Q. Let P, Q and R be three atomic prepositional assertions.

Let X denote $(P \vee Q) \rightarrow R$ and Y denote $(P \rightarrow R) \vee (Q \rightarrow R)$. Which one of the following is a tautology? **GATE – 2005**

(a) $X \equiv Y$

(b) $X \rightarrow Y$

(c) $Y \rightarrow X$

(d) $\sim Y \rightarrow X$

Q. Consider the following propositional statements:

$X \quad \cong \quad Y$

$$P1 : ((A \wedge B) \rightarrow C)) \equiv ((A \rightarrow C) \wedge (B \rightarrow C))$$

$$P2 : ((A \vee B) \rightarrow C)) \equiv ((A \rightarrow C) \vee (B \rightarrow C))$$

GATE – 2006

(a) P1 is a tautology, but not P2

(b) P2 is a tautology, but not P1

(c) P1 and P2 are both tautologies

(d) Both P1 and P2 are not tautologies

$P_1 : (A \wedge B) \rightarrow C \equiv (A \rightarrow C) \wedge (B \rightarrow C)$

$RHS = (A \rightarrow C) \wedge (B \rightarrow C)$

$\equiv (\sim A \vee C) \wedge (\sim B \vee C)$

$\equiv (\sim A \wedge \sim B) \vee C$

$\equiv \sim (A \vee B) \vee C$

$\equiv (A \vee B) \longrightarrow C$

$\not\equiv LHS$

$P_2 : (A \vee B) \rightarrow C \equiv (A \rightarrow C) \vee (B \rightarrow C)$

$RHS \equiv (A \rightarrow C) \vee (B \rightarrow C)$

$\equiv (\sim A \vee C) \vee (\sim B \vee C)$

$\equiv \sim A \vee C \vee \sim B \vee C$

$\equiv (\sim A \vee \sim B) \vee (C \vee C)$

$\equiv (\sim A \vee \sim B) \vee C$

$\equiv \sim (A \wedge B) \vee C$

$\equiv (A \wedge B) \longrightarrow C \quad \not\equiv LHS$

P_1 & P_2 Both are NOT Tautologies

Q. A logical binary relation Θ, is defined as follows: $\odot$

A	B	A Θ B
True	True	True
True	False	True
False	True	False
False	False	True

$\sim$

Let be the unary negation (NOT) operator, with higher precedence than Θ. Which one of the following is equivalent to $\boxed{A \wedge B}$? **GATE – 2006**

(A) $(\sim A \;\Theta\; B)$

(B) $\sim(A \;\Theta \sim B)$

(C) $\sim (\sim A \;\Theta \sim B)$

(D) $\sim (\sim A \;\Theta\; B)$

$\sim A \odot B$

$\sim A$

$\sim A \odot B$

Q. P and Q are two propositions. Which of the following logical expressions are equivalent?

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I. $P \vee \sim Q$

II. $\sim(\sim P \wedge Q) \equiv P \vee \sim Q$

III. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

IV. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge Q)$

(a) Only I and II

(b) Only I, II and III

(c) Only I, II and IV

(d) All of I, II, III and IV

III. $(P \wedge Q) \vee (P \wedge \sim Q) \vee (\sim P \wedge \sim Q)$

$\equiv [P \wedge (Q \vee \sim Q)] \vee (\sim P \wedge \sim Q)$

$\equiv [P \wedge (t)] \vee (\sim P \wedge \sim Q)$

$\equiv P \vee (\sim P \wedge \sim Q)$

$\equiv (P \vee \sim P) \wedge (P \vee \sim Q)$

$\equiv (t) \wedge (P \vee \sim Q)$

$\equiv P \vee \sim Q$

IV $(P\wedge Q)\vee(P\wedge\sim Q)\vee(\sim P\wedge Q)$

$$\equiv [P\wedge(Q\vee\sim Q)]\vee(\sim P\wedge Q)$$

$$\equiv [P\wedge(t)]\vee(\sim P\wedge Q)$$

$$\equiv P\vee(\sim P\wedge Q)$$

$$\equiv (P\vee\sim P)\wedge(P\vee Q)$$

$$\equiv (t)\wedge(P\vee Q)$$

$$\equiv P\vee Q$$

[ ]

$(P\wedge Q)\vee(P\wedge\sim Q)\vee(\sim P\wedge Q)$

$[P\wedge(Q\vee\sim Q)]$

$[P\wedge(t)]$

$(P)\vee(\sim P\wedge Q)$

Q. Which one of the following propositional logic formulas is TRUE when exactly two of p, q and r are **TRUE?** **(GATE-14-Set1)**

