Q. Let Graph(x) be a predicate which denotes that x is a graph. Let Connected(x) be a predicate which denotes that x is connected. Which of the following first order logic sentences **DOES NOT** represents the statement: "Not every graph is connected"? **GATE – 07**

(a) $\sim\forall x(\text{Graph}(x) \Rightarrow \text{Connected}(x))$ = Not every graph is connected

(b) $\exists x(\text{Graph}(x) \wedge \sim \text{Connected}(x))$ = Some graphs are connected

⇒ Some graphs are NOT connected

(c) $\sim\forall x(\sim\text{Graph}(x) \vee \text{Connected}(x))$

$\equiv \exists x\,[G(x) \wedge \sim C(x)]$

(d) $\forall x(\text{Graph}(x) \Rightarrow \sim \text{Connected}(x))$

Q. The **CORRECT** formula for the sentence, "not all rainy days are cold" is

GATE-14-Set3

(a) $\forall d\,(\text{Rainy}(d) \wedge \sim \text{Cold}(d))$

(b) $\forall d(\sim \text{Rainy}(d) \rightarrow \text{Cold}(d))$

(c) $\exists d(\sim \text{Rainy}(d) \rightarrow \text{Cold}(d))$

(d) $\exists d(\text{Rainy}(d) \wedge \sim \text{Cold}(d))$

= Some rainy days are cold

$\Rightarrow$ Some rainy days are not cold

(b) $\forall d\,[\sim R(d) \longrightarrow C(d)]$

$\forall d\,[R(d) \vee C(d)]$

Which one of the first order predicate calculus statements given below correctly express following English statement?

Tigers and lions attack if they are hungry or threatened. GATE – 06

40 + 20

(a) $\forall x[(tiger(x) \wedge lion(x)) \rightarrow \{(hungry(x) \vee threatened(x)) \rightarrow attacks(x)\}]$

(b) $\forall x[(tiger(x) \wedge lion(x)) \rightarrow \{(hungry(x) \vee threatened(x)) \rightarrow attacks(x)\}]$

(c) $\forall x[(tiger(x) \vee lion(x)) \rightarrow \{attacks(x) \rightarrow (hungry\ (x) \vee threatened(x))\}]$

(d) $\forall x[(tiger(x) \vee lion(x)) \rightarrow \{(hungry(x) \vee threatened(x)) \rightarrow attacks(x)\}]$

Verify the following logical relationship

I. $[\forall x \; p(x) \vee \forall x \; q(x)] \longrightarrow \forall x \; [p(x) \vee q(x)]$

Let us consider:

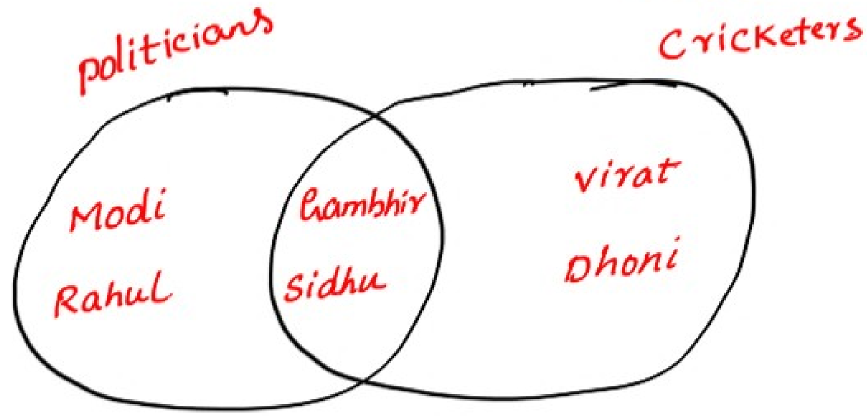
$p(x)$: x is a politician.

