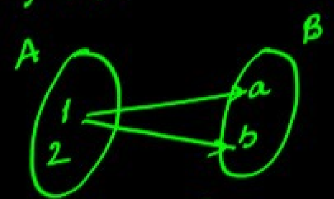
Functions

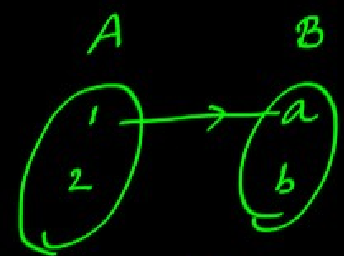
Functions: A relation is said to be a function, If every element of its domain has unique order pair

$f : A \longrightarrow B$



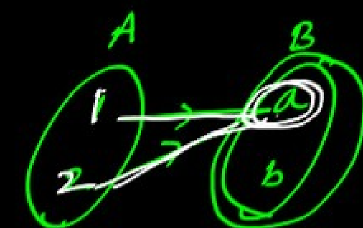
$f = \{(1,a),(1,b)\}$

NOT function



$f = \{(1,a)\}$

NOT function



$f = \{(1,a),(2,a)\}$

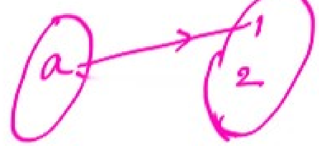
functions

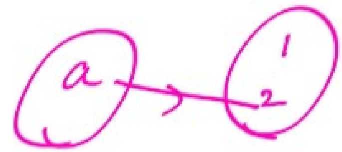
Domain $f = A = \{1,2\}$

Co domain $f = B = \{a,b\}$ ✓

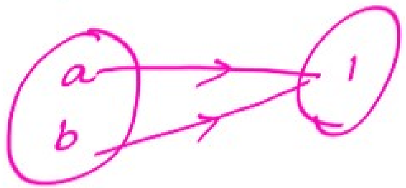
Range $f = f(A) = f\{1,2\} = \{a\}$

i) No. of function 1-element set to 2-element = 2



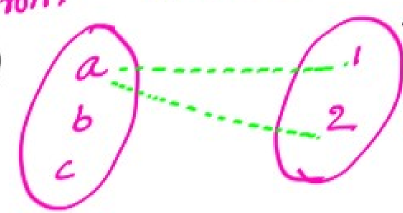


ii) No. of function from 2-element to one element set = 1



iii) No. of function from 3-element set to 2-element set $= 2^3 = 8 = n^m$

m



n

$$\frac{2}{a} * \frac{2}{b} * \frac{2}{c} = 2^3$$

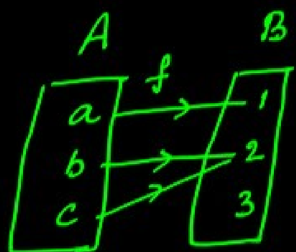
NOTE

If $|A| = m$ and $|B| = n$ Then the possible number of functions from A to B $= n^m$

Classification of Functions:

i) One-one (Injective):

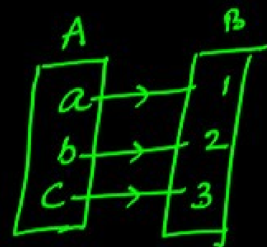
A function in which different elements of domain have different images in co-domain, is known as one-one function.



