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① Design electrical conductivity of semiconductors. Derive expression for conductivity in terms of impurity ionization energy. Give a plot to discuss "Theoretical temp^r dependence of the electrical conductivity of an impurity semiconductor".

⇒ A semiconductor has two types of charge carriers (i.e. the electrons and the holes). The total conductivity of semiconductor is the sum of the conductivity due to electrons and holes.

i.e.,

$$\sigma = \sigma_e + \sigma_h \quad \text{--- (1)}$$

where,

σ_e = conductivity of electrons

σ_h = conductivity of holes.

The current density for electrons and holes is given by

$$J_e = \sigma_e E \quad \text{--- (2)}$$

$$\text{and } J_h = \sigma_h E \quad \text{--- (3)}$$

where, $J_e = \frac{I_e}{A}$ and $J_h = \frac{I_h}{A}$ current

densities and E is electric field.

Again, the current density in terms of drift velocity (v_d) is

$$J_e = n e v_d e$$

$$J_e = n e \mu_e E \quad \text{--- (4)} \quad [\because v_d \propto E, v_d = \mu_e E]$$

where,

μ_e = mobility of electron,

n = no. of free electron per unit volume

Similarly,

$$J_h = peVd_h$$

$$J_h = pe \mu_h E \quad \text{--- (IV)}$$

where, b is a constant and μ_h is hole mobility.

p = no. of holes per unit volume

μ_h = mobility of holes

Now, comparing eqn (I), (II) and (IV) we get,

$$\sigma_e E = ne \mu_e E \Rightarrow \sigma_e = n e \mu_e \quad \text{--- (V)}$$

$$\sigma_h E = pe \mu_h E \Rightarrow \sigma_h = pe \mu_h \quad \text{--- (VI)}$$

Now,

putting these values in (I)

$$\sigma = n e \mu_e + p e \mu_h$$

$$\text{or, } \sigma = e (n \mu_e + p \mu_h)$$

At thermal equilibrium, for intrinsic semiconductor,

$$n = p = n_i$$

i.e. no. of electrons = no. of holes = no. of intrinsic carrier density

and

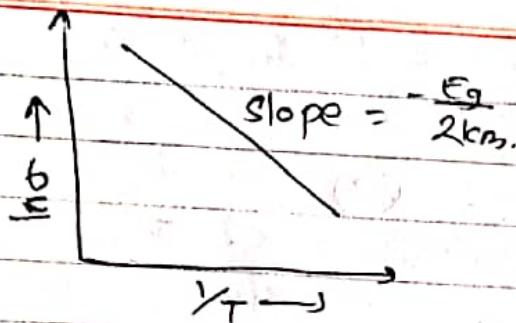
$$n_i = \sqrt{N_c N_v} e^{-E_g/2k_B T}$$

Now,

$$\sigma = e \times n_i (\mu_e + \mu_h)$$

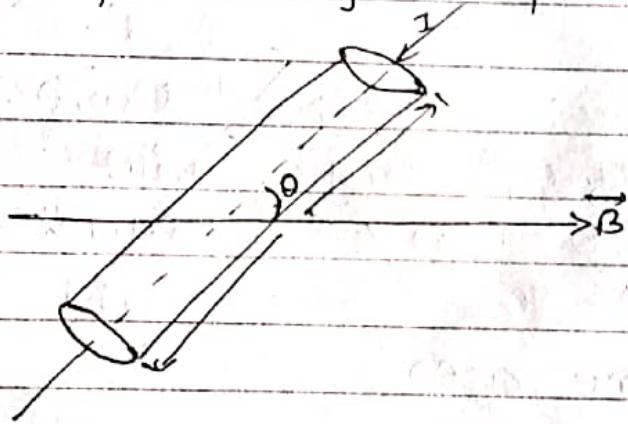
$$= e \cdot \sqrt{N_c N_v} e^{-E_g/2k_B T} (\mu_e + \mu_h)$$

This is the expression for electrical conductivity of semiconductor.



This above conclusion is shown that the electrical conductivity of an intrinsic semiconductor increases as the temperature rises.

- Q) Find expression for force on a current carrying wire in a magnetic field to find the force experienced by a single charge.



If q be the charge carried by drifting electrons then force is

$$F = Bqv_d \sin\theta \quad \text{--- (1)}$$

If I be the current when the electrons drift with the velocity v_d .

$$I = nev_d A$$

Also,

$$v_d = \frac{l}{t}$$

$$t = l/v_d \quad \text{--- (2)}$$

Now, we know,

$$q = It \\ = I \times \frac{1}{V_d} \quad \text{--- } ③$$

Again, eqn ① becomes -

$$F = \frac{Ie}{V_d} \times V_d \times B \sin\theta.$$

$$\boxed{F = BIl \sin\theta} \quad \text{--- } 4$$

In vector form,

$$\vec{F} = I(\vec{I} \times \vec{B})$$

This is the reqd' expression for force on a current carrying wire
some cases

- ① If $\theta = 0^\circ$ $\rightarrow F = 0$ (parallel to B),
- ② If $\theta = 90^\circ \rightarrow F_{max} = BIl$. (\perp to B)

some cases:

- ① When $\theta = 0^\circ$ or 180° , then $F = 0$ i.e., current carrying conductor is placed parallel to the magnetic field, the conductor experiences no force.
- ② When $\theta = 90^\circ$ then $F_{max} = BIl$ i.e., current carrying conductor is placed at right angle to the magnetic field, the conductor experiences maximum force.

④ Give spectrum of Hydrogen atom and discuss the lines.

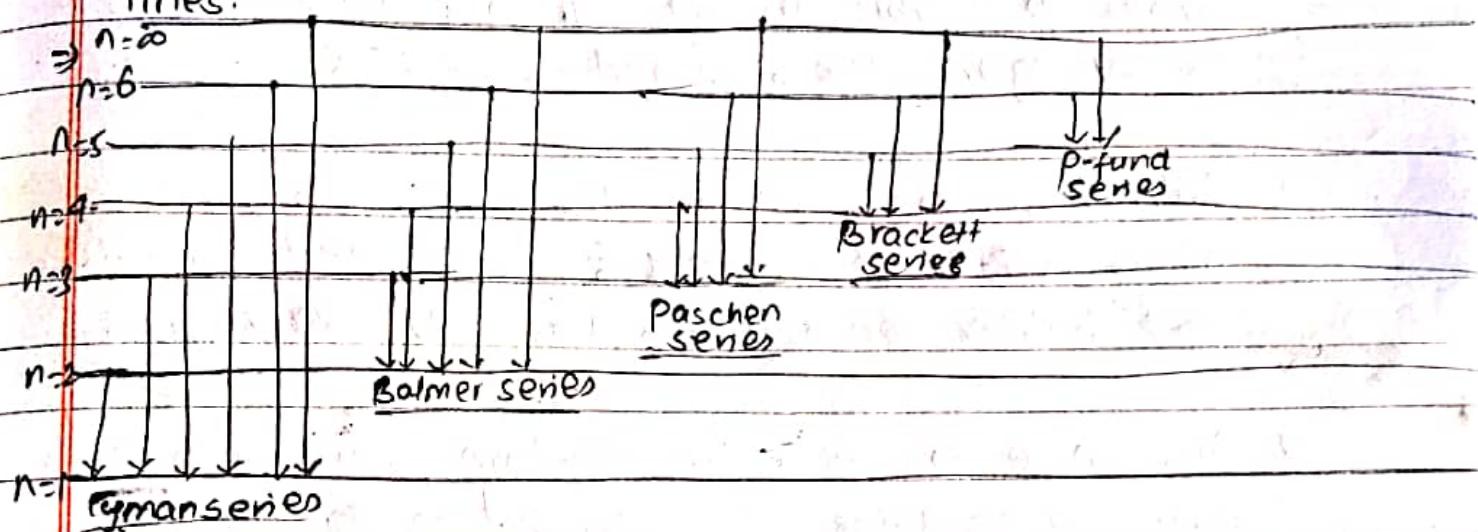


fig:- Spectral series of Hydrogen atom.

(LBP B P)

Horizontal line = energy level.

Vertical with arrow = transition from one band.

$n=1$, (ground state) has least energy.

$n \uparrow, E \uparrow \downarrow$

- ① Lyman series: When an electron jumps from ^{higher} second orbit to 1st orbit, we get Lyman series which lies in UV region. Here, $n_1=1$ and $n_2 = 2, 3, 4, 5, \dots$
- ② Balmer series: When an electron jump from higher orbits i.e $n=3, 4, 5, \dots$ to 2nd orbit, we get Balmer series which lies in visible region.
- ③ Paschen series: When an electron jump from higher orbits i.e $n=4, 5, 6, \dots$ to 3rd orbit we get Paschen series which lies in IR region.
- ④ Brackett series: When an electron jump from higher orbits i.e $n=5, 6, \dots$ to 4th orbit we get Brackett series which lies in IR region.
- ⑤ P-fund series: When an electron jumps from higher orbit i.e $n=6, 7, \dots$ to 5th orbit, we get p-fund series which lies in IR region.

