Eg: Which of the following is NOT a Tautology

a) $\sim(p \to q) \to p$

b) $\sim(p \to q) \to \sim q$

c) $[\sim p \wedge (p \vee q)] \to q$

d) $(p \to q) \wedge q \to p$

I. Truth table

II. Logical Approach

III. Properties (or) laws

a) $\overbrace{\sim(p\to q)}^{\text{True}} \to \overbrace{p}^{\text{False}} = \text{False}$

$\sim(\text{False}\to q) \to \text{False}$

$\sim(T) \to F$

$F \to F = T =$ NO False

Tautology

b) $\overbrace{\sim(p\to q)}^{\text{True}} \to \overbrace{\sim q}^{\text{False}}$

$\sim(p\to T)\to F$

$\sim(T)\to F$

$F\to F$

T Tautology

c) $\overbrace{[\sim p\wedge(p\vee q)]}^{T}\to \overbrace{q}^{F}$

i) $[F\wedge(T\vee F)]\to F = \text{True}$

ii) $[T\wedge(F\vee F)]\to F = \text{True}$

Tautology

d) $\overbrace{[(p\to q)\wedge q]}^{\text{True}} \to \overbrace{p}^{\text{False}}$

$[(F\to T)\wedge T]\to F$

$[T\wedge T]\to F$

$T\to F = \text{False}$

NOT a Tautology

example

prove : $X \longrightarrow Y$ is NOT a Tautology

Sol If $X =$ True and $Y =$ False then $X \rightarrow Y =$ False

= NOT a Tautology

Approach-I : consider X is TRUE and then Try to show Y is False

Approach-II : consider Y is False and then Try to show X is TRUE

Example

Prove : $X \longrightarrow Y$ is a Tautology (always True)

Sol Idea : No case Exists like when $X = TRUE$ and $Y = False$

Approach - I : Consider $X = TRUE$ and try to show $Y = TRUE$

Approach - II : Consider $Y = False$ and try to show $X = False$

Eg: Which of the following is NOT a tautology

a) $(p \wedge q) \rightarrow (p \vee q)$

c) $\sim p \rightarrow (p \rightarrow q)$

b) $(p \wedge q) \rightarrow (p \rightarrow q)$

d) $p \rightarrow (p \wedge q)$

a) $$\overbrace{(p\wedge q)}^{T} \longrightarrow \overbrace{(p\vee q)}^{F}$$
$$(T\wedge T) \longrightarrow (T\vee T)$$
$$T \longrightarrow T \quad = \text{True}$$
Tautology

b) $$\overbrace{(p\wedge q)}^{T} \longrightarrow \overbrace{(p\to q)}^{F}$$
$$T \to F$$
$$(T\wedge F) \longrightarrow (T\to F)$$
$$F \longrightarrow F \quad = \text{True}$$
Tautology

c) $$\underbrace{\overbrace{\sim p}^{T}}_{T} \to \overbrace{(p\to q)}^{F}$$
$$T \to (F\to q)$$
$$T \to T \quad = \text{True}$$
Tautology

d) $$\underbrace{\overbrace{p}^{T}}_{T} \to \overbrace{(p\wedge q)}^{F}$$
$$T \longrightarrow (T\wedge F)$$
$$T \longrightarrow F \quad = \text{False} \quad = \text{NOT Tautdogy}$$

Conditional Statements:

p → q

hypothesis → consequence

The statement $p \rightarrow q$ is called a conditional statement, because $p \rightarrow q$ asserts that 'q' is True on the condition that 'p' holds.

Politicians win. promised → Laptop

False → True.

The conditional statement $p \rightarrow q$ is the proposition "If p, then q". The conditional statement $p \rightarrow q$ is false when 'p' is true and 'q' is false, otherwise True.