(a) $((p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$

(b) $(\sim (p \leftrightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$

(c) $((p \rightarrow q) \wedge r) \vee (p \wedge q \wedge \sim r)$

(d) $(\sim (p \leftrightarrow q) \wedge r) \wedge (p \wedge q \wedge \sim r)$

Exactly two of p, q, r are TRUE

p	q	r	
T	T	F	✓
T	F	T	✓
F	T	T	✓

(a) $[(p \longleftrightarrow q) \wedge r] \vee (p \wedge q \wedge \sim r)$

$[(T \leftrightarrow T) \wedge F] \vee (T \wedge T \wedge T) = T$

$[(T \leftrightarrow F) \wedge T] \vee (T \wedge F \wedge F) = F$

$[(F \longleftrightarrow T) \wedge T] \vee (F \wedge T \wedge F) = F$

(b) $[\sim(p \leftrightarrow q) \wedge r] \vee (p \wedge q \wedge \sim r)$

$[\sim(T \leftrightarrow T) \wedge F] \vee (T \wedge T \wedge T) = T$

$[\sim(T \leftrightarrow F) \wedge T] \vee (T \wedge F \wedge F) = T$

$[\sim(F \leftrightarrow T) \wedge T] \vee (F \wedge T \wedge F) = T$

Which one of the following Boolean expressions is **NOT** a tautology?

(GATE-14-Set2)

(a) $((a \rightarrow b) \wedge (b \rightarrow c)) \rightarrow (a \rightarrow c)$

(b) $(a \leftrightarrow c) \rightarrow (\sim b \rightarrow (a \wedge c))$

(c) $(a \wedge b \wedge c) \rightarrow (c \vee a)$

(d) $a \rightarrow (b \rightarrow a)$

(d) $\overset{T}{a} \longrightarrow \overset{F}{(b \longrightarrow a)}$

$\overset{T}{T} \longrightarrow \underbrace{(b \rightarrow T)}_{T}$ = Tautology

Which one of the following is **NOT** equivalent to $p \leftrightarrow q$?

(a) $(\sim p \vee q) \wedge (p \vee \sim q)$

(b) $(\sim p \vee q) \wedge (q \rightarrow p)$

(c) $(\sim p \wedge q) \vee (p \wedge \sim q)$

(d) $(\sim p \wedge \sim q) \vee (p \wedge q)$

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$\equiv (\sim p \vee q) \wedge (q \rightarrow p)$ = option (b)

$\equiv (\sim p \vee q) \wedge (\sim q \vee p)$ = option (a)

$\equiv$

we know that

$p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$ = option (d)

Q. Let p, q, r, s represent the following propositions. **(GATE-16-Set1)**

ACE

p: $x \in \{8, 9, 10, 11, 12\}$

q: x is a composite number

r: x is a perfect square

s: x is a prime number

The integer $x \geq 2$ which satisfies $\sim((p \Rightarrow q) \wedge (\sim r \vee \sim s))$ is _______.

p : $x \in \{8, 9, 10, 11, 12\}$

q : composite $= \{4, 6, 8, 9, 10, 12, 14, 15, \ldots\}$, $\sim q = \{1, 2, 3, 5, 7, 11, 13, \ldots\}$

r : perfect square $= \{1, 4, 9, 16, 25, \ldots\}$

s : prime number $= \{2, 3, 5, 7, 11, 13, \ldots\}$

$x \geq 2$, $\sim[(p \rightarrow q) \wedge (\sim r \vee \sim s)]$ is $\underline{11}$

$$\equiv \sim[(\sim p \vee q) \wedge (\sim r \vee \sim s)]$$

$$\equiv \underline{(p \wedge \sim q)} \vee (r \wedge s)$$

$$\equiv \{11\} \vee \phi$$

$$= \{11\}$$

Q. The statement $(\sim p) \Rightarrow (\sim q)$ is logically equivalent to which of the statements below? **(GATE-17-Set1)**

I. $p \Rightarrow q$

II. $q \Rightarrow p$

III. $(\sim q) \vee p$

IV. $(\sim p) \vee q$

(a) I only

(b) I and IV only

(c) II only

(d) II and III only

Given

$\sim p \to \sim q$

$\sim p \to \sim q \equiv q \to p$ ✓

(By contra positive law)

