Q. Let $S = \{1, 2, 3, \ldots, m\}$, $m > 3$. Let $X_1, X_2, \ldots, X_n$ be subsets of S each of size $\boxed{3}$. Define a function f from S to the set of natural numbers as, f(i) is the number of sets X_j that contains the element i. That is $f(i) = |\{j \mid i \in X_j\}|$.

Then $\sum_{i=1}^{m} f(i)$ is (2006 : 2 Marks)

a) 3m

b) 3n

c) 2m + 1

d) 2n + 1

$m > 3$

$m = 5$

$S = \{1, 2, 3, 4, 5\}$

$5C_3 = 10$

$X_1 = \{1,2,3\}$

$X_2 = \{1,2,4\}$

$X_3 = \{1,2,5\}$

$X_4 = \{1,3,4\}$

$X_5 = \{1,3,5\}$

$X_6 = \{2,3,4\}$

$X_7 = \{2,3,5\}$

$X_8 = \{2,4,5\}$

$X_9 = \{3,4,5\}$

$X_{10} = \{1,4,5\}$

$\boxed{n = 10}$

$f : S \to N$

$f(i) = |\{j : i \in X_j\}|$

$f(1) = 6$

$f(2) = 6$

$f(3) = 6$

$f(4) = 6$

$f(5) = 6$

$m > 3$

$m = 4$

$4C_3 = 4 = n$

$$\sum_{i=1}^{m} f(i) = f(1) + f(2) + f(3) + f(4) + f(5)$$

$$= 6 + 6 + 6 + 6 + 6$$

$$= 30$$

$$= 3 \times 10$$

$$= 3n$$

Q. The number of onto functions (surjective functions) from set X = {1, 2, 3, 4} to set Y = {a, b, c} is ______. **(2015 (Set-2) : 2 Marks)**

$n^m - nc_1 (n-1)^m + nc_2 (n-2)^m + \cdots$

$m = \cancel{3}\,4, \quad n = 3$

36

Q. A function $f : N+ \rightarrow N^+$, defined on the set of positive integers N^+, satisfies the following properties:

$f(n) = f(n/2)$ if n is even

$f(n) = f(n + 5)$ if n is odd

Let $R = \{i | \exists\, j : f(j) = i\}$ be the set of distinct values that f takes. The maximum possible size of R is 2.

(2016 (Set-1) : 2 Marks)

