MTH 112-2079✡

Tribhuvan University
Institute of Science and Technology
2079

✡

Bachelor Level / First Year/ First Semester/ Science
Computer Science and Information Technology (MTH 112)
(Mathematics I)
(NEW COURSE)

Full Marks: 80
Pass Marks: 32
Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.
The figures in the margin indicate full marks.

Group A $(10 \times 3 = 30)$

Attempt any THREE questions.

1. (a) If a function is defined by [1+1+1+2]

$$f(x) = \begin{cases} 1+x, \text{ if } x \leq -1 \\ x^2, \text{ if } x > -1, \end{cases}$$

evaluate $f(-3), f(-1)$ and $f(0)$ and sketch the graph.

(b) Prove that $\lim_{x \to 0} \frac{|x|}{x}$ does not exist. [5]

2. (a) Sketch the curve $y = x^2 + 1$ with the guidelines of sketching [5]

(b) If $z = xy^2 + y^3, x = \sin t, y = \cos t$, find $\frac{dz}{dt}$ at $t = 0$. [5]

3. (a) Estimate the area between the curve $y = x^2$ and the lines $x = 0$ and $x = 1$, using rectangle method, with four sub intervals. [5]

(b) A particle moves along a line so that its velocity v at time t is [5]

$$v = t^2 - 2t + 10$$

(i) Find the displacement of the particle during the time period $1 \leq t \leq 4$.
(ii) Find the distance traveled during this time period.

4. (a) Define initial value problem. Solve: [5]

$$y'' + y' - 6y = 0, y(0) = 1, y'(0) = 0$$

(b) Find the Taylor's series expansion for $\cos x$ at $x = 0$. [5]

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Group B $(10 \times 5 = 50)$

Attempt any TEN questions.

5. (a) Dry air is moving upward. If the ground temperature is 20^o and the temperature at a height of $2km$ is 10^oC, express the temperature T in ^oC as a function of the height h (in kilometers), assuming that a linear model is appropriate. (b) Draw the graph of the function and find the slope. Hence, give the meaning of the slope. (c) What is the temperature at a height of $2km$? [5]

6. Find the equation of tangent at $(1, 3)$ to the curve $y = 2x^2 + 1$.

7. State Rolle's theorem and verify the theorem for $f(x) = x^2 - 9, x \in [-3, 3]$.

8. Starting with $x_1 = 1$, find the third approximation x_3 to the root of the equation $x^3 - x - 5 = 0$.

9. Show that the integral

$$\int_0^3 \frac{dx}{x-1},$$

diverges.

10. Use Trapezoidal rule to approximate the integral $\int_1^2 \frac{dx}{x}$, with $n = 5$.

11. Find the derivative of $\mathbf{r}(t) = t^2\mathbf{i} - te^{-t}\mathbf{j} + \sin 2t\mathbf{k}$ and find the unit tangent vector at $t = 0$.

12. What is a sequence? Is the sequence

$$a_n = \frac{n}{5+n}$$

convergent?

13. Find the angle between the vectors $\mathbf{a} = (2, 2, -1)$ and $\mathbf{b} = (1, 3, 2)$.

14. Find the partial derivative f_{xx} and f_{yy} of $f(x, y) = x^2 + x^3y^2 - y^2 + xy$, at $(1, 2)$.

15. Evaluate

(a) $\int_0^3 \int_1^2 x^2y\, dxdy$

(b) $\int_0^\pi \int_1^2 y \sin(xy) dxdy$

2079 Question Solution

BSc.CSIT 1st Sem — Pravin Gupta

Subject:- Mathematics-I

Group 'A'

1(a) If a function is defined by

$$f(x) = \begin{cases} 1+x, & \text{if } x \leq -1 \\ x^2, & \text{if } x > -1 \end{cases}$$

evaluate $f(-3)$, $f(-1)$ and $f(0)$ and sketch the graph

Sol Given,

$$f(x) = \begin{cases} 1+x, & \text{if } x \leq -1 \\ x^2, & \text{if } x > -1 \end{cases}$$

Then,

$f(-3) = 1+x$, being $x \leq -1$ i.e $-3 \leq -1$

$= 1+(-3)$

$= -2$

$f(-1) = 1+x$, being $x \leq -1$ i.e $-1 \leq -1$

$= 1+(-1)$

$= 0$

$f(0) = x^2$, being $x > -1$ i.e $0 > -1$

$= 0$

For graph, $x \leq -1$

let $y = 1+x$ ---(1)