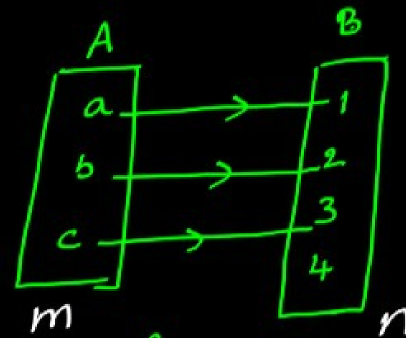
$f(b)=2$

$f(c)=2$

It is a function,
But NOT one-one



f_1



f_2

f_1, f_2 are one-one functions

$\boxed{m \leq n}$ ✓

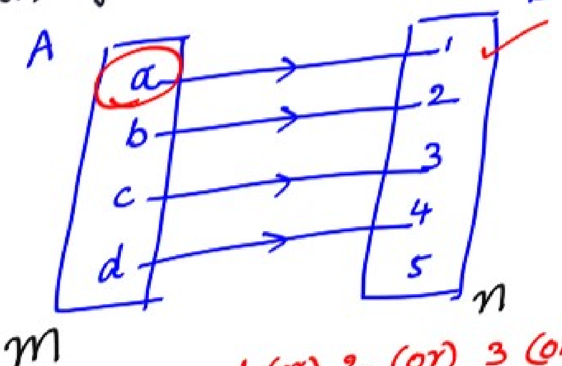
(or)

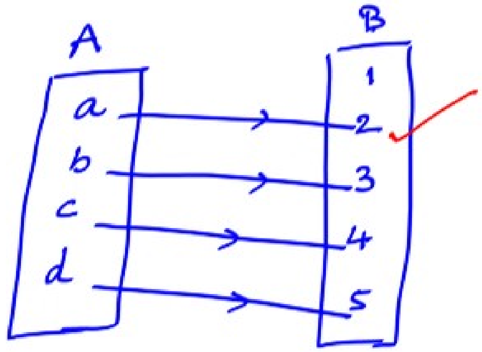
$n \geq m$

Q

Let $A = \{a, b, c, d\}$ and $B = \{1, 2, 3, 4, 5\}$ Find possible No. of one-one function from A to B

Sol





$a \rightarrow 1 \text{ (or) } 2 \text{ (or) } 3 \text{ (or) } 4 \text{ (or) } 5$

$$\underline{5} * \underline{4} * \underline{3} * \underline{2} = {}^5P_4$$

$${}^nP_r = \frac{n!}{(n-r)!} \quad ; \quad {}^nC_r = \frac{n!}{r!\,(n-r)!}$$

$|A| = m, \quad |B| = n$

$A \rightarrow B = {}^nP_m$

$(n \geq m)$

No. of one-one functions from m-element set to n-element $= {}^nP_m$

Q: Let $n(A) = 3$ and $n(B) = 5$ then Find

i) No. of elements in $A \times B = 3 \times 5 = 15$

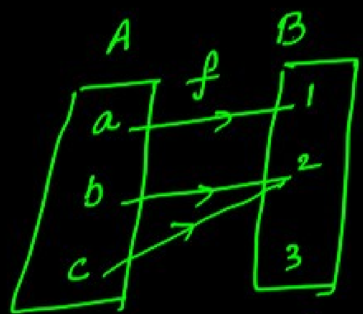
ii) No. of relation from A to B $= 2^{m \times n} = 2^{3 \times 5} = 2^{15} = 32768$

iii) No. of functions from A to B $= n^m = 5^3 = 125$

iv) No. of one-one functions from A to B $= {}^nP_m = {}^5P_3 = 60$

ii) Onto (Surjective):-

A function in which its range is same as co-domain is known as onto function.

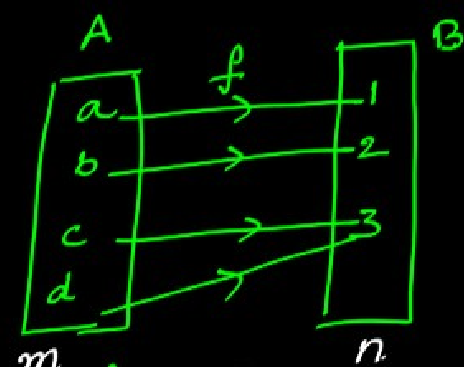


Range $f = f(A) = \{1,2\}$

Co-domain $= B = \{1,2,3\}$

Range $\neq$ co-domain

NOT onto



$f(A) = B$

onto

$\boxed{m \geq n}$

* Number of onto functions from m-element set to n-element set

$$= n^m - {}^nC_1.(n-1)^m + {}^nC_2.(n-2)^m - {}^nC_3.(n-3)^m + \ldots\ldots\ldots + {}^nC_{n-1}(1)^m$$

