Tribhuvan University
Institute of Science and Technology
2080
✡

Bachelor Level / First Year/ First Semester/ Science
Computer Science and Information Technology (MTH117)
(Mathematics I)
(NEW COURSE)

Full Marks: 60
Pass Marks: 24
Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.
The figures in the margin indicate full marks.

Section A (2 × 10 = 20).

Attempt any TWO questions:

1. (a) If $\vec{a} = (4, 0, 3)$ and $\vec{b} = (-2, 1, 5)$, find $|\vec{a}|, 3\vec{b}, \vec{a} + \vec{b}$ and $2\vec{a} + 5\vec{b}$. [1+1+1+2]
(b) Estimate the value of $\lim_{x \to 0} \frac{\sqrt{x^2+9}-3}{x^2}$ [5]

2. (a) The area of the parabola $y = x^2$ from $(1, 1)$ to $(2, 4)$ is rotated about the y-axis. Find the area of the resulting surface. [5]
(b) Find the solution of the equation $y^2dy = x^2dx$ that satisfies the initial condition y(0)= 2. [5]

3. (a) As dry air moves upward, it expands and cools. If the ground temperature is 20^0C and the temperature at height of $1\,km$ is 10^0C, express the temperature T(in ^0C) as a function of the height h (in kilometer), assuming that linear model is appropriate.
(b) Draw a graph of the function in part (a). What does the slope represent?
(c) What is the temperature at a height of $2.5\,km$? [5+3+2]

Section B (8 × 5 = 40).

Attempt any EIGHT questions:

4. Integrate $\int_0^1 x^2\sqrt{x^3+1}\,dx$. GUPTA TUTORIAL [5]
5. Find the Maclaurin series expansion of $f(x) = e^x$ at $x = 0$. [5]
6. Find where the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is increasing and where it is decreasing. [5]
7. Find y' if $x^3 + y^3 = 6xy$. [5]
8. Show that $y = x - \frac{1}{x}$ is a solution of the differential equation $xy' + y = 2x$. [5]
9. Sketch the graph and find the domain and range of the function $f(x) = 2x - 1$. [5]
10. Determine whether the series $\sum_{n=1}^{\infty} \frac{n^2}{5n^2+4}$ converges or diverges. [5]
11. If $f(x, y) = x^3 + x^2y^3 - 2y^2$, find $f_x(2, 1)$ and $f_y(2, 1)$. [5]
12. Show that the function $f(x) = x^2 + \sqrt{7-x}$ is continuous at $x = 4$. [5]

2080 (New Course) Questions Solution

Subj:- Mathematics-I

— Pravin Gupta

GOOD MORNING
PAGE NO.
DATE: / /

1 (a) Give two vectors are

$\vec{a} = (4, 0, 3)$ &

$\vec{b} = (-2, 1, 5)$

find

$|\vec{a}| = ?$

$3\vec{b} = ?$

$\vec{a} + \vec{b} = ?$

$2\vec{a} + 5\vec{b}$

Now,

for $|\vec{a}| = \sqrt{(4)^2 + (0)^2 + (3)^2}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

$\therefore |\vec{a}| = 5$ Ans

for $3\vec{b} = ?$

we know

$\vec{b} = (-2, 1, 5)$

then

$3\vec{b} = 3(-2, 1, 5)$

$\therefore 3\vec{b} = (-6, 3, 15)$ Ans

for $\vec{a} + \vec{b} =$

we know,

$\vec{a} = (4, 0, 3)$ & $\vec{b}\ (-2, 1, 5)$

then $\vec{a} + \vec{b} = (4, 0, 3) + (-2, 1, 5)$

$= (4 + (-2), 0 + 1, 3 + 5)$

$\therefore \vec{a} + \vec{b} = (2, 1, 8)$ Ans

for $2\vec{a} + 5\vec{b} = 2(4, 0, 3) + 5(-2, 1, 5)$

$= (8, 0, 6) + (-10, 5, 25)$

$= (8 + (-10)\ 0 + 5\ 6 + 25)$

$\therefore 2\vec{a} + 5\vec{b} = (-2, 5, 31)$ Ans

GOOD MORNING
PAGE NO.:
DATE: / /

b. Given,

$$\lim_{x \to 0} \frac{\sqrt{x^2+9} - 3}{x^2}$$

Here in this equation if we put $x=0$ then we obtain $\frac{0}{0}$ which is indeterminant. So, we use L-Hospital use.

