (n < x < 2n)\}]$

38. Consider the following mathematical statement in number theory:

"For every integer *n* bigger than 1, there is a prime strictly between n and 2n":

If the Universe of discourse is set of all integers and p(x) denotes "x is a prime number", then which of the following first order logic sentences correctly represents ***negation*** of the above statement?

(a) $\exists n\ [(n > 1) \wedge \forall x\ \{p(x) \to ((x \le n) \vee (x \ge 2n))\}]$

(b) $\forall n\ [(n > 1) \wedge \exists x\ \{p(x) \to ((x \le n) \vee (x \ge 2n))\}]$

(c) $\exists n\ [(n > 1) \to \forall x\ \{p(x) \wedge ((x \le n) \vee (x \ge 2n))\}]$

(d) $\forall n\ [(n > 1) \to \exists x\ \{p(x) \wedge ((x \le n) \vee (x \ge 2n))\}]$

39. Consider the following arguments

I) All doctors are college graduates
Some doctors are not golfers
Hence, some golfers are not college graduates

II) No mothers are males
Some males are politicians
Hence, some politicians are not mothers

Which of the following is ***true***?

(a) Both arguments are valid

(b) Both arguments are invalid

(c) Only argument I is valid

(d) Only argument II is valid

40. Let P and Q be two predicates. Consider the following statements:

S_1: $\exists x\ [P(x) \vee Q(x)] \Leftrightarrow (\exists x\ P(x) \vee \exists x\ Q(x))$

S_2: $\forall x\ [P(x) \wedge Q(x)] \Leftrightarrow (\forall x\ P(x) \wedge \forall x\ Q(x))$

Which of the following is true?

(a) Only S_1

(b) Only S_2

(c) Both S_1 and S_2

(d) Neither S_1 nor S_2

41. Let P and Q be two predicates.

Consider the following statements:

S_1: $(\forall x\, P(x) \vee \forall x\, Q(x)) \Rightarrow \forall x[P(x) \vee Q(x)]$

S_2: $\forall x[P(x) \vee Q(x)] \Rightarrow (\forall x\, P(x) \vee \forall x\, Q(x))$

Which of the following is true?

(a) Only S_1 (b) Only S_2

(c) Both S_1 and S_2 (d) Neither S_1 nor S_2

42. Let S(x) = x is a student,

M(x) = x likes mathematics,

H(x) = x likes history.

Consider the statement,

"There is a student who likes mathematics but not history".

Which of the following first order logic sentences correctly represents the negation of the above statement?

(a) $\forall x\, \{\{S(x) \wedge M(x)\} \to H(x)\}$

(b) $\exists x\, \{\sim S(x) \vee \sim M(x) \vee H(x)\}$

(c) $\forall x\, \{S(x) \to \{\sim M(x) \wedge H(x)\}\}$

(d) $\exists x\, \{S(x) \to \{\sim M(x) \wedge H(x)\}\}$

43. Consider the following:

I. $\{\exists x\, P(x) \wedge \exists x\, Q(x)\} \Rightarrow \exists x\{P(x) \wedge Q(x)\}$

II. $\forall x\{P(x) \vee Q(x)\} \Rightarrow \{\forall x\, P(x) \vee \forall x\, Q(x)\}$

Which of the following is true?

(a) Only I is valid

(b) Only II is valid

(c) Both I and II are valid

(d) Both I and II are invalid

44. Consider the following arguments

I. $\{\forall x[P(x) \to \{Q(x) \wedge S(x)\}],$
$\forall x\{P(x) \wedge R(x)\}\} \Rightarrow \forall x\{R(x) \to S(x)\}$

II. $\{\forall x\{P(x) \vee Q(x)\}, \forall x[\{\sim P(x) \wedge Q(x)\}$
$\to R(x)]\} \Rightarrow \forall x\{\sim R(x) \to P(x)\}$

Which of the following is true?

(a) Only argument I is valid

(b) Only argument II is valid

(c) Both arguments are valid

(d) Both arguments are invalid

45. Consider the following assumptions.

S1: All mathematicians are interesting people.

S2: Only uninteresting people become sales persons.

S3: Every genius is a mathematician.

Which of the following conclusions is ***not*** valid?

(a) Sales people are not mathematicians

(b) Some geniuses are sales persons

(c) Some geniuses are interesting people

(d) Geniuses are interesting people

46. Let D(x) denote x is a driver, and S(x) denote x obeys the speed limits.

Consider,

Some drivers do not obey speed limits.

Which of the following first order logic sentences correctly represents the negation of the above statement?

(a) $\forall x\ \{(D(x) \rightarrow S(x)\}$

(b) $\exists x\ \{D(x) \rightarrow S(x)\}$

(c) $\forall x\ \{(D(x) \wedge S(x)\}$

(d) $\exists x\ \{(D(x) \wedge S(x)\}$

47. Consider the following statements:

S1: $\exists x\ \{P(x) \rightarrow Q(x)\}$

$\rightarrow \{\forall x\ P(x) \rightarrow \exists x\ Q(x)\}$

S2: $\exists x\ \forall y\ P(x, y) \rightarrow \forall y\ \exists x\ P(x, y)$

Which of the following is valid?

(a) only S_1 is true

(b) only S_2 is true

(c) both S_1 and S_2 are true

(d) both S_1 and S_2 are false

48. Which of the following is valid?

(a) $\forall_x \exists_y P(x, y) \rightarrow \exists_x \forall_y P(x, y)$

(b) $\exists_x \forall_y P(x, y) \rightarrow \forall_x \exists_y P(x, y)$

(c) $\exists_y \forall_x P(y, x) \rightarrow \forall_y \exists_x P(x, y)$

(d) $\exists_y \forall_x P(x, y) \rightarrow \forall_y \exists_x P(x, y)$

49. Determine the truth value of each of the following statements if the universe of discourse of each variable is the set of all integers

S_1: $\forall n\ \exists m\ (n + m = 5)$

S_2: $\exists\ n\ \forall m\ (nm = m)$

S_3: $\forall m\ \exists n\ (mn = 1)$

S_4: $\exists m\ \forall n\ (m + n = 0)$

(a) S_1 and S_3 are true (b) S_2 and S_4 is true

(c) S_3 and S_4 are true (d) S_1 and S_2 are true

50. Which of the following equivalences is/are true?

S_1: $\exists x(A(x)\rightarrow B(x)) \Leftrightarrow (\forall x)A(x) \rightarrow (\exists x)\ B(x)$

S_2: $\exists x\ A(x) \rightarrow (\forall x)B(x) \Leftrightarrow (\forall x)(A(x) \rightarrow B(x))$

S_3: $\exists x(A(x) \vee B(x)) \Leftrightarrow (\exists x\ A(x) \vee \exists x\ B(x))$

S_4: $\exists x\ (A(x) \wedge B(x)) \Leftrightarrow (\exists x)A(x) \wedge \exists x\ B(x)$

(a) Only S_1 and S_3 (b) Only S_1 and S_4

(c) Only S_1 and S_2 (d) Only S_2 and S_4

51. Which of the following is not valid?

(a) $\forall_x(P(x) \rightarrow Q(x)) \Rightarrow \{\forall_x P(x) \rightarrow \forall_x Q(x)\}$

(b) $\{\forall_x P(x) \rightarrow \forall_x Q(x)\} \Rightarrow \forall_x \{P(x) \rightarrow Q(x)\}$

(c) $\forall_x \{P(x) \vee Q(x)\} \Rightarrow \{\forall_x P(x) \vee \exists_x Q(x)\}$

(d) $\{\forall_x P(x) \vee \forall_x Q(x)\} \Rightarrow \forall_x \{P(x) \vee Q(x)\}$

52. Let P (x, y) denote the open statement ‘x is a divisor of y’ where the universe of discourse is set of all integers.

Consider the following statement:

S_1: $\forall_x \exists_y P(x, y)$ **S_2:** $\forall_y \exists_x P(x, y)$

S_3: $\exists_x \forall_y P(x, y)$ **S_4:** $\exists_y \forall_x P(x, y)$

Which one of the following options is ***correct***?

(a) S_1 and S_2 are true

(b) S_2 and S_3 are true

(c) S_3 and S_4 are true

(d) S_1 and S_4 are true

53. Let the domain be the set of animals. Let B(x) be that x is a bear.

What is the correct predicate translation for "There is at most one bear."?

(a) $(\neg(\exists x\ (\exists y\ ((B(x) \wedge B(y)) \wedge (x \neq y)))))$

(b) $(\exists x\ (\exists y\ ((B(x) \wedge B(y)) \wedge x \neq y)))$

(c) $(\forall x\ (\forall y\ ((B(x) \wedge B(y)) \rightarrow (x = y))))$

(d) $(\forall x\ (B(x) \wedge (\forall y\ (B(y) \rightarrow (x = y)))))$

54. An island has two kinds of inhabitants, knights, who always tell the truth, and knaves, who always lie. You encounter two people A and B. What are A and B if : A says "The two of us are both knights" and B says "A is a knave"

(a) A is knight.

(b) B is Knight.

(c) A is knave.

(d) B is knave.

02. Combinatorics

Practice Questions

01. In a class of 50 students, 11 students can speak both French and German, and 8 students can speak neither German nor French. Number of students who can speak only one of the two languages is _____.

02. A, B and C are 3 sets such that
$n(A \cup B \cup C) = 31$, $n(A) = 20$, $n(B) = 16$, $n(C) = 8$, $n(A \cap B \cap \overline{C}) = 3$, $n(A \cap \overline{B} \cap C) = 4$,
$n(\overline{A} \cap B \cap C) = 2$,
then $n(A \cap B \cap C) =$ ______?

03. The number of integers between 1 and 400 inclusive that are not divisible by 5, 6 or 8 is ______.

04. At a construction site, George is the incharge of hiring skilled workers for the project. Out of 80 candidates that he interviewed, he found that
45 of them are painters,
50 of them are electricians,
50 of them are plumbers;
15 of them skilled in all 3 areas;
all of them had skills in atleast one of these areas. Which of the following is ***not*** true?

(a) Number of candidates with atleast two skills = 50
(b) Number of candidates with atmost two skills = 65
(c) Number of candidates with exactly two skills = 35
(d) Number of candidates with only one of the three skills = 30

05. Let A, B, C, D are sets so that $n(A) = 42$, $n(B) = 36$, $n(C) = 28$, $n(D) = 24$, Intersection of any two of these 4 sets contain 12 elements, intersection of any three of these three contain 8 elements
and $n(A \cap B \cap C \cap D) = 4$. then number of elements in $(A \cup B \cup C \cup D)$ is _____.

06. A palindrome is a string whose reversal is identical to the string. How many bit strings of length n are palindromes?

(a) 2^{n-1} (b) $2^{\frac{n}{2}}$
(c) $2^{\left\lceil \frac{n}{2} \right\rceil}$ (d) $2^{\left\lfloor \frac{n}{2} \right\rfloor}$

07. In how many integers between 1 and 10,000 (both integers inclusive) does the digit 7 appear?

08. In a binary matrix each element is 0 or 1.
How many symmetric binary matrices of order 3×3 are possible?

09. Suppose we have 6 different English movies, 8 different Telugu movies and

10 different Hindi movies. How many ways we can choose 2 movies of different languages?

10. Number of integers in the set $\{1, 2, 3, \ldots, 1000\}$ with atleast one digit repeated is ______.

11. How many 4-digit integers are there with digit 0 appearing exactly once?

12. How many integers in the set $\{1, 2, 3, \ldots, 1,00,000\}$ contains exactly one 3, exactly one 4 and exactly one 5 in their decimal representation ?

13. Suppose n players were enrolled in a singles elimination tennis tournament. How many matches are to be conducted to decide the winner?

    (a) $n - 1$ (b) $n + 1$

    (c) 2^n (d) 2^{n-1}

14. A set S has n elements. How many ways we can choose subsets P and Q of S. So, that $(P \cap Q) = \phi$?

    (a) 3^{n-1} (b) $3^n - 1$

    (c) 3^n (d) $3^n + 1$

15. In how many ways 6 different books can be distributed among 10 persons so that no person can take more than one book and maximum number of books are to be distributed?

16. In how many ways 4 boys and 4 girls can sit in a row so that no two girls are sitting side by side?

17. In how many ways 4 boys and 4 girls can sit in a row so that no two children of same gender are side by side?

18. In how many signals can be generated using 5 different colored flags, if any number of the flags can be hoisted at a time in row?

19. In how many ways can we distribute 6 different books among 10 persons?

    (a) 10^6 (b) 6^{10}

    (c) P(10, 6) (d) C(10, 6)

20. How many 5-digits positive integers are possible with the digits 2, 3 and 5?

21. How many 10 letter permutations are possible with the letters {a,a,b,b,b,c,c,c,c} if all the letters of the set is used at a time?

22. Number of strings possible with seven 0's, two 1's and one 2 is ______.

23. In how many ways 10 persons can be divided into 3 teams, so that

    Team 1 contains 3 members and

    Team 2 contains 2 members and

    Team 3 contains 5 members?

24. In how many ways 10 persons can be divided into 5 teams of 2 each?

25. Number of ways we can assign 5 persons into 3 different rooms, so that each room contains atleast one person is _______.

26. A tennis club is selecting a team of 4 mixed pairs, for a tournament. There are 6 men and 6 women for selection. The total number of possible selections are ______.

27. The maximum number of points of intersection possible with 10 straight lines in a plane is _____.

28. How many binary sequences of length 10 are possible with exactly 3 zeros?

29. How many binary sequences of length 10 are possible with exactly 4 zeros and no two zeros are consecutive?

30. How many 5 digit integers are possible so that in each of these integers, every digit is less than the digit on its right?