$$= n^m - nC_1 (n-1)^m + nC_2 (n-2)^m - nC_3 . (n-3)^m + \ldots\ldots$$

Q. Find the no.of surjective functions from $\{1, 2, 3, 4\}$ to $\{a, b, c\}$

$m = 4, \quad n = 3$

No. of Surjective functions from m-element set to n-element set

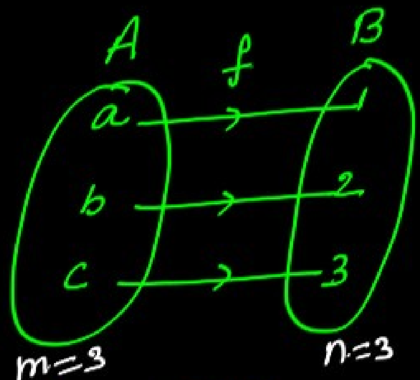
$$= n^m - {}^nC_1 \cdot (n-1)^m + {}^nC_2 \cdot (n-2)^m - {}^nC_3 \cdot (n-3)^m + \cdots$$

$$= 3^4 - {}^3C_1 \cdot (3-1)^4 + {}^3C_2 \cdot (3-2)^4$$

$$= 81 - 48 + 3$$

$$= 36$$

Bijection: A function which is both one-one and onto is called as a Bijective function.



f is one-one

f is also onto

Hence f is Bijective

$\boxed{m=n}$

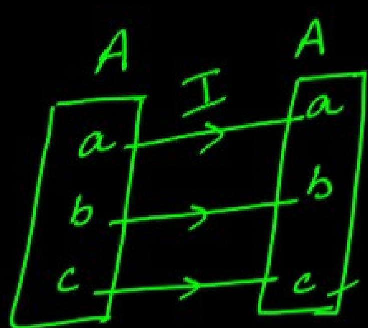
No.of Bijective functions on n element set $(m=n)$

$$= {}^{n}P_{m} = {}^{n}P_{n} = n!$$

Identity:

A function $I : A \to A$ is called an identity function

iff $\boxed{I(x) = x, \forall x \in A}$



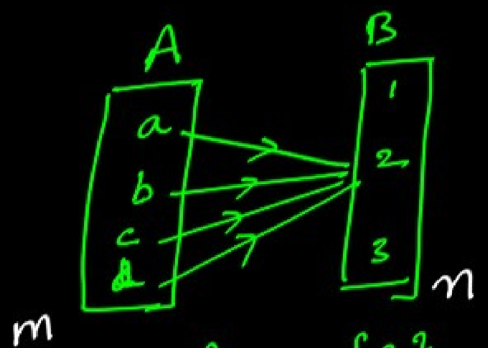
Is It one-one ? yes

Is It onto ? yes

Is It Bijective ? yes.

* Every Identity function is Bijective

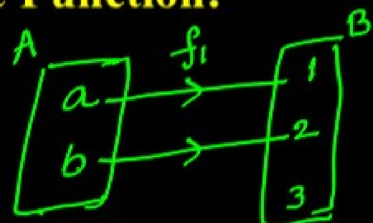
Constant Function: A function in which range is a singleton set is known as constant function.



$$\text{Range} = f(A) = \{2\}$$
$$= \text{Singleton.}$$

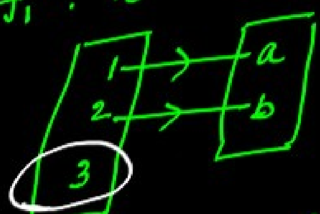
Inverse Function:

(1)