$$\lim_{x \to 0} \frac{\sqrt{x^2+9} - 3}{x^2}$$

$$= \lim_{x \to 0} \frac{\frac{1}{2}(x^2+9)^{\frac{1}{2}-1} \times 2x - 0}{2x}$$

$$= \lim_{x \to 0} \frac{\frac{1}{2}(x^2+9)^{-\frac{1}{2}} \times \cancel{2x}}{\cancel{2x}}$$

$$= \lim_{x \to 0} \frac{1}{2}(x^2+9)^{-\frac{1}{2}}$$

$$= \lim_{x \to 0} \frac{1}{2\sqrt{x^2+9}}$$

If we put $x=0$ then, $\frac{1}{2\sqrt{0+9}}$

$$= \frac{1}{2 \times 3}$$

$$= \frac{1}{6}$$

$\therefore$ The value of $\lim_{x \to 0} \frac{\sqrt{x^2+9} - 3}{x^2} = \frac{1}{6}$ Ans

2(a). Given

The area of the parabola $y = x^2$ --- (i)

$\cancel{x = \sqrt{y}}$

And the points are

(1,1) to (2,4)

we have formula

$$A = \int 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \quad --- (ii)$$

for limiting we have to ~~po~~ choose 1 & 2 as x-axis point

for $\frac{dy}{dx} \Rightarrow$

$$y = x^2$$

$$\frac{dy}{dx} = 2x$$

GUPTA TUTORIAL

then eq^n (ii) becomes

$$A = \int_1^2 2\pi x \sqrt{1 + (2x)^2}\, dx$$

$$A = 2\pi \int_1^2 x\sqrt{1 + 4x^2}\, dx \quad --- (iii)$$

let

$u = 1 + 4x^2$ at $x = 1$, $u = 1 + 4 = 5$

$du = 8x\,dx$ at $x = 2$, $u = 1 + 4 \times 2^2 = 17$

$\frac{1}{8} du = x\,dx$ then eq^n (iii) becomes

$$A = 2\pi \int_5^{17} \frac{1}{8} \sqrt{u}\, du$$

$$A = \frac{\pi}{4} \int_5^{17} \sqrt{u}\, du$$

$$= \frac{\pi}{4}\left[\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_5^{17}$$

$$= \frac{\pi}{4}\left[\frac{u^{3/2}}{3/2}\right]_5^{17}$$

$$= \frac{\pi}{\cancel{4}\,2} \times \frac{\cancel{2}}{3}\left[u^{3/2}\right]_5^{17}$$

$$= \frac{\pi}{6}\left[(17)^{3/2} - (5)^{3/2}\right]$$

$$= \frac{\pi}{6}\left(17\sqrt{17} - 5\sqrt{5}\right)$$ Ans

$\therefore$ The area of the resulting surface is $\frac{\pi}{6}\left(17\sqrt{17} - 5\sqrt{5}\right)$ Ans

