Q. which of the following are well-formed formulea ?

a) $\sim P \wedge Q$: WFF

b) $\sim (P \wedge Q)$: WFF

c) $\underline{(P \wedge Q) \Longrightarrow Q}$: WFF

d) $(P \Longrightarrow Q) \Longrightarrow (\wedge Q)$: NOT WFF

e) $(P \wedge Q) \Longrightarrow Q)$: NOT WFF

$a \vee b \rightarrow c$ ✗ $(a \wedge b) \rightarrow c$

$(a \vee b) \rightarrow c$ ✓

$a \vee (b \rightarrow c)$ ✓

(e)

$(P \wedge Q) \longrightarrow Q$

$(P \wedge Q) \longrightarrow (Q)$ ✓

$[(P \wedge Q) \rightarrow (Q)]$

$[(P \wedge Q) \rightarrow Q]$ ✓

$[P \wedge Q \longrightarrow Q]$

Normal Forms:

The standardization of given propositional formulea is known as Normal Forms. It is very difficult to compare logical expressions like P and Q, when there are too many propositional variables. Normal forms are helpful to compare logical expressions either they tautology or contradiction or equivalent, etc.

① $(p \wedge q \wedge \sim r) \vee (\sim p \wedge q \wedge r) = P$

② $[(p \wedge q) \to r] \wedge [(p \wedge q) \wedge \sim r] = Q$

Different types of Normal forms are (canonical forms)

i. DNF (Disjunctive Normal Form)

ii. CNF (Conjunctive Normal Form)

iii. PDNF (Principle Disjunctive Normal Form)

iv. PCNF (Principle conjunctive Normal Form)

elementary Sum : Disjunction of propositional variables or their Negations

p, $\sim p$, $p \vee q$, $\sim p \vee \sim q$, $\sim p \vee q$, $p \vee \sim q$

elementary product : conjunction of propositional variables or their negations

p, $\sim p$, q, $\sim q$, $p \wedge q$, $\sim p \wedge \sim q$, $\sim p \wedge q$, $p \wedge \sim q$, $p \wedge q \wedge \sim r$,

$$P \equiv P \vee P \equiv P \wedge P \equiv (P \wedge P) \vee (P \wedge P) \equiv (P \vee P) \wedge (P \vee P)$$

DNF: A logical expression expression is said to in disjunctive normal form if it is a Sum of elementary product.

eg: P,

$(p \wedge q) \vee (q \wedge r)$,

$(\sim p \wedge r) \vee (\sim q \wedge \sim r)$,

$(p \wedge \sim q) \vee (p \wedge \sim r) \vee (q \wedge r)$

$$\begin{array}{ccc} P & & Q \\ \downarrow & & \downarrow \\ PDNF & \equiv & PDNF \end{array}$$

PDNF: A DNF is said to be PDNF iff It consists all propositional variables in each term

eg: If there are three variables in given formula, (p, q, r)

SOP i) $(p \wedge q) \vee (q \wedge r)$: DNF, But NOT PDNF

ii) $(p \wedge q \wedge r) \vee (\sim p \wedge \sim q \wedge \sim r)$: PDNF

iii) $(p \wedge q \wedge \sim r) \vee (p \wedge \sim q \wedge r) \vee (\sim p \wedge \sim q \wedge r)$: PDNF

iv) $(p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge \sim r) \vee (p \wedge \sim q \wedge \sim r)$

CNF : A logical expression is said to be in conjunctive normal form, If It is a product of elementary Sums

eg: P, ✓

$(p \vee q) \wedge (\sim p \vee \sim q)$ ✓ , $(p \vee q) \wedge (p \vee r) \wedge (q \vee \sim r)$ ✓

PCNF : A CNF is called as PCNF If It consists all propositional variables in every term of It's formula.

eg: If there are three variables, Then

$(p \vee q \vee r) \wedge (\sim p \vee \sim q \vee \sim r) \wedge (\sim p \vee q \vee r)$

Q obtain the DNF of the formula $P \wedge (P \Rightarrow Q)$

Sol $P \wedge (P \rightarrow Q) \equiv P \wedge (\sim P \vee Q)$

$$\equiv (P \wedge \sim P) \underbrace{\vee}_{\text{Sum}} (P \wedge Q)$$

Q: obtain DNF of propositional formula $p \wedge (p \to q)$

Sol

$$p \wedge (p \to q) \equiv p \wedge (\sim p \vee q)$$

$$\equiv (p \wedge \sim p) \vee (p \wedge q)$$

Above formula is in the form of Sum of products

It is Required DNF

$$p \wedge (p \to q) = p \wedge (\sim q \to \sim p)$$

$$\equiv p \wedge (\sim p \vee q) \quad = p \wedge (\sim(\sim p) \to \sim(\sim q))$$

