Q. What is the minimum number of ordered pairs of non-negative numbers that should be chosen to ensure that there are two pairs(a,b) and (c,d) in the chosen set such that a ≡ c mod 3 and b ≡ d mod 5

I

0,1,2,3,4 **(GATE-CS-05)**

$a \equiv c \bmod 3$

a) 4

3)a (

$\boxed{c} \rightarrow 0,1,2$

b) 6

c) 16

d) 24

Q. The number of permutations of the characters in LILAC so that no character appears in its original position, if the two L's are indistinguishable, is ______. (GATE-20)

$\underset{1}{\underline{L}}\ \underset{2}{\underline{I}}\ \underset{3}{\underline{L}}\ \underset{4}{\underline{A}}\ \underset{5}{\underline{C}}$

I, A, C are arranged $= 3P_2$

the third letter can be $= 2$

I A C = 1 (or) 3

$\underset{1}{\underline{I}}\ \underset{2}{\underline{C}}\ \underset{3}{\underline{A}}\ \underset{4}{\underline{\ \ }}\ \underset{5}{\underline{\ \ }}$

1	3
I	A
A	I
I	C

$3P_2 \times 2$

C L_1 A I $= (3\times2)\times2 = 12$ ways L_2

$\underset{1}{\square}\ \underset{2}{\square}\ \underset{3}{\square}\ \underset{4}{\square}\ \underset{5}{\square}$

C L_2 A I L_1

I, A, C = 2, 4, 5

$3P_2 \times 2$

Division and Distribution:

I. Group sizes are fixed (unequal)

II. Group sizes equal (fixed)

III. Group are not fixed

Non-identical

$$\frac{10!}{4!\,6!} * 2!$$

(Distribution)

I. Group sizes are fixed & unequal

Let us consider 10 non-identical objects (balls)

(1)

$$\begin{array}{ccc} & 10 & \\ \swarrow & & \searrow \\ \text{group A} & & \text{group B} \\ 4 & & 6 \\ \cancel{6} & & \cancel{4} \end{array}$$

$$= {}^{10}C_4 * {}^{6}C_6$$

$$= \frac{10\times 9\times 8\times 7}{4\times 3\times 2\times 1} \times 1$$

$$= 210$$

$$\frac{10!}{4!\,6!} \rightarrow \text{Division}$$

$$= \frac{10\times 9\times 8\times 7\times 6\times 5\times 4\times 3\times 2\times 1}{4\times 3\times 2\times 1\times 6!}$$

$$= 210$$

(2)

$$10 \rightarrow 2,\ 3,\ 5$$

A	B	C
2	3	5

$$= {}^{10}C_2 * {}^{8}C_3 * {}^{5}C_5 \quad \text{(or)} \quad \frac{10!}{2!\,3!\,5!} \text{ Division}$$

$$\frac{10!}{2!\,3!\,5!} * 3!$$

A	B	C
2	3	5
2	5	3
3	2	5
3	5	2
5	2	3
5	3	2

(3)

$$14 \rightarrow 2,\ 3,\ 4,\ 5$$

14

$$\frac{14!}{2!\,3!\,4!\,5!} \text{ Division}$$

$$\frac{14!}{2!\,3!\,4!\,5!} * 4! \text{ Distribution.}$$

II. Group sizes are fixed & Equal :

① 4 → 2, 2

$$= \frac{4!}{2!\,2! * 2!} = \text{Division}$$

$$= \frac{4!}{2!\,2!\,2!} * 2! = \text{Distribution}$$

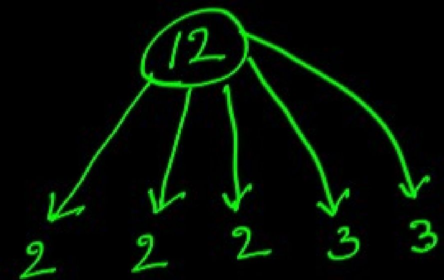
Division $\{a, b, c, d\}$

A	B
$\{a,b\}$,	$\{c,d\}$ ✓
$\{a,c\}$,	$\{b,d\}$
$\{a,d\}$,	$\{b,c\}$
~~$\{b,c\}$,~~	~~$\{a,d\}$~~
~~$\{b,d\}$~~	~~$\{a,c\}$~~
~~$\{c,d\}$,~~	~~$\{a,b\}$~~ ✓