9(b) Given equation is

$$y^2 dy = x^2 dx \quad \cdots (i)$$

$$\frac{dy}{dx} = \frac{x^2}{y^2} \quad \cdots (i)$$

initial condition $y(0) = 2$

from eq^n (i)

$$y^2 dy = x^2 dx$$

on integrating bothside

$$\int y^2 dy = \int x^2 dx$$

$$\frac{y^3}{3} = \frac{x^3}{3} + \frac{C}{3} \quad \cdots (ii)$$

we have $y(0) = 2$

i.e $x = 0, y = 2$

then eq^n (ii) becomes

$$\frac{2^3}{3} = 0 + C$$

$$\therefore C = \frac{8}{3}$$

∴ Substute the value of C in eq^n (ii). we get

$$\frac{y^3}{3} = \frac{x^3}{3} + \frac{8}{3}$$

$$\therefore y^3 = x^3 + 8$$

$$\therefore y = \sqrt[3]{x^3 + 2}$$

$$\therefore y = \sqrt[3]{x^3 + 2} \quad \text{Ans}$$

3 (a) Given.

Ground Temperature $= 20°C$

Temperature at height $1km = 10°C$

Height $= 1km$

Consider the linear model for this, temperature and height are independent

$$T = mh + b \quad \text{---} \quad (i)$$

Since we know, at ground level $h = 0$

so, $T = m \times 0 + b$

$20 = 0 + b$

$\therefore b = 20$

then eq^n (i) becomes

$$T = mh + 20 \quad \text{---} \quad (ii)$$

Also given $h = 1km$, $T = 10°C$ then eq^n (ii) becomes

$T = mh + 20$

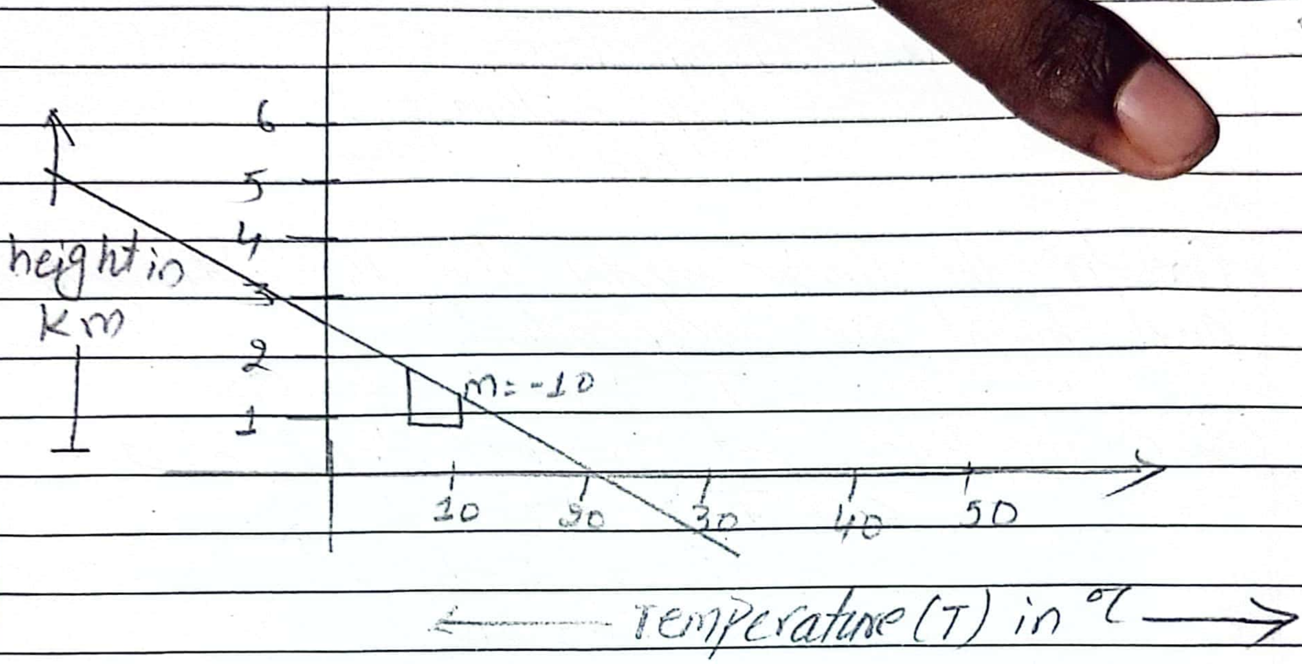
or, $10 = m \times 1 + 20$

$\therefore m = -10$

Therefore, appropriate linear model is

$$T = -10h + 20 \quad \text{---} \quad (iii) \text{ is req}^n \text{ model}$$

(b) For the given of the function



The slope ($m = -10$) represent, if height is increased by 1km then the temperature is decreased by $-10^{\circ}C$