$\sim p \to \sim q \equiv q \to p$

$\equiv \sim q \vee p$

$$a \to b \equiv \sim a \vee b \equiv \sim b \to \sim a$$

$$\sim p \to \sim q \equiv \sim(\sim q) \to \sim(\sim p)$$

$$\equiv q \to p \quad (\text{II})$$

$$\equiv \sim q \to p \quad (\text{III})$$

Q. Let p and q be propositions. Decide whether $(p \leftrightarrow q)$ does not imply $(p \rightarrow \sim q)$ is true or false GATE – 94

Sol Approach

Example: A implies B

$A \longrightarrow B$

when A is TRUE Then B will be TRUE

= when A is TRUE Then B cannot be FALSE

$p \leftrightarrow q$

$\therefore p \rightarrow \sim q$

Sol

$(p \longleftrightarrow q) \implies (p \longrightarrow \sim q)$ [checking]

$(p \longleftrightarrow q)$ $\quad$ $(p \longrightarrow \sim q)$

$T \longleftrightarrow T = \text{True}$ $\quad$ $T \longrightarrow F = \text{False}$

$(p \longleftrightarrow q)$ doesnot implies $(p \longrightarrow \sim q)$

$(p \longleftrightarrow q) \not\Longrightarrow (p \longrightarrow \sim q)$

example : X implies Y

$X \Longrightarrow Y$

when $X =$ True then $Y =$ True

Question : check $(P \longleftrightarrow q) \Longrightarrow (P \longrightarrow \sim q)$

$(P \longleftrightarrow q)$

$\underbrace{T \longleftrightarrow T}_{T}$

True

$(P \longrightarrow \sim q)$

$\underbrace{(T \longrightarrow F)}_{F}$

False

True $\not\Longrightarrow$ False

$(P \longleftrightarrow q)$ does not implies $(P \longrightarrow \sim q)$: True

Well-Formed Formula (WFF):

A well-Formed Formula (WFF) can be defined recursively as follows:

$p = (p \wedge p)$

1. If 'p' is propositional variable then it is a WFF.
2. If 'x' is WFF Then $\sim x$ is also WFF.
3. If 'x' and 'y' are WFF then $x \vee y$, $x \wedge y$, $x \rightarrow y$, $x \leftrightarrow y$ are also WFF.
4. Any string of Symbol, obtained by finitely many applications of rule (1) to rule (3) above is a WFF.

$[(\quad t)] \wedge (p \rightarrow q)$

$(p \wedge q \wedge r \wedge s) = p$

$[\wedge (p \rightarrow q)$

$p \sim q$ ✗

Q. which of the following are well-formed formulea ?

a) $\sim P \wedge Q$: WFF ✓

b) $\sim (P \wedge Q)$: WFF ✓

c) $(P \wedge Q) \Longrightarrow Q$: WFF ✓

d) $(P \Longrightarrow Q) \Longrightarrow (\wedge Q)$: NOT WFF ✓

e) $(P \wedge Q) \Longrightarrow Q)$: NOT WFF ✓

$a \vee b \to c$ ✗ $(a \wedge b) \to c$

$(a \vee b) \to c$ ✓

$a \vee (b \to c)$ ✓

$(P \wedge Q) \longrightarrow Q$

$(P \wedge Q) \longrightarrow (Q)$ ✓

$[(P \wedge Q) \longrightarrow (Q)]$

$[(P \wedge Q) \longrightarrow Q]$ ✓

$[P \wedge Q \longrightarrow Q]$

Normal Forms:

The standardization of given propositional formulea is known as Normal Forms. It is very difficult to compare logical expressions like P and Q, when there are too many propositional variables. Normal forms are helpful to compare logical expressions either they tautology or contradiction or equivalent, etc.

Different types of Normal forms are (canonical forms)

i. DNF (Disjunctive Normal Form)

ii. CNF (Conjunctive Normal Form)

iii. PDNF (Principle Disjunctive Normal Form)

iv. PCNF (Principle Conjunctive Normal Form)

elementary Sum : Disjunction of propositional variables or their Negations

$p, \sim p, p \vee q, \sim p \vee \sim q, \sim p \vee q, p \vee \sim q$

elementary product : conjunction of propositional variables or their negations

$p, \sim p, q, \sim q, p \wedge q, \sim p \wedge \sim q, \sim p \wedge q, p \wedge \sim q, p \wedge q \wedge \sim r,$