Describe the term "space quantization".

⇒ The discreteness of the possible spatial orientations of the angular momentum of an atom or simply the definite magnitude and direction of one component of angular momentum is known as space quantization. Zeeman effect can be considered as best example of space quantization.

Suppose

$$E'' - E' = h\nu_0$$

when an electron falls from E'' to E' the energy released in absence of magnetic field is

$$E'' - E' = h\nu_0$$

$$\nu_0 = \frac{E'' - E'}{h}$$

Here, ν_0 is the frequency of spectral line emitted in absence of magnetic field.

If magnetic field is applied then emitted frequency of spectral line will be,

$$\nu = \nu_0 + \Delta\nu \quad \textcircled{1}$$

where, $\Delta\nu$ = change in frequency of spectral line due to magnetic field.

$$\therefore \text{Energy} (E) = -\vec{m}_e \cdot \vec{B}$$

As we know Energy (E)

$$\Delta\nu = \frac{eB}{4\pi m_e} \quad \textcircled{2}$$

Using ① and ②

$$\gamma = V_0 + \frac{eB}{4\pi m} \Delta M_e$$

Using selection rule,

$$\Delta M_e = \pm 1, 0$$

$$\therefore V = V_0 + \frac{eB}{4\pi m} \quad \text{for } 1$$

$$V = V_0 \quad \text{for } 0$$

$$V = V_0 - \frac{eB}{4\pi m} \quad \text{for } -1$$

This describes space quantization.

- ④ A given spring stretches 0.1m when a force of 20N pulls on it. A 2kg block attached to it on a frictionless surface is pulled to the right 0.2m and released.

⑤ What is the frequency of oscillation of the block?

⑥ What are the velocity and accn when $x = 0.12\text{m}$, at the block's first passing this point?

sof

Given,

$$\text{stretches (x)} = 0.1\text{m}$$

$$F = 20\text{N}$$

$$M = 2\text{kg}$$

$$\text{distance (r)} = 0.2\text{m}$$

Now,

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

we know,

$$F = kx$$

$$\text{or, } k = \frac{F}{x} = \frac{20}{0.1} = 200$$

then,

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{2}}$$

$$= 1.6 \text{ Hz}$$

Again,

velocity when $x = 0.12 \text{ m}$ passing this point

$$v = \omega \sqrt{x^2 - r^2}$$

$$= 2\pi f \sqrt{(0.2)^2 - (0.12)^2}$$

$$= 2\pi \times 1.6 \sqrt{(0.2)^2 - (0.12)^2}$$

$$= 1.6 \text{ m/s}$$

Also,

$$\text{accel } (a) = \omega^2 x$$

$$= (2\pi \times 1.6)^2 \times 0.12$$

$$= 12.12 \text{ m/s}^2$$

- ⑧ A proton is moving with a velocity $v = (3 \times 10^5 \text{i} + 7 \times 10^5 \text{k}) \text{ m/sec}$ in a region where there is a magnetic field $B = 0.4 \text{j T}$. Find the force experienced by the proton.

Sol:

Given,

$$\text{velocity} = 3 \times 10^5 \text{i} + 7 \times 10^5 \text{k}$$

$$B = 0.4 \text{j T}$$

$$\vec{V} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 \times 10^5 & 0 & 7 \times 10^5 \\ 0 & 0.4 & 0 \end{vmatrix}$$

$$= \hat{i} (2.8 \times 10^5) - \hat{k} 1.2 \times 10^5$$

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we know,

$$\text{mass of proton } (m) = 1.67 \times 10^{-27} \text{ kg.}$$

$$\text{charge " " } (q) = 1.6 \times 10^{-19} \text{ C}$$

$$F = ?$$

Now,

$$\begin{aligned} \vec{F} &= \vec{q} (\vec{V} \times \vec{B}) \\ &= 1.6 \times 10^{-19} [(3 \times 10^5 \hat{i} + 7 \times 10^5 \hat{k}) \times 0.4 \hat{j}] \\ &= 1.6 \times 10^{-19} (-1.2 \times 10^5 \hat{k} + 2.8 \times 10^5 \hat{i}) \\ &= 1.6 \times 10^{-19} (-1.2 \hat{k} + 2.8 \hat{i}) 10^5 \\ &= (-1.92 \hat{k} + 4.48 \hat{i}) \times 10^{14} \text{ N.} \end{aligned}$$

- ⑨ A neutron spectroscopy a beam of mono energetic neutrons is obtained by reflecting reactor neutron from a beryllium crystal. If separation between the atomic planes of the beryllium crystal is 0.732 Å° , what is the angle between the incident neutron beam and the atomic planes that will yield a monochromatic beam of neutrons of wavelength 0.1 Å° .

Hence,

$$\text{separation } (d) = 0.732 \text{ Å}^\circ = 0.732 \times 10^{-10} \text{ m}$$

$$\text{wavelength } (\lambda) = 0.1 \text{ Å}^\circ = 0.1 \times 10^{-10} \text{ m}$$

$$\theta = ?$$

We know, from Bragg's law,

$$2d \sin \theta = n\lambda$$

$$\text{or, } \sin \theta = \frac{n\lambda}{2d} \quad (\text{since } n=1 \text{ for 1st order})$$

$$\theta = \sin^{-1} \left(\frac{a}{2d} \right)$$

$$= \sin^{-1} \left(\frac{0.1 \times 10^{-10}}{2 \times 0.732 \times 10^{-10}} \right)$$

$$\therefore \theta = 3.92^\circ$$

Hence, reqd angle between the incident neutron beam and the atomic plane is 3.92° .

(a) How many atomic states are there in hydrogen with $n = 3$?

(b) How are they distributed among the subshells? Label each state with the appropriate set of quantum numbers n, l, m_l .

(c) Show that the number of states in a shell that is states having the same n is given by $2n^2$

so?

$$\frac{n(n+1)}{2}$$

for a,

Principle Quantum number (n) = 3.

$$\text{No. of atomic states} = \frac{n(n+1)}{2}$$

$$= \frac{3(3+1)}{2} = 6$$

They are

① 1s

② 2s 2p

③ 3s 3p 3d

(5)

State	n	l	m_l	m_s	No. of states in shell
For $1s$ state	1	0	0	$\pm \frac{1}{2}$	1
For $2s$ state	2	0	0	$\pm \frac{1}{2}$	1
For $2p$ state	2	1	$\pm 1, 0$	$\pm \frac{1}{2}$	3
For $3s$ state	2	0	0	$\pm \frac{1}{2}$	1
for $3p$ state	3	1	$\pm 1, 0$	$\pm \frac{1}{2}$	3
for $3d$ state	3	2	$0, \pm 1, \pm 2$	$\pm \frac{1}{2}$	5

For (6).

No. of states in shell having quantum number $1s^2, 2s^2, 2p^6, 3s^2, 3p^6$; i.e. is 18 same as $2 \times 3^2 (t8)$.

$$\frac{2 \times r^2}{2 \times 3^2} = 18$$

- (i) The density of aluminium is 2.70 g/cm^3 and its molecular wt. is 26.98 g/mole .
 @ calculate the fermi energy.
 b) If the experimental value of E_f is 12 eV , what is the electron effective mass in aluminium?
 (Al is trivalent).

So?