31. Suppose n couples are in a party. If each person shakes hands with every other person except his/her spouse, then number of different hand shakes possible in the party is ______.
(a) $n(2n - 1)$ (b) $2n(n - 1)$
(c) $2n(n + 1)$ (d) $2n(2n + 1)$

32. How many rectangles are there in a chess board which are not squares?

33. A medical student has to work in a hospital for five days in January. However, he is not allowed to work two consecutive days in the hospital. In how many different ways can he choose the five days he will work in the hospital?
(a) C (27, 5) (b) C(26, 5)
(c) C(27, 4) (d) C(26, 4)

34. In how many ways can we distribute 6 similar books among 10 persons so that no person can take more than one book and maximum number of books are to be distributed?

35. A company has 20 employees, 12 males and 8 females. The number of ways to form a committee of 5 employees that contains atleast one male and atleast one female is _____.

36. Consider a convex polygon with n vertices where $n \geq 3$. Suppose you want to draw a triangle by connecting 3 vertices but the triangle cannot share any sides of the polygon. Number of ways you can draw is
(a) $C(n, 3) - n(n + 4) + n$
(b) $C(n, 3) - n(n - 3) - n$
(c) $C(n, 3) - n(n - 4) - n$
(d) $C(n, 3) - n(n - 2) - n$

37. In how many ways can we distribute 10 similar books among 5 persons?

38. In how many ways can we distribute 16 similar balls in 4 numbered boxes so that each box should contain at least 1 ball?

39. Number of non negative solutions to the following inequality $12 \le w + x + y + z \le 14$ is ____.

40. Number of integral solutions to the equation $x_1 + x_2 + x_3 = 8$ where $x_1 \ge 3$, $x_2 \ge -2$ and $x_3 \ge 4$ is _______.

41. The number of integers between 1 and 1000 (inclusive) that have sum of digits equal to 10 is _______.

42. In how many ways k students can sit in a row of n seats (n > 2k) satisfying both of the following restrictions
   (i) from left to right, they are seated in alphabetical order and
   (ii) each student has an empty seat immediately to his/her right?

(a) C(n + k, k) (b)C(n – k, k)
(c) C(n + k, n – k) (d) C(n – k, k – 1)

43. If x, y, z and w are positive integers then number of solutions possible to the inequality

$(x + y + z + w) \le 10$ is _______.

44. Six distinct symbols are transmitted through a communication channel. A total of 12 blanks are to be inserted between the symbols with atleast 2 blanks between every pair of symbols. The number of ways we can arrange the symbols and blanks is ______.

45. If 410 letters were distributed in 50 apartments. Which of the following are always necessarily true?

S_1: Some apt. received at least 9 letters
S_2: Some apt. received at most 8 letters
S_3: Some apt. received at least 10 letters
S_4: Some apt. received at most 7 letters
S_5: Some apt. received at least 8 letters
S_6: Some apt. received at most 10 letters
(a) S_1, S_2, S_3 and S_6
(b) S_1, S_2, S_3 and S_4
(c) S_2, S_4, S_5 and S_6
(d) S_1, S_2, S_5 and S_6

46. Suppose thirty buses are used to transport 2000 students and each bus has 80 seats. Consider the following statements.
S_1: one of the buses will carry at least 67 passengers.
S_2: one of the buses will have at least 14 empty seats.

Which of the following is true?
(a) S_1 is true and S_2 is false
(b) S_1 is false and S_2 is true
(c) both S_1 and S_2 are true
(d) both S_1 and S_2 are false

47. What is the minimum number of persons we have to choose randomly so that atleast 9 persons were born in the same month?

48. A bag contains 6 Red balls, 8 Blue balls, 10 green balls, 15 White balls, 20 Yellow balls. What is the minimum number of balls we have to choose randomly from the bag to ensure that we get atleast 6 balls of same color?

49. In the previous question, what is the minimum number of balls we have to choose from the bag to ensure that we get at least 9 balls of same color?

50. A box contains 12 Red, 7 Blue and x green balls. If the minimum number of balls we have to choose randomly from the box to guarantee that we have 6 balls of same color is 15, then x = _______.

51. Let S = {0, 1, 2 … 9}. What is the smallest positive integer K such that any subset of S of size K contain two distinct subset of size two, $\{x_1, x_2\}$ and $\{y_1, y_2\}$, such that $x_1 + x_2 = y_1 + y_2 = 9$?

52. The minimum number of non negative integers, we have to choose randomly, so that there will be atleast two integers x and y such that x + y is divisible by 10 is_____.

53. The minimum number of cables required to connect 8 computers to 4 printers to ensure that any 4 computers can directly access 4 different printers is ______.

54. Number of positive integers which are less than 210 and relatively prime to 210 is ______.

55. Number of positive integers which are less than 1368 and coprime to 1368 is _____.

56. Number of positive integers which are less than 317 and coprime to 317 is _____.

57. The formula for the number of positive integers m which are less than p^k and relatively prime to p^k, where p is a prime number and k is a positive integer is ________.

(a) $p^k (p-1)$ (b) $p^{k-2} (p-1)$

(c) $p^k (p-2)$ (d) $p^{k-1}(p-1)$

58. How many 1-1 functions are possible on a set with 6 elements, so that no element is mapped to itself?

59. A party was attended by n guests. When the guests arrived, they left their hats in the same coatroom. After the party ended, there was an electrical power failure, so each guest took a hat from the coatroom at random. When the guests were back on the street, they were amused to find out that none of them got his back. In how many different ways could that happen?

(a) $\sum_{i=0}^{n}(-1)^i \frac{n!}{i!}$ (b) $\sum_{i=0}^{n}(-1)^{i-1} \frac{n!}{i!}$

(c) $\sum_{i=0}^{n}(-1)^i \frac{(n+1)!}{i!}$ (d) $\sum_{i=0}^{n}(-1)^{i-1} \frac{(n+1)!}{i!}$

60. In how many ways can we put 5 letters L_1, L_2, L_3, L_4, L_5, in 5 envelopes e_1, e_2, e_3, e_4, and e_5 (at 1 letter per envelope) so that
    i. no letter is correctly placed.
    ii. at least 1 letter is correctly placed
    iii. exactly 2 letters are correctly placed
    iv. at most 1 letter is correctly placed
    v. at least 1 letter is wrongly placed
    vi. exactly 1 letter is wrongly placed ?

61. Number of derangements possible for the sequence {a, b, c, d, e, f, g, h, i, j} so that
    i) The first 5 letters of the sequence are in first 5 places
    ii) None of the first 5 letters of the sequence are in first 5 places?

62. Suppose 4 different books are distributed among 4 students (@ 1 book per student). Further suppose, the books were returned by the students and again distributed among the students later on. In how many ways this can be done so that no student can take the same book twice?

63. The recurrence relation for the maximum number of pieces of a pizza made by *n* straight cuts is

(a) $f(n) = f(n-1) + n$

(b) $f(n) = f(n-1) + 2n$

(c) $f(n) = f(n-1) + n^2$

(d) $f(n) = f(n-1) + (n-1)$

64. The recurrence relation for the maximum number of nodes in a binary tree of depth d is

(a) $n(d) = d(n-1) + 2^d$

(b) $n(d) = n(d-1) + 2^d$

(c) $n(d) = n(d-1) + 2^n$

(d) $n(d) = d(n-1) + 2^n$

65. The recurrence relation for the number of n-digit quaternary sequences that have an even number of zeros. (quaternary sequences use only 0, 1, 2, 3 for digits) is _______.

(a) $a_n = a_{n-1} + 4^{n-1}$ (b) $a_n = 3a_{n-1} + 4^{n-1}$

(c) $a_n = 2a_{n-1} + 4^{n-1}$ (d) $a_n = a_{n-1} + 4^{n-2}$

66. If a_n = number of bit strings of length n with 3 consecutive zeros, then recurrence relation for a_n is __________.

(a) $a_n = a_{n-1} + a_{n-2} + a_{n-3} + 2^{n-3}$

(b) $a_n = a_{n-1} + a_{n-2} + a_{n-3}$

(c) $a_n = a_{n-1} + a_{n-2} + a_{n-3} - 2^{n-3}$

(d) $a_n = (n-4)\, a_{n-3}$

67. Let a_n be the number of bit strings of length n that do not contain 01. The recurrence relation for a_n is_____.

(a) $a_n = a_{n-1} + 1$ (b) $a_n = a_{n-1} + 2^{n-2}$

(c) $a_n = 3a_{n-2} + 2^{n-2}$ (d) $a_n = a_{n-1} + a_{n-2}$

68. The solution of the recurrence relation $a_n = a_{n-1} + 2(n-1)$ where $a_1 = 2$ is _________.

(a) $n^2 - n + 2$ (b) $n(n+1)$

(c) $n^2 - 2n + 3$ (d) $n^2 + 2n - 1$

69. Consider the recurrence relation

$a_n - a_{n-1} = 3n^2$, where $n \geq 1$ and $a_0 = 7$

$a_{20} =$ _______.

70. The solution of $a_n = n\, a_{n-1}$, where $a_0 = 1$ is

(a) n^2 (b) 2^n

(c) $n!$ (d) $\frac{n(n+1)}{2}$

71. The solution of $a_n = a_{n-1} + (2n+1)$ where $a_0 = 1$ is

(a) n^2 (b) $(n+1)^2$

(c) $(2n+1)^2$ (d) $(2n-1)^2$

72. The solution of $a_n = a_{n-1} + \frac{1}{n(n+1)}$ where $a_0 = 1$ is

(a) $\frac{2n+1}{n+1}$ (b) $\frac{2n-1}{n+1}$

(c) $\frac{2n+1}{n-1}$ (d) $\frac{2n-1}{n-1}$

73. The solution of the recurrence relation: for $n \in N$,

$$f(n) = \begin{cases} 1 & \text{if } n = 1; \\ 3f\left(\left\lceil \frac{n}{3} \right\rceil\right) & \text{if } n \geq 2 \end{cases}$$ is

(a) $3^{\lceil \log_n 3 \rceil}$ (b) $3^{\lceil n \log 3 \rceil}$

(c) $3^{\lceil \log_3 n \rceil}$ (d) $3^{\lceil 3 \log n \rceil}$

74. The solution of the recurrence relation $a_n - a_{n-1} - a_{n-2} = 0$ with initial conditions $a_0 = 0$ and $a_1 = 1$ is______.

(a) $\frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right]$

(b) $\frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{n} - \left(\frac{1-\sqrt{5}}{2}\right)^{n}\right]$

(c) $\frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{n} + \left(\frac{1-\sqrt{5}}{2}\right)^{n}\right]$

(d) $\frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{n+1} + \left(\frac{1-\sqrt{5}}{2}\right)^{n+1}\right]$