Q obtain CNF of $P \vee (\sim P \wedge Q)$

Sol $P \vee (\sim P \wedge Q) \equiv \underbrace{(P \vee \sim P) \wedge (P \vee Q)}$

$\downarrow$

product

Q obtain the CNF of $P \vee (\sim P \wedge Q)$

Sol $P \vee (\sim P \wedge Q) \equiv (P \vee \sim P) \wedge (P \vee Q)$

$\equiv$ The above formula is in product of sums

It is required CNF.

Q obtain PDNF of the formula $p \wedge (q \vee r)$

Sol Given $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$
T	T	T	T	T *
T	T	F	T	T *
T	F	T	T	T *
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

$(p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (p \wedge \sim q \wedge r)$

$\equiv (p \wedge q \wedge r) \vee (p \wedge q \wedge r') \vee (p \wedge q' \wedge r)$

Method-II :

Given : $p \wedge (q \vee r)$

$\equiv \boxed{(p \wedge q)} \vee (p \wedge r)$

Here we have two terms, In first term there is no 'r', In the second term there is no 'q'

Take first term $= (p \wedge q)$

$= (p \wedge q \wedge t)$

$= [p \wedge q \wedge (r \vee \sim r)]$

$= [(p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r)]$

Take Second term

$(p \wedge r) = (p \wedge r \wedge t)$

$= [p \wedge r \wedge (q \vee \sim q)]$

$= (p \wedge r \wedge q) \vee (p \wedge r \wedge \sim q)$

Given : $p \wedge (q \vee r)$

$= (p \wedge q) \vee (p \wedge r)$

$= [(p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r)]$

$\vee [(p \wedge r \wedge q) \vee (p \wedge r \wedge \sim q)]$

$\equiv (p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (p \wedge \sim q \wedge r)$

Q obtain PCNF of the formula $p \vee (q \wedge r)$

Sol $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F ✓	T ✓	F ✓	F	F *
F ✓	F ✓	T ✓	F	F *
F ✓	F ✓	F ✓	F	F *

Required PCNF is

$(p \vee \sim q \vee r) \wedge (p \vee q \vee \sim r) \wedge (p \vee q \vee r)$

Method - II :

$$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$$

$$= (p \vee q \vee f) \wedge (p \vee r \vee f)$$

$$= [p \vee q \vee (r \wedge \sim r)] \wedge [p \vee r \vee (q \wedge \sim q)]$$

$$= (p \vee q \vee r) \wedge (p \vee q \vee \sim r) \wedge (p \vee r \vee q) \wedge (p \vee r \vee \sim q)$$

$$= (p \vee q \vee r) \wedge (p \vee q \vee \sim r) \wedge (p \vee \sim q \vee r)$$

Q: Find Disjunctive Normal form and conjunctive Normal form of formula $f(P, Q, R)$ defined as

P	Q	R	f(P, Q, R)
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

The Required PDNF

$(P' \wedge Q' \wedge R') \vee (P' \wedge Q' \wedge R)$
$\vee (P \wedge Q' \wedge R') \vee (P \wedge Q \wedge R')$

The Required PCNF is

$(P \vee Q' \vee R) \wedge (P \vee Q' \vee R') \wedge$
$(P' \vee Q \vee R') \wedge (P' \vee Q' \vee R')$

Q Obtain DNF of formula:

$$p \to [(p \to q) \wedge \sim(\sim q \vee \sim p)]$$

$$\equiv p \to [(\sim p \vee q) \wedge (q \wedge p)]$$

$$\equiv \sim p \vee [(\sim p \vee q) \wedge q \wedge p]$$

$$\equiv \sim p \vee [(\sim p \wedge q \wedge p) \vee (q \wedge q \wedge p)]$$

$$\equiv \sim p \vee [(\sim p \wedge p \wedge q) \vee (q \wedge q \wedge p)]$$

$$\equiv \sim p \vee [f \vee (q \wedge p)]$$

$$\equiv (\sim p) \vee [q \wedge p]$$

$$\equiv \sim p \vee (p \wedge q)$$

sum

$$\equiv (\sim p \wedge \sim p) \vee (p \wedge q)$$ ✓

Obtain DNF of the formula

$$p \to [(p \to q) \wedge \sim(\sim q \vee \sim p)]$$