Set Theory

A well defined unordered collection of distinct objects is called a set. The objects are called the elements of the set.

Example:

Subset Set: Let A and B are any two sets. If every element of A is an element of B, then A is called a subset of B.

$$A \subseteq B$$

If $A = \{1, 3, 5\}$ $B = \{1, 2, 3, 4, 5\}$ then $A \subseteq B$

* For any set A, $A \subseteq A$ and $\phi \subseteq A$

* For any set A, ϕ and A are called Trivial subsets.

Proper Subset: If $A \subseteq B$ and $A \neq B$ then A is called proper subset of B

$$A \subset B$$

Power Set: Let A be any set, then the set of all subsets of A is called power set of A. Its is denoted by P(A).

$|A| = n$

$|P(A)| = 2^n$

Q. The cardinality of the powerset of $\{0, 1, 2, 3, 4, \ldots\ldots, 10\}$ is ______

(GATE)

$A = \{0, 1, 2, 3, 4, \ldots, 10\}$

$n(A) = 11$

$n[P(A)] = 2^{11} = 2048$

Q. For a set A, the power set of A is denoted by 2^A. If A = $\{5, \{6\}, \{7\}\}$. Which of the following options are True? **(GATE)**

I. $\phi \in 2^A$ ✓

II. $\phi \subseteq 2^A$ ✓

III $\{5, \{6\}\} \in 2^A$ ✓

IV $\{5, \{6\}\} \subseteq 2^A$ ✗

a) I and III only

b) II and III only

c) I, II and III only ✓

d) I, II and IV only

$A = \{5, \{6\}, \{7\}\}$

$P(A) = 2^A = \{\phi, \{5\}, \{\{6\}\}, \{\{7\}\}, \{5, \{6\}\}, \{5, \{7\}\}, \{\{6\}, \{7\}\}, \{5, \{6\}, \{7\}\}\}$

$\{5, \{6\}\} \in 2^A$

$\{\{5, \{6\}\}\} \subseteq 2^A$

Equal Sets:

Two set are equal if they have same elements.

Two sets A and B are equal $\Leftrightarrow A \subseteq B$ and $B \subseteq A$.

Empty Set: A set with no elements is called empty set

$\phi = \{ \}$

Universal Set: The set of all objects under consideration (or) discussion is called universal set, 'U' (μ).

Singleton: A set with only one element is called singleton

$A = \{5\}$

Operations on sets:

Example: A = {1, 2, 3, 4} B = {3, 4, 5, 6}

μ = {1, 2, 3, 4, 5, 6, 7, 8}

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

$A \cap B = \{3, 4\}$

$A - B = \{1, 2\}$

$B - A = \{5, 6\}$

$A \Delta B = A \oplus B$

$= (A \cup B) - (A \cap B)$

$= (A - B) \cup (B - A)$

$= \{1, 2, 5, 6\}$

$A^c = \mu - A = \{5, 6, 7, 8\}$

Set Union: $A \cup B = \{x / x \in A \text{ or } x \in B\}$ = $\{x : x \in A$ or $x \in B$ or $x \in$ Both$\}$

Set Intersection: $A \cap B = \{x/x \in A \text{ and } x \in B\}$

Set Difference: $A - B = \{x/x \in A \text{ and } x \notin B\}$

$B - A = \{x/x \in B \text{ and } x \notin A\}$

Symmetric Difference (Boolean Sum):

$A \Delta B = A \oplus B = \{x/x \in A \text{ (or) } x \in B$ (But not both)

$A \oplus B =$

Set Complement:

$A^C = \{x/x \notin A \text{ and } x \in U\}$ ✓

$A \cup A^c = \mu$

$A \cap A^c =$

Cardinality of Set: The number of elements in a set is called the cardinality of that set.

Example:

$A = \{1, 2, 3, 4, 5, 6\}$ ✓

$n(A) = |A| = \text{size} = 6$ ✓

Properties: The following properties hold good for any three sets A, B, C.

Idempotent laws:

$A \cup A = A$ ✓

$A \cap A = A$ ✓

$A = \{1,2\}$

$A \cup A = \{1,2\} \cup \{1,2\}$

$= \{1,2\}$

$= A$

Commutative laws:

$A \cup B = B \cup A$ ✓

$A \cap B = B \cap A$ ✓

$A \Delta B = B \Delta A$ ✓

$2+3 = 3+2$

Associative laws:

$A \cup (B \cup C) = (A \cup B) \cup C$ ✓

$A \cap (B \cap C) = (A \cap B) \cap C$ ✓

$A \cup (B \cup C) = (A \cup B) \cup C$

Distributive laws:

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Demorgan's Laws:

$(A \cup B)^C = A^C \cap B^C$

$(A \cap B)^1 = A^1 \cup B^1$

$A - (B \cup C) = (A - B) \cap (A - C)$

$A - (B \cap C) = (A - B) \cup (A - C)$

Modular laws:

$A \cup (B \cap C) = (A \cup B) \cap C \Leftrightarrow A \subseteq C$

$A \cap (B \cup C) = (A \cap B) \cup C \Leftrightarrow C \subseteq A$

Absorption laws:

$A \cup (A \cap B) = A$

$A \cap (A \cup B) = A$

Distributive

$A \cup (B \cap C) = (A \cup B) \cap \overset{C}{(A \cup C)}$

if $A \subseteq C$

Identity laws:

$A \cup \phi = A$

$A \cap \mu = A$

$A \cup \phi = A$

$A \cap \mu = A$

Complement's laws: $A \cup A^C = \mu$ ✓

$A \cap A^C = \phi$ ✓

Involutory law: $(A^C)^C = A$ ✓

Dominant laws: $A \cup \mu = \mu$

$A \cap \phi = \phi$

$A = A$

$A \cup \mu = \mu$

$A \cap \phi = \phi$

$A = \{1,2\}$ $B = \{2,3\}$ $C = \{3,4\}$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

