Graph Theory

A graph G is defined as a pair of sets (V, E) where

V = set of all vertices (nodes) in G and

E = set of all edges in G

|V(G)| = Number of vertices in graph G

= Order of G $= v = n$

|E(G)| = Number of edges in graph G

= Size of the graph $= e$

I. Basic concepts

II. planar graphs

III Graph coloring

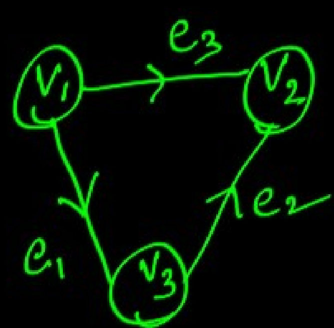
IV Isomorphism

V Matchings, coverings, Independent sets

VI Connectivity

VII etc.

Directed graph (diagraph): The elements of E are ordered pairs of vertices. In this case an edge (u, v) is said to be from u to v.



an edge

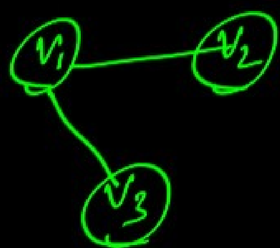
(x, y)

(u, v)

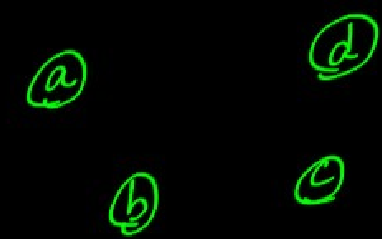
$\{u, v\}$ undirected

Non-directed graph:

The elements of E are unordered pairs (sets) of vertices. In this case an edge {u, v} is said to join u and v (or) to be between u and v.



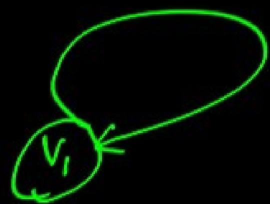
Null graph: A null graph of order n is a graph with n vertices and no edges.



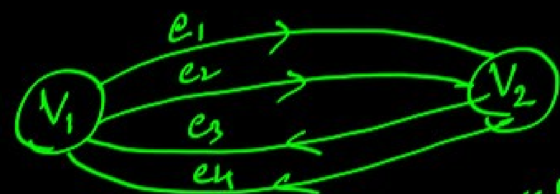
Trivial graph: A null graph with only one vertex is called trivial graph.

v_1

Loop: An edge drawn from a vertex to itself is called a loop.



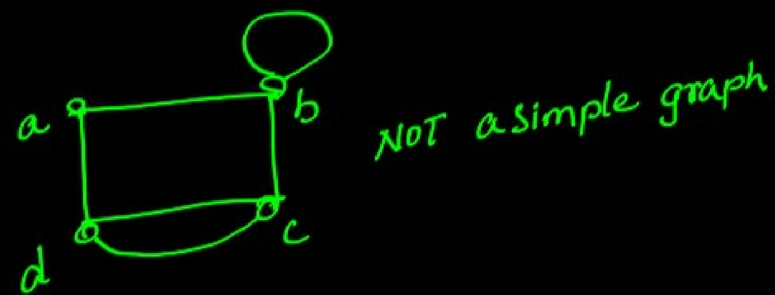
Parallel edges: In a graph, if a pair of vertices are joined by more than one edge then those edges are called parallel edges.

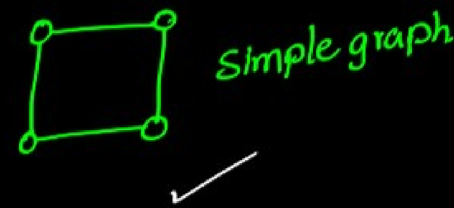


e_1, e_2, e_3, e_4 are parallel edges

Multi-graph: If one allows more than one edge to join a pair of vertices, the resulting graph is then called a multi graph.

Simple graph: A graph with no loops and no parallel edges is called a simple graph.

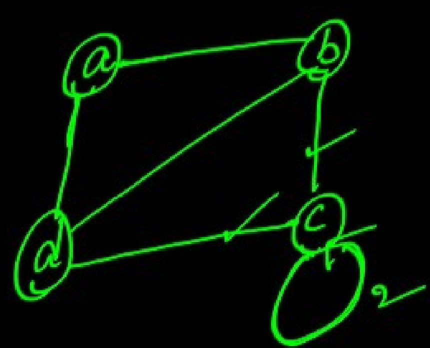




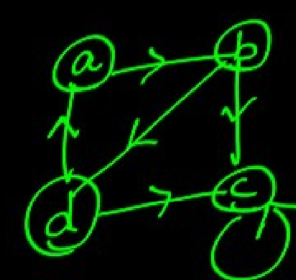
Degree: Degree of a vertex in an undirected graph is the number of edges incident with it, except that a loop at a vertex contributes twice to the degree of that vertex.

The degree of the vertex 'v' is denoted by deg(v).

Note: In a simple graph with n vertices, degree of any vertex cannot exceed (n – 1)



deg(a) = 2
deg(b) = 3
deg(c) = 4
deg(d) = 3



vertex	IN	OUT
a →	1	1
b →	1	2
c →	3	1
d →	+ 1	+ 2
	6	= 6

In degree an out degree: In a directed graph an edge (u, v) is said to be incident from u, and to be incident to 'v'.

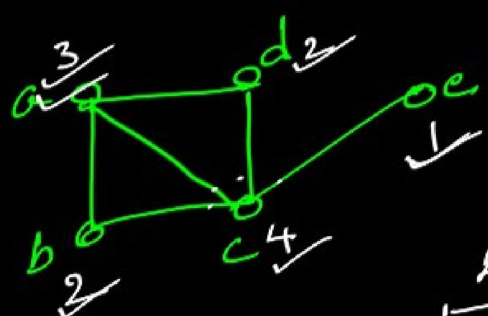
Within a particular diagraph, the number of edges incident on a vertex is called the **in-degree** of the vertex and the number of edges emerging from the vertex is called its **out-degree.**

- The in-degree of a vertex 'v' in a graph G is denoted by $deg^{+}(v)$. The out-degree of a vertex v is denoted by $deg^{-}(\cancel{G})$. $deg^{-}(v)$
- A loop in a diagraph is counted once, for both indegree and outdegree of the vertex.

Degree Sequence: If v_1, v_2, ……, v_n are the vertices of a graph G, then the sequence $\{d_1, d_2, \ldots, d_n\}$

Where

d_i = degree of v_i is called the degree sequence of G.Usually we order the degree sequence so that the degree sequence is monotonically decreasing.



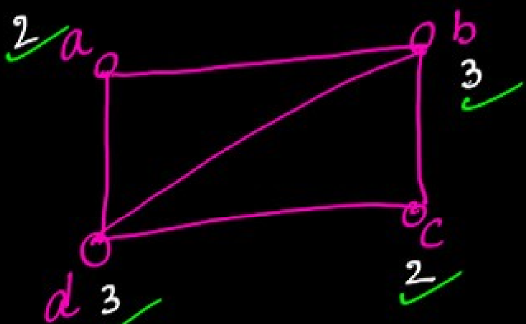
4, 3, 2, 2, 1

$\boxed{4, 3, 2, 2, 1}$

Sum of Degree Theorem:

If $V = \{v_1, v_2, \ldots, v_n\}$ is the vertex set of a non directed graph G then

$$\sum_{i=1}^{n} \deg(v_i) = 2|E|$$



$\overset{2}{\deg(a)} + \overset{2}{\deg(b)} + \overset{2}{\deg(c)} + \overset{2}{\deg(d)}$

