Tribhuvan University
Institute of Science and Technology
2080
✡

Bachelor Level / First Year/ First Semester/ Science
Computer Science and Information Technology (MTH112)
(Mathematics I)
**(OLD COURSE)**

Full Marks: 80
Pass Marks: 32
Time: 3 hours.

*Candidates are required to give their answers in their own words as for as practicable.*
The figures in the margin indicate full marks.

Group A (3 × 10 = 30).

Attempt any THREE questions:

1. (a) If $f(x) = 3x^2 - x + 2$, then find $f(2)$, $f(-2)$, $f(a)$, $f(-a)$ and f(0). [5]
   (b) Find $\lim_{x \to -2} \frac{x^3 + 2x^2 - 1}{5 - 3x}$ [5]

2. (a) Find the derivative of $y = \frac{x^2 + x - 2}{x^3 + 6}$. [5]
   (b) Estimate the area between the parabola $y = x^2$ and the line $y = x$. [5]

3. (a) Verify the Mean Value Theorem for the function $f(x) = 3x^2 + 2x + 5$ $x \in [-1, 1]$. [4]
   (b) Define initial value problem. Solve the equation $xy' + 2y = 3x$, $y(1) = 2$. [4]
   (c) Find the volume of a sphere of radius $a$. [2]

4. (a) Find the local maximum and minimum values and saddle points of $f(x, y) = x^4 + y^4 - 4xy + 1$. [5]
   (b) Find the curvature of the parabola $y = x^2$ at the points $(0, 0)$, $(0, 1)$ and $(2, 4)$. [5]



Group B (10 × 5 = 50).

Attempt any TEN questions:

5. Verify Rolle's theorem for $f(x) = x^2 - 4$, $x \in [-2, 2]$. [5]

6. Find the Maclaurin series expansion of $\sin x$ at $x = 0$. [5]

7. If $f(x) = \sqrt{x - 2}$ and $g(x) = \sqrt{x}$, find $(fog)(x)$ and $(gof)(x)$. [5]

8. Show that the absolute value function $f(x) = |x|$ is continuous everywhere. [5]

9. Find $\int_0^{\frac{1}{2}} \frac{x}{\sqrt{1 - 4x^2}} dx$. [5]

10. Sketch the curve $y = x^3$. [5]

11. Find the solution of $y'' + 4y' + 4 = 0$, $y(0) = 2$, $y'(0) = 1$. [5]

12. Test the series $\sum_{n=1}^{\infty} \frac{n^2}{5n^2 + 4}$ diverges or converges. [5]

13. Define cross product of two vectors. If $\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}$ and $\vec{b} = 2\vec{i} + 3\vec{j} - 4\vec{k}$, find $\vec{a} \times \vec{b}$ and $\vec{b} \times \vec{a}$. [5]

14. Find the second partial derivatives of $f(x, y) = x^3 + x^2y^3 - 2y^2$. [5]

15. Find the length of the arc of the semi-cubical parabola $y^2 = x^3$ between the points $(1, 1)$ and $(4, 8)$. [5]

# 2080 (Old Syllable) Questions Solution

Subj:- Mathematics-I

— Pravin Gupta



1(a) Given

$$f(x) = 3x^2 - x + 2 \quad \text{---} (1)$$

find $f(2) = ?$ $f(-2) = ?$ $f(a) = ?$

$f(-a) = ?$ $f(0) = ?$

Now,

for $f(2) = ?$

if we put $x = 2$ in eq$^n$ (1)

$$f(2) = 3x^2 - x + 2$$
$$= 3\times 2^2 - 2 + 2$$
$$\therefore f(2) = 12$$



for $f(-2)$; put $x = -2$ then,

$$f(-2) = 3\times(-2)^2 - (-2) + 2$$
$$= 12 + 2 + 2$$
$$\therefore f(-2) = 16$$

for $f(a)$; put $x = a$ then

$$f(a) = 3\times a^2 - a + 2$$
$$\therefore f(a) = 3a^2 - a + 2$$

for $f(-a)$; put $x = -a$ then.

