Tribhuvan University
Institute of Science and Technology
2075
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Bachelor Level / First Year/ First Semester/ Science
Computer Science and Information Technology (MTH. 112)
(Mathematics I)
(NEW COURSE)

Full Marks: 80
Pass Marks: 32
Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.
The figures in the margin indicate full marks.

Attempt any three questions: (3×10=30)

1. (a) A function is defined by $f(x) = |x|$, calculate f(-3), f(4), and sketch the graph. (5)

   (b) Prove that the $\lim_{x\to 2} \dfrac{|x-2|}{x-2}$ does not exist. (5)

2. (a) Find the domain and sketch the graph of the function $f(x) = x^2 - 6x$. (5)

   (b) Estimate the area between the curve $y = x^2$ and the lines $y = 1$ and $y = 2$. (5)

3. (a) Find the Maclaurin series for $\cos x$ and prove that it represents $\cos x$ for all x. (4)

   (b) Define initial value problem. Solve that initial value problem of $y' + 2y = 3$, $y(0) = 1$. (4)

   (c) Find the volume of a sphere of radius a. (2)

4. (a) If $f(x,y) = \dfrac{y}{x^4}$ does $\lim_{(x,y)\to(0,0)} f(x, y)$ exist? Justify. (5)

   (b) Calculate $\iint_R f(x,y)\,dA$ for $f(x,y) = 100 - 6x^2y$ and R: $0 \le x \le 2$, $-1 \le y \le 1$. (5)

Attempt any ten questions: (10×5=50)

5. If $f(x) = \sqrt{2-x}$ and $g(x) = \sqrt{x}$, find fog and fof. (5)

6. Define continuity on an interval. Show that the function $f(x) = 1 - \sqrt{1-x^2}$ is continuous on the interval [-1, 1]. (5)

7. Verify Mean value theorem of $f(x) = x^3 - 3x + 2$ for [-1, 2]. (5)

8. Starting with $x_1 = 2$, fine the third approximation x_3 to the root of the equation $x^3 - 2x - 5 = 0$. (5)

9. Evaluate $\int_0^{\infty} x^3\sqrt{1-x^4}\,dx$. (5)

10. Find the volume of the resulting solid which in enclosed by the curve y = x and y = x^2 is rotated about the x – sxis. (5)

11. Find the solution of $y'' + 4y' + 4 = 0$

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12. Determine whether the series $\sum_{n=1}^{\infty} \frac{n^2}{5n^2+4}$ converges or diverges. (5)

13. If **a** = (4, 0, 3) and **b** = (-2, 1, 5) find |**a**|, the vector **a** – **b** and 2**a** + 5**b**. (1+2+2)

14. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if z is defined as a function of x and y by the equation $x^3 + y^3 + z^3 + 6xyz = 1$. (5)

15. Find the extreme values of the function $f(x, y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$ (5)

2075 Question Solution

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BSC.CSIT 1^{st} Sem — Pravin Gupta

Subject:- Mathematics-I

Group 'A'

1(a) A Function is defined by $f(x) = |x|$, calculate $f(-3)$, $f(4)$ and sketch the graph

Sol

Given,

$$f(x) = |x|$$

Since,

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

then

$f(-3)$ i.e $-3 < 0$ so,

$$f(-3) = -x$$
$$= \cancel{x} -(-3)$$
$$= 3$$

And

$f(4)$ i.e $4 \geq 0$ so,

$$f(4) = x$$
$$= 4$$

For graph,

$$x \geq 0$$

let $y = x$

if $y = 4$, $x = 4$

$y = 1$, $x = 1$ $\therefore (x,y) = (4,4), (1,1), (0,0)$

$y = 0$, $x = 0$

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For,

$$x < 0$$

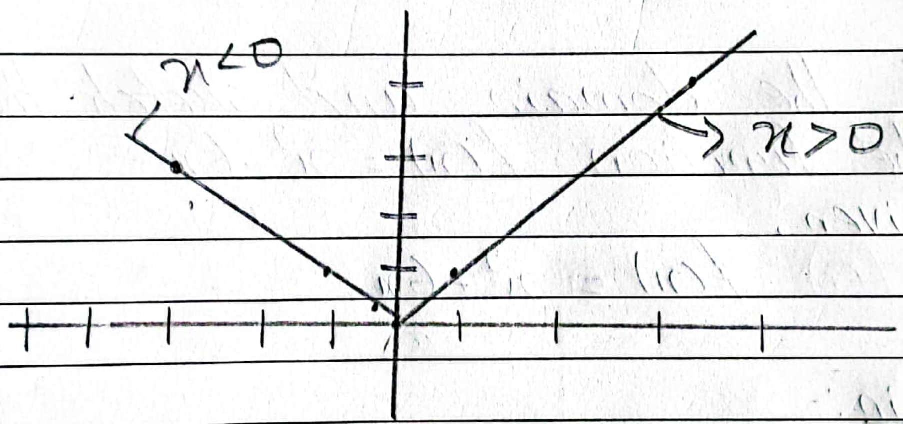
let, $y = -x$

if $x = -3$, $y = -(-3) = 3$

$x = -1$, $y = -(-1) = 1$

$x = -0.1$, $y = -(-0.1) = 0.1$

Now, plotting these point in graph.