at $x = -3$, $y = 1+(-3) = -2$

at $x = -1$, $y = 1+(-1) = 0$

$x = -2$, $y = 1+(-2) = -1$

$\therefore$ eq^n (i) passes through the point $(-3,-2)$ $(-2,-1)$ and $(-1,0)$

Again,

$$f(x) = x^2 \quad \text{--- (ii)} \qquad x > -1$$

Suppose $x = 1, 2, \ldots$

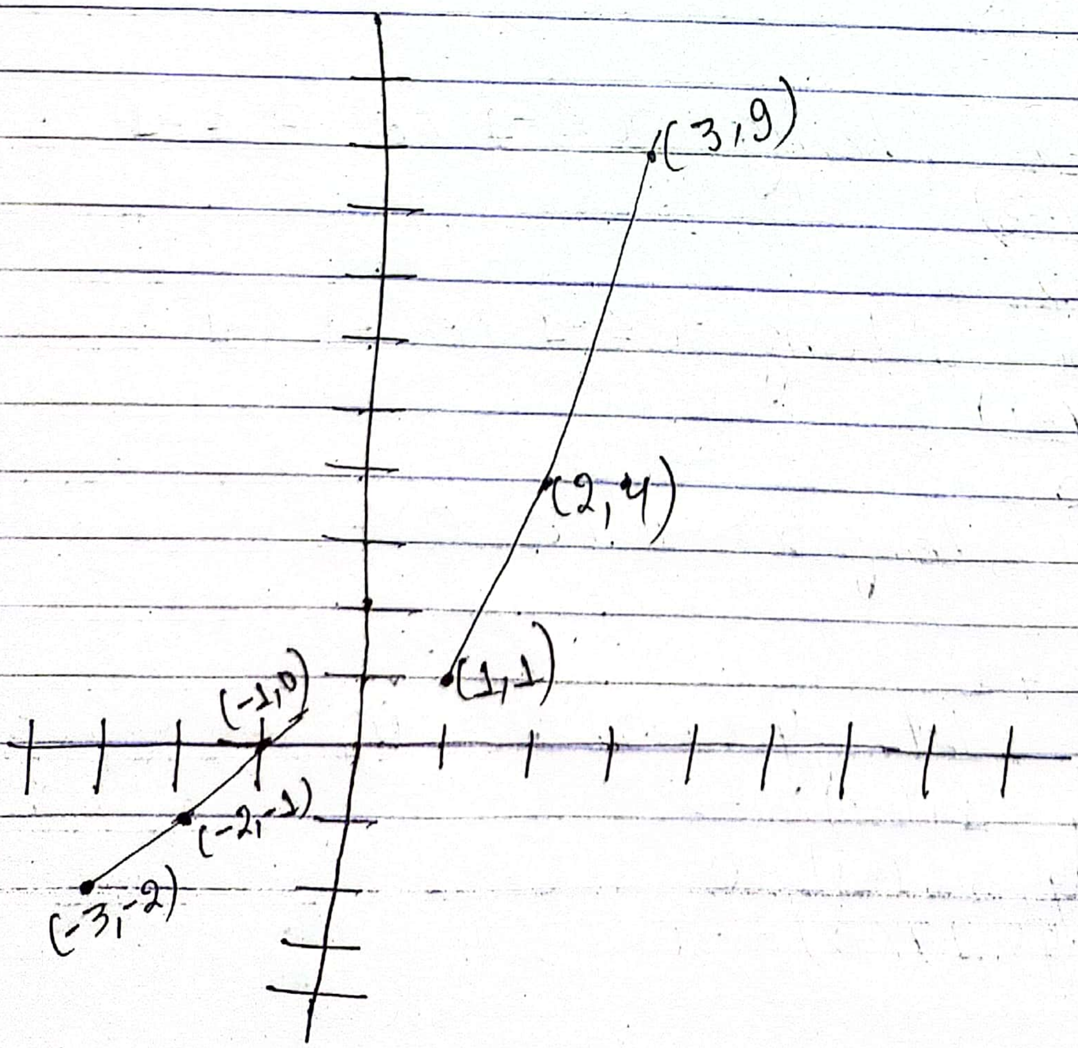
let $y = x^2$

if $x = 1$, $y = 1$

$x = 2$, $y = 4$

$x = 3$, $y = 9$

$\therefore$ eq^n (ii) passes through the point $(1,1)$, $(2,4)$, $(3,9)$



(b) Prove that $\lim\limits_{x \to 0} \frac{|x|}{x}$ doesn't exist

Sol Here,

Given function

$$\lim_{x \to 0} \frac{|x|}{x}$$

we know that

$$|x| = \begin{cases} x & ; \text{ if } x \geq 0 \\ -x & , \text{ if } x < 0 \end{cases}$$

then,

$$LHL = \lim_{x \to 0^-} \frac{|x|}{x} = \lim_{x \to 0} = \frac{-x}{x} = -1$$

$$RHL = \lim_{x \to 0^+} \frac{|x|}{x} = \lim_{x \to 0} = \frac{x}{x} = 1$$

$$\therefore LHL \neq RHL$$

Hence, the limit doesn't exist

2(a) Sketch the curve $y = x^2+1$ with the guidelines of sketching.