$$f(-a) = 3\times(-a)^2 - (-a) + 2$$
$$\therefore f(-a) = 3a^2 + a + 2$$

for $f(0)$; put $x = 0$ then,

$$f(0) = 3\times 0 - 0 + 2$$
$$\therefore f(0) = 2$$

Therefore,

$f(2) = 12$ $\qquad f(-a) = 3a^2 + a + 2$

$f(-2) = 16$ $\qquad f(0) = 2$

$f(a) = 3a^2 - a + 2$

Ans

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b Given,

$$\lim_{x \to -2} \frac{x^3 + 2x^2 - 1}{5 - 3x}$$

$$\frac{(-2)^3 + 2\times(-2)^2 - 1}{5 - 3\times(-2)}$$

$$= \frac{-8 + 8 - 1}{5 + 6}$$

$$= \frac{-1}{11}$$

$$\therefore \lim_{x \to -2} \frac{x^3 + 2x^2 - 1}{5 - 3x} = \frac{-1}{11} \quad \text{Ans}$$



2(a) Given,

$$y = \frac{x^2 + x - 2}{x^3 + 6} \quad \text{---} \; (1)$$

Diffⁿ eqⁿ (1) w. r. to 'x'

$$\frac{d}{dx}\left(\frac{x^2 + x - 2}{x^3 + 6}\right) = \frac{(x^3+6)\frac{d}{dx}(x^2+x-2) - (x^2+x-2)\frac{d}{dx}(x^3+6)}{(x^3+6)^2}$$

$$= \frac{(x^3+6)(2x+1) - (x^2+x-2)(3x)}{(x^3+6)^2}$$

$$= \frac{2x^4 + x^3 + 12x + 6 - 3x^3 - 3x^2 + 6x}{(x^3+6)^2}$$

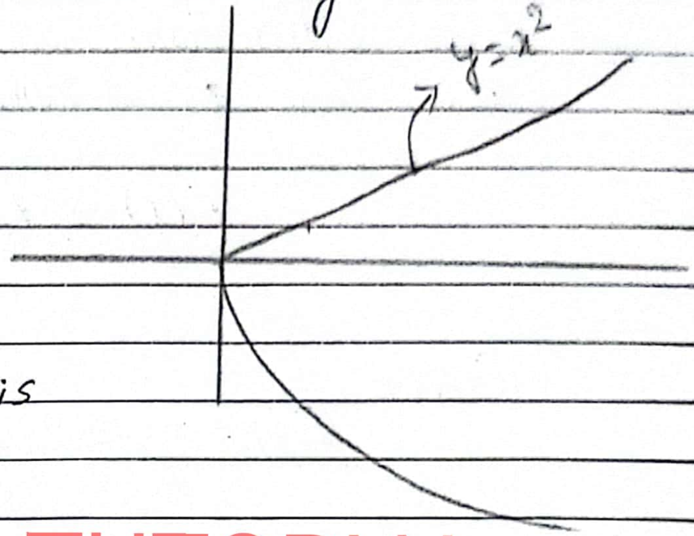
$$= \frac{2x^4 - 2x^3 - 3x^2 + 18x + 6}{(x^3+6)^2}$$

$\therefore$ The derivative of $y = \frac{x^2+x-2}{x^3+6}$ is $\frac{(2x^4 - 2x^3 - 3x^2 + 18x + 6)}{(x^3+6)^2}$

2(b) Given,

the parabola is $y = x^2$ —(i) and the line is $y = x$ —(ii)

Solving eq$^n$ (i) & (ii) we get limits of integration




$$x^2 = x$$

or, $x^2 - x = 0$

or, $x(x-1) = 0$

$\therefore$ $x = 0$ & $x = 1$

The area between the curves is

$$A = \int_a^b (f(x) - g(x))\,dx$$

$$= \int_0^1 (x^2 - x)\,dx$$

$$= \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_0^1$$

$$= \frac{1}{3} - \frac{1}{2}$$

$$= \frac{2-3}{6}$$

$$= \frac{-1}{6} \text{ squnit}$$

$\therefore$ The area can't be negative so $\frac{1}{6}$ squnit Ans

3 (a) Statement of Mean Value Theorem

let f be a function that Satisfies the following conditions.