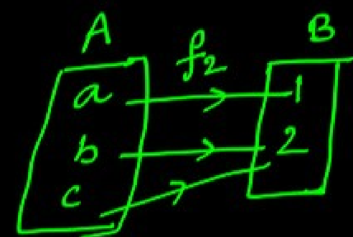
$f_1 = \{(a,1),(b,2)\}$

$f_1^{-1} : B \longrightarrow A$



$f_1^{-1} = \{(1,a),(2,b)\}$

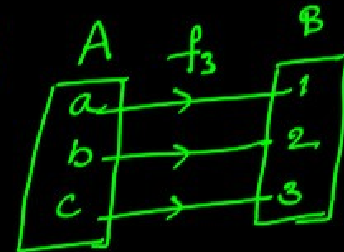
(2)



$f_2 = \{(a,1),(b,2),(c,2)\}$

$f_2^{-1} : B \rightarrow A = \{(1,a),(2,b),(2,c)\}$

(3)



$f_3 = \{(a,1),(b,2),(c,3)\}$

$f_3^{-1} = \{(1,a),(2,b),(3,c)\}$

f_1 is one-one fun, But f_1^{-1} is not a function

f_2 is onto fun, But f_2^{-1} is not a function

* f_3 is Bijective, then f_3^{-1} is also fun (Bijective).

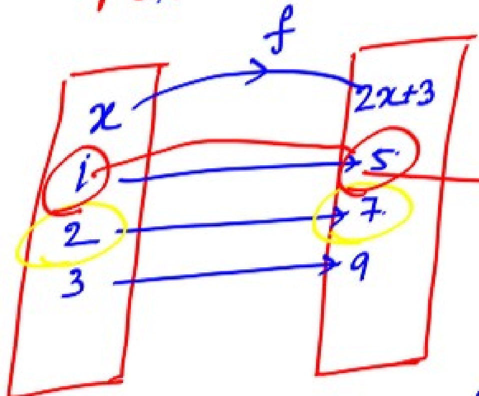
f is Invertable $\Longleftrightarrow$ f is Bijective.

[Inverse fun exists]

① If $f(x) = 2x+3$, $x \in N$, then find inverse of f

If $f(x) = y$ then $x = f^{-1}(y) \longrightarrow$ ①

Sol $f(x) = 2x+3$



Here $f(2) = 7 \implies f^{-1}(7) = 2$

$f(x) = y$

$2x+3 = y$

$x = \frac{y-3}{2}$

$f^{-1}(y) = \frac{y-3}{2}$

put $y = x$

$$\boxed{f^{-1}(x) = \frac{x-3}{2}}$$

$f^{-1}(x) = \frac{x-3}{2}$

$f^{-1}(5) = \frac{5-3}{2} = 1$

② $f(x) = 3x+5$

$f(x) = X$

$3x+5 = X$

$x = \frac{X-5}{3}$

$f^{-1}(x) = \frac{x-5}{3}$ ✓

③ $f(x) = 5x-6$

$5x-6 = X$

$x = \frac{X+6}{5}$

④ $f(x) = 3x^2+2 = X$

$= \sqrt{\frac{X-2}{3}}$

⑤ $f(x) = \frac{x+5}{6} = X$

$6X-5$

⑥ $f(x) = 3\sqrt{x}+5 = X$

$\left(\frac{X-5}{3}\right)^2$

Composite Function:

Q. Suppose X and Y are sets and |X| and |Y| are their respective cardinalities. It is given that there are exactly ~~97~~ 64 functions from X to Y. From this one can conclude that **(GATE-96)**