Sol Given,

$$y = x^2+1$$

A. Domain

$$y = x^2+1$$

The domain of given function is; all possible value of x $D = (-\infty, \infty)$

B. Intercept

For X-intercept

put $y=0$, $y = x^2+1$

$$0 = x^2+1$$

$$x^2 = -1$$

no real number

For Y-intercept

put $x=0$,

$$y = x^2+1$$

$$y = 0+1$$

$$y = 1$$

It passes through $(x,y) = (0,1)$

C. Symmetry

let

$$f(x) = x^2 + 1$$

put $x = -x$,

$$f(-x) = (-x)^2 + 1$$
$$= x^2 + 1$$
$$= f(x)$$

$\therefore$ $f(x) = f(-x)$

Symmetric about y-axis

D. Asymptote :-

Horizontal asymptotes

$$\lim_{x \to \infty} f(x) = x^2 + 1 = \infty + 1 = \infty$$

There is no horizontal asymptotes.

Vertical asymptotes

put $x = a$ in vertical asymptote of $y = f(x)$ if either $\lim_{x \to a^+} f(x) = \pm\infty$ or $\lim_{x \to a^-} f(x) = \pm\infty$.

$$y = x^2 + 1$$

if we put any value of x we cannot get ∞ so, there is no vertical asymptote.

Hence the given $f(x) = x^2 + 1$ has no asymptote.

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E. Interval of increasing and decreasing

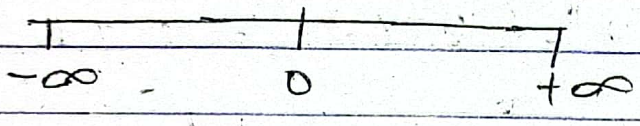
$$f(x) = x^2 + 1 \quad \cdots (1)$$

diff^n eq^n (1) w.r. to 'x'

$$f'(x) = 2x$$

For critical point

$$f'(x) = 0$$
$$2x = 0$$
$$\therefore x = 0$$



Interval	$(-\infty, 0)$	$(0, \infty)$
Sign of $f'(x)$	$-ve$	$+ve$
Nature of $f(x)$	Decreasing	Increasing

