Mahadev Awasthi

Atomic Physics

Page No.:
Date: / /

Bohr's theory of Hydrogen atom

Postulates

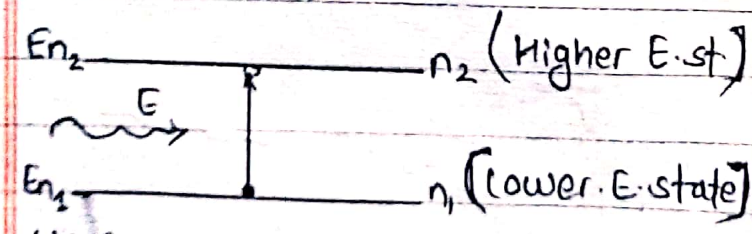
1) Electrons revolve around nucleus in a fixed circular orbit, known as stationary orbit. Angular momentum of electron must be equal to the integral multiple of $\frac{h}{2\pi}$.

~~I.e.~~ i.e. angular momentum,

$$\boxed{mvr = n \cdot \frac{h}{2\pi}} \quad ; \quad n = 1, 2, 3 \ldots$$

where, h = planck's constant (6.6×10^{-34} Js)
m = mass of electron (9.1×10^{-31} kg)
v = speed of electron
r = radius of orbit

II) Electron absorbs or emits radiation only if it jumps from one stationary orbit to another orbit.



Here,

$$E = E_{n_2} - E_{n_1}$$

$$hf = E_{n_2} - E_{n_1}$$

where, $E = hf = \frac{hc}{\lambda}$ = energy absorbed
E_{n_1} = (Energy of lower state)
E_{n_2} = Energy of higher state.

Radiation
(Electromagnetic wave)

$\lambda_{min}, f_{max}, E_{max}$ ↑

- γ-ray
- X-ray
- U.V-ray
- visible light (VIBGYOR)
- IR
- microwaves
- Radiowaves

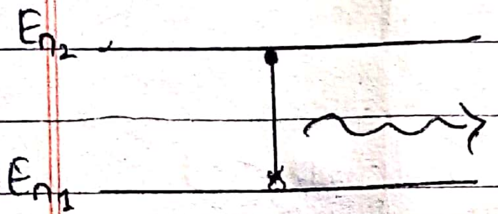
$\lambda_{max}, f_{min}, E_{min}$

→ speed in vaccum of all radiation (c) = 3×10^8 m/s.
→ particles called photon.
→ Energy of radiation $E = hf$

$$E = \frac{hc}{\lambda} \quad [\because \underset{(c)}{v} = f \times \lambda]$$

$\Rightarrow E \propto f \propto \frac{1}{\lambda}$

CamScanner



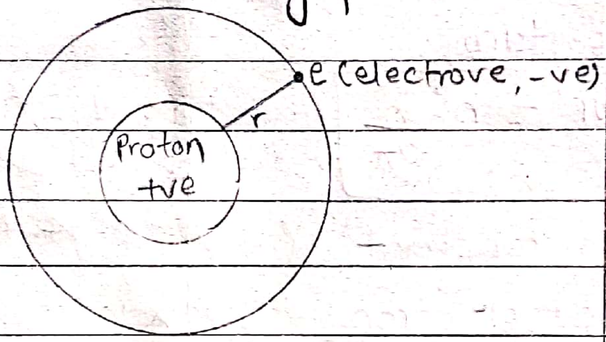
$$E = hf = \frac{hc}{\lambda}$$

$$E = \mathcal{E}_{n_2} - \mathcal{E}_{n_1}$$

where,

E = energy emitted

*



Felectrostatic force,

$$F = \left| \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \right|$$

electrostatic P.E,

$$P.E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}$$

Consider hydrogem atom and an electron is revolving around nucleus in a circularpath of radius r is shown in fig.

