

Welcome to ACE Engineering Academy - online live class

Subject: **Computer Organization and Architecture**

Faculty: **Y.V. Ramaiah**

Subject

Computer organization & Architecture

Chapters (Topics)

I. Computer Arithmetic ✓

II. Memory Organization

III. Secondary Memories

IV. Basic processor organization and Design

V. Pipeline organization

VI. Control unit Design

VII. IO Organization

Recommended books for COA subject

<u>Chapter (Topic)</u>	<u>Author</u>
I. Computer Arithmetic	CARL Hamachar & william stallings
II. Memory Organization	Nicholas CARTER & PAUL Chaudhuri
III. Secondary Memories	CARL Hamachar & William stalling
IV. Basic processor organization and Design	Morris mono & Hamachar
V. Pipeline organization	Nicholas carter
VI. Control unit Design	Nicholas carter, Morris mono, Paul Chaudhuri
VII. IO Organization	Morris Mono

<u>Author Name</u>	<u>Title of the Book</u>
CARL Hamachar	Computer Organization
Nicholas Carter	Computer Architecture
Morris mono	Computer System Architecture
William stallings	Computer Organization and Architecture (6 th edition old)
PAUL chaudhuri	Computer Organization and Design

C.O.A. subject weightage in marks (chapter-wise) in GATE

from 2016 to 2021 ~~2022~~

Name of the Chapter	2016		2017		2018	2019	2020	2021		2022
	Set I	Set II	Set I	Set II				Set I	Set II	
I. Computer Arithmetic	1	0	1	1	1	0	1	2	2	2
II. Memory Organization	1	1	2	3	2	3	3	1	2	3
III. Secondary Memories	0	0	1	0	1	0	0	0	0	0
IV. Basic Processor Organization & Design	0	2	2	0	2	0	2	1	0	0
V. Pipeline Organization	1	2	1	0	1	0	1	1	1	1
VI. Control Unit Design	0	0	1	0	0	0	1	0	0	0
VII. IO Organization	1	0	0	0	0	0	1	0	1	1
Total No. of Questions	4	5	8	4	7	3	9	5	6	7

Chapter - 1 Computer Arithmetic

1. Signed binary data representation (Integer)
2. Overflow concept in signed 2's complement representation
3. Different Arithmetical operations
4. Booth's Algorithm
5. Floating point representation
6. IEEE standards for floating point representation
7. Ripple carry Adder
8. Carry lookahead generator and Adder

Q. Let R1 and R2 be two 4-bit registers that store numbers in 2's complement form. For the operation R1+R2, which one of the following values of R1 and R2 gives an arithmetic overflow?

- (A) $R1 = \begin{smallmatrix} -9 \\ 1011 \end{smallmatrix}$ and $R2 = \begin{smallmatrix} 12 \\ 1110 \end{smallmatrix} = -7$ No OVFL
- (B) $R1 = \begin{smallmatrix} -4 \\ 1100 \end{smallmatrix}$ and $R2 = \begin{smallmatrix} -1 \\ 1010 \end{smallmatrix} = \begin{smallmatrix} -10 \\ -10 \end{smallmatrix}$ Yes
- (C) $R1 = \begin{smallmatrix} +3 \\ 0011 \end{smallmatrix}$ and $R2 = \begin{smallmatrix} +4 \\ 0100 \end{smallmatrix} = \begin{smallmatrix} +7 \\ +7 \end{smallmatrix}$ No
- (D) $R1 = \begin{smallmatrix} -7 \\ 1001 \end{smallmatrix}$ and $R2 = \begin{smallmatrix} -1 \\ 1111 \end{smallmatrix} = \begin{smallmatrix} -8 \\ -8 \end{smallmatrix}$ ND

-8 to +7

-8 to +7

on Dec 26th 2021.

Expected gate 2022

questions live session on

Youtube

Started with OVFL Question





Signed Binary Data Representation (Integer)

unsigned
25

10110₂
Value

$$\begin{array}{r} \text{Signed} \\ +25 \\ -25 \end{array}$$

$$\begin{array}{r} (26) 825 \\ \hline \text{INT M} \end{array}$$

MSB
0 110 5 bit Value
↓ Sign bit
Let MSB = 0, +ve
MSB = 1, -ve

unsigned Integer Conversions

$$25 = \begin{array}{cccc} \checkmark & \checkmark & \checkmark & \checkmark \\ 16 & 8 & 4 & 1 \\ \hline 1 & 1 & 0 & 0 & 1 \end{array}$$

$$25 = (11001)_2$$



$$\begin{array}{r} 2 \overline{) 25} \\ 12-1 \\ \hline 2 \overline{) 13} \\ 12-0 \\ \hline 2 \overline{) 3} \\ 2-0 \\ \hline 1 \end{array}$$

$$64 \ 32 \ 16 \ 8 \ 4 \ 2 \ 1 \\ 78 = 1001110_2$$

$$139 = 1000101_2$$



Binary to Decimal Conversion

$$\begin{array}{r} 1 \ 1 \ 0 \ 1 \ 1 \\ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0 \\ (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) \\ = 27 = 11011_2 \end{array}$$