This shows the minimum occur at $x = 0$

$$f(x) = x^2 + 1$$
$$y = f(0) = 0 + 1 = 1$$

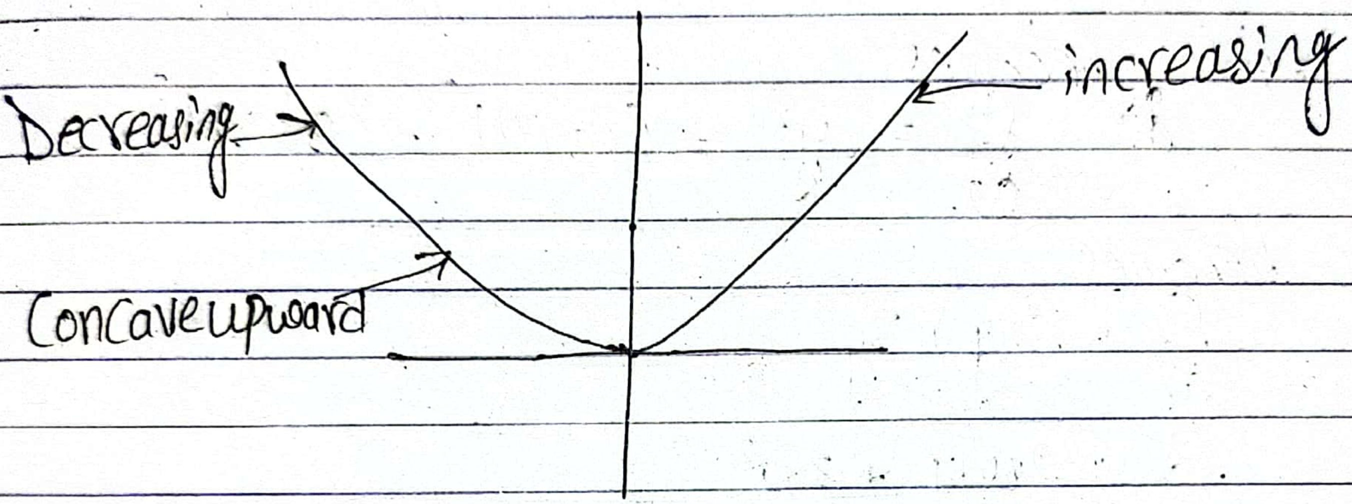
$$\therefore (x, y) = (0, 1)$$

Again,

$$f''(x) = 2$$

the 2^{nd} derivative is constant

So, the function is concave upward



⑤ If $z = xy^2 + y^3$, $x = \sin t$, $y = \cos t$, find $\frac{dz}{dt}$ at $t=0$

Sol: Given,

$$z = xy^2 + y^3 \text{ --- (*)}, \quad x = \sin t, \quad y = \cos t$$

The partial derivative of z with respect to x is

$$\frac{\delta z}{\delta x} = y^2 \text{ --- (i)}$$

Again, the partial derivative of z with respect to y is

$$\frac{\delta z}{\delta y} = x \cdot 2y + 3y^2 = 2xy + 3y^2 \text{ --- (ii)}$$

Now,

$x = \sin t$ $\qquad$ $\frac{dx}{dt} = \cos t$

$t = 0$, $x = \sin(0) = 0$ $\qquad$ at $t=0$, $\frac{dx}{dt} = \cos(0) = 1$

$y = \cos t$ $\qquad$ $\frac{dy}{dt} = -\sin t$

$t = 0$, $y = \cos(0) = 1$ $\qquad$ at $t=0$, $\frac{dy}{dt} = -\sin(0) = 0$

Then eqn (i) becomes

$$\frac{\delta z}{\delta x} = y^2 = (\cos(t))^2 = ((\cos(0))^2 = 1^2$$
$$= 1$$

eqn (ii) becomes

$$\frac{\delta z}{\delta y} = 2xy + 3y^2$$

$$= 2 \times 0 \times 1 + 3 \times 1^2$$

$$= 0 + 3$$

$$\therefore \frac{\delta z}{\delta y} = 3$$

Finally eqn (1) becomes

$$z = xy^2 + y^3$$

using chainrules

$$\frac{dz}{dt} = \frac{\delta z}{\delta x} * \frac{dx}{dt} + \frac{\delta z}{\delta y} * \frac{dy}{dt}$$

$$= 1 * 1 + 3 * 0$$

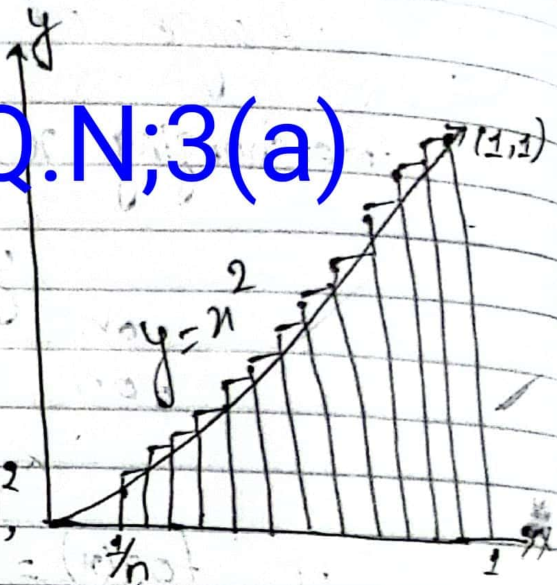
$$= 1$$

$$\therefore \frac{dz}{dt} = 1 \text{ at } t = 0$$

Sol.

Let R_n be the Sum of the area of the n rectangles as shown in figure. Each rectangle has width $1/n$ and the heights are the values of the function $f(x) = x^2$ at the Points $\frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \dots \frac{n}{n}$

Q.N;3(a)



i.e the heights are $\left(\frac{1}{n}\right)^2, \left(\frac{2}{n}\right)^2, \left(\frac{3}{n}\right)^2, \dots \left(\frac{n}{n}\right)^2$

Thus,

$$R_n = \frac{1}{n}\left(\frac{1}{n}\right)^2 + \frac{1}{n}\left(\frac{2}{n}\right)^2 + \dots + \frac{1}{n}\left(\frac{n}{n}\right)^2$$

$$= \frac{1}{n^3}\left(1 + 2^2 + 3^2 + \dots + n^2\right)$$

$$= \frac{1}{n^3} \times \frac{n(n+1)(2n+1)}{6}$$

$$R_n = \frac{(n+1)(2n+1)}{6n^2}.$$

Thus, we have,

$$\text{Area}(A) = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \frac{(n+1)(2n+1)}{6n^2}$$

$$= \lim_{n\to\infty} \frac{1}{6}\left(\frac{n+1}{n}\right)\left(\frac{2n+1}{n}\right) \quad (\because 1/\infty = 0)$$

$$= \lim_{n\to\infty} \frac{1}{6}\left(1 + 1/n\right)\left(2 + 1/n\right) = \frac{1}{6} \times 1 \times 2 = 1/3$$

3(b) A particle moves along a line so that its velocity v at time t is

$$V = t^2 - 2t + 10$$

(i) find the displacement of the particle during the time period $1 \le t \le 4$.