$3 + 3 + 3 + 3$

$= 2 + 3 + 2 + 3$

$= 10$

$= 2(5)$

$= 2e$

$\underline{4(2)}$ $\quad 4(3)$

$8 \quad 12$

$8 \leq 10 \quad 12 \geq 10$

$8 \leq 10 \leq 12$

$\Sigma \deg(v_i) = 2e$

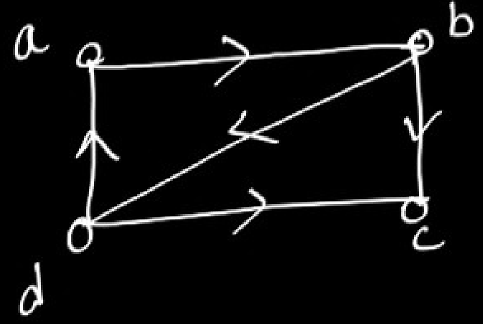
$\text{Min} \deg(a) = \delta(a) = K$

$K + K + \text{---} \ (n \text{ times}) \leq 2e$

$\boxed{K \cdot n \leq 2e}$

$\text{Max.} \deg(a) = \Delta(a) = K$

$\boxed{K \cdot n \geq 2e}$



vertex	IN deg	OUT deg
$a \longrightarrow$	1	1
$b \longrightarrow$	1	2
$c \longrightarrow$	2	0
$d \longrightarrow$	$+\ 1$	$+\ 2$
	5	$=\ 5\ =\ e$

$$\boxed{\sum_{i=1}^{n} \deg^{+}(v_i) = \sum_{i=1}^{n} \deg^{-}(v_i) = |E|}$$

Corrolories:

1. For directed graph

$$\sum_{i=1}^{n} \deg^{+}(v_i) = \sum_{i=1}^{n} \deg^{-}(v_i) = |E| \quad e$$

2. ~~Every~~ undirected graph has an even number of vertices of odd degree. = No. of odd degree vertices in a non-directed graph is even

3. If G is a k-regular graph, then

   $k.|V| = 2.|E|$ $\quad k \cdot n = 2e$

4. If $k = \Delta(G)$ is the maximum degree of all vertices in a undirected graph G, then $k.|V| \geq 2.|E|$

   $k \cdot n \geq 2e$

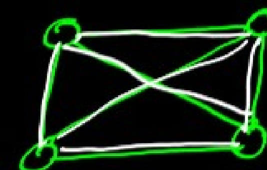
5. If $k = \delta(G)$ is the minimum degree of all vertices in a undirected graph G, then $k.|V| \leq 2.|E|$

   $k \cdot n \leq 2e$

6. $\delta(G).\ |V| \leq 2|E| \leq \Delta(G)\ .\ |V|$

$\delta(G) \cdot |V| \leq 2e \leq \Delta(G) \cdot |V|$

$8 \leq 10 \leq 12$



$4C_2 = nC_2$

3, 3, 2, 2 = 2

3, 3, 3, 3 = 4

k = 3

1, 1, 0, 0 =	No. odd degree 2
1, 1, 1, 1	4
2, 2, 1, 1	2
2, 2, 2, 2	0

Q. What is the number of vertices in an undirected connected graph with 27 edges, 6 vertices of degree 2, 3 vertices of degree 4 and remaining of degree 3?

6 3 x

(GATE)

a) 10 $e = 27$

b) 11

c) 18

d) 19 ✓

6 vertices → 2 deg
3 vertices → 4 deg
Remaing (x) → 3 deg

Sum of deg, $\sum deg(v_i) = 2e$

$6(2) + 3(4) + x(3) = 2(27)$

$12 + 12 + 3x = 54$

$3x = 30$

$x = 10$

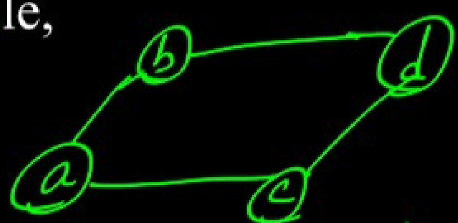
$v = n = |V(G)| = 6 + 3 + x$

$= 19$

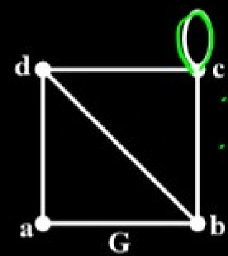
Neighbors: If there is an edge incident from u to v, or incident on u and v, then u and v are said to be **adjacent** (or to be neighbors).

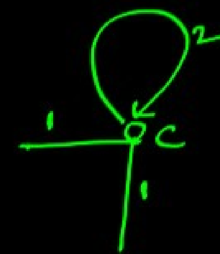
Minimum of all the degrees of vertices in a graph G is denoted by $\boldsymbol{\delta(G)}$. *

Maximum of all the degrees of vertices in a graph G is denoted by $\boldsymbol{\Delta(G)}$. * For example,



Neighbours (a) = b, c





For the graph shown above $|V| = 4$ and $|E| = 5$

Degree of a = 2

Degree of b = 3

Degree of c = 4

Degree of d = 3

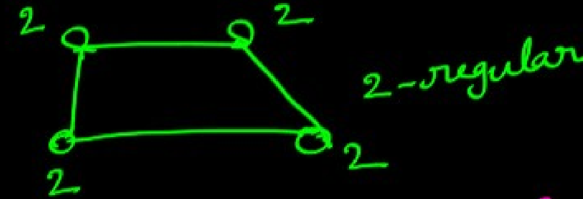
There is a loop at vertex c

$\delta(G) = 2$ and $\Delta(G) = 4$

Regular graph: If each vertex of G has degree k, then G is said to be a **regular graph** of degree k (k-regular). For example,

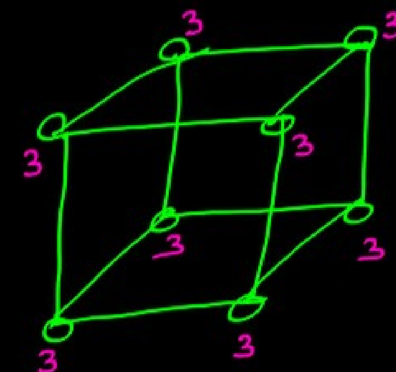
A polygon is a 2-regular graph

And A 3-regular graph is a cubic graph.



In a graph G, if $\delta(G) = \Delta(G) = k$ then G is a regular graph. For example,

The following graphs are regular



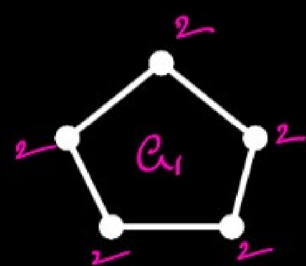
$\boxed{K \cdot n = 2e}$

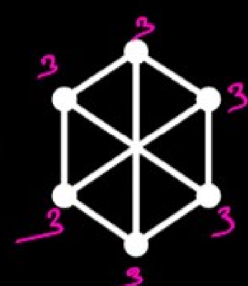
G_1

$2+2+2+2+2$

$2 \cdot 5$

10

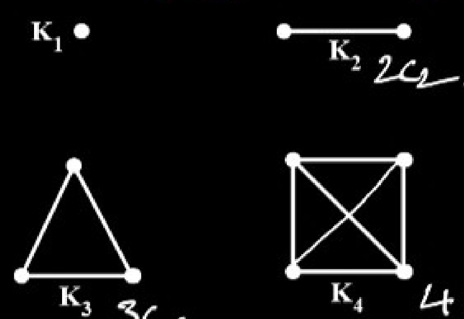


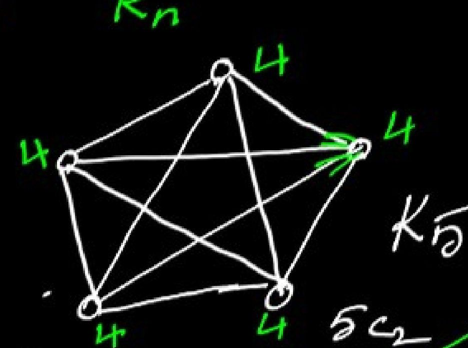


Complete Graph: A simple non directed graph with 'n' mutually adjacent vertices is called a completed graph on n vertices and it is denoted by K_n.

