Rules of Inference:

Sl. No	Rule	Name
1	p $\therefore p \vee q$	Addition (or) Disjunctive amplification
2	$p \wedge q$ $\therefore p$	Simplification (or) Conjunctive simplification
3	p q $\therefore p \wedge q$	Conjunction a b c $\therefore a \wedge b \wedge c$

$\dfrac{p}{\therefore p \vee r}$ $\quad \dfrac{p}{\therefore p \vee s}$

$\dfrac{a \wedge b}{\therefore a}$ $\quad \dfrac{a \wedge b}{\therefore b}$

Rules of Inference:

Sl. No	Rule	Name
4 ✓	$p \vee q$ $\sim p$ ――― $\therefore q$	Disjunctive Syllogism
5 ✓	$p \vee q \quad = T$ $\sim p \vee r \quad = T$ ――― $\therefore q \vee r$	Resolution
6 ✓	$p \rightarrow q$ p ――― $\therefore q$	Modus Ponens

$p \vee q$

$$\begin{array}{ll} p \rightarrow q & = \text{True} \\ p & = \text{True} \\ \hline \therefore q & = \text{True} \end{array}$$

Disjunctive Syllogism:

$$\begin{array}{l} T = \overset{F}{p} \vee \overset{T}{q} \\ T = \sim p \\ \hline \therefore q \end{array} \quad \equiv \quad \begin{array}{c} \text{Father} \vee \text{Mother} \\ \sim\text{Father} \\ \hline \therefore \text{Mother} \end{array} \quad = \quad \begin{array}{c} \text{Student} \vee \text{Faculty} \\ \sim\text{Student} \\ \hline \therefore \text{Faculty} \end{array}$$

$$\begin{array}{c} p \vee q \\ \sim q \\ \hline \therefore p \end{array}$$

Resolution:

$$\begin{array}{l} F \quad \overset{T}{p} \vee \overset{T}{q} = T \\ T \quad \overset{F}{\sim p} \vee \overset{T}{r} = T \ ✓ \\ \hline \quad \overset{T}{q} \vee \overset{T}{r} = \end{array}$$

$$\begin{array}{l} a \vee b = T \\ \sim a \vee c = T \\ \hline \therefore b \vee c \end{array}$$

Modus ponens :

$p \to q$	GATE Rank $\to$ Car.	* Rohit hits cent then he will get car.
p	GATE Rank	* Rohit hits century
$\therefore q$	$\therefore$ will get car	$\therefore$ Rohit will get new car.

Modus Tollens :

$p \to q$

$\sim q$

$\sim p$

* virat hit Cent. then he will get new car.
* virat will not get new car ✓ =

$\therefore$ virat has not century ✓

Argument : ✗

$p \to q$ ✓ $\equiv$ virat hits century then he will get new car

$\sim p$ ✓ $\equiv$ virat has not Century

$\therefore \sim q$ ✓ $\quad \therefore$ virat will not get new car. ✓ ✗ Not valid

Rules of Inference:

Sl. No	Rule	Name
7	$p \to q$ $\sim q$ $\therefore \sim p$	Modes Tollens
8	$p \to q$ $q \to r$ $\therefore p \to r$	Hypothetical Syllogism
9	$p \vee Q$ $p \to r$ $Q \to r$ $\therefore R$	Dilema

$P_1 = T$

$P_2 = T$

$P_3 = T$

$\overset{F}{P} \to \overset{F}{R} = T$

$\overset{F}{Q} \to \overset{F}{R} = T$

$\overset{F}{P} \vee \overset{F}{Q} = T$

$\therefore R$

Father $\longrightarrow$ You. = True

Mother $\longrightarrow$ You. = True

Father $\vee$ Mother. = True

You

Hypothetical

$$\begin{array}{ll} a \longrightarrow b = \text{True} & \equiv \text{Salman is brother of Ameer} \\ b \longrightarrow c = \text{True} & \equiv \text{Ameer is brother of Abhishek} \\ \hline \therefore a \longrightarrow c = \text{True} & \therefore \text{Salman is brothu of Abhishek} \\ \hline \end{array}$$

Rules of Inference:

Sl. No	Rule	Name
10	$p \to Q$ $R \to S$ $p \vee R$ $\therefore Q \vee S$	Constructive Dilemma
11	$p \to Q$ $R \to S$ $\sim Q \vee \sim S$ $\therefore \sim p \vee \sim R$	Destructive Dilemma $P \to Q$ $R \to S$ $\sim Q \vee \sim S$ $\therefore \sim P \vee \sim R$

$P \to Q$
$R \to S$
$P \vee R$
$Q \vee S$

$a \to b$
$c \to d$
$a \vee c$
$b \vee d$

$a \to b$
$c \to d$
$\sim b \vee \sim d$
$\therefore \sim a \vee \sim c$

Consider the following logical inferences

I_1 : If it rains then the cricket match will not be played.

The cricket match was played.

Inference : There was no rain

p : It rains

q : match will not be played

$$\begin{array}{l} p \rightarrow q \\ \sim q \\ \hline \therefore \sim p \end{array}$$

I_1 is valid by Modus Tollen's

I_2 : If it rains then the cricket match will not be played.

It did not rain.

Inference : The cricket match was played.

$$\begin{array}{l} p \rightarrow q \\ \sim p \\ \hline \therefore \sim q \end{array}$$

X NOT valid

$$
\begin{array}{l}
p \rightarrow \sim q \\
\quad q \\
\hline
\therefore \sim p \\
\hline
\end{array}
$$

p : It rains

q : Match will be played.

Modus Tollen's :

$$
\begin{array}{l}
\overset{p}{a} \rightarrow b \;\; \sim q \\
\quad \sim b \;\; q \\
\hline
\therefore \sim a \;\; \sim p \\
\hline
\end{array}
$$

I_2 : If it rains then the cricket match will not be played.

It did not rain.

Inference : The cricket match was played.

Which of the following is TRUE?

a) Both I_1 and I_2 are correct inferences

b) I_1 is valid, But NOT I_2

c) I_2 is valid, But NOT I_1

d) Neither I_1 nor I_2 is valid

Q. Check the validity of the following argument

$$\{p \rightarrow (r \rightarrow s), \sim r \rightarrow \sim p, p\} \Rightarrow s$$

$x \rightarrow y \equiv \sim y \rightarrow \sim x$

Argument : $\{P_1 \wedge P_2 \wedge P_3 \wedge \cdots \wedge P_n\} \Longrightarrow q$

Given : $\{ p \rightarrow (r \rightarrow s), \sim r \rightarrow \sim p, p \} \Longrightarrow s$

premises $P_1 = p \rightarrow (r \rightarrow s)$, $P_2 = \sim r \rightarrow \sim p$, $P_3 = p$, conclusion, $q = s$

$$P_2 \equiv \sim r \rightarrow \sim p \equiv p \rightarrow r$$
$$P_3 \equiv p$$
$$\therefore r$$

$$P_1 \equiv p \rightarrow (r \rightarrow s)$$
$$P_3 \equiv p$$
$$\therefore r \rightarrow s$$
$$r$$
$$\therefore s$$

Q. Check the validity of the following argument

$$\{\sim t \rightarrow \sim r, t \rightarrow w, r \vee s, \sim s\} \Rightarrow w$$

Given : $\left\{ \underbrace{\sim t \rightarrow \sim r}_{P_1},\ \underbrace{t \rightarrow w}_{P_2},\ \underset{P_3}{\overset{*}{r \vee s}},\ \underset{P_4}{\overset{*}{\sim s}} \right\} \Longrightarrow w$

$$\begin{array}{l} r \vee s \\ \sim s \\ \hline \therefore r \ \checkmark \\ \hline \end{array}$$

$$\begin{array}{l} P_1 \equiv \sim t \rightarrow \sim r \equiv r \rightarrow t \\ P_2 \equiv t \rightarrow w \\ \hline \therefore r \rightarrow w \ \checkmark \\ \hline \end{array}$$

(Hypothetical syllogism)

$$\begin{array}{l} r \rightarrow w \\ r \\ \hline \therefore w \\ \hline \end{array}$$

Q. Check the validity of the following arguments

$I_1 : \{R \rightarrow S, P \rightarrow Q, R \vee P\} \Rightarrow (S \vee Q)$

I_1 is valid by Rule of constructive dilemma.