(ii) find the distance travelled during this time period.

Sol. Given,

$$V = t^2 - 2t + 10$$

(i) Now, the displacement of particle during time period $1 \le t \le 4$ is

$$S = \int_1^4 V dt = \int_1^4 (t^2 - 2t + 10) = \left[\frac{t^3}{3} - \frac{\cancel{2}t^2}{\cancel{2}} + 10t\right]_1^4$$

$$= \left(\frac{64}{3} - 16 + 40\right) - \left(\frac{1}{3} - 1 + 10\right)$$

$$= \frac{64 - 72}{3} - \left(\frac{1 - 27}{3}\right)$$

$$= \frac{-8}{3} + \frac{26}{3} = \frac{18}{3} = 6$$

(ii) Given that the particle moves along a line for the time period $1 \le t \le 4$. That is the total traveled time duration is $(4-1) = 3$ second. Now the distance travelled during this period is

$$h = \frac{S}{t} = 6 \times \frac{1}{3} = 2 \text{ Ans}$$

Q.N;4(a)

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Sol: 1st part

A differential equation together with initial condition(s) is called the initial value problem. For example:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0 \, , \, y(0) = -1 \, , \, y'(0) = 1$$

Here, $y(0) = -1$ and $y'(0) = 1$ is an initial condition

Sol The given 2^{nd} order homogenous linear eq^n is

$$y'' + y' - 6y = 0 \quad \text{---(i)}$$

Let

$y = e^{mx}$ be the req^n sol^n of eq^n(i)

Now,

we get an auxilarly eq^n of (i) by replacing y'' by m^2, y' by m and y by 1.

2nd Part

$$m^2 + m - 6 = 0$$

or, $m^2 + 3m - 2m - 6 = 0$

or, $m(m+3) - 2(m+3) = 0$

$(m-2)(m+3) = 0$

Either or

$m = 2$, $m = 3$

$\therefore m_1 = 2$, $m_2 = -3$ which is real and unequal

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The req^n sol^n is;

$$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

$$y = c_1 e^{2x} + c_2 e^{-3x} \quad \text{----(ii)}$$

Since, $y(0) = 1$, i.e $x = 0, y = 1$

from eq^n(ii)

$$1 = c_1 e^0 + c_2 e^0$$

$$1 = c_1 + c_2 \quad \text{---(iii)}$$

Again,

diff^n eq^n (ii) with respect to x

$$y' = 2c_1 e^{2x} - 3c_2 e^{-3x} \quad \text{--- (iv)}$$

since,

$y'(0) = 0$ i.e $y' = 0,\ x = 0$

from eq^n (iv)

$$0 = 2c_1 - 3c_2 \quad \text{--- (v)}$$

from eq^n (iii) and (v)

$2 \times$ eq^n (iii) $-$ (v)

$$\begin{array}{r} \cancel{2c_1} + 2c_2 = 2 \\ -\cancel{2c_1} \mp 3c_2 = 0 \\ \hline 5c_2 = 2 \\ c_2 = 2/5 \end{array}$$

if $c_2 = 2/5$ then,

$$c_1 + c_2 = 1$$
$$c_1 = 1 - c_2$$
$$c_1 = 1 - \frac{2}{5} = 3/5$$

Substuting the value of c_1 and c_2 in eq^n (ii)

$$y = c_1 e^{2x} + c_2 e^{-3x}$$
$$y = \frac{3}{5} e^{2x} + \frac{2}{5} e^{-3x} \quad \underline{\text{Ans.}}$$

Sol Let

$f(x) = \cos x$

Q.N;4(b)

the given function is

$f(x) = \cos x$ --- ①

we have to find the taylor series generated by Function $f(x) = \cos x$ about $x = a = 0$ For

diff eqn ① w.r.to 'x' successively we get

$f'(x) = -\sin x$

$f''(x) = -\cos x$

$f'''(x) = \sin x$

$f^{IV}(x) = \cos x$

$f^{V}(x) = -\sin x$

$f^{VI}(x) = -\cos x$

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rearrangement all the above derivative

$f'(x) = -\sin x$ $\quad f'(0) = 0$ $\quad f''(x) = -\cos x$ $\quad f''(0) = -1$

$f'''(x) = \sin x$ $\xRightarrow{x=a=0}$ $f'''(0) = 0$ $\quad f^{IV}(x) = \cos x$ $\xRightarrow{x=a=0}$ $f^{IV}(0) = 1$

$f^{V}(x) = -\sin x$ $\quad f^{V}(0) = 0$ $\quad f^{VI}(x) = -\cos x$ $\quad f^{VI}(0) = -1$