75. The solution of $a_n - 2a_{n-1} = 3(2)^n$ where $a_0 = 1$ is ____.

(a) $C_1 2^n + 3\, n2^n$

(b) $C_1 3^n + 2\, n2^{n-1}$

(c) $C_1 2^n - 3\, n2^{n-1}$

(d) $C_1 2^n + 2\, n3^{n-1}$

76. The solution of $a_n - 3a_{n-1} + 2a_{n-2} = 2^n$ is ______.

(a) $C_1 + C_2 n + 2n(2^n)$

(b) $C_1 - C_2 n - 2n(2^n)$

(c) $C_1 + C_2 n + n(2^n)$

(d) $C_1 + C_2 n + 3n(2^n)$

77. The solution of $a_n - 6a_{n-1} + 9a_{n-2} = 3^n$ is _____.

(a) $(C_1 + C_2\, n)\, 3^n + n(n-1)3^n$

(b) $(C_1 + C_2\, n)\, 3^n + n(n+1)3^n$

(c) $(C_1 + C_2\, n)\, 3^n + n(n-1)3^{n-1}$

(d) $(C_1 + C_2\, n)\, 3^n + n(n-1)3^{n-2}$

78. The solution of the recurrence relation $a_n = 2a_{n-1} - a_{n-2} + 2^n$ is ______.

(a) $C_1 + C_2 n + 2^{n-2}$

(b) $C_1 n + C_2 n^2 + 2^{n-1}$

(c) $C_1 n + C_2 + 2^n$

(d) $C_1 + C_2 n + 2^{n+2}$

79. The solution of $a_n - 3a_{n-1} = n + 3$ is ______.

(a) $C_1 3^n - \frac{n}{2} - \frac{9}{4}$

(b) $C_1 3^n + \frac{n}{2} + \frac{7}{4}$

(c) $C_1 3^n + \frac{n}{2} - \frac{7}{4}$

(d) $C_1 3^n - \frac{n}{2} + \frac{7}{4}$

80. The solution of $a_n - 2a_{n-1} + a_{n-2} = 3n + 5$ is ______.

(a) $C_1 + C_2 n + 4n^2 + \frac{n^3}{2}$

(b) $C_1 + C_2 n + 2n^2 + \frac{n^3}{2}$

(c) $C_1 + C_2 n + n^2 + \frac{n^3}{2}$

(d) $C_1 + C_2 n + 3n^2 + \frac{n^3}{2}$

81. The solution of the recurrence relation

$a_n = 4a_{n-1} + 3n\, 2^n$ where $a_0 = 4$ is

(a) $a_n = 10(4^n) - (3n + 6)\, 2^n$

(b) $a_n = 7(4^n) + (3n + 4)\, 2^n$

(c) $a_n = 8(4^n) + (2n + 3)\, 2^n$

(d) $a_n = 5(4^n) - (2n + 6)\, 2^n$

82. The solution of $a_{n+2} - 2a_{n+1} + a_n = n^2\, 2^n$ is

(a) $a_n = (C_1 + C_2\, n) + 2^n(n^2 - 8n + 20)$

(b) $a_n = (C_1 + C_2\, n) - 2^n(n^2 - 8n + 20)$

(c) $a_n = (C_1 - C_2\, n) + 2^n(n^2 - 8n + 20)$

(d) $a_n = (C_1 + C_2\, n) + 2^n(n^2 - 8n - 20)$

83. The solution of $\sqrt{a_n} - 2\sqrt{a_{n-1}} + \sqrt{a_{n-2}} = 0$ where $a_0 = 1$ and $a_1 = 2$ is

(a) $a_n = \left[\frac{2^{n+1} + (-1)^n}{3}\right]^2$ (b) $(n + 1)^2$

(c) $(n - 1)^3$ (d) $(n - 1)^2$

84. The solution of

$$T(n) = 7T\left(\frac{n}{3}\right) + 2n$$

Where $T(1) = \frac{5}{2}$ is

(a) $a_n = (-3/2)\, n + 4\,.7^{\log_3 n}$

(b) $a_n = 3n/2 - 4\,.\,7 \log_3{}^n$

(c) $a_n = -3n/2 - 4\,.\,7 \log_3{}^n$

(d) none of these

85. The generating function of the sequence

$\{1, -2, 4, -8, 16, \ldots\ldots\ldots\ldots\ldots\ldots\infty\}$ is

(a) $(1 + 2x)^{-1}$ (b) $(1 - 2x)^{-1}$

(c) $(1 + x)^{-2}$ (d) $(1 - x)^{-2}$

86. The generating function of the sequence $\{0, 1, 3, 9, 27, \ldots\ldots \infty\}$ is ________.

(a) $x(1 - 3x)^{-1}$ (b) $x(1 + 3x)^{-1}$

(c) $(1 - 3x)^{-1}$ (d) $(1 + 3x)^{-1}$

87. The generating function of the sequence

$\{a_0, a_1, a_2, \ldots\ldots\ldots\ldots\ldots\}$

where $a_n = (n + 1)(n + 2)$ is

(a) $2(1 - x)^{-3}$ (b) $2x(1 - x)^{-3}$

(c) $2x(1 + x)^{-3}$ (d) $2(1 + x)^{-3}$

88. The generating function of the sequence $\{0, 0, 1, -2, 3, -4, \ldots \infty\}$ is_______.

(a) $\dfrac{x^2}{(1-x)^2}$ (b) $\dfrac{x}{(1-x)^2}$

(c) $\dfrac{2x}{(1+x)^2}$ (d) $\dfrac{x^2}{(1+x)^2}$

89. The generating function of the sequence

$\{1, 0, 1, 0, \ldots, (-1)^n + 1, \ldots\ldots \infty\}$

is ______.

(a) $(1 - x)^{-2}$ (b) $(1+x)^{-2}$

(c) $(1-x^2)^{-1}$ (d) $(1+x^2)^{-1}$

90. The coefficient of x^{27} in the expansion of

$$(x^4 + 2x^5 + 3x^6 + 4x^7 + \ldots \infty)^5$$

is _______

(a) C(16, 9) (b) C(15, 8)

(c) C(17, 10) (d) C(14, 7)

91. Number of ways we can distribute 15 similar balls among three distinct boxes so that no box will contain more than 7 balls is ________.

(a) 24 (b) 28

(c) 32 (d) 36

92. There are seven copies of one book, eight of a second book and nine of a third book.

In how many ways can two people divide them if each takes 12 books?

93. Let $0 \le S(0) \le T(0)$, and suppose we have the recurrences

$S(n + 1) = aS(n) + f(n)$

$T(n + 1) = bT(n) + g(n)$,

where $0 \le a \le b$ and $0 \le f(n) \le g(n)$ for all $n \in N$.

Which of the following are necessarily true ?

(a) $S(n) \le T(n)$ for all $n \in N$.

(b) $T(n) \le S(n)$ for all $n \in N$.

(c) $T(n) < S(n)$ for some $n \in N$.

(d) $0 \le S(n) \le T(n)$ for all $n \in N$.

94. Let $n \in N$ be a natural number and let $X \subseteq N$ be a subset with $n + 1$ elements.

Then which of the following is/are necessarily true?

(a) there exist two distinct natural numbers $x, y \in X$ such that $x - y$ is divisible by n.

(b) there exist two distinct natural numbers $x, y \in X$ such that both x, y are divisible by n.

(c) there exist two distinct natural numbers $x, y \in X$ such that $x + y$ is divisible by n.

(d) there exist two distinct natural numbers $x, y \in X$ such that xy is divisible by n.

03. Graph Theory

Practice Questions

01. Let G be a simple graph with 11 vertices. If degree of each vertex is atleast 3 and atmost 5, then the number of edges in G should lie between _______ and _______.

(a) 18 and 28 (b) 17 and 27
(c) 18 and 27 (d) 17 and 25

02. A simple graph is called regular if all its vertices have the same degree. Let G be a connected regular graph with 22 edges. A possible number of vertices in G is ____.

(a) 44 (b) 2
(c) 4 (d) 11

03. If G be simple graph with 38 edges and degree of each vertex is 4, then number of vertices in G is ________.

04. Suppose all vertices in a graph G have degree k, where k is an odd number. The number of edges in G is

(a) even (b) odd
(c) a multiple of k (d) a divisor of k

05. Which of the following degree sequences represent a simple non directed graph?

(a) {2, 3, 3, 4, 4, 5} (b) {2, 3, 4, 4, 5}
(c) {1, 3, 3, 4, 5, 6, 6}
(d) {0, 1, 2, …., n–1}
(e) {2, 3, 3, 3, 3}

06. Which of the following degree sequences represent a simple non directed graph?

S_1 = {6, 6, 6, 6, 4, 3, 3, 0}
S_2 ={6, 5, 5, 4, 3, 3, 2, 2, 2}

(a) Only S_1
(b) Only S_2
(c) Both S_1 and S_2
(d) Neither S_1 nor S_2

07. If G is a simple graph with degree sequence {5, 2, 2, 2, 2, 1} then number of edges in the complement $\overline{G}$ = _____.

08. Let G be a simple connected graph with minimum number of edges. If G has n vertices with degree 1, 2 vertices of degree 2, 4 vertices of degree 3 and 3 vertices of degree 4, then n = _______.

09. If G is a simple connected graph with 10 vertices and minimum number of edges, then sum of the degrees of all vertices in G = _____.

10. If a graph G has 9 vertices, all but one of odd degree, then number of vertices with odd degree in $\overline{G}$ is____.

11. If G is a simple graph with 11 vertices and degree of each vertex is at most 5. Then maximum number of edges possible in G is ____.

12. Which of the following is a simple connected graph?

(a) A graph with 12 vertices, 28 edges and degree of each vertex is 3 or 4

(b) A graph with 10 vertices and 46 edges

(c) A bipartite graph with 9 vertices and 21 edges

(d) A connected graph with n vertices and n – 1 edges ($n \geq 2$)

13. If degree sequence of a simple graph G is {3, 2, 2, 1, 0} then degree sequence of $\overline{G}$ is ______.

(a) {5, 4, 3, 3, 2} (b) {4, 3, 2, 2, 1}
(c) {5, 4, 3, 3, 0} (d) {4, 3, 2, 2, 0}

14. Number of simple graphs possible with 6 vertices and 12 edges is _______.

15. If a simple graph G has p vertices and q edges, then the number of edges in $\overline{G}$ is ______.

(a) $\frac{p(p-1)}{2}-q$ (b) $\frac{p(p+1)}{2}-q$
(C) $\frac{(p-1)(p-2)}{2}-q$ (d) $\frac{(p+1)(p+2)}{2}-q$

16. Let G be a simple connected graph with n vertices on which the adjacency relation is transitive. Whenever there is an edge uv and an edge vw, there is also an edge uw. Number of edges in G is

(a) $\frac{n^2}{4}$ (b) $\frac{n^2+n}{2}$
(c) $\frac{n^2-n}{2}$ (d) $\frac{n^2-3n+2}{2}$

17. If W_n is a wheel graph with ‘n’ vertices then, the number of edges in the complement of W_n is

(a) $\frac{n(n-3)}{2}$

(b) $\frac{n(n-2)}{2}$

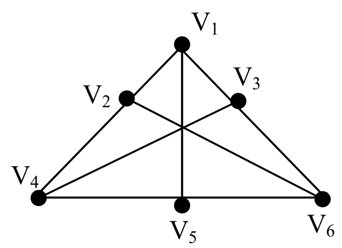
(c) $\frac{(n-1)(n-3)}{2}$

(d) $\frac{(n-1)(n-4)}{2}$

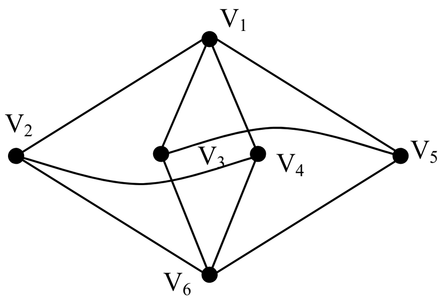
18. A tree has 14 vertices of degree 1 and degree of each of the remaining vertices is 4 or 5. If the tree has ‘n’ vertices then number of vertices with degree 5 is

(a) (40 – 2n) (b) (3n – 54)
(c) (54 – 2n) (d) (3n – 40)

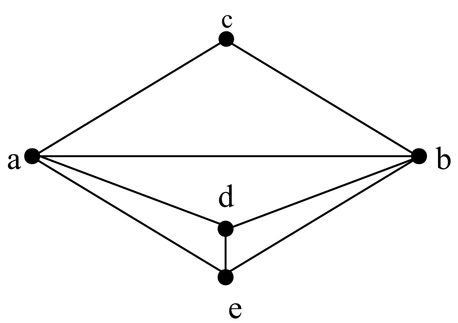
19. The chromatic number of the graph shown below is ________.



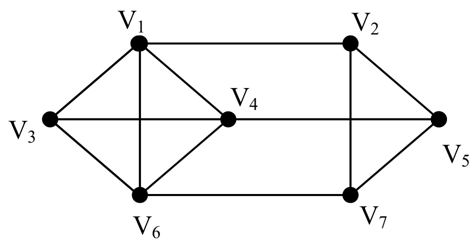
20. For the graph shown below, the chromatic number is ______.



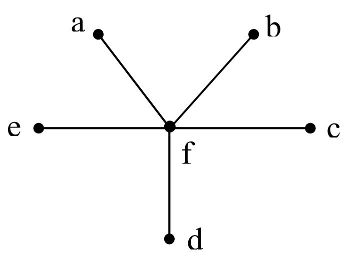
21. For the graph shown below, the chromatic number is ______.



22. For the graph shown below the chromatic number is ________.

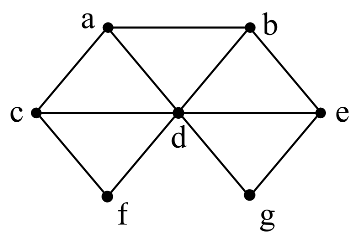


23. For the graph G shown below.

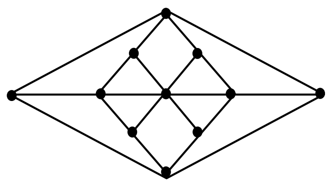


Chromatic number of G + chromatic number of $\overline{G}$ = ________.

24. Chromatic number of the graph G shown below is __________.



25. For the graph shown below.



The minimum number of colours required for vertex coloring of G is ______.

26. If G is a star graph with 6 vertices, then the chromatic number of $\overline{G}$ is ______.

27. Let n be a positive integer and $n \geq 4$. Let C_n is a cycle graph with n vertices and W_n is a wheel graph with n vertices. If α and β are chromatic numbers of C_n and W_n respectively, Then $\alpha + \beta =$

(a) 5 (b) 6

(c) 7 (d) 8

28. Let G be a complete graph with 10 mutually adjacent vertices. If we delete an edge in G then chromatic number of resulting graph is ________.

(a) 4 (b) 8

(c) 9 (d) 10

29. If G is a bipartite graph with 6 vertices and 9 edges, then the chromatic number of $\overline{G}$ = _____.

30. Let C_n be a cycle graph with n vertices ($n \geq 3$) if matching number of C_n is α and chromatic number of C_n is β, then $2\alpha + \beta$ =

(a) n (b) n + 1

(c) n + 2 (d) n + 3

31. Which of the following is ***not*** true?

(a) The vertex chromatic number of complete graph $K_n = n$

(b) The vertex chromatic number of cycle graph $C_n = n - 2\left\lfloor \frac{n}{2} \right\rfloor + 2$

(c) The vertex chromatic number of wheel graph $W_n = n - 2\left\lceil \frac{n}{2} \right\rceil + 4$

(d) The vertex chromatic number of the complete bipartite graph $K_{n,n} = n$

32. Which of the following is ***not*** true?

(a) The vertex chromatic number of bipartite graph (with atleast one edge) = 2

(b) The vertex chromatic number of star graph with n vertices ($n \geq 2$) = 2

(c) The vertex chromatic number of tree with n vertices ($n \geq 2$) = 2

(d) If G is a simple graph in which all the cycles are of even length, then the vertex chromatic number of G = 3

33. Which of the following is ***not*** true?

(a) Number of perfect matchings in $K_{2n} = \frac{2n\,!}{2^n\ n!}$

(b) Number of perfect matchings in $K_{n,n}$=n !

(c) Number of perfect matchings in C_n (n is even) = 2

(d) Number of perfect matchings in W_{2n} = 2n

34. Which of the following is true?

(a) Number of perfect matchings in a tree with n vertices = 2

(b) Number of perfect matchings in a star graph with n vertices = 1

(c) Number of perfect matchings in $K_{m,n}$ = Minimum of {m, n}

(d) Number of perfect matchings in $K_{3,3}$ = 6

35. Consider the following:

S_1: $K_{m,n}$ has a perfect matching $\Leftrightarrow$ (m = n)

S_2: A graph G has a perfect matching $\Leftrightarrow$ Number of vertices in G is even

S_3: A bipartite graph G with vertex partition $\{V_1, V_2\}$ has a complete matching $\Leftrightarrow |V_1| < |V_2|$

Which of the following is true?

