Multiple Quantifiers:

A different ordering of the quantifiers may yield a different statement.

* The statement $\exists x \; \forall y \; \boxed{p(x, y)}$ and $\forall y \; \exists x \; \boxed{p(x, y)}$ are not logically equivalent.

There are 8 ways to apply the two quantifiers.

Example:

p(x, y) : x likes y

x = { Boys }   y = { Girls }

$P(x,y)$: x likes y

1) $\forall x \, \forall y \; P(x,y)$: Every boy likes Every girl.

2) $\boxed{\forall x \, \exists y \; P(x,y)}$ * : Every boy likes Some girls

3) $\exists x \, \forall y \; P(x,y)$: Some boys likes all girls

4) $\exists x \, \exists y \; P(x,y)$: Some boys likes Some girls

5) $\forall y \, \forall x \; P(x,y)$: Every girl is liked by Every boy

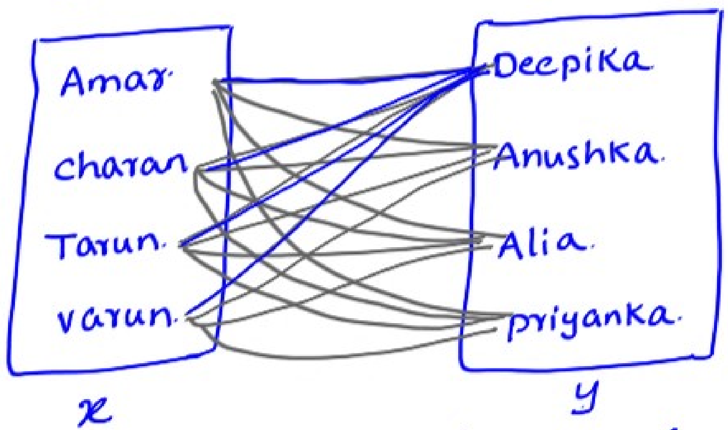
6) $\forall y \, \exists x \; P(x,y)$: Every girl is liked by Some boys

7) $\boxed{\exists y \, \forall x \; P(x,y)}$ * : Some girls are liked by all boys.

8) $\exists y \, \exists x \; P(x,y)$: Some girls are liked by Some boys.

① $\forall x \forall y \; P(x,y)$: Every boy likes Every girl

DIS-I



$4 \times 4 = 16$ mappings

Conclusion: $\overset{\text{True}}{a} \longrightarrow \overset{\text{can be True}}{b}$

(i) $\forall x \forall y \; P(x,y) \longrightarrow \forall y \forall x \; P(x,y)$

$b \longrightarrow a$

(ii) $\forall y \forall x \; P(x,y) \longrightarrow \forall x \forall y \; P(x,y)$

(iii) $\forall x \forall y \; P(x,y) \longleftrightarrow \forall y \forall x \; P(x,y)$

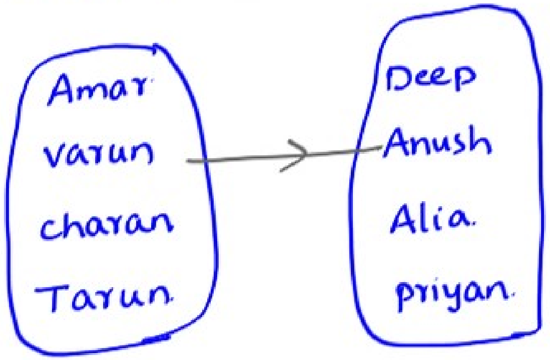
$a \longleftrightarrow b$

From the above mapping, we can conclude that
"Every girl is liked by Every boy"

$= \forall y \forall x \; P(x,y)$

4 $\exists x \exists y \; P(x,y)$: Some boys likes some girls

DIS-II



$\Rightarrow$ Some girls are liked by Some boys

Anushk varun

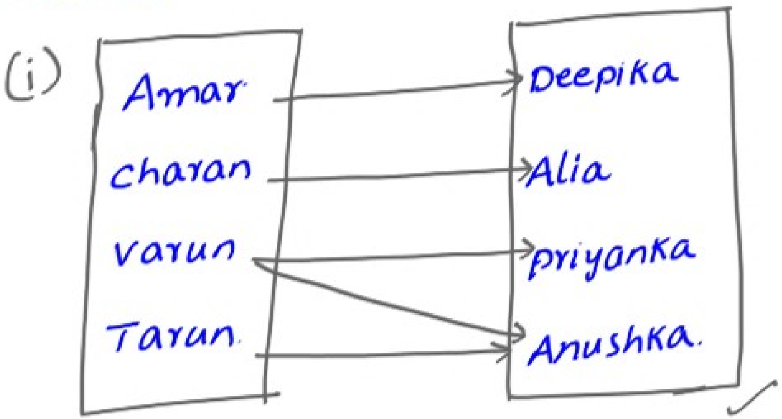
$\Rightarrow \; \exists y \exists x \; P(x,y)$