Given,

$$\text{Density } (\rho) = 2.70 \text{ g/cm}^3$$

$$\text{Mol. wt. } (m) = 26.98 \text{ g/mole}$$

we know,

$$N = \frac{V \rho N_A}{m}$$

where $v = \text{Valency of atom}$

$N_A = \text{Avogadro's no.}$

$$\therefore N = \frac{3 \times 2.70 \times (6.023 \times 10^{23})}{20.98}$$

$$= 1.807 \times 10^{23} \text{ electron m}^{-3}$$

then,

We know,

$$E_f = \frac{\hbar^2}{2m_e} (3N\pi^2)^{2/3}$$

$$= \frac{(6.626 \times 10^{-34})^2}{2\pi} \left(3 \times 1.807 \times 10^{23} \times \pi^2 \right)^{2/3}$$

$$= 1.867 \times 10^{-18} \text{ J}$$

$$= 11.66 \text{ eV.}$$

Again, for effective mass (m_e^*)

$$m_e^* = \frac{\hbar^2}{2E_f} (3N\pi^2)^{2/3}$$

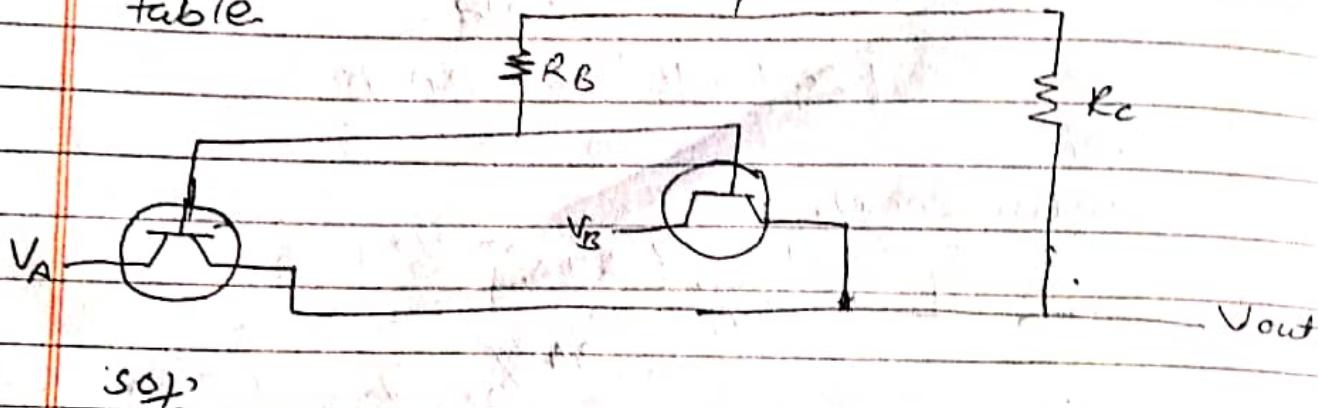
$$= \frac{(6.62 \times 10^{-34})^2}{2\pi} \left(3 \times 1.807 \times 10^{23} \times \pi^2 \right)^{2/3}$$

$$(E_f = 12 \text{ eV})$$

$$= 8.897 \times 10^{-31} \text{ kg.}$$

$$\therefore m_e^* \approx 0.97 m_e$$

- (12) Analyze the circuit in the figure below. Determine the logic function performed by the circuit by making and justifying the appropriate truth table.



The logic function performed by the given circuit.

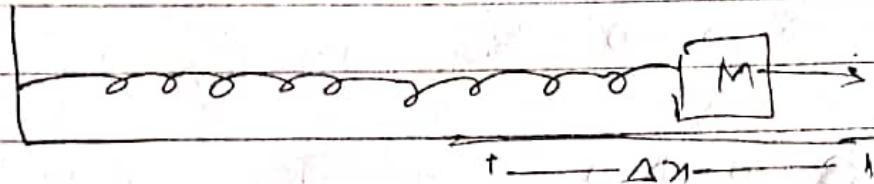
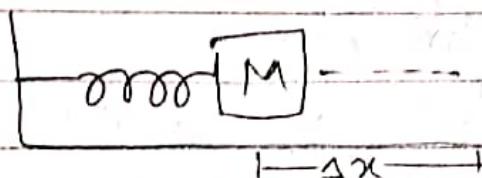
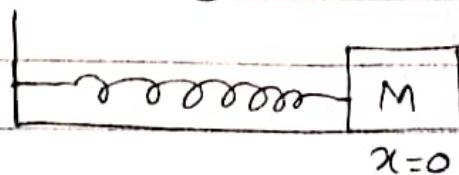
output $Y = V_A \cdot V_B$ i.e., AND operation

Truth table

Input		Output
$V_A(V)$	$V_B(V)$	$V = V_A(V) \cdot V_B(V)$
0	0	0
0	5	0
5	0	0
5	5	5

2077

- ② Set up differential equation for an oscillation of a spring using Hooke's and Newton's second law, find the general solution of this equation and hence the expressions for period, velocity and acceleration of oscillation.



Consider a mass m connected to massless spring at one end and the other end is fixed to the rigidize support. Let the mass m is displaced to maximum value of $y(x)$ and left, so that it vibrates in simple harmonic motion horizontally.

According to Hooke's law, the restoring force in the spring is directly proportional to the displacement.

i.e., $F \propto x$

$$F = -kx \quad \text{--- (1)}$$

where k is proportionality constant called spring constant and negative sign means the

internal force opposes x .

Also, from Newton's second law,

$$F = ma \quad \text{--- (II)}$$

from (I) and (II)

$$ma = -kx$$

$$\text{or, } a = -\frac{kx}{m}$$

$$\text{or, } \frac{d^2x}{dt^2} + \frac{kx}{m} = 0$$

$$\text{i.e., } \frac{d^2x}{dt^2} + \omega^2 x = 0 \quad \text{--- (III)}$$

$$\text{where, } \omega^2 = \frac{k}{m}, \omega = \sqrt{\frac{k}{m}}, T = \frac{2\pi}{\omega} = \frac{2\pi\sqrt{m}}{k}$$

The solution of eqn (III) is given by,

$$x = x_m \sin(\omega t + \phi)$$

(a) velocity

$$\text{we have, } x = x_m \sin(\omega t + \phi)$$

$$\text{velocity; } v = \frac{dx}{dt} = \omega x_m \cos(\omega t + \phi)$$

$$= \omega x_m \sqrt{1 - \sin^2(\omega t + \phi)}$$

$$= \omega x_m \sqrt{1 - x^2}$$

$$= \omega x_m \sqrt{\frac{x_m^2 - x^2}{x_m^2}}$$

$$= \omega \sqrt{A^2 - x^2} \quad (\text{where } x_m = A)$$

(b) Acceleration,

we have, $v = \omega x_m \cos(\omega t + \phi)$

$$a = \frac{dv}{dt} = -\omega^2 x_m \sin(\omega t + \phi)$$

$$\therefore a = -\omega^2 x$$

where -ve sign shows a' is directed towards mean position.

(c) Time period

$$\omega^2 = \frac{k}{m}$$

$$\left(\frac{2\pi}{T}\right)^2 = \frac{k}{m}$$

$$\text{or, } \frac{4\pi^2}{T^2} = \frac{k}{m}$$

$$\text{or, } T^2 = \frac{4\pi^2 m}{k}$$

$$\text{or, } T = 2\pi \sqrt{\frac{m}{k}}$$

Similarly,

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad \underline{\text{Ans}}$$

- ③ Describe frank Hertz experiment. Interpret how the results of this experiment advocate atomic model proposed by Bohr?

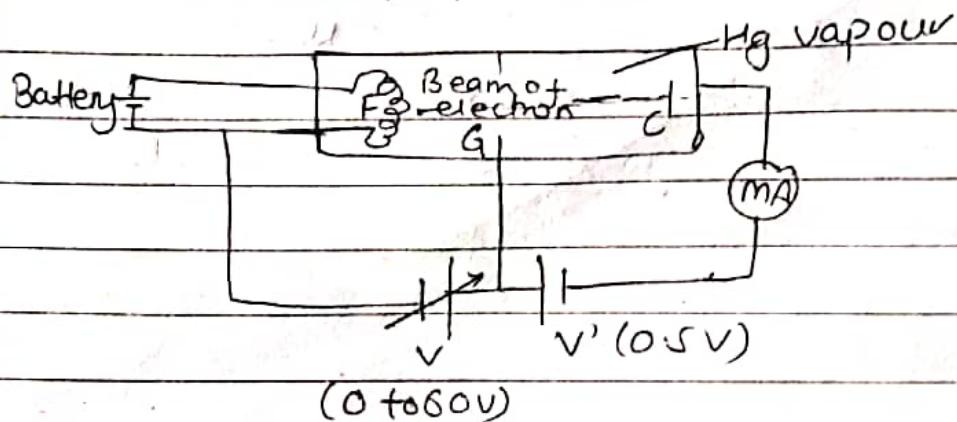
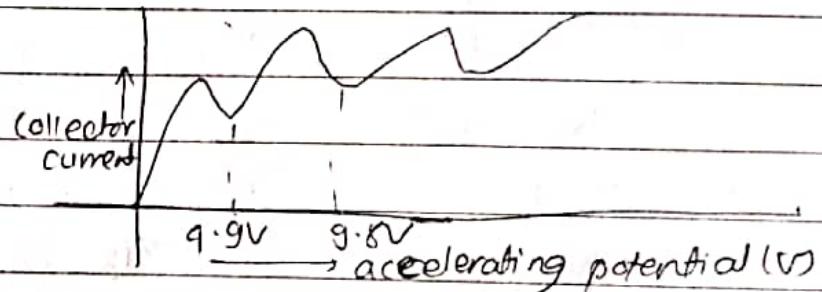


fig:- frank Hertz experiment to explain discrete energy levels.