Now,

electrostatic force of attraction between proton and electron should provide centripetal force.

i.e, $$\left| \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \right| = \frac{mv^2}{r}$$

$$\left| \frac{1}{4\pi\varepsilon_0} \frac{(-e)^2}{r^2} \right| = \frac{mv^2}{r}$$

or, $$\frac{1}{4\pi\varepsilon_0} \frac{e^2}{r^2} = \frac{mv^2}{r}$$

or, $$\boxed{v^2 = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{mr}} \quad \text{——— (1)}$$

Page No.:
Date: / /

Also, we have angular momentum,

$$mvr = \frac{nh}{2\pi} \quad ; n = 1, 2, 3 \ldots$$

$$v = \frac{nh}{2\pi mr} \quad \text{——— (2)}$$

Using eqⁿ (2) in eqⁿ (1), we get

$$\frac{n^2h^2}{4\pi^2 m^2 r^2} = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{mr}$$

or, $$\frac{n^2h^2}{\pi mr} = \frac{e^2}{\varepsilon_0}$$

or, $$\boxed{r = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2}} \quad ; n = 1, 2, 3 \ldots \quad \text{——— (3)}$$

~~⇒ r ∝~~

This gives radius of n^{th} orbit of hydrogen atom.

⇒ $r \propto n^2$; n = principal quantum number

This shows that the radius of orbit is directly proportional to the square of n.

Using eqⁿ (3) in eqⁿ (2), we get

$$v = \frac{n \cdot h}{2\pi m} \cdot \frac{\pi m e^2}{\varepsilon_0 n^2 h^2}$$

or, $$\boxed{v = \frac{e^2}{2\varepsilon_0 n h}} \quad \text{——— (4)}$$

This is the velocity of an electron in nth orbit.

⇒ $v \propto \frac{1}{n}$

This shows that speed of electron is inversely proportional to principal quantum number (n).

Page No.:
Date: / /

Total energy of electron,

$$E = K.E + P.E \quad \text{—— (5)}$$

calculation ~~of~~ for K.E.

$$\because KE = \frac{1}{2} m v^2$$

Using eqⁿ (4)

$$KE = \frac{1}{2} \cdot m \cdot \frac{e^4}{4 \varepsilon_0^2 n^2 h^2}$$

$$K.E = \frac{me^4}{8 \varepsilon_0^2 n^2 h^2} \quad \text{—— (6)}$$

Calculation for P.E.

$$P.E = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{(+e)(-e)}{r}$$

$$= -\frac{1}{4\pi\varepsilon_0} \frac{e^2}{r}$$

Using eqⁿ (3)

$$P.E = -\frac{1}{4\pi\varepsilon_0} \cdot e^2 \cdot \frac{\pi m e^2}{\varepsilon_0 n^2 h^2}$$

$$P.E = \frac{-me^4}{4 \varepsilon_0^2 n^2 h^2} \quad \text{—— (7)}$$

Using eqⁿ (6) & (7), we get

$$E = \frac{me^4}{8 \varepsilon_0^2 n^2 h^2} - \frac{me^4}{4 \varepsilon_0^2 n^2 h^2}$$

or, $$E = \frac{me^4}{4 \varepsilon_0^2 n^2 h^2} \left(\frac{1}{2} - 1 \right)$$

or, $E = \frac{me^4}{4\varepsilon_0^2 n^2 h^2}\left(\frac{1-2}{2}\right)$

or, $$\boxed{E = -\frac{me^4}{8\varepsilon_0^2 n^2 h^2}} \quad \text{—— (8)}$$

This is the energy of an electron in nth orbit and negative sign shows that it is the binding energy.

Using, m = mass of electron = 9.1×10^{-31} kg.
e = charge of electron = 1.6×10^{-19} C.
ε_0 = permitivity of free space (vaccum)
= 8.85×10^{-12} C^2/Nm^2 or F/m.
h = plank's constant ⇒ 6.6×10^{-34} JS.

Eqn 8 becomes,

$$\boxed{E_n = -\frac{13.6ev}{n^2}}$$

$-\frac{13.6ev}{n^2}$

For ground state, n = 1
$E_1 = -13.6ev$

for 1st excited state, n = 2.
$E_1 = -\frac{13.6ev}{2^2} = -3.4ev$

For 2nd excited state, n = 3.
$E_1 = -\frac{13.6ev}{3^2} = -1.51ev$.

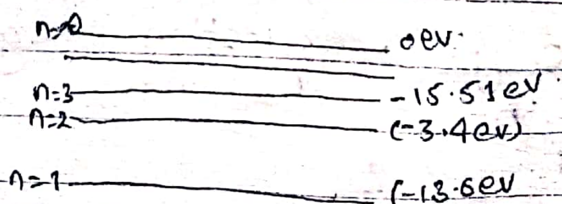


fig: Energy level diagram for hydrogen atom

* **Ionization energy**: It is defined as the energy required to eject electron from ground state ($n=1$) to $n=\infty$.