n bit unsigned

$$\begin{array}{r} (a_{n-1} \ a_{n-2} \ a_{n-3} \dots \ a_1 \ a_0)_2 = (\quad)_{10} \\ 2^{n-1} \ 2^{n-2} \ 2^{n-3} \dots \ 2^1 \ 2^0 \\ (a_{n-1}) * (2^{n-1}) + \dots + (a_2 * 2^2) + (a_1 * 2^1) + (a_0 * 2^0) \\ \sum_{i=0}^{n-1} a_i * 2^i \end{array}$$



unSigned Fraction Conversions

FIFO

26. $81_{10} = (11010 \cdot 1101)_2$

Self check

42-8125
Fra
ction

$$\begin{array}{r}
 8125 \times 2 \\
 16250 \times 2 \\
 1250 \times 2 \\
 0.50 \times 2 \\
 \hline
 1.0
 \end{array}$$

$$\begin{array}{r}
 11010 \cdot 1101 \\
 168421 \\
 \hline
 26.8125
 \end{array}$$

$$\frac{8+4+0+1}{16} = \frac{13}{16}$$

$(a_1 a_2 a_3 \dots a_n)_{10} = (\quad)_{10}$

$\frac{1}{2} + \frac{1}{8} + \dots + \frac{1}{2^n}$

$\frac{1}{2^1} + \frac{1}{2^2} + \frac{1}{2^3} + \dots + \frac{1}{2^n}$

$(a_1 \times \frac{1}{2^1}) + (a_2 \times \frac{1}{2^2}) + \dots + (a_n \times \frac{1}{2^n})$

$\sum_{i=1}^n a_i \times \frac{1}{2^i}$



$$0.\overset{1}{\cancel{1}}\overset{1}{\cancel{1}}_2 = (\quad)_{10}$$

$$\frac{4+2+1}{8} = \frac{7}{8} = 1 - \underset{\text{MSB}}{\cancel{\frac{1}{8}}} \approx 1$$

= 1 - LSB Value .

Note:- When Fraction field is filled with all '1's, then its Decimal Value ≈ 1 but accurate value = 1 - LSB

$$0.\overset{1}{\cancel{1}}\overset{1}{\cancel{1}}\overset{1}{\cancel{1}}_2 = (\quad)_{10}$$

$$\frac{16+8+4+2+1}{32} = \frac{31}{32}$$

$$= 1 - \frac{1}{32} \approx 1$$

= 1 - LSB Value

$$0.\overset{1}{\cancel{1}}\overset{1}{\cancel{1}}\overset{1}{\cancel{1}}\overset{1}{\cancel{1}}_2$$

MSB LSB

- Q) The nbit fixed point representation of an unsigned Real Number (x) uses ' f ' bits for the fraction field. Let $i = (n-f)$. The Decimal Value Range for (x) in this notation is
- a) $(-\frac{1}{2^f} \text{ to } \frac{1}{2^i})$ b) $\frac{1}{2^f} \text{ to } (\frac{i}{2} - \frac{1}{2^f})$
- c) $0 \text{ to } \frac{i}{2^i}$ d) $0 \text{ to } (\frac{i}{2} - \frac{1}{2^f})$
- Int* *Fraction* \rightarrow Real Number = x



$x \dots x \ x \ x \ x \ x \ x$ $\frac{x \ x \ x \ x \dots x}{2^1 \ 2^2 \ f \ \dots \ 2^f}$ fixed point notation

$i = (n-f)$

$(0 \text{ to } \frac{i}{2^{i-1}}) \quad 0 \text{ to } (1 - \frac{f}{2^i})$

Total Range

$0 \text{ to } \frac{i}{2^{i-1}} + 1 - \frac{f}{2^i} = \approx 1$

$0 \text{ to } (1 - \frac{f}{2^i})$



Integer Range (n bit)

min: 0 0 0 0 ... 0 : 0

; to

max 1 1 1 1 ... 1 : $(2^n - 1)$

When Integer is filled with all '1's, its Decimal value = $(2^n - 1)$



Fixt
 $(25, 525)$ fraction.
fixed point notation

$$= 25.525 \times 10^0$$

Floating Point
Representation



Signed Data can be represented in

3 ways

- (i) Signed magnitude Notation
- (ii) Signed 1's Complement Notation
- (iii) Signed 2's Complement Notation
most of systems use this.



Signed Q's complement Notation

- It is free from Negative zero
- It provides more range for Negative values than other notations.
- It permits 8421 weighted code while finding its Decimal Value; in this process only MSB weight is negative. → **It permits Sign bit Extension technique**

S		<u>Ubit data</u>	
+ve	{ 0 0 0 0	+ 0 to	<u>- 8 to + 7</u>
	0 1 1 1	+ 7	<u>- 3 to + 2 - 1</u>
-ve	{ 1 0 0 0	- 8	<u>- 2 ⁿ⁻¹ to + 2 ⁿ⁻¹ - 1</u>
	1 1 1 1	(-1)	<u>- 2 ⁿ⁻¹ to + 2 ⁿ⁻¹ - 1</u>
for nbit, Range ; $-(2^{n-1})$ to $+(2^{n-1} - 1)$			

The following values are in 2's Complement notation, Their decimal Values are



$$\begin{array}{ll}
 \text{(i) } 1101 & \text{(ii) } 01110 \\
 \text{MSB=0} \\ \text{+ve} \\ \text{MSB=1} \\ \text{-ve} \\
 \text{1. } \underline{101} & \text{+ve} \\
 \text{S } \underline{101+1} & \text{(+)} \underline{1110} = +14 \\
 \text{(-) } 010+1 & \text{MSB} \\
 \text{= (-) } 011 = -3 & \underline{01110} = +14 \\
 & \hline
 & \underline{1101} = -8+5 = -3
 \end{array}$$