1(b) Prove that the $\lim\limits_{x \to 2} \frac{|x-2|}{x-2}$ does not exist

Sol Given,

$$\lim_{x \to 2} \frac{|x-2|}{x-2}$$

As we know,

$$|x-2| = \begin{cases} (x-2) & x > 0 \\ -(x-2) & x < 0 \end{cases}$$

Here,

$$LHL = \lim_{x \to 2^+} \frac{(x-2)}{(x-2)} = \lim_{x \to 2^+} 1 = 1$$

RHL,

$$\lim_{x \to 2^-} \frac{-(x-2)}{(x-2)}$$

$$= \lim_{x \to 2^-} -1$$

$$= -1$$

$\therefore$ LHL $\neq$ RHL

Hence, limit doesn't exist.

2(a) Find the domain and sketch the graph of the function $f(x) = x^2 - 6x$

Sol Given,

$$f(x) = x^2 - 6x$$

(A) Domain:-

For domain,

$$y = x^2 - 6x$$

$$y = x(x-6)$$

clearly, this function exists for all value of x i.e $(-\infty, \infty)$

So, domain of $y = (-\infty, \infty)$

(B) Intercept:-

For x-intercept put $y=0$,

$$y = x^2 - 6x$$

$$y = x(x-6)$$

or, $0 = x(x-6)$

$\therefore x = 0, 6$

$\therefore$ The X-intercept is $x=0$ and $x=6$

For Y-intercept, put $x=0$

$$y = x^2 - 6x$$
$$y = 0 - 0$$
$$y = 0$$

$\therefore$ The Y-intercept is $y=0$

$\therefore$ The curve meets the axes only at $(0,0)$ and $(6,0)$

(c) Symmetry :- Curve Symmetric about y-axis $f(x) = f(-x)$ and for curve symmetric about origin $f(-x) = -f(x)$ then,

$$f(x) = x^2 - 6x$$

if $x = -x$, $f(-x) = (-x)^2 - 6\times(-x)$

$$f(-x) = x^2 + 6x$$
$$f(-x) \neq -f(x) \neq f(x)$$

That is y is neither symmetrical about the axes.

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(D) Asymptote

Horizontal asymptote:-

If either $\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$ then $y = L$ is horizontal asymptote of curve $y = f(x)$

So,

$$\lim_{x\to\infty} x^2 - 6x = (\infty)^2 - 6\times(\infty) = \infty$$

Vertical asymptote:-

A line $x = a$ is vertical asymptote of $y = f(x)$ if either $\lim_{x\to a^+} f(x) = \pm\infty$ or $\lim_{x\to a^-} f(x) = \pm\infty$

So,

$$\lim_{x\to a} (x^2 - 6x) = a^2 - 6a \neq \infty$$ for any finite value of a

$\therefore$ Thus, the curve has no horizontal and vertical asymptote

(E) Interval of increasing and decreasing

Here,

$$f(x) = x^2 - 6x$$

So, $f'(x) = 2x - 6 = 2(x-3)$

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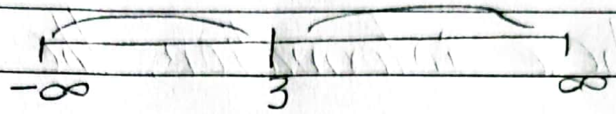
for critical point,

$$f'(x) = 0$$
$$2(x-3) = 0$$
$$\therefore x = 3$$



Interval	$(-\infty, 3)$	$(3, \infty)$
Sign of $f'(x)$	$-ve$	$+ve$
Nature of curve	Decrease	Increase

(F) Concavity:-

$$f(x) = x^2 - 6x$$
$$f'(x) = 2x - 6$$
$$f''(x) = 2 \neq 0 \quad \text{not defined for all } x$$

So there is no point of inflection $f''(x) > 0$ for all x so the curve of $f(x)$ is concave up for all x