Distribution

A	B
$\{a,b\}$	$\{c,d\}$
$\{a,c\}$	$\{b,d\}$
$\{a,d\}$	$\{b,c\}$
$\{c,d\}$	$\{a,b\}$
$\{b,d\}$	$\{a,c\}$
$\{b,c\}$	$\{a,d\}$

(2)



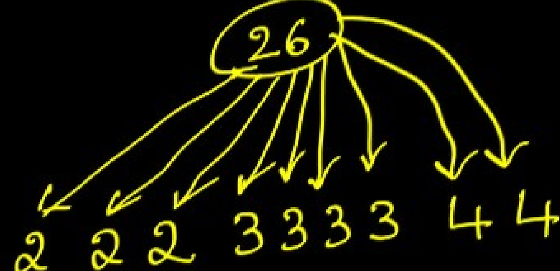
Division

$$\frac{12!}{2!\,2!\,2!\,3!\,3!\,(3!\,2!)}$$

Distribution

$$\frac{12!}{2!\,2!\,2!\,3!\,3!\,3!\,2!} * 5!$$

(3)



$$\frac{26!}{2!\,2!\,2!\,3!\,3!\,3!\,3!\,4!\,4!\,(3!\,4!\,2!)} * 9!$$

(Distribution)

III. Group sizes are not fixed

objects (chocolates) $= c_1, c_2, c_3, \ldots, c_{10}$

groups (girls) $= g_1, g_2, g_3$

g_1	g_2	g_3
1	1	8
8	1	1
10	0	0
5	0	5
5	5	0

$\boxed{\begin{matrix} c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_{10} \end{matrix}}$

objects (chocolates)

$\boxed{\begin{matrix} g_1 \\ g_2 \\ g_3 \end{matrix}}$

groups (girls)

10 chocolates (Identical)

first chocolate (c_1) can be distributed to either g_1 (or) g_2 (or) g_3 = 3 ways

$c_2 \longrightarrow 3$ ways

$c_3 \longrightarrow 3$ ways

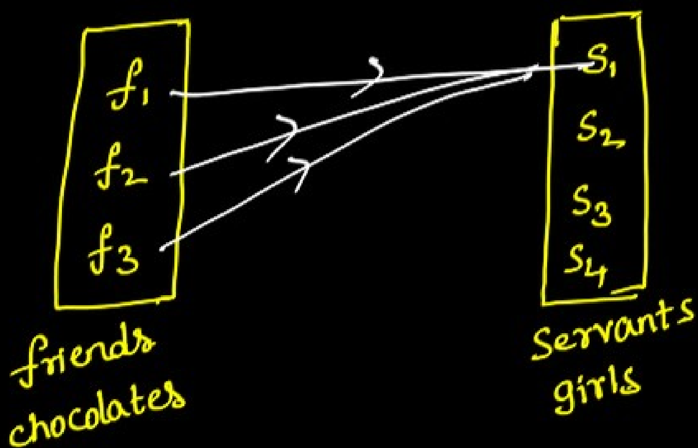
$3 \times 3 \times 3 \times \cdots \times 3$ (10 times)