$R = \{ i / \exists j : f(j) = i \}$

$f(j) = i$

$R = \{ i \}$

$f(n) = f\left(\frac{n}{2}\right)$; n is even

$f(n) = f(n+5)$; n is odd

$f(1) = f(6)$ ✓

$f(2) = f(1)$ ✓

$f(3) = f(8)$

$f(4) = f(2)$ ✓

$\boxed{f(5) = f(10)}$

$f(6) = f(3)$ ✓

$f(7) = f(12)$ ✓

$f(8) = f(4)$ ✓

$f(9) = f(14)$ ✓

$\boxed{f(10) = f(5)}$

$f(11) = f(16)$

$f(12) = f(6)$ ✓

$f(13) = f(18)$

$f(14) = f(7)$ ✓

$\boxed{f(15) = f(20)}$

$f(16) = f(8)$

$f(17) = f(22)$

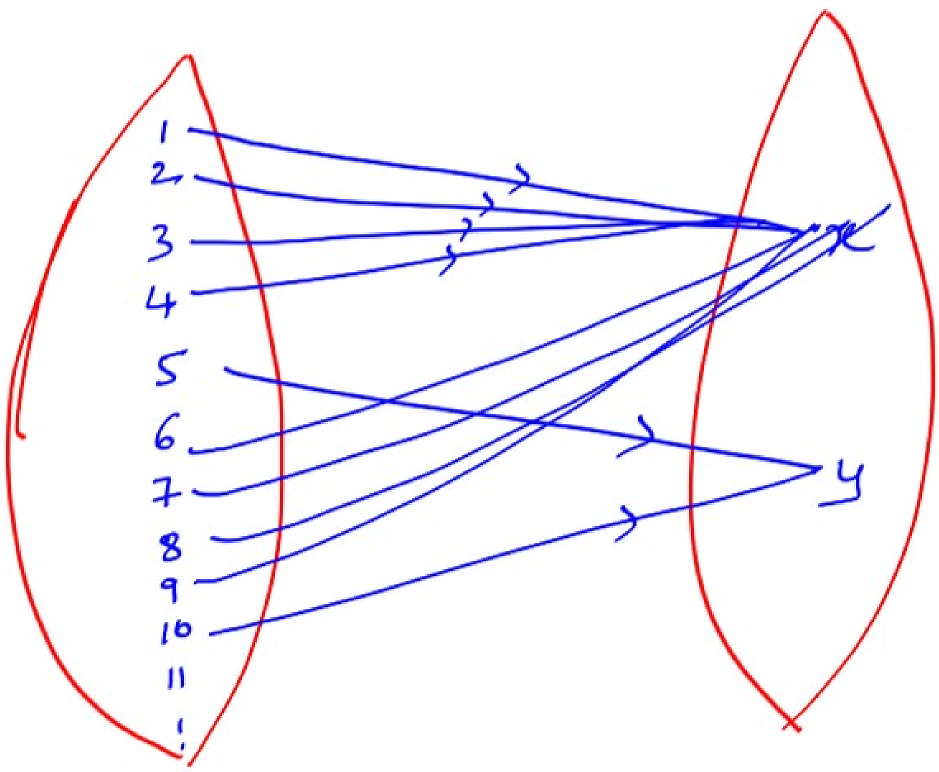
$f(18) = f(9)$

$f(19) = f(24)$

$\boxed{f(20) = f(10)}$

$f(1) = f(2) = f(3) = f(4) = f(6) = f(7)$
$= f(8) = f(9) = f(11) = f(12) = f(13) = f(14)$
$= f(16) = f(17) = f(18) = f(19) = f(21) - - - -$

$f(5) = f(10) = f(15) = f(20) = - - - -$



Max possible size $R = 2$

Q. How many onto (or subjective) functions are there from an n-element ($n \geq 2$) set to a 2-element set? **(GATE-12)**

a) 2^n

b) 2^n-1

c) 2^n-2 ✓

d) $2(2^n-2)$

n-element $\longrightarrow$ 2-element

$2^n - 2$

possible functions

(Not onto)
[constant]

$A \longrightarrow A$

Q. Let R denote the set of ~~real numbers~~. Let f: $(R \times R) \rightarrow R \times R$ be a bijective function defined by $f(x, y) = (x + y, x - y)$. The inverse function of f is given by **(GATE-96)**