(G) Extreme :-

$$f(x) = x^2 - 6x$$

if $x = 3$, $f(3) = 3^2 - 6 \times 3 = 9 - 18 = -9$

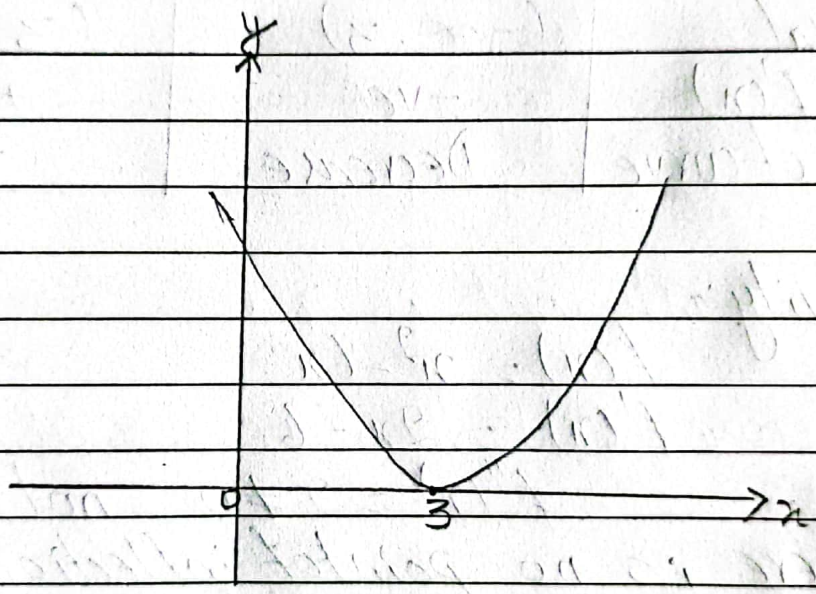
$\therefore$ $f(3) = 0$

So, the curve $f(x)$ has manimum at $(3, 0)$

(H) Summary

Interval	$(-\infty, 3)$	$(3, \infty)$
Nature of curve	Decrease Concave up	Increase Concave up

With these information, the sketch of the curve is;



9(b) Estimate the area between the curve $y = x^2$ and the line $y = 1$ and $y = 2$

Sol^n

Given,

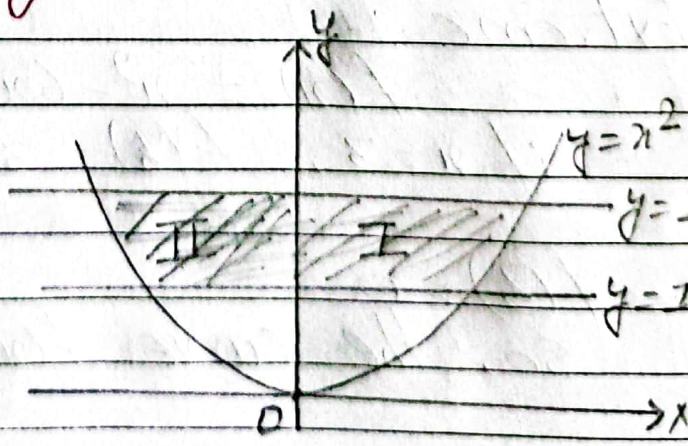
$y = x^2$

$x = \sqrt{y}$ --- (1)

and the line are

$y = 1$ and

$y = 2$



Now,

The area of bounded region is

$$A = 2\int_1^2 x\,dy$$

$$= 2\int_1^2 \sqrt{y}\,dy$$

$$= 2\int_1^2 y^{1/2}\,dy$$

$$= 2\left[\frac{y^{1/2+1}}{1/2+1}\right]_1^2$$

$$= \cancel{\frac{2\times 2}{3}\left[y^{3/2}\right]_1^2} \qquad 2\times\frac{2}{3}\cdot\left[y^{3/2}\right]_1^2$$

$$= \cancel{\frac{4}{3}\left[(2)^{3/2} - 1^{3/2}\right]} \qquad \frac{4}{3}\left[(2)^{3/2} - 1^{3/2}\right]$$

$$= \frac{4}{3}\left((2^3)^{1/2} - 1\right)$$

$$= \frac{4}{3}\left[\sqrt{8} - 1\right]$$

$$= \frac{4}{3}\left(\sqrt{2^2\times 2} - 1\right)$$

$$= \frac{4}{3}\left(2\sqrt{2} - 1\right) \text{ sq. units}$$

Thus, the area of the region is $\frac{4}{3}(2\sqrt{2} - 1)$ sq. units.