(a) S_1 and S_2 (b) S_1 and S3

(c) Only S_1 (d) Only S_2

36. Which of the following is ***not*** true?

(a) Matching number of $K_n = \left\lfloor \frac{n}{2} \right\rfloor$

(b) Matching number of $C_n = \left\lfloor \frac{n}{2} \right\rfloor$

(c) Matching number of $W_n = \left\lfloor \frac{n}{2} \right\rfloor$

(d) Matching number of $K_{m,n} = \left\lfloor \frac{m+n}{2} \right\rfloor$

37. Which of the following is ***not*** true?

(a) Matching number of a star graph with n vertices ($n \geq 2$) = 1

(b) Matching number of bipartite graph $K_{m,m} = m$

(c) Matching number of bipartite graph $K_{m,n}$ = maximum of {m, n}

(d) Matching number of a tree with n vertices = 1

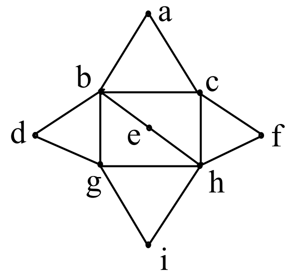
38. If G is a complete bipartite graph with n vertices ($n \geq 2$) and minimum number of edges, then matching number of G is _____.

(a) 1 (b) $n - 1$

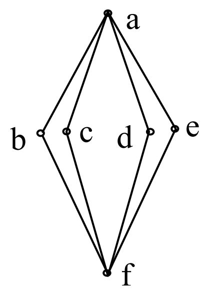
(c) $\left\lfloor \frac{n}{2} \right\rfloor$ (d) $\left\lceil \frac{n}{2} \right\rceil$

39. If G is a disconnected graph with 10 vertices and maximum number of edges, then matching number of G + chromatic number of G = _________.

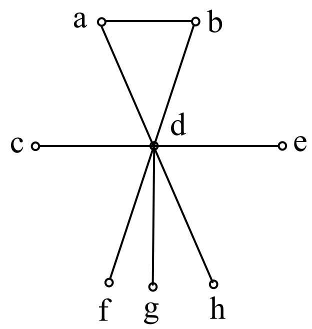
40. Matching number of the graph shown is ____.



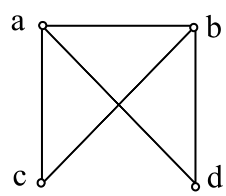
41. Matching number of the graph shown below is _____ .



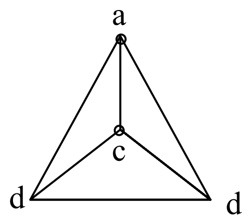
42. Matching number of the graph shown below is _____.



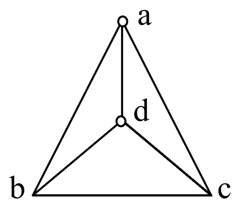
43. Number of maximal matchings in the graph shown below is _______.



44. Number of maximal matchings in the graph shown below is _______.



45. Number of matchings in the graph shown below is _______.



46. If G is a bipartite graph with n vertices and maximum number of edges. Then matching number of G is _____.

(a) $\left\lfloor \frac{n}{2} \right\rfloor$ (b) $\left\lceil \frac{n}{2} \right\rceil$

(c) $\left\lfloor \frac{n-1}{2} \right\rfloor$ (d) $\left\lceil \frac{n+1}{2} \right\rceil$

47. If G is a simple graph with 10 vertices and 3 components, then number of edges in G should lie between _______ and _______.

(a) 7 and 28 (b) 13 and 31

(c) 8 and 45 (d) 6 and 27

48. If G is a bipartite graph with 9 vertices and maximum number of edges, then vertex connectivity of G = _____.

49. If G is not a simple connected graph with n vertices, then maximum number of edges possible in G is _______.

(a) $\frac{(n)(n-2)}{2}$ (b) $\frac{(n-1)(n)}{2}$

(c) $\frac{(n-1)(n-2)}{2}$ (d) $\frac{(n-1)(n-2)}{4}$

50. If ‘G’ is a simple connected graph with 10 vertices in which degree of every vertex is ‘2’, then the number of cut edges in ‘G’ is _______.

51. Let G is a simple connected graph in which every vertex has even degree.

Consider the following statements.

S_1: G has no cut vertex

S_2: G has no cut edge

Which of the following is true?

(a) S_1 is true and S_2 is false

(b) S_1 is false and S_2 is true

(c) Both S_1 and S_2 are true

(d) Both S_1 and S_2 are false

52. Let G be a simple graph with n vertices. If number of edges in G lies between (n –1) and $\frac{(n-1)(n-2)}{2}$ then which of the following is ***true***?

(a) G is necessarily connected

(b) G is necessarily disconnected

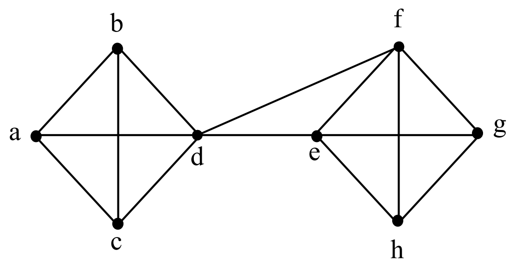
(c) G is traversable (Euler path exists in G)

(d) The graph G may or may not be connected

53. If G is a connected graph with 6 vertices and maximum number of edges, then which of the following is true?

(a) Euler path exists, but Euler circuit does not exist in G
(b) Only Euler path exists in G
(c) Euler circuit does not exists in G
(d) G is not traversable

54. For the graph G shown below



Vertex connectivity of G + Edge connectivity of G = _______.

55. If G is a simple graph with 20 vertices and 5 components, then maximum number of edges possible in G is ________.

56. If G is any graph having 10 vertices and $\delta(G) \geq 5$, then G is _________.

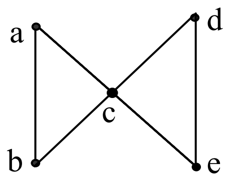
(a) Connected
(b) disconnected with two components
(c) disconnected with three components
(d) may or may not be connected

57. Let G is a simple graph with 4 components and these components have 5, 6, 7, 8 vertices respectively. The maximum number of edges possible in G is _______.

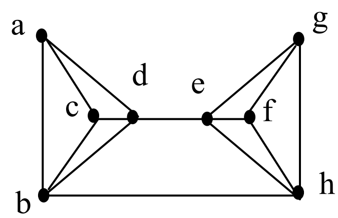
(a) 54 (b) 74
(c) 84 (d) 90

58. Number of cut sets, possible on a tree with 10 vertices is _______.

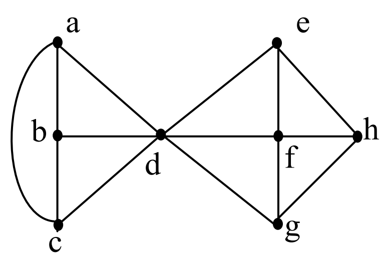
59. For the graph shown below, vertex connectivity is ______. and edge connectivity is _____.



60. For the graph shown below, vertex connectivity is _____ and edge connectivity is _____.



61. For the graph shown below, find vertex connectivity is _____and edge connectivity _____.



62. If G is a simple graph on *n* vertices then which of the following statements is true?

(a) Atleast one of G and its complement is connected.

(b) If G is connected then its complement is also connected.

(c) If G is connected then its complement is disconnected.

(d) If complement $\overline{G}$ is connected then G is disconnected.

63. Which of the following statements is/are true?

S_1: If a simple graph G is not connected then its complement $\overline{G}$ is connected .

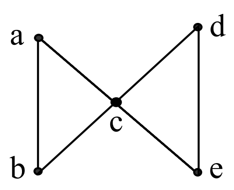
S_2: If a simple graph G is connected then its complement $\overline{G}$ is not connected

S_3: A simple graph with n vertices is necessarily connected if $\delta(G) = \frac{(n-1)}{2}$

S_4: If a simple graph has exactly two vertices of odd degree then there exists a path between the two vertices of odd degree.

(a) S_1, S_3 and S_4 (b) S_1, S_2 and S_3
(c) S_1, S_2 and S_4 (d) S_1, S_3 and S_2

64. For the graph shown below:



Which of the following statements is/are true?

(S_1) Euler path exists

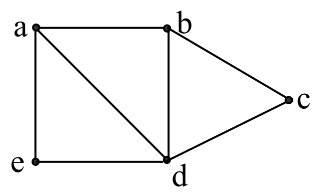
(S_2) Euler circuit exists

(S_3) Hamilltonian cycle exists

(S_4) Hamilltonian path exists

(a) S_1, S_3 and S_4 (b) S_1, S_2 and S_3
(c) S_1, S_2 and S_4 (d) S_1, S_3 and S_2

65. For the graph shown below



Which of the following statements is/are true?

(S_1) Euler path exists

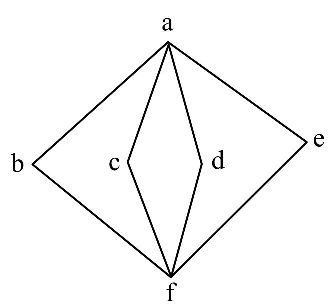
(S_2) Euler circuit exists

(S_3) Hamilltonian cycle exists

(S_4) Hamilltonian path exists

(a) S_1, S_3 and S_4 (b) S_1, S_2 and S_3
(c) S_1, S_2 and S_4 (d) S_1, S_3 and S_2

66. For the graph shown below



Which of the following statements is/are true?

(S_1) Euler path exists

(S_2) Euler circuit exists

(S_3) Hamilltonian cycle exists

(S_4) Hamilltonian path exists

(a) S_1, S_3 and S_4 (b) S_1 and S_2

(c) S_1, S_2 and S_4 (d) S_1, S_3 and S_2

67. Consider the following

S_1: If a connected graph G has a cut point, then G has a cut edge

S_2: If a connected graph G has a cut edge, then G has a cut point

Which of the following is ***true***?

(a) S_1 and S_2 are true

(b) S_1 and S_2 are false

(c) S_1 is true and S_2 is false

(d) S_1 is false and S_2 is true

68. Let G be a connected graph with 7 connected components and each component is a tree. If G has 26 edges then number of vertices in G is _________.

69. A forest is a disconnected graph in which each component is a tree. Let F be a forest on n vertices with k connected components. Then number of edges in G is _______.

(a) n – k + 1 (b) n – k – 1

(c) n – k (d) n + k

70. If a 2-regular graph G has a perfect matching then which of the following is/are ***true***?

(S_1) G is a cycle of even length

(S_2) Chromatic number of G is 2

(S_3) G is connected

(S_4) Every component of G is an even cycle

(a) S_1 and S_2 are true

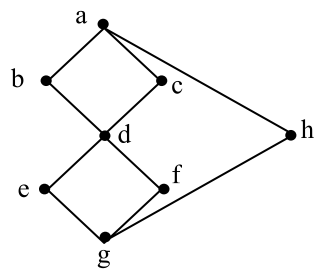
(b) S_2 and S_4 are true

(c) S_3 and S_4 are true

(d) S_1 and S_4 are true

71. If G is a simple disconnected graph with 12 vertices and 4 components, then maximum number of edges possible in G is ______.

72. For the graph shown below



Consider the following

S_1: Euler circuit exists

S_2: Euler path exists

S_3: Hamiltonian cycle exists

S_4: Hamiltonian path exists

Which of the following is **true**?

(a) S_2 and S_4 are true

(b) Only S_1 and S_3 are true

(c) Only S_4 is true

(d) Only S_2 is true

73. Consider the following statements:

S1: If a graph has a closed Eulerian walk, then it has an even number of edges.

S2: If G be a simple graph on 9 vertices, and the sum of all degrees in G is atleast 27, then G has a vertex of degree atleast four.

Which of the following is true?

(a) Only S1 is true

(b) Only S2 is true

(c) Both S1 and S2 are true

(d) Both S1 and S2 are false

74. Consider the following statements:

S1: If a simple graph has a closed Eulerian walk, then it has a Hamiltonian cycle.

S2: If a simple graph has a Hamiltonian cycle, then it has a closed Eulerian walk.

Which of the following is true?

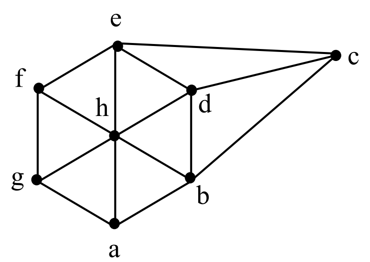
(a) Only S1 is true

(b) Only S2 is true

(c) Both S1 and S2 are true

(d) Both S1 and S2 are false

75. For the graph is shown below



Chromatic number of G + matching number of G = ________.

(a) 6 (b) 7

(c) 5 (d) 8

76. Consider the statements given below:

S_1. In a simple graph with 6 vertices, if degree of each vertex is 2, then the graph is connected.

S_2. In a simple graph G with 6 vertices, if degree of each vertex is 2, then Euler circuit exists in G.

S_3. In a simple graph G with 6 vertices, if degree of each vertex is 3 then the graph G is connected.

Which of the following is ***true*** ?

(a) only S_1 is true (b) S_1 and S_2 are true

(c) only S_3 is true (d) S_2 and S_3 are true

77. If G is a connected graph with 10 vertices and vertex connectivity 3, then minimum number of edges necessary in G is _______.

(a) 15 (b) 7 (c) 30 (d) 21

78. Let G = (V, E) be a simple non-empty connected undirected graph, in which every vertex has degree 4. For any partition V into two non-empty and non-overlapping subsets S and T. Which of the following is true?