a) $f^{-1}(x,y) = \left(\frac{1}{x+y}, \frac{1}{x-y}\right)$

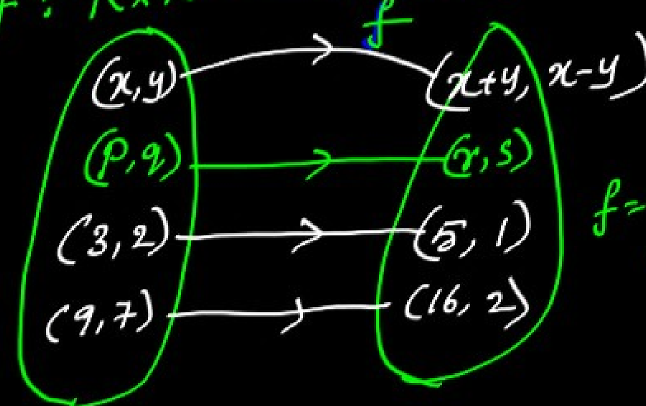
b) $f^{-1}(x,y) = (x - y, x + y)$

c) $f^{-1}(x,y) = \left(\frac{x+y}{2}, \frac{x-y}{2}\right)$ ✓

d) $f^{-1}(x,y) = (2(x - y), 2(x + y))$

$f: R \longrightarrow R$

$f: R \times R \longrightarrow R \times R$



$f = \{((p,q),(r,s)),$

a) $f^{-1}(x,y) = \left(\frac{1}{x+y}, \frac{1}{x-y}\right)$

$f^{-1}(5,2) = \left(\frac{1}{7}, \frac{1}{3}\right) \quad \times$

b) $f^{-1}(x,y) = (x-y, x+y)$

$f^{-1}(5,1) = (4, 6)$

c) $f^{-1}(x,y) = \left(\frac{x+y}{2}, \frac{x-y}{2}\right)$

$f^{-1}(5,1) = \left(\frac{5+1}{2}, \frac{5-1}{2}\right)$

$= (3, 2)$

$$(5,1) \xrightarrow{f^{-1}} (3,2)$$

Q. Let X and Y be finite sets and $f : X \to Y$ be a function. Which one of the following statements is **TRUE?** **(GATE-14-Set3)**

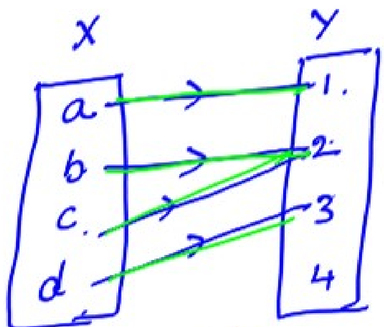
a) For any subsets A and B of X, $|f(A \cup B)| = |f(A)| + |f(B)|$

b) For any subsets A and B of X, $|f(A \cap B)| = |f(A) \cap f(B)|$

c) For any subsets A and B of X, $|f(A \cap B)| = \min\{|f(A)|, |f(B)|\}$

d) For any subsets S and T of Y, $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$

$f : X \longrightarrow Y$



$X = \{a, b, c, d\}$

ⓐ $\boxed{A = \{a, b\} \quad B = \{c, d\}}$ $\quad A \cup B = \{a, b, c, d\}$

$|f(A \cup B)| \neq |f(A)| + |f(B)|$

$|\{1,2,3\}| \neq |\{1,2\}| + |\{2,3\}|$

$3 \neq 2 + 2$

$3 \neq 4$

ⓑ $f(A \cap B) \neq f(A) \cap f(B)$

$f(\phi) \neq \{1,2\} \cap \{2,3\}$

$\phi \neq \{2\}$ ✓

ⓒ $|f(A \cap B)| \neq \min\{|f(A)|, |f(B)|\}$

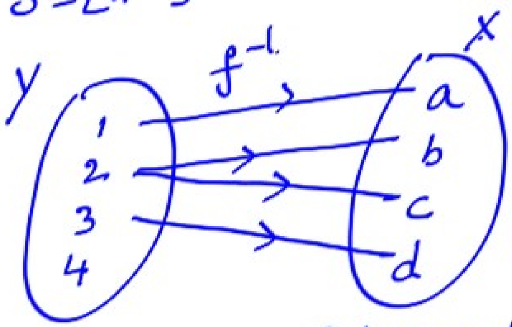
$|\cancel{\{1,2,3\}}| \neq \min\{2, 2\}$ — $f(\phi)$

$\cancel{3}\ 0 \neq 2$

$0 \neq 2$

(d) $Y = \{1,2,3,4\}$

$S = \{1,2\}$ $\quad T = \{3,4\}$



$$f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$$

$$f^{-1}(\phi) = \{a,b\} \cap \{d\}$$

$$\phi = \phi$$

Q. The number of functions from an m element set to an n element set is

(GATE-98)