$\because$ Ground state energy of electron $= -13.6\,eV$.

$$\text{Ionization energy} = E_\infty - E_1$$
$$= 0-(-13.6)$$
$$= +13.6\,eV.$$

* **Ionization Potential**: It is defined as potential difference applied to an electron at which accelerated electron can ionize Hydrogen atom when it collides with atom.

In other words, potential difference corresponding to ionization energy is called ionization potential.

Ionization potential for H-atom is $13.6\,V$.

* **Excitation energy**: It is defined as a energy required to excite an electron from lower energy state to higher energy state (except infinity, $n=\infty$)

And excitation potential is defined as potential applied to an electron at which accelerated electron can excite hydrogen atom when collides with atom / the potential difference corresponding to excitation energy is called excitation potential.

Q. What is excitation energy to excite an electron of Hydrogen atom from n=1 to n=2 state?

Weknow,

$$E_n = \frac{-13.eV}{n^2}$$

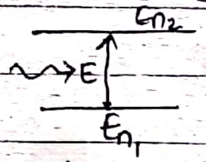
$$E_1 = -13.6eV$$

$$E_2 = -3.4eV$$

$$\text{Excitation energy} = E_2 - E_1$$
$$= -3.4eV - (-13.6eV)$$
$$= 10.2eV.$$

Q. A photon of energy 10.2eV incident in an atom of hydrogen at ground state. Find the excited state.

Given,



$$E_{n_1} + E = E_{n_2} \quad \text{—} \quad (1)$$

$$E_n = \frac{-13.6eV}{n^2}$$

$$E_{n_1} = \frac{-13.eV}{n_1^2} \quad , \quad E_{n_2} = \frac{-13.6eV}{n_2^2}$$

(1) becomes,

$$\frac{-13.eV}{n_1^2} + 10.2 = \left(\frac{-13.6eV}{n_2^2}\right)$$

$$\text{or,} \quad -3.4eV = \frac{-13.6eV}{n_2^2}$$

$$\text{or,} \quad n_2^2 = \frac{-13.6eV}{-3.4eV} \Rightarrow 4$$

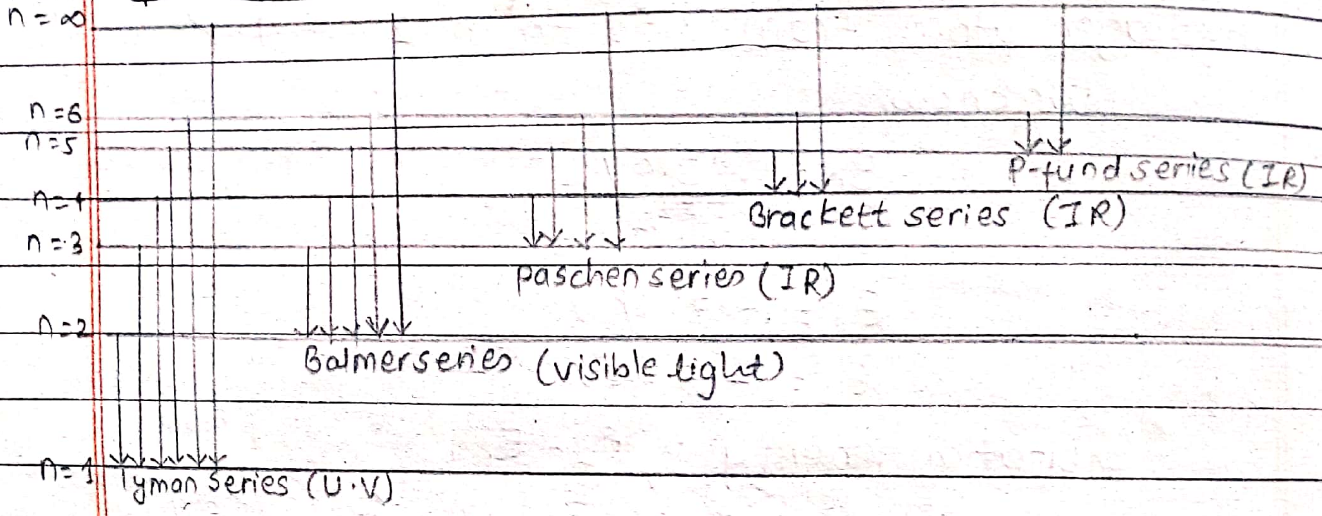
$$\therefore n_2 = 2.$$

It is the 1st excited state.