3(a)

Find the Maclaurin's series for $\cos x$ and prove that it represents $\cos x$ for all x.

Given,

the given function is,

$f(x) = \cos x$ ---- (1)

we have to find Maclaurin's series generated by the function $f(x) = \cos x$ for this we need to find Taylor's series generated by the function.

$$f(x) = \cos x \quad \text{about } x=0 \quad (\text{i.e } x = a = 0)$$

Diff^n eq^n (1) with respect to 'x' successively.

$f'(x) = -\sin x$
$f''(x) = -\cos x$
$f'''(x) = \sin x$
$f^{iv}(x) = \cos x$
$f^{v}(x) = -\sin x$
$f^{vi}(x) = -\cos x$

rearrangement all derivative as bellow

$f'(x) = -\sin x$	$f''(x) = -\cos x$
$f'''(x) = \sin x$	$f^{iv}(x) = \cos x$
$f^{v}(x) = -\sin x$	$f^{vi}(x) = -\cos x$
⋮	⋮
$f^{2n+1}(x) = (-1)^n \sin x$	$f^{2n}(x) = (-1)^n \cos x$

at $x=0$,

$$f(0) = \cos(0) = 1$$

$f'(0) = -\sin 0 = 0$	$f''(0) = -\cos 0 = -1$
$f'''(0) = \sin 0 = 0$	$f^{iv}(0) = \cos 0 = 1$
$f^{v}(0) = -\sin 0 = 0$	$f^{vi}(0) = -\cos 0 = -1$
$\vdots$	$\vdots$
$f^{2n+1}(0) = (-1)^n \times 0 = 0$	$f^{2n}(0) = (-1)^n$

We know that,

$$f(x) = f(0) + \frac{f'(0)(x)}{1!} + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{iv}(0)x^4}{4!} + \cdots + \frac{f^n(0)x^n}{n!} \cdots$$

$$\cos x = 1 + 0 - \frac{1 \times x^2}{2!} + 0 + \frac{1 \times x^4}{4!} + \cdots + (-1)^n \times \frac{x^{2n}}{(2n)!} + \cdots$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{(-1)^n \times x^{2n}}{(2n)!}$$

Since, the cosine function (i.e $f(x) = \cos x$) and all the derivative of cosine function (i.e $f'(x), f''(x), f'''(x), \ldots$) have value less than or equal to 1 (i.e value ≥ 1)

So, by Taylor's inequality

$$|R_n(x)| \leq M \frac{|x-a|^{n+1}}{(n+1)!}$$

$$|R_{2n}(x)| \leq \frac{|x|^{2n+1}}{(2n+1)!}$$

Here, we have to take limit $n \to \infty$ then,

$$\lim_{n\to\infty} \frac{|x|^{2n+1}}{(2n+1)!} = 0$$

i.e

$$\lim_{n\to\infty} R_{2n}(x) = 0 \quad \text{for all } x$$

Which shows that given series is converges to $\cos x$ for every value of x.

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3(b) Define initial value problem. Solve that initial value problem of $y'+2y=3$, $y(0)=1$

Sol

1st part,

A differential equation together with initial condition(s) is called the initial value.

For example:-

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0, \quad y(0) = -1, \quad y'(0) = 1$$

Here, $y(0) = -1$ and $y'(0) = 1$ is an initial condition

2nd Part,

Given,

$$y' + 2y = 3 \qquad , \qquad y(0) = 1$$

$$\frac{dy}{dx} + 2y = 3 \; ----(1)$$

comparing eqn (1) with $\frac{dy}{dx} + Py = Q$

where, $P = 2$, $Q = 3$

Now,

Integrating factor (I.f) $= e^{\int P dx} = e^{\int 2 dx} = e^{2x}$

Multiplying by I.f in eqn (1) on bothside

$$e^{2x}\left(\frac{dy}{dx} + 2y\right) = 3 \times e^{2x}$$

or, $$d(e^{2x}.y) = 3 \times e^{2x}$$

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on integrating

$$\int d(e^{2x}\cdot y) = \int 3\times e^{2x}$$

$$e^{2x}\cdot y = 3\times\frac{e^{2x}}{2} + c \quad \text{----(11)}$$

As we know,

$$y(0) = 1$$

$y = 1$, $x = 0$, then eq^n (11) becomes

or, $$e^{0}\cdot 1 = 3\times\frac{e^{0}}{2} + c$$

or, $$1 = \frac{3}{2} + c$$

or, $$c = 1 - \frac{3}{2}$$

$$\therefore c = -\frac{1}{2}$$

Then eq^n (11) becomes

$$e^{2x}\cdot y = 3\times\frac{e^{2x}}{2} - \frac{1}{2}$$

or, $$y = \frac{3\times e^{2x}}{2\times e^{2x}} - \frac{1}{2e^{2x}}$$

or, $$y = \frac{3}{2} - \frac{1}{2}e^{-2x}$$

$$\therefore y = \frac{1}{2}(3 - e^{-2x})$$ Ans

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(c) Find the volume of sphere of radius a