(c) Given height = 2.5km , T = ?

from eq^n (iv)

$$T = -10h + 20$$
$$T = -10 \times (2.5) + 20$$
$$T = -25 + 20$$
$$T = -5^{\circ}C$$

∴ Therefore at 2.5km temperature becomes $-5^{\circ}C$ Ans

4 Given

$$\int_0^1 x^2\sqrt{x^3+1}\;dx \quad \cdots (1)$$

let ~~$u = \sqrt{x^3+1}$~~ $\quad u = x^3+1 \quad \cdots (ii)$

~~$du =$~~ $\quad du = 2x^2\,dx$

$$\frac{1}{2}du = x^2\,dx$$

~~Then,~~ if $x = 0$, then eqⁿ (1) becomes $u = 1$

if $x = 1$ " " " " $u = 2$

then eqⁿ (1) becomes

$$\int_1^2 \sqrt{u}\;\frac{1}{2}du$$

$$= \frac{1}{2}\left[\frac{u^{1/2+1}}{1/2+1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{u^{3/2}}{3/2}\right]_1^2$$

$$= \frac{1}{\cancel{2}} \times \frac{\cancel{2}}{3}\left[u^{3/2}\right]_1^2$$

$$= \frac{1}{3}\left(2^{3/2} - 1^{3/2}\right)$$

$$= \frac{1}{3}\left(2\sqrt{2} - 1\right)$$

$$\therefore \int_0^1 x^2\sqrt{x^3+1}\;dx = \frac{1}{3}\left(2\sqrt{2}-1\right) \quad \text{Ans}$$

Sol

The given function is

$$f(x) = e^x \quad \text{----} \; (1)$$

Q.N;5

we have to find a Maclaurin series generated by the function $f(x) = e^x$ for this we need to find Taylor's series generated by the function

$$f(x) = e^x \text{ about } x = 0 \quad (\text{i.e } x = a = 0)$$

diff^n eq^n (1) successively,

$$\begin{aligned}
f(x) &= e^x & f(0) &= e^0 = 1 \\
f'(x) &= e^x & f'(0) &= e^0 = 1 \\
f''(x) &= e^x & f''(0) &= e^0 = 1 \\
f'''(x) &= e^x \quad \xrightarrow{\text{at } x=0} & f'''(0) &= e^0 = 1 \\
f^{iv}(x) &= e^x & f^{iv}(0) &= e^0 = 1 \\
&\vdots & &\vdots \\
f^n(x) &= e^x & f^n(0) &= e^0 = 1 \\
&\vdots & &
\end{aligned}$$

Date

Page

we know that,

the req^n Maclaurian Series is

$$f(x) = f(0) + \frac{f'(0)}{1!}(x-0)^1 + \frac{f''(0)}{2!}(x-0)^2 + \cdots + \frac{f^n(0)(x-0)^n}{n!}$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!} + \cdots$$