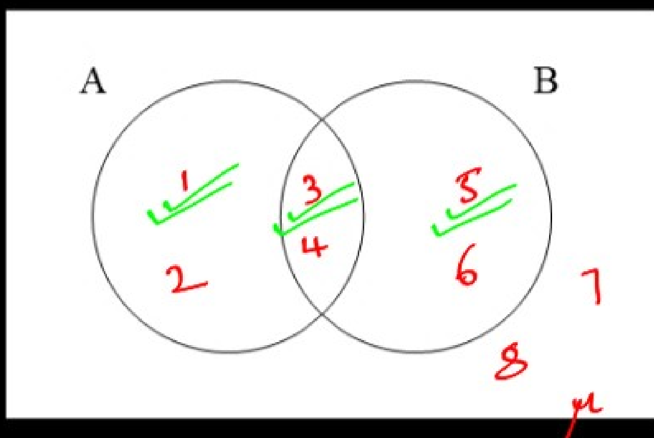
$\{1,2\} \cap \{2,3,4\}$ $\{2\} \cup \{\ \}$

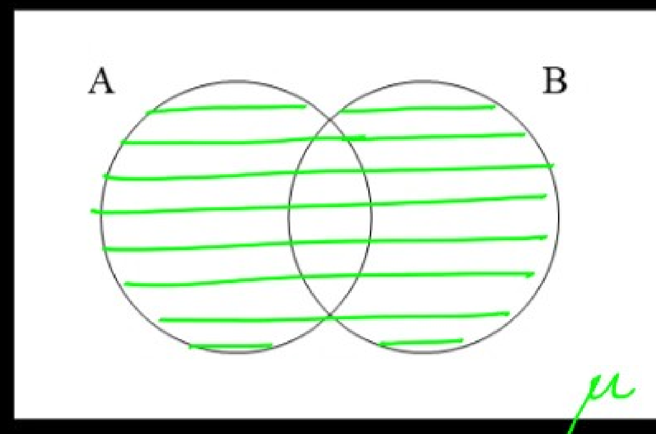
$\{2\}$ ✓ $\{2\}$ ✓

Venn diagram: The diagram which represents the logical relationship among given sets by using closed curves is known as venn diagram