Let 111011 is in Signed 2's Complement Notation; its Decimal Value is _____.



$$\begin{array}{r}
 \text{111011} \\
 \text{-32 } 16 \text{ } 8 \text{ } 4 \text{ } 2 \text{ } 1 \\
 \hline
 \end{array}
 \begin{array}{l}
 11 = -32 + 27 \\
 = -5
 \end{array}$$



(Ex) Let 1111111 is in
Signed 2's Complement Notation;
It's Decimal Value is _____.

$$\begin{array}{r} 1 \ 1 \ 1 \ . \ 1 \ | \ 1 \\ -64 \quad 32 \quad 16 \quad 8 \ 4 \quad 2 \ 1 \end{array} = -64 + 63 \\ (-7) = 1001 \quad \text{(-8 + 1)}$$

(Ex) What is the signed 2's Complement
Notation for -28, -14 and -7



For (-7): $s=1$, $7=111$, $\bar{7}=000$, $\bar{7}+1=001$

$$(-7) = \begin{array}{r} 1001 \\ -8 \\ \hline 1 \end{array} = -7$$

$$(-7) = -8 + 1 = \begin{array}{r} -8 \ 4 \ 2 \ 1 \\ 1 \ 0 \ 0 \ 1 \\ \hline \end{array}$$



$$\begin{array}{r} -14 = -16 + 2 = \\ \hline -28 = -32 + 4 = \end{array}$$

$\begin{array}{r} -16 & 8 & 4 & 2 & 1 \\ | & 0 & 0 & 1 & 0 \\ \hline -32 & & & 4 \\ | & 0 & 0 & 1 & 0 \end{array}$

$$\begin{array}{r} -126 = -128 + 2 \\ = \end{array}$$

$\begin{array}{r} -128 \\ | 0 0 0 0 0 1 0 \\ \hline \end{array}$

(Ex) 2's Complement notation for -49 is

$$-49 = -64 + 15 = -64 + 8 + 4 + 2 + 1$$

$$\begin{array}{r} -64 & 32 & 16 & 8 & 4 & 2 & 1 \\ | & 0 & 0 & 1 & 1 & 1 & 1_2 \\ \hline \end{array}$$



$$-49 = \begin{smallmatrix} -64 \\ 1001111 \end{smallmatrix}$$

(Ep) 2's complement Notation for -22 is

$$\begin{smallmatrix} -32 & 8 & 2 \\ 101010 \end{smallmatrix} = -22$$

Sign bit extension:- It is the process of converting the smaller size signed Data to Larger size by padding the sign bit to left. All these additional padded bits are known as Guard bits or Dummy bits.

The below data are represented in Signed 2's Complement notation



$$\begin{array}{l}
 \begin{array}{rcl}
 \begin{array}{c} -8u^2 \\ | 001 \end{array} = -7 & \text{(4 bit)} & \\
 \begin{array}{c} s \\ | 11001 \end{array} = -7 & \text{(6 bit)} & \\
 \begin{array}{c} -32 \\ | 1111001 \end{array} = -7 & \text{(8 bit)} & \\
 \begin{array}{c} 128 \\ | 1111001 \end{array} = -7 & \text{(8 bit)} &
 \end{array} \\
 \left. \begin{array}{l} \text{Same} \\ \text{No} \\ \text{Change} \\ \text{in its} \\ \text{Decimal} \\ \text{Value} \end{array} \right\}
 \end{array}$$

Let $x_3x_2x_1x_0$ data is in Signed 2's Complement Notation, it's 8 bit format
is

$x_3x_3x_3x_3$ $\overset{(5)}{\cancel{x_3x_2x_1x_0}}$

a	0 0 0 0 $x_3x_2x_1x_0$
b	1 1 1 1 $x_3x_2x_1x_0$
c	$x_3x_3x_3x_3x_3x_2x_1x_0$
d	$\overline{x_3}\overline{x_3}\overline{x_3}\overline{x_3}x_3x_2x_1x_0$



4 bit 2's complement Notation Range.

