Discrete Mathematics

DMS = Discrete Mathematic Structures

DS = Discrete Structures

D.M = D. Maths = Discrete Maths

MFCS = Mathematical Foundations for Computer Science

DSGT = Discrete Structures & Graph Theory

Discrete Maths	Engg. Maths
I. Mathematical Logic	1. Linear alg
II. Set theory & Algebra	2. calculus
III Combinatorics	3. Probability
IV. Graph Theory	

Discrete Maths (6 Marks — 15 Marks)

8 Marks

UNITS

I. Mathematical Logic

II. Set Theory & Algebraic structures

III. Combinatorics

IV. Graph Theory

Chapterwise

1. propositional logic (10)
2. First-order logic (10)
3. sets (4)
4. Relations (4)
5. Partial-ordering & Lattices (4)
6. Functions (4)
7. Algebraic structures (Groups) (6)
8. Combinatorics (20)
9. Graph Theory (20)

80 hrs

Reference Books

** 1. Discrete Mathematical Structure with Applications to Computer Science – Tremblay & Manohar
2. Discrete Mathematics for Computer Scientist & Mathematicians – Mott, Kandell & Baker.
3. Discrete Mathematics – Kenneth Rosen
4. Discrete Mathematics – C.L.Liu

LOGIC

Logic is the basis of all mathematical reasoning, and of all automated reasoning.

It has the practical applications

- To the design of computing machines
- To the specification of systems
- To artificial intelligence
- To programming languages and to other areas of computer science.

Propositions:-

A proposition is a declarative sentence (that is, a sentence that declares a fact) that is either true or false, But not both.

proposition = T (or) F

Eg:-

1. Washington D.C. is the capital of the united states of America. (T)

2. Bombay is the capital of India. (F)

3. $1 + 1 = 2$ (T)

4. $2 + 2 = 3$ (F)

Eg:-

1. What time is it? NOT proposition

2. Read this carefully "

3. $x + 1 = 2$ "

4. $x + y = z$ "

Propositional Variables : p, q, r, s, ...

Truth Values : True → T

False → F

No.of prop. → No. of possi

$1 \longrightarrow 2$

$2 \longrightarrow 4$

$3 \longrightarrow 8$

$n \longrightarrow 2^n$

Propositional Calculus : The area of logic that deals with propositions is called as propositional logic (or) propositional calculus.

Compound Propositions : Many Mathematical statements are constructed by combining one or propositions by using logical operators, known as compound propositions.

Logical Operators:

$\wedge$ (and) - Conjunction

$\vee$ (or) - Disjunction

$\rightarrow$ (Implies) - Implication

$\leftrightarrow$ (Bi-implies) - Bi-conditional

Negation:

p : Mumbai is the capital of Maharashtra

~p : Mumbai is NOT the capital of Maharashtra

Conjunction: when two or more simple propositions are combined with logically connective "and ($\wedge$)" then the obtaine new proposition is known as conjunction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Truth Table: Truth table for compound proposition

p	q	$p \wedge q$	$p \vee q$	$p \to q$	$p \leftrightarrow q$	$q \to p$	$p \downarrow q$ NOR	$p \uparrow q$ NAND	$p \oplus q$ Boolean Sum
T	T	T	T	T	T	T	F	F	F
T	F	F	T	F	F	T	F	T	T
F	T	F	T	T	F	F	F	T	T
F	F	F	F	T	T	T	T	T	F

Construct Truth Tables of the following compound proposition:

1. $\sim(p \wedge q) \vee (\sim p \rightarrow \sim q) = TTTT$ satisfiable

2. $\sim(p \vee q) = FFFT$
3. $\sim p \wedge \sim q = FFFT$

contingency & satisfiable.

4. $\sim p \vee q = TFTT$
5. $\sim(\sim p) = TF$

Contingency

6. $(\sim p \wedge \sim q) \vee (p \wedge q) = TFFT$
7. $(\sim p \vee q) \wedge (p \vee \sim q) = TFFT$

8. $p \vee (q \rightarrow r) = TTTTTFTT$
9. $p \vee \sim p = TT$ = always True
10. $p \wedge \sim p = FF$ = always False

$$\sim(p \vee q) \cong \sim p \wedge \sim q$$

$$p \rightarrow q \cong \sim p \vee q$$

$$\sim(\sim p) \cong p$$

$$p \leftrightarrow q \equiv (\sim p \wedge \sim q) \vee (p \wedge q) \equiv (\sim p \vee q) \wedge (p \vee \sim q)$$

⑧ $p \vee (q \rightarrow r)$

p	q	r	$q \rightarrow r$	$p \vee (q \rightarrow r)$
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

$\sim q \rightarrow \sim p$

$X \leftrightarrow Y$

p	q	$p \rightarrow q$	$\sim q$	$\sim p$	$\sim q \rightarrow \sim p$	$(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$
T	T	T	F	F	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

$$p \rightarrow q \cong \sim q \rightarrow \sim p$$

Logical Equivalence: Compound propositions that have the same truth values in all possible cases are called logically equivalent.

P $\quad$ Q

The compound propositions 'p' and 'q' are called logically equivalent if $p \leftrightarrow q$ is a tautology

$\boxed{P \longleftrightarrow Q}$ is a Tautology

$$p \cong q$$

$P \cong Q$

$X \cong Y$ if and only if $\overset{X}{p} \longleftrightarrow \overset{Y}{q}$ Tautology

GATE: The binary equation □ is defined as follows

p	q	p □ q
T	T	T
T	F	T
F	T	F
F	F	T

Then compound proposition $(p \vee q) \Leftrightarrow$ $\cong$

a) ~ q □ ~ p   b) p □ ~ q   c) ~ q □ p   d) p □ q

Δ $\quad p \bigcirc q \quad p \odot q \quad p * q$

$p \vee q \cong$

a)

b)

c)

d)

p	q	$p \square q$	$p \vee q$	$\sim q$	$\sim p$	$\sim q \square \sim p$	$p \square \sim q$
T	T	T	T	F	F	T	T
T	F	T	T	T	F	T	T
F	T	F	T	F	T	F	T
F	F	T	F	T	T	T	F

$$p \vee q \cong p \square \sim q$$

Tautology, Contradiction, Contingency, Satisfiable and Unsatisfiable:

Always TRUE = Tautology

Always False = Contradiction

Neither Tautology
Nor contradiction } = Contingency

at least one TRUE = Satisfiable

NOT Satisfiable = unsatisfiable.

* Every contingency is satisfiable,
But converse need not be TRUE

Eg: Which of the following is NOT a Tautology

a) $\sim (p \rightarrow q) \rightarrow p$

b) $\sim (p \rightarrow q) \rightarrow \sim q$

c) $[\sim p \wedge (p \vee q)] \rightarrow q$

d) $(p \rightarrow q) \wedge q \rightarrow p$

I. Truth table

II. Logical Approach

III. Properties (or) laws