In this experiment, filament emits electrons when it is heated with the help of battery and these emitted electrons are accelerated with the help of variable battery of V volt. And the accelerated electrons undergo inelastic collision with mercury atoms present inside vacuum tube. Grid (G) is at positive potential with respect to collector (C) as shown in figure.

Experimental results shows that collector current is zero when $V < V'$ and current increases with increase in V for $V > V'$. But surprising result occur at $4.9V$ and

9.8V accelerating potential. Collector current sharply decrease at these potential and we can conclude that this is possible due to absorption of energy of 4.9eV and 9.8eV by mercury atoms.

This experimental result indicates that atom has discrete energy levels and supports Bohr's atomic model.

Further, we observe that wavelength of radiation emitted during transition of electron of Hg-atom from 1st excited state to ground state is 2536A'

Then,

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{2536 \times 10^{-10}} = 4.9 \text{ eV}$$

Limitation: This experiment cannot distinguish between excitation potential and ionization potential.

- ④ Discuss magnetic dipole moment. What is its effect on atoms and on molecules? Explain.
- ⑤ The magnetic dipole moment is defined as - the product of current of the loop and the area of loop.

$$\mu = IA \quad \text{--- (1)}$$

The expression for torque can now be written as

$$\tau = \mu B \sin \theta \quad \text{--- (2)} \quad I = B I \cancel{A} \sin \theta$$

$$= \mu B \sin \theta$$

Eqn (ii) give the magnitude of torque but they don't specify the direction of τ . The direction of the torque can be obtained as

$$\tau = \vec{\mu} \times \vec{B} \quad \text{--- (iii)}$$

Because a magnetic dipole experience a torque when placed in an external field, work must be done to change its orientation. This work done is also be referred as energy of dipole

$$U = \int_{\theta_1}^{\theta_2} \tau d\theta.$$

$$= \int_{90^\circ}^{\theta} \mu B \sin \theta d\theta.$$

$$= -\mu B \cos \theta \quad \text{--- (iv)}$$

This can be expressed in dot product as

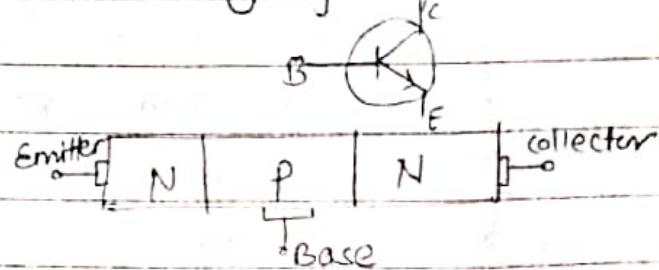
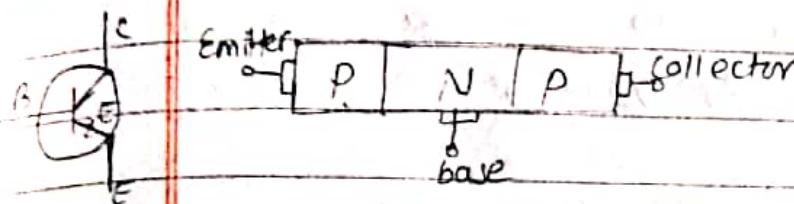
$$U = -\vec{\mu} \cdot \vec{B} \quad \text{--- (v)}, \text{ we conclude that}$$

$U_{\max} = +\mu B$, when $\theta = 0^\circ$, that is when μ and B are aligned anti

$U_{\min} = -\mu B$, when $\theta = 90^\circ$, that is when μ & B are aligned.

Electrons revolving around atomic nuclei, electrons spinning on their axes, and rotating positively charged atomic nuclei, all are magnetic dipoles. The sum of these effects may not be a magnetic dipole. If they don't fully cancel, the atom is a permanent magnetic dipole as iron atoms. The same alignment to form ferromagnetic domain also constitute a magnetic dipole.

⑥ Explain the construction and working of bipolar junction transistor (BJT)



The construction and circuit symbols for both npn and pnp bipolar transistor are given in figure. The arrow in the circuit symbol always showing the direction of "conventional current flow" between the base terminal and its emitter terminal.

The direction of the arrow always points from the positive p-type region to the negative n-type region for both transistor types, exactly the same as for the standard diode symbol we can tabulate the emitter, base and collector current, as well as the emitter-base, collector-base and collector-emitter voltage for npn and pnp transistor.

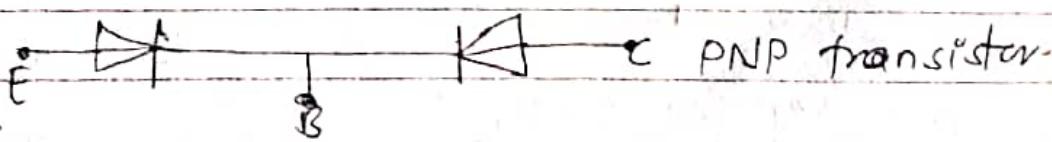
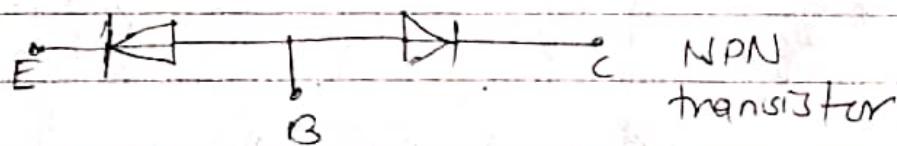


fig: Two diode analogy

Bipolar transistor is a 3-terminal device,

there are basically 3 possible ways to connect it with in electronic circuit, with one terminal being

common to both input and output. Each method of connection responding differently to its input signal within a circuit as the static characteristics of the transistor vary with each circuit arrangement.

(i) Common Base configuration - has voltage gain but no current gain.

(ii) Common collector configuration - has current gain but no voltage gain.

(iii) Common emitter configuration - has both current and voltage gain.

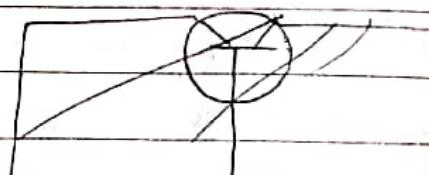
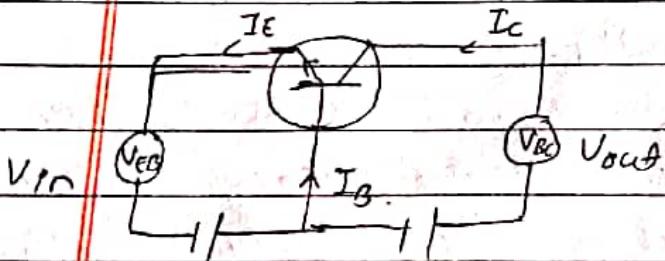


fig: common base configuration

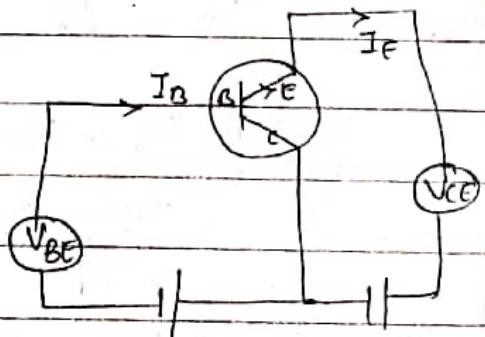
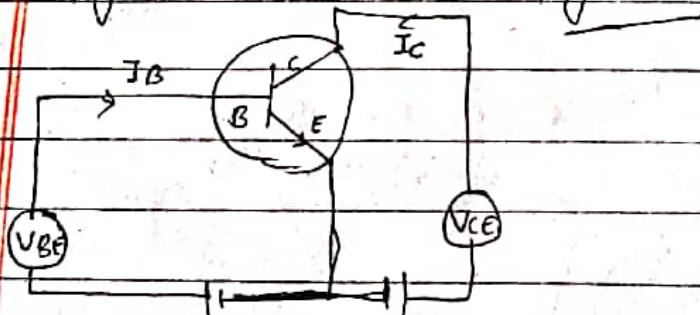


fig:- common collector configuration

fig: common emitter configuration

- ⑦ A roulette wheel with moment of inertia $I = 0.5 \text{ kgm}^2$ rotating initially at 2 rev/sec coasts to a stop from the constant friction torque of bearing. If the torque of bearing is 0.4 Nm, how long does it take to stop?

