Laws (or) Properties (or) Logical Equivalences:

	Rules	Name
1	$p \vee p \cong p$	Idempotent law
	$p \wedge p \cong p$	
2	$p \vee q \cong q \vee p$	Commutative law
	$p \wedge q \equiv q \wedge p$	
3	$p \vee (q \vee r) \equiv (p \vee q) \vee r$	Associative law
	$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$	
4	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	Distributive Law
	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	
5	$\sim (p \vee q) \equiv \sim p \wedge \sim q$	De Morgan's Law
	$\sim (p \wedge q) \equiv \sim p \vee \sim q$	
6	$p \rightarrow q \equiv \sim p \vee q$	Implication law

Laws (or) Properties (or) Logical Equivalences:

	Rules	Name
7	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$	Bi-implication law $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$
8	$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption law
9	$p \wedge t \equiv p$ $p \vee f \equiv p$	Identity law
10	$\sim(\sim p) \equiv p$	Double Negation (or) Involutary law
11	$p \vee \sim p \equiv t$ $p \wedge \sim p \equiv f$	Negation law (or) Complements (or) Inverse law
12	$p \vee t \equiv t$ $p \wedge f \equiv f$	Domination law

Eg:

Q. The propositional function $p \vee (q \vee \sim p)$ is

a) tautology

b) contradiction

c) contingency

d) $p \wedge q$

Q. The propositional function $(p \rightarrow q) \leftrightarrow (p \wedge \sim q)$ is

a) tautology

b) contradiction

c) contingency

d) None

Q. If A and B are two propositions and '*' is a binary operator defined as

A	B	A * B
T	T	T
T	F	T
F	T	F
F	F	T

Then the compound proposition $(A \wedge B) \equiv$

a) $\sim A * B$

b) $\sim A * \sim B$

c) $\sim (A * \sim B)$

d) $\sim (\sim A * B)$

Q. The proposition $p \wedge (\sim p \vee q)$ is GATE – 93

(a) a tautology

(b) $\Leftrightarrow (p \wedge q)$

(c) $\Leftrightarrow (p \vee q)$

(d) a contradiction

$$p \wedge (\sim p \vee q) \equiv (p \wedge \sim p) \vee (p \wedge q)$$

$$\equiv f \vee (p \wedge q)$$

$$\equiv p \wedge q$$

$$\Leftrightarrow p \wedge q$$

Q. $\boxed{p \to (q \to r)} \Leftrightarrow \cong$

a) $(p \wedge q) \to \sim r$

b) $(p \vee q) \to r$

c) $(p \vee q) \to \sim r$

d) $(p \wedge q) \to r$

$p \to (q \to r) \cong p \to (\sim q \vee r)$

$\cong \sim p \vee (\sim q \vee r)$

$\cong (\sim p \vee \sim q) \vee r$

$\cong \sim (p \wedge q) \vee r.$

$\cong (p \wedge q) \to r$ $\quad \sim x \vee y \equiv x \to y$

Q. Which of the following is/are tautology? **GATE - 92**

(a) $(a \vee b) \rightarrow (b \wedge c)$

(b) $(a \wedge b) \rightarrow (b \vee c)$

(c) $(a \vee b) \rightarrow (b \rightarrow c)$

(d) $\overbrace{(a \rightarrow b)}^{T} \rightarrow \overbrace{(b \rightarrow c)}^{F}$

$(a \rightarrow T) \rightarrow (T \rightarrow F)$

$T \rightarrow F$

F

NOT a tautology

(b)

$\overbrace{(a \wedge b)}^{\text{True}} \longrightarrow \overbrace{(b \vee c)}^{\text{False}}$

$T \wedge T$

$(T \wedge T) \longrightarrow (T \vee T/F)$

$T \longrightarrow T = \text{True} = \text{Tautology}$

(a) $\overbrace{(a \vee b)}^{\text{True}} \longrightarrow \overbrace{(b \wedge c)}^{\text{False}}$