$I_2 : \{\sim R \rightarrow (S \rightarrow \sim T), \sim R \vee w, \sim P \rightarrow S, \sim w\} \Rightarrow (T \rightarrow P)$

$I_2 : \{ \underbrace{\sim R \to (S \to \sim T)}_{P_1}, \underbrace{\sim R \vee W}_{P_2}, \underbrace{\sim P \to S}_{P_3}, \underbrace{\sim W}_{P_4} \} \vdash (T \to P)$ ✓

$$\begin{array}{l} \sim R \vee W \\ \sim W \\ \hline \therefore \sim R \\ \hline \end{array}$$ ✓

$$\begin{array}{l} \sim P \to S \\ S \to \sim T \\ \hline \therefore \sim P \to \sim T \\ \hline \end{array} \equiv T \to P$$ ✓

$$\begin{array}{l} \sim R \\ \sim R \to (S \to \sim T) \\ \hline \therefore (S \to \sim T) \\ \hline \end{array}$$ ✓

Quantifiers:

Quantifiers are the words the refers to Quantities such as some or all,

and indicates how frequently a certain statement is true.

Types:

1. "Universal" (∀) quantifier $\forall$ = for all

2. "There existential" (∃) quantifier $\exists$ = Their Exists

Universal Quantifier:

Let P(x) be statement defined on universe of discourse A, then the universal quantification P(x) is the statement P(x) is true for all x belongs to A

P(x) is true

eg-1: For every real number x, $x^2 \geq 0$

$\forall x, x^2 \geq 0$

eg-2: All Students are good

= Every student is good

= Each student is good

Let $S(x)$: x is a student

$g(x)$: x is good

$\forall x \, [S(x) \longrightarrow g(x)]$

eg-3 : All Boys are Innocent

$B(x)$: x is a Boy

$I(x)$: x is innocent

$\forall x \; [B(x) \longrightarrow I(x)]$

Existential Quantifier:

Let $P(x)$ be a statement defined on universe of discourse A, the Existential Quantification $P(x)$, such that $P(x)$ is TRUE for Some 'x' belongs to 'A' (Domain).

eg-1: For some real number x, $x+5=0$

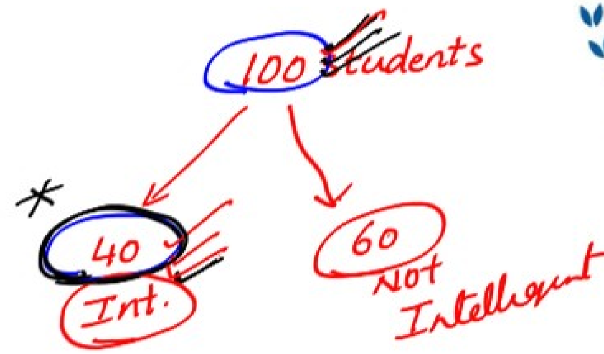
$$\exists x\,[\,x+5=0\,]$$

eg-2: Some students are Intelligents ✓

Let $S(x)$: x is a student

$I(x)$: x is Intelligent

$$\exists x\,[S(x) \wedge I(x)]$$



$S(x) \longrightarrow I(x)$

eg-3: Some parallelograms are rectangles

$p(x)$: x is a parallelogram

$r(x)$: x is a rectangle

$$\exists x\,[P(x) \wedge r(x)]$$

100 Quatrilatral

4 Paralle ..

NOTE :

I. Universal quantfeor ($\forall$) is usually followed by Implication ($\longrightarrow$)

II. Existential quantifier ($\exists$) is usually followed by and ($\wedge$)

III $\forall x\, P(x) = \text{True}$

$= P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge \ldots\ldots \wedge P(x_n) = \text{TRUE}$

IV $\exists x\, P(x) = \text{True}$

$= P(x_1) \vee P(x_2) \vee P(x_3) \vee \ldots\ldots \vee P(x_n) = \text{TRUE}$