Sol

Given,

$$\text{inertia } (I) = 0.5 \text{ kgm}^2$$

$$\text{Torque } (T) = 0.4 \text{ Nm}$$

$$\text{frequency } (f_0) = 2 \text{ rev/sec}$$

angular

Now,

$$T = I \cdot \alpha$$

$$0.4 = I \left(\frac{\omega_f - \omega_0}{t} \right)$$

$$\text{or, } 0.4 = 0.5 \left(\frac{2\pi f_f - 2\pi f_0}{t} \right) \quad (\because f_f = 0)$$

$$\text{or, } 0.4 = 0.5 \times \left(-\frac{2\pi \times 2}{t} \right)$$

$$\text{or, } t = \frac{-0.5 \times 2\pi \times 2}{0.4}$$

$$\text{or, } t = 15.07 \text{ sec.}$$

- ⑧ Two large parallel plates are separated by a distance of 5 cm. The plates have equal but opposite charges that create an electric field in the region between the plates. An α -particle ($q = 3.2 \times 10^{-19} \text{ C}$) is released from the

positively charged plate and it strikes the negatively charged plate 2×10^{-6} second later. Assuming that the electric field b/w the plates is uniform and perpendicular to the plates. What is the strength of the electric field?

so?

$$\text{Distance b/w plates } (s) = 5\text{cm} = 5 \times 10^{-2}\text{m.}$$

$$\text{charge of } \alpha \text{ particle } (q) = 3.2 \times 10^{-19}\text{C}$$

$$\text{mass of } \alpha \text{ particle } (m) = 6.68 \times 10^{-27}\text{kg}$$

$$\text{time taken } (t) = 2 \times 10^{-6}\text{ sec.}$$

$$\text{Electric field b/w the plates } (E) = ?$$

Now,

$$s = ut + \frac{1}{2}at^2$$

$$s = \cancel{ut} + \frac{1}{2}at^2 \quad (\because \text{initially at rest, } u=0)$$

$$a = \frac{2s}{t^2} \quad \text{--- (1)}$$

Again from Newton second law of motion

$$F = ma \quad \text{--- (2)}$$

and electrostatic force,

$$F = qE \quad \text{--- (3)}$$

from eqn (1), (2) & (3)

$$m \times \frac{2s}{t^2} = qE$$

$$\text{or, } E = \frac{2ms}{t^2 \times q}$$

$$= \frac{2 \times 6.68 \times 10^{-27}}{3.2 \times 10^{-19} / (2 \times 10^{-6})^2} \times 5 \times 10^{-2}$$

$$= 521.875$$

$$\therefore E = 522 \text{ Nc}^{-1}$$

(10) What is the probability of finding a particle in a well of width a at a position $\frac{a}{4}$ from the wall if $n=1$, if $n=2$, if $n=3$. Use the normalized wavefunction

$$\Psi(n, t) = \left(\frac{2}{a}\right)^{1/2} \sin\left(\frac{n\pi x}{a}\right) e^{-iEt/\hbar}$$

Given,

$$\text{Position } (x) = \frac{a}{4}$$

$$\text{Normalized wave function, } \Psi(x, t) = \left(\frac{2}{a}\right)^{1/2} \sin\left(\frac{n\pi x}{a}\right) e^{-iEt/\hbar}$$

We know that probability of finding a particle,

$$P = \Psi \times \Psi^*$$

Then, at $x = \frac{a}{4}$,

$$P = \left(\sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a} \cdot \frac{a}{4}\right) e^{-iEt/\hbar} \right) \left(\sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a} \cdot \frac{a}{4}\right) e^{-iEt/\hbar} \right)$$

$$P = \frac{2}{a} \sin^2 \frac{n\pi}{4} \quad \text{--- (1)}$$

$$\text{If } n=1, P_1 = \frac{2}{a} \sin^2 \frac{\pi}{4} = \frac{2}{a} \times \frac{1}{2} = \frac{1}{a}$$

$$\text{If } n=2, P_2 = \frac{2}{a} \sin^2 \frac{2\pi}{4} = \frac{2}{a}$$

And for $n=3$, $P_3 = \frac{2}{a} \sin^2 \frac{3\pi}{4} = \frac{2}{a} \sin^2 105^\circ = \frac{2}{a} \left(\frac{1}{2}\right)^2$

$$\therefore P_3 = \frac{1}{a}$$

Hence, probability of finding a particle in a well of width of a position $a = \frac{a}{4}$ form the wall for $n=1$, $n=2$ and $n=3$ are $\frac{1}{a}$, $\frac{2}{a}$ and $\frac{1}{a}$ respectively.

- (ii) The energy gap in silicon is 1.1 eV, whereas in diamond it is 6 eV. What conclusion can you draw about the transparency of two materials to visible light (4000 \AA° to 7000 \AA°)?

Sol:

Here,

$$\text{Energy gap in silicon } (E_g)_{Si} = 1.1 \text{ eV} = 1.1 \times 1.6 \times 10^{-19} \text{ J}$$

$$\text{Energy gap in diamond } (E_g)_{Dia} = 6 \text{ eV} = 6 \times 1.6 \times 10^{-19} \text{ J}$$

wavelength of visible light = 4000 \AA° to 7000 \AA°

We know,

$$\text{Band gap energy } (E_g) = \frac{hc}{\lambda c}$$

For Si,

$$(E_g)_{Si} = \frac{hc}{(\lambda c)_{Si}}$$

$$(\lambda_{Si}) = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{1.1 \times 1.6 \times 10^{-19}} = 1.128 \times 10^{-6} \text{ m}$$

Hence all visible lights are absorbed since $(\lambda_c)_{\text{sp}} < \lambda_{\text{visible}}$
but can transmit infrared light having $\lambda \approx 1.7 \times 10^{-6} \text{ m}$

For diamond,

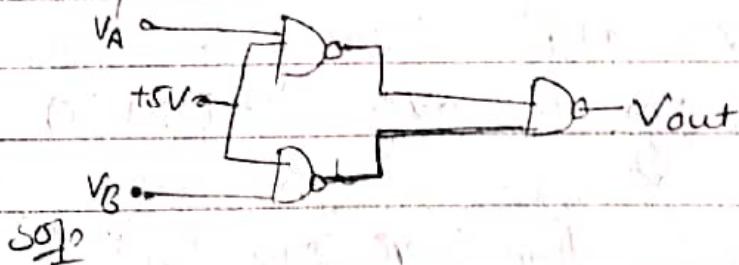
$$(\lambda_c)_{\text{dia}} = \frac{hc}{E_g}_{\text{dia}}$$

$$= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{6 \times 1.6 \times 10^{-19}} \\ = 2.066 \times 10^{-7} \text{ m}$$

$\therefore (\lambda_c)_{\text{dia}} < \lambda_{\text{visible}}$ i.e. $E_{\text{photon}} < E_g$.

Hence, in case of diamond all visible lights are transmitted by such type of non metallic materials.
Thus, Diamond appears transparent & colorless.

- Q12) Find the truth table for the circuit shown in figure.
What logic function will the circuit perform if the constant +5V input to the first two gates is changed to ground potential?



Truth table for given circuit of logic function

$$Y = \overline{V_A} \cdot \overline{V_B}$$

$$Y = V_A + V_B \quad (\text{OR operation})$$

Inputs (v)		Output
V_A	V_B	$\frac{1}{2} (V_A + V_B)$
0	0	0
0	5	2.5
5	0	2.5
5	5	5

In 1-D SWE for an electron moving in the constant potential V_0 is:

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} [E - V_0] \psi = 0$$

$$\Rightarrow \frac{d^2\psi(x)}{dx^2} + k^2 \psi(x) = 0 \quad \text{--- (1)}$$

where, $k^2 = \frac{2m}{\hbar^2} [E - V_0]$

The sol' of eqn (1) is -

$$\psi(n) = e^{\pm ikx} \quad \text{--- (2)}$$

for an electron moving in 1-D periodic potential

$$V(x) = V(n+a), \text{ the SWE is}$$

$$\frac{d^2\psi(n)}{dx^2} + \frac{2m}{\hbar^2} [E - V(n)] \psi(n) = 0 \quad \text{--- (3)}$$

Since periodic functions are equal at a distance

$$\psi(n) = \psi_k(x) e^{\pm ikx} \quad \text{--- (4)}$$

where, $\psi_k(n) = \psi_k(x+a) \quad \text{--- (5)}$

2075

⑤ Explain Bloch theorem? Discuss its use in the Kronig-Penney model and hence in band theory.

