

Let P, Q and R be three atomic prepositional assertions. Q.

> Let X denote $(P \lor Q) \to R$ and Y denote $(P \to R) \lor (Q \to R)$. Which one of the following is a tautology? **GATE - 2005**

- (a) $X \equiv Y$
- $(b) \stackrel{\checkmark}{X} \rightarrow Y$ $(c) Y \rightarrow X$
- $(d) \sim Y \rightarrow X$



Consider the following propositional statements:

P1:
$$((A \land B) \to C)) \equiv ((A \to C) \land (B \to C))$$

P2:
$$((A \lor B) \to C)) \equiv ((A \to C) \lor (B \to C))$$
 GATE – 2006

- (a) P1 is a tautology, but not P2
- (b) P2 is a tautology, but not P1
- (c) P1 and P2 are both tautologies
- (d) Both P1 and P2 are not tautologies

$$P_1: (A \land B) \rightarrow C \cong (A \rightarrow C) \land (B \rightarrow C)$$

$$RHS = (A \rightarrow C) \wedge (B \rightarrow C)$$

$$\equiv (AVB) \longrightarrow C$$

$$P_2: (AvB) \rightarrow c \equiv (A \rightarrow c) \lor (B \rightarrow c)$$



P, & P2 Both are NOT Tautologies Q. A logical binary relation Θ , is defined as follows:

| | | - | ı |
|---|---|---|---|
| / | | | ١ |
| 1 | | | ı |
| v | _ | 9 | |



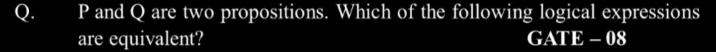
| A | В | A O B |
|-------|-------|-------|
| True | True | True |
| True | False | True |
| False | True | False |
| False | False | True |

Let be the unary negation (NOT) operator, with higher precedence than O. Which one of the following is equivalent to $A \wedge B$? GATE – 2006

$$(C) \sim (\sim A \odot \sim B)$$

~A O B







$$I.P \lor \sim Q$$

$$II_{\sim} (\sim P \wedge Q) \equiv P \vee \sim 6$$

III.
$$(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$$

IV.
$$(P \land Q) \lor (P \land \sim Q) \lor (\sim P \land Q)$$

are equivalent?

I.P
$$\vee \sim Q$$

II. $(\sim P \wedge Q) \equiv P \vee \sim Q$

III. $(\sim P \wedge Q) = P \vee \sim Q$

III. $(\sim P \wedge Q) \vee (\sim P \wedge \sim Q) = [\sim P \wedge (\sim P \wedge \sim Q)] \vee (\sim P \wedge \sim Q)$

IV. $(\sim P \wedge Q) \vee (\sim P \wedge \sim Q) = [\sim P \wedge (\sim P \wedge \sim Q)] \vee (\sim P \wedge \sim Q)$

(a) Only I and II

(b) Only I, II and III

(c) Only I, II and IV

(d) All of I, II, III and IV

$$= [(-1)] \times (-p - 0)$$



$$\overline{V} \quad (P \wedge Q) \quad V(P \wedge \neg Q) \quad V \leftarrow P \wedge Q)$$

$$\equiv \left[P \wedge (Q \vee \neg Q)\right] \quad V(\neg P \wedge Q)$$

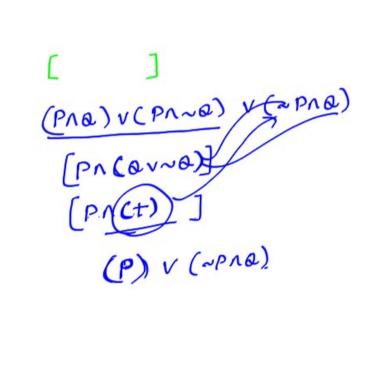
$$\equiv \left[P \wedge (t)\right] \quad V(\neg P \wedge Q)$$