<u>4 bit</u>	<table border="0" style="width: 100%;"> <tr> <td style="text-align: right; vertical-align: bottom;">+ve</td><td style="text-align: center; vertical-align: bottom;"> $\begin{array}{c} S \\ \{ \\ 0 & 0 & 0 & 0 \\ \text{to} & & & +0 \end{array}$ </td><td style="text-align: center; vertical-align: bottom;"> $\begin{array}{c} \text{to} \\ +7 \end{array}$ </td><td style="text-align: center; vertical-align: bottom;"> $\begin{array}{c} -8 \text{ to } -1 \text{ and} \\ +0 \text{ to } +7 \end{array}$ </td></tr> </table>	+ve	$\begin{array}{c} S \\ \{ \\ 0 & 0 & 0 & 0 \\ \text{to} & & & +0 \end{array}$	$\begin{array}{c} \text{to} \\ +7 \end{array}$	$\begin{array}{c} -8 \text{ to } -1 \text{ and} \\ +0 \text{ to } +7 \end{array}$
+ve	$\begin{array}{c} S \\ \{ \\ 0 & 0 & 0 & 0 \\ \text{to} & & & +0 \end{array}$	$\begin{array}{c} \text{to} \\ +7 \end{array}$	$\begin{array}{c} -8 \text{ to } -1 \text{ and} \\ +0 \text{ to } +7 \end{array}$		
-ve	<table border="0" style="width: 100%;"> <tr> <td style="text-align: right; vertical-align: bottom;">-</td><td style="text-align: center; vertical-align: bottom;"> $\begin{array}{c} S \\ \{ \\ 1 & 0 & 0 & 0 \\ \text{to} & & & -8 \end{array}$ </td><td style="text-align: center; vertical-align: bottom;"> $\begin{array}{c} \text{to} \\ -1 \end{array}$ </td><td style="text-align: center; vertical-align: bottom;"> $\begin{array}{c} +7 \\ +2 \\ +1 \\ 0 \\ -1 \\ -6 \\ -7 \end{array}$ </td></tr> </table>	-	$\begin{array}{c} S \\ \{ \\ 1 & 0 & 0 & 0 \\ \text{to} & & & -8 \end{array}$	$\begin{array}{c} \text{to} \\ -1 \end{array}$	$\begin{array}{c} +7 \\ +2 \\ +1 \\ 0 \\ -1 \\ -6 \\ -7 \end{array}$
-	$\begin{array}{c} S \\ \{ \\ 1 & 0 & 0 & 0 \\ \text{to} & & & -8 \end{array}$	$\begin{array}{c} \text{to} \\ -1 \end{array}$	$\begin{array}{c} +7 \\ +2 \\ +1 \\ 0 \\ -1 \\ -6 \\ -7 \end{array}$		

(-8) to (+7)



4 bit Range -8 to +7

4 bit Range
Value

$$-2^3 \text{ to } +2^3 - 1$$

$$-2^{4-1} \text{ to } +2^{4-1} - 1$$

$$-2^{n-1} \text{ to } +2^{n-1} - 1$$

For
8 bit
Range.

-128 to
+127

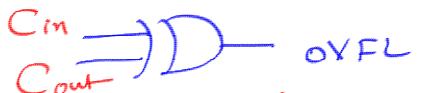
For 'n' bitsize, the Range is

Overflow (OVFL) in Signed 2's complement



- Overflow is also Considered as Signed Carry.
- After adding $2^{no\ of\ nbit\ data}$, the OVFL occurs when ' n ' bits are not sufficient to store the Result (in Decimal).
- OVFL doesn't occur after adding one positive data and one Negative data.
- Occurrence of OVFL changes the Target Sign bit in the Result.

Overflow Detection



C_{in} = It is the carry generated by adding data that enters in Sign bit position -

C_{out} = It is the carry generated by adding both sign bits.

- presence of both C_{in} and C_{out} is needed to detect OVFL.
- C_{out} participates in final Result; when OVFL has occurred only.

C_{in}	C_{out}	XOR OIP
0	0	0
1	1	
0	1	1
1	0	



Decimal
Unsigned Decimal
Signed

$ \begin{array}{r} \text{(Ex1)} \\ \begin{array}{r} 9 \\ + 3 \\ \hline 12 \end{array} \\ \text{carry} \downarrow \text{Sum} \end{array} $	$ \begin{array}{r} +9 \\ +3 \\ \hline +12 \end{array} \\ \text{OVFL Sum} \end{array} $
--	---

Let data size is 4 bit; Range permitted
~~S -8 4 2 1~~ databits ; -8 to +7.

$ \begin{array}{r} \text{(Ex1)} \\ \begin{array}{r} 1111 \quad (-1) \\ + 110 \quad (-2) \\ \hline 1101 \quad (-3) \end{array} \\ \text{Cout} \rightarrow D_0 \\ \text{NO OVFL} \end{array} $	$ \begin{array}{r} \text{(Ex2)} \\ \begin{array}{r} S \text{ data} \\ \begin{array}{r} 1010 \quad -6 \\ 0111 \quad +7 \\ \hline 1001 \end{array} \\ \text{Cin} \rightarrow D_0 \\ \text{Cout} \rightarrow D_0 \\ \text{NO OVFL} \end{array} \end{array} $
--	---

RANGE



$(Ex3)$ $ \begin{array}{r} 001 (-7) \\ -842 \\ \hline 1010 (-6) \end{array} $ $ \begin{array}{r} C_{in} @ \\ \hline 0011 \end{array} $ <p><u>Cout</u></p> <p>$\Rightarrow D_1$ Yes OVFL Occurred</p>	$(Ex4)$ $ \begin{array}{r} 111 (+7) \\ +5 \\ \hline 01100 \end{array} $ $ \begin{array}{r} C_{in} @ \\ \hline 111 \end{array} $ <p><u>Cout</u></p> <p>$\Rightarrow D_1$ Yes Overflow occurred</p>	$(Ex5)$ $ \begin{array}{r} 111 (-1) \\ -842 \\ \hline 111 (-1) \end{array} $ $ \begin{array}{r} C_{in} @ \\ \hline 1110 \end{array} $ <p><u>Cout</u></p> <p>$\Rightarrow D_0$ No No OVFL</p>
---	--	---

Cout participates in final result when OVFL has occurred only.