$\vdots$

$f^{2n+1}(x) = (-1)^n \sin x$ $\quad f^n(0) = 0$ $\quad f^{2n}(x) = (-1)^n \cos x$ $\quad f^n(1) = (-1)^n$

we know,

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a)^1 + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 +$$

$$\frac{f^{IV}(a)}{4!}(x-a)^4 + \frac{f^{V}(a)}{5!}(x-a)^5 + \frac{f^{VI}(0)}{6!}(x-a)^6 + \cdots + \frac{f^n(0)}{n!}(x-a)^n + \cdots$$

or, $$\cos x = 1 + 0 - \frac{1}{2!}(x-0)^2 + 0 + \frac{1}{4!}x^4 + 0 + \frac{(-1)}{6!}(x)^6 + \cdots$$

$$\cdots + \frac{(-1)^n}{(2n)!}(x-0)^{2n} + \cdots$$

$$\cos x = 1 - \frac{(x)^2}{2!} + \frac{x^4}{4!} - \frac{(x)^6}{6!} + \cdots + \frac{(-1)^n}{(2n)!}(x)^{2n} + \cdots$$

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Sol Given,

(a) Ground temperature $= 20^{\circ}C$

Temperature at height 2km $= 10^{\circ}C$

Height $= 2km$

Consider the linear model for this, temperature and height are independent

$$T = mh + b \quad \text{---} (1)$$

Since we know, at ground level $h = 0$

$$\text{so, } T = m \times 0 + b$$

$$20 = 0 + b$$

$$\therefore b = 20$$

Therefore the eq^n (1) becomes

$$T = mh + 20 \quad \text{---} (11)$$

Also given: $h = 2km$, $T = 10^{\circ}C$ then eq^n (11) becomes

$$10 = m \times 2 + 20$$

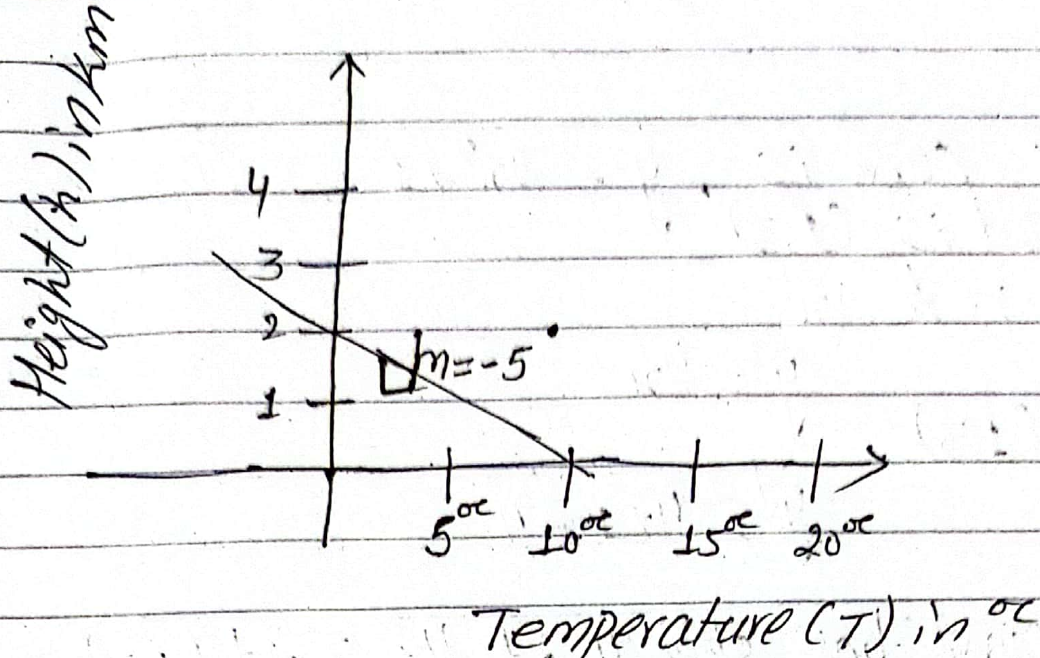
$$-10 = 2m$$

$$\therefore m = -5$$

Therefore, appropriate linear model is

$$T = -5h + 20 \quad \text{---} (111)$$ is req^n model

(b) For the graph of the function.



The slope (m = -5) represent, if height is increased by 2km then the temperature is decreased by -5 °C

(c) Given height = 2km, T = ?
from eq (iii)

$$T = -5h + 20$$
$$T = -5 \times 2 + 20$$
$$T = 10\,^{\circ}C$$

∴ Therefore at 2km temperature become 10 °C

6.