Sol Since, we know that intersection of sphere of radius 'a' and the plane is of a circle of radius 'a'

then circle is

$$x^2+y^2=a^2$$

$$y^2=a^2-x^2 \quad \text{---} (1)$$

clearly the circle has end $x=-a$ to $x=a$ Now, the volume of the solid that is generated by revolving the circle (1) about x-axis is (i.e $y \le 0$)

$$\text{Volume} = \pi\int_{-a}^{a} y^2 dx$$

$$= \pi\int_{-a}^{a} (a^2-x^2)dx$$

$$= \pi\left[a^2x - \frac{x^3}{3}\right]_{-a}^{a}$$

$$= \pi\left\{\left(a^2\cdot a - \frac{a^3}{3}\right) - \left(a^2\cdot(-a) - \frac{(-a)^3}{3}\right)\right\}$$

$$= \pi\left\{\left(a^3-\frac{a^3}{3}\right) - \left(-a^3+\frac{a^3}{3}\right)\right\}$$

$$= \pi\left\{\frac{3a^3-a^3}{3} - \left(\frac{-3a^3+a^3}{3}\right)\right\}$$

$$= \pi\left(\frac{2a^3}{3}+\frac{2a^3}{3}\right) = \pi\times\frac{4a^3}{3} = \frac{4\pi a^3}{3} \text{ Ans}$$

$\therefore$ The volume of the sphere of radius 'a' is $\frac{4\pi a^3}{3}$ Ans

1(a) If $f(x,y) = \frac{y}{x}$, does $\lim_{(x,y)\to(0,0)} f(x,y)$ exist? Justify

Sol: Here, when we put $x=0$ and $y=0$ the function $f(x,y)$ takes an indetermant form $0/0$. Also it can't be simplified

So, we use two path ways of testing limit for this let $y=kx$ be helping curve which moves the given curve to the limiting point we have,

$$y = kx$$

if $y \to 0 \Rightarrow k = 0$

$$\lim_{(x,y)\to(0,0)} \frac{y}{x}$$

$$\lim_{(x,y)\to(0,0)} \frac{kx}{x}$$

$$\lim_{(x,y)\to(0,0)} = k$$

Taking, if $k = 1$

$\Rightarrow 1$

if $k = 2 \Rightarrow 2$

Hence,

$1 \neq 2$

Hence, By 2 pathway the value of the function doesn't remain same for different value of k. so, that the given function doesn't exist at limit $(x,y)\to(0,0)$.

4(b) Calculate $\iint_R f(x,y)\,dA$ for $f(x,y) = 100 - 6x^2y$ and

$R: 0 \le x \le 2,\ -1 \le y \le 1$

Sol^n Given,

$f(x,y) = 100 - 6x^2y \quad ; \ R: 0 \le x \le 2,\ -1 \le y \le 1$

Here,

$$= \int_{-1}^{1}\int_{0}^{2} (100 - 6x^2y)\,dx\,dy$$

$$= \int_{-1}^{1} \left[100x - \frac{6x^3}{3}y\right]_0^2 dy$$

$$= \int_{-1}^{1} \left[100x - 2x^3y\right]_0^2 dy$$

$$= \int_{-1}^{1} \left[100\times 2 - 2\times(2)^3y\right] dy$$

$$= \int_{-1}^{1} (200 - 16y)\,dy$$

$$= \left[200y - \frac{16y^2}{2}\right]_{-1}^{1}$$

$$= \left[200y - 8y^2\right]_{-1}^{1}$$

$$= \left(200\times(1) - 8\times(1)^2\right) - \left(200\times(-1) - 8\times(-1)^2\right)$$

$$= (200 - 8) - (-200 - 8)$$

$$= 200 - 8 + 200 + 8$$

$$= 400 \text{ Ans}$$

5 If $f(x) = \sqrt{2-x}$ and $g(x) = \sqrt{x}$, find $f \circ g$ and $f \circ f$

Sol: Given,

$$f(x) = \sqrt{2-x}$$
$$g(x) = \sqrt{x}$$
$$f \circ g = ?$$
$$f \circ f = ?$$

Now,

$f \circ g$

$= f(\sqrt{x})$

$= \sqrt{2-\sqrt{x}}$ Ans

$f \circ f$

$= f(\sqrt{2-x})$

$= \sqrt{2-(\sqrt{2-x})}$ Ans

$\therefore$ $\left.\begin{array}{l} f \circ g = \sqrt{2-\sqrt{x}} \\ f \circ f = \sqrt{2-(\sqrt{2-x})} \end{array}\right\}$ Ans