$(Ex1)$ $ \begin{array}{r} 0110 +6 \\ +1001 -7 \\ \hline 01111 (-1) V \end{array} $ <p><u>Cout</u></p> <p>$\boxed{\text{NO OVFL}}$ <u>final Result</u> $-842 = -1$</p>	$(Ex2)$ $ \begin{array}{r} 1001 (-7) \\ +1001 (-7) \\ \hline 0010 -14 \end{array} $ <p><u>Cout</u></p> <p><u>final Result</u> $-10010 = -14$</p>
---	---

MSQ



- Q) After adding two of signed
2's Complement Notation data, the overflow
occurs, when $Cin = 1$ * False
b) occurs, when $Cout = 1$ False
c) occurs, when $Cin \neq Cout$ Yes
d) Does not occur after adding
one positive data and one
negative Data Yes
C and D

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Q. Booth's coding in 8 bits for the decimal number -57 is:

~~(a) $0 - 1 \overset{64}{0} + 1 \overset{32}{0} 0 \overset{16}{+} 1 \overset{8}{0} 0 \overset{4}{+} 2 \overset{2}{1} = -56$~~

(b) $0 - 1 \overset{64}{0} + 1 \overset{32}{0} 0 \overset{16}{+} 1 \overset{8}{0} 0 \overset{4}{-} 1 = -57$

(c) $0 - 1 + 1 \overset{32}{0} 0 - 1 \overset{16}{0} + 1 \overset{8}{0} 0 - 1$

(d) $0 \overset{64}{0} - 1 \overset{32}{0} + 1 \overset{16}{0} 0 - 1 \overset{8}{0} 0 - 1$

$(-57) = \boxed{10001110}$

\downarrow

$1 \overset{64}{0} 0 0 0 1 \overset{16}{1} 1 1 0$

$0 - 1 \overset{32}{0} 0 + 1 \overset{16}{0} 0 - 1$

$0 - 1 \overset{64}{0} + 1 \overset{32}{0} 0 - 1$

- Q. Sign extension is the step in
- Floating point multiplication
 - Signed 16 bit integer addition
 - Arithmetic left shift
 - Converting a signed integer from one size to another



Overflow in Signed &'s Complement Notation

→ Occurrence of OVFL changes the Target sign bit in the Result.

Data I : $x \ x \ x \ x \ x \ x \ x \ x$
Data II : $+ . c_{in} \ x \ x \ x \ x \ x \ x \ x$
 $\text{Cout} (z) \ x \ x \ x \ x \ x \ x \ x$

x = Sign bit in first data
 y = Sign bit in 2nd data ; z = Sign bit in the Result

(Ex)

$$\begin{array}{r}
 -72 \\
 -121 \\
 + \\
 \hline
 -193
 \end{array}
 \quad
 \begin{array}{c}
 | 0 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0 \\
 128 \ 64 \ 32 \ 16 \ 8 \ 4 \ 2 \ 1 \\
 | \ 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 1 \\
 \text{cin} \\
 \hline
 | 0 \ 0 \ 1 \ 1 \ 1 \ 1 \ 1
 \end{array}
 \quad
 \begin{array}{l}
 = A \ (8 \text{bit}) \\
 = B \ (8 \text{bit}) \text{ after} \\
 A + B, \\
 \text{status of} \\
 \text{Arithmetic} \\
 \text{Overflow?}
 \end{array}$$

1 → D → 1 Yes occurs ✓

→ -2^{8-1} to $+2^{8-1}$
 $= -128 \text{ to } +127$



Q) Let x is the sign bit in first data, y is the sign bit in 2nd data and z is the sign bit in Result. The SOP expression for occurrence of overflow.

- a) $x+y+z$
- b) $xy+yz+xz$
- c) ~~$xy\bar{z}+\bar{x}\bar{y}z$~~
- d) $\bar{x}\bar{y}\bar{z}$

xy	$x y z$	OVFL
++	00 0	0 ✓
++	00 1	1 ✓
+ -	01 0	0
+ -	01 1	0
- +	10 0	0
- +	10 1	0
--	00 0	1 ✓
--	11 1	0

SOP Expression

$$\bar{x}\bar{y}z + xy\bar{z} \checkmark$$

Different Arithmetical operations performed by the C.P.U.



→ multiplication operation can be performed in 3 methods

(i) Repetitive Addition

(ii) Shift and Add technique

(iii) Booth's multiplication

→ For (i) and (ii), one operand is compulsory unsigned.

→ while adding 2 no. of signed data both operands size is Compulsory same

(Ex)

$\begin{array}{r} -16 \quad 8 \quad 4 \quad 2 \quad 1 \\ 0 \quad 0 \quad 1 \quad 0 \quad (5\text{bit}) \\ -14 \\ \hline 1 \quad 0 \quad 1 \quad 3 \text{bit} \\ \hline \end{array}$	-3	$\begin{array}{r} -17 \\ \hline \end{array}$	$\begin{array}{r} 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad I \\ + \quad 1 \quad 1 \quad 0 \quad 1 \quad II \\ \hline 0 \quad 1 \quad 1 \quad 1 \quad 1 \quad \\ \hline \end{array}$
		cout	$\begin{array}{r} 1 \quad 0 \quad 1 \quad 1 \quad 1 \\ -32 \quad -16 \quad 8 \quad 4 \quad 2 \\ \hline -32 + 15 = -17 \quad II \\ \hline \end{array}$

Doesn't satisfy $\boxed{-9} \times$

Satisfies



VVVIMP.