Hence, it is Maclaurian series of e^x

GOOD MORNING
PAGE NO. :
DATE : / /

6 Given,

$$f(x) = 3x^4 - 4x^3 - 12x^2 + 5 \quad --- (1)$$

for increasing and decreasing we have to diff^n eq(1) w. r. to 'x'

$$f'(x) = 12x^3 - 12x^2 - 24x$$

$$f'(x) = 0$$

$$12x^3 - 12x^2 - 24x = 0$$

$$12x(x^2 - x - 2) = 0$$

Either $x = 0$ or, $x^2 - x - 2 = 0$

$$x^2 - x(2-1) - 2 = 0$$

$$x^2 - 2x + x - 2 = 0$$

$$x(x-2) + 1(x-2) = 0$$

$$\therefore x = -1, x = 2$$

$$\therefore x = 0, -1, 2$$

Interval	$(-1, 0)$	$(0, 2)$
sign of $f'(x)$	+ve	−ve
Nature of $f(x)$	Increasing	Decreasing

Increasing at $(-1, 0)$ &
Decreasing at $(0, 2)$

GOOD MORNING
PAGE NO.
DATE: / /

7 Given,

$$x^3 + y^3 = 6xy \quad \cdots (1)$$

Diffⁿ eqⁿ (1) w.r. to 'x'

or, $3x^2 + 3y^2\frac{dy}{dx} = 6x\cdot\frac{dy}{dx} + 6y \times 1$

or, $3x^2 + 3y^2\frac{dy}{dx} = 6x\frac{dy}{dx} + 6y$

or, $3y^2\frac{dy}{dx} - 6x\frac{dy}{dx} = 6y - 3x^2$

or, $\cancel{3}\frac{dy}{dx}\left(y^2 - 2x\right) = \cancel{3}(2y - \cancel{3}x^2)$

$\therefore \frac{dy}{dx} = \frac{2y - x^2}{y^2 - 2x}$

$\therefore \frac{dy}{dx} = \frac{(2y - x^2)}{(y^2 - 2x)}$ Ans

GOOD MORNING
PAGE NO. :
DATE : / /

8 Here,

$$xy' + y = 2x$$

$$x\frac{dy}{dx} + y = 2x$$

dividing by x on both side

$$\frac{dy}{dx} + \frac{1}{x}\cdot y = 2 \quad \text{---} (1)$$

Now, eq^n (1) is in linear and comparing with

$$\frac{dy}{dx} + Py = Q$$

where, $P = \frac{1}{x}$, $Q = 2$

Now,

$$I.F = e^{\int p\,dx} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$$

Then,

$$y \times I.F = \int Q \times I.F\, dx$$

or, $y \times x = \int 2 \times x\, dx$

or, $y \times x = \frac{2x^2}{2} + c$

$\therefore \; yx = x^2 + c$ Proved

To prove

$$y = x - \frac{1}{x}$$

$$y = \frac{x^2 - 1}{x}$$

$yx = x^2 - 1$ where -1 is constant

Hence, it is proved.