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6\. Define continuity on an interval. Show that the function $f(x) = 1-\sqrt{1-x^2}$ is continuous on the interval $[-1,1]$

Sol

1^{st} part,

A function $f(x)$ is continuous at $x=a$ if $\lim\limits_{x\to a} f(x) = f(a)$ (functional value = limiting value)

Precisely, a function $f(x)$ is continuous at $x=a$ if

(a) $f(a)$ is defined (i.e a is in the domain of f)

(b) $\lim\limits_{x\to a} f(x)$ exists

(c) $\lim\limits_{x\to a} f(x) = f(a)$

2^{nd} Part

Given function,

$$f(x) = 1-\sqrt{1-x^2} \quad \text{on } [-1,1] = [a,b]$$

(a) for $-1 < c < 1$ (i.e $a < c < b$)

$$\therefore \lim_{x\to c} f(x) = \lim_{x\to c}\left(1-\sqrt{1-x^2}\right)$$

$$= 1-\sqrt{1-c^2} \text{ exist on } [-1,1]$$

Also,

$$f(c) = 1-\sqrt{1-c^2}$$

$$\therefore \lim_{x\to c} f(x) = f(c)$$

$\therefore f(x)$ is continuous at $c \in (-1,1)$

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(b) At left end $a = -1$

$$\lim_{x \to (-1)^+} f(x)$$

$$= \lim_{x \to (-1)^+} 1 - \sqrt{1-(-1)^2}$$

$$= 1 - \sqrt{1-1^2}$$

$$1$$

(c) Right end point $b = 1$

$$\lim_{x \to 1^-} f(x)$$

$$= \lim_{x \to 1^-} 1 - \sqrt{1 - 1^2}$$

$$= 1 - 0$$

$$= 1$$

$\therefore$ from above condition (b) and (c), it shows that $f(x)$ is continuous on close interval of $[-1,1]$ Ans

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7 Verify Mean Value Theorem of $f(x) = x^3 - 3x + 2$ for $[-1, 2]$

Sol^n Statement of Mean Value Theorem

Let f be a function that satisfies the following conditions.

(i) f is continuous on the closed interval $[a, b]$

(ii) f is ~~con~~ differentiable on the open interval (a, b)

Then there is a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Now, Given, function is

$$f(x) = x^3 - 3x + 2 \quad \text{for } [-1, 2]$$

(I) Since $f(x)$ is a polynomial function. So, it is continuous on the closed interval $[-1, 2]$

(II) $f'(x) = 3x^2 - 3$ which exists for all $x \in (-1, 2)$

$\therefore$ $f(x)$ is differentiable on the open interval

all the condition of Mean Value Theorem are satisfied hence, there exist a number c in $(-1, 2)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

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$f'(x) = 3x^2 - 3$
$f'(c) = 3c^2 - 3$
$f(x) = x^3 - 3x + 2$
$f(a) = f(-1) = (-1)^3 - 3\times(-1) + 2 = -1 + 3 + 2 = 4$
$f(b) = f(2) = 2^3 - 3\times2 + 2 = 8 - 6 + 2 = 4$
Then,

$$3c^2 - 3 = \frac{4-4}{2-(-1)}$$

$$\text{or, } 3(c^2-1) = \frac{0}{3}$$

$$\text{or, } 3(c^2-1) = 0$$

$$\text{or, } c^2 - 1 = 0$$

$$\text{or, } c^2 = 1$$

$$\therefore c = \pm 1$$

Hence, $c = 1 \in (-1, 2)$
$\therefore$ Mean Value theorem verify.