Example: A = {1, 2, 3, 4} B = {3, 4, 5, 6} μ = {1, 2, 3, 4, 5, 6, 7, 8}

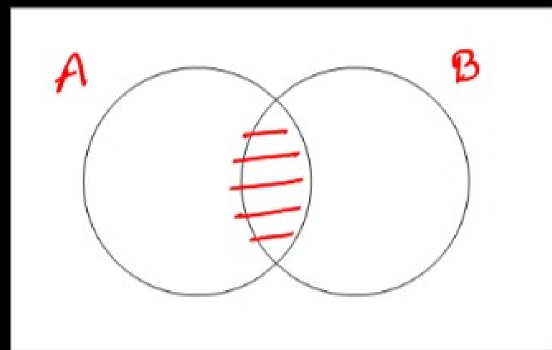
$A \cup B = \{1, 2, 3, 4, 5, 6\}$



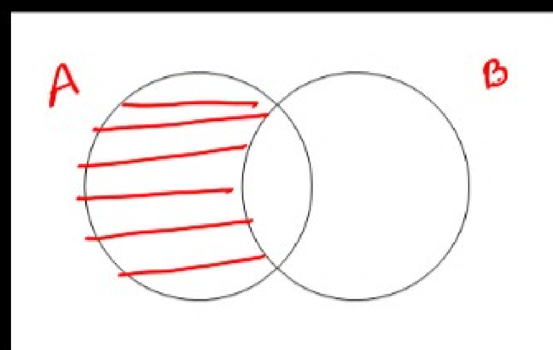


A∪B

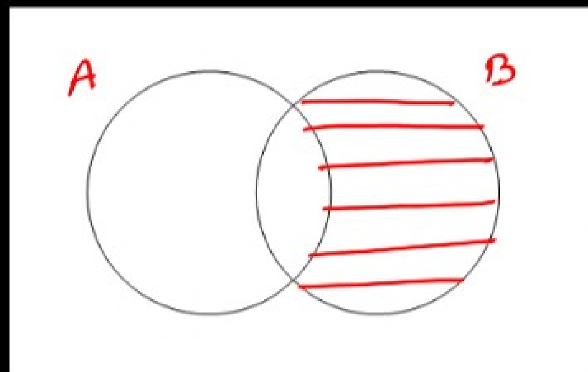
$A \cap B$



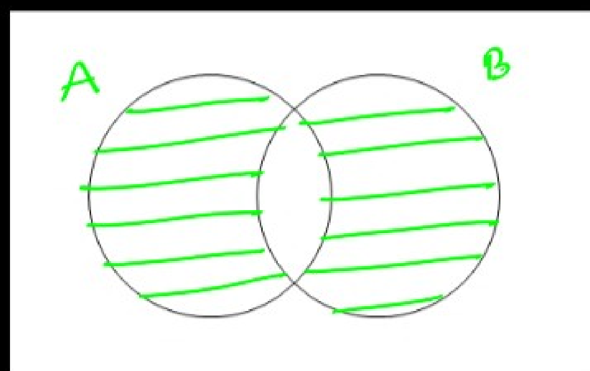
$A - B$



B – A

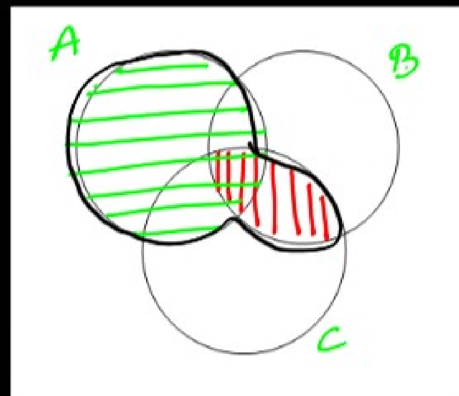


A Δ B



$$A \Delta B = (A \cup B) - (A \cap B)$$
$$= (A - B) \cup (B - A)$$

$A \cup (B \cap C)$

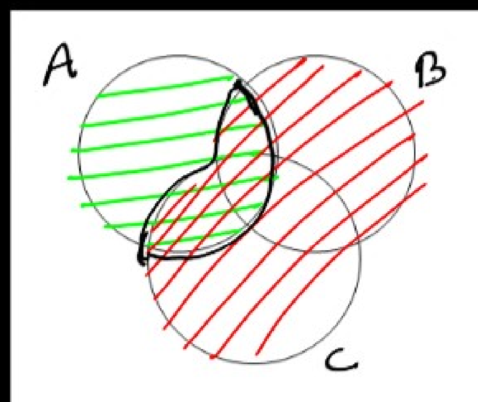


$|||$ = $B \cap C$

$\equiv$ = A

$A \cup (B \cap C) = \equiv + |||$

$A \cap (B \cup C)$

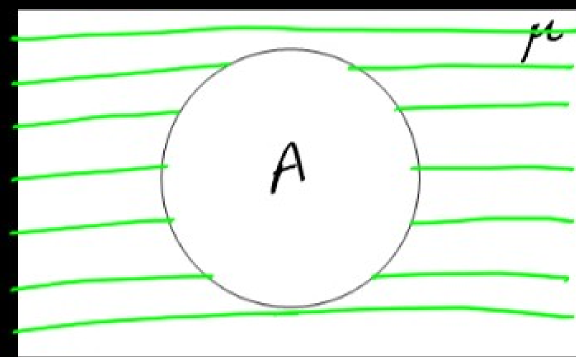


$\equiv$ = A

$///$ = $B \cup C$.

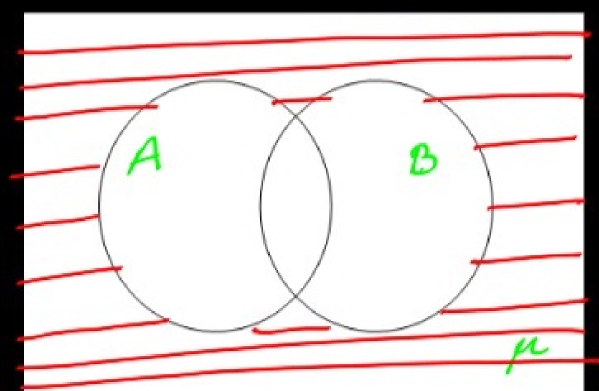
$A \cap (B \cup C) = \#$

A^C



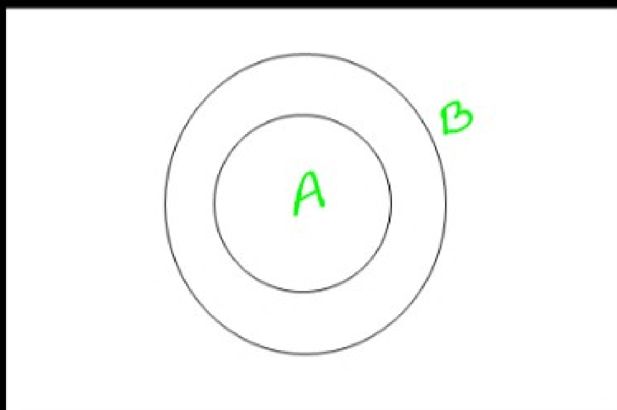
$A^c = \mu - A$

$(A \cup B)^C$

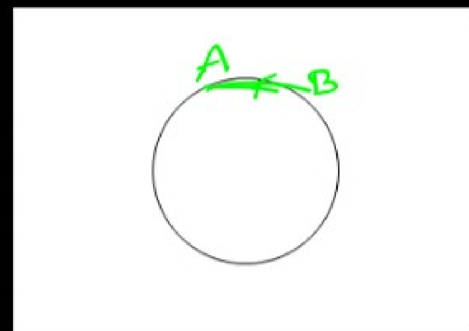


$(A \cup B)^c = \mu - (A \cup B)$

$A \subseteq B$



$A \subset B$



$A = B$

Multisets: Multi sets are un ordered collections of elements where an element can occur as a member more than once. The Notation $\{m_1.a_1,\ m_2.a_2,\ \ldots\ldots,\ m_n.a_n\}$ denotes the multiset with element 'a_1' occurring 'm_1' times and so on.

m_i = multiplicity of a_i

$M = \{m_1 \cdot a_1,\ m_2 \cdot a_2,\ m_3 \cdot a_3, \ldots\ldots, m_n \cdot a_n\}$

eg $M = \{2, 2, 2, 3, 3, 3, 3, 5, 5, 5\}$

$= \{3 \cdot 2,\ 4 \cdot 3,\ 3 \cdot 5\}$

$M = \{a, a, a, b, b, c, c, c\}$

$= \{3 \cdot a,\ 2 \cdot b,\ 3 \cdot c\}$

Operations on Multisets

Let $M_1 = \{3.a, 4.b, 5.c\}$ $M_2 = \{2.a, 3.b, 6.d\}$

$M_1 \cup M_2 = \{3.a, 4.b, 5.c, 6.d\}$

$M_1 \cap M_2 = \{2.a, 3.b,\}$

$M_1 - M_2 = \{1.a, 1.b, 5.c\}$

$-1 \times a$ ×

$M_2 - M_1 = \{6.d\}$

$M_1 \Delta M_2 = (M_1 \cup M_2) - (M_1 \cap M_2) = (M_1 - M_2) \cup (M_2 - M_1)$

$= \{1.a, 1.b, 5.c, 6.d\}$

$M_1 + M_2 = \{5.a, 7.b, 5.c, 6.d\}$

$M_1 = \{5.p, 4.q, 6.r\}$

$M_2 = \{6.p, 3.q, 7.s\}$

$M_1 \cup M_2$

$M_1 \cap M_2$

$M_1 - M_2$

$M_2 - M_1$

$M_1 \Delta M_2$

$M_1 + M_2$

Q. Find the all possible number of Multisets of size '4' from n-elements in which atleast one element appears exactly twice?