$= 3^{10}$

Q. How many ways we can distribute $\underbrace{\text{100 distinct letters}}_{\text{chocolates}}$ among $\underbrace{\text{10 boxes}}_{\text{girls}}$

$$= \text{girls}^{\text{chocolates}} = 10^{100}$$

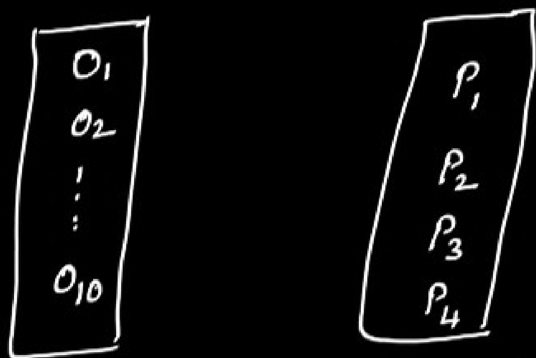
Q. How many ways $\underbrace{\text{four servants}}_{\text{girls}}$ can invite $\underbrace{\text{three friends}}_{\text{chocolates}}$ in any manner?



$4 \times 4 \times 4$

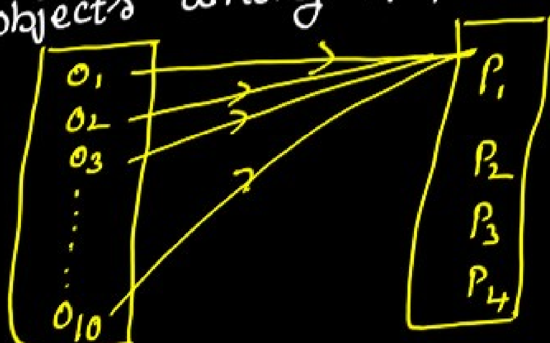
$= 4^3$

Q. How many ways we can distribute 10 distinct objects among 4 persons such that each person gets at most 9 objects.



Required $= 4^{10} - 4$

10 objects among 4-persons $= 4^{10}$



NOT allowed

4 cases

Q. There are 6 jobs with distinct difficulty levels, and 3 computers with distinct processing speeds. Each job is assigned to a computer such that:

- The fastest computer gets the toughest job and the slowest computer gets the easiest job

at least one job

- Every computer gets the ~~easiest~~ job

The number of ways in which this can be done is ________

(GATE-21-Set1)

6 Jobs = $J_1, J_2, J_3, J_4, J_5, J_6$

3 computers = C_1, C_2, C_3

J_1 = Tough Job $\xrightarrow{\text{assign}}$ C_1 = Fastest

J_2 = Easiest Job $\xrightarrow[\text{assign}]{}$ C_2 = slowest

Remaing Jobs = 4 (J_3, J_4, J_5, J_6)

4 Jobs	3 computers
J_3	$\rightarrow C_1$
J_4	$\rightarrow C_2$
J_5	C_3
J_6	

4 Jobs canbe distri among 3 comp $= 3^4$ ways

All Jobs (J_3, J_4, J_5, J_6) distributed 2 comp $(C_1, C_2) = 2^4$ ways

Required $= 3^4 - 2^4$

$= 81 - 16$

$= 65$

Q. n couples are invited to a party with the condition that every husband should be accompanied by his wife. However, a wife need not be accompanied by her husband. The number of different gathering possible at the party is **(GATE-CS-03)**

a) $\binom{2n}{n} * 2^n$

b) 3^n

c) $\frac{(2n)!}{2^n}$

d) $\binom{2n}{n}$

n-couples

Each couple can attend the party in 3 ways

[Hus & wife (or) wife (or) Noone]

Required $= 3 * 3 * 3 * \cdots * 3$ (n times) $= 3^n$

Integral Solutions:

Q. How many positive integral solutions are there for the equation $x_1 + x_2 = 4$

$x_1 + x_2 = 4$

positive integral solutions = 1, 2, 3, 4, ...

$$x_1 + x_2 = 4$$

3 solutions
$$\left\{\begin{array}{l} 1 + 3 = 4 \\ 2 + 2 = 4 \\ 3 + 1 = 4 \end{array}\right.$$