$T \vee F$

$(T \vee F) \longrightarrow (F \wedge T/F)$

$T \longrightarrow F = \text{False}$

$= \text{NOT a Tautology}$

ACE

Q. If the proposition $\sim p \Rightarrow q$ is true, then the truth value of the proposition $\sim p \vee (p \Rightarrow q)$, where '$\sim$' is negation, '$\vee$' is inclusive or and '$\Rightarrow$' is implication is

GATE – 95

(a) true

~~(b)~~ multiple-valued

(c) ~~(b)~~ false

(d) cannot be determined

$\sim p \rightarrow q = \text{True}$

$\sim(\sim p) \vee q = \text{True}$

$\boxed{p \vee q = \text{TRUE}}$

$\sim p \vee (p \rightarrow q)$

$\equiv \sim p \vee (\sim p \vee q)$

$\equiv (\sim p \vee \sim p) \vee q$

$\equiv \boxed{\sim p \vee q}$

$p \vee q = \text{True}$

$T \vee F$ ✓

$F \vee T$

$T \vee T$

$\sim p \vee q$

$F \vee F$

p	q	$p \vee q$	$\sim p \vee q$
T	T	T	$\boxed{T}$
T	F	T	$\boxed{F}$
F	T	T	T

when $p \vee q$ = True then

$\sim p \vee q$ canbe True (or) False

so we cannot determine one truth value

Q. Let p, q and r be propositions and the expression $(p \rightarrow q) \rightarrow r$ be a contradiction. Then, the expression $(r \rightarrow p) \rightarrow q$ is **(GATE-17-Set1)**

MCQ

(a) a tautology

(b) a contradiction

(c) always TRUE when p is FALSE

(d) always TRUE when q is TRUE

$(r \rightarrow p) \rightarrow q$

$T/F \rightarrow T = \text{True}$

$(p \to q) \to r$ = contradiction

$\underbrace{(p \to q)}_{\text{True}} \to \underbrace{r}_{\text{False}} = f$

$T \to T$ ✓ $\quad F$

$F \to F$ ✓ $\quad F$

$F \to T$ ✓ $\quad F$

* $(r \to p) \to q$

$(F \to T) \to T$ ✓ = True ✓

$(F \to F) \to F$ = False

$(F \to F) \to T$ ✓ = True ✓

Q. Choose the correct choice(s) regarding the following propositional logic assertions:

$S: \left[((P \wedge Q) \rightarrow R)\right] \rightarrow \left[((P \wedge Q) \rightarrow (Q \rightarrow R))\right]$ **(GATE-21-Set2)**

(a) S is a tautology

(b) The antecedent of S is logically equivalent to the consequence of S

(c) S is a contradiction

(d) S is neither a tautology nor a contradiction

$$\underbrace{[(P\wedge Q)\rightarrow R]}_{\text{antecedent}} \longrightarrow \underbrace{[(P\wedge Q)\rightarrow (Q\rightarrow R)]}_{\text{Consequent}}$$

Consequent : $(P\wedge Q)\rightarrow (Q\rightarrow R)$

$$\equiv (P\wedge Q)\rightarrow (\sim Q\vee R)$$

$$\equiv \sim (P\wedge Q)\vee (\sim Q\vee R)$$

$$\equiv (\sim P\vee \sim Q)\vee (\sim Q\vee R)$$

$$\equiv \sim P\vee (\sim Q\vee \sim Q)\vee R$$

$$\equiv \sim P\vee \sim Q\ \vee R$$

$$\equiv \sim (P\wedge Q)\vee R$$

$$\equiv (P\wedge Q)\rightarrow R$$

$\equiv$ antecedent ✓

Tautology

$$X\longrightarrow Y$$

X	Y
T	T
F	F

Q. "If X then Y unless Z" is represented by which of the following formulas in propositional logic? ("~" is negation, "∧" is conjunction, and "→" is implication) **GATE – 2002**