$q(x)$: x is a cricketer.

politicians only = { Modi, Rahul }

cricketers only = { Virat, Dhoni }

politicians and cricketers = { Goutam Gambhir, Sidhu }



I. $[\forall x\, P(x) \vee \forall x\, q(x)] \longrightarrow \forall x\, [P(x) \vee q(x)]$

Here we should consider domain in such away that LHS is TRUE

Let us consider Domain = $\{\overset{x_1}{\text{Modi}}, \overset{x_2}{\text{Rahul}}\}$

$$LHS = \forall x\, P(x) \vee \forall x\, q(x)$$
$$= [P(x_1) \wedge P(x_2)] \vee [q(x_1) \wedge q(x_2)]$$
$$= [P(Modi) \wedge P(Rahul)] \vee [q(Modi) \wedge q(Rahul)]$$
$$= (T \wedge T) \vee (F \wedge F)$$
$$= T \vee F$$
$$= T$$

$$RHS = \forall x\, [P(x) \vee q(x)]$$
$$= [P(x_1) \vee q(x_1)] \wedge [P(x_2) \vee q(x_2)]$$
$$= [P(Modi) \vee q(Modi)] \wedge [P(Ra) \vee q(Ra)]$$
$$= (T \vee F) \wedge (T \vee F)$$
$$= T \wedge T$$
$$= T$$

Yes It is logically implied

II. Check following logical implication

$\forall x\,[P(x) \vee q(x)] \longrightarrow [\forall x\, P(x) \vee \forall x\, q(x)]$

Let us consider Domain = $\{\underset{x_1}{\text{Modi}}, \underset{x_2}{\text{virat}}\}$

LHS = $\forall x\,[P(x) \vee q(x)]$

$= [P(x_1) \vee q(x_1)] \wedge [P(x_2) \vee q(x_2)]$

$= [P(\text{Modi}) \vee q(\text{Modi})] \wedge [P(\text{virat}) \vee q(\text{virat})]$

$= (T \vee F) \wedge (F \vee T)$

$= T \wedge T$

$= T$

RHS = $[\forall x\, P(x) \vee \forall x\, q(x)]$

$= [P(x_1) \wedge P(x_2)] \vee [q(x_1) \wedge q(x_2)]$

$= [P(\text{Modi}) \wedge P(\text{virat})] \vee [q(\text{Modi}) \wedge q(\text{virat})]$

$= (T \wedge F) \vee (F \wedge T)$

$= F \vee F$

$= F$

NOT logically implied.

III check following logical implication.

$$[\forall x\, P(x) \wedge \forall x\, q(x)] \longrightarrow \forall x\, [P(x) \wedge q(x)]$$

Domain = {Gambhir, siddhu}

$LHS = \forall x\, P(x) \wedge \forall x\, q(x)$

$= [P(Gam) \wedge P(sidd)] \wedge [q(Gam) \wedge q(sidd)]$

$= (T \wedge T) \wedge (T \wedge T)$

$= T$

$RHS = \forall x\, [P(x) \wedge q(x)]$

$= [P(Gam) \wedge q(Gam)] \wedge [P(sid) \wedge q(sid)]$

$= [T \wedge T] \wedge [T \wedge T]$

$= T$

Yes It is logically implied

IV check the following logical implication

$$\forall x\,[P(x) \wedge q(x)] \longrightarrow [\forall x\,P(x) \wedge \forall x\,q(x)]$$

Let us Domain = {Gambhir, siddhu}

$LHS = \forall x\,[P(x) \wedge q(x)]$

$= [P(Gam) \wedge q(Gam)] \wedge [P(sid) \wedge q(sid)]$

$= (T \wedge T) \wedge (T \wedge T)$

$= T$

$RHS = \forall x\,P(x) \wedge \forall x\,q(x)$

$= [P(Gam) \wedge P(sid)] \wedge [q(Gam) \wedge q(sid)]$

$= (T \wedge T) \wedge (T \wedge T)$

$= T$

yes It is Logically implied

Conclusion :

I. $[\forall x\, P(x) \wedge \forall x\, q(x)] \longrightarrow \forall x\, [P(x) \wedge q(x)]$

II. $\forall x\, [P(x) \wedge q(x)] \longrightarrow [\forall x\, P(x) \wedge \forall x\, q(x)]$

From above I & II

$$[\forall x\, P(x) \wedge \forall x\, q(x)] \longleftrightarrow \forall x\, [P(x) \wedge q(x)]$$