a) $|X| = 1, |Y| = 97$ ✓

b) $|X| = 97, |Y| = 1$

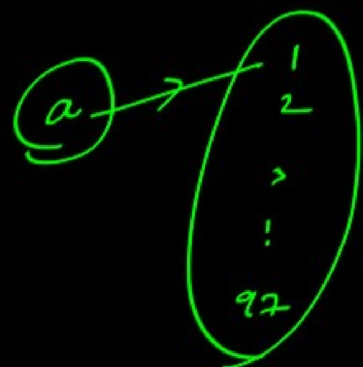
c) $|X| = 97, |Y| = 97$

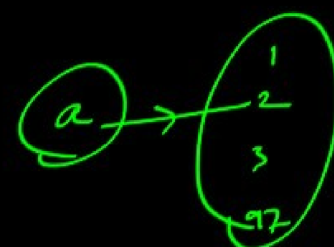
d) None of the above

No. of functions from X to Y $= 97 = (97)^1 = n^m$

$\therefore n = 97 = |Y|$

$m = 1 = |X|$





97 functions $\longrightarrow$ 64 functions

$$64 = (64)^1 = (8)^2 = (4)^3 = 2^6 = n^m$$

Q. Let $f : A \to B$ be a function and let E and F be subsets of A. Consider the following statements about images. **(GATE-01)**

S1: $f(E \cup F) = f(E) \cup f(F)$

S2: $f(E \cap F) = f(E) \cap f(F)$

Which of the following is true about S1 and S2?

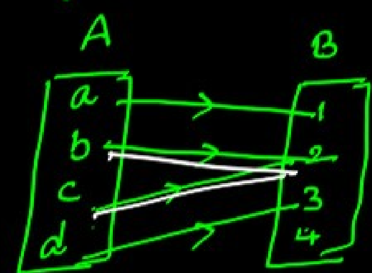
a) Only S1 is correct

b) Only S2 is correct

c) Both S1 and S2 are correct

d) None of S1 and S2 is correct

$f : A \to B$



$E \subseteq A, \; F \subseteq A$

$E = \{a, b\}, \; F = \{b, c\}$ ✓

$E = \{a, b\} \; F = \{c, d\}$ ✓

Q. Let R and S be any two equivalence relations on a non-empty set A. which one of the following statements is TRUE? **(GATE-05)**

a) $R \cup S$, $R \cap S$ are both equivalence relations

b) $R \cup S$ is an equivalence relation

c) $R \cap S$ is an equivalence relation

d) Neither $R \cup S$ nor $R \cap S$ is an equivalence relation

Q. Consider the set S= {a, b, c, d}: consider the following 4 partitions π_1, π_2, π_3, π_4 on S: $\pi_1 = \{\overline{abcd}\}$, $\pi_2 = \{\overline{ab}, \overline{cd}\}$, $\pi_3 = \{\overline{abc}, \overline{d}\}$, $\pi_4 = \{\overline{a}, \overline{b}, \overline{c}, \overline{d}\}$, Let $\prec$ be the partial order on the set of partitions $S = \{\pi_1, \pi_2, \pi_3, \pi_4\}$ defined as follows: $\pi_i \prec \pi_j$ if and only if π_i refines π_j. The poset diagram for $(S, \prec)$ is **(GATE-07)**