$$^{n}C_r = \frac{n!}{r!(n-r)!}$$

$$^{5}C_2 = \frac{5!}{2!\,3!} = 10$$





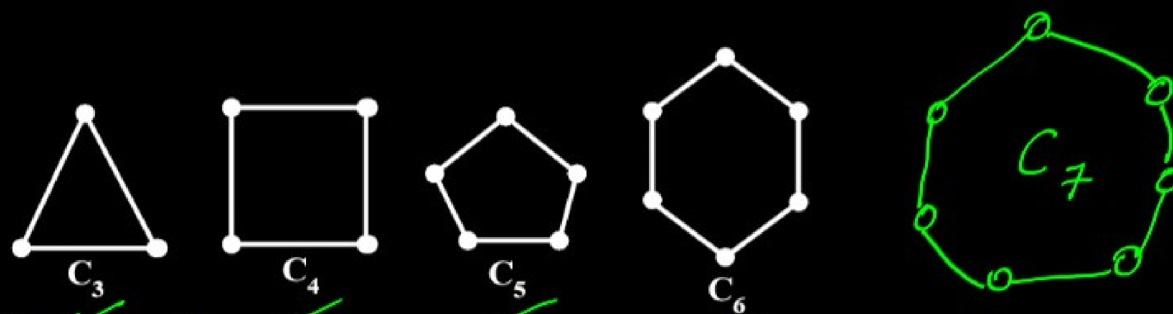
Note: Every complete graph K_n is a regular graph and each of its vertices has degree '$n - 1$'.

Note: Number of edges in a complete graph $K_n = C(n, 2) = n(n - 1)/2$

Note: A completed graph K_n is a simple graph with n vertices and maximum number of edges.

Cycle graph: A cycle graph of order $n (n \geq 3)$ is a simple connected graph whose edges form a cycle of length n.

Its denoted by C_n.



even cycles = $C_4, C_6, C_8, ---$

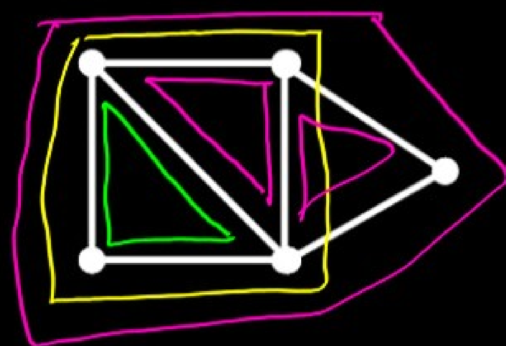
odd cycles = $C_3, C_5, C_7, ---$

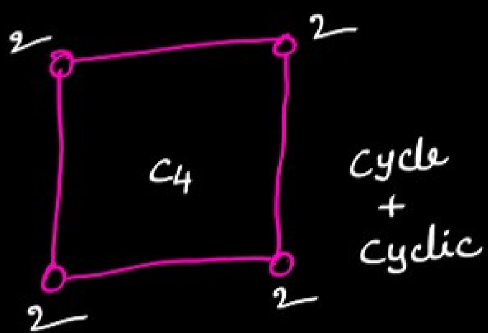
n vertices with n-edges

exactly one cycle

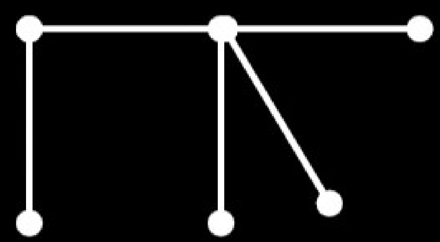
Note: A cycle graph 'C_n' of order n has n vertices and n edges.

Cyclic graph: A graph with atleast one cycle is called cyclic graph.





Acyclic graph: A simple graph having no cycles is called acyclic graph.



Connected graph: An undirected graph G is called connected if there is a path between every pair of distinct vertices in G.

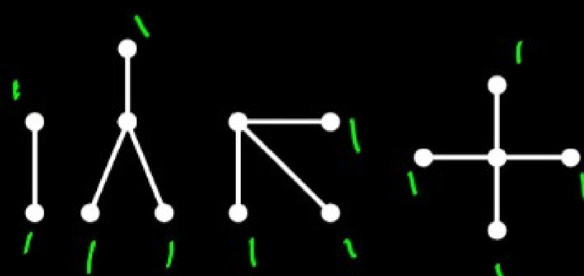
Note: A graph which is not connected has atleast two connected components.

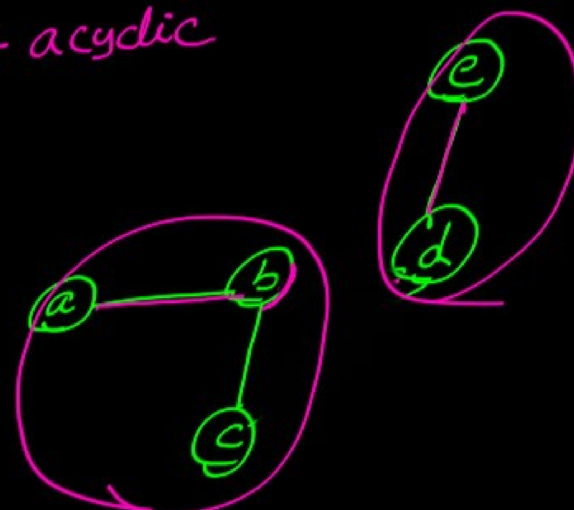
Tree: A connected graph with no cycles is called a tree. connected + acyclic

Note: A tree with n vertices has (n – 1) edges.

Note: A tree with n(n > 1) vertices has atleast two vertices of degree 1.

Ex:

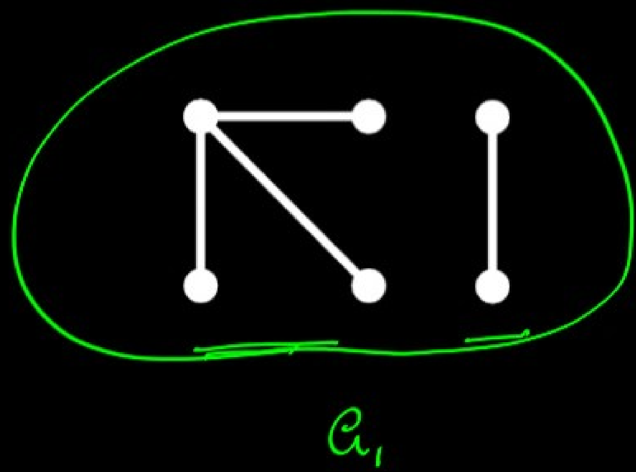




a, e

Note: An acyclic graph which is not connected is called a **forest.**

Ex:



Example:

Q. What is the maximum number of edges in an acyclic undirected graph with n vertices? (GATE)

a) n – 1 ✓

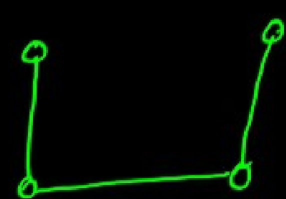
b) n

c) n + 1

d) 2n – 2

acyclic = No cycle

n=4



3 edges

Wheel graph: A wheel graph of order $\boxed{n(n \geq 4)}$ can be obtained from a cycle graph C_{n-1} by adding a new vertex (the hub) which is adjacent to all vertices of C_{n-1}.

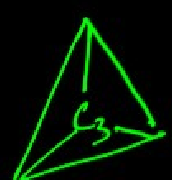
It is denoted by W_n.

$\omega_{100} \longrightarrow C_{99}$

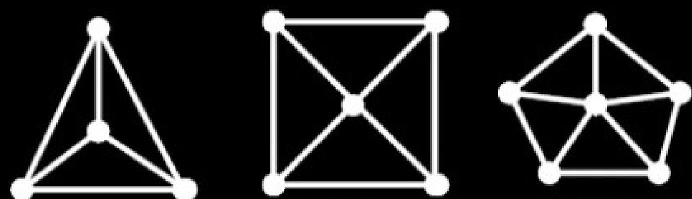
$\omega_7 \longrightarrow C_6$

Note: A wheel graph W_n has 'n' vertices and $2(n-1)$ edges.