a) $m + n$

b) m^n

c) n^m

d) $m*n$

Q. Consider the binary relation:

$S = \{(x, y) \mid y = x+1 \text{ and } x, y \in \{0, 1, 2, \ldots.\}\}$

The reflexive transitive colure of S is **(GATE-04)**

a) $\{(x, y) \mid y > x \text{ and } x, y \in \{0, 1, 2, \ldots\}\}$

b) $\{(x, y) \mid y \geq x \text{ and } x, y \in \{0, 1, 2, \ldots\}\}$

c) $\{(x, y) \mid y < x \text{ and } x, y \in \{0, 1, 2, \ldots\}\}$

d) $\{(x, y) \mid y \leq x \text{ and } x, y \in \{0, 1, 2, \ldots\}\}$

Q. The inclusion of which of the following sets into (GATE-04)

$S = \{\{1,2\}, \{1,2,3\}, \{1,3,5\}, \{1,2,4\}, \{1,2,3,4,5\}\}$ is necessary and sufficient to make S a complete lattice under the partial order defined by set containment?

a) $\{1\}$

b) $\{1\}, \{2,3\}$

c) $\{1\}, \{1,3\}$

d) $\{1\}, \{1,3\}, \{1,2,3,4\}, \{1,2,3,5\}$

Q. Consider the following sets, where $n \geq 2$:

S_1: Set of all $n \times n$ matrices with entries from the set (a, b, c)

n^2

S_2: Set of all functions from the set $\{0, 1, 2, \ldots, n^2-1\}$ to the set $\{0, 1, 2\}$

Which of the following choice(s) is/are correct? **(GATE-21-Set2)**

a) There exists a surjection from S_1 to S_2.

b) There does not exist an injection from S_1 to S_2

c) There does not exist a bijection from S_1 to S_2

d) There exists a bijection from S_1 to S_2

S_1 : Set of all $n \times n$ matrices with entries from $\{a, b, c\}$

$$3 \times 3 \times 3 \times 3$$

$$\begin{bmatrix} a_1 & & \\ a, b, c \;\; 3\times & & \\ & & \end{bmatrix}_{n \times n}$$

n^2 elements

$$3 \times 3 \times 3 \times \dots (n^2 \text{ times}) = 3^{n^2}$$

$a_1 = \underline{\quad}$: a, b, c

$n = 2$ $\begin{bmatrix} a & b \\ c & a \end{bmatrix}$

S_2 : set all functions

$\{0, 1, 2, \dots, n^2-1\} \to \{0, 1, 2\}$

$m = n^2$ $\quad$ $n = 3$

$$= n^m = 3^{n^2}$$

$$|S_1| = |S_2|$$

Algebraic Structure

Algebraic Structure: A non-empty set which is equipped with some operations and some properties is known as algebraic structure.

$$\boxed{(S, *)}$$

- Groupoid ✓
- Semi-group ✓
- Monoid ✓
- Group ✓
- Abelian Group ✓

Some Properties:

I. Closure: Let 'S' be the given algebraic structure, '*' is the binary operation and a, b are any two elements in S,

If $a * b \in S$ then we can say (S, *) follows closure property.

$$\boxed{\forall\, a, b \in S, \quad a * b \in S}$$

II. Associative:

$$\forall\, a, b, c \in S, \qquad \boxed{a * (b * c) = (a * b) * c}$$

III. Identity:

$$\forall\, a \in S, \quad \boxed{\exists\, e \in S,} \ni$$

$$\boxed{a * e = e * a = a}$$

$\mathbb{Z} = S = \{ \ldots -3, -2, -1, 0, 1, 2, \ldots \}$

Binary operation "+"