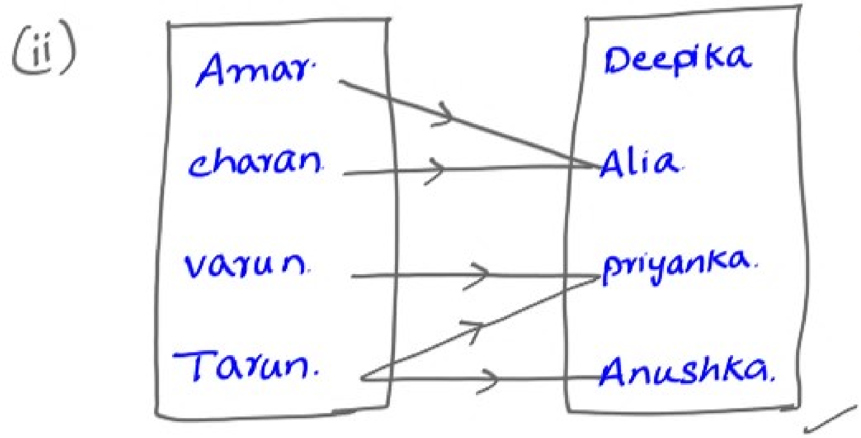
Conclusion:

$$\exists x \exists y \; P(x,y) \longleftrightarrow \exists y \exists x \; P(x,y)$$

Discussion - III

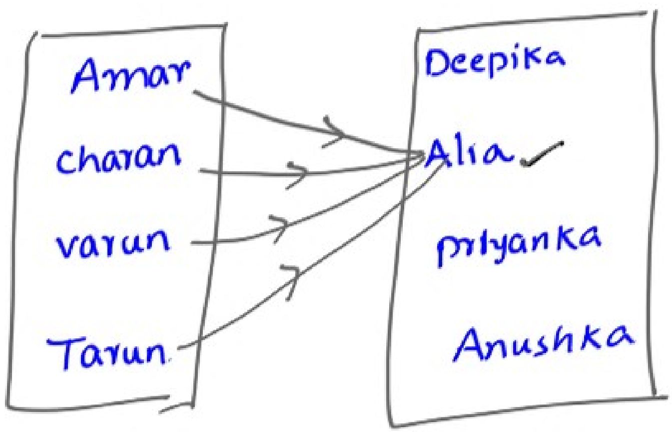
(2) $\forall x \, \exists y \; P(x,y)$: Every boy likes some girls

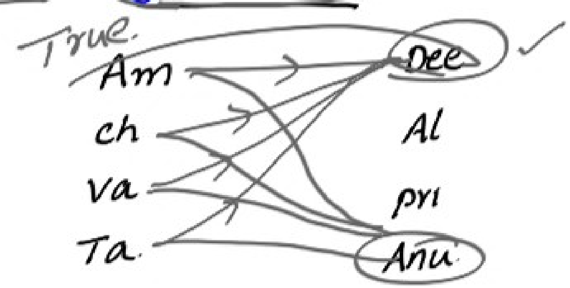




⑦ $\exists y \; \forall x \; P(x,y)$: There exists some y such that for every x $P(x,y)$ is TRUE

= Some girls are liked by all boys





Am → D, A
ch → D, A
V → D, A
T → D, A

② True

True

$\forall x \exists y \; P(x,y) \;\not\longrightarrow\; \exists y \; \forall x \; P(x,y)$

② NOT implied ⑦

Conclusion :

i) $\forall x\, \exists y\; P(x,y) \not\longrightarrow \exists y\, \forall x\; P(x,y)$

NOT logically implied

= if $\forall x\, \exists y\; P(x,y)$ is TRUE Then $\exists y\, \forall x\; P(x,y)$ may be TRUE or may not be TRUE (No gurantee)

(ii) $\exists y\, \forall x\; P(x,y) \longrightarrow \forall x\, \exists y\; P(x,y)$

= if $\exists y\, \forall x\; P(x,y)$ is Then $\forall x\, \exists y\; P(x,y)$ will be TRUE (always)

Logical Relationship diagram

$$
\begin{tikzcd}
\forall x \forall y \arrow[rr, leftrightarrow] \arrow[d] & & \forall y \forall x \arrow[d] \\
\exists x \forall y \arrow[d] & P(x, y) & \exists y \forall x \arrow[d] \\
\forall y \exists x \arrow[d] & & \forall x \exists y \arrow[d] \\
\exists x \exists y \ \text{True} \arrow[rr, leftrightarrow] & & \exists y \exists x \ \text{True}
\end{tikzcd}
$$

$\underline{\forall x \forall y}\ P(x,y) \longrightarrow \underline{\exists y \exists x}\ P(x,y)$ ✓

$\forall a \forall b$

$\exists a \forall b$

$\downarrow \quad \uparrow \times$

$\forall b \exists a$

Q. Consider the first-order logic sentence F : $\forall x[\exists y\ R(x, y)]$. Assuming non-empty logical domain, which of the sentences below are implied by F?