(i) f is continuous on the closed interval $[a,b]$

(ii) f is differentiable on the open interval $(a,b)$

then there is a number c in $(a,b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b-a}$$



Given,

$$f(x) = 3x^2+2x+5 \quad x \in [-1,1]$$

(i) Since $f(x)$ is a polynomial function. so, it is continuous on the closed interval $[-1,1]$

(ii) $f'(x) = 6x+2$ which exists for all $x \in (-1,1)$

$\therefore$ $f(x)$ is differentiable on the open interval

All the condition of Mean Value Theorem are Satisfied hence, there exist a number C in $(-1,1)$ Such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$f'(x) = 6x+2$

$f'(c) = 6c+2$

$f(x) = 3x^2+2x+5$

$f(a) = f(1) = 3\times1^2+2\times1+5 = 10$

$f(b) = f(-1) = 3\times(-1)^2+2\times(-1)+5 = 3-2+5 = 6$

then,

$$6c+2 = \frac{10}{6}$$

or, $2(3c+1) = 2\left(\frac{5}{3}\right)$

or, $18c+6 = 5$

or, $18c = 5-6$

or, $18c = -1$

$\therefore c = \frac{-1}{18} \in (-1,1)$

Hence, $c = \frac{-1}{18} \in (-1,1)$

$\therefore$ Mean Value theorem verify.

Q.N;3(b)

Sol? $1^{st}$ part,

A differential equation together with initial condition (s) is called the initial value.

For example:-

$$\frac{d^2y}{dx^2} - \frac{2dy}{dx} + y = 0, \quad y(0) = -1, \quad y'(0) = 1$$

Here, $y(0) = -1$ and $y'(0) = 1$ is an initial condition.

3b 2nd Part,

Given

$$xy' + 2y = 3x \quad , \quad y(1) = 2$$

$$x\frac{dy}{dx} + 2y = 3x$$



dividing $x$ on bothside, we get

$$\frac{dy}{dx} + \frac{2}{x}\cdot y = 3 \quad \text{---} \textcircled{1}$$

Comparing $eq^n$ ① with $\frac{dy}{dx} + Py = Q$

where, $P = 2/x$, $Q = 3$

Now,

Integrating factor (I.F) $= e^{\int P dx} = e^{\int \frac{2}{x} dx} = e^{2\int \frac{1}{x} dx} = e^{2\log x} = x^2$

Multiplying $eq^n$ ① by I.F

$$x^2\left[\frac{dy}{dx} + \frac{2}{x}\cdot y\right] = 3x^2$$

$$d.(y.x^2) = 3x^2$$

on integrating both side

$$\int d(y.x^2) = \int 3x^2$$

or, $y.x^2 = 3\frac{x^3}{3}$

or, $y \times x^2 = \frac{3x^3}{3}$

$$y = \frac{x^3 + c}{x^2} \quad \text{---(ii)}$$

since,

$y(1) = 2$

$y = 2, \quad x = 1$

then eq$^n$ (ii) becomes

$$2 = \frac{1 + c}{1^2}$$

$$2 = 1 + c$$

$$\therefore c = 1$$

Substuting the value of c in eq$^n$ (ii)

$$y = \frac{x^3 + 1}{x^2}$$

or, $y = \frac{x^3}{x^2} + \frac{1}{x^2}$

$\therefore y = x + \frac{1}{x^2}$ Ans

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3 (c) Find the volume of Sphere of radius a

Sol Since, we know that intersection of sphere of radius 'a' and the plane is of a circle of radius 'a'

Then circle is

$$x^2+y^2=a^2$$
$$y^2=a^2-x^2 \quad \text{---} \; (1)$$

clearly the circle has end $x=-a$ to $x=a$ Now, the Volume of the solid that is generated by revolving the circle (1) about $x$-axis is (i.e $y=0$)



$$\text{Volume} = \pi\int_{-a}^{a} y^2\,dx$$
$$= \pi\int_{-a}^{a} (a^2-x^2)\,dx$$
$$= \pi\left[a^2x - \frac{x^3}{3}\right]_{-a}^{a}$$
$$= \pi\left\{\left(a^2\cdot a - \frac{a^3}{3}\right) - \left(a^2\cdot(-a) - \frac{(-a)^3}{3}\right)\right\}$$
$$= \pi\left\{\left(a^3-\frac{a^3}{3}\right) - \left(-a^3+\frac{a^3}{3}\right)\right\}$$
$$= \pi\left\{\frac{3a^3-a^3}{3} - \left(\frac{-3a^3+a^3}{3}\right)\right\}$$
$$= \pi\left(\frac{2a^3}{3}+\frac{2a^3}{3}\right) = \pi\times\frac{4a^3}{3} = \frac{4\pi a^3}{3} \text{ Ans}$$

$\therefore$ The volume of the sphere of radius 'a' is $\frac{4\pi a^3}{3}$ Ans

Q.N;4(a)