- After adding 2 no. of n bit data,
the longest Result size is $(n+1)$ bits
- After multiplying 2 no. of n bit data,
the longest Result size = $2n$ bits.
- After multiplying one m bit size and
another n bit size data; $(m+n)$ bits



Repetitive Addition.

$$\begin{array}{r} (-7) \times 4 \\ \hline -7 \\ -7 \\ -7 \\ -7 \\ \hline -28 \end{array}$$



Shift and Add technique

Shift left is used to perform multiplication. Binary (Signed)

$$\begin{array}{c}
 \text{Decimal} - \\
 26 \times 1000_{10} \\
 = 26 \times 10^3 \\
 = 26000
 \end{array}
 \quad
 \begin{array}{l}
 (-7) \times 8 = -56 \\
 -7 = \overset{-8}{1}001 \times 8 = 1001 \times 2^3 \\
 -64 \underset{-64}{\cancel{32}} \underset{16}{\cancel{16}} \underset{8}{\cancel{8}} \underset{4}{\cancel{4}} \underset{2}{\cancel{2}} \underset{1}{\cancel{1}} = -64 + 8 \\
 = -56
 \end{array}$$

(Ex)

$$\begin{aligned}
 (-7) \times 9 &= \overset{-63}{(-63)} = (-7) \times \left(\frac{3}{2} + 1\right) \\
 &\text{Signed Unsigned} \\
 -7 &= \overset{-8}{1}001, \quad 1001 \times \left(\frac{3}{2} + 1\right) \\
 &= (1001 \times \frac{3}{2}) + 1001 = 1001000 \\
 &\quad \text{Adding} + 1111001 \\
 &\quad \begin{array}{r}
 110000001 \\
 -128 \underset{-64}{\cancel{64}} \underset{32}{\cancel{32}} \underset{16}{\cancel{16}} \underset{8}{\cancel{8}} \underset{4}{\cancel{4}} \underset{2}{\cancel{2}} \underset{1}{\cancel{1}}
 \end{array} \\
 &= -128 + 65 = -63
 \end{aligned}$$



Booth's multiplication

- It permits only signed operands.
- It provides excellent performance when the multiplier is having string of zeros or string of 1's and it provides worst performance when the multiplier is having more no. of bit changes.
- In this technique the Booth's code is generated for multiplier only.



- For generating Booth's code; the multiplier is scanned from LSB and always one special bit is taken for LSB scanning (0)
- while scanning moves from 0 to 0 or 1 to 1 no arithmetical operation is required, just the partial result is shifted.
- while scanning moves from 0 to 1; the shifted multiplicand is added for (-1) times and while scanning moves from 1 to 0, the shifted multiplicand is added for +1 times.

M = multiplicand
 Q = multiplier } Standard Result
 size = $2n$ bits



(EX) $(-7) \times (+4) = -28$

$M = -7 = 1001$

$Q = +4 = 0100$

$(-1) * M = +7 = 0111$

$M = 1001, \bar{M} = 0110.$

$\bar{M} + 1 = 0111 = (+7)$

$Q = +4$
 $= 0100$

$0 1 0 0$
 $\underline{\quad \quad \quad \quad}$
 $+1 (-1) 0 0$

Booth's Code for +4

$+1 -1 0 0$
 $\underline{8 \quad 4 \quad 2 \quad 1}$
 $= 8 - 4 = 4$

$(-7) * (+4) = -28$

$-7 = 1001$ (ubit)

Booth's code for +4

$\rightarrow +1 -1 0 0$

Standard Result

size is 8 bit

$M = -7 = 1001$

$(-1)M = +7 = 0111$

$* 1001 - (M)$
 $+1 \underline{00}$

$0 0 0 0 0 0 0 0$
 $0 0 0 0 0 0 0 -$
 $0 0 0 1 1 1 - -$
 $+ 1 1 0 0 1 - -$

$\underline{-128 \quad 64 \quad 32 \quad 16 \quad 8 \quad 4 \quad 2 \quad 1} = -128 + 100$
 $= -28$

Result = $\frac{+100100}{-32 \quad 16 \quad 8 \quad 4 \quad 2 \quad 1} = -34 + 4 = -28$

(Ex)

Generate the Booth's Code for multipliers -

$(-37)_{10}, \quad (+65)_{10}$

$-37 = \begin{array}{r} -64 \\ -1011011 \end{array}$

$\begin{array}{r} -73 \\ -37 \end{array}$

$\begin{array}{r} 1011011 \\ \text{---} \\ -1+10-1+10-1 \\ \hline -1+10-1+10-1 \end{array}$

$\begin{array}{r} 64 \\ 32 \\ 8 \\ 4 \\ 1 \end{array}$

$+65 = 64 + 1$

010000010

$\begin{array}{r} 128 \\ 64 \\ 32 \\ 16 \\ 8 \\ 4 \\ 2 \\ 1 \end{array}$

$+130 - 65 = +65$



Find the Booth's Code for

(i) 11111110 (ii)

Best performance

$\begin{array}{r} 11111110 \\ \text{---} \\ 0000000-10 \end{array}$

one operation ①

101010

Worst performance

$\begin{array}{r} 101010 \\ \text{---} \\ -1+1-1+1-1-10 \end{array}$

$\begin{array}{r} -1+1-1+1-1+1-10 \\ \text{---} \\ \text{सत्ता} \end{array}$



Q) In the given below Booth's multipliers, which one gives worst performance?