Page No.:
Date: / /

(LBPBP)

* Spectral ~~line~~ series



We have,

Energy of hydrogen atom in nth state

$$E_n = -\frac{m \cdot e^4}{8\varepsilon_0^2 n^2 h^2} \quad \text{———} \quad (1)$$

Suppose, E_{n_2} is energy of higher energy state

E_{n_1} = lower energy state.

According to Bohr's postulate,

When an electron jumps from higher energy state to lower energy state, radiation is emitted.

Energy of emitted radiation (E) = $E_{n_2} - E_{n_1}$

Now,

$$E = \frac{-me^4}{8\varepsilon_0^2 n_2^2 h^2} - \left(-\frac{me^4}{8\varepsilon_0^2 n_1^2 h^2}\right)$$

$$\text{or, } hf = \frac{me^4}{8\varepsilon_0^2 n_1^2 h^2} - \frac{me^4}{8\varepsilon_0^2 n_2^2 h^2}$$

$$\text{or, } hf = \frac{me^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\text{or, } \frac{hc}{\lambda} = \frac{me^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Page No.:
Date: / /

$$\frac{1}{\lambda} = \frac{me^4}{8\varepsilon_0^2 ch^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \quad \text{--- (2)}$$

where, R = Rydberg constant

$$= \frac{me^4}{8\varepsilon_0^2 ch^3}$$

$$= 1.097 \times 10^7 m^{-1}.$$

$\frac{1}{\lambda}$ = wave number.

Case I, Lymann Series; $n_1 = 1$ & $n_2 = 2, 3, 4, 5, \ldots \infty$.
eq^n (2) becomes,

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n_2^2}\right), \text{ where } n_2 = 2, 3, 4, 5, \ldots$$

This series lies in ultraviolet region.

Q. What is the minimum wavelength of lyman series?

$n_1 = 1, \quad n_2 = \infty$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$E_{max} \Rightarrow hf_{max} =$

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{\infty}\right)$$

$$\frac{1}{\lambda} = R$$

$$\therefore \lambda_{min} = \frac{1}{R} \Rightarrow \frac{1}{1.097 \times 10^7} m.$$

$$f_{max} \Rightarrow \frac{c}{\lambda} = .$$

Q. What is the maximum wavelength of lymann series?

$n_1 = 1$, $n_2 = 2$.

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$\frac{1}{\lambda} = R \times \frac{3}{4}$$

$$\therefore \lambda = \frac{4}{3R} = \frac{4}{3 \times 1.097 \times 10^7} \text{ m.}$$

case II Balmer Series; $n_1 = 2$, $n_2 = 3, 4, 5, 6, \ldots \infty$

Eq^n ② $\Rightarrow$ $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_2^2}\right)$$

First line of Balmer series,

$n_1 = 2$, $n_2 = 3$

for 2nd line of Balmer series,

$n_1 = 2$, $n_2 = 4$.

Balmer series lies in visible region.

Q. What is the shortest wavelength of Balmer series?

$n_1 = 2$, $n_2 = \infty$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Page No.:
Date: / /

$$\frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{\infty}\right)$$

$$\frac{1}{\lambda} = \frac{R}{4} \quad \therefore \lambda = \frac{4}{R}.\ m$$

Q. The wavelength of first line of Balmer series is 5800Å then find wavelength of second line of Balmer series.

For first line of Balmer series.

$$\frac{1}{\lambda_1} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\frac{1}{\lambda_1} = R\left(\frac{9-4}{36}\right)$$

$$\frac{1}{\lambda_1} = \frac{5R}{36}$$

$$\lambda_1 = \frac{36}{5R} \quad \text{—— (I)}$$

For 2nd line of Balmer series,

$n_1 = 2$, $n_2 = 4$.

$$\frac{1}{\lambda_2} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\frac{1}{\lambda_2} = R\left(\frac{1}{4} - \frac{1}{16}\right)$$

$$\frac{1}{\lambda_2} = \frac{R.3}{16}$$

$$\lambda_2 = \frac{16}{3R} \quad \text{—— (II)}$$

Page No.:
Date: / /

Dividing (ii) and (i)

$$\frac{\lambda_1}{\lambda_2} = \frac{36}{5R} \times \frac{3R}{16}$$

$$\frac{\lambda_1}{\lambda_2} = \frac{36 \times 3}{5 \times 16}$$

$$\lambda_2 = \frac{\lambda_1 \times 5 \times 16}{36 \times 3}$$

$$\lambda_2 = \frac{5800 \times 5 \times 16}{36 \times 3}$$

$$\lambda_2 = 4296.3 \text{ m}.$$

case III Paschen Series; $n_1 = 3$, $n_2 = 4, 5, 6, ---\infty$.