Sol<sup></sup>

Sol$^n$ Given Function,

$$f(x,y) = x^4 + y^4 - 4xy + 1$$

so,

$f_x = 4x^3 - 4y$ $\quad$ $f_y = 4y^3 - 4x$

$f_{xx} = 12x^2$ $\quad$ $f_{yy} = 12y^2$

$f_{xy} = -4$ $\quad$ $f_{yx} = -4$

For stationary point

$f_x = 0$ $\quad$ $f_y = 0$

$4x^3 - 4y = 0$ $\quad$ $4y^3 - 4x = 0$

$x^3 = y$ --- (i) $\quad$ $y^3 = x$ ---- (ii)



from eq$^n$ (i) and (ii)

$(x^3)^3 = x$ $\quad (\because y = x^3)$

or, $x^9 = x$

or, $x^9 - x = 0$

or, $x(x^8 - 1) = 0$

$\therefore x = 0, x = 1$

substitute the ~~point~~ value of y in (i)

when $x = 0, y = 0$

" $x = 1, y = 1$

At point (0,0)

$f_{xx} = 0$

$f_{yy} = 0$

Then,

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$$

$$= \begin{vmatrix} 0 & -4 \\ -4 & 0 \end{vmatrix} = 0 - 16 = -16 < 0$$

so, it has saddle point at (0,0)



At point (1,1)

$f_{xx} = 12$ , $f_{yy} = 12$

$$D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yx} \end{vmatrix} = \begin{vmatrix} 12 & -4 \\ -4 & 12 \end{vmatrix} = 144 - 16$$

$$= 128 > 0$$

so, it has minimum value at ~~(1,1)~~ (1,1)

$\therefore$ Minimum value is

$$f(1,1) = 1^4 + 1^4 - 4 \times 1 \times 1 + 1$$

$$= 1 + 1 - 4 + 1$$

$$= -1$$

4b Given,

Parabola is $y = x^2$ --- (i)

and the points are $(0,0)$, $(0,1)$ & $(2,4)$

For curvature we are using formula as;

$$K = \frac{1}{|\vec{v}|}\left|\frac{d\vec{T}}{dt}\right| \quad \text{--- (ii)}$$

diffⁿ eqⁿ (i) w.r. to 'x'

i.e

$$\vec{v} = \frac{d\vec{y}}{dx} = 2x \quad \text{--- (iii)}$$

$$|\vec{v}| = \sqrt{2^2} = 2$$

| at (0,0) | at (0,1) | at (2,4) |
|---|---|---|
| $\vec{v} = 0$ | $\vec{v} = 0$ | $\vec{v} = 2 \times 2 = 4$ |
| $\lvert\vec{v}\rvert = 0$ | $\lvert\vec{v}\rvert = 0$ | $\lvert\vec{v}\rvert = \sqrt{4^2} = 4$ |



Again, diffⁿ eqⁿ (iii) w.r. to 'x'

$$a(x) = 2$$

then,

at (2,4) $K = \frac{1}{4} \times 2 = \frac{1}{2}$

at (0,0) & (0,1)

$$K = \frac{1}{0} \times 2$$

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5. Given,

$$f(x) = x^2 - 4, \quad x \in [-2,2]$$

(i) Since $f(x)$ is a polynomial function. So, it is continuous on the closed interval $[-2,2]$

(ii) $f'(x) = 2x$ which exists for all $x \in (-2,2)$

$\therefore f(x)$ is differentiable in $(-2,2)$



(iii) $f(-2) = (-2)^2 - 4 = 0$

$f(2) = 2^2 - 4 = 0$

$\therefore f(-2) = f(2)$

$\therefore$ All the condition of Rolle's theorem are satisfied Hence there exists at least a point $c \in (0,3)$ such that

$$f'(c) = 0$$

or, $2c = 0$

$\therefore c = 0 \in (-2,2)$

Hence, Rolle's theorem verified an $0 \in (-2,2)$ Ans

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Sol^n

The given Function is Q.N;6

$$f(x) = \sin x \quad \text{---} (1)$$

we have to Find the Taylor's series generated by the Function $f(x) = \sin x$ at $x = a = 0$

· diff eq^n (1) w.r.to 'x' successively we get·

$f(x) = \sin x$
$f'(x) = \cos x$
$f''(x) = -\sin x$
$f'''(x) = -\cos x$
$f^{iv}(x) = \sin x$
$f^{v}(x) = \cos x$
$f^{vi}(x) = -\sin x$
$\vdots$

rearrangement all derivative as below.