$x_1 + x_2 = 4$

$\square \quad \square \qquad 0\,0\,0\,0$

1	3
2	2
3	1

$0 \_ 0 \_ 0 \_ 0$

one plus symbol in 3 gaps $= {}^{3}C_1 = 3$

Q. How many Non-negative integral solutions are possible for the equation $x_1 + x_2 = 4$

Non-negative integral solutions (0, 1, 2, 3, ----)

$$x_1 + x_2 = 4$$

$$5 \left\{ \begin{array}{l} 1 + 3 \\ 2 + 2 \\ 3 + 1 \\ 0 + 4 \\ 4 + 0 \end{array} \right.$$

Q. Find the possible number of positive integral solutions to the equation

$x_1 + x_2 + x_3 = 8$

No. of positive integral solutions to

$x_1 + x_2 + \cdots + x_r = n$ is ${}^{n-1}C_{r-1} = {}^{8-1}C_{3-1}$

$= {}^{7}C_{2}$

$= 21$

Q. How many Non-negative solutions are exists for equation $x_1 + x_2 + x_3 = 8$

No. of non-negative solutions to $x_1 + x_2 + \cdots + x_r = n$ is

$$= {}^{n+r-1}C_{r-1}$$

$$= {}^{8+3-1}C_{3-1}$$

$$= {}^{10}C_2$$

$$= \underline{\underline{45}}$$

Note:

*• Number of ways of distributing 'n'-identical objects among r-persons each person gets at least one object = $\boxed{{}^{(n-1)}C_{(r-1)}}$ = Number of ways of distributing n-identical balls among r-boxes, each box contains at least one ball =
= No. of positive integral Integral solutions to $x_1+x_2+x_3+\cdots+x_r=n$

* Number of ways of distributing n-identical object among r-persons = ${}^{n+r-1}C_{r-1}$ =

= No. of ways of distributing n-identical balls among r-boxes =
= No. of non-negative integral solutions to $\boxed{x_1+x_2+x_3+\cdots\cdots+x_r=n}$ = ${}^{n+r-1}C_{r-1}$

Q. In how many ways can b blue balls and r red balls be distributed in n distinct boxes? **(GATE-IT-08)**

a) $\frac{(n+b-1)!(n+r-1)!}{(n-1)!b!(n-1)!r!}$ b) $\frac{(n+(b+r)-1)!}{(n-1)!(n-1)!(b+r)!}$

c) $\frac{n!}{b!r!}$ d) $\frac{(n+(b+r)-1)!}{n!(b+r-1)!}$

Normal: No. of ways of distributing n-balls among r-boxes = ${}^{n+r-1}C_{r-1}$

Here: No. of ways of distributing b-blue balls among n-boxes = ${}^{b+n-1}C_{n-1}$

No. of ways of distributing r-red balls among n-boxes = ${}^{r+n-1}C_{n-1}$

Required = ${}^{b+n-1}C_{n-1} * {}^{r+n-1}C_{n-1} = \frac{(b+n-1)!}{(n-1)! * b!} * \frac{(r+n-1)!}{(n-1)!\, r!}$

Q. m identical balls are to be placed in n distinct bags. You are given that $\boxed{m \geq kn}$, where k is a natural number ≥ 1. In how many ways can the balls be placed in the bags if each bag must contain at least k balls? **(GATE-CS-03)**

a) $\binom{m-k}{n-1}$

b) $\binom{m-kn+n-1}{n-1}$

c) $\binom{m-1}{n-k}$

d) $\binom{m-kn+n+k-2}{n-k}$

n - different bags

Each bag contains k-balls = kn

Remaining balls = $m-kn$

General: n-balls among r-boxes

$= {}^{n+r-1}C_{r-1}$

Here: (m-kn) balls among n-bags

$= {}^{(m-kn)+n-1}C_{n-1}$

Pigeon-hole principle:

If n pigeonholes are occupied by n+1 or more pigeons, then at least one pigeonhole is occupied by greater than one pigeon.