a) $\frac{n(n+1)}{2}$ $= \frac{\cancel{2}\times 3}{\cancel{2}} = 3 \neq 1$

b) $\frac{n(n+1)(2n+1)}{6}$ $= \frac{2(3)(5)}{6} = 5 \neq 1$

c) $\left[\frac{n(n+1)}{2}\right]^2$ $= \left[\frac{2(3)}{2}\right]^2 = 9 \neq 1$

d) $\frac{n(n-1)^2}{2}$ $= \frac{2(1)^2}{2} = 1$

$n=2$

$\{a,b\}$

No. of Multisets $= 1$

$\{a,b,a,b\}$

$\{a, b, c, \ldots, n\}$

Size 4 ✓

at least one element appears exactly twice

$= \{a, b, c, a\}$ (or) $\{a, b, a, b\}$ (or) $\cancel{\{a, b, c, a, b, c\}}$

$= [nC_3 * 3C_1] + [nC_2 * 2C_2]$

$= \dfrac{n(n-1)(n-2)}{\cancel{3} \times 2 \times 1} * \cancel{3} + \dfrac{n(n-1)}{2} \times 1$

$= \dfrac{n(n-1)}{2} [(n-2) + 1]$

$= \dfrac{n(n-1)^2}{2}$

Partitions: Let ‘S’ be a non-empty set and $P_1, P_2, P_3, P_4, \ldots, P_n$ non empty sets are said to be partitions of ‘S’ $\Leftrightarrow$

i) $P_1 \cup P_2 \cup P_3 \cup \text{- - - - - - - - -} \cup P_n = S$, $\sum_{i=1}^{n} P_i \cup = S$

ii) $P_i \cap P_j = \phi, \forall i, j$

$P_1 \cup P_2 \cup P_3 \cup \text{--} = S$

$P_1 \cap P_2 = \Phi$

$P_1 \cap P_3 = \Phi$

Q. {a, b, c}

which of the following are partitions of $\{1,2,3,4\}$

I. $\{1\}, \{2\}, \{3\}, \{4\}$ ✓

II. $\{1,2\}, \{3,4\}$ ✓

III. $\{1,2,3\}, \{4\}$ ✓

IV. $\{1,2,3,4\}$ ✓

× V. $\{1,2,3\}, \{3,4\}$ $\cdot\ P_i \cap P_j \neq 0$

× VI. $\{1\}, \{2,3\}$ $\cdot\ \Sigma P_i \cup = S$

× VII $\{1,2\}, \{3,4\}, \phi$

Q: List out all possible partitions of $\{1,2,3,4\}$

$A = \{1,2,3,4\}$

1. $\{1\}, \{2\}, \{3\}, \{4\} \longrightarrow 1$
2. $\{1,2\}, \{3,4\} \longrightarrow 3$
3. $\{1,2,3\}, \{4\} \longrightarrow 4$
4. $\{1,2,3,4\} \longrightarrow 1$
5. $\{1,2\}, \{3\}, \{4\} \longrightarrow 6$

$+$

15

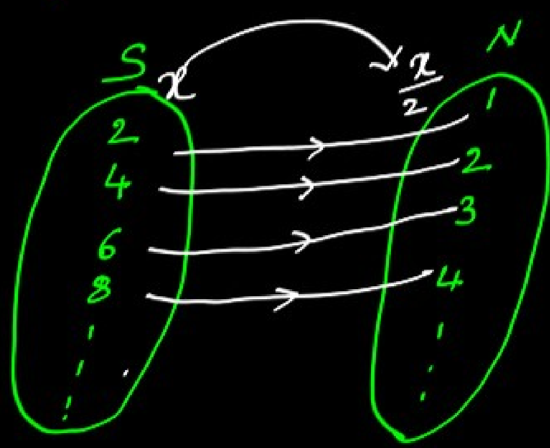
Countably Infinite: A set is said to be countably Infinite if there is a Bijective function f : S → N (Where 'N' is set of Natural Numbers), otherwise It is uncountable.