| | |
|---|---|
| $f'(x) = \cos x$ | $f''(x) = -\sin x$ |
| $f'''(x) = -\cos x$ | $f^{iv}(x) = \sin x$ |
| $f^{v}(x) = \cos x$ | $f^{vi}(x) = -\sin x$ |
| $\vdots$ | $\vdots$ |
| $f^{2n+1}(x) = (-1)^n \cos x$ | $f^{2n}(x) = (-1)^n \sin x$. |



at $x=0$

| | |
|---|---|
| $f'(0) = \cos 0 = 1$ | $f''(0) = -\sin 0 = 0$ |
| $f'''(0) = -\cos 0 = -1$ | $f^{iv}(0) = \sin 0 = 0$ |
| $f^{v}(0) = \cos 0 = 1$ | $f^{vi}(0) = -\sin 0 = 0$ |
| $\vdots$ | |
| $f^{2n+1}(0) = (-1)^n$ | $f^{2n}(0) = 0$ |

we know,

$f(x) = f(a) + \frac{f'(a)}{1!}(x-a)^1 + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots + n^{th}\text{term} + \dots$

$\sin x = 0 + \frac{1}{1!}(x-0)^1 + 0 + \frac{(-1)}{3!}x^3 + \dots + n^{th}\text{term} + \dots$

$\sin x = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \dots (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \dots$ #

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7\. Given,

$$f(x) = \sqrt{x-2} \quad \& \quad g(x) = \sqrt{x}$$

then,

$$fog(x) = ? \qquad gof(x) = ?$$

we know,

$$fog(x) = f(g(x))$$
$$= f(\sqrt{x})$$
$$= \sqrt{\sqrt{x}-2}$$

$$\therefore fog(x) = \sqrt{\sqrt{x}-2}$$

Again for gof(x)



$$gof(x) = g(f(x))$$
$$= g(\sqrt{x-2})$$
$$= \sqrt{\sqrt{x-2}}$$

$$\therefore gof(x) = \sqrt{\sqrt{x-2}}$$

$$\left.\begin{aligned} \therefore fog(x) &= \sqrt{\sqrt{x}-2} \\ gof(x) &= \sqrt{\sqrt{x-2}} \end{aligned}\right\} \text{Ans}$$

8

A function $f(x)$ is continuous at a point $x=a$ if the following conditions are satisfied.

(i) $f(a)$ is defined

(ii) $\lim_{x \to a} f(x)$ exists

(iii) $\lim_{x \to a} f(x) = f(a)$



For the absolute value function $f(x) = |x|$

(i) $f(a)$ is defined for all real numbers

(ii) The limit of the function as $x$ approaches $a$ exists for all real numbers $a$. since the function is piecewise defined as $f(x) = x$ for $x \geq 0$ and $f(x) = -x$ for $x < 0$, the limit from both the right and the left as $x$ approaches $a$ is $a$.

(iii) The limit of the function as $x$ approaches $a$ is equal to the value of the function at $x=a$ for all real numbers $a$. This is true because the function's value at $x=a$ is either $a$ or $-a$, depending on the sign of $a$.

$\therefore$ The absolute value satisfies all three conditions for continuity at every point so, it is continuous everywhere on the real number.

$\therefore$ $f(x) = |x|$ is continuous everywhere.

9. Given,

$$\int_0^{1/2} \frac{x}{\sqrt{1-4x^2}}\,dx$$

let,

$$y = 1-4x^2$$
$$dy = -8x\,dx$$
$$\frac{-1}{8}dy = x\,dx$$

then,

$$\int_0^{1/2} \frac{-1/8\,dy}{\sqrt{y}} = \frac{-1}{8}\int_0^{1/2} \frac{dy}{\sqrt{y}} = \frac{-1}{8}\int_0^{1/2} y^{-1/2}\,dy$$

$$= \frac{-1}{8}\left[\frac{y^{-1/2+1}}{-1/2+1}\right]_0^{1/2}$$



$$= \frac{-1}{8} \times \frac{2}{1}\left[y^{1/2}\right]_0^{1/2}$$

$$= \frac{-1}{4}\left[\sqrt{y}\right]_0^{1/2}$$

$$= -\frac{1}{4}\left[\sqrt{1-4x^2}\right]_0^{1/2}$$

$$= \frac{-1}{4}\left[\left(\sqrt{1-4\times(1/2)^2}\right) - \sqrt{1-0}\right)$$

$$= \frac{-1}{4}\left(\sqrt{0} - 1\right)$$

$$= \frac{1}{4}$$ Ans

10 Given,

$f(x) = y = x^3 \quad \text{---} \; (1)$

(i) Domain:-

For domain; The function exists for all $x$. So, the domain is

$$\text{Domain} = (-\infty, \infty)$$

diff$^n$ eq$^n$(1) w.r.to '$x$'

$$f'(x) = 3x^2$$

for critical point

$$f'(x) = 0$$
$$3x^2 = 0$$
$$\therefore x = 0$$



| Interval | $(-\infty, 0)$ | $(0, \infty)$ |
|---|---|---|
| sign of $f'(x)$ | +ve | +ve |
| Nature of $f$ | increasing | increasing |

