Composite Function:

Let $f : A \longrightarrow B$ and $g : B \longrightarrow C$ are two functions then

$g o f : A \rightarrow C$ is known as a composite function

$f : A \longrightarrow B$

$g : B \longrightarrow C$

$g o f : A \longrightarrow C$

gof

The condition for defining composite function gof is: Domain of g = co-domain of f.

gof

$f : A \longrightarrow B$

$g : B \rightarrow C$

$g o f : A \rightarrow C$

Let $A = \{1, 2, 3, 4\}$ $\quad B = \{a, b, c, d\}$ $\quad C = \{5, 6, 7, 8\}$

$f: A \to B = \{(1,a), (2,b), (3,c), (4,d)\}$

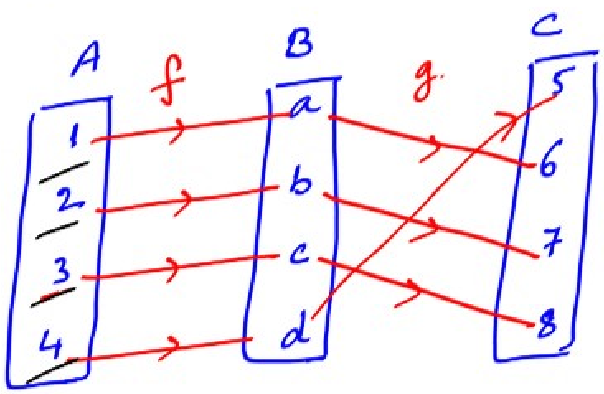
$g: B \to C = \{(a,6), (b,7), (c,8), (d,5)\}$

$gof = ?$ $\quad fog = ?$

$\cancel{fo}$

$gof(3) = g[f(3)] = g(c) = 8$

$gof: A \to C = \{(1,6), (2,7), (3,8), (4,5)\}$



$fog = fog(a) = f[g(a)]$

$= f(6)$

$=$ does not Exists

fog doesnot Exists

Composite

$xoy = ?$

Domain x = cod. y

$y : \to$

$x : \to$

Let $f(x) = 3x+4$, $g(x) = 2x-1$, $h(x) = x^2$

Find
i) fog ✓
ii) gof
iii) $fog(3)$ ✓
iv) $gof(-5)$
v) $fo(goh)$

$f : A \to A$
$g : A \to A$
$h : A \to A$

$$fog = f[g(x)]$$
$$= \cancel{f(3x+4)}\ f(2x-1)$$
$$= 3(2x-1)+4$$
$$= 6x+1$$

$$fog(3) = 6x+1$$
$$= 6(3)+1$$
$$= 19$$

$gof = 6x+7$. ✓

$gof(-5) = -23$

$fo(goh) = 6x^2+1$

Q. Suppose X and Y are sets and |X| and |Y| are their respective cardinalities. It is given that there are exactly ~~97~~ 64 functions from X to Y. From this one can conclude that **(GATE-96)**

a) |X| = 1, |Y| = 97

b) |X| = 97, |Y| = 1

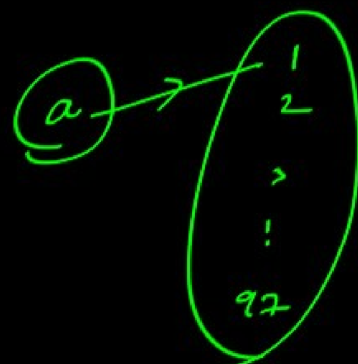
c) |X| = 97, |Y| = 97

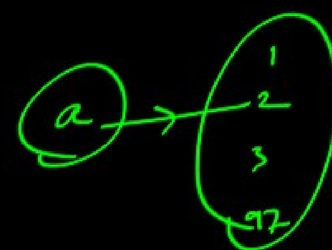
d) None of the above

No. of functions from X to Y $= 97 = (97)^1 = n^m$

$\therefore n = 97 = |Y|$

$m = 1 = |X|$





97 functions $\longrightarrow$ 64 functions

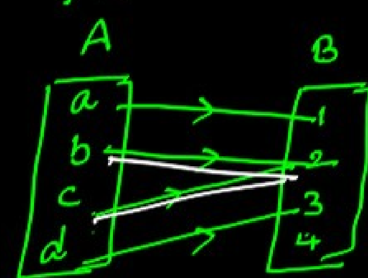
$$64 = (64)^1 = (8)^2 = (4)^3 = 2^6 = n^m$$

Q. Let $f : A \to B$ be a function and let E and F be subsets of A. Consider the following statements about images. **(GATE-01)**

$f : A \to B$

S1: $f(E \cup F) = f(E) \cup f(F)$ ✓

S2: $f(E \cap F) = f(E) \cap f(F)$ ✓



Which of the following is true about S1 and S2?