$$\equiv P \vee (\neg P \wedge Q)$$

$$\equiv (P \vee \neg P) \quad \wedge (P \vee Q)$$

$$\equiv (t) \quad \wedge (P \vee Q)$$

$$\equiv P \vee Q$$











(a)
$$((p \leftrightarrow q) \land r) \lor (p \land q \land \sim r)$$

(b)
$$(\sim (p \leftrightarrow q) \land r) \lor (p \land q \land \sim r)$$

(c)
$$((p \rightarrow q) \land r) \lor (p \land q \land \sim r)$$

(d)
$$(\sim (p \leftrightarrow q) \land r) \land (p \land q \land \sim r)$$

Exactly two of P, q, on are TRUE

| | Þ | 9 | 37 | | |
|---|---|---|----|---|--|
| ١ | T | T | F | / | |
| | T | F | T | 5 | |
| | F | T | T | 5 | |
| L | | | | | |

(a)
$$[(P \leftrightarrow q) \land \sigma] \lor (P \land q \land \neg \sigma)$$

 $[(T \leftrightarrow T) \land F] \lor (T \land T \land T) = T$
 $[(T \leftrightarrow F) \land T] \lor (T \land F \land F) = F$
 $[(F \leftrightarrow T) \land T] \lor (F \land T \land F) = F$

(B)
$$[\sim (P \leftrightarrow 9) \land 91] \lor (P \land 9 \land \sim 91)$$

$$[\sim (T \leftrightarrow T) \land F] \lor (T \land T \land T) = T \quad ACE$$

$$[\sim (T \leftrightarrow F) \land T] \lor (T \land F \land F) = T$$

$$[\sim (F \leftrightarrow T) \land T] \lor (F \land T \land F) = T$$





Which one of the following Boolean expressions is **NOT** a tautology?

(GATE-14-Set2)

(a)
$$((a \rightarrow b) \land (b \rightarrow c)) \rightarrow (a \rightarrow c)$$

(b)
$$(a \leftrightarrow c) \rightarrow (\sim b \rightarrow (a \land c))$$

$$(c) (a \land b \land c) \rightarrow (c \lor a)$$

(d)
$$a \rightarrow (b \rightarrow a)$$

$$\begin{array}{ccc}
 & T & F \\
\hline
 & A & (b \rightarrow a) \\
 & T & (b \rightarrow T) & = Tautology
\end{array}$$



Which one of the following is **NOT** equivalent to $p \leftrightarrow q$?



(a)
$$(\sim p \lor q) \land (p \lor \sim q)$$

(b)
$$(\sim p \lor q) \land (q \rightarrow p)$$

$$(c) (\sim p \land q) \lor (p \land \sim q)$$

(d) (
$$\sim p \land \sim q$$
) $\lor (p \land q)$

(a)
$$(\sim p \lor q) \land (p \lor \sim q)$$

$$(b) (\sim p \lor q) \land (q \to p)$$

$$(c)(\sim p \land q) \lor (p \land \sim q)$$

$$(c)(\sim p \land q) \lor (p \land \sim q)$$

$$(c)(\sim p \land q) \lor (p \land \sim q)$$

$$(c)(\sim p \land q) \lor (p \land \sim q)$$

$$(c)(\sim p \land q) \lor (p \land \sim q)$$

we know that

$$P \longleftrightarrow 9 \equiv (P \land 9) \lor (\sim P \land \sim 9) = option (d)$$



Q. Let p, q, r, s represent the following propositions. (GATE-16-Set1)



p: $x \in \{8, 9, 10, 11, 12\}$

q: x is a composite number

r: x is a perfect square

s: x is a prime number

The integer $x \ge 2$ which satisfies $\sim ((p \Rightarrow q) \land (\sim r \lor \sim s))$ is _____.