8 Starting with $x_1 = 2$, find the third approximation x_3 to the root of the equation $x^3 - 2x - 5 = 0$

Sol^n Given,

$$f(x) = x^3 - 2x - 5 \quad \text{--- (1)} \qquad x_1 = 2$$

diff^n eq^n(1) w.r.to 'x'

$$f'(x) = 3x^2 - 2$$

By Newton's method formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

When $x_1 = 2$,

$$x_{1+1} = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 2 - \frac{(2^3 - 2\times 2 - 5)}{(3\times 2^2 - 2)}$$

$$x_2 = 2 - \frac{(8-4-5)}{(10)}$$

$$x_2 = 2 - \frac{(-1)}{10}$$

$$\therefore x_2 = 2.1$$

Again,

$$x_{2+1} = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 2.1 - \frac{(9.261 - 4.2 - 5)}{3\times(2.1)^2 - 2}$$

$$x_3 = 2.1 - \frac{0.061}{11.23}$$

$$x_3 = 2.1 - 0.0054$$

$$\therefore x_3 = 2.0946 \text{ Ans}$$

$\therefore$ The third approximation x_3 is 2.0946 Ans

9 Evaluate $\int_0^{\infty} x^3\sqrt{1-x^4}\, dx$

Sol^n Given,

$$\int_0^{\infty} x^3\sqrt{1-x^4}\, dx$$

$$\lim_{h\to\infty} \int_0^{h} x^3\sqrt{1-x^4}\, dx$$

Let $z = 1-x^4$

$dz = -4x^3 dx$

$\frac{-dz}{4} = x^3 dx$

if $x=0$, $z = 1$

$x=h$, $z = 1-h^4$

Then,

$$\lim_{h\to\infty} \int_1^{(1-h^4)} \sqrt{z} \times \frac{-dz}{4}$$

$$= \lim_{h\to\infty} \left(-\frac{1}{4}\right)\left[\frac{z^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_1^{(1-h^4)}$$

$$= -\frac{1}{4} \lim_{h\to\infty} \frac{2}{3} \left[z^{3/2}\right]_1^{(1-h^4)}$$

$$= \frac{-1}{6} \lim_{h\to\infty} \left[(1-h^4)^{3/2} - 1\right]$$

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$= \frac{-1}{6}\left[(1-\infty)^{3/2}-1\right]$

$= \frac{-1}{6}\times\infty$

$= \infty$ Ans

10 Find the volume of the resulting Solid which in enclosed by the curve $y=x$ and $y=x^2$ is rotated about the x-axis

Sol Here,

Given curve are

$y=x$ --- (i)

$y=x^2$ --- (ii)

$\Rightarrow x=\sqrt{y}$ --- (ii)

Solving these curve to obtained the limiting value of y

$y=\sqrt{y}$

S.b.S

$y^2=y$

or, $y^2-y=0$

or, $y(y-1)=0$

Either or,

$y=0$ $y-1=0$

$\therefore y=0,1$

Hence,

$\text{Volume}(V) = \int_a^b \pi.(y-\sqrt{y})\,dy$

$$= \pi \int_0^1 (y - \sqrt{y})\, dy$$

$$= \pi \left\{ \left[\frac{y^2}{2} \right]_0^1 - \left[\frac{y^{1/2+1}}{1/2+1} \right]_0^1 \right\}$$

$$= \pi \left\{ \frac{1}{2} - \frac{2}{3} \left[y^{3/2} \right]_0^1 \right\}$$

$$= \pi \left\{ \frac{1}{2} - \frac{2}{3} \times (1 - 0) \right\}$$

$$= \pi \left(\frac{1}{2} - \frac{2}{3} \right)$$

$$= \pi \left(\frac{3-4}{6} \right)$$

$$= \frac{-\pi}{6}$$

$\therefore$ Thus, the volume of the solid is $\frac{\pi}{6}$ cubic unit.

11 Find the solution of $y'' + 4y' + 4 = 0$

Sol The given 2^{nd} order homogenous linear eqn is

$$y'' + 4y' + 4 = 0 \quad \text{--- (1)}$$

let

$y = e^{mx}$ be the reqn soln of eqn (1) we get an auxilarly eqn of (1) by replacing y'' by m^2 and y' by m

$$m^2 + 4m + 4 = 0$$

$$m^2 + 2 \cdot m \cdot 2 + (2)^2 = 0$$

$\cancel{(m+1)^2 = 0}$

$\therefore$ ~~m = -1, -1~~

$(m+2)^2 = 0$

$\therefore$ m = -2 which is real and equal

$\therefore$ the reqⁿ solⁿ is

$$y = e^{mx}(C_1 + C_2 x)$$

$$\therefore y = e^{-2x}(C_1 + C_2 x) \quad \text{Ans}$$

12. Determine whether the series $\sum_{n=1}^{\infty} \frac{n^2}{5n^2+4}$ converges or diverges

Sol Given series is

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{n^2}{5n^2+4}$$

where,

$$a_n = \frac{n^2}{5n^2+4}$$

$$a_n = \frac{n^2}{n^2(5+4/n^2)}$$

$$a_n = \frac{1}{\left(5+\frac{4}{n^2}\right)}$$

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Now,

$$\lim_{n\to\infty} a_n$$

$$\lim_{n\to\infty} \frac{1}{(5+4/n^2)}$$

$$= \frac{1}{5+4/\infty}$$

$$= \frac{1}{5+0}$$

$$= \frac{1}{5} \neq 0$$

$\therefore$ The given series is divergent by condition for divergency.