→ The eigen function of the wave eqⁿ for a periodic potential are the product of plane wave e^{ikx} and a function $u_k(x)$ with periodicity of the crystal lattice.

i.e., $\Psi(x) = u_k(x) e^{ikx}$ — ① which is Bloch function.
If the periodicity repeats after 'a' distance then the Bloch function can be written as.

$$\Psi(x+a) = u_k(x+a) e^{ik(x+a)} \quad \text{— ②}$$

Since periodic functions are equal at a distance then eqⁿ ② becomes.

$$\Psi(x+a) = \Psi(x) e^{ika} \quad \text{which is Bloch function.}$$

or, $\Psi(x+a) = \lambda \Psi(x)$, where $\lambda = e^{ika}$

○ Bloch waves are important because of Bloch's theorem, which states the energy eigenstates for an electron in a crystal can be written as Bloch waves. This fact underlies the concept of electronic band structures.

○ where $u_k(x) = u_k(x+a)$ represents periodic function and e^{ikx} represents plane wave. The above statement is known as Bloch theorem.

The Kronig-Penney model demonstrate that a simple one-dimensional periodic potential yield energy bands as well as energy band gaps.

⑦ A large wheel of radius 0.4m and moment of inertia 1.2 kgm^2 , pivoted at the center, is free to rotate without friction. A rope is wound around it and a 2kg weight is attached to the rope, when the weight has descended 1.5m from its starting position, (a) what is its downward velocity?

(b) what is the rotational velocity of the wheel?
Sol:

Given,

$$\text{radius } (r) = 0.4 \text{ m}$$

$$\text{Inertia } (I) = 1.2 \text{ kgm}^2$$

$$\text{mass } (m) = 2 \text{ kg}$$

$$\text{height } (h) = 1.5 \text{ m}$$

$$\text{downward velocity } (v) = ?$$

$$\text{rotational velocity } (\omega) = ?$$

Now, from conservation of energy.

$$\text{P.E. of wt} = \text{K.E. of wt} + \text{Rotational K.E. of wheel}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$\text{or, } mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\left(\frac{v}{r}\right)^2 \quad [\because \omega = v/r]$$

$$\text{or, } 2mgh = \left(m + \frac{I}{r^2}\right)v^2$$

$$\text{or, } v = \sqrt{\frac{2mgh}{m + I/r^2}} = \sqrt{\frac{2 \times 2 \times 9.8 \times 1.5}{2 + 1.2/0.4^2}}$$

$$\text{or, } v = 2.5 \text{ m/s}$$

Also,



$$V = r\omega$$

$$\text{or, } \omega = \frac{V}{r} = \frac{2.5}{0.4} = 6.2 \text{ rad/sec}$$

- ② An electron is placed midway between two fixed charges, $q_1 = 2.5 \times 10^{-10} \text{ C}$ and $q_2 = 5 \times 10^{-10} \text{ C}$. If the charges are 1m apart, what is the velocity of the electron when it reaches a point 40cm from q_2 ?

~~so far~~

Given,

$$q_1 = 2.5 \times 10^{-10} \text{ C}$$

$$q_2 = 5 \times 10^{-10} \text{ C}$$

Distance betn q_1 and q_2 (r) = 1m \Rightarrow 100cm

Distance betn e and $q_1, q_2 = r_1 = r_2 = 0.5 \text{ m} \Rightarrow$ 50cm

Distance travelled by e towards q_2 from the initial position (s) = $(50 - 10) \text{ cm} = 40 \text{ cm} = 0.4 \text{ m}$

We know,

$$v^2 = u^2 + 2as$$

$$v = \sqrt{2as} \quad \text{--- (1)} \quad [\because u = 0 \text{ m/s}]$$

Again,

Electrostatic force betn q_1 and e at rest condn

$$F_1 = \frac{q_1 e}{4\pi\epsilon_0 r_1^2}$$

Electrostatic force between q_2 and e,

$$F_2 = \frac{q_2 e}{4\pi\epsilon_0 r_2^2}$$

Now,

$$\epsilon_0 = 8.86 \times 10^{-12}$$

Net force (F) = $F_2 - F_1$

$$= \frac{q_2 e}{4\pi\epsilon_0 r_2^2} - \frac{q_1 e}{4\pi\epsilon_0 r_1^2}$$

$$= \frac{e}{4\pi\epsilon_0} \left(\frac{q_2}{r_2^2} - \frac{q_1}{r_1^2} \right)$$

$$= \frac{1.6 \times 10^{-19}}{9 \times 3.14 \times 8.86 \times 10^{-12}} \left(\frac{5 \times 10^{-10} - 2.5 \times 10^{-10}}{(0.5)^2} \right)$$

$$= 1.44 \times 10^{-18} N$$

Again, from Newton's second law of motion

$$F = ma$$

$$\text{or, } a = \frac{1.44 \times 10^{-18}}{9.1 \times 10^{-31}}$$

$$\therefore m = m_e = 9.1 \times 10^{-31}$$

$$\therefore a = 1.58 \times 10^{12} \text{ m/s}^2$$

Now, eqn ① becomes,

$$v = \sqrt{2 \times 1.58 \times 10^{12} \times 0.4}$$

$$\therefore v = 1.124 \times 10^6 \text{ m/s}$$

Hence, reqd velocity of electron when it reaches a point 30 cm from q_2 is $1.124 \times 10^6 \text{ m/s}$.

Q. A small particle of mass 10^{-6} gm moves along the x -axis; its speed is uncertain by 10^{-6} m/s .

① What is the uncertainty in the x coordinate of the particle?

② Repeat the calculation for an electron assuming

that the uncertainty in its velocity is also 10^{-6} m/s. Use the known values for electrons and Planck's constant.

so?

Given,

$$\text{mass } (m) = 10^{-6} \text{ gm} = 10^{-9} \text{ kg.}$$

$$\text{speed } (\nu) = 10^{-6} \text{ m/s}$$

Uncertainty in x-coordinate (Δx) = ?

Uncertainty in velocity ($\Delta \nu$) = 10^{-6} m/s

Now, we know that,

$$\Delta x \Delta p \geq \frac{h}{2\pi}$$

$$\text{or, } \Delta x \Delta p \geq \frac{h}{2\pi}$$

$$\therefore \Delta x = \frac{h}{2\pi \Delta p} \quad (\text{Here } \Delta p = m_{\text{particle}} \Delta v)$$

$$= \frac{6.62 \times 10^{-34}}{2\pi \times m \Delta v}$$

$$= \frac{6.62 \times 10^{-34}}{2\pi \times 10^{-9} \times 10^{-6}}$$

$$= 1.05 \times 10^{-19} \text{ m}$$

$$\therefore \Delta x = 1.05 \times 10^{-19} \text{ m}$$

⑥ $m_e = 9.1 \times 10^{-31} \text{ kg.}$

$$\Delta x = \frac{h}{2\pi \Delta p}$$

$$= \frac{6.62 \times 10^{-34}}{2\pi \times 9.1 \times 10^{-31} \times 10^{-6}}$$

$$= 115.768 \text{ m}$$

SCC, $r = \frac{a}{2}$, 52%

FCC, no. = 4, $r = \frac{a}{\sqrt{2}}$, 74%

BCC, no. = 2, $r = \frac{\sqrt{3}}{4}a$, 68%

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- (ii) Assuming that atoms are in a crystal structure and arranged as close-packed spheres, what is the ratio of the volume of the atoms to the volume available for the simple cubic structure? Assume a one-atom basis.

So/2

Each corner atom in a cubic unit cell is shared by a total no. of eight unit cells so that the corner atom contributes only $\frac{1}{8}$ of its effective part to a unit cell. Since there are in all 8 corners their total contribution is equal to $\frac{8}{8} = 1$.