$\pi_1 = \{ \overline{abcd} \}$

$= \{ \{a, b, c, d\} \}$

$\pi_2 = \{ \overline{ab}, \overline{cd} \}$

$= \{ \{a, b\}, \{c, d\} \}$

all π_i are subset to any π_j

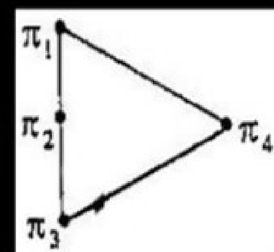
Every $\pi_i \in$ one element π_j

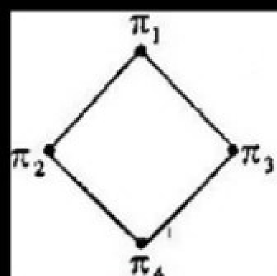
$A \longleftrightarrow A$

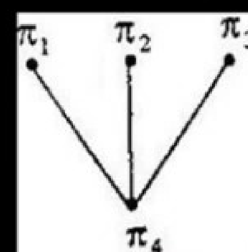
$A = \{1, 2, 3, 4\}$

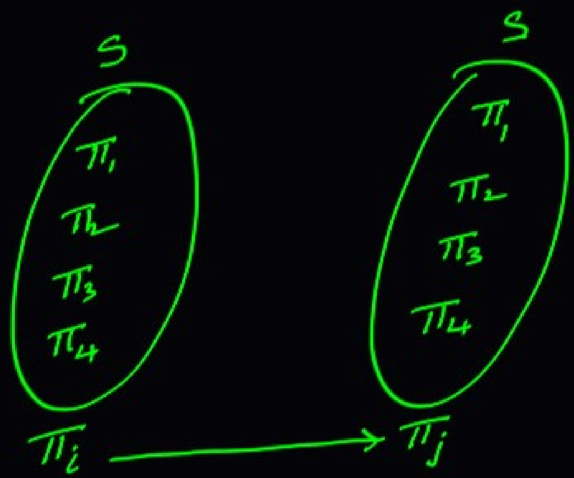
$\{\pi_1, \pi_2, \pi_3, \pi_4\}$

a)


b)


c)


d)




π_i refines π_j

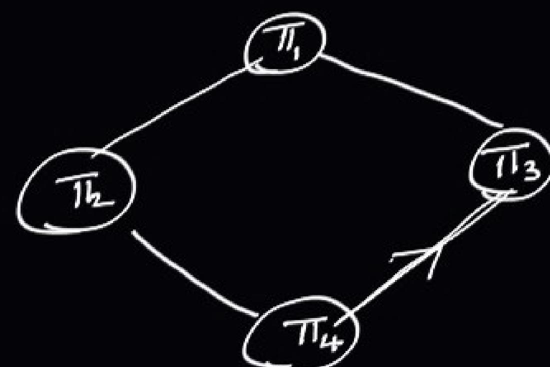
Every element of π_i is subset of one of elements π_j

$\pi_1 = \{\{a,b,c,d\}\}$

$\pi_2 = \{\{a,b\}, \{c,d\}\}$

$\pi_3 = \{\{a,b,c\}, \{d\}\}$

$\pi_4 = \{\{a\}, \{b\}, \{c\}, \{d\}\}$



π_2 π_1

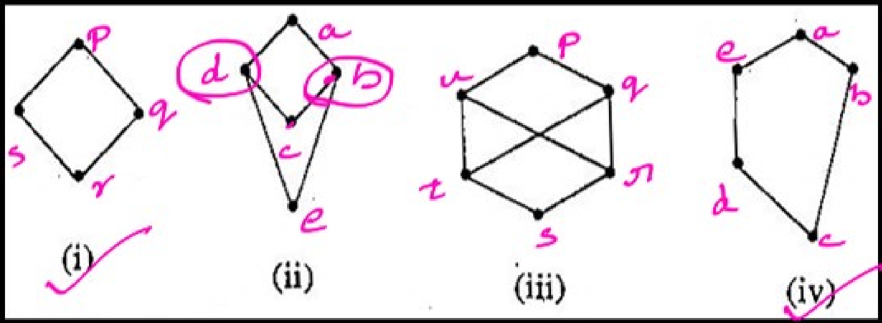
$\{a,b\} \subseteq \{a,b,c,d\}$

$\{c,d\} \subseteq \{a,b,c,d\}$

π_2 refines π_1

π_4 refines π_3

Q. Consider the following Hasse diagrams. (GATE-07)



Which all of the above represent a lattice?

a) i & iv only

b) ii & iii only

c) iii only

d) i, ii & iv only

Structure (ii)

$UB(c,e) = d, b, a$

$LUB(c,e) = c \vee e = ?$

$LB(d,b) = c, e$

$GLB(d,b) = c \wedge e = ?$

Structure iii

$t \vee r = ?$

$u \wedge q = ?$

Q. Let R be a relation on the set of ordered pairs of positive integers such that $((p, q), (r, s)) \in R$. if and only if $p - s = q - r$. Which one of the following is true about R? **(GATE-15-Set3)**