Generalized pigeonhole principle is: - If n pigeonholes are occupied by kn+1 or more pigeons, where k is a positive integer, then at least one pigeonhole is occupied by k+1 or more pigeons.

10-pigeons
9-pigeonholes

$\boxed{1}$ $\boxed{2}$ - - - $\boxed{9}$

1 2 9

Birth month

Theorem-

I) If "A" is the average number of pigeons per hole, where A is not an integer then

- At least one pigeon hole contains **ceil[A]** (smallest integer greater than or equal to A) pigeons
- Remaining pigeon holes contains at most **floor[A]** (largest integer less than or equal to A) pigeons

20-pigeonholes
21-pigeon

Average pig = $\frac{21}{20} = 1.0$

$\lceil 1.3 \rceil = 2$

$\lfloor 1.3 \rfloor = 1$

what is the minimum number students that we can take which ensure (gurantees) that at least two students born in the same Month ?

Month :	Jan	Feb	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
Student :	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}
	S_{13}											

students = pigeons

Months = pigeon-holes

13

Deadlocks

processes

resources

Q. **Example-1:** If (Kn + 1) pigeons are kept in n pigeon holes where K is a positive integer what is the average no. of pigeons per pigeon hole?

ACE

No. of pigeons = $Kn+1$

No. of pigeon-holes = $\cancel{K}n$

Average number of pigeons per hole = $\frac{Kn+1}{\cancel{K}n}$ = $\cancel{n+\frac{1}{K}}$ $K+1/n$ (not integer)

$\lceil K+\frac{1}{n}\rceil = K+1$ $\quad$ $\cancel{\lceil n+\frac{1}{K}\rceil = n+1}$ (at least one hole)

$\lfloor K+\frac{1}{n}\rfloor = K$ $\quad$ $\cancel{\lfloor n+\frac{1}{K}\rfloor = n}$ (atmost by remaing holes)

formula: Minimum pigeons required to occupi n-holes such each at least two holes consider more than K-pigeons $= Kn+1$

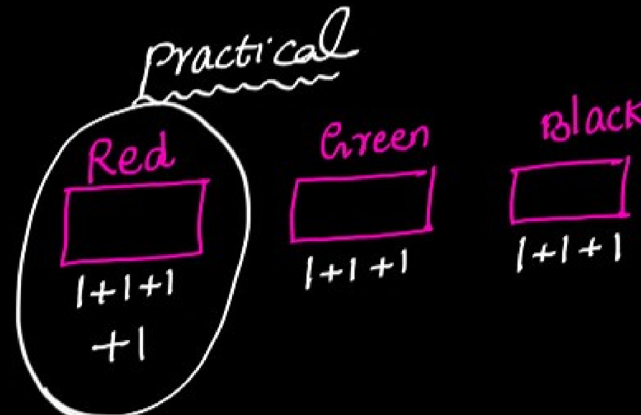
Q. A bag contains 4 red balls, 5 green balls, 6 blacks. Find the minimum no.of balls need to take from the bag, that guarantees 4 balls are of the same colour?

No.of colours = No.of holes = 3 (Red, Green, Black)

$$\text{Average No.of pigeons} = \frac{\text{No.of pigeons}}{\text{No.of holes}}$$

$$= \left\lceil \frac{x}{3} \right\rceil = 4$$

$$x = 10$$



Q. The minimum number of cards to be ~~dealt~~ drawn from an arbitrarily shuffled deck of 52 cards to guarantee that three cards are from some same suit is:

(GATE-CS-2000)

a) 3

b) 8

c) 9

d) 12

Cards(52)
- Red(26)
  - Diamonds(13)
  - HeartS(13)
- Black(26)
  - Spades(13
  - Clubs(13)

No.of Suits = No.of holes = 4.

No.of Cards = No.of piges = ?

$$Avg = \left\lceil \frac{x}{4} \right\rceil = 3$$

$$= 9$$