(a) $(X \wedge \sim Z) \rightarrow Y$

(b) $(X \wedge Y) \rightarrow \sim Z$

(C) $X \rightarrow (Y \wedge \sim Z)$

(d) $(X \rightarrow Y) \wedge \sim Z$

If x then y unless z

If x is TRUE and z is False then y will be TRUE

$\Rightarrow$ If x is TRUE and z is False

$= (X \wedge \sim Z) \rightarrow Y$

Q. Determine whether each of the following is a tautology a contradiction, or neither ("$\vee$" is disjunction, "$\wedge$" is conjunction. "$\rightarrow$" is implication, '$\sim$' is negation and "$\leftrightarrow$" is biconditional (if and only if). **GATE – 2002**

(i) $A \leftrightarrow (A \vee A)$

(ii) $(A \vee B) \rightarrow B$

(iii) $A \wedge (\sim (A \vee B))$

① $A \leftrightarrow (A \vee A)$

$A \leftrightarrow A$

$T \leftrightarrow T = t$

$F \leftrightarrow F = t$

Tautology

② $(A \vee B) \longrightarrow B$

$T \vee T \longrightarrow T = T$ ✓

$T \vee F \longrightarrow F = F$ ✓

Contingency

(or)

$(A \vee B) \longrightarrow B$

$\sim (A \vee B) \vee B$

$\equiv (\sim A \wedge \sim B) \vee B$

$\equiv (\sim A \vee B) \wedge (\sim B \vee B)$

$\equiv (\sim A \vee B) \wedge t$

$\equiv \sim A \vee B$

$=$ Contingency

③ $A \wedge (\sim A \wedge \sim B)$

$\equiv (A \wedge \sim A) \wedge \sim B$

$\equiv f \wedge \sim B = f$ contradiction

Q. (a) Show that the formula **GATE - 99**

$[(\sim p \vee q) \Rightarrow (q \Rightarrow p)]$ is not a tautology.

(b) Let A be tautology and B be any other formula, Prove that $(A \vee B)$ is a tautology.

(a) $[(\sim p \vee q) \rightarrow (q \rightarrow p)]$

$\overbrace{(p \rightarrow q)}^{T} \rightarrow \overbrace{(q \rightarrow p)}^{F}$

$T \rightarrow F$

$(F \rightarrow T) \longrightarrow (T \rightarrow F)$

$T \longrightarrow F$ = False.
Not a tautology

(b) A = Tautology = True = t

$A \vee B$

$T \vee T/F$ = True
= Tautology

Q. Let a, b, c, d be propositions. Assume that the equivalences $a \leftrightarrow (b \vee \sim b)$ and $b \leftrightarrow c$ hold. Then the truth value of the formulae $(a \wedge b) \rightarrow ((a \wedge c) \vee d)$ is always **GATE – 2000**

(a) True

(b) False

(C) Same as truth value of b

(d) Same as truth value of d

$X \cong Y$

$X \leftrightarrow Y$

Given equivalences

i) $a \leftrightarrow (b \vee \sim b)$

$a \cong b \vee \sim b$

$a \cong$ True

ii) $b \leftrightarrow c$

$\boxed{b \cong c}$

$(a \wedge b) \rightarrow [(a \wedge c) \vee d]$

$(a \wedge b) \rightarrow [(a \wedge b) \vee d]$

$\underbrace{(a \wedge b)}_{x} \longrightarrow [\underbrace{(a \wedge b)}_{x} \vee d]$

$x \longrightarrow x \vee d$

$T \longrightarrow T \vee d = T$

$=$ Tautology

$$\overbrace{X}^{\text{True}} \longrightarrow \overbrace{Y}^{\text{False.}}$$

$$T \longrightarrow T$$

$$F \longrightarrow T/F$$