Sol Given,

the curve is, $y = 2x^2+1$ --- (1)

Given point is $(1,3)$

diff^n eq^n (1) w.r. to 'x'

$$y' = 4x$$

At point $(1,3)$,

$$y' = 4\times1 = 4$$

This show the curve has slope 4 at the point $(1,3)$ since the point $(1,3)$ is the common point of the curve and the tangent line $y = 2x^2+1$

Therefore the slope of the tangent line is 4

Thus, the eq^n of tangent at $(1,3)$ to the curve $y = 2x^2+1$ is

$$y - y_1 = m(x - x_1)$$
$$y - 3 = 4(x-1)$$
or, $$y - 3 = 4x - 4$$
$$y = 4x - 4 + 3$$
$$y = 4x - 1 \text{ Ans}$$

$\therefore$ The eq^n of tangent is $y = 4x-1$ Ans

Sol

Q.N;7

1st Part,

Let f be a function that Satisfies the following three conditions;

(i) f is continuous on the closed interval [a,b]

(ii) f is differentiable on the open interval (a,b)

(iii) $f(a) = f(b)$

Then there is a number C in (a,b) such that $f'(c) = 0$

2nd Part

Given,

$$f(x) = x^2 - 9 \quad in\ [-3, 3]$$

(i) Since $f(x)$ is a polynomial function. So, it is continuous on closed interval $[-3,3]$

(ii) $f'(x) = 2x$ which exists for all $x \in (-3,3)$

$\therefore f(x)$ is differentiable in $(-3,3)$

(iii) $f(-3) = (-3)^2 - 9 = 9 - 9 = 0$

$f(3) = (3)^2 - 9 = 9 - 9 = 0$

$\therefore f(-3) = f(3)$

All the Conditions of Rolle's theorem are satisfied then there exists at least a point $c \in (-3,3)$ such that

$$f'(c) = 0$$

$f'(x) = 2x$ i.e $f'(c) = 2c$

so,

$$2c = 0$$

$\therefore c = 0 \in (-3, 3)$

Hence, Rolle's theorem verified an $0 \in (-3,3)$ Ans

8

Sol. Given,

$f(x) = x^3 - x - 5 \quad \cdots \text{①}, \quad x_1 = 1$

$x_3 = ?$

diff^n eq^n ① w.r.to 'x'

$f'(x) = 3x^2 - 1$

By Newton's method, formula,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

when, $x_1 = 1$

$$x_{1+1} = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 1 - \frac{(-5)}{2}$$

$$x_2 = 7/2 = 3.5$$

Again, $x_{2+1} = x_2 - \frac{f(x_2)}{f'(x_2)}$

$$x_3 = 3.5 - \frac{\{(3.5)^3 - (3.5) - 5\}}{\{3 \times (3.5)^2 - 1\}}$$

$$= 3.5 - \frac{34.375}{35.75}$$

$$= 3.5 - 0.961$$

$$x_3 = 2.539$$

$\therefore$ The third approximation x_3 is 2.539 Ans

9 Given, series is

$$\int_0^3 \frac{dx}{x-1}$$

$$f(x) = \frac{1}{x-1}$$

for integral test

$$\int_0^3 f(x)\,dx = \lim_{K \to 3} \int_0^K \frac{1}{x-1}\,dx$$

$$= \lim_{K \to 3} [\ln(x-1)]_0^K$$

$$= \lim_{K \to 3} [\ln(x-1)]_0^K$$

$$= \lim_{K \to 3} [\ln(K-1) - \ln(0-1)]$$

$$= \ln(3-1) - \ln(-1)$$

$$= \ln(2) - \ln(-1)$$

$$= \infty \text{ which is infinite}$$

Thus the given $\int_0^3 \frac{1}{x-1}\,dx$ is diverges

10

Sol Given,

$$\int_1^2 \frac{1}{x}\,dx \quad \text{with } n=5$$

$a=1, \quad b=2, \quad f(x)=\frac{1}{x}$

$$\Delta x = \frac{b-a}{n}$$

$$\Delta x = \frac{2-1}{5} = 0.2$$

Using Trapezoidal formula

$$\int_a^b f(x)\,dx = \frac{\Delta x}{2}\left[f(x_0)+2f(x_1)+2f(x_2)+\cdots\cdots+2f(x_{n-1})+f(x_n)\right]$$

$$= \frac{0.2}{2}\left[f(1)+2f(1.2)+2f(1.4)+2f(1.6)+2f(1.8)+f(2)\right]$$

$$= 0.1\left(\frac{1}{1}+2\times\frac{1}{1.2}+2\times\frac{1}{1.4}+2\times\frac{1}{1.6}+2\times\frac{1}{1.8}+\frac{1}{2}\right)$$

$= 0.695635$ Ans

$\therefore$ The integral $\int_1^2 \frac{dx}{x}$ with $n=5$ is 0.695635 Ans

11.