Thus, the ~~maxima~~ minima occur at $x = 0$

$f(0) = 0$ is ~~maximum~~ minimum point

Again,

$$f''(x) = 6x$$

for Point of inflection,

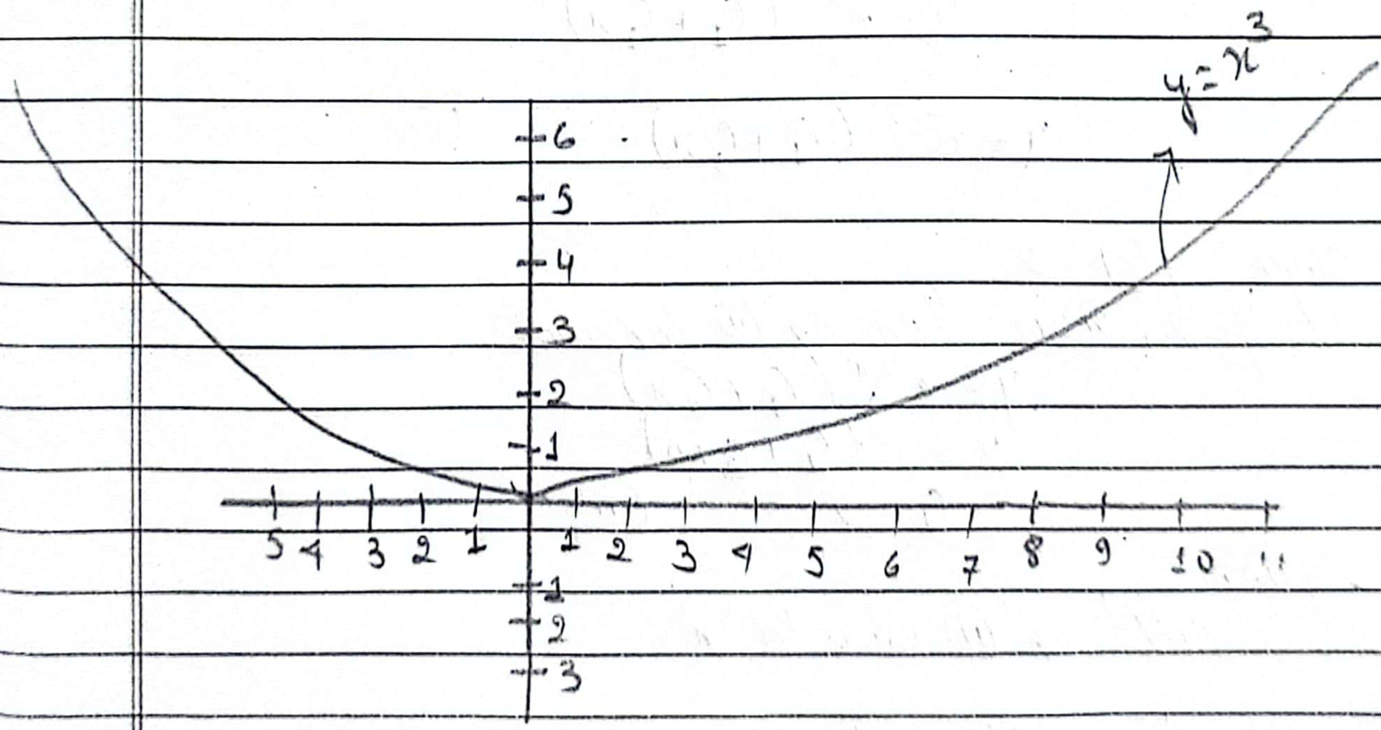
$$f''(x) = 0$$
$$6x = 0$$
$$\therefore x = 0$$

| Interval | $(-\infty, 0)$ | $(0, \infty)$ |
|---|---|---|
| sign of $f''$ | $-ve$ | $+ve$ |
| Nature of $f$ | Concave down | Concave up |

Thus, point of inflection occur at $x=0$.
so,
$f(0) = 0$ is the point of inflection.