Eq^n (2) becomes,

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n_2^2}\right), \; n_2 = 4, 5, ---\infty$$

case IV Brackett series; $n_1 = 4$, $n_2 = 5, 6, ---\infty$.

Eq^n (2) becomes,

$$\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n_2^2}\right), \; n_2 = 5, 6, ---\infty.$$

case V P-fund series; $n_1 = 5$, $n_2 = 6, 7, 8, ---\infty$.

Eq^n (2) becomes,

$$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n_2^2}\right).$$

Q. Find the energy of H-atom in its first excited state.

$n = 2.$

$$E_n = \frac{-13.6\,ev}{n^2} = \frac{13.6\,ev}{4} = 3.4\,ev.$$

* Limitation of Bohr's Atomic Theory:

i) It can't explain the elliptical orbit of electron around nucleus.

ii) It can't explain fine structure of spectra.

iii) It can't explain atomic spectra of multi-electron system

iv) It can't explain wave nature of electron.

Page No.:
Date: / /

* de'Broglie wave/matter wave

According to de'Broglie hypothesis, every moving particle is always associated with wave and this wave is known as de'Broglie wave or matter wave. The wavelength of matter wave is given by

$$\boxed{\lambda = \frac{h}{p}}$$

where,

h = planck's constant ⇒ 6.6×10^{-34} Js

P = momentum of particle ⇒ mv

Matter wave will be detectable only if de'Broglie wavelength is comparable to the dimension of particle. (or region where particle is located)

For example;

Suppose a body of mass 2kg, density 8000 kg/m³ is moving with a speed of 500 m/s then de'Broglie wavelength is,

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.6 \times 10^{-34}}{2 \times 500}$$

$$\lambda = 6.6 \times 10^{-37} \text{ m}$$

This de'Broglie wavelength is very small in comparision to size of the body. And Hence, its matter wave is not detectable.

CamScanner

Page No.:
Date: / /

Q. why matter wave is not detectable in our real life?
→ Due to matter wave will be detectable only if de'Broglie wavelength is comparable to the dimension of particle.

* Note: de'Broglie wavelength of electron in Bohr's orbit (first orbit) is

$$\lambda = \frac{h}{mv}$$

Using $m = 9.1 \times 10^{-31}$ kg

$v = \frac{1}{137} c$ $\left[\because v = \frac{e^2}{2\varepsilon_0 n h} ; n = 1 \right]$

$$\lambda = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times \frac{1}{137} \times 3 \times 10^8}$$

$$\lambda = 3.312 \times 10^{-10} \text{ m}$$

Now,

Circumference of Bohr's first orbit $= 2\pi r$ $\left[\because r = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2} \right]$

$$= \frac{2\varepsilon_0 n^2 h^2}{me^2}$$

This shows that de'Broglie wavelength of electron in Bohr's orbit is comparable to the circumference and hence, its matter wave is detectable.

Page No.:
Date: / /

* Derivation of de-Broglie wavelength

We have,

Energy of photon,

$$\boxed{E = hf} \quad \text{——} \quad (1)$$

and also from Einstein's mass-energy relation

$$\boxed{E = mc^2} \quad \text{——} \quad (2)$$

from eqⁿ (1) and (2) we get,

$$mc^2 = hf$$

$$mc^2 = \frac{hc}{\lambda}$$

$$\text{or, } mc = \frac{h}{\lambda}$$

$$\text{or, } p = \frac{h}{\lambda}$$ where, $p = mc \Rightarrow$ momentum of photon

$$\therefore \boxed{\lambda = \frac{h}{p}}$$

* de-Broglie wavelength

$$\lambda = \frac{h}{p} \quad \text{——} \quad (1)$$

case I: Non-relativistic expression. ($v << c$)