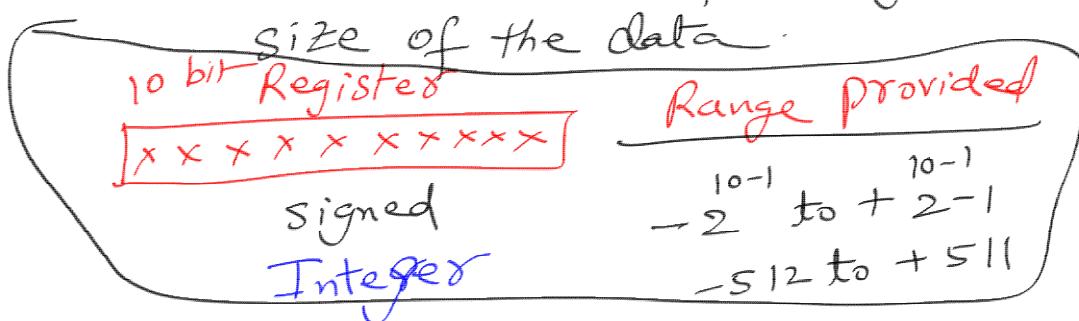


a	1	1	1	1	1	1	1999
b	1	1	0	0	1	1	<u>GATE</u>
c	1	0	1	0	1	0	more no. of operations
d	0	0	0	1	1	1	ISRO.

Floating Point Representation -



It provides more range than Integer representation for the given





Fixed point Notation	REAL number	Floating point Notation
$\frac{625.73}{10}$ INT Mantissa Fraction Significand	$=$	625.73×10^0 $10 = \text{base} = 2$ $0 = \text{Exponent}$
11001.0101_2	$=$	11001.0101×2^0

Floating point Representation requires
 Mantissa (M), Base (Binary Base),
 Exponent, sign of the Mantissa and
 Exponent.

Sign of the Exponent -

$$S \pm M \times 2^e$$

S = sign of the Exponent
 M = Mantissa
 e = exponent

VVRIMP
 → only sign of
 the Mantissa
 gives the sign
 of the data
 but not Exponent sign.



Ex1 $625_{10} = 625.0 = 625.0 \times 10^0$

$$= +0. \underline{625} \times \frac{10}{m}^{\textcircled{+3}} e$$

Ex2 $0.00528 = 0.00528 \times 10^0$

$$= 0.528 \times 10^{\textcircled{-2}} e$$

(Ex) $+0. \underline{110101} \times 2^{\textcircled{-4}} e =$

$$\checkmark +0.00000110101 \times 2^{\textcircled{1}} +$$

Sign of the Exponent gives the Relative
position of the decimal point in the
fractional Mantissa



Mantissa range: M_{range}
unsigned - n bit

$$M_{min} = 0.0000 \dots 0 = 0$$

$$M_{max} = 0.\underset{\substack{MSB \\ 1}}{1}\underset{\substack{LSB \\ 0}}{1}\dots1 = \frac{15}{16} = 1 - \frac{1}{16} = 1 - \frac{1}{2^n}$$

In mantissa, when all bits are '1' then
its value ≈ 1 , accurate value
 $= 1 - LSB = 1 - \frac{1}{2^n}$

Expression Value for Floating Data Representation



S = sign of
the mantissa

$$(\pm) 0.M \times \frac{1}{2^e}$$

$$(-) 0.M \times \frac{1}{2^e}$$

$$S=0 \text{ +ve}$$

$$S=1 \text{ -ve}$$

Representation of Floating point data in a Register



s = Sign of the 'm' 0 for +ve, '1' for -ve.

Biasing: It is the process of converting signed exponent to unsigned exponent by adding some fixed value to the TRUE exponent.

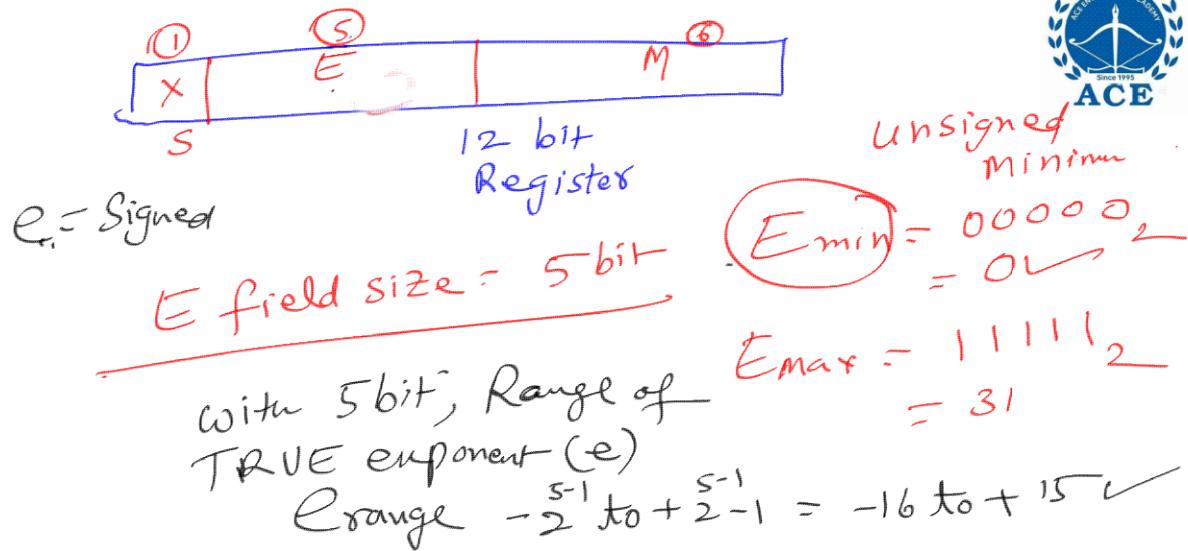
E = Biased Exponent (unsigned)