(a) There are atleast two edges that have one end point in S and one end point in T

(b) There are atleast three edges that have one end point in S and one end point in T

(c) There are exactly two edges that have one end point in S and one end point in T

(d) There are exactly one edge that have one end point in S and one end point in T

79. Which of the following is ***not*** true?

(a) In a complete graph K_n ($n \geq 3$), Euler circuit exists $\Leftrightarrow$ n is odd

(b) In a complete bipartite graph $K_{m,n}$ ($m \geq 2$ and $n \geq 2$), Euler circuit exists $\Leftrightarrow$ m and n are even

(c) In a cycle graph C_n ($n \geq 3$), Euler circuit exists for all n

(d) In a wheel graph W_n ($n \geq 4$), Euler circuit exists $\Leftrightarrow$ n is even

80. Which of the following is ***not*** true?

(a) In a complete graph K_n ($n \geq 3$), Hamiltonian cycle exists for all n

(b) In a complete bipartite graph $K_{m,n}$ ($m \geq 2$ and $n \geq 2$), Hamiltonian cycle exists $\Leftrightarrow$ $m = n$

(c) In a cycle graph C_n ($n \geq 3$), Hamiltonian cycle exists for all n

(d) In a wheel graph W_n ($n \geq 4$), Hamiltonian cycle exists $\Leftrightarrow$ n is even

81. Which of the following is ***not*** true?

(a) Number of edge disjoint Hamiltonian cycles in K_7 is 3

(b) If G is a simple graph with 6 vertices and degree of each vertex is atleast 3, then Hamiltonian cycle exists in G

(c) Number of Hamiltonian cycles in $K_{4,4}$ = 72

(d) If G is a simple graph with 5 vertices and 7 edges, then Hamiltonian cycle exists in G

82. Consider a complete bi-partite graph K(10,10) where two partitions are A,B and each has 10 vertices. We add two more edges into this graph and obtain G(10,10). Consider the following statements about G(10,10).

Which of the following statements is/are necessarily true?

(a) G(10,10) is 3-colorable i.e. G(10,10) can be properly colored using 3 colors such that no two adjacent vertices have the same color.

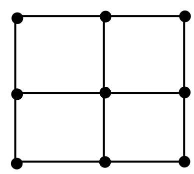
(b) G(10,10) has an independent set of size 9.

(c) G(10,10) has a vertex cover of size 11.

(d) The maximum size matching in G(10,10) is of size 10.

83. Which of the following is/are true ?

(a) The graph given below is bipartite



(b) A simple graph G with 13 vertices has 4 vertices of degree 3, 3 vertices of degree 4 and 6 vertices of degree 1. The graph G must be a tree:

(c) A spanning tree for a simple graph of order 24 has 23 edges

(d) The number of spanning trees in the complete graph K^8 is 8^6

04. Set Theory

Practice Questions

01. Let X be any set and let a, b $\notin$ X. If X has n subsets, and Y = X $\cup$ {a, b}, then number of subsets Y has

(a) 4n (b) n + 4

(c) 2n (d) 2^{n+2}

02. Let A, B and C be sets.

Consider the following:

S_1: If A $\in$ B and B $\subseteq$ C then A $\subseteq$ C

S_2: If A $\subseteq$ B and B $\in$ C then A $\subseteq$ C

Which of the following is true?

(a) S_1 is true and S_2 is false

(b) S_1 is false and S_2 is true

(c) Both S_1 and S_2 are true

(d) Both S_1 and S_2 are false

03. Consider the following:

S1: $A \cup (B - C) = (A \cup B) - (C - A)$

S2: $A \cap (B - C) = (A \cap B) - (A \cap C)$

Which of the following is true?

(a) Only S1 (b) Only S2

(c) Both S1 and S2

(d) Neither S1 nor S2

04. Suppose that S is a set with n elements. How many ordered pairs (A, B) are there such that A and B are subsets of S and A $\subseteq$ B?

(a) 2^n (b) 3^n

(c) n^2 (d) C(n, 2)

05. Let X = { 1, 2, 3, ……, 2n}. If A and B are sub sets of the set X and (A Δ B) denotes the set of all elements of X which belong to exactly one of A or B, then number of ways we can choose A and B so that (A Δ B) = {2, 4, 6, ….., 2n} is

(a) 2^n (b) 2^{n+1}

(c) 2^{2n-1} (d) 2^{2n}

06. Which of the following statements is not correct?

(a) If $A \oplus B = A$ then $B = \phi$

(b) $(A \oplus B) \oplus B = A$

(c) If $A \oplus C = B \oplus C$ then A = B

(d) $A \oplus B = (A \cup B) \cap (A - B)$

07. If A, B, C are any three sets, then which of the following is ***true***?

(a) If $(A \cap C) = (B \cap C)$ then A = B

(b) If $(A \cup C) = (B \cup C)$ then A = B

(c) If $(A \Delta C) = (B \Delta C)$ then A = B

(d) If $(A - C) = (B - C)$ then A = B

08. Let U = {1, 2, …., n}.

Let A = {(x, X) | x $\in$ X, X $\subseteq$ U}.

Consider the following two statements on |A|.

I. $|A| = n2^{n-1}$

II. $|A| = \sum_{k=1}^{n} k\binom{n}{k}$

Which of the above statements is/are true?

(a) Neither I nor II (b) Only II

(c) Both I and II (d) Only I

09. Let R be the following equivalence relation on the set A = {1, 2, 3, 4, 5, 6}:

R = {(1, 1), (1, 5), (2, 2), (2, 3), (2, 6), (3, 2), (3, 3), (3, 6), (4, 4), (5, 1), (5, 5), (6, 2), (6, 3), (6, 6)}.

The number of distinct equivalence classes on A with respect to R is _____.

10. If R is an anti-symmetric relation on a set A and S is any relation on A, then which of the following is ***not*** true?

(a) $(R \cap S)$ is always anti-symmetric

(b) $(R - S)$ is always anti-symmetric

(c) R^{-1} is always anti-symmetric

(d) $\overline{R}$ is always anti-symmetric

11. Let A = {1, 2, 3, 4}. A relation R on A is defined by R = {(1, 1), (2, 2), (2, 3), (3, 2), (4, 2), (4, 4)}, then number of ordered pairs in the transitive symmetric closure of R is ______.

12. Let D_{36} = set of all positive divisors of 36. A relation R on the set D_{36} is defined by

$a^{R}b \Leftrightarrow$ a is a divisor of b.

The number of edges in the Hasse diagram of the poset is _____.

13. Consider the following statements:

S1: If R is irreflexive, then the transitive closure of R is irrflexive.

S2: If R is symmetric, then the transitive closure of R is symmetric.

Which of the following is true?

(a) Only S1 is true

(b) Only S2 is true

(c) Both S1 and S2 are true

(d) Both S1 and S2 are false

14. Let R be the relation on the set of real numbers such that "xRy if and only if x and y are real numbers that differ by less than 1, that is $|x - y| < 1$". Which of the following is true?

(a) R is not an equivalence relation

(b) R is symmetric and transitive

(c) R is reflexive and transitive

(d) R is reflexive but not symmetric

15. Let S be a set with n elements. How many relations on S are symmetric, anti-symmetric and transitive?

(a) 2^n (b) $\frac{n(n-1)}{2}$

(c) 0 (d) 1

16. Let N be the set all positive integers, and R be a relation on N defined as follows:

Definition of R: For all a, b $\in$ N, (a, b)$\in$ R iff $\frac{a}{b} = 2^i$ for some integer $i \geq 0$.

Which of the following is ***not*** true?

(a) R is reflexive

(b) R is anti-symmetric

(c) R is transitive

(d) R is symmetric

17. Let A be a set having a total of n elements, where $n \geq 1$. Let R be any equivalence relation on A.

Consider the following statements:

S_1: The total number of ordered pairs in R is odd if n is odd

S_2: The total number of ordered pairs in R is even if n is even

Which of the following is true?

(a) Only S_1

(b) Only S_2

(c) Both S_1 and S_2

(d) Neither S_1 nor S_2

18. Let R and S be two relations both on A.

Consider the following statements:

S_1: If R and S are both reflexive, is $R \cup S$ also reflexive

S_2: If R and S are both symmetric, is $R \cup S$ also symmetric

S_3: If R and S are both transitive, is $R \cup S$ also transitive

Which of the following is true?

(a) Only S_1 and S_2 (b) Only S_2 and S_3

(c) Only S_1 and S_3 (d) S_1, S_2 and S_3

19. Let S = {1, 2, ..., 10}.

Define the following four sets as:

$P_1 = \{\{1, 3, 8\}, \{2, 4, 6\}, \{5, 7, 10\}, \{9\}\}$.

$P_2 = \{\{7, 4, 3, 8\}, \{1, 5, 10, 3\}, \{2, 6\}\}$.

$P_3 = \{\{1, 5, 9\}, \{2, 10, 4, 7\}, \{8, 3, 6\}\}$.

$P_4 = \{\{4, 2\}, \{3, 8\}, \{6\}, \{10, 7\}, \{1\}, \{5\}, \{9\}\}$.

Which of the sets above is a partition of S?

(a) P_1 (b) P_2

(c) P_3 (d) P_4

20. Let S = {1, 2, ..., 10}.

Define the following four sets as:

$P_1 = \{\{1, 3, 8\}, \{2, 4, 6\}, \{5, 7, 10\}, \{9\}\}$.

$P_2 = \{\{7, 4, 3, 8\}, \{1, 5, 10, 3\}, \{2, 6\}\}$.

$P_3 = \{\{1, 5, 9\}, \{2, 10, 4, 7\}, \{8, 3, 6\}\}$.

$P_4 = \{\{4, 2\}, \{3, 8\}, \{6\}, \{10, 7\}, \{1\}, \{5\}, \{9\}\}$.

Which of the following is true?

(a) P_4 is a refinement of both P_2 and P_3

(b) P_4 is a refinement of both P_1 and P_2

(c) P_4 is a refinement of both P_1, P_2 and P_3

(d) P_4 is a refinement of both P_1 and P_3

21. The set {1, 2, 3, 4, 5, 6} is partitioned as

$P\{\{1, 2, 3\}, \{4, 5\}, \{6\}\}$.

The number of refinements of P is _____.

22. Let R be a relation on a set A.

Consider the following statements.

S_1: R is anti-symmetric, if $(R \cap R^{-1}) \subseteq \Delta_A$ where Δ_A is diagonal relation on A.

S_2: R is transitive, if $(R \text{ o } R) \subseteq R$ where R o R is composite relation.

Which of the following is true?

(a) S_1 is true and S_2 is false

(b) S_1 is false and S_2 is true

(c) Both S_1 and S_2 are true

(d) Both S_1 and S_2 are false

23. Let R be the relation {(a, b) | a divides b} on the set of integers. The symmetric closure of R is

(a) {(a, b) | b divides a}

(b) {(a, b) | a divides b or b divides a}

(c) {(a, b) | a divides b}

(d) {(a, b) | a divides b and b divides a}

24. Let A = {1, 2, 3, 4, 5}, R = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4), (5, 5)} and S = {(1, 1), (2, 2), (3, 3), (4, 4), (4, 5), (5, 4), (5, 5)}. If R and S are equivalence relations then the partition corresponding to the smallest equivalence relation containing R and S is

(a) { {1}, {2}, {3, 4, 5} }

(b) { {1, 2}, {3, 4, 5} }

(c) { {1, 2}, {3}, {4, 5} }

(d) { {1, 3}, {2, 4, 5} }

25. Let R be an equivalence relation on a set A. For any two elements x, y $\in$ A, consider the following statements.

S_1: [x] = [y]

S_2: $[x] \cap [y] = \{\ \}$

where, [x] and [y] are equivalence classes of x and y respectively.

Which of the following is **true**?

(a) S_1 is true and S_2 is false

(b) S_1 is false and S_2 is true

(c) Both S_1 and S_2 are true

(d) Either S_1 or S_2 is true

26. Let A = {1, 2, 3}, then number of relations on A that are equivalence and a partial order at the same time is ______.

27. The relation R on set of all integers Z is defined by

"$x \overset{R}{\ } y$ iff $(x – y)$ an even integer".

Consider the following statements.

S_1: R is an equivalence relation

S_2: R is a partial order

Which of the following is true?

(a) S_1 is true and S_2 is false

(b) S_1 is false and S_2 is true

(c) Both S_1 and S_2 are true

(d) Both S_1 and S_2 are false

28. Let A = {1, 2, 3}. Number of relations on A which are neither reflexive nor irreflexive, but symmetric is _____.

29. Let A = {1, 2, 3} and Δ_A is the diagonal relation on A.

Consider the following statements.

S_1: Δ_A is a partial order

S_2: Δ_A is a linear order (total order)

S_3: Δ_A is an equivalence relation

Which of the following is **true**?

(a) S_1 and S_2

(b) S_2 and S_3

(c) S_1 and S_3

(d) S_1, S_2 and S_3

30. Let R and S are partial orders on a set A.

Consider the following statements.

S_1: $(R \cup S)$ is a partial order

S_2: $(R \cap S)$ is a partial order

Which of the following is **true**?

(a) S_1 is true and S_2 is false

(b) S_1 is false and S_2 is true

(c) Both S_1 and S_2 are true

(d) Either S_1 or S_2 is true

31. Let A = {1, 2}. Number of relations on A which are reflexive and symmetric, but not transitive is _______.

32. For the poset [{1, 2, 4, 6, 12}; R] where R is defined as $a^R b \Leftrightarrow$ a is a divisor of b. Which of the following is ***true***?

(a) The poset is not lattice

(b) The poset is a distributive lattice

(c) The poset is a complemented lattice

(d) The poset is a lattice but not bounded

33. Let A = {1, 2, 3}. A relation R on A × A is defined by

$$"(a, b) \, ^R \, (c, d) \Leftrightarrow (a \leq c \text{ and } b \leq d)"$$

Consider the following statements.

S_1: R is a partial order

S_2: The poset [A × A; R] is a lattice.

Which of the following is **true**?

(a) S_1 is true and S_2 is false

(b) S_1 is false and S_2 is true

(c) Both S_1 and S_2 are true

(d) Either S_1 or S_2 is true

34. The poset $[D_{36}; |\,]$ is a lattice.

Which of the following is true?