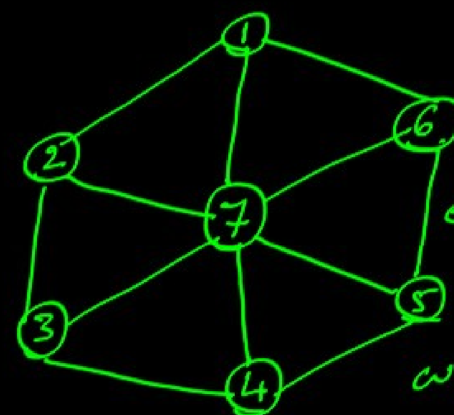
Ex:



$2(3)$ ω_4



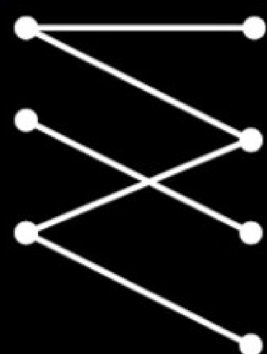
W_4 ✓ W_5 ✓ W_6 ✓



$e = 2(n-1)$
$= 2(7-1)$
$= 12$

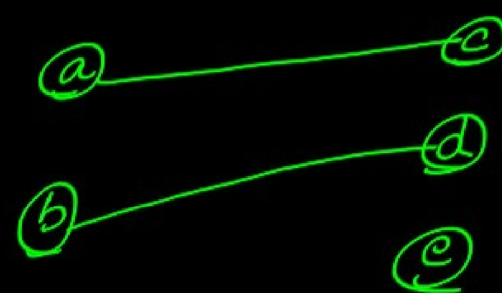
$\omega 7$

Bipartite graph: A bipartite graph G is a simple non directed graph whose set of vertices can be partitioned in to two sets M and N in such a way that each edge of G joins a vertex in M to a vertex in N.



$V(G) = \{a, b, c, d, e\}$

$M = \{a, b\}$ $N = \{c, d, e\}$



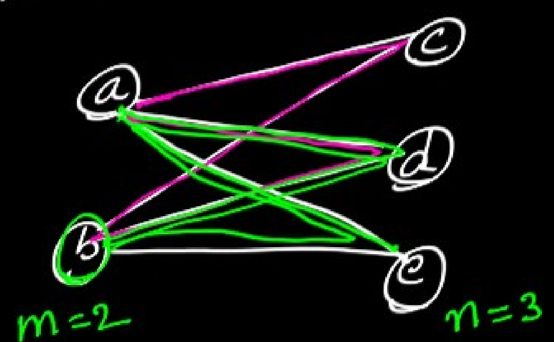
Complete bi-partiate : $[K_{m,n}]$

A bi-partiate graph which is complete is known as complete bi-partiate graph.

Let G be a graph of order 5.

$V(G) = \{a, b, c, d, e\}$

$M = \{a, b\}$ $\quad N = \{c, d, e\}$



K_5

$e = m \times n = 2 \times 3 = 6$

Properties:

- If $|M| = m$ and $|N| = n$ then the complete bipartite graph is denoted by $K_{m,n}$.
- $K_{m,n}$ has '$m + n$' vertices and 'mn' edges.
- In general, a complete bipartite graph is not a complete graph. $K_{2,3} \not\equiv K_5$
- $K_{m,n}$ is a complete graph $\Leftrightarrow$ $(m = n = 1)$

  $K_{1,1} \equiv K_2$
- G is a bipartite graph $\Leftrightarrow$ G has no cycles of odd length.

- Maximum number of edges possible in a bipartite graph with n vertices is $\left\lfloor \frac{n^2}{4} \right\rfloor$.

$\left\lfloor \frac{n^2}{4} \right\rfloor$ $n = 7$

$n = v = 7$

$e = m \times n.$

$m = 1$

$n = 6 \quad = 1 \times 6 = 6$

$m = 2$

$n = 5 \longrightarrow 2 \times 5 = 10$

$3 \qquad 4 \longrightarrow 3 \times 4 = 12.$

$4 \qquad 3$

$\frac{n}{2} \times \frac{n}{2}$

$5 \qquad 2$

$6 \qquad 1$

$\left\lfloor \frac{n^2}{4} \right\rfloor$

Q. The maximum number of edges in a bipartite graph on 12 vertices is ________ (GATE)

Max. No. of edges $= \left\lfloor \frac{n^2}{4} \right\rfloor = \left\lfloor \frac{12^2}{4} \right\rfloor = \left\lfloor \frac{12 \times 12}{4} \right\rfloor = 36$

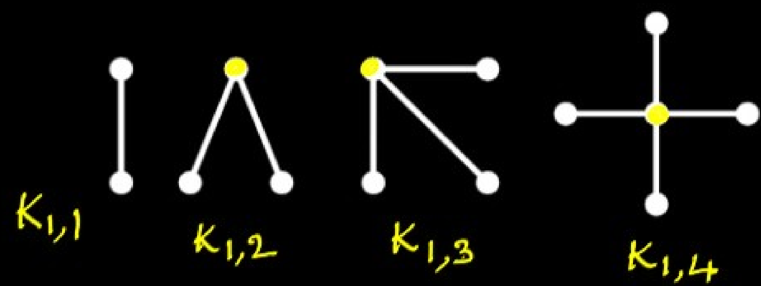
(or)

12

$e = m \times n$

$\lvert M \rvert = m = 1$	$\times$	$\lvert N \rvert = n = 11$	$= 11$ ✓
2	$\times$	10	$= 20$ ✓
3	$\times$	9	$= 27$ ✓
4	$\times$	8	$= 32$ ✓
5	$\times$	7	$= 35$ ✓
6	$\times$	6	$= 36$ ✓

Star graph: A complete bipartite graph of the form $K_{1,\,n-1}$ is called a star graph.
For example,

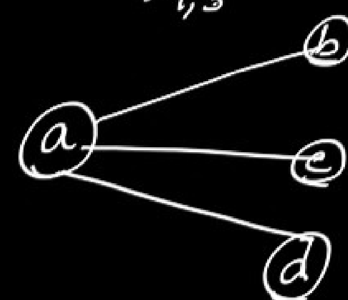


$K_{m,n}$

$K_{1,\,n-1}$

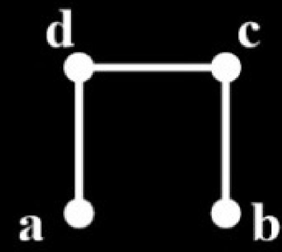
$K_{1,4}$

$K_{1,3}$

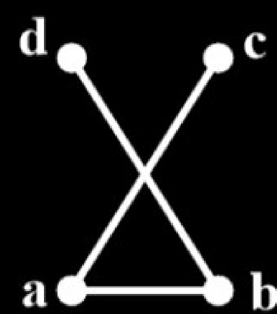


Note: Every star graph is a tree

Complement of a graph: If G is a simple graph then **complement** of G is a simple graph $\overline{G}$ with the same vertices as G and an edge exists in $\overline{G}$ iff it does not exist in G.