$S \longrightarrow N$

$N \longrightarrow S$

Q. S = {2, 4, 6, 8, 10, …… }

$f : S \longrightarrow N \qquad f(x) = \frac{x}{2}$



$f : S \rightarrow N$ is one-one & onto

Hence $f : S \rightarrow N$ is a Bijective

$\therefore$ S is countably infinite

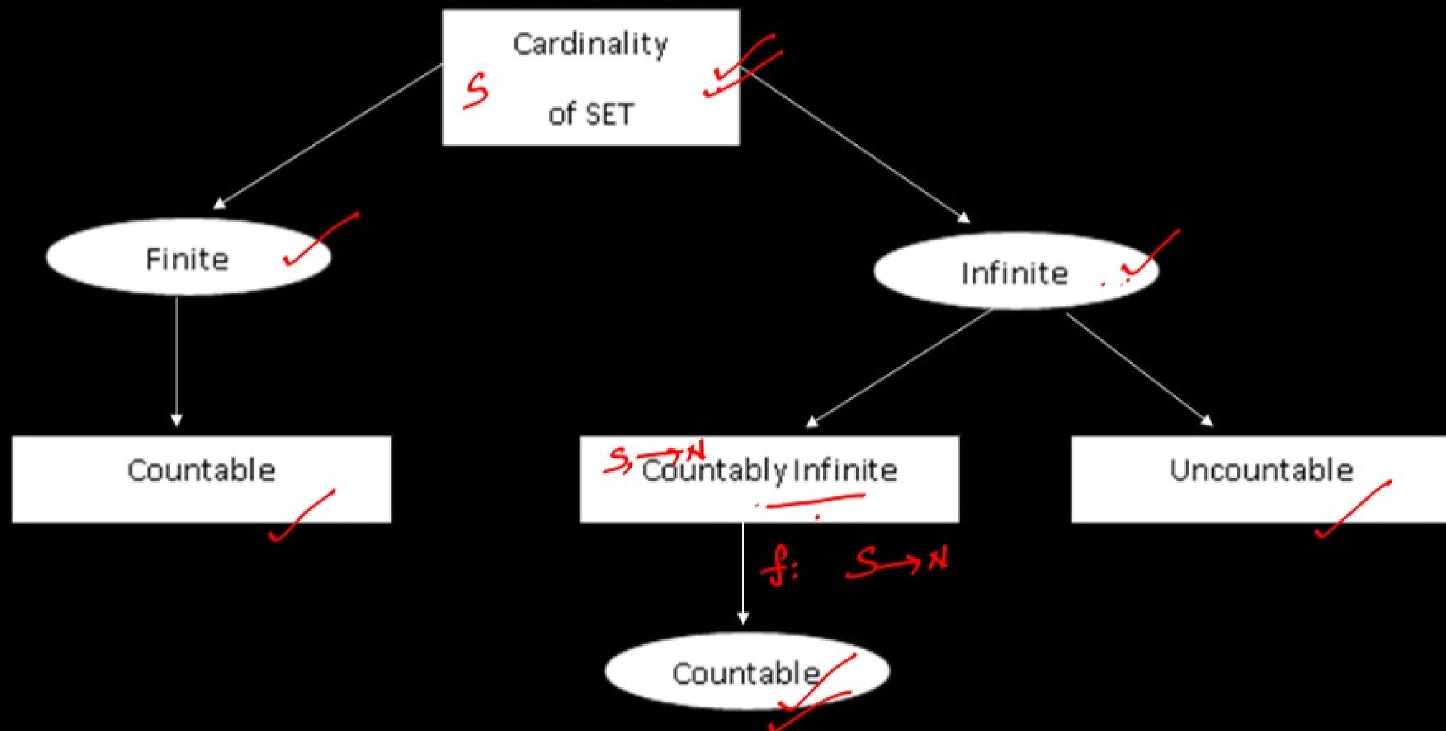
NOTE

A function which is both one-one and onto is known as Bijective function.

one-one : Different elements of domain has different images in co-domain

onto : Range is Same as co-domain

Countable:



Cardinality of SET
S
Finite
Infinite
Countable
S → N
Countably Infinite
Uncountable
f: S → N
Countable

Q. S = {…….., -2, -1, 0, 1, 2, ……..}

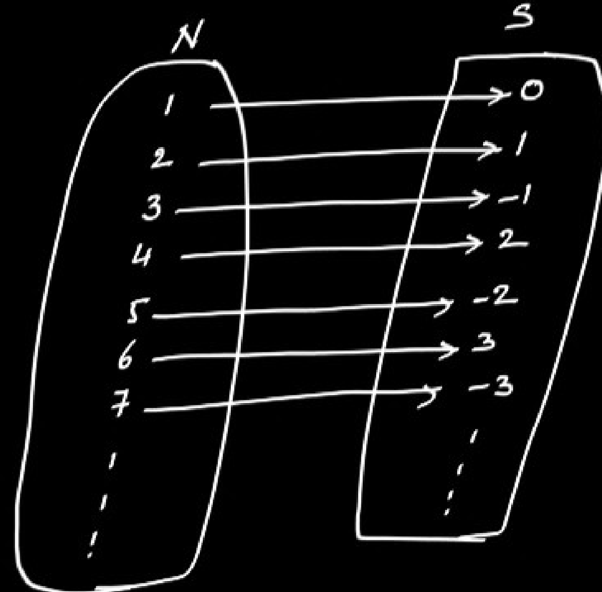
Set of Integers is countable / uncountable

$$\mathbb{Z} = S = \{ \ldots, \underline{-3, -2, -1, 0}, \underline{1, 2, 3, \ldots} \}$$

$$N = \{ \underline{1}, 2, \underline{3}, 4, \ldots \}$$

$$f : N \longrightarrow S$$

$$f(x) = \begin{cases} \dfrac{x}{2}, & \text{when } x \text{ even} \\ \dfrac{1-x}{2}, & \text{when } x \text{ odd} \end{cases}$$



Countably Infinite

Q. The symmetric difference of sets $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $B = \{1, 3, 5, 6, 7, 8\}$

ISRO-2017

a) $\{1, 3, 5, 6, 7, 8\}$

b) $\{2, 4, 3\}$

c) $\{2, 4\}$ ✓

d) $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$$A \Delta B = (A \cup B) - (A \cap B)$$
$$= (A - B) \cup (B - A)$$

Q. The number of elements in the power set of $\{\{1, 2\}, \{2, 1, 1\}, \{2, 1, 1, 2\}\}$ is

ISRO-2017

ACE

a) 3

b) 8

c) 4

d) 2

$$A = \{\{1,2\}, \{2,1,1\}, \{2,1,1,2\}\}$$

$$\{1,2\} = \{2,1,1\} = \{2,1,1,2\}$$

$$A = \{\{1,2\}, \{2,1,1\}, \{2,1,1,2\}\} = \{\{1,2\}\}$$

$$n(A) = |A| = 1 \quad \therefore \quad |P(A)| = 2^n = 2^1 = 2$$

Q. If A = {x, y, z} and B = {u, v, w, x}, and the universe is {s, t, u, v, w, x, y, z}. Then $(A \cup \overline{B}) \cap (A \cap B)$ is equal to ISRO 2020

a) {u, v, w, x}

b) {x}

c) {u, v, w, x, y, z}

d) {u, v, w}

Q. Let A, B & C be non-empty sets and let (GATE-05)

$X = (A - B) - C$ and $Y = (A - C) - (B - C)$ ✓

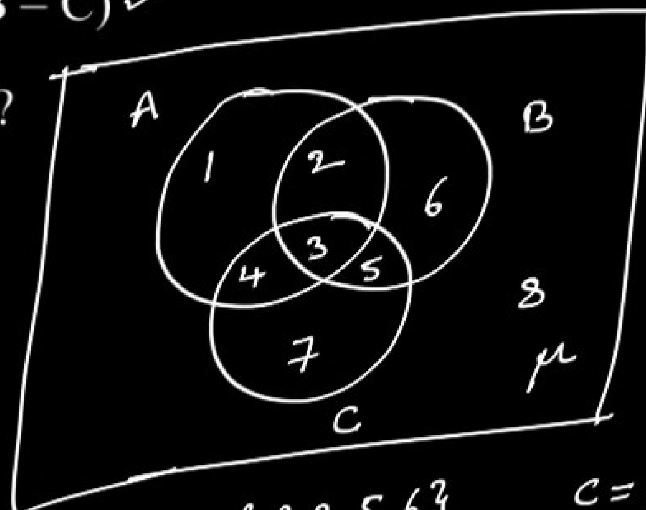
Which one of the following is TRUE?