(a) Complement of 2 = 18

(b) Complement of 3 = 12

(c) Complement of 4 = 9

(d) Complement of 6 = 1

35. Let A = $\{2^n$ | n is a positive integer}. A relation R on A is defined by $a^R b \Leftrightarrow$ a is a divisor of b.

Then the set A with respect to R is _______.

(a) a poset but not a lattice

(b) a lattice but not a distributive lattice

(c) a distributive lattice but not a bounded lattice

(d) not a poset

36. Let A = {1, 2, 3, ...10} and X is the power set of A. X is a lattice with meet defined as set intersection and join defined as set union. If B is the set of all prime numbers in A, then the complement of B is

(a) {2, 3, 5, 7} (b) {1, 4, 6, 8, 9, 10}

(c) {1, 4, 6, 8} (d) {4, 6, 8, 10}

37. Recall that, if x is minimum element of S then x **is related to** y $\forall y \in S$. Let [S; R] be a partially ordered set. If every non-empty subset of S has a minimum element, then

(a) S is totally ordered set

(b) S is a bounded lattice

(c) S is a complemented lattice

(d) S is a Boolean algebra

38. Let A = {1, 2, 3, ..., 10}. A relation R on A is defined by

"a ^{R} b iff a is a divisor of b".

Number of edges in the Hasse diagram of the poset [A; R] is _______.

39. Let R be a partial ordering relation on the set $A = \{1, 2, 3, 4\}$ such that $R \cup R^{-1} = A \times A$.

Consider the following statements.

S₁: The poset [A; R] is a distributive lattice.

S₂: The poset [A; R] is a complemented lattice.

Which of the following is true?

(a) S_1 is true and S_2 is false

(b) S_1 is false and S_2 is true

(c) Both S_1 and S_2 are true

(d) Both S_1 and S_2 are false

40. Which of the following statements is/are true?

S_1: In a lattice L, if each element has atmost one complement, then L is a distributive lattice

S_2: A sub lattice of a complemented lattice is also complemented.

Which of the following is true?

(a) S_1 is true and S_2 is false

(b) S_1 is false and S_2 is true

(c) Both S_1 and S_2 are true

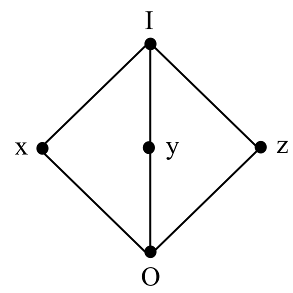
(d) Both S_1 and S_2 are false

41. In any lattice, $(x \wedge y) \vee y =$

(a) x (b) y

(c) $x \wedge y$ (d) $x \vee y$

42. In the lattice



Consider the following statements

S_1: $x \vee (y \wedge z) = (x \vee y) \wedge z$

S_2: $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$

Which of the following is true?

(a) only S_1 is true

(b) only S_2 is true

(c) both S_1 and S_2 are true

(d) both S_1 and S_2 are false

43. Which of the following is a bijection on set of all real numbers.

(a) $f(x) = x^2$ (b) $g(x) = |x|$

(c) $h(x) = \lfloor x \rfloor$ (d) $\phi(x) = x^3$

44. Let A, B and C are k element sets and let S be an n element set, where $k \le n$. How many triples of functions $f: A \to S$, $g: B \to S$ and $h : C \to S$ are there such that f, g and h are all injective and

$f(A) = g(B) = h(C) = ?$

(a) 3P(n, k). k! (b) $P(n, k). (k!)^3$

(c) $C(n, k). (k!)^3$ (d) 3 C(n, k). k!

45. Let A and B are any two sets such that $A \subseteq B$. Consider the following.

S_1: There exists an injection $f: A \to B$

S_2: There exists a surjection g: B $\rightarrow$ A
Which of the following is true?

(a) S_1 is true and S_2 is false

(b) S_1 is false and S_2 is true

(c) Both S_1 and S_2 are true

(d) Both S_1 and S_2 are false

46. Let A = {1, 3, 5, ..., 99} and B = {2, 4, 6, ... 100}. If the function f : A $\rightarrow$ B is one-to-one, then consider the statements

S_1: f is on-to function

S_2: f^{-1} exists

Which of the following is true?

(a) S_1 is true and S_2 is false

(b) S_1 is false and S_2 is true

(c) Both S_1 and S_2 are true

(d) Both S_1 and S_2 are false

47. Let f: X $\rightarrow$ Y be bijection. Let S and T are subsets of Y.

Consider the following

S1: $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$

S2: $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$

Which of the following is true?

(a) Only S1 (b) Only S2

(c) Both S1 and S2 (d) Neither S1 nor S2

48. Let f(x, y) = (2x – y, x – 2y) then $f^{-1}(x, y)$ =

(a) (2x–y , x–2y)

(b) $\left(\frac{2x-y}{2}, \frac{x-2y}{2}\right)$

(c) $\left(\frac{y-2x}{3}, \frac{x-2y}{3}\right)$

(d) $\left(\frac{2x-y}{3}, \frac{x-2y}{3}\right)$

49. Determine whether f is a function from the set of all bit strings to the set of all integers if

i) f(S) is the position of a 0 bit in S

ii) f(S) is the number of 1 bits in S

Which of the following is true?

(a) In case (i) f is a function and in case (ii) f is not a function

(b) In case (i) f is not a function and in case (ii) f is a function

(c) In both cases (i) and (ii), f is a function

(d) In both cases (i) and (ii), f is not a function

50. Consider the following functions from Z to Z

S_1: $f(x) = x^3$ is one-to-one but not on-to

S_2: $f(n) = \lceil n/2 \rceil$ is on-to but not one-to-one

Which of the following is true?

(a) S_1 is true and S_2 is true

(b) S_1 is true and S_2 is false

(c) S_1 is false and S_2 is true

(d) S_1 is false and S_2 is false

51. Let A = R–{3}, B=R-{1}. Let f : A$\rightarrow$B is defined by f(x) = (x – 2)/(x – 3), then which of the following is not true?

(a) f is a bijection

(b) f is one-to-one

(c) f is on-to

(d) one-to-one but not on-to

52. Let f and g are functions defined by
f(x) = x/(x+1) and g(x) = x/(1–x) then
$(fog)^{-1}x$ = ______.
(a) x (b) 1/x
(c) 1 (d) not defined

53. Which of the following is true?
(a) Every function can be represented graphically
(b) The functions f(x) = x and $g(x)=\sqrt{x^2}$ are identical
(c) The functions $f(x) = \log x^2$ and $g(x) = 2 \log x$ are identical
(d) The domain of $f(x)=\dfrac{1}{\sqrt{|x|-x}}$ is $(-\infty,0)$

54. Let f : A → B is a bijection then which of the following is false ?
(a) $f \circ f^{-1} = I_A$ (b) $f^{-1} \circ f = I_A$
(c) $I_B \circ f^{-1} = f$ (d) $f \circ I_A = f$

55. Let f: A → B and g: B → A are functions such that gof = I_A, where f is identity function.
S_1: f is one to one function
S_2: f is on-to function
Which of the following is true?
(a) S_1 is true and S_2 is false
(b) S_1 is false and S_2 is true
(c) Both S_1 and S_2 are true
(d) Both S_1 and S_2 are false

56. Consider the group G = {1, 2, 3, 4, 5, 6} under multiplication modulo 7.
Which of the following is ***not*** true?
(a) Order of the element 1 = 1
(b) Order of the element 2 = 3
(c) Order of the element 3 = 5
(d) Order of the element 4 = 3

57. Let S be a non empty set and P(S) is power set of S.
A binary operation * on P(S) is defined by
$A * B = A \oplus B$
= Symmetric difference of A and B
Now, P(S) with respect to * is ______.
(a) a semi group but not a monoid
(b) a monoid but not a group
(c) a group
(d) not a semi group

58. Let G be the set of all non-zero real numbers and let $a * b = \dfrac{ab}{2}$. If G is abelian group, then inverse of 4 is ______.

59. Let S denote set of all non-zero relational numbers. A binary operation * is defined by a* b = 2 a b. The set S is a group w.r.t.*.
The inverse of $\dfrac{2}{3}$ is
(a) $\dfrac{3}{2}$ (b) $\dfrac{3}{4}$
(c) $\dfrac{3}{8}$ (d) $\dfrac{3}{16}$

60. Let R be the set of all real numbers and * is a binary operation defined by
$a * b \equiv a + b + ab$.

Which of the following is false?

(a) Identity element is 0

(b) The inverse of –1 is 1

(c) The inverse of a is –a/(a + 1)

(d) R is not a group

61. Which of the following is ***not*** true?

(a) Every group of order 3 is abelain

(b) G = {1, –1} is an abelian group under multiplication

(c) In a group with 2 elements, each element is its own inverse

(d) G = {1, –1, i, –i} is not a group w.r.t. multiplication.

62. Which of the following is false?

(a) If (G, *) is a group and $a \in G$ such that $a * a = a$ then $a = e$

(b) If every element of a group is its own inverse, then the group must be abelian

(c) In a group of even order there will be atleast one element (other than identity element) which is its own inverse

(d) In a group of order 3 each element is its own inverse

63. The set G = {0, 1, 2, 3, 4, 5} is a group with respect to addition modulo 6.

Which of the following is false?

(a) The inverse of 2 is 4

(b) The inverse of 3 is 3

(c) The inverse of 5 is 2

(d) The inverse of 1 is 5

64. G ={1, –1, i, –i} is a group w.r.t multiplication. The order –i is

(a) 2 (b) 3 (c) 4 (d) 1

65. Which of the following is false?

(a) The union of any two sub groups of a group G is also a sub group of G

(b) The intersection of any two sub groups of a group G is also a sub group of G

(c) Let G = {0, ±k, ±2k, ±3k, ±4k, ….∞} where k is any fixed integer is a group w.r.t. addition.

(d) Every sub group of an abelian group is also an abelian group

66. Which of the following is false?

(a) every cyclic group is an abelian group

(b) if 'a' is generator of a cyclic group then a^{-1} is also a generator of G.

(c) the order of a cyclic group is equal to the order of its generating element.

(d) A sub group of a cyclic group need not be cyclic

67. Which of the following is false?

(a) Every group of prime order is cyclic.

(b) If (G, *) is a group of even order then there is an element a in G ($a \neq e$), such that $a * a = e$.

(c) Every finite group of order less than 6 must be abelian.

(d) The group ({1, 2, 3, 4, 5, 6}, $\otimes_7$) is cyclic with 2 as its generator.

68. If G = {0, 1, 2, 3, 4, 5} is a group w.r.t. $\oplus_6$ then which of the following is a subgroup of G?

(a) {0, 4} (b) {1, 5}

(c) {0, 2, 4} (d) {1, 3, 5}

69. If G = {1, 2, 3, ..., 10} is group w.r.t. $\otimes_{11}$, then number of generators in G = _____.

70. Let G = {2, 4, 6, 8} and the binary operation is $\otimes_{10}$. Which of the following is ***not*** true?

(a) G is a cyclic group

(b) G is a group with identity element 6

(c) G is a cyclic group with generators 2 and 8

(d) G is not a group

71. Let G = {a, b, c, d} and G is a group with respect to the binary operation *.

The incomplete composition table of the group (G, *) is shown below.

*	a	b	c	d
a	b	d	a	c
b	d	c	b	a
c	×	×	×	×
d	×	×	×	×

Which of the following is true?

(a) The identity element is c

(b) The inverse of a is b

(c) The inverse of b is d

(d) The inverse of d is d

72. Which of the following is true ?

(a) if m is an integer, then $\lfloor x \rfloor + \lfloor m - x \rfloor$ = m − 1 if x is not an integer.

(b) if m is an integer then $\lfloor x \rfloor + \lfloor m - x \rfloor$ = m, if x is an integer.

(c) Let A, B, and C be sets. Then (A−B)− C = (A − C) − (B − C).

(d) Let A, B, and C be sets. Then (A − B) − C = (A − B) − (B − C).

73. Which of these functions from Z to Z is Onto?

(a) f(n) = n – 1;

(b) $f(n) = n^2 + 1$;

(c) $f(n) = n^3$;

(d) $f(n) = \left\lfloor \frac{n}{2} \right\rfloor$

05. Probability&Statistics(Revised & Updated)

Practice Questions

01. Two points are selected at random in the interval $0 \le x \le 1$. Determine the probability that the sum of their squares is less than 1?

(a) $\pi/4$ (b) $\pi/16$

(c) $\pi/8$ (d) $\pi/6$

02. A sequence $<a_1, a_2, ..., a_n>$ is non-decreasing if $a_i \le a_j$ when $i \le j$. Suppose we generate a sequence $<a_1, a_2, ..., a_n>$ of n values from {0, 1, 2} uniformly at random. The probability that the sequence is non-decreasing is

(a) $\frac{n^2+3n+2}{2(3^n)}$ (b) $\frac{n^2+n}{2(3^n)}$

(c) $\frac{n^2-3n+2}{2(3^n)}$ (d) $\frac{n^2-n}{2(3^n)}$

03. Consider the following speedy but inaccurate algorithm for guessing the median of a set S, where $n = |S|$ is odd and $n \ge 3$. Choose a 3 element subset R of S uniformly at random, then return the median of R. What is the probability that the median of R is infact the median of S?

(a) $\frac{3(n+1)}{2n(n+2)}$ (b) $\frac{2(n-1)}{3n(n-2)}$

(c) $\frac{2(n+1)}{3n(n+2)}$ (d) $\frac{3(n-1)}{2n(n-2)}$

04. Suppose you flip a fair coin n times, where $n \ge 1$. What is the probability of the event that both of the following hold:

(i) the coin comes up heads atleast once and

(ii) once it comes up heads, it never comes up tails on any later flip?