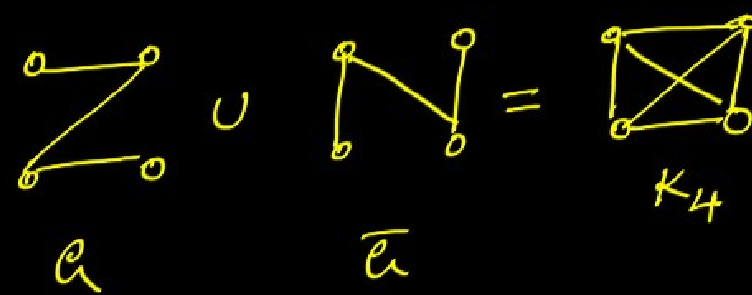


G



$\overline{G}$

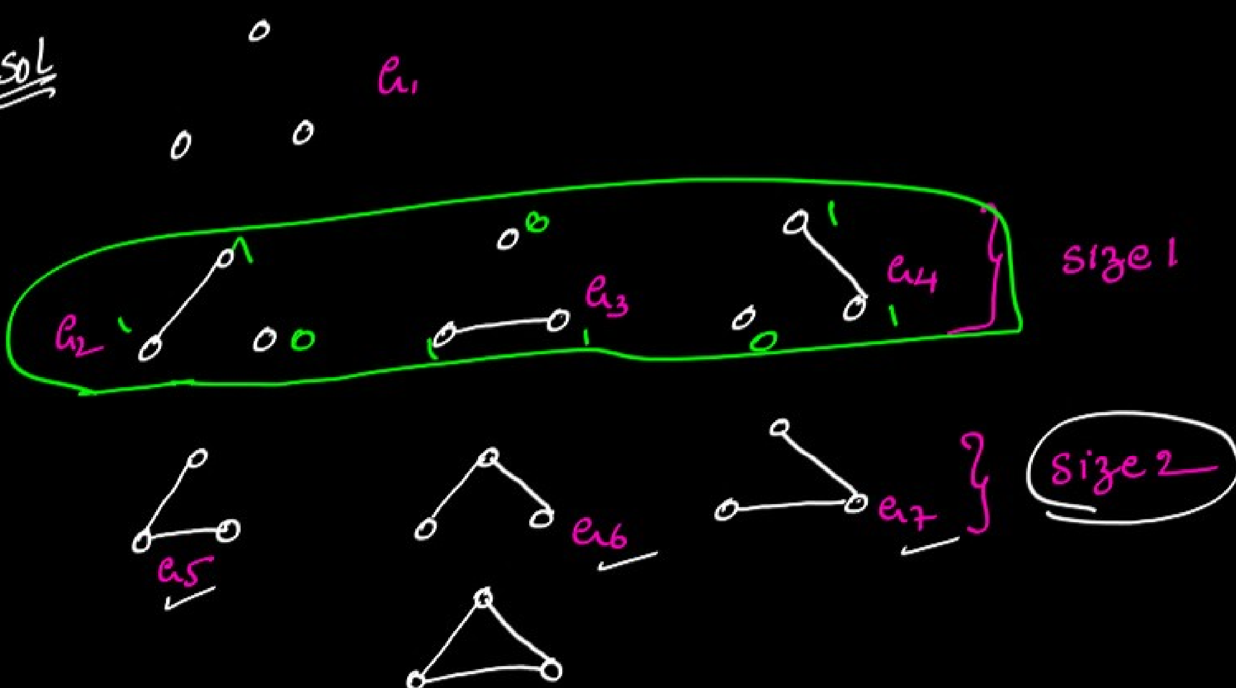
Note: $|E(G)| + |E(\overline{G})| = |E(K_n)|$ where $n = |V(G)|$



Concept- Building Example:

* Draw all possible Simple graph with 3 nodes

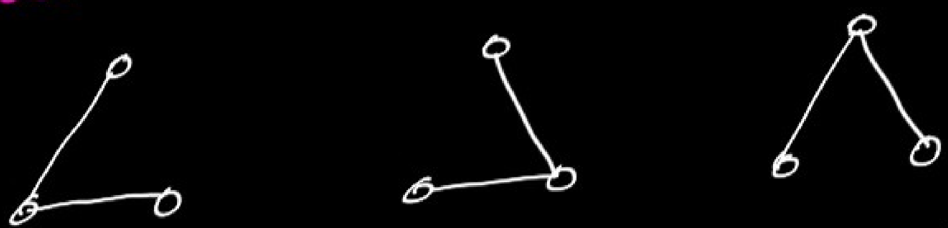
Sol



possible ways $= {}^3C_2$

$$\text{Total} = 2^{{}^3C_2} = 2^{{}^nC_2}$$

* Draw possible Number different Simple graphs of size 2 with 3 nodes



when there 3 nodes, Max possible edges = $nC_2 = 3C_2 = 3$

If required size is 2, then it will be $3C_2$

* How many simple graphs are possible with order 5 and size 3

Max. possible edges with 5 nodes = $5C_2 = 10$

No. of size 3 graphs = $10C_3$

* Number of Simple graphs possible with n-nodes of size 'm'

order = n
size = m

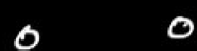
$$\boxed{{}^{{}^{n}C_2}C_m}$$

* Find the possible number of different simple graphs up to 3 nodes

<u>case-1:</u> $n=1$

G_1

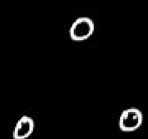
<u>case-2:</u> $n=2$



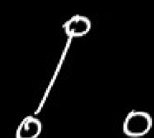
G_2

G_3

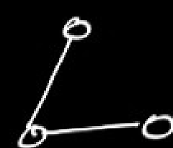
<u>case-3:</u> $n=3$



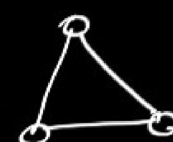
G_4



G_5



G_6



G_7

7

Havel-Hakimi Result:

Consider the following two sequences and assume that the sequence (i) is in descending order

i) s, $t_1, t_2, \ldots\ldots t_s, d_1, d_2, \ldots\ldots, d_n$ ✓ 10

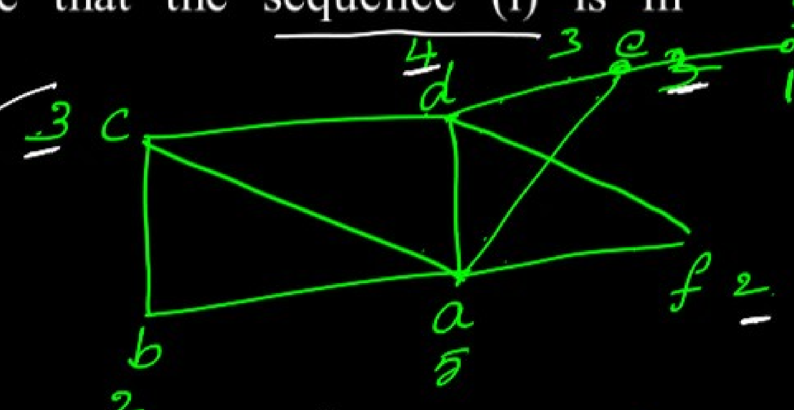
ii) $t_1 - 1, t_2 - 1, \ldots., t_s - 1, d_1, d_2, \ldots., d_n$ ✓ 9

Sequence (i) is graphic $\Leftrightarrow$ sequence (ii) is graphic.

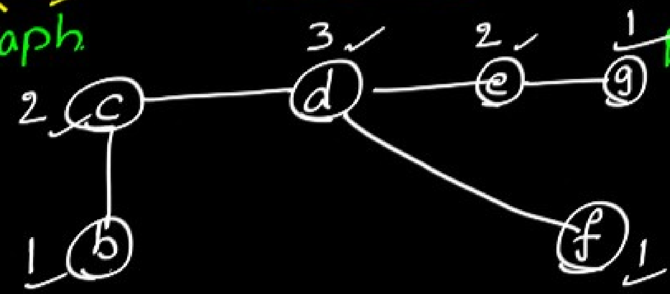
$\Longleftrightarrow$

forms a graph

(i)



(ii)



Deg seq: 5, 4, 3, 3, 2, 2, 1

5, 4, 3, 3, 2, 2, 1

$-1 \quad -1 \quad -1 \quad -1 \quad -1$

$\Rightarrow$ 3, 2, 2, 1, 1, 1

Vamsi Krishna

12 yrs

10 yrs

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Q. The degree sequence of a simple graph is the sequence of the degrees of the nodes in the graph in decreasing order. Which of the following sequences can not be the degree sequence of any graph? **(GATE)**