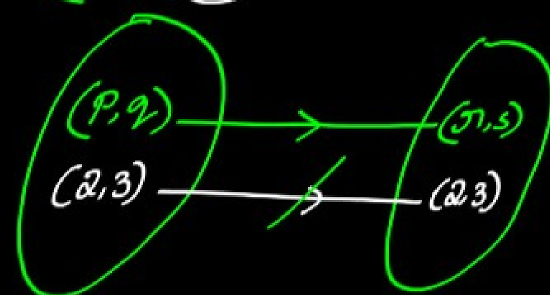
a) Both reflexive and symmetric

b) Reflexive but not symmetric

c) Not reflexive but symmetric

d) Neither reflexive nor symmetric

$((p,q),(r,s)) \in R \iff p-s = q-r$



$(2,3) \not R (2,3) \iff 2-3 \neq 3-2$

$-1 \neq 1$

$\therefore$ NOT reflexive

Symmetric $xRy \Rightarrow yRx$

let $(p,q) R (r,s) \Rightarrow p-s = q-r$

$\Rightarrow s-p = r-q$

$\Rightarrow r-q = s-p$

$(r,s) R (p,q)$ Symmetric

$r-q = s-p$

$(r,s) R (p,q)$

example

$$\big((4, 2), (1, 3)\big) \in R$$

$$\Rightarrow \big((1, 3), (4, 2)\big) \in R$$

Q. A binary relation R on N × N is defined as follows: (a, b) R (c, d) if $a \le c$ or $b \le d$. Consider the following propositions: **(GATE-16-Set2)**

P: R is reflexive

Q: R is transitive

Which one of the following statements is TRUE?

a) Both P and Q are true.

b) P is true and Q is false.

c) P is false and Q is true.

d) Both P and Q are false.

(3,4) (4,1) (2,3)

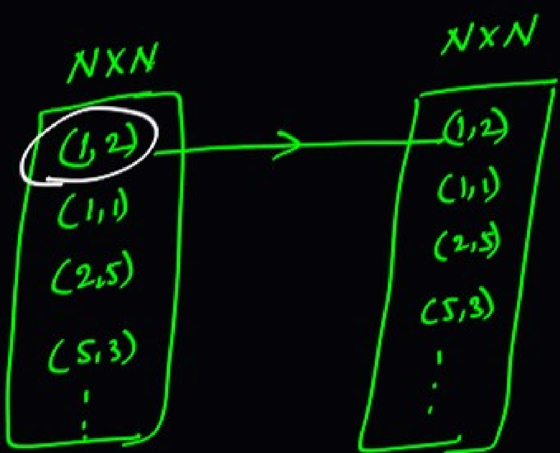
(3,6) (7,4) (1,5)

(9,8) (1,10) (2,7)

R defined on NXN

Rel is defined on set A $\quad R : A \to A$

$$\boxed{(a,b)\, R\, (c,d) \Longleftrightarrow a \le c \text{ (or) } b \le d}$$



Reflexive: $(1,2)\, R\, (1,2) \Longleftrightarrow 1 \le 1$ (or) $2 \le 2$.

Reflexive.

$(a,b)\, R\, (a,b) \Longleftrightarrow a \le a$ (or) $b \le b$

Transitive: If $\underline{xRy}$, $\underline{yRz}$ Then $\underline{xRz}$

If $(a,b)\, R\, (c,d)$ and $(c,d)\, R\, (e,f)$ Then $(a,b)\, R\, (e,f)$

$(3,6), (7,4), (1,5)$

Here $(3,6)\, R\, (7,4)$ Because $3 \le 7$ (or) $6 \le 4$

True False

$(7,4)\, R\, (1,5)$ Because $7 \le 1$ (or) $4 \le 5$

False True

Hence $(3,6) \cancel{R} (1,5) \Longleftrightarrow 3 \le 1$ (or) $6 \le 5$

False False

NOT Transitive