13 If $a = (4,0,3)$ and $b = (-2,1,5)$ find $|a|$, the vector $a-b$ and $2a+5b$

Sol Given,

$$\vec{a} = (4,0,3)$$
$$\vec{b} = (-2,1,5)$$

then,

$$|a| = \sqrt{4^2+0^2+3^2} = \sqrt{16+0+9} = \sqrt{25} = 5 \text{ Ans}$$

$$\vec{a}-\vec{b} = (4,0,3) - (-2,1,5)$$
$$= (4-(-2),\ 0-(1),\ 3-5)$$
$$= (4+2,\ -1,\ -2) = (6,-1,2) \text{ Ans}$$

$$2\vec{a}+5\vec{b} = 2(4,0,3) + 5(-2,1,5)$$
$$= (8,0,6) + (-10,5,25)$$
$$= (8+(-10),\ 0+5,\ 6+25) = (-2,5,31) \text{ Ans}$$

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14. Find $\frac{\delta z}{\delta x}$ and $\frac{\delta z}{\delta y}$ if z is defined as a function of x and y by the equation $x^3+y^3+z^3+6xyz=1$

Sol: Given,

$$x^3+y^3+z^3+6xyz=1 \quad \text{--- (1)}$$

diff^n eq^n (1) with respect to 'x'

$$3x^2+3z^2\frac{\delta z}{\delta x}+\frac{\delta(6x.y.z)}{\delta x}=0$$

or, $$3x^2+3z^2\frac{\delta z}{\delta x}+6yz+6xy\frac{\delta z}{\delta x}=0$$

or, $$\frac{\delta z}{\delta x}(3z^2+6xy)=-3x^2-6yz$$

$$\therefore \frac{\delta z}{\delta x}=\frac{3(-x^2-2yz)}{3(z^2+2xy)}=\frac{(-x^2-2yz)}{(z^2+2xy)} \text{ Ans}$$

Again,

diff^n eq^n (1) with respect to 'y'

$$3y^2+3z^2\frac{\delta z}{\delta y}+\frac{\delta(6y.x.z)}{\delta y}=0$$

or, $$3y^2+3z^2\frac{\delta z}{\delta y}+6xz+6xy\frac{\delta z}{\delta y}=0$$

$$\frac{\delta z}{\delta y}(3z^2+6xy)=-3y^2-6xz$$

$$\frac{\delta z}{\delta y}=\frac{3(-y^2-2xz)}{3(z^2+2xy)}=\frac{(-y^2-2xz)}{(z^2+2xy)} \text{ Ans}$$

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15. Find the extreme value of the function $f(x,y) = x^2 + 2y^2$ on the circle $x^2+y^2 = 1$

Sol^n Given,

$$f(x,y) = x^2 + 2y^2 \quad --- \text{(i)}$$
$$\nabla f = 2x + 4y$$

and

let

$$g(x,y) = x^2+y^2-1 = 0 \quad --- \text{(ii)}$$
$$\nabla g = 2x+2y$$

By method of lagrange's multiplier for some scalar λ

$$\nabla f = \lambda \nabla g$$

or, $2x+4y = \lambda(2x+2y)$

or, $2x+4y = 2x\lambda + 2y\lambda$

Comparing

$2x = 2x\lambda$

$2x - 2x\lambda = 0$

$2x(1-\lambda) = 0$

$\therefore x = 0, \ \lambda = 1$

$2y\lambda = 4y$

$2y\lambda - 4y = 0$

$2y(\lambda-2) = 0$

$\therefore y = 0, \ \lambda = 2$

if $x = 0$, then eq^n (ii) becomes

$$x^2+y^2 = 1$$
$$y^2 = 1$$
$$y = \pm 1$$

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If $y=0$, then eqn (ii) becomes

$$x^2+y^2=1$$
$$x^2=1$$
$$\therefore x=\pm 1$$

$\therefore (x,y) = (0,\pm 1)\ (\pm 1, 0)$

Point	$f = x^2+2y^2$
$(0,-1)$	2
$(0,1)$	2
$(-1,0)$	1
$(1,0)$	1

$\therefore$ Maximum at $(0,\pm 1)$ is 2
Minimum is 1 at $(\pm 1, 0)$ } Ans

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