I. 7, 6, 5, 4, 4, 3, 2, 1

II. 6, 6, 6, 6, 3, 3, 2, 2

III. 7, 6, 6, 4, 4, 3, 2, 2

IV. 8, 7, 7, 6, 4, 2, 1, 1

a) I and II

b) III and IV

c) IV only

d) II and IV

I. 7, 6, 5, 4, 4, 3, 2, 1

Deg sequence

$\cancel{7}, \underset{-1}{6}, \underset{-1}{5}, \underset{-1}{4}, \underset{-1}{4}, \underset{-1}{3}, \underset{-1}{2}, \underset{-1}{1}$

$\Rightarrow \cancel{5}, 4, 3, 3, 2, 1, 0$

$\Rightarrow \cancel{3}, 2, 2, 1, 0, 0$

$\Rightarrow \cancel{1}, 1, 0, 0, 0$

$\Rightarrow 0, 0, 0, 0$ Graphic

II. 6, 6, 6, 6, 3, 3, 2, 2

$\cancel{6}, 6, 6, 6, 3, 3, 2, 2$

$\Rightarrow 5, 5, 5, 2, 2, 1, 2$

$= \cancel{5}, 5, 5, 2, 2, 2, 1$ (Decreasing)

$\Rightarrow \cancel{4}, 4, 1, 1, 1, 1$

$\Rightarrow 3, 0, 0, 0, 1$

$\Rightarrow \cancel{3}, 1, 0, 0, 0$ (Decreasing)

$\Rightarrow 0, -1, -1, 0$

NOT graphic

III. $\cancel{7}, 6, 6, 4, 4, 3, 2, 2$

$\Rightarrow \cancel{5}, 5, 3, 3, 2, 1, 1$

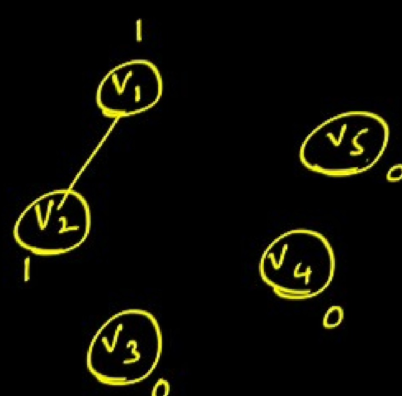
$\Rightarrow 4, 2, 2, 1, 0, 1$

$= \cancel{4}, 2, 2, 1, 1, 0$

$\Rightarrow \cancel{1}, 1, 0, 0, 0$

$\Rightarrow 0, 0, 0, 0$

Graphic.

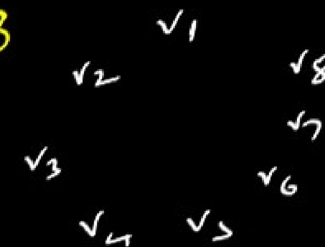


IV. $8, 7, 7, 6, 4, 2, 1, 1$

$= \cancel{8}, 7, 7, 6, 4, 2, 1, 1$

$|V(G)| = \vartheta = n = 8$

NOT graphic



I, II form a graph

II, IV doesnot forms a graph.

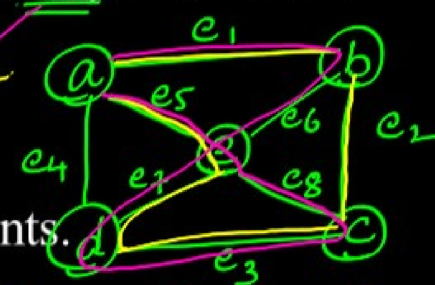
Cycle (Vs) Circuit

Note:

Path = 6 P^2 $P6$

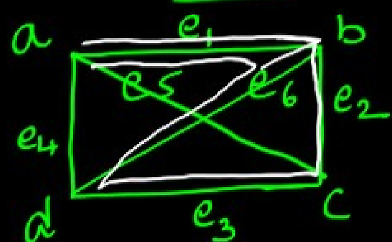
- A path of length ≥ 1 with no repeated edges and whose end points are equal is called a **circuit.**
- A circuit may have repeated vertices other than its end points.
- A **cycle** is a circuit with no other repeated vertices except its end points.
- A **loop** is a cycle of length 1.
- In a simple graph, a cycle that is not a loop must have length at least 3.

circuit ✓
NOT cycle



$a-b-e-d-c-e-a$ ✓

$[e_1-e_6-e_7-e_3-e_8-e_5]$



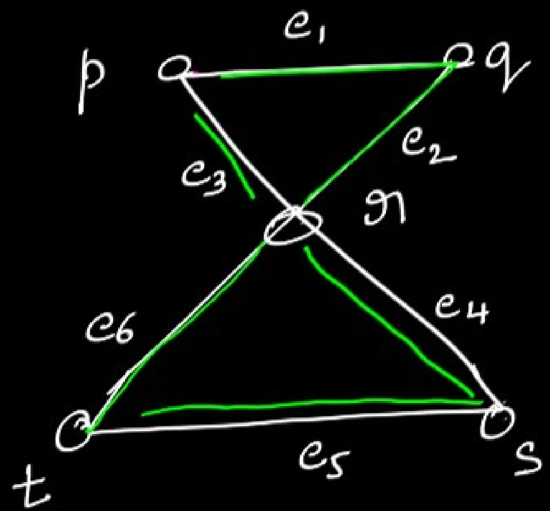
$a-b-c-d-b-a$ = ~~Circuit~~ NOT a circuit

$[e_1-e_2-e_3-e_6-e_1]$

e_1 - edge repeated

NOT circuit

$a-b-c-d-a$ = cycle



p - q - r - p = circuit = cycle

p - q - r - t - s - r - p = Circuit
NOT cycle.

Eulerian (Vs) Hamiltonian

Euler & Hamiltonian:

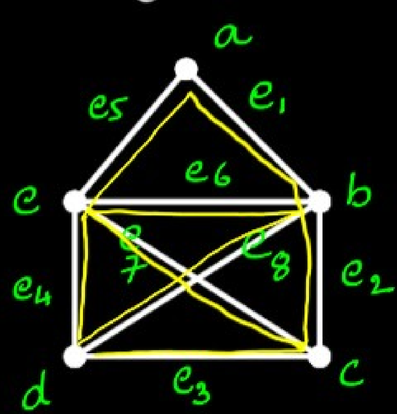
Euler Graph: If a graph consist Euler path then it is known as Euler graph.

Euler Path: Each edge exactly once, each vertex atleast once.

Euler Circuit: Starting and end vertices same

* In a graph if every vertex has even degree then their must exist Euler path

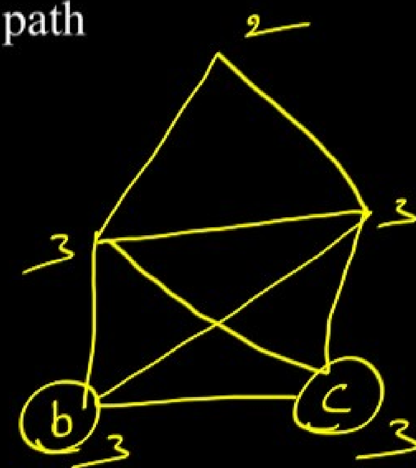
path (or) walk consists all edges of given graph [edge repetition is NOT allowed]



Starting - d

ending - c

Eulerian.