(a) $n. 2^{-n}$ (b) $(n+1). 2^{-n}$

(c) $(n-1) . 2^{-n}$ (d) $(n+2). 2^{-n}$

05. Consider a biased coin that comes up heads with probability $\frac{1}{3}$ and tails with probability $\frac{2}{3}$. If the coin is tossed 2n times, then the probability that at some time during this experiment two consecutive coin flips come up both heads or both tails is

(a) $\frac{3^n-2^n}{3^{2n}}$ (b) $\frac{3^n-2^{n+1}}{3^{2n}}$

(c) $\frac{3^{n+1}-2^n}{3^{2n}}$ (d) $\frac{3^n+2^{n+1}}{3^{2n}}$

06. Three integers are chosen at random from the set{1, 2, 3, ..., 20} . The probability that their product is even is

(a) $\frac{2}{19}$ (b) $\frac{3}{29}$

(c) $\frac{17}{19}$ (d) $\frac{4}{19}$

07. If A & B are two events such that the conditional probability $P(A|B) = 1$, then $P(B^C|A^C)$ is

(a) $\frac{1}{4}$ (b) $\frac{1}{5}$

(c) 1 (d) $\frac{3}{4}$

08. The probability that a person will get an electric contract is $\frac{2}{5}$ and the probability that he will not get plumbing contract is $\frac{4}{7}$. If the probability of getting atleast one contract is $\frac{2}{3}$, then the probability that he will get both the contracts is

(a) $\frac{17}{105}$ (b) $\frac{3}{7}$

(c) $\frac{4}{7}$ (d) $\frac{5}{7}$

09. An integer is selected from the set {1, 2, 3, ..., 100}. Let A be the event that this number is divisible by 3 and B is the event that the integer is divisible by 7. Consider the following statements:

S_1: A and B are mutually inclusive

S_2: A and B are independent

Which of the following is true?

(a) only S_1 is true

(b) only S_2 is true

(c) both S_1 and S_2 are true

(d) both S_1 and S_2 are false

10. Suppose we want to pick two numbers from {1, 2, ..., 100} randomly. The probability that sum of the two numbers is divisible by 5 is ____.

11. In a housing society, half of the families have a single child per family, while the remaining half have two children per family. The probability that a child picked at random, has a sibling is ______.

12. When you throw three fair dice, probability that the sum of the numbers on the top faces is 10 is _____.

13. Let A and B be disjoint events in a sample space. If A and B are independent then

(a) Pr(A) = 0 or Pr(B) = 0

(b) Pr(A) = 0 and Pr(B) = 0

(c) Pr(A) = 0 or Pr(B) = 1

(d) Pr(A) = Pr(B)

14. Let a random variable X denote the outcome of throwing a fair die. The variance of X = ____.

15. A bag has

one counter marked as 1,

two counters marked as 4,

three counters marked as 9,

....

n counters marked as n^2.

If you draw one counter and are paid the amount shown on it (in Rs), then your expectation is ___.

(a) n^2 (b) $\frac{n(n-1)}{2}$

(c) $\frac{n(n+1)}{2}$ (d) $\frac{(n-1)(n-2)}{2}$

16. A women's health clinic has four doctors and each patient is assigned to one of them. If a patient gives birth between 8 am and 4 pm, then her chance of being attended by her assigned doctor is $\frac{3}{4}$, otherwise it is $\frac{1}{4}$. What is the probability that a patient is attended by the assigned doctor when she gives birth?

(a) $\frac{25}{144}$ (b) $\frac{5}{12}$

(c) $\frac{7}{12}$ (d) $\frac{1}{12}$

17. Let X be a random variable having density function $f(x)=\begin{cases} cx & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$

$P(X > 1) =$ ______.

18. Suppose a pair of six sided fair dice were tossed. Let X and Y be the numbers appearing on the dice. If $Z = |X - Y|$, then expectation of Z is _____.

19. Flip n independent fair coins, and let X be a random variable that counts how many coins come up heads. Let a be the constant. The expectation of a^x is

(a) $\left(\frac{a-1}{2}\right)^n$ (b) $\left(\frac{a+1}{3}\right)^n$

(c) $\left(\frac{a+1}{2}\right)^n$ (d) $\left(\frac{a-1}{3}\right)^n$

20. If X is a random variable with mean 1 and variance 5, then $E[(2 + X)^2]$ is

(a) 4 (b) 8

(c) 12 (d) 14

21. Suppose that the random variable X has possible values 1, 2, 3, ……. and that

$P(X = x) = K(1-\beta)^{x-1}, \quad 0 < \beta < 1.$

Now the constant K= __________.

(a) β (b) $1 - \beta$

(c) 1 (d) -1

22. Let X be a random variable with the following probability distribution.

x	-3	6	9
P(X = x)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Expectation of $(2X + 1)^2$ is ______.

23. You throw a fair die. If an odd number turns up you get that number of rupees. If an even number occurs you pay that number of rupees. What is the value of your expectation in rupees for each throw

(a) 0 (b) 1

(c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

24. Let V be the wind velocity (mph) and suppose that V is uniformly distributed over the interval [0, 10]. The pressure, say W (in *lb/ft*^{2}), on the surface of an aeroplane

wing is given by the relationship: $W = 0.003 V^2$. The expected value of W is ____ *lb/ft*^{2}.

25. Let X be any random variable with expected value μ and standard deviation σ. Then, for any constant $k > 1$,

$$\Pr(\mu - k\sigma < X < \mu + k\sigma) \geq \_\_\_\_\_.$$

(a) $\frac{1}{k^2}$ (b) $1-\frac{1}{k^2}$

(c) $\frac{k^2}{k^2+1}$ (d) $1-\frac{1}{k^3}$

26. You throw two dice. If the sum of the faces is even, you win that many rupees, if odd you lose that many rupees. Your expectation in rupees is _____.

27. Throwing two dice, you win $|x - y|$ rupees, where x and y are the face numbers. Your expected amount in rupees is ______.

(a) $\frac{25}{36}$ (b) $\frac{25}{18}$

(c) $\frac{35}{18}$ (d) $\frac{15}{9}$

28. A binary source generates digits 1 and 0 randomly with probabilities 0.6 and 0.4 respectively. What is the probability that 2 ones and 3 zeros will occur in a 5-digit sequence?

29. A random variable X follows binomial distribution with
mean = 2(variance) and

mean + variance = 3.
$P(X = 3) =$ ______.

30. In an experiment, positive and negative values are equally likely to occur. The probability of obtaining at most one negative value in five trials is

(a) $\frac{1}{32}$ (b) $\frac{2}{32}$

(c) $\frac{3}{32}$ (d) $\frac{6}{32}$

31. If the probability of winning a tennis game is p, then the probability that after six games of tennis you lead you opponent four games to two is

(a) $15(p^4 - 2p^5 - p^6)$ (b) $15(p^4 + 2p^5 + p^6)$

(c) $15(p^4 + 2p^5 - p^6)$ (d) $15(p^4 - 2p^5 + p^6)$

32. Over a large set of inputs a program runs twice as often as it aborts. The probability that of the next 6 attempts, 4 or more will run is ______.

33. A telephone exchange receives an average of 180 calls per hour. The probability that it will receive only two calls in a given minute is _____.

34. On the average, 15 out-of-state cars pass a certain point on a road per hour. The probability that exactly four out-of-state cars pass that point in a 12 minute period is ______.

35. If X has uniform distribution in the interval form 0 to 10, then $P\left\{\left(X+\frac{10}{X}\right)\geq 7\right\}$ is ______.

36. Suppose the average waiting time for a customer's call to be answered by a company representative is five minutes. The probability that a call is answered during the first minute is

(a) 0.1813 (b) 0.1718

(c) 0.1638 (d) 0.1931

37. Suppose that the random variable X has possible values 1, 2, 3, ……. and that

$P(X = r) = k(1-\beta)^{r-1}, 0 < \beta < 1.$

The mode of the distribution is ______.

(a) 1 (b) 2

(c) 3 (d) 4

38. For the sample 27, 35, 40, 35, 36, 36, 29, find mean, median, mode and standard deviation.

39. A discrete random variable X takes value from 1 to 5 with probabilities as shown in the table.

k	1	2	3	4	5
P(X = k)	0.1	0.2	0.4	0.2	0.1

Find mean, median, mode and standard deviation.

40. If the probability density function of a random variable X is given by

$$f(x)=\begin{cases} kx(1-x), & 0\leq x\leq 1\\ 0, & \text{otherwise}\end{cases}$$

then find k, mean, median, mode and standard deviation.

41. A random variable X has normal distribution with mean 100. If $P(100 < X < 120) = 0.3$, then $P(X < 80) =$ ______.

42. A certain type of missile hits its target with probability 0.3. The minimum number of missiles that should be fired so that probability of hitting the target is greater than 75% is _____.

43. 2000 cashew nuts are thoroughly mixed in flour. The entire mixture is divided into 1000 equal parts and each part is used to make a biscuit. Assume that no cashew is broken in the process. A biscuit is picked at random. The probability that it contains atleast one cashew nut is_________.

44. An urn contains 3 red marbles and 7 white marbles. A marble is drawn from the urn and a marble of the other colour is then put into the urn. A second marble is drawn from the urn. If both marbles were of the same colour. What is the probability that they were both red?

(a) 5/6 (b) 1/8

(c) 3/8 (d) 9/10

45. Three machines A, B and C produce respectively 50%, 30% and 20% of total number of items of a factory. The percentages of defective output of these machines are 3%, 4% and 5% respectively. If an item is selected at random and is found to be defective then the probability that it is produced by machine C is

(a) 15/37 (b) 16/37

(c) 14/34 (d) 10/37

46. If P and Q are two random events, ten which of the following are false?

(a) Independence of P and Q implies that probability $(P \cap Q) = 0$

(b) Probability $(P \cap Q)$= probability (p) + probability (Q)

(c) If P and Q are naturally exclusive then they must be independent.

(d) Probability $((P \cap Q) \leq$ probability (p)

05. Linear Algebra(Revised & Updated)

Practice Questions

01. Let $A = \begin{bmatrix} 3 & P & P \\ P & 3 & P \\ P & P & 3 \end{bmatrix}$. If rank of A is 1, then

P = ______.

02. The maximum value of the determinant among all 2×2 real symmetric matrices with trace 10 is _____.

03. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = A^{-1}$, then the element in the second row and third column of B = ____.

(a) 0

(b) $\frac{1}{2}$

(c) $-\frac{1}{2}$

(d) 1

04. Suppose that $A_{n\times n}$ is upper triangular matrix such that $a_{ii} = 0$, i = 1, 2, ..., n. Then rank of A^n = _______.

(a) 0 (b) n – 1

(c) 1 (d) n

05. If $\text{adj } A = \begin{bmatrix} -18 & -11 & -10 \\ 2 & 14 & -4 \\ 4 & 5 & -8 \end{bmatrix}$, then absolute value of determinant of A = ______.

06. If A and B are symmetric matrices of order 3 × 3, then consider the following:

S_1: AB = BA

S_2: AB – BA is singular

Which of the following is true?

(a) S_1 is true and S_2 is false

(b) S_1 is false and S_2 is true

(c) Both S_1 and S_2 are true

(d) Both S_1 and S_2 are false

07. The number of n × n symmetric matrices possible with the entries chosen from the set {0, 1, 2, ..., q – 1} is

(a) $q^{n\left(\frac{n+1}{2}\right)}$ (b) $q^{n\left(\frac{n-1}{2}\right)}$

(c) $(q-1)^{(n^2)}$ (d) $q^{(n^2)}$

08. Let A be a square matrix of order n – 1. The elements of A are defined by

$$a_{ij} = \begin{cases} n-1 & \text{for } i=j \\ -1 & \text{for } i \neq j \end{cases}.$$

The determinant of A = ______.

(a) n^{n-1} (b) n^{n-2}

(c) $(n-1)^{n-2}$ (d) $(n-1)^{n-3}$

09. Consider the following statements:

S1: If x_1, x_2, x_3, x_4 is a linearly independent sequence of vectors, then the sequence x_1, x_2, x_3 is linearly independent.

S2: If x_1, x_2, x_3, x_4 is a linearly dependent sequence of vectors, then the sequence x_1, x_2, x_3 is linearly dependent.

Which of the following is true?

(a) Only S1 (b) Only S2

(c) Both S1 and S2 (d) Neither S1 nor S2

10. Let $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ where a, b, c are non-zero real numbers. Then Rank of A =

(a) 0 (b) 1

(c) 2 (d) 3

11. Consider the linear equations

$x - 2y + z = 3,$

$2x + \alpha z = -2,$

$-2x + 2y + \alpha z = 1.$

In order to have unique solution to this linear system of equations the value of α should not be equal to

(a) $\frac{-2}{3}$ (b) $\frac{2}{3}$

(c) $\frac{4}{3}$ (d) $\frac{-4}{3}$

12. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 4 & 3 & 10 \end{bmatrix}$, then which of the following is ***not*** true?

(a) Rank of A = 2

(b) The system AX = O has infinitely many solutions, where $X = [x\ y\ z]^T$

(c) The system AX = B has a unique solution

(d) A^{-1} does not exist

13. Consider the system $\begin{cases} kx + y + z = 1 \\ x + ky + z = 1 \\ x + y + kz = 1 \end{cases}$.

If the system has a unique solution then which of the following is true?

(a) k = 1 and k = –2

(b) $k \neq 1$ and $k \neq -2$

(c) $k = 1$ and $k \neq -2$

(d) $k \neq 1$ and $k = -2$

14. Let AX = B be a system of three equations in three variables x, y and z. The augmented matrix of the system is given by

$[A \mid B] = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$.

Which of the following is ***not*** true?

(a) Rank of [A | B] = 2

(b) The system AX = B has infinitely many solutions

(c) The system AX = O has unique solution

(d) Rank of A = 2

15. The following system of equations

$$x + y + z = 3,$$
$$x + 2y + 3z = 4,$$
$$x + 4y + kz = 6$$

has infinitely many solutions when

(a) $k \neq 0$ (b) $k = 0$

(c) $k = 7$ (d) $k \neq 7$

16. Consider the matrix $A = \begin{bmatrix} k & k & k \\ 0 & k-1 & k-1 \\ 0 & 0 & k^2-1 \end{bmatrix}$.

If the system AX = O has only one independent solution then k = ______.