GATE Rank → gift Car

True   True

False → T/F

Variety of Terminology to express p → q:

$\boxed{p \longrightarrow q}$

"if p then q"	"p implies q"
"if p, q"	$\boxed{\text{"p only if q"}}$
"p is sufficient for q"	"a sufficient condition for q is p"
$X \rightarrow Y$ "q if p" Y if X	"q when ever p"
"q when p"	"q is necessary for p"
"a necessary condition for p is q"	"q follows from p"
"q unless ~ p"	"q provided that p"

If ~p is NOT TRUE
P = True $\longrightarrow$ q

'a' only if 'b'

$\equiv a \longrightarrow b$

$X \longleftrightarrow Y$

Converse, Inverse and contra positive:

Let Implication : $p \longrightarrow q$: $a \longrightarrow b$: $x \longrightarrow y$

Converse : $q \longrightarrow p$: $b \longrightarrow a$: $y \longrightarrow x$

Inverse : $\sim p \longrightarrow \sim q$: $\sim a \longrightarrow \sim b$: $\sim x \longrightarrow \sim y$

Contra positive : $\sim q \longrightarrow \sim p$: $\sim b \longrightarrow \sim a$: $\sim y \longrightarrow \sim x$

$$\boxed{p \rightarrow q \cong \sim q \longrightarrow \sim p}$$

Contra positive law

conditional :

Eg: In Triangle ABC, If $\underbrace{AB = AC}_{p}$ then $\underbrace{\angle B = \angle C}_{q}$

converse : $q \rightarrow p$: In Triangle ABC, If $\angle B = \angle C$ then $AB = AC$

Inverse : $\sim p \rightarrow \sim q$: In Triangle ABC, If $AB \neq AC$ then $\angle B \neq \angle C$.

contrapositive : $\sim q \rightarrow \sim p$: In Triangle ABC, If $\angle B \neq \angle C$ then $AB \neq AC$

Eg: "The $\underbrace{\text{home Team wins}}_{q}$ whenever $\underbrace{\text{it is raining}}_{p}$" [q whenever $p \equiv p \rightarrow q$]

*Conditional: $p \rightarrow q \equiv$ If It is raining then the home team wins

Converse: $q \rightarrow p \equiv$ If the home team wins then It is raining

Inverse: $\sim p \rightarrow \sim q \equiv$ If It is not raining then the home team does not win

Contra positive: $\sim q \rightarrow \sim p \equiv$ If the home team doesnot win then It is not raining

GATE:

What is the converse of the following assertion?

"I stay only if you go"

a) I stay if you go ✓

b) If I stay then you go ✗

c) If you do not go then I do not stay ✗

d) If I do not stay then you go ✗

(p only if q $\equiv$ $p \to q$)

I stay only if you go ✓

$\equiv$ If I stay then you go

Converse : $q \to p \equiv$ If $\underbrace{\text{you go}}_{X}$ then $\underbrace{\text{I stay}}_{Y}$ ✓

$\equiv$ Y if X

$\equiv$ I stay if you go ✓

Laws (or) Properties (or) Logical Equivalences:

	Rules	Name
1	$p \vee p \cong p$ $\vee$	Idempotent law
	$p \wedge p \cong p$ $\wedge$	
2	$p \vee q \cong q \vee p$ $2+3 = 3+2$	Commutative law
	$p \wedge q \equiv q \wedge p$	
3	$p \vee (q \vee r) \equiv [(p \vee q)] \vee r$	Associative law
	$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$	
4	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	Distributive Law
	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	
5	$\sim (p \vee q) \equiv \sim p \wedge \sim q$	De Morgan's Law
	[illegible]	

p	p	p∨p
T	T	T
F	F	F

$p \vee p \cong p$

1	$p \vee p \cong p$	Idempotent law
	$p \wedge p \cong p$	
2	$p \vee q \cong q \vee p$	Commutative law
	$p \wedge q \equiv q \wedge p$	
3	$p \vee (q \vee r) \equiv (p \vee q) \vee r$	Associative law
	$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$	
4	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$ $\vee \leftrightarrow \wedge$ Distributive Law $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ Dual
	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	
5	$\sim (p \vee q) \equiv \sim p \wedge \sim q$	De Morgan's Law
	$\sim (p \wedge q) \equiv \sim p \vee \sim q$	
6 *	$p \rightarrow q \equiv \sim p \vee q$ $a \rightarrow b \equiv \sim a \vee b$	Implication law