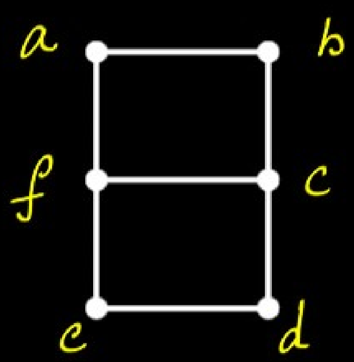


Hamiltonian graph: A graph with Hamiltonian cycle is known as Hamiltonian

Hamiltonian cycle: A cycle which covers all vertices of the graph

p-q-r-p

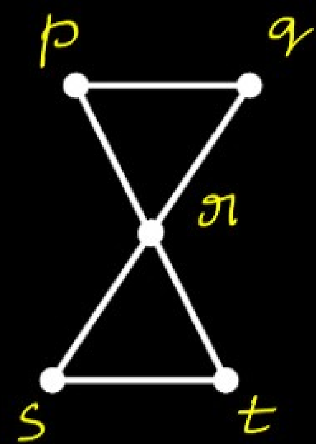
Ex:



CBOT

OR

TSP



path = a-b-c-d-e-f-a = circuit = Cycle

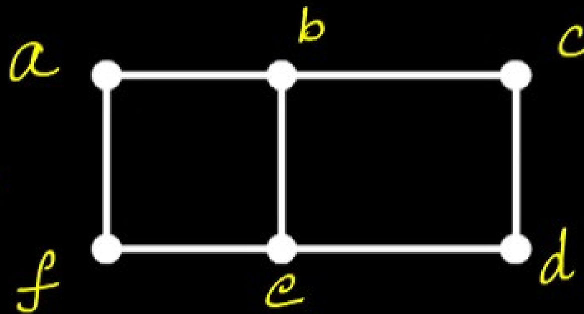
Hamiltonian

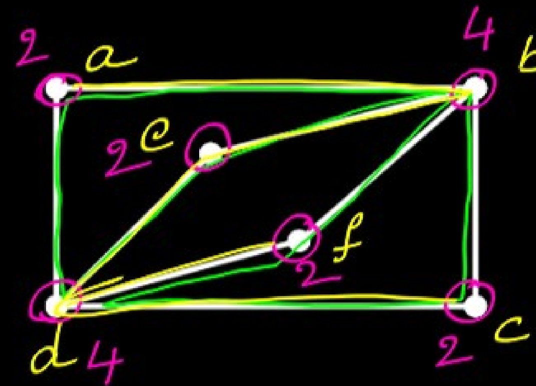
No Hamiltonian cycle

⇒ Hamitonian

Which of the following are Hamiltonian and Euler path.

G_1 :





G_2

degree of vertex is even.

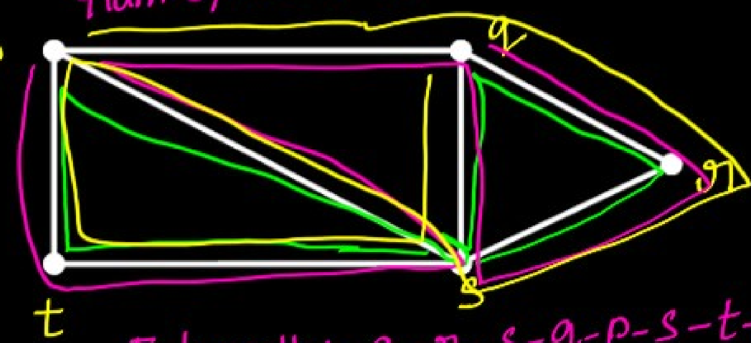
∴ Eulerian.

Euler path: b-c-d-f-b-e-d-a-b

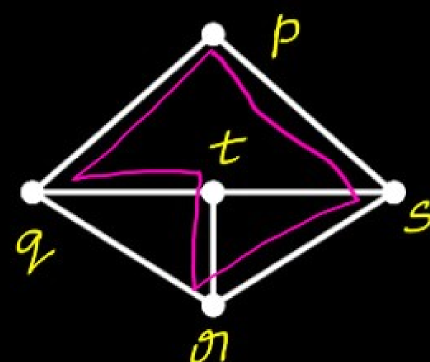
Not Hamiltonian.

Ham. cycle = p-q-r-s-t-p

G_3



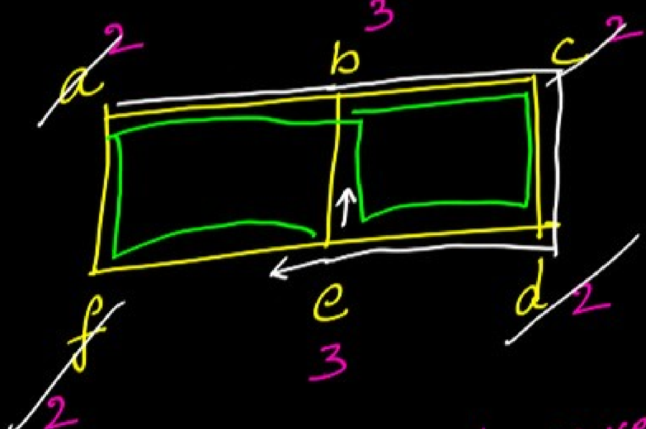
Euler path: q-r-s-q-p-s-t-p



: G_4

Ham. cycle: p-q-t-r-s-p

G_1



There are two 3-degree vertices
and four 2-degree vertices

path = b-c-d-e-b-a-f-e
= Euler-path
= Euler graph

Hamiltonian cycle
= a-b-c-d-e-f-a.

G_1 is both Eulerian & Hamiltonians ✓

G_2 is Eulerian, but NOT Hamiltonian. ✓

G_3 is Eulerian, & Hamiltonian. ✓

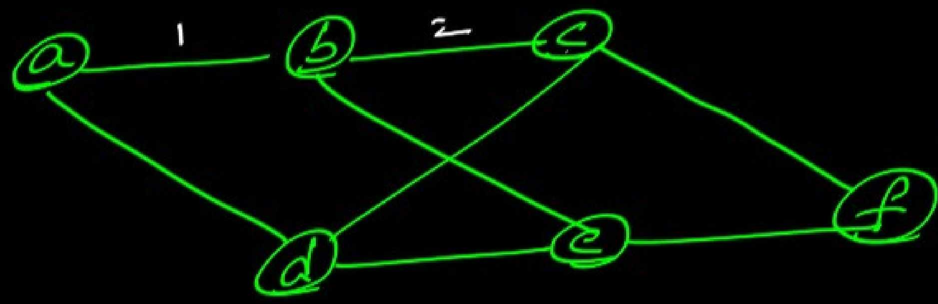
G_4 is Hamiltonian, but not Eulerian. ✓

MINIMAL SPANNING TREE (MST)

Tree : A connnected graph without a cycle
[acyclic & connected]
n-vertices with (n-1) edges

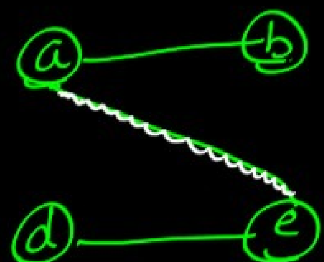
Spanning Tree : Let G be graph, and T is a subgraph of G
and T is called Spanning if It consists all vertices of 'G'
ord(G) = ord(T)

Minimal Spanning Tree : A spanning tree which has Minimum cost



G.

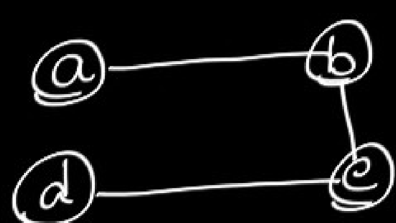
T_1 :



T_1 is NOT subgraph

adjacency NOT preserved

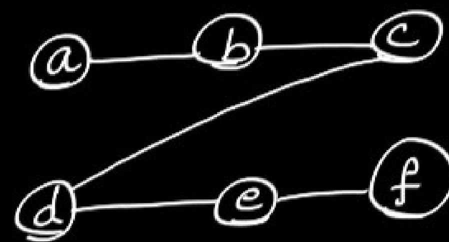
T_2 :



Tree,

NOT spanning

T_3 :

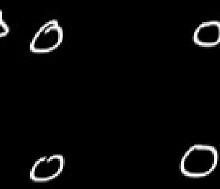


Tree

Spanning Tree

* No. of spanning trees possible on n-nodes $= n^{n-2}$
(ISRO)

$n=4$

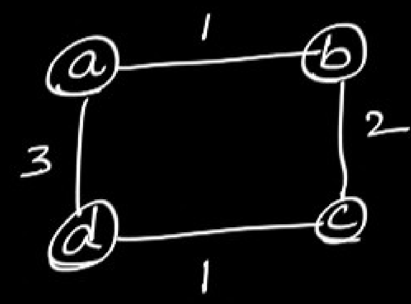


* There are two algorithms to find Minimal spanning trees

i) Krushka's algorithm

ii) Prim's algorithm

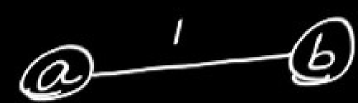
Kruskal's algorithm:



Step-1: Select the edge which consists Minimum of all edge weight.