Sol Given,

$$r(t) = t^2 i - te^{-t} J + \sin 2t k \quad \cdots (1)$$

diffn eqn (1) w.r. to 't'

$$\frac{d(\vec{r}(t))}{d(t)} = \frac{d\{t^2 i\}}{dt} - \frac{d(t \cdot e^{-t})J}{dt} + \frac{d(\sin 2t)k}{dt}$$

$$= 2ti - (t \cdot (-1) \cdot e^{-t} + e^{-t} \cdot 1)J + \left\{\frac{d(\sin(2t))}{d(2t)} \times \frac{d(2t)}{dt}\right\}k$$

$$= 2ti + (te^{-t} - e^{-t})J + \cos 2t \times 2k$$

$$= 2ti + e^{-t}(t-1)J + 2\cos 2t k$$

$$\therefore \frac{d\vec{r}}{dt} = 2ti + (t-1)e^{-t}J + 2\cos 2t k$$

At $t=0$,

$$r(0) = 0i - 0J + 0K = 0$$

$$\frac{d\vec{r}}{dt} = r'(0) = 0 + (0-1) \cdot e^{0} J + 2 \times 1 K = -1J + 2K$$

since by using formula

$$T(t) = \frac{r'(t)}{|r'(t)|}$$

$$T(0) = \frac{-1J + 2K}{\sqrt{(-1)^2 + (2)^2}} = \frac{-1J + 2K}{\sqrt{5}} = \frac{-1}{\sqrt{5}}J + \frac{2}{\sqrt{5}}K \quad \text{Ans}$$

Sol 1st Part, Q.N;12

A Sequence is an ordered list of things. Such thinks may finite or infinite. In a sequence, the terms are separated by commas(,)

2^{nd} Part

Given Sequence is

$$\{a_n\} = \left\{\frac{n}{5+n}\right\}$$

To check convergent,

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left(\frac{n}{5+n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{\cancel{n}}{\cancel{n}(5/n+1)}\right)$$

$$= \lim_{n\to\infty}\left(\frac{1}{5/n+1}\right)$$

$$= \lim_{n\to\infty}\frac{1}{5/\infty+1}$$

$$= \frac{1}{0+1} = 1$$

Hence, the given Series $\{a_n\}$ is 1 i.e convergent.

13. Given,

$$a = (2, 2, -1)$$
$$b = (1, 3, 2)$$

Then,

$$\vec{a} \cdot \vec{b} = (2, 2, -1)\,(1, 3, 2)$$
$$= 2 + 6 + (-1) \times 2$$
$$= 8 - 2$$
$$= 6$$

Also,

$$|\vec{a}| = \sqrt{2^2 + 2^2 + (-1)^2} = \sqrt{4+4+1} = \sqrt{9} = 3$$

$$|\vec{b}| = \sqrt{1^2 + 3^2 + 2^2} = \sqrt{1+9+4} = \sqrt{14}$$

By formula,

$$\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}|\,|\vec{b}|}\right)$$
$$= \cos^{-1}\left(\frac{\cancel{6}\,2}{\cancel{3} \times \sqrt{14}}\right)$$
$$= \cos^{-1}\left(\frac{2}{\sqrt{14}}\right)$$
$$= 57.68^\circ \text{ Ans}$$

The angle between the vector is 57.68° Ans

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Sol Given, $f(x,y) = x^2 + x^3y^2 - y^2 + xy$ ... (I)

diff^n eq^n (I) w.r. to 'x'

$$f_x = \frac{\delta f}{\delta x} = 2x + 3x^2y^2 + y \quad \text{...(II)}$$

Again, diff eq^n (II) w.r. to 'x'

$$f_{xx} = 2 + 6xy^2$$

Now at Point (1,2)

$$f_{xx} = 2 + 6 \times 1 \times 2^2$$
$$= 2 + 24$$
$$= 26$$

Again, diff^n eq^n (I) w.r. to 'y'

$$f_y = 2x^3y - 2y + x \quad \text{...(III)}$$

Again, diff^n eq^n (III) w.r. to 'y'

$$f_{yy} = 2x^3 - 2$$

At Point (1,2)

$$f_{yy} = 2 \times 1^3 - 2 = 0$$

$\therefore f_{xx} = 26$ and $f_{yy} = 0$ Ans

15 Evaluate

(a) $\int_0^3 \int_1^2 x^2 y \, dx \, dy$

sol Given,

$$= \int_0^3 \int_1^2 x^2 y \, dx \, dy$$

$$= \int_0^3 \left[\frac{x^3}{3} y\right]_1^2 dy$$

$$= \int_0^3 \left(\frac{8y}{3} - \frac{1y}{3}\right) dy$$

$$= \int_0^3 \frac{7}{3} y \, dy$$

$$= \frac{7}{3}\left[\frac{y^2}{2}\right]_0^3$$

$$= \frac{7}{\cancel{3}} \times \frac{\cancel{9}3}{2} = \frac{21}{2}$$