∴ no. of atoms per unit cell = 1

from figure,

$$a = 2r$$

$$\text{or, } r = a/2$$



∴ Volume occupied by atom in unit cell is

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi \left(\frac{a}{2}\right)^3$$

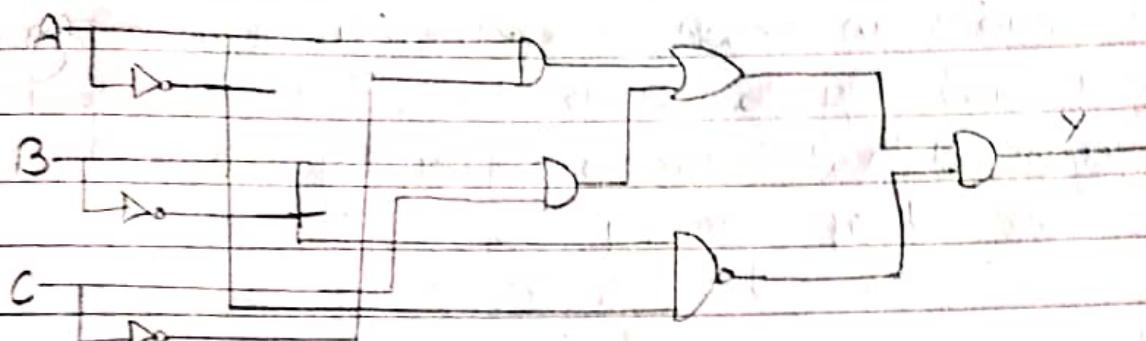
$$\text{Volume of unit cell} = a^3$$

Thus,

$$\begin{aligned} \text{Packing fraction} &= \frac{1 \times \frac{4}{3}\pi \left(\frac{a}{2}\right)^3}{a^3} = 0.152 \\ &= 52\% \end{aligned}$$

(12) The output of a digital circuit (y) is given by this expression: $y = (CB + CA)(\bar{B}A)$ where A, B and C represent inputs. Draw a circuit of the above eqn using OR, AND and NOT gate and hence find its truth-table.

SoE



Truth table

A	B	C	BA	$\bar{B}A$	CB	CA	$C'A$	y
0	0	0	0	1	1	0	0	0
0	0	1	0	0	1	0	0	0
0	1	0	0	1	1	0	0	0
0	1	1	0	0	1	1	0	1
1	0	0	0	1	1	0	0	1
1	0	1	0	0	1	0	1	0
1	1	0	1	1	0	0	1	0
1	1	1	1	0	0	1	0	0

2074

- ② Describe moment of inertia and torque for a rotating rigid body. Find the expression for rotational kinetic and discuss the conditions for conservation.
- ⇒ The property of a body to change its state from rest to rotation or rotation to rest by itself known as moment of inertia. A rotating body has a tendency to be rotating even if a torque is applied to it. This property of the body is known as moment of inertia.

$$\text{i.e. } I = \sum_{i=1}^n m r_i^2$$

. And Torque is the rotational or turning effect of force in a body.

$$\text{i.e., } \tau = I\alpha = r \times F$$

In linear motion, the work done dW by a force F in moving an object through displacement dx is $dW = F \cdot dx$.

In similar way, rotational motion F is replaced by τ and dx is replaced by $d\theta$.

$$\text{or, } dW = I\alpha d\theta$$

$$\text{or, } dW = I \frac{d\omega}{dt} d\theta \quad (\because \alpha = \frac{d\omega}{dt})$$

$$\text{or, } dW = I \omega \frac{dt}{d\theta} d\theta \quad (\because \frac{d\theta}{dt} = \omega)$$

Here, total energy worked in rotating body from 0 to ω_0 angular velocity is,

$$W = \int_0^{\omega} dw$$

$$= \int_0^{\omega} I w dw$$

$$= I \left[\frac{w^2}{2} \right]_0^{\omega}$$

$$= \frac{I}{2} (\omega^2 - 0^2)$$

$$\therefore W = \frac{1}{2} I w^2$$

From work energy theorem, total work done is equal to K.E of rotating body.

$$\text{So, } K.E = \frac{1}{2} I w^2$$

If no net external torque acts on a system, the total angular momentum of system remains constant.

$$\text{i.e. } I\omega = \text{constant}$$

Now,

$$\text{we know, } L = I\omega$$

Diffr. w.r.t. time t'

$$\frac{dL}{dt} = \frac{d(I\omega)}{dt}$$

$$\text{or, } \frac{dL}{dt} = I \frac{d\omega}{dt}$$

$$\text{or, } \frac{dL}{dt} = I\alpha = 0$$

Also, from definition of torque (τ)

$$\tau = I\alpha \quad \text{--- (1)}$$

from (1) and (1)

$$\tau = \frac{dL}{dt}$$

If $\tau = 0$

$$\frac{dL}{dt} = 0$$

or, $L = \text{constant}$

$$\text{i.e., } L_1 = L_2$$

$$\text{or, } I_1 w_1 = I_2 w_2$$

This the expression for conservation of angular momentum

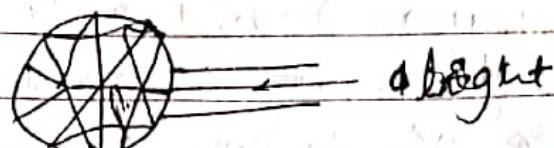
③ Explain the theory of Black body radiation.

Why this theory needs quantum mechanical interpretation? How this interpretation become experimentally successful? Explain.

⇒ If a body absorbs all the radiation incident on it then it appear black and known as black body. Actually, black body absorbs all wavelength of incident radiation and according to Kirchhoff's law of radiation, black body is a good emitter of radiation as it is good absorber of radiation.

Nobody is perfectly black body but for practical purpose a copper empty vessel

having small hole painted black to its inner surface is considered as black body.



black body.

Stefan's law: Statement: Energy distributed radiated per unit area per unit time by a black body is directly proportional to the fourth power of absolute temp.

$$\text{ie, } \frac{\text{Energy}}{\text{Area} \times \text{time}} \propto T^4$$

$$E \propto T^4$$

$$E = \sigma T^4$$

$$\text{where, } E = \frac{E}{Axt}$$

where, σ = Stefan's constant

Stefan's law indicates that radiation emitted by a black body only depends upon the temp of a black body.

The main features of the spectrum emitted by a black body are:-

- i) The spectrum is continuous with a broad maximum.

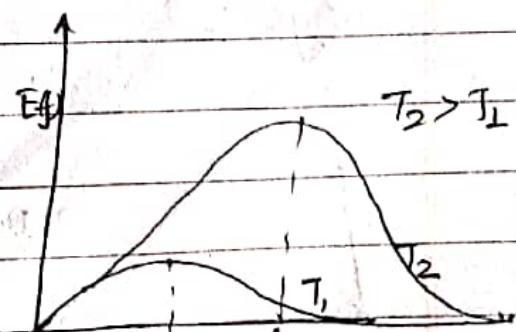
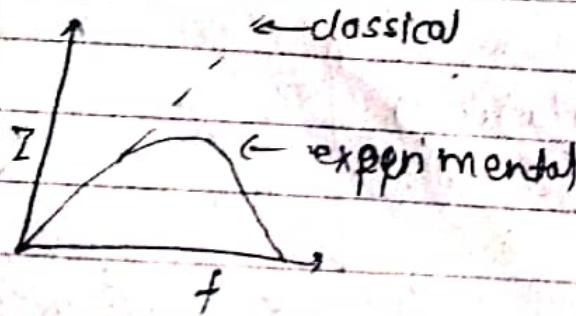


fig: Intensity of black body vs frequency

- i) The energy and intensity depends on fourth power of its temperature.
- ii) This graph shows that at spectrum shifts towards right increasing the frequency as the temp' increases.
i.e. $\nu_{\text{max}} \propto T$

④ Classical mechanics could not explain black body because the human eye cannot perceive light waves at lower frequencies. So, quantum mechanics is necessary to explain this theory. In quantum mechanics, Planck's radiation law explains the black body radiation.

According to classical physics, electromagnetic wave is produced when charge particles like electron vibrates, speed up, slowdown etc. In the hot body, the electron vibrate to any energy level and comes back to ground state releasing energy in the form of radiation. According to classical theory, the radiation of lower frequency has lower intensity and higher frequency has higher intensity which is the failure of classical physics.



Plank's formulated a mathematics to explain the spectral energy distribution of radiation emitted by black body. He assumed that atom oscillator cannot have any value of energy but it has energy which is equal to integral multiple of hf i.e.

$$E = nhf \text{ where } n = 1, 2, 3, \dots$$

The atomic oscillator emit energy which is equal to the difference in two energy level.

$$\text{i.e., } \Delta E = E_2 - E_1 = hf$$

He also suggested that most of the electron jumps to the lower energy level in discrete way.

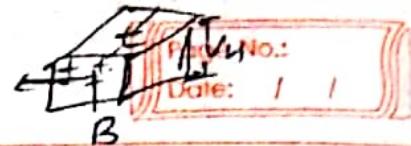
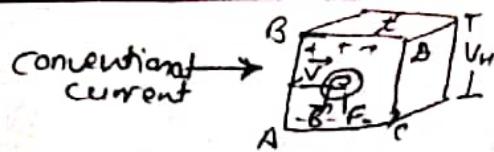
Also, plank's derived an expression of $I(F)_{\lambda\lambda}$

$$I_f = \frac{2\pi h f^3}{c^3} \left(\frac{1}{e^{hf/k_B T} - 1} \right)$$

This expression on plotting in graph matches with experimental result.

④ Explain Hall effect and discuss the importance of hall voltage while manufacturing electronic devices.