III $[\forall x\, P(x) \vee \forall x\, q(x)] \longrightarrow \forall x\, [P(x) \vee q(x)]$

IV $\forall x\, [P(x) \vee q(x)] \not\longrightarrow \forall x\, P(x) \vee \forall x\, q(x)$

(NOT logically implied)

Q5 Check the following implication

$$[\exists x\, P(x) \vee \exists x\, q(x)] \longrightarrow \exists x\, [P(x) \vee q(x)]$$

valid

Q6 Check the following implication

$$\exists x\, [P(x) \vee q(x)] \longrightarrow \exists x\, P(x) \vee \exists x\, q(x)$$

valid

Q7 Check the following logical implication.

$$[\exists x\, P(x) \wedge \exists x\, q(x)] \longrightarrow \exists x\, [P(x) \wedge q(x)]$$

NOT valid

Q8: Check: $\exists x\, [P(x) \wedge q(x)] \longrightarrow [\exists x\, P(x) \wedge \exists x\, q(x)]$

valid

Q7 $[\exists x\; P(x) \wedge \exists x\; q(x)] \longrightarrow \exists x\, [P(x) \wedge q(x)]$

Let Domain $= \{ \overset{x_1}{\text{Modi}}, \overset{x_2}{\text{virat}} \}$

LHS $= \exists x\; P(x) \wedge \exists x\; q(x)$

$= [P(x_1) \vee P(x_2)] \wedge [q(x_1) \vee q(x_2)]$

$= [P(\text{Modi}) \vee P(\text{virat})] \wedge [q(\text{Modi}) \vee q(\text{virat})]$

$= (T \vee F) \wedge (F \vee T)$

$= T \wedge T$

$= T$

RHS $= \exists x\, [P(x) \wedge q(x)]$

$= [P(x_1) \wedge q(x_1)] \vee [P(x_2) \wedge q(x_2)]$

$= [P(\text{Modi}) \wedge q(\text{Modi})] \vee [P(\text{virat}) \wedge q(\text{virat})]$

$= (T \wedge F) \vee (F \wedge T)$

$= F \vee F$

$= F$

NOT logically implied.

Conclusions :

I. $[\exists x\, P(x) \vee \exists x\, q(x)] \longleftrightarrow \exists x\, [P(x) \vee q(x)]$

II. $\exists x\, [P(x) \wedge q(x)] \longrightarrow [\exists x\, P(x) \wedge \exists x\, q(x)]$

III. $[\exists x\, P(x) \wedge \exists x\, q(x)] \not\longrightarrow \exists x\, [P(x) \wedge q(x)]$

(NOT implied)

Different ways of considering Domain

$Domain_1 = \{Modi\}$

$Domain_2 = \{virat\}$

$Domain_3 = \{Gambhir\}$

$Domain_4 = \{Modi, virat, Gambhir\}$

$Domain_5 = \{Modi, virat\}$

Q. Suppose the predicate F(x, y, t) is used to represent the statement that person x can fool person y at time t. Which one of the statements below expresses best the meaning of the formula $\forall x \exists y \exists t (\sim F(x, y, t))$?

$\sim$

GATE – 10

(a) Everyone can fool some person at some time

(b) No one can fool everyone all the time

$\sim \exists x \; \forall y \; \forall t \; F(x, y, t)$

(c) Everyone cannot fool some person all the time

$\sim$ [Some one everyone all time]

No one.

(d) No one can fool some person at some time

$F(x,y,t)$: Person 'x' can fool person 'y' at time t

$\sim F(x,y,t)$: Person 'x' cannot fool person 'y' at time t

$\underline{\underline{\forall x}}\ \underline{\underline{\exists y}}\ \exists t \ \sim F(x,y,t)$: Every person cannot fool Some person at Some time

= No one can fool Some person at Some time.