Laws (or) Properties (or) Logical Equivalences:

	Rules	Name
7	$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$	Bi-implication law
8	$p \vee (p \wedge q) \equiv p$	Absorption law
	$p \wedge (p \vee q) \equiv p$	
9	$p \wedge t \equiv p$	Identify law
	$p \vee f \equiv p$	
10	$\sim(\sim p) \equiv p$	Double Negation (or) Involutary law
11	$p \vee \sim p \equiv t$	Negation law (or) Complements (or) Inverse law
	$p \wedge \sim p \equiv f$	

Dual

$1+0=1$
$2+0=2$
$3+0=3$

$p \leftrightarrow q \equiv (\sim p \wedge \sim q) \vee (p \wedge q)$

$p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$

p	q	$p \wedge q$	$p \vee p(\wedge q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

$p \vee (p \wedge q)$

$p \equiv p$

(9)

p	t	$p\wedge t$	f	$p\vee f$	$p\vee t$	$p\wedge f$
T	T	T	F	T	T	F
F	T	F	F	F	T	F

Identity $\left\{ \begin{array}{l} p\wedge t \cong p \\ p\vee f \cong p \end{array} \right.$

$\left. \begin{array}{l} p\vee t \equiv t \\ p\wedge f = f \end{array} \right\}$ Dominant

$\left. \begin{array}{l} p \vee (p\wedge q) \cong p \\ p \wedge (p\vee q) \cong p \end{array} \right\}$ Law of absorption

p	$\sim p$	$\sim(\sim p)$
T	F	T
F	T	F

$\therefore \sim(\sim p) \equiv p$

$\cong$

7	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$	Bi-implication law
8	$p \vee (p \wedge q) \equiv p$	Absorption law
	$p \wedge (p \vee q) \equiv p$	
9	$p \wedge t \equiv p$	Identify law ✓
	$p \vee f \equiv p$	
10 ✓	$\sim(\sim p) \equiv p$	Double Negation (or) Involutary law
11	$p \vee \sim p \equiv t$	Negation law (or) Complements (or) Inverse law $a \vee \sim a = t$ $a \wedge \sim a = f$
	$p \wedge \sim p \equiv f$	
12	$p \vee t \equiv t$ ✓	Domination law (13) contrapositive. $p \rightarrow q \cong \sim q \rightarrow \sim p$
	$p \wedge f \equiv f$	

Eg:

Q. The propositional function $p \vee (q \vee \sim p)$ is

a) tautology

b) contradiction

c) contingency

d) $p \wedge q$

Method I

Given $p \vee (q \vee \sim p)$

$\equiv p \vee (\sim p \vee q)$ ($\because$ Commutative)

$\equiv (p \vee \sim p) \vee q$ ($\because$ Associative)

$\equiv (t) \vee q$ ($\because$ complement's law)

$\equiv t$ ($\because$ Dominant)

Method II : Truth table

p	q	$\sim p$	$q \vee \sim p$	$p \vee (q \vee \sim p)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

$\therefore p \vee (q \vee \sim p) \equiv t$

Method III :

Logical approach

$p \vee (q \vee \sim p)$

i) $\mathbf{T} \vee (\quad\quad) = \text{True}$

ii) $\mathbf{F} \vee (q \vee \mathbf{T}) = \text{True}$

Q. The propositional function $(p \rightarrow q) \leftrightarrow (p \wedge \sim q)$ is

a) tautology

b) contradiction

c) contingency

d) None

$$(p \rightarrow q) \longleftrightarrow (p \wedge \sim q)$$

$$\equiv (\sim p \vee q) \longleftrightarrow (p \wedge \sim q) \qquad (\because a \rightarrow b \equiv \sim a \vee b)$$

$$\equiv \boxed{(\sim p \vee q)} \longleftrightarrow \sim(\sim p \vee q)$$

$$\equiv X \longleftrightarrow \sim X = \text{false}$$

$$= \text{contradiction}$$