a) $X = Y$ ✓

b) $X \subset Y$

c) $Y \subset X$

d) none of these



$A = \{1,2,3,4\}$ $\quad B = \{2,3,5,6\}$ $\quad C = \{3,4,5,7\}$

$X = (A-B) - C$

$= \{1,4\} - \{3,4,5,7\}$

$= \{1\}$

$Y = (A-C) - (B-C)$

$= \{1,2\} - \{2,6\}$

$= \{1\}$

$X = Y$

Q. If $A = \{x, y, z\}$ and $B = \{u, v, w, x\}$, and the universe is $\{s, t, u, v, w, x, y, z\}$. Then $(A \cup \overline{B}) \cap (A \cap B)$ is equal to ISRO 2020

ACE

a) $\{u, v, w, x\}$

b) $\{x\}$

c) $\{u, v, w, x, y, z\}$

d) $\{u, v, w\}$

Q. Let A and B be sets and let A^c and B^c denote the complements of the sets A and B. the set

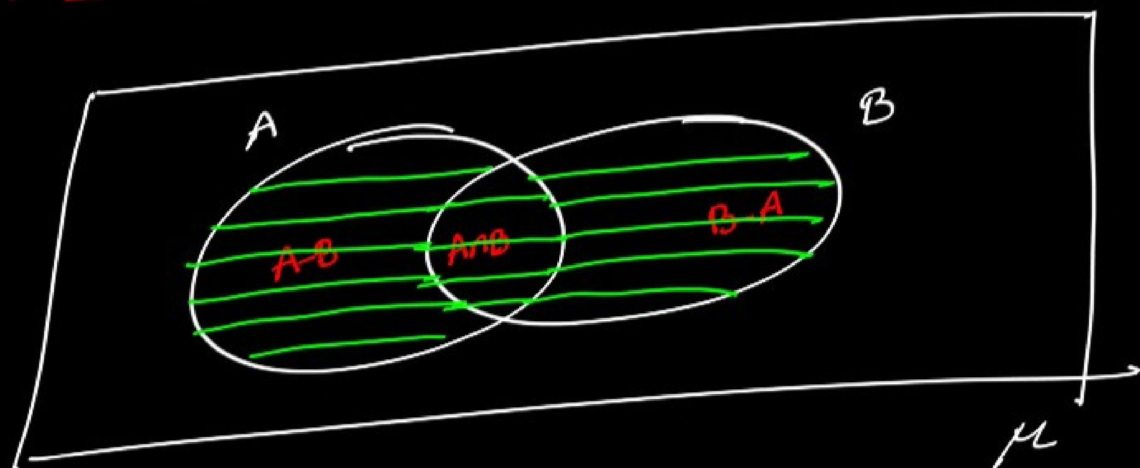
$(A - B) \cup (B - A) \cup (A \cap B)$ is equal to (GATE-96)

a) $A \cup B$

b) $A^c \cup B^c$

c) $A \cap B$

d) $A^c \cap B^c$



Q. Let P, Q and R be the sets. Let Δ denote the symmetric difference operator defined as $P\Delta Q = (P \cup Q) - (P \cap Q)$. Using venn diagrams, determine which of the following is/are TRUE?

(I) $P\Delta(Q\cap R) = (P\Delta Q)\cap(P\Delta R)$

(II) $P\cap(Q\Delta R) = (P\cap Q)\Delta(P\cap R)$ **(GATE-06)**

a) I only

b) II only

c) Neither I nor II

d) Both I and II

Q. Let S be an infinite set and $S_1, S_2 \ldots S_n$ be sets such that $S_1 \cup S_2 \cup \ldots\ldots \cup S_n = S$. then (GATE-93)

a) At least one of the sets S_i is a finite set

b) not more than one of the sets S_i can be finite

c) At least one of sets S_i is ~~finite~~ infinite

d) not more than one the sets S_i is an a infinite set

Q. Let E, F and G be finite sets. (GATE-06)

Let X=$(E \cap F) - (F \cap G)$ and $Y = (E - (E \cap G)) - (E - F)$. which one of the following is true?