I. $\exists y(\exists x\ R(x, y))$

II. $\exists y\ (\forall x\ R(x, y)$

III. $\forall y(\exists x\ R(x, y))$

IV. $\sim \exists x(\forall y \sim R(x, y))$

a) IV only

b) I and IV only

c) II only

d) II and III only

$F : \forall x\ \exists y\ R(x,y)$

Let $R(x,y)$: x likes y , $x = \{\text{boys}\}$, $y = \{\text{girls}\}$

$F : \forall x\ \underline{\exists y}\ R(x,y)$: $\boxed{\text{Every boy likes Some girl}}$

I. ✓ $\exists y\ \exists x\ R(x,y)$: Some girls are liked by Some boy

II. ✗ $\exists y\ \forall x\ R(x,y)$: Some girls are liked by every boy

III. ✗ $\forall y\ \exists x\ R(x,y)$: Every girl is liked by some boy

IV. ✓ $\sim \exists x\ (\forall y \sim R(x,y)) = \forall x\ \exists y\ R(x,y) \equiv F$

$$\overbrace{\forall x\ \exists y\ R(x,y)}^{\text{True}} \longrightarrow$$

I

II.

III.

IV ✓

Method-II $\quad \forall x\, \exists y\; R(x,y)$ (Vs) $\exists y\, \forall x\; R(x,y)$

Let $R(x,y) = x+y = 10$

Here if $x=1$, Then $y=9$

$x=2 \qquad y=8$

$x=3 \qquad y=7$

$x=4 \qquad y=6$

$x=5 \qquad y=5$

$\forall x\, \exists y\; R(x,y)$

For every x There is some 'y' such that $R(x,y)$ is TRUE

Let $R(x,y) = x \times y = 0$

Here if $x=1$ then $y=0$

$x=2 \qquad y=0$

$x=3 \qquad y=0$

$x=4 \qquad y=0$

$\exists y\, \forall x\; R(x,y)$

There is some 'y' for every 'x' such that $R(x,y)$ is TRUE

Rules of Inference for Quantified Statements:

All the rules of inference for proposition formulas are also applicable for predicate calculus.

propositional formula.

i) $p \vee q \equiv q \vee p$

ii) $$\begin{array}{l} p \rightarrow q. \\ p \\ \hline q \end{array}$$

predicate formula.

ii) $p(x) \vee q(x) \equiv q(x) \vee p(x)$

$\forall x \, [p(x) \vee q(x)] \equiv \forall x \, [q(x) \vee p(x)]$

$$\begin{array}{l} \forall x \, [p(x) \longrightarrow q(x)] \\ \forall x \, p(x) \\ \hline \therefore \; \forall x \, q(x) \end{array}$$

we have four more rules for Quantified statements

I. Universal Instantiation

II. Universal Generalization

III. Existential Instantiation

IV. Existential Generalization

I. Universal Instantiation:

If a statement $\forall x\ p(x)$ is true then the universal quantifier can be dropped to for an arbitrary element 'C' from universe of discourse

$$\frac{\forall x\ p(x)}{\therefore\ p(C) \text{ for all } C}$$

for any C.

All Boys are Indians.

{ Niran, Ayush, Sharma, .. }

Sharama is an Indian

Example:

$$\frac{\textit{every man is mortal}, \quad \textit{socrates is a man}}{\therefore\ \textit{socrates is mortal}}$$

$\forall x P(x)$

$\Rightarrow P(x_1)$

II Universal Generalization:

If a statement P(C) is true for every element in the universe of discourse, Then $\forall x\, P(x)$ is true. The element 'C' is an arbitrary, not a specific element.

$$\frac{P(C) \text{ for all } C}{\therefore \forall (x)\, P(x)}$$

{ Niran, Vamsi, Sharma, Ayush ... }

$$\frac{P(c) \text{ is TRUE for every 'c'}}{\therefore \forall x\, P(x)}$$

ACE

III. Existential Insta~~llation~~: Instantiation (Specification)

ACE

If $\exists x\, P(x)$ is TRUE, Then we can conclude that there is an element 'C' in the universe of discourse for which P(C) is true.

$$\exists x\, P(x) \checkmark$$

$$\therefore P(C) \text{ for some } C \checkmark$$

Some Boys are Indians

{Niran, Sharma, Ayush, ... }

P(Ayush) = TRUE

IV Existential Generalization:

When a particular element ‘C’ with P(C) TRUE is known, then we can conclude that $\exists x\, P(x)$ is TRUE.

$$P(C) \text{ for some } C$$

$$\therefore \exists x\, P(x)$$