⇒ Hall effect is the production of a voltage difference (the Hall voltage) across an electrical conductor, transverse to an electric current in the conductor and a magnetic field perpendicular to the current.



Hall voltage (V_H): - let d be width of the slab and t be the thickness of slab and A be the area of slab and v be the velocity of each electron.

Then electric field setup betw two layers of charges is

$$E = \frac{V_H}{d} \quad \text{--- (1)}$$

at equilibrium condn,

$$F_e = F_m \quad \text{--- (2)}$$

but electric force (F_e) = eE and magnetic force (F_m) = Bev

Now, eqn (2) becomes,

$$eE = Bev$$

$$E = Bv$$

$$\frac{V_H}{d} = Bv$$

$$V_H = Bvd$$

$$\text{Also, } I = VenA$$

$$V = \frac{I}{nea}$$

$$\text{Now, } V_H = B \left(\frac{I}{nea} \right) d$$

$$= \frac{BId}{nea}$$

$$\text{Also, Area} = t \times d$$

$$V_H = \frac{BI}{n e}$$

4 \rightarrow This gives Hall voltage.

Hall resistance (R)

The Hall resistance is given by,

$$R = \frac{V_H}{I} \quad \text{and Hall coefficient}$$

$$R = \frac{B I_{\text{net}}}{n e I}$$

$$R = \frac{B}{n e}$$

$$V_H = \frac{B I d}{n o A} = \left(\frac{1}{n e}\right) \frac{I}{A} \times B$$

$$\underline{\underline{R_H}} = R_H J B$$

$$R_H = \frac{1}{n e} = \frac{E}{J B} = \frac{V_H}{J B}$$

This gives Hall resistance. (It is the characteristic of the material from which the conductor is made, since its value depends on the type, number, and properties of the charge carriers that constitute the current.)

Application of Hall voltage/effect:

- i) Nature of charge carrier is known from the Hall coeff ~~(R_H)~~.
- ii) Concentration of carrier (n) can be calculated.
- iii) Mobility of charge carrier can be determined.

⑤ Discuss effective mass of electrons and holes.

⇒ The electrons in the crystal are not completely free but interact with the periodic potential of crystal lattice. Thus the motion of electron is different from that of free electron and mass of alters. This altered value of mass is called effective mass.

We know, group velocity of particle is,

$$V_g = \frac{d\omega}{dk} \quad \text{--- (i)}$$

we know, $E = \hbar\omega$

$$\omega = \frac{E}{\hbar} \quad \text{--- (ii)}$$

diff eqn (ii) w.r.t. k , we get,

$$\frac{d\omega}{dk} = \frac{1}{\hbar} \frac{dE}{dk} \quad \text{--- (iii)}$$

from (i) and (iii), we get

$$V_g = \frac{1}{\hbar} \frac{dE}{dk}$$

Again, diff it w.r.t. time t .

$$\frac{dV_g}{dt} = \frac{1}{\hbar} \frac{d}{dt} \left(\frac{dE}{dk} \right)$$

$$\text{or, } a = \left(\frac{1}{\hbar^2} \frac{d^2 E}{dk^2} \right) \left(\frac{d\omega}{dt} \right) \quad \text{--- (iv)}$$

According to Newton's second law of motion.

$$F = ma$$

$$\Rightarrow a = \frac{F}{m} \quad \text{--- (v)}$$

on comparing (iv) and (v), we get.

$$\boxed{\frac{1}{m^*} = \frac{1}{\hbar^2} \frac{d^2 E}{dE^2}} \quad \text{--- (vi)}$$

This eqn gives the effective mass of electron on crystal.

- (B) An oscillating block of mass 250g takes 0.15 sec to move between the endpoints of motion, which are 40cm apart.
- (a) What is the frequency of the motion?
 - (b) What is the amplitude of the motion?
 - (c) What is the force constant of the spring?

Given,

$$\text{mass (m)} = 250\text{g} = 250 \times 10^{-3} \text{kg.}$$

$$\text{time (t)} = 0.15 \text{sec}$$

$$\text{Displacement (x)} = 40\text{cm} = 40 \times 10^{-2} \text{m.}$$

$$\text{frequency (f)} = ?$$

$$\text{amplitude (A)} = ?$$

$$\text{force constant (k)} = ?$$

Now,

$$f = \frac{1}{t} = \frac{1}{0.15} \doteq 6.67 \text{ Hz/sec}.$$

$$A (\text{m}) = \frac{40 \times 10^{-2}}{2} = 20 \times 10^{-2} \text{m.}$$

We know that,

$$\sigma f = \sqrt{\frac{k}{m}}$$

$$4\pi^2 f^2 = \frac{k}{m}$$

$$\text{or, } k = 4\pi^2 f^2 m$$

$$\text{or, } k = 4\pi^2 (6.67)^2 \times 250 \times 10^{-3}$$

$$\text{or, } k = 439.08 \text{ N/m. } \cancel{*}$$

- (8) A current of 50A is established in a slab of copper 0.5cm thick and 2cm wide. The slab is placed in a magnetic field ~~perpendicular~~^B of 1.5T. The magnetic field is perpendicular to the plane of the slab and to the current. The free electron concn in copper is 8.4×10^{28} electrons/m³. What will be the magnitude of the Hall voltage across the width of the slab?

SOP

Given,

$$\text{current (I)} = 50\text{A}$$

$$\text{thickness of copper (a)} = 0.5\text{cm.}$$

$$\text{width of } (b) = 2\text{cm} = 2 \times 10^{-2}\text{m}$$

$$\text{magnetic field (B)} = 1.5\text{T}$$

$$\text{No. of free electron (N)} = 8.4 \times 10^{28}$$

$$\text{magnitude of Hall voltage (V_H)} = ?$$

We know,

7

t=1KA

$$\Delta V = \frac{IBb}{NqA}$$

$$= \frac{50 \times 1.5 \times 2 \times 10^{-2}}{8.4 \times 10^{28} \times 1.6 \times 10^{-19} \times 2 \times 10^{-2} \times 0.5 \times 10^{-2}}$$

$$= 1.12 \times 10^{-6} \text{ V}$$

- ③ The uncertainty in the position of a particle is equal to the de Broglie wavelength of the particle. Calculate the uncertainty in the velocity of the particle in terms of the velocity of the de Broglie wave associated with the particle.

so,

Given,

Uncertainty in position (Δx) = de-Broglie wavelength (λ)Uncertainty in the velocity (Δv) = ?

we know,

$$\Delta x \Delta p \geq \frac{\hbar}{2\pi}$$

$$\text{or, } \Delta x = \frac{\hbar}{2\pi m \Delta v} \quad \text{--- (1)}$$

Again, from de-Broglie wavelength.

$$\lambda = \frac{\hbar}{mv} \quad \text{--- (2)}$$

from (1) and (2), we get.

$$\frac{\hbar}{2\pi m \Delta v} = \frac{\hbar}{mv}$$

$$\text{or, } \Delta v = \frac{v}{2\pi}$$

Hence uncertainty in velocity is equal to the $\frac{1}{2\pi}$ times velocity of the de-Broglie wave.

- (ii) Copper has a face-centered cubic structure with a one-atom basis. The density of copper is 8.96 g cm^{-3} and its atomic wt. is 63.5 g/mole . What is the length of the unit cube of the structure?

SOL:

Given,

$$\text{density of copper } (\rho) = 8.96 \text{ g cm}^{-3}$$

$$\text{No. of atom per cell (CN)} = 4$$

$$\text{Avargado Number (NA)} = 6.023 \times 10^{23}$$

$$\text{length of unit cube (a)} = ?$$

$$\text{At wt of Cu (m)} = 63.5 \text{ g/mole}$$

we know,

$$\frac{N}{V} = \frac{nNA}{m\rho}$$

$$\text{or, } \frac{N}{V} = \frac{sNA}{m}$$

$$\text{or, } \frac{N}{a^3} = \frac{sNa}{m}$$

$$\text{or, } a^3 = \frac{Nm}{sNa}$$

$$= \frac{4 \times 63.5}{8.96 \times 6.023 \times 10^{23}}$$

$$\text{or, } a^3 = 4.71 \times 10^{-23}$$

$$a = 3.61 \times 10^{-7} \text{ cm.}$$

Hence, reqd length of unit cubic of the structure is $3.61 \times 10^{-7} \text{ cm.}$

$$\begin{aligned} x &= \sqrt[3]{4.71 \times 10^{-23}} \\ &= \sqrt[3]{\frac{4.71}{m}} \times 10^{-7} \\ &\approx 3.602^3 \times 10^{-7} \end{aligned}$$