a) $X \subset Y$

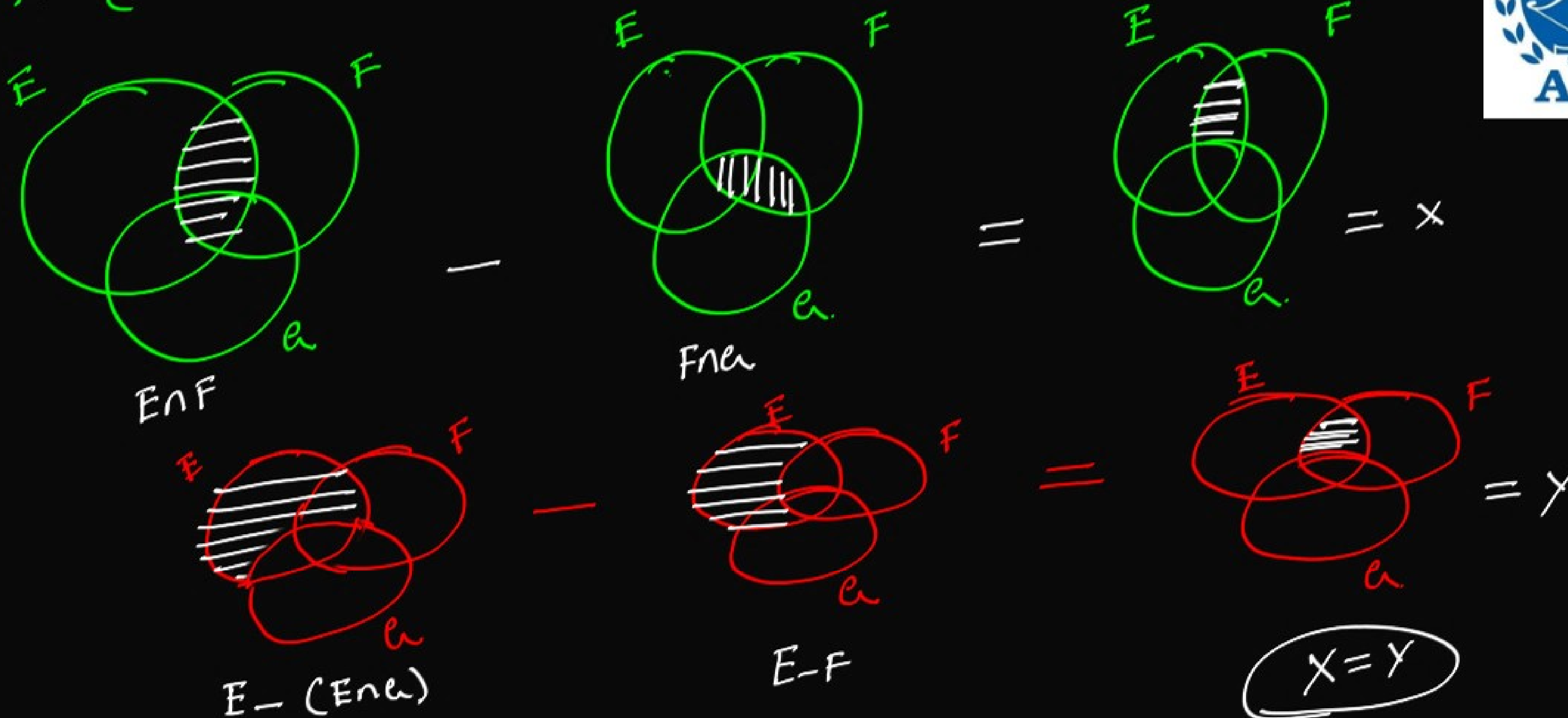
b) $X \supset Y$

c) $X = Y$

d) $X - Y \neq \varnothing$, $Y - X \neq \varnothing$

$X = (E \cap F) - (F \cap G)$

$Y = [E - (E \cap G)] - (E - F)$



Q. let A be a set with n elements. Let C be a collection of distinct subsets of A such that for any two subsets S_1 and S_2 in C, either $S_1 \subset S_2$ or $S_2 \subset S_1$. What is the maximum cardinality of C? **(GATE-05)**

a) n

b) n+1

c) $2^{n-1}+1$

d) n!

$A = \{a, b, c\}$ 3

All possible subsets = $P(A)$

$= \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$

For any S_1 and S_2 either $S_1 \subset S_2$ (or) $S_2 \subset S_1$

$C = \{\phi, \{a\}, \{a,b\}, \{a,b,c\}\}$

$C = \{\phi, \{b\}, \{b,c\}, \{a,b,c\}\}$

$C = \{\phi, \{c\}, \{a,c\}, \{a,b,c\}\}$

$|C| = 4$

ACE

Q. Consider the following statements:

S1: There exist infinite sets A, B, C such that $A \cap (B \cup C)$ is finite.

S2: There exist two irrational numbers x and y such that $(x + y)$ is rational.

Which of the following is true about S1 and S2? **(GATE-01)**

a) only S1 is correct

b) only S2 is correct

c) Both S1 and S2 are correct

d) none of S1 and S2 is correct

Q. Let P(S) denote the power set of a S. which of the following is always true? **(GATE-00)**

a) $P(P(S)) = P(S)$

b) $P(S) \cap P(P(S)) = \{\varnothing\}$

c) $P(S) \cap S = P(S)$

d) $S \notin P(S)$

Q. The number of elements in the power set P(S) of the set S = { {ϕ},1,{2,3}} is **(GATE-95)**

a) 2

b) 4

c) 8

d) None of these

Q. Suppose U is the power set of the set S = {1, 2, 3, 4, 5, 6}. For any $T \in U$, let $|T|$ denote the number of elements in T and T^1 denote the complement of T. For any T, $R \in U$, let $T|R$ be the set of all elements in T which are not in R. Which one of the following is true? $T-R$ (GATE-15-Set3)

a) $\forall X \in U\ (|X| = |X^1|)$ ×

b) $\exists X \in U\ \exists Y \in U\ (|X| = 5, |Y| = 5 \text{ and } X \cap Y = \phi)$ ×

c) $\forall X \in U\ \forall Y \in U\ (|X| = 2, |Y| = 3 \text{ and } X/Y = \phi)$ ×

d) $\forall X \in U\ \forall Y \in U\ (X|Y = Y^1|X^1)$ ✓

(d) ✓ $\overline{X/Y} = X-Y$

$Y^1/X^1 = Y^1 - X^1$

$= (U-Y) - (U-X) = X-Y$

$S = \{1,2,3,4,5,6\}$

U = powerset of $S = P(S)$

$|U| = |P(S)| = 2^6 = 64$

a) $\forall X \in U, \quad |X| = |X'|$

Let $X = \{1,2,3\} \subset S$

$X' = U - S$

(b) $S = \{1,2,3,4,5,6\}$

$|X| = 5 \qquad |Y| = 5$

$X = \{1,2,3,4,5\} \qquad Y = \{2,3,4,5,6\}$

$X \cap Y \neq \phi$

(c) $X = \{1,2\} \quad Y = \{3,4,5\}$

$X / Y = X - Y$

$= \{1,2\} \neq \phi$

$X \in U$

$X \in P(S)$

Relations

Cartesian Product: Cartesian product of two sets A and B, written as ‘$A \times B$’ and is defined as

$$A \times B = \{(a, b) \,/\, a \in A \text{ and } b \in B\}$$

$$A \times B \times C = \{(a, b, c) \,/\, a \in A,\ b \in B \text{ and } c \in C\}$$

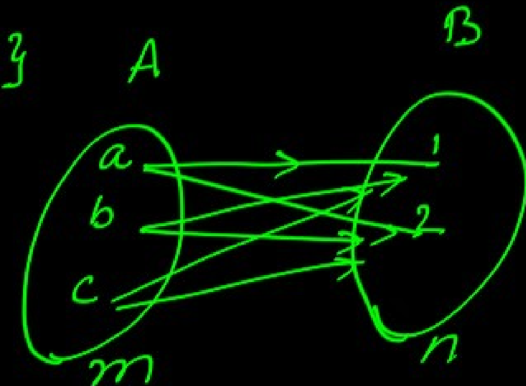
$A = \{a, b, c\}$ $\quad$ $B = \{1, 2\}$

$A \times B = \{(a,1), (a,2), (b,1), (b,2), (c,1), (c,2)\}$

$n(A) = 3$ $\quad$ $n(B) = 2$

$n(A \times B) = 3 \times 2 = 6$

$m \times n$



Relation: A (binary) relation ‘R’ from A to B is a subset of A × B. If A = B, then we can say ‘R’ is a relation on A

$R \subseteq A \times B$

$2^{m \times n}$

$|S| = 3$

$|P(S)| = 2^3$

$|A| = m.$ $|B| = n.$

Relations A to B = $2^{m \times n}$

Note:

* If $|A| = m$ and $|B| = n$ then the number of relations possible from A to B $= 2^{mn}$.