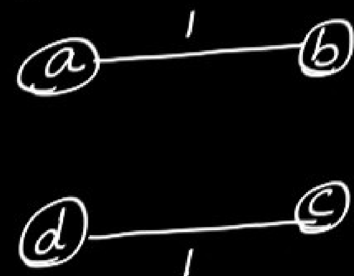
cost $\{a,b\} = 1^{*}$; cost $\{b,c\} = 2$; cost $\{c,d\} = 1^{*}$, cost $\{a,d\} = 3$

Take any edge $\{a,b\}$ or $\{c,d\}$



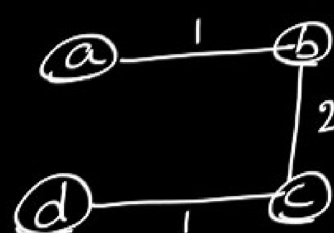
Step-2: Select the edge which Minimum cost of are remaining edges

cost $\{b,c\} = 2$; cost $\{c,d\} = 1^*$; cost $\{a,d\} = 3$



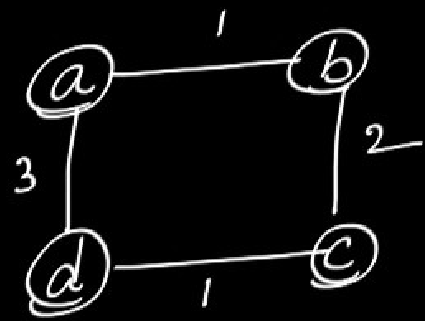
(Disconnected)

Step-3: Repeat step-2 until we reach $(n-1)$ edges



$\text{Cost}(MST) = 1+1+2 = 4$ units

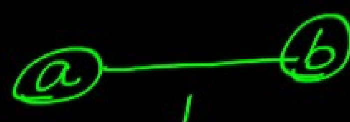
Prim's algorithm:



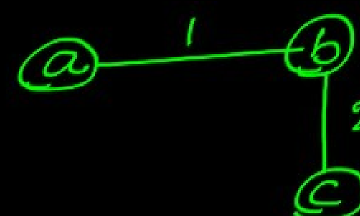
Pr Kr

Step-1: Select the edge which consists Minimum cost of all edges

cost $\{a,b\} = 1^{*}$, cost $\{b,c\} = 2$, cost $\{c,d\} = 1^{*}$, cost $\{a,d\} = 3$

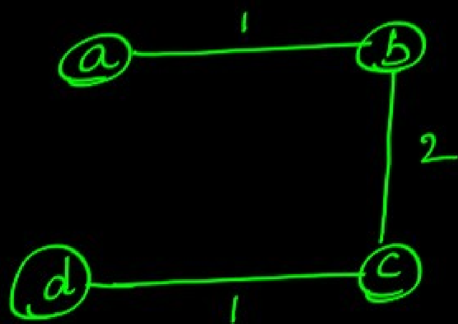


Step-2: Expand the existing nodes in the previous step and select the edge which has Minimum cost



(connected)

Step-3: Repeat Step-2 untill we get (n-1) edges



$MST_{cost} = 1+2+1 = 4$ units

Cost (MST) is unique, But MST is NOT unique

Common Data

Consider a complete undirected graph with vertex set {0, 1, 2, 3, 4}. Entry W_{ij} in the matrix W below is the weighted of the edge {i, j} **(GATE)**

$$
W = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array}
\begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 8 & 1 & 4 \\ 1 & 0 & 12 & 4 & 9 \\ 8 & 12 & 0 & 7 & 3 \\ 1 & 4 & 7 & 0 & 2 \\ 4 & 9 & 3 & 2 & 0 \end{bmatrix} \end{array}
\qquad \text{cost}\{2,3\} = 7
$$

Q. What is the minimum possible weight of a spanning tree T in this graph such that vertex 0 is a leaf node in the tree T?

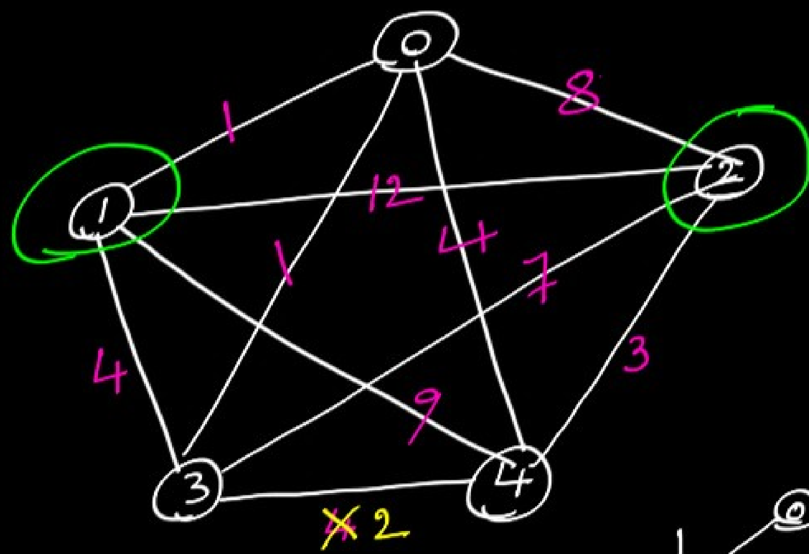
a) 7

b) 8

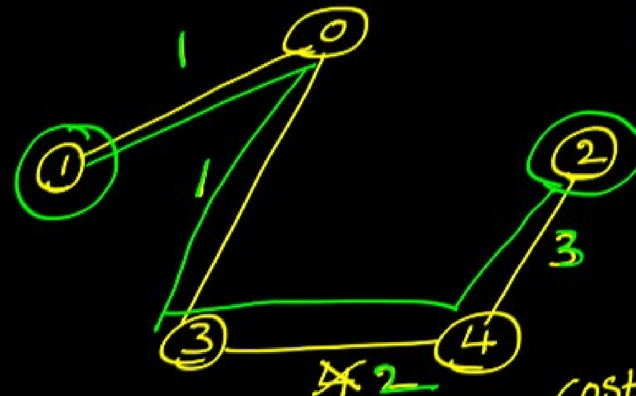
c) 9

d) 10

$V = \{0, 1, 2, 3, 4\}$



with out any condition

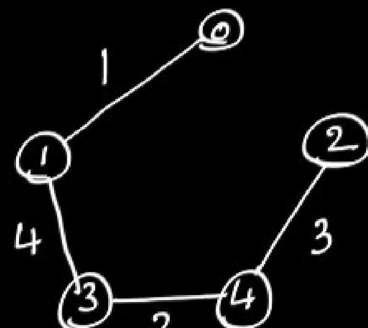


leaf 1, 2

$cost(MST) = 1 + 1 + \cancel{4}^{2} + 3$

$= 7$ units

with condition



$Cost(MST)_0 = 1 + 2 + 3 + 4 = 10$ units

Q. What is the minimum possible weight of a path P from vertex 1 to vertex 2 in this graph such that P contains at least 3 edges?

a) 7

b) 8

c) 9

d) 10

path from vertex '1' to vertex '2' such that

path length $(P^L) \geq 3$

path $= 1-0-3-4-2 = 1+1+2+3 = \underline{7 \text{ units}}$