b = biasing value

e = TRUE Exponent (original exponent)

$$E = e + b$$

→ Always Mantissa is expressed in
Signed Magnitude notation only



E field size = 5 bit, with 5 bits,

$(00000) 0 \leq E \leq 31 (11111)$

$E_{\min} = 0$

$E_{\max} = 31 = (2^5 - 1)$

$E = e + b$

$E_{\min} = e_{\min} + b$

$0 = -16 + b$

$b = 16$

with 5 bit e_{range}

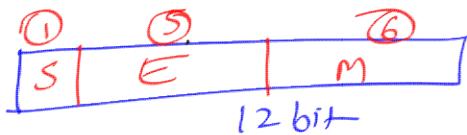
$+15 \leq e - 16$

$e_{\min} = -16$

$e_{\max} = +15$

$E_{\max} = e_{\max} + b$

$31 = 15 + b; b = 16$



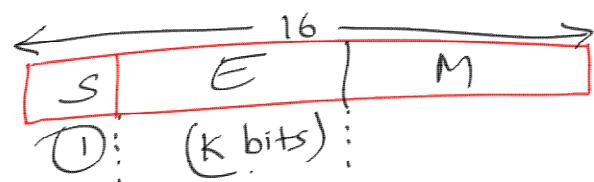
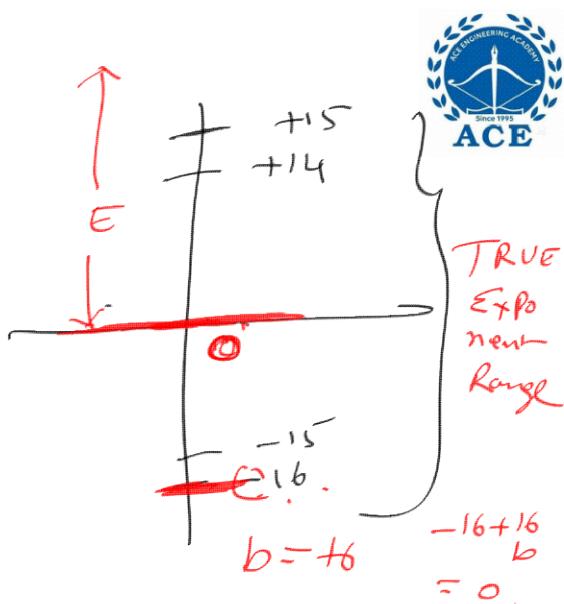
with 5 bit; E range

$$-2^{5-1} \text{ to } +2^{5-1}$$

$$-16 \text{ to } +15$$

$$E_{\min} = -16$$

$$E_{\max} = 0$$



with K bit; E range, $E_{\min} = 0$, $E_{\max} = 2^k - 1$

$$E_{\min} = -2^{k-1} \text{ to } +2^{k-1}$$

$$E_{\min} = e_{\min} + b$$

$$0 = -2^{k-1} + b, b = 2^{k-1}$$

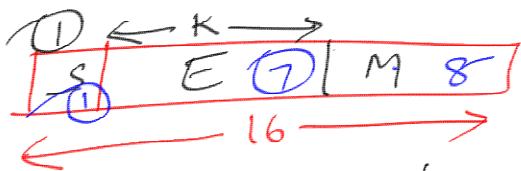
$$E_{\max} = e_{\max} + b$$

$$E_{\max} = 0$$



Some times Biasing Value is also known as Excess Value.

- (Ex) A 16 bit Register is aimed to store Floating data with Excess 64 technique.
- 1) No. of bits needed for 'M' field is 8.
 - 2) No. of bits needed for 'E' field is 7.



$$b = 64 = 2^{k-1} = 2^6$$

$$k-1 = 6$$

$$k = 7$$



(Ex) A 12 bit Register is aimed to store a floating data with Excess 16 technique.

18) No. of bits needed for 'M' field is 6
 " " for 'E' field is 5

29) " in the above Register

30) Store -12.75_{10} in the above Register and Express the Value in Hexadecimal and Octal Notations -

$$b = 16 = 2^{k-1} = 2^4$$

$k=5$

$$\begin{aligned} -12.75 \\ S=1, \quad 12.75 &= 1100.11 \times 2^0 \\ &= 0. \underline{\underline{110011}} \times 2^4 \\ E &= 4 \quad M = 110011 \\ b &= 16 \\ E &= 20 \\ &= 10100 \end{aligned}$$

1.75×2
 1.5×2
 $\downarrow 1.0$

$S \oplus$	$E \oplus$	$M \oplus$
1	10100	110011
D	3	3
$(D33)_{16}$		



$$1 \text{ } | \text{ } 10 \text{ } | \text{ } 100 \text{ } | \text{ } 110 \text{ } | \text{ } 0 \text{ } | \text{ } 11 = ()_8$$

$$(6463)_8$$