$2+3 = 5 \in \mathbb{Z}$

$-2+5 = 3 \in \mathbb{Z}$

$0+(-7) = -7 \in \mathbb{Z}$

"+" closed

$2+(3+4) = (2+3)+4$

$2+(7) = (5)+4$

$9 = 9$

For every $a \in S$, There Exists some $e \in S$ such that

$a * e = a$

$2 + 0 = 2$

$3 + 0 = 3$

$4 + 0 = 4$

$\boxed{e = 0}$

IV. Inverse:

$\forall a \in S, \quad \exists b \in S, \quad \ni$

$$\boxed{a * b = b * a = e}$$

$a * b = e$

$-2 + (2) = 0$

$-5 + (5) = 0$

$9 + (-9) = 0$

$0 + 0 = 0$

$\boxed{e^{-1} = e}$

$(Z, +) \subseteq (R, +)$

$Z \subseteq R$

$(H, *) \subseteq (G, *)$

V. Commutative:

$\forall a, b \in S$

$$\boxed{a * b = b * a}$$

$2 + 3 = 3 + 2$

Classification of Algebraic Structure

Groupoid (1)	Semi-group (2)	Monoid (3)	Group * (4)	Sub-Group (5)	Abelian (5)
1) Closure	1) Closure	1) Closure	1) Closure	1) Closure	1) Closure
	2) Associative	2) Associative	2) Associative	2) Associative	2) Associative
		3) Identity	3) Identity	3) Identity	3) Identity
			4) Inverse	4) Inverse	4) Inverse
					5) Commutative
					commutative

Sub-Group:

Let (G, *) be a group, H is a subset of 'G' and (H, *) is also group then we can say (H, *) is a subgroup of (G, *)

$$(H, *) \subseteq (G, *)$$

$$Z \subseteq R$$

$$(Z, +) \subseteq (R, +)$$

Q. Check the properties of commutative and associative on binary operation '*' is defined by $a * b = a^b, \forall\ a, b \in N$

Given binary operation $*$ defined by

$$\boxed{a * b = a^b}$$

Commutative:

Take x, y (or) $2, 3$

consider $2*3 = 2^3 = 8$

$3*2 = 3^2 = 9$

$2*3 \neq 3*2$

'$*$' is NOT commutative

Associative:

Take $2, 3, 4$.

$2*(3*4) = 2*(3^4)$

$= 2*(81) = 2^{81}$

$(2*3)*4 = (2^3)*4$

$= 8*4$

$= 8^4 = 2^{12}$

$\therefore 2*(3*4) \neq (2*3)*4$

NOT associative

Q. Show that the set of all rational number Q – {0} forms an abelian group under composition '*' defined by $a * b = \frac{ab}{2}$

ACE

Sol Given set $= \mathbb{Q} - \{0\} = S$ (say)

Binary operation $\boxed{a * b = \frac{ab}{2}}$

I. closure: x, y.

$x * y = \frac{xy}{2} \in S$

$(S, *)$ is closed

II. Associative : x, y, z.

$x * (y * z) = x * \frac{(yz)}{2}$

$= \frac{x\left(\frac{yz}{2}\right)}{2} = \frac{xyz}{4}$

$(x * y) * z = \left(\frac{xy}{2}\right) * z$

$= \frac{xyz}{4}$

$\therefore (S, *)$ is associative

III. Identity: Let $e \in S$ such that

$a * e = a$

$\frac{\cancel{a}e}{2} = \cancel{a}$

$\boxed{e = 2}$

IV **Inverse** : Let us suppose there is some $b \in S$ such that

$$a * b = e$$

$$\frac{ab}{2} = 2$$

$$\boxed{b = \frac{4}{a}}$$

Inverse of 'a' $= \frac{4}{a}$.

V **Commutative** :

Take $x * y = \frac{xy}{2}$

$$= \frac{yx}{2}$$

$$= y * x$$

$(S, *)$ is commutative

$(S, *)$ is an abelian group

S.T. set of rational numbers and some conditions with binary operation '$*$' defined by $\boxed{a*b = a+b-ab}$ is an abelian group

Closure:

Associative:

Identity: $e \in S$

$a*e = a$

$\cancel{a}+e-ae = \cancel{a}$

$e(1-a) = 0$

$\boxed{e=0}$

Inverse:

$a*b = e$

$a+b-ab = 0$

$a+b(1-a) = 0$

$b = \frac{-a}{1-a} = \frac{a}{a-1}$

commutative: $a*b = a+b-ab$

$= b+a-ba$

$= b*a$

Q. Show that $[G, +_6]$ is a group where $G = \{0, 1, 2, 3, 4, 5\}$

ACE