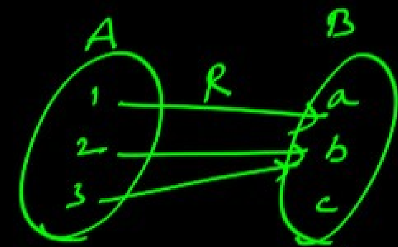
* If $|A| = n$ then the number of relations possible on set A $= 2^{n \times n} = 2^{n^2}$

$A \longrightarrow A$

$|A \times A| = n^2$

Domain and Range:

If ‘R’ is a relation from ‘A’ to ‘B’. Then Domain of R = $\{x/x \in A$ and $(x, y) \in R$ for some $y \in B\}$



Range of

$R = \{y/y \in B$ and $(x, y) \in R$ for some $x \in A\}$

Domain $R = \{1,2,3\}$

Inverse Relation:

$$R^{-1} = \{(b, a) \,/\, (a, b) \in R\}$$

Complementary relation: If R is a relation from A to B then the complement of ‘R’ is

$$\overline{R} = \{(a, b) \,/(a, b) \notin R\}$$

$\overline{A} = A' = A^c = \mu - A$

$\overline{R} = (A \times B) - R$

$\overline{R} =$ Cartesian $- R$

Types of Relations

Reflexive: A relation 'R' on set A is said to be reflexive if, x R x, $\forall$ x $\in$ A

$$\text{i.e., } (x, x) \in R \ \forall x \in A$$

Examples:

$$A = \{1, 2, 3, 4\}$$

$R : A \rightarrow A$

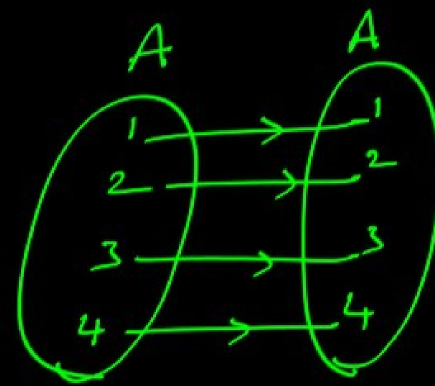
$R_1 = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$ ✓

$R_2 = \{(1, 1), (2, 2), (3, 3)\}$ ✗

$R_3 = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (1, 3)\}$ ✓

$R_4 = \{\ \}$ ✗

✗



Symmetric Relation: A relation R on a set A is said to be symmetric

if x R y then y R x $\forall$ x, y $\in$ A

True

if (x, y) $\in$ R then (y, x) $\in$ R $\forall$x, y $\in$ A

False

Examples:

A = {a, b, c}

R_1 = {(a, b), (b, a)}

R_2 = {(a, b), (b, a), (a, c)} X

R_3 = { }

R_4 = A × A

{(a, c), (c, a)}

{(b, c), (c, b)}

{(a, a)}

$(c, a) \notin R_2$

$(a, c) \in R_2$ but $(c, a) \notin R_2$

R_2 is NOT Symmetric

Transitive Relation: A relation 'R' on a set A is said to be transitive if (xRy) and (yRz) then (xRz)

$\forall\ x, y, z \in A$

If (x, y) and $(y, z) \in R$. Then $(x, z) \in R$

Examples:

Let $A = \{a, b, c, d\}$

$R_1 = \{(a, b), (b, c), (a, c)\}$ = $\{(a,b), (b,c), (a,c)\}$

$R_2 = \{(a, a), (b, b)\}$ = $\{\underset{x\,y}{(a,a)}, \underset{y\,z}{(a,a)}, \underset{x\,z}{(a,a)}$

$R_3 = \phi$

$R_4 = A \times A$

Brother

Salman → Sharuk → Abhi

brother

Equivalence: A relation (R, ≤) is said to be an equivalence relation if it is reflexive, symmetric and Transitive.

$A = \{1, 2, 3, 4\}$

$R_1 = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1)\}$

$R_2 = \{(1, 1), (2, 2), (3, 3)\}$

$R_3 = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$

$R_4 = A \times A$

$[R, \preceq]$

preceeding to

preceeds to

Q. State whether the following statements are TRUE (or) FALSE

S_1: The union of Two equivalence relations is also an equivalence relation. False

S_2: The intersection of two equivalence relations is also an equivalence relation. TRUE

$A = \{1, 2, 3, 4\}$

$R_1 : A \to A = \{(1,1), (2,2), (3,3), (4,4), (1,2), (2,1)\}$

$R_2 : A \to A = \{(1,1), (2,2), (3,3), (4,4), (1,3), (3,1)\}$

$R_1 \cup R_2 = \{(1,1), (2,2), (3,3), (4,4), (1,2), \boxed{(2,1), (1,3)}, (3,1)\}$

$(2,1)$ and $(1,3) \in R_1 \cup R_2$

But $(2,3) \notin R_1 \cup R_2$

$R_1 \cup R_2$ is NOT Transitive

$R_1 \cup R_2$ NOT equivalence

$R_1 \cap R_2 = \{(1,1), (2,2), (3,3), (4,4)\}$

= equivalence

$A \times A$

$= \{(1,1), (1,2), (1,3), (1,4),$
$(2,1), (2,2), (2,3), (2,4),$
$(3,1), (3,2), (3,3), (3,4),$
$(4,1), (4,2), (4,3), (4,4)\}$

(5) Compatable relation:

If a relation is both reflexive and Symmetric.
then It is known as compatable relation.

Q. The number of equivalence relations on set $\{1, 2, 3, 4\}$ is

a) 15

b) 16

c) 24

d) 24

ACE