Summarizing above two tables,

| $(-\infty, 0)$ | $(0, \infty)$ |
|---|---|
| ~~+ve~~ Increasing | increasing |
| Concave down | concave up |



11 The given $2^{nd}$ order homogenous linear $eq^n$ is

$$\cancel{y''+4y'+4y=0} \text{ ——— } (I)$$

$$y''+4y'+4y=0 \text{ - - - - } (I)$$

Let

$y=e^{mx}$ be the $req^n$ $sol^n$ of $eq^n$ (I)

We get an auxilarly $eq^n$ of (I) by replacing $y''$ by $m^2$, $y'$ by $m$ and $y$ by $1$.

Then,

$$m^2+4m+4=0$$

$$(m+2)^2=0$$

$$m=-2 \text{ which is real \& equal.}$$

The $req^n$ $sol^n$ is

$$y=e^{mx}(C_1+C_2x)$$

$$y=e^{-2x}(C_1+C_2x) \text{ - - - - } (II)$$



Since, $y(0)=2$

if $y=2$, $x=0$ then $eq^n$ (II) becomes

$$y=e^{-2x}(C_1+C_2x)$$

$$2=e^{0}(C_1+C_2\times 0)$$

$$C_1=2 \text{ - - - } (III)$$

Again,

diff$^n$ $eq^n$ (II) w.r. to '$x$'

$$y'=e^{-2x}(0+C_2)+(C_1+C_2x)(-2)\times e^{-2x}$$

$$y'=e^{-2x}\times C_2-2e^{-2x}(C_1+C_2x) \text{ ——— } (IV)$$

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Since, $y'(0)=1$
if $y'=1$, $x=0$ then eq$^n$ (iv) becomes,

$$y' = e^{-2x}C_2 - 2e^{-2x}(C_1+C_2x)$$

or, $1 = e^0 \times C_2 - 2\times e^0 (C_1 + C_2\times 0)$
or, $1 = C_2 - 2\times C_1$ (from eq$^n$ (iii))
or, $1 = C_2 - 2\times 2$
or, $C_2 = 1+4$
$\therefore C_2 = 5$

Substuting the value of $C_1$ & $C_2$
then eq$^n$ (ii) becomes.

$$y = e^{-2x}(2+5x)$$

Ans

Given Series is Q.N;12

$$\sum_{n=1}^{\infty} \frac{n^2}{5n^2+4}$$

i.e $\sum_{n=1}^{\infty} a_n$

where,

$$a_n = \frac{n^2}{5n^2+4}$$

Now,

$$\lim_{n\to\infty} \frac{n^2}{5n^2+4}$$

$$= \lim_{n\to\infty} \frac{n^2}{n^2(5+4/n^2)}$$



$$= \lim_{n\to\infty} \frac{1}{5+4/n^2}$$

$$= \frac{1}{5+4/\infty}$$

$$= \frac{1}{5+0}$$

$$= \frac{1}{5}$$

$\therefore$ The Given Series $\sum_{n=1}^{\infty} \frac{n^2}{5n^2+4}$ is divergent by $n^{th}$ Root Test.

Sol

Q.N;13

1st part

let $\vec{a}$ and $\vec{b}$ are two non-zero vectors. Then the cross product of $\vec{a}$ and $\vec{b}$ is denoted by $\vec{a}\times\vec{b}$ and is defined as

$$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$$

where $\theta$ be the angle between $\vec{a}$ and $\vec{b}$ and $\hat{n}$ is unit vector along $(\vec{a}\times\vec{b})$

13 $2^{nd}$ Part,

Given,

$$\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}$$
$$\vec{b} = 2\vec{i} + 3\vec{j} - 4\vec{k}$$

Then,

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 3 \\ 2 & 3 & -4 \end{vmatrix}$$

$$= \vec{i}(-8-9) - \vec{j}(-4-6) + \vec{k}(3-4)$$
$$= -17\vec{i} + 10\vec{j} - 1\vec{k}$$