a) Only S1 is correct ✓

✗ b) Only S2 is correct

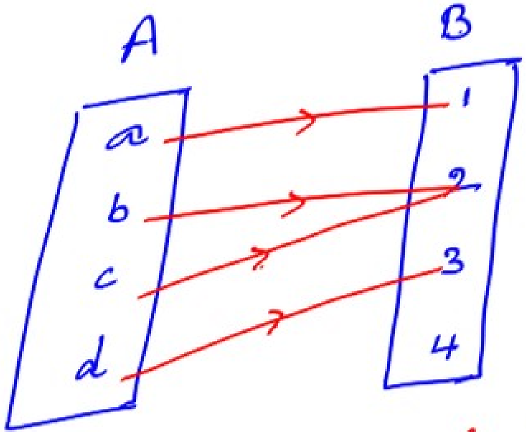
✗ c) Both S1 and S2 are correct

✗ d) None of S1 and S2 is correct

$E \subseteq A,\ F \subseteq A$

~~$E = \{a, b\}$, $F = \{b, c\}$~~

✗ $E = \{a, b\}$ $F = \{c, d\}$



$E = \{a, b\}$ $\quad$ $F = \{c, d\}$

$E \cup F = \{a, b, c, d\}$

$f(E \cup F) = f(\{a, b, c, d\})$

$= \{1, 2, 3\}$

$f(E) \cup f(F) = \{1, 2\} \cup \{2, 3\}$

$= \{1, 2, 3\}$

$S_1 : f(E \cup F) = f(E) \cup f(F)$

$S_2 : f(E \cap F) = f(E) \cap f(F)$

$f(E \cap F) = f(\phi)$

$= \phi$

$f(E) \cap f(F) = f(\{a, b\}) \cap f(\{c, d\})$

$= \{1, 2\} \cap \{2, 3\}$

$= \{2\}$

$f(E \cap F) \neq f(E) \cap f(F)$

Q_1 $f: A \to B$ and $g: B \to C$ are two functions ~~then~~ and the composite function gof is onto then

a) f is onto

b) g is onto

c) f & g are onto

d) None

If TRUE

gof onto $\Longrightarrow$

accept

a) f is onto

b) g is onto

c) f & g onto

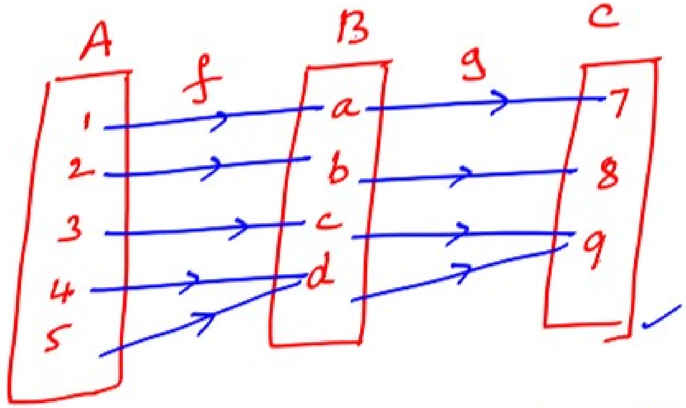
d) None

f, g onto $\Longrightarrow$ gof onto

gof onto $\Longrightarrow$

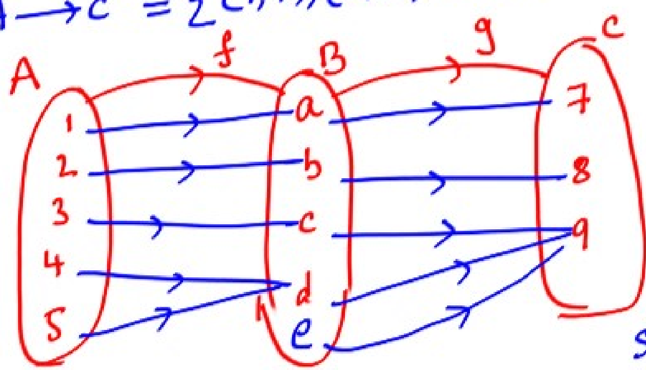
gof onto:

I.



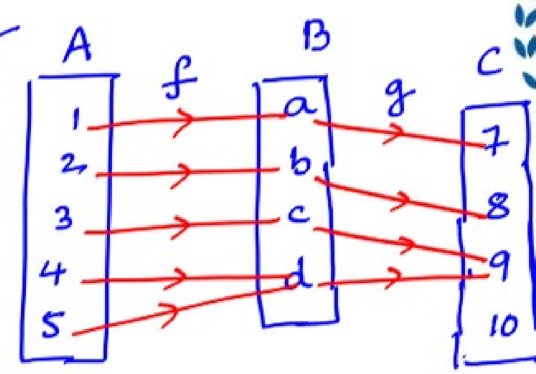
$gof = A \longrightarrow C = \{(1,7),(2,8),(3,9),(4,9),(5,4)\}$