$$K.E\ (E_K) = \frac{1}{2}mv^2$$

$$E_K = \frac{(mv)^2}{2m}$$

$$E_K = \frac{P^2}{2m} \quad (\because P = mv \Rightarrow \text{momentum})$$

$$\boxed{P = \sqrt{2mE_K}} \text{ ——— } (2)$$

using eqⁿ (2) in eqⁿ (1) we get,

$$\boxed{\lambda = \frac{h}{P} = \frac{h}{\sqrt{2mE_K}}} \text{ ——— } (3)$$

For an electron of charge 'e' is accelerated through potential difference of V volt, then

$$\boxed{E_K = eV} \text{ ——— } (4)$$

Now, eqⁿ (3) becomes

$$\boxed{\lambda = \frac{h}{\sqrt{2meV}}}$$

for a particle in thermal equilibrium,

$$\boxed{E_K = \frac{3}{2} KT}$$

where, K = Boltzman constant = 1.38×10^{-23} JK^{-1}.
T = Temperature in kelvin.

eqⁿ (3) becomes,

$$\lambda = \frac{h}{\sqrt{2m \times \frac{3}{2} KT}}$$

$$\boxed{\lambda = \frac{h}{\sqrt{3mKT}}}$$

$E = mc^2$
$\downarrow$
moving mass

$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$

case II: Relativistic expression
(v is comparable to c)

We have relativistic expression for energy and momentum.

$$E^2 = p^2c^2 + m_0^2 c^4 \quad \text{——} \quad (1)$$

where,

m_0 = rest mass , E = Total energy

Also,

$$\boxed{E = E_K + m_0 c^2} \quad \text{——} \quad (2)$$

Using eqn (2) in eqn (1) we get,

$$(E_K + m_0c^2)^2 = p^2c^2 + m_0^2c^4.$$

or, $E_K^2 + 2m_0c^2E_K + \cancel{m_0^2c^4} = p^2c^2 + \cancel{m_0^2c^4}$.

or, $E_K^2 + 2m_0c^2E_K = p^2c^2$

or, $p^2 = \frac{E_K^2 + 2m_0c^2E_K}{c^2}$

$$\therefore p = \sqrt{\frac{E_K^2 + 2m_0c^2E_K}{c^2}}$$

$$= \sqrt{\frac{2m_0E_K\left(c^2 + \frac{E_K}{2m_0}\right)}{c^2}}$$

$$\boxed{p = \sqrt{2m_0E_K\left(1 + \frac{E_K}{2m_0c^2}\right)}}$$

Page No.:
Date: / /

Now, de'Broglie wavelength for relativistic case is.

$$\lambda = \frac{h}{p}$$

$$\boxed{\lambda = \frac{h}{\sqrt{2m_0 E_K\left(1+\frac{E_K}{2m_0c^2}\right)}}} \quad \text{—— (3)}$$

If $E_K << (2m_0c^2)$, $\frac{E_K}{2m_0c^2} << 1$.

Eqn (3) can be written as,

$$\boxed{\lambda = \frac{h}{\sqrt{2m_0E_K}}}$$

$\because 1 + \frac{E_K}{2m_0c^2} \approx 1$.

This shows that relativistic expression of de'Broglie wavelength reduces to non-relativistic expression if $E_K << (2m_0c^2)$.

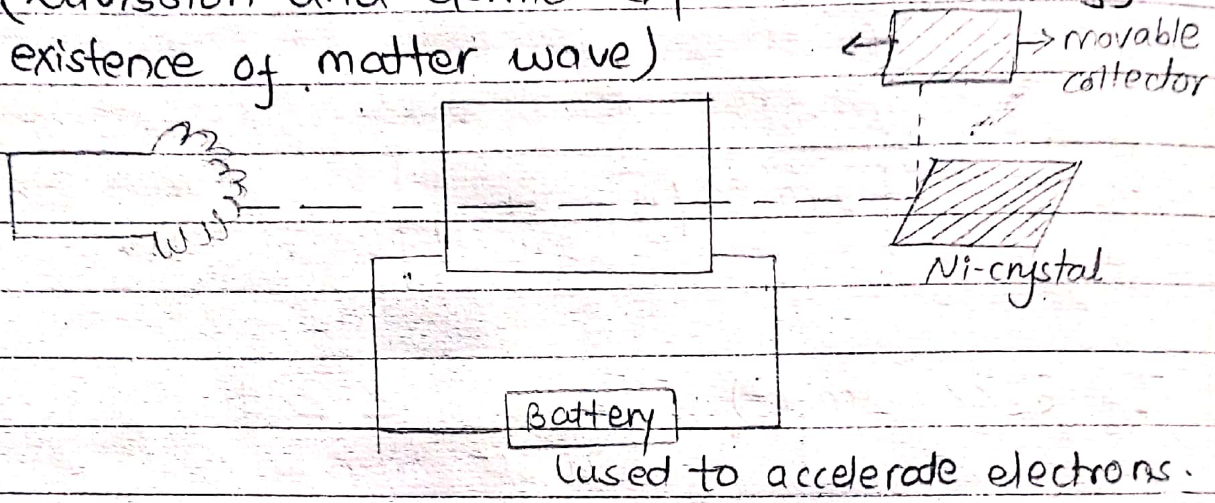
Formula:

(i) $\lambda = \frac{h}{p}$

(ii) $p = \sqrt{2mE_K}$ for non-relativistic.

(iii) $p = \sqrt{2m_0E_K\left(1+\frac{E_K}{2m_0c^2}\right)}$ for relativistic.

* Experimental verification for matter wave
(Verification of de'Broglie ~~wavelength~~ hypothesis)
(Davission and Germer experiment to verify existence of matter wave)



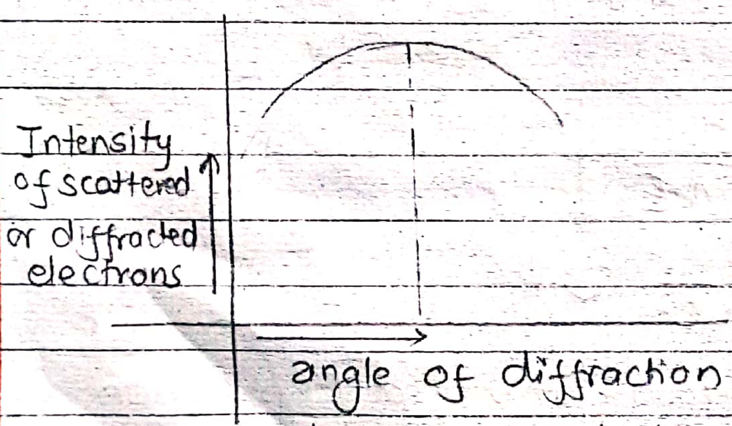


fig:- wave like behaviour of electrons.

Suppose a filament is heated to emit electrons and electrons emitted by filament is accelerated by a ~~particle~~ potential difference of different values. Now, these electrons are allowed to incident on a Ni-crystal and intensity of scattered electron is measured with the help of movable collector at different position.
Experimentally it is found that intensity distribution of scattered electron is wave like and is

Page No.:
Date: / /

prominent at 54 volt.

From de' Broglie hypothesis,

$$\lambda = \frac{h}{p}$$

$$\lambda = \frac{h}{\sqrt{2mE_K}}$$

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$$\lambda = \frac{6.6\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times54\times e}}$$

$h = 6.6\times10^{-34}$, $m = 9.1\times10^{-31}$ kg

$V = 54$ V

$$\boxed{\lambda = 1.66\,\text{Å}} \text{ —— } (1)$$

Also, we have from Bragg's law of diffraction,

$$2d\sin\theta = n\lambda$$

where,

d = interplanar spacing = 0.91 Å (for Ni-crystal)

for 1^{st} order,

$$n = 1$$

for glancing angle,

$$\theta = 65°$$

then,

$$2d\sin\theta = n\lambda$$

or, $2\times0.91\times\sin65° = 1\times\lambda$

or, $\boxed{\lambda = 1.65\,\text{Å}}$ —— (2)

From eqⁿs (1) & (2), it is clear that electron undergoes diffraction similar to the X-ray.

Page No.:
Date: / /

Thus due to observed diffraction phenomena of electrons it is clear that electron has wave nature.

Q. Find de'Broglie wavelength of electron if it is accelerated through a potential difference of 50 Volt.

Soln:

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 50}}$$

$$= 1.73 \times 10^{-10} \, m \Rightarrow 1.73 \, A^{\circ}.$$

Q. Find de'Broglie wavelength