And we know,

$$(\vec{a} \times \vec{b}) = -(\vec{b} \times \vec{a})$$

So,

$$\vec{b} \times \vec{a} = -(\vec{a} \times \vec{b})$$
$$= -\{-17\vec{i} + 10\vec{j} - 1\vec{k}\}$$
$$= 17\vec{i} - 10\vec{j} + 1\vec{k}$$

$$\left.\begin{aligned} \therefore (\vec{a} \times \vec{b}) &= (-17\vec{i} + 10\vec{j} - 1\vec{k}) \\ (\vec{b} \times \vec{a}) &= (17\vec{i} - 10\vec{j} + 1\vec{k}) \end{aligned}\right\} \text{Ans}$$

14 Given,

$$f(x,y) = x^3 + x^2y^3 - 2y^2 \quad \text{---} (I)$$

diffⁿ eqⁿ (I) w.r.to 'x' partially,

$$\frac{\delta f}{\delta x} = 3x^2 + 2xy^3 \quad \text{Ans} \quad \text{---} (II)$$

diffⁿ eqⁿ (I) w.r.to 'y' partially

$$\frac{\delta f}{\delta y} = 3y^2x^2 - 4y \quad \text{Ans} \quad \text{---} (III)$$

diffⁿ eqⁿ (II) w.r.to 'x' partially

$$\frac{\delta^2 f}{\delta x^2} = 6x + 2y^3 \quad \text{Ans}$$



diffⁿ eqⁿ (III) w.r.to 'y' partially

$$\frac{\delta^2 f}{\delta y^2} = 6yx^2 - 4 \quad \text{Ans}$$

diffⁿ eqⁿ (II) w.r.to 'y' partially

$$\frac{\delta^2 f}{\delta x \delta y} = 6xy^2 \quad \text{Ans}$$

diffⁿ eqⁿ (III) w.r.to 'x' partially

$$\frac{\delta^2 f}{\delta y \delta x} = 6xy^2 \quad \text{Ans}$$

Q.N;15

Sol: Given

$$y^2 = x^3$$

~~S.b.s~~ taking root on bothsides

$$\sqrt{y^2} = \sqrt{x^3}$$

or, $y = \sqrt{x^3}$

$\therefore y = x^{3/2}$ ---- (1)

diff$^n$ eq$^n$(1) w.r.to $x$

$$\frac{dy}{dx} = \frac{3}{2} x^{3/2 - 1}$$

$$\therefore \frac{dy}{dx} = \frac{3}{2} x^{1/2}$$

Now, using arc length formula,

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$



$$L = \int_1^4 \sqrt{1 + \left(\frac{3}{2} x^{1/2}\right)^2}\, dx$$

$$L = \int_1^4 \sqrt{1 + \frac{9}{4} x^{1/2 \times 2}}\, dx$$

$$\int_1^4 \sqrt{1 + \frac{9}{4} x}\, dx.$$

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Let

$$z = 1 + \frac{9}{4}x$$

$$dz = \frac{9}{4}dx \Rightarrow \frac{4}{9}dz = dx$$

changing limit

$\Rightarrow$ if $x = 1$, $z = 1 + \frac{9}{4} = \frac{13}{4}$

if $x = 4$, $z = 1 + \frac{9}{4} \times 4 = 10$

Now,

$$L = \int_{\frac{13}{4}}^{10} \sqrt{z} \times \frac{4}{9} dz = \int_{\frac{13}{4}}^{10} (z)^{\frac{1}{2}} \times \frac{4}{9} dz$$

$$= \frac{4}{9} \left[ \frac{z^{\frac{1}{2}+1}}{\frac{1}{2}+1} \right]_{\frac{13}{4}}^{10}$$

$$= \frac{4}{9} \times \frac{2}{3} \left[ z^{3/2} \right]_{13/4}^{10}$$

$$= \frac{8}{27} \left[ 10^{3/2} - \left(\frac{13}{4}\right)^{3/2} \right)$$

$$= \frac{1}{27} \left( 8 \times \sqrt{10^3} - 8 \times \sqrt{\left(\frac{13}{4}\right)^3} \right)$$

$$= \frac{1}{27} \left( 8 \times 10\sqrt{10} - \cancel{8}^{2} \times \frac{13}{\cancel{4}} \sqrt{\frac{13}{4}} \right)$$

$$= \frac{1}{27} \left( 80\sqrt{10} - 26\sqrt{\frac{13}{4}} \right)$$ Ans

or,

$$= \frac{1}{27} \left( 80\sqrt{10} - \cancel{26}^{13} \times \frac{1}{\cancel{2}} \sqrt{13} \right) \Rightarrow \frac{1}{27} \left( 80\sqrt{10} - 13\sqrt{13} \right)$$ Ans