9: composite = f(4), 6, 8, 9, 10, 12, 14, 15, ---3, f(4) = f(1), 2, 3, 5, 7, 11, 13, ---3b x ε 2 8 9 0 11, 23

S: prime number =
$$2 2 16$$
, $x \ge 2$,





The statement $(\sim p) \Rightarrow (\sim q)$ is logically equivalent to which of the Q. statements below? (GATE-17-Set1)



$$\underline{I}.\ p \Longrightarrow q$$

$$JI \cdot q \Rightarrow p$$

III.
$$(\sim q) \vee p$$

$$\underline{IV}$$
. $(\sim p) \vee q$

CBy contra positive law)

$$a \rightarrow b \equiv \sim avb \equiv \sim b \rightarrow \sim a$$

 $\sim P \rightarrow \sim 2 \equiv \sim (\sim 2) \rightarrow \sim (\sim P)$
 $\equiv 9 \rightarrow P \qquad (\blacksquare)$
 $\equiv \sim 2 \rightarrow P \qquad (\blacksquare)$





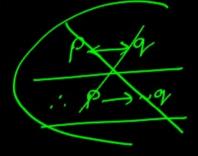
Let p and q be propositions. Decide whether $(p \leftrightarrow q)$ does not Q. imply $(p \rightarrow \sim q)$ is true or false **GATE** – **94**



Example: A implies B

when A is TRUE Then B will be TRUE

= when A is TRUE Then B cannot be FALSE



$$(P \longrightarrow 9) = (P \longrightarrow \sim 9)$$
 [checking]

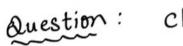


$$(P \longleftrightarrow 9)$$
 $(P \longrightarrow -9)$
 $T \longleftrightarrow T = True$ $T \longrightarrow F = False$

$$(P \leftrightarrow 9)$$
 does not implies $(P \rightarrow \sim 9)$
 $(P \leftrightarrow 9)$ \Rightarrow $(P \rightarrow \sim 9)$

$$x = y$$

when
$$X = Torue$$
 then $Y = Torue$



check
$$(P \longleftrightarrow q) \Longrightarrow (P \to \sim q)$$

$$(P \longleftrightarrow 9)$$

$$T \longleftrightarrow T_{Y}$$

$$T \longleftrightarrow T_{\nu}$$

$$(P \longrightarrow \sim 9)$$

$$(T \longrightarrow F)$$

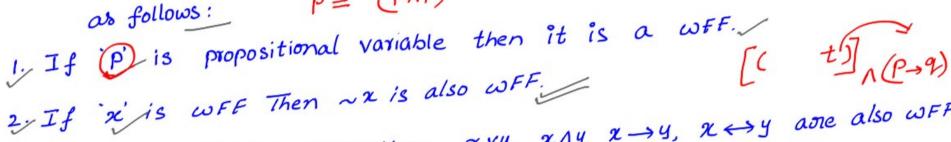
$$F$$



(P > 9) does not implies (P->~9): Torue

well-Formed Formula (WFF):

A well-Formed Formula (wff) can be defined necurisively as follows: P = (PAP)



3. If 'x' and 'y' are ωFF then $\underline{\chi} \nu y$, $\underline{\chi} \wedge y$, $\underline{\chi} \to y$, $\underline{\chi} \leftrightarrow y$ and also ωFF .

4. Any string of Symbol, obtained by finitely many applications of stule (1)

to onule (3) above is a wff.

$$(p \land q \land \sigma \land s) = p \left(\land (p \rightarrow q) \right)$$

$$p \sim q \times$$

De which of the following are well-formed formulea?

$$avb \rightarrow c/$$
 $(anb) \rightarrow c$
 $(avb) \rightarrow c/$
 $(avb) \rightarrow c/$

$$\left((PNQ) \rightarrow (Q) \right]$$

$$[(PAQ) \rightarrow Q]$$

$$[PAQ \rightarrow Q]$$

Normal Forms:

The standardization of given propositional formulea is known ar Normal Forms. It is very difficult to compare logical expressions like P and Q, when there are too many propositional variables. Normal forms are helpful to compare logical expressions either they tautology or contradiction or equivalent, etc.

Different types of Normal forms are (canonical forms)

Since 1995

- 1. DNF (Disjunctive Normal Form)
- ii. CNF (Conjunctive Normal Form)
- III. PONF (Principle Disjunctive Normal Form)
- IV. PCNF. (Principle conjunctive Normal Form)

elementary Sum: Disjunction of propositional variables or their Negations
P, ~P. PV9, ~PV~9, ~PV9, PV~9

elementary product: conjunction of propositional variables or their negations, P, ~P, 9, ~9, PA9, ~PA9, PA9, PA9, PA9A, PA9A,