Q. Let p, q, r and s be four primitive statements. Consider the following arguments:

GATE – 2004

P : $[(\sim p \vee q) \wedge (r \rightarrow s) \wedge (p \vee r)] \rightarrow (\sim s \rightarrow q)$

Q : $[(\sim p \wedge q) \wedge [q \rightarrow (p \rightarrow r)]] \rightarrow \sim r$

R : $[[(q \wedge r) \rightarrow p] \wedge (\sim q \vee p)] \rightarrow r$

S : $[p \wedge (p \rightarrow r) \wedge (q \vee \sim r)] \rightarrow q$

Which of the above arguments are valid?

(a) P and Q only (b) P and R only

(C) P and S only (d) P, Q, R and S

$P: \{(\sim p \vee q) \wedge (r \to s) \wedge (p \vee r)\} \longrightarrow (\sim s \to q)$

Sol

$$\begin{array}{l} \sim p \vee q \\ r \to s \\ p \vee r \\ \hline \therefore \sim s \to q \\ \hline \end{array} \quad \equiv \quad \begin{array}{l} p \to q \\ r \to s \\ p \vee r \\ \hline \therefore q \vee s \\ \hline \end{array}$$

(constructive dilemma)

$q \vee s \equiv s \vee q \equiv \sim s \to q$

Q:

$$\begin{array}{l} \sim p \wedge q \\ q \to (p \to r) \checkmark \\ \hline \therefore \sim r \\ \hline \end{array}$$

$$\begin{array}{l} \sim p \wedge q \\ \hline \therefore \sim p \\ \hline \end{array} \qquad \begin{array}{l} \sim p \wedge q \\ \hline \therefore q \checkmark \\ \hline \end{array}$$

$$\begin{array}{l} q \to (p \to r) \\ q \\ \hline \therefore p \to r \\ \sim p \\ \hline \therefore \end{array}$$

$R:$

$$\frac{\begin{array}{c}(q \wedge r) \to p \\ \sim q \vee p\end{array}}{\therefore r}$$

$\equiv$

$$\frac{\begin{array}{c}\overset{T\ \ T\ \ T/F}{(q \wedge r)} \to p^{T} \\ q \to p\end{array}}{\therefore \cancel{r}}$$

$\equiv \sim q \vee p = T$ $\quad \sim a \vee b \equiv a \to b$

F	T
T	F
T	T

$S:$

$$\frac{p \wedge (p \to r)}{\begin{array}{c} q \vee \sim r \\ \hline q \end{array}}$$

$$\frac{a \wedge b}{\therefore a} \qquad \frac{a \wedge b}{b}$$

$$\frac{p \wedge (p \to r)}{\therefore p} \qquad \frac{p \wedge (p \to r)}{\therefore p \to r}$$

$$\frac{\begin{array}{c} p \\ p \to r \end{array}}{\therefore r}$$ (Modusponens)

$$\frac{\begin{array}{c} q \vee \sim r \\ r \end{array}}{\therefore q}$$ (Disjunctive)

Q. Identify the correct transition into logical notation of the following assertion.

Some boys in the class are taller than all the girls **GATE – 04**

Note: taller(x, y) is true if x is taller than y.

True True

(a) $(\exists\, x)\,(\text{boy}\,(x) \rightarrow (\forall y)\,(\text{girl}(y) \wedge \text{taller}(x, y)))$

(b) $(\exists\, x)\,(\text{boy}(x) \wedge (\forall y)\,(\text{girl}(y) \wedge \text{taller}(x, y)))$

T T T

(c) $(\exists\, x)\,(\text{boy}(x) \rightarrow (\forall y)(\text{girl}(y) \rightarrow \text{taller}(x, y)))$

(d) $(\exists\, x)(\text{boy}(x) \wedge (\forall y)(\text{girl}(y) \rightarrow \text{taller}(x, y)))$ ✓

True

* Some ($\exists$) followed by and ($\wedge$)
* All ($\forall$) followed by implies ($\longrightarrow$)

100 students

40 Boys 60 Girls

4 Boys taller than all 60 Girls.

Some Boys in the class taller than all girls.

$boy(x) \wedge$