9. Given function is;

$$f(x) = 2x-1$$

For domain, for all real value x is exist. So, domain is set of all real number

i.e domain is $(-\infty, \infty)$

For range,

$$y = 2x-1$$

$$y+1 = 2x$$

$$x = \frac{y+1}{2}$$

for all possible value of ~~y ≥ 0~~ y x ~~is defined~~. Thus

$\therefore$ Range $(-\infty, \infty)$

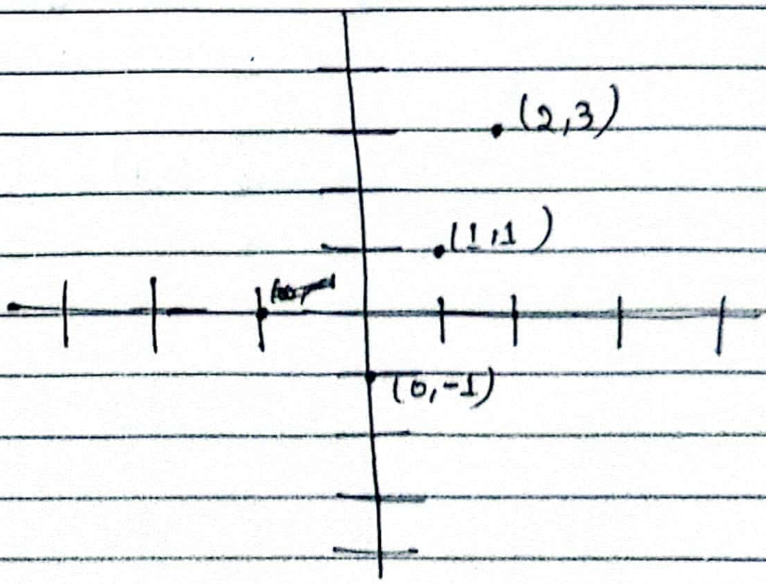
$\therefore$ Domain $(-\infty, \infty)$

Range $(-\infty, \infty)$

For the graph,

$f(x) = 2x-1$

let x	2	1	0
$f(x)$	3	1	-1



GOOD MORNING
PAGE NO. :
DATE : / /

10 Given Series is

$$\sum_{n=1}^{\infty} \frac{n^2}{5n^2+4}$$

i.e $\sum_{n=1}^{\infty} a_n$

where,

$$a_n = \frac{n^2}{5n^2+4}$$

Now,

$$\lim_{n\to\infty} \frac{n^2}{5n^2+4}$$

$$= \lim_{n\to\infty} \frac{\cancel{n^2}}{\cancel{n^2}(5+4/n^2)}$$

$$= \lim_{n\to\infty} \frac{1}{5+4/n^2}$$

$$= \frac{1}{5+4/\infty}$$

$$= \frac{1}{5+0}$$

$$= \frac{1}{5}$$

$\therefore$ The Given Series $\sum_{n=1}^{\infty} \frac{n^2}{5n^2+4}$ is divergent by n^{th} Root Test.

11 Given

$$f(x,y) = x^3 + x^2y^3 - 2y^2 \quad \text{---} \; (1)$$

find $f_x(2,1) = ?$ & $f_y(2,1) = ?$

diff^n eq^n (1) w.r. to 'x'

$$f_x = 3x^2 + 2xy^3$$

at point (2,1)

$$f_x(2,1) = 3\times 2^2 + 2\times 2\times 1^3$$
$$= 12 + 4$$
$$= 16$$

$\therefore \; f_x(2,1) = 16$ Ans

Again,

diff^n eq^n (1) w.r. to 'y'

$$f_y = 3y^2x^2 - 4y$$

at point (2,1)

$$f_y(2,1) = 3\times 1^2\times 2^2 - 4\times 1$$
$$= 12 - 4$$
$$= 8$$

$\therefore \; f_y(2,1) = 8$ Ans

GOOD MORNING
PAGE NO. :
DATE : __/__/____

12 Given,

$$f(x) = x^2 + \sqrt{7-x}$$

is continuous at $x = 4$

then,

$$\lim_{x\to4^-} f(x)$$

$$\lim_{x\to4^-} x^2 + \sqrt{7-\cancel{4}x}$$

$$\lim_{x\to4^-} (4)^2 + \sqrt{7-(4)}$$

$$= \cancel{16 + \sqrt{7-4}} \quad 16 + \sqrt{7-(4)}$$

$$= \cancel{16 + \sqrt{113}} \quad 16 + \sqrt{3}$$

Again,

GUPTA TUTORIAL

$$\lim_{x\to4^+} f(x)$$

$$\lim_{x\to4^+} \{x^2 + \sqrt{7-x}\}$$

$$= \lim_{x\to4^+} x^2 + \lim_{x\to}(4)^2 + \sqrt{7-4x}$$

$$= \cancel{16+\sqrt{3}}$$

$$= 4^2 + \sqrt{7-4}$$

$$= 16 + \sqrt{3}$$

$$\therefore \lim_{x\to4^-} f(x) = \lim_{x\to4^+} f(x)$$

So, Given function is continuous at $x = 4$