17. Let AX = B be a system of three equations in three variables x, y and z. If A has three linearly independent columns and B is a linear combination of the columns of A, then which of the following is true?

(a) The system has unique solution

(b) The system has infinitely many solutions

(c) The system has no solution

(d) The system AX = O has non-zero solution

18. Consider the following system of equations:

$3x + 2y = 1$, $4x + 7z = 1$,

$x + y + z = 3$, $x - 2y + 7z = 0$,

Which of the following is true?

(a) The system has no solution

(b) The system has infinitely many solutions

(c) The system has unique solution

(d) The rank of the augmented matrix of the system is 2

19. If the matrix $A_{3\times3}$ has 3 distinct eigen values, then which of the following is true?

(a) A^{-1} exists

(b) Rank of A = 3

(c) A has 3 linearly independent eigen rows

(d) A has 3 linearly independent eigen vectors

20. If $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ then the eigen values of A are

(a) 8, 7, 3

(b) 0, 3, 15

(c) 1, 2, 3

(d) 1, –1, 2

21. The distinct eigen values of

$A = \begin{bmatrix} 1 & 1 & \cdots\cdots & 1 \\ 1 & 1 & \cdots\cdots & 1 \\ \vdots & & & \\ 1 & 1 & \cdots\cdots & 1 \end{bmatrix}_{n\times n}$ are

(a) 0, n (b) 1, n – 1

(c) 0, n – 1 (d) 1, n

22. If 1, 2 and 3 are the eigen values of the matrix $A_{3\times3}$, then $6A^{-1} =$

(a) $A^2 + 6A - 11\,I$

(b) $A^2 - 6A - 11\,I$

(c) $A^2 - 6A + 11\,I$

(d) $A^2 + 6A + 11\,I$

23. An eigen vector of the matrix $\begin{bmatrix} 10 & -4 \\ 18 & -12 \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 \\ 9 \end{bmatrix}$

(c) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$

24. An eigen vector corresponding to the smallest eigen value of the matrix $\begin{bmatrix} 1 & 1 & -1 & 2 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is

(a) $[1\ \ 0\ \ 0\ \ 0]^T$ (b) $[1\ \ 1\ \ 0\ \ 0]^T$

(c) $[1\ \ 0\ \ 2\ \ 0]^T$ (d) $[-1\ \ -1\ \ 2\ \ 2]^T$

25. Let $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & 6 \\ -1 & -2 & 0 \end{bmatrix}$. If – 3 and –3 are two eigen values of A then the eigen vector corresponding to the third eigen value is

(a) $[1\ \ 2\ \ 1]^T$ (b) $[1\ \ 2\ \ -1]^T$

(c) $[1\ \ -2\ \ 1]^T$ (d) $[-1\ \ 2\ \ 1]^T$

26. If $[2\ \ -2\ \ 1]^T$ is an eigen vector of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & x & -4 \\ 2 & -4 & 3 \end{bmatrix}$, then x = ____.

27. Consider the matrix $A = \begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{pmatrix}$ whose eigen values are 1, –1 and 3.

Then trace of $(A^4 - 3A^3)$ is ________.

28. Let $A = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ and

$B = A^3 - A^2 - 4A + 6I$,

where I is the 3 × 3 identity matrix.

The determinant of B is _______.

29. If $(1, 2, 0)^T$ is an eigen vector of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$, then the corresponding eigen value is _____.

30. If A is a skew-symmetric matrix of order n, then number of linearly independent eigen vectors of $(A + A^T)$ = _____.

(a) 0 (b) 1

(c) n – 1 (d) n

31. Let A be a non-zero upper triangular matrix all of whose eigen values are zero.

Then (I + A) is

(a) invertible

(b) singular

(c) symmetric

(d) skew-symmetric

32. If the characteristic polynomial of a 3×3 matrix M over R (the set of real numbers) is $\lambda^3 - 12\lambda^2 + a\lambda - 32$. $a \in R$, and one eigen value of M is 2, then the largest among the absolute values of the eigen values of M is ________.

33. Suppose $A = \begin{bmatrix} 1 & 2 & -3 \\ -3 & -4 & 13 \\ 2 & 1 & -5 \end{bmatrix}$. If A = L.U where L is lower triangular matrix with diagonal elements as unity, then U = ______ and L = _______.

(a) $\begin{bmatrix} 1 & 2 & -3 \\ 0 & -2 & 4 \\ 0 & 0 & 7 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 2 & \frac{-3}{2} & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & -3 \\ 0 & 2 & 4 \\ 0 & 0 & 7 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 2 & \frac{-3}{2} & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 & -3 \\ 0 & 2 & 4 \\ 0 & 0 & -7 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & \frac{3}{2} & 1 \end{bmatrix}$

(d) $\begin{bmatrix} -1 & 2 & -3 \\ 0 & 2 & 4 \\ 0 & 0 & 7 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & \frac{3}{2} & 1 \end{bmatrix}$

34. Consider the following systems of linear equations:

$$5x_1 - x_2 + 2x_3 = 9$$
$$-2x_1 + 6x_2 + 9x_3 = 9$$
$$-7x_1 + 5x_2 - 2x_3 = 1$$

Which of the following is true?

(a) The system is consistent and has infinite number of solutions.

(b) The system is inconsistent.

(c) The following system is consistent and has unique solution.

(d) The system is consistent and $(x_1,x_2,x_3) = (3, 4, -1)$ is a solution.

35. Which of the following matrices is in row-echelon form?

(a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 5 \\ 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

07. Calculus

Practice Questions

01. $\underset{x\to 1}{\text{Lt}}\left\{\left(\frac{1+x}{2+x}\right)^{\left(\frac{1-\sqrt{x}}{1-x}\right)}\right\} =$ _______.

02. The current in a coil containing a resistance R, and inductance, L, and a constant electromotive force, E, at time *t* is given by $i = \frac{E}{R}\left(1-e^{-Rt/L}\right)$. The formula for estimating *i* when R is very close to 0 is

(a) $\frac{Et}{L}$ (b) $\frac{-Et}{L}$

(c) $\frac{Et}{2}$ (d) $\frac{E-t}{2L}$

03. Let f(x) = $\frac{\sqrt{2a^3x - x^4} - a\left(a^2x\right)^{\frac{1}{3}}}{a - \left(ax^3\right)^{\frac{1}{4}}}$,

where a > 0. $\underset{x\to a}{\text{Lt}}\, f(x) =$

(a) $\frac{4a}{3}$ (b) $\frac{8a}{3}$

(c) $\frac{16a}{9}$ (d) 0

04. Which of the following functions is differentiable in the domain [–1, 1]?

(a) $f(x) = |x|$

(b) $f(x) = \cot x$

(c) $f(x) = \sec x$

(d) $f(x) = \operatorname{cosec} x$

05. If $f(x) = \begin{cases} x, & x \le 1 \\ 2x-1, & \text{when } x > 1 \end{cases}$

then at x = 1 which of the following is **true**?

(a) f(x) is continuous but not differentiate

(b) f(x) is continuous and differentiable

(c) f(x) is neither continuous nor differentiable

(d) f(x) is differentiable but not continuous

06. Let $f(x) = \begin{cases} x^2 & \text{if } x \le 2 \\ mx+b & \text{if } x > 2 \end{cases}$.

If f(x) is differentiable every where then

(a) m = 4 and b = –4

(b) m = 4 and b = 4

(c) m = –4 and b = –4

(d) m = –4 and b = 4

07. The number C that satisfy the conclusion of mean value theorem for f(x) = x + (4/x) in the interval [1, 8] is

(a) 4.5 (b) 3.5

(c) $2\sqrt{2}$ (d) 5

08. A function f(x) = $3x^2 + 4x - 5$ is defined over an open interval (1, 3). At least at one point in this interval, $f'(x)$ is exactly

(a) 16 (b) 8

(c) 2 (d) 4

09. The value C of Cauchy's mean value theorem for f(x) = e^x and g(x) = e^{-x} in the interval (2, 3) is _______.

10. The mean value c of Cauchy's theorem for the functions f(x) = $\frac{1}{x}$ and g(x) = $\frac{1}{x^2}$ in the interval [2, 3] is

(a) 2.4 (b) 2.5

(c) 2.6 (d) 2.8

11. The function f(x) = cosh x + cos x has _______.

(a) a minimum at x = 0

(b) a maximum at x = 0

(c) neither maximum nor minimum at x = 0

(d) a minimum at x = $\frac{\pi}{2}$

12. The area bounded by the curve $y = x^4 - 2x^3 + x^2 + 3$, the x-axis and two ordinates corresponding to the points of minimum of this function is

(a) $\frac{1}{30}$ (b) $\frac{91}{30}$

(c) $\frac{-758}{480}$ (d) $\frac{728}{480}$

13. $\int_0^{\frac{\pi}{4}} \frac{\sin 2x}{\cos^4 x + \sin^4 x} dx$ = ______.

14. The area enclosed by the loop of the curve $y^2 = x (x - 1)^2$ is _____.

15. The area of region bounded by the parabola $y^2 = 4x$ and the line y = 2x – 4 is ________.

16. The value of the integral $\int_0^1 \frac{x^\alpha - 1}{\log x} dx, (\alpha > 0)$ is_______.

(a) log(1 – α) (b) log α

(c) log(1 + α) (d) –log α

17. If x sin(πx) = $\int_0^{x^2} f(t)dt$, where f is a continuous function, then f(4) = ______.

(a) $\frac{\pi}{2}$ (b) $\frac{\pi^2}{4}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi^2}{2}$

18. $\int_0^1 x^m (\log x)^n \ dx$ (m > 0 and n is a natural number) is equal to

(a) $\frac{n!}{(m+1)^{n+1}}$ (b) $\frac{(-1)^n n+1!}{(m+1)^{n+1}}$

(c) $\frac{(-1)^n \ n\,!}{(m+1)^{n+1}}$ (d) $\frac{(-1)^n \ n\,!}{(m-1)^{n+1}}$

19. The value of the integral $\int_{-\infty}^{\infty} \frac{dx}{\left(1 + a^2 + x^2\right)^{\frac{3}{2}}}$ is

(a) $\frac{1}{1+a^2}$ (b) 0

(c) $\frac{2}{1+a^2}$ (d) $\frac{1}{\left(1+a^2\right)^{\frac{3}{2}}}$

20. The value of the integral $\int_{-\infty}^{0} e^{x+e^x}\,dx$ is

(a) e
(b) e^{-1}
(c) e + 1
(d) e – 1

21. The value of the integral $\int_{0}^{\pi} x\sin^2 x\,dx$ is

(a) $\frac{\pi^2}{4}$
(b) $\frac{\pi^2}{2}$
(c) $\frac{\pi^2}{8}$
(d) 0

22. Let $g(x) = \begin{cases} -x, & x \le 1 \\ x+1, & x \ge 1 \end{cases}$ and

$$f(x) = \begin{cases} 1-x, & x \le 0 \\ x^2, & x > 0 \end{cases}.$$

Consider the composition of f and g, i.e., $(f \circ g)(x) = f(g(x))$.

The number of discontinuities in $(f \circ g)(x)$ present in the interval $(-\infty, 0)$ is

(a) 0
(b) 1
(c) 2
(d) 4

23. The value of $\lim_{x\to\infty} (1+x^2)^{e^{-x}}$ is

(a) 0
(b) $\frac{1}{2}$
(c) 1
(d) ∞

24. If f(x) = x(x – 1)(x –2), then mean value c of Rolle's theorem in the interval [1, 2] is

(a) 1.577
(b) 1.377
(c) 0.577
(d) 0.423

25. $\underset{x\to 0}{Lt} \dfrac{\int_0^{x^2} \sin\sqrt{x}\,dx}{x^3} =$ ______.

(a) $\frac{1}{2}$
(b) $\frac{2}{3}$
(c) $\frac{1}{3}$
(d) $\frac{4}{3}$

26. The minimum value of the function $f(x) = x^3 - 3x^2 - 24x + 100$ in the interval [–3, 3] is

(a) 20
(b) 28
(c) 16
(d) 32

27. The maximum area (in square units) of a rectangle whose vertices lie on the ellipse $x^2 + 4y^2 = 1$ is ______.

28. Let f:[–1, 1] → R, where $f(x) = 2x^3 - x^4 - 10$. The minimum value of f(x) is _________.

29. The range of values of k for which the function $f(x) = (k^2 - 4)x^2 + 6x^3 + 8x^4$ has a local maxima at point x = 0 is

(a) k < –2 or k > 2
(b) k ≤ –2 or k ≥ 2
(c) –2 < k < 2
(d) –2 ≤ k ≤ 2

30. If $f(x) = \int_0^x \frac{\sin t}{t}\,dt$ (x > 0), then f(x) has _______.

(a) a maximum at x = nπ, where n is even
(b) a minimum at x = nπ, where n is odd
(c) a maximum at x = nπ, where n is odd
(d) a minimum at $x = \frac{n\pi}{2}$, where n is odd

31. For the function

$$f(x)=\frac{50}{3x^4+8x^3-18x^2+60}.$$

Which of the following is ***not*** true?

(a) f(x) has a local maximum at $x = -3$

(b) f(x) has a local minimum at $x = 0$

(c) f(x) has a local maximum at $x = 1$

(d) f(x) has a local maximum at $x = 0$

32. $\int_0^{\pi} x \ \sin^4 x \ \cos^6 x \ dx =$

(a) $3\pi^2 / 512$ (b) $5\pi^2 / 256$

(c) $3\pi^2 / 128$ (d) none of these

33. If $\int_0^{2\pi} |x \sin x| \, dx = k\pi$, then the value of

k = _______.

34. If f(x) = x|x| then f(x) at x = 0 is _________

(a) Continuous

(b) Not continuous

(c) Differentia

(d) Not differentia