II



f not onto
still gof onto

II



g is NOT onto $\Longrightarrow$ gof NOT onto

Q_2: $f: A \to B$, $g: B \to C$

$g \circ f$ is one-one $\Longrightarrow$ True

a) g is one-one ✗

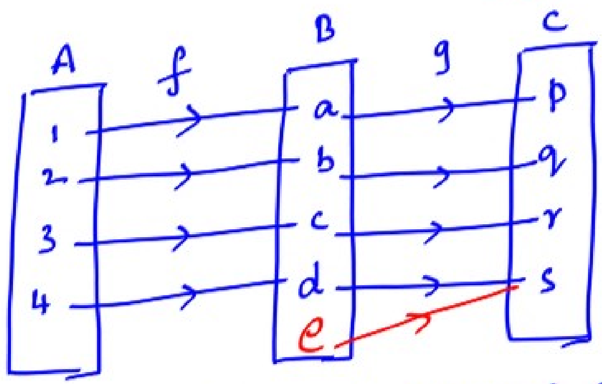
b) f is one-one True

c) f & g are one-one

d) None.

I

II

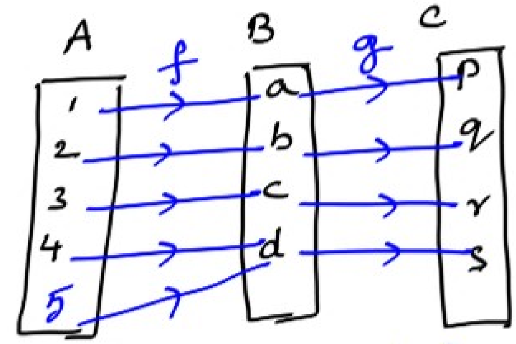


$g \circ f = \{(1,p), (2,q), (3,r), (4,s)\}$

g is not one-one

But still gof is one-one

III



$g \circ f = \{(1,p), (2,q), (3,r), \boxed{(4,s), (5,s)}\}$

Q. Let f: B → C and g: A → B be two functions and let h = f o g. Given that h is an onto function. Which one of the following is TRUE?

(2005 : 2 Marks)

a) f and g should both be onto functions

b) f should be onto but g need not be onto

c) g should be onto but f not be onto

d) both f and g need not be onto

Normal

gof is onto $\Longrightarrow$ g is onto

$f : A \longrightarrow B$

$g : B \longrightarrow C$

$gof : A \longrightarrow C$

$g : A \longrightarrow B$

$f : B \longrightarrow C$

$fog : A \longrightarrow C$

fog is onto $\Longrightarrow$ f is onto

Q. Let f be a function from a set A to a set B, g a function from B to C, and h a function from A to C, such that $h(a) = g(f(a))$ for all $a \in A$. Which of the following statements is always true for all such functions f and g?

$\boxed{h = gof}$

(IT-2005 : 2 Marks)

a) g is onto $\Rightarrow$ $\overset{gof}{h}$ is onto

b) $\overset{gof}{h}$ is onto $\Rightarrow$ f is onto

c) $\overset{gof}{h}$ is onto $\Rightarrow$ g is onto

d) $\underset{gof}{h}$ is onto $\Rightarrow$ f and g are onto

Method-II

$f : A \to B$

$* \; g : B \to \boxed{C}$ covered

$\therefore \; gof : A \to \boxed{C}$ covered

gof onto $\Longrightarrow$ co-domain of gof covered

$gof, A \to C$

g onto

$* \; f \; \boxed{A} \to B$

$g : B \to C$

$gof : \boxed{A} \to C$

gof one-one $\Longrightarrow$

$A \to C$

Domain

f one-one

Q. Let, X, Y, Z be sets of sizes x, y and z respectively. Let $W = X \times Y$ and E be the set of all subsets of W. The number of functions from Z to E is **(2006 : 1 Mark)**

a) z

b) $z \times 2^{xy}$

c) 2^z

d) 2^{xyz}

$|W| = |X \times Y| = xy$

$E = P(W)$

$|E| = |P(W)| = 2^{xy}$

Number of functions from Z to E

$= \underset{m}{z} \longrightarrow \underset{n}{2^{xy}} = n^m = (2^{xy})^z$

$= 2^{xyz}$

Q. Let $S = \{1, 2, 3, \ldots\ldots, m\}$, $m > 3$. Let $X_1, X_2, \ldots., X_n$ be subsets of S each of size 3. Define a function f from S to the set of natural numbers as, f(i) is the number of sets X_j that contains the element i. That is $f(i) = |\{j \mid i \in X_j\}|$.

Then $\sum_{i=1}^{m} f(i)$ is **(2006 : 2 Marks)**

a) 3m

b) 3n

c) 2m + 1

d) 2n + 1