### Laws (or) Properties (or) Logical Equivalences:



	Rules	Name	
1	$p \lor p \cong p$	Idempotent law	
	$p \wedge p \cong p$	raempotent law	
2	$p \lor q \cong q \lor p$	Commutative law	
	$p \wedge q \equiv q \wedge p$	Commutative law	
3	$p \lor (q \lor r) \equiv (p \lor q) \lor r$	Associative law	
	$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$		
4	$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$	Distributive Law_	
	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive Eury	
5	$\sim (p \lor q) \equiv \sim p \land \sim q$	Do Morgan's Law	
	$\sim (p \land q) \equiv \sim p \lor \sim q$	De Morgan's Law	
6	$p \to q \equiv  \sim p \lor q$	Implication law	

### Laws (or) Properties (or) Logical Equivalences:



	Rules	Name
7	$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$	Bi-implication law $P \leftrightarrow q = (P \land q) \lor (\sim P \land \neg q)$
8	$p \lor (p \land q) \equiv p$ $p \land (p \lor q) \equiv p$	Absorption law
9	$p \wedge t \equiv p$ $p \vee f \equiv p$	Identity law
10	~(~ p) ≡ p	Double Negation (or) Involutary law
	$p \lor \sim p \equiv t$ $p \land \sim p \equiv f$	Negation law (or) Complements (or) Inverse law
12	$p \lor t \equiv t$ $p \land f \equiv f$	Domination law

## Eg:

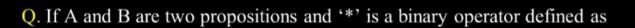


- Q. The propositional function  $p \lor (q \lor \sim p)$  is
  - a) tautology
  - b) contradiction
  - c) contingency
  - d)  $p \wedge q$





- Q. The propositional function  $(p \rightarrow q) \leftrightarrow (p \land \sim q)$  is
  - a) tautology
  - b) contradiction
  - c) contingency
  - d) None





A	В	A * B
T	Т	T
T	F	Т
F	T	F
F	F	Т

Then the compound proposition  $(A \land B) \equiv$ 

- a)  $\sim$  A \* B
- b)  $\sim$  A \*  $\sim$  B
- $c) \sim (A \ * \sim B)$
- d)  $\sim$  ( $\sim$  A \* B)





#### Q. The proposition $p \land (\sim p \lor q)$ is

(a) a tautology

$$(b) \Leftrightarrow (p \land q)$$

$$(c) \Leftrightarrow (p \lor q)$$

(d) a contradiction

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$$P \wedge (\sim P \vee 9) \equiv (P \wedge \sim P) \vee (P \wedge 9)$$
  
 $\equiv f \vee (P \wedge 9)$   
 $\equiv P \wedge 9$ 





$$Q. p \to (q \to r) \Leftrightarrow_{l} \cong$$

a) 
$$(p \wedge q) \rightarrow \sim r$$

b) 
$$(p \lor q) \rightarrow r$$

c) 
$$(p \lor q) \rightarrow \sim r$$

d) 
$$(p \land q) \rightarrow r$$

$$P \rightarrow (q \rightarrow \sigma r) \equiv P \rightarrow (-q v \sigma r)$$

$$\cong (P \land Q) \longrightarrow D$$
  $\sim x \lor y \equiv x \rightarrow y$ 

$$\sim x v y = x \rightarrow y$$



Q. Which of the following is/are tautology?

(a) 
$$(a \lor b) \rightarrow (b \land c)$$

$$(b) (a \wedge b) \to (b \vee c)$$

$$(c)\ (a\lor b)\to (b\to c)$$

$$(d)(a \rightarrow b) \rightarrow (b \rightarrow c)$$

$$(a \rightarrow T) \rightarrow (T \rightarrow F)$$

$$T \rightarrow F$$

$$b$$

$$F$$

$$a + a w to looks$$

re tautology?

$$True$$
 $(avb) \longrightarrow (bnc)$ 
 $TvF$ 

$$(T \vee F) \longrightarrow (F \wedge 1)F$$

$$F = False$$

$$(T \cap T) \longrightarrow (T \vee T/F)$$
  
 $(T \cap T) \longrightarrow T = True = Tautology$ 



- Q. If the proposition  $\sim p \Rightarrow q$  is true, then the truth value of the proposition  $\sim p \lor (p \Rightarrow q)$ , where ' $\sim$ ' is negation, ' $\vee$ ' is inclusive or and ' $\Rightarrow$ ' is implication is GATE 95
  - (a) true

- (c) (b) false
  - (d) cannot be determined

$$\sim P \rightarrow 2 = Tonue$$
  
 $\sim (\sim P) \vee 9 = Tonue$   
 $p \vee 9 = TRUE$ 

Р	9	prq.	~P v 9.
T	T	Tノ	丁
T	F	Т	F
T	_	T.	T
F	1		



when Prq = True then

~ Prq canbe True (or) False

so we cannot determine one truth value.



# Q. MCO

Let p, q and r be propositions and the expression  $(p \rightarrow q) \rightarrow r$  be a contradiction. Then, the expression  $(r \rightarrow p) \rightarrow q$  is **(GATE-17-Set1)** 

- (a) a tautology
- (b) a contradiction



- (c) always TRUE when p is FALSE
- (d) always TRUE when q is TRUE

$$(P \rightarrow 9) \rightarrow \mathfrak{N} = contradiction$$
 $True \qquad False$ 
 $(P \rightarrow 9) \rightarrow \mathfrak{N} = f$ 
 $(P \rightarrow 9) \rightarrow \mathfrak{N} = f$ 
 $(F \rightarrow T) \rightarrow T = True$ 
 $(F \rightarrow F) \rightarrow F = False$ 
 $(F \rightarrow F) \rightarrow T = True$ 





Q. Choose the correct choice(s) regarding the following propositional logic assertions:

$$S: \left[ ((P \land Q) \to R)) \right] \to \left[ ((P \land Q) \to (Q \to R)) \right]$$
 (GATE-21-Set2)

- (a) S is a tautology
- (b) The antecedent of S is logically equivalent to the consequence of S
- (c) S is a contradiction
- (d) S is neither a tautology nor a contradiction



Consequent: 
$$(P \land a) \rightarrow (a \rightarrow R)$$

$$= (P \land Q) \longrightarrow R$$

$$X \longrightarrow Y$$
 $T = T$ 



Q. "If X then Y unless Z" is represented by which of the following formulas in propositional logic? (" $\sim$ " is negation, " $\wedge$ " is conjunction, and " $\rightarrow$ " is implication)

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(a) 
$$(X \land (Z)) \rightarrow (Y)$$

(b) 
$$(X \wedge Y) \rightarrow \ \ \sim Z$$

$$(C) \: X \to (Y \land {\scriptstyle \sim} \: Z)$$

(d) 
$$(X \rightarrow Y) \land \sim Z$$

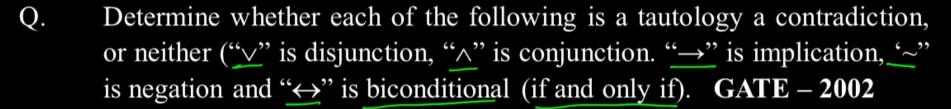
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If x then y unless 
$$\Xi$$

$$\Rightarrow \text{ If } x \text{ is } \text{TRUE and } \Xi \text{ is } \text{False then. y will be } \text{TRUE}$$

$$= (x \land \sim \Xi) \rightarrow Y$$





(i) 
$$A \leftrightarrow (A \lor A)$$

$$(A \leftrightarrow (A \lor A))$$

(ii) 
$$(A \lor B) \to B$$

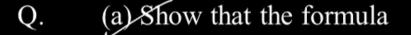
$$A \longleftrightarrow A$$

$$T \longleftrightarrow F = t$$

(iii) 
$$A \wedge (\sim (A \vee B))$$
 — Tautology

(3) 
$$A \cap (\sim A \cap \sim B)$$
  
 $\equiv (A \cap \sim A) \cap \sim B$   
 $\equiv f \cap \sim B = f$  contradiction







 $[(\sim p \lor q) \Rightarrow (q \Rightarrow p)]$  is not a tautology.

(b) Let A be tautology and B be any other formula, Prove that (A v B)

is a tautology.

(a) 
$$[(\sim PVQ) \longrightarrow (Q \rightarrow P)]$$
  
 $T \longrightarrow F$   
 $T \longrightarrow F$   
 $T \longrightarrow F$   
 $T \longrightarrow F = False$   
 $T \longrightarrow F = False$   
 $T \longrightarrow F = False$ 



- Let a, b, c, d be propositions. Assume that the equivalences  $a \leftrightarrow b (b \lor \sim b)$ Q. and b  $\leftrightarrow$  c hold. Then the truth value of the formulae  $(a \land b) \rightarrow ((a \land c) \lor a)$ d) is always GATE - 2000
  - (a) True

X = Y (b) False X や Y (C) S (C) Same as truth value of b

(d) Same as truth value of d

Qiven equivalences
$$a \leftrightarrow (bv \sim b)$$

$$a \cong bv \sim b$$

$$a \cong bv \sim b$$

$$a \cong Tonue$$

$$(anb) \rightarrow [(anb) \lor d]$$

$$a \cong Tonue$$

$$(anb) \rightarrow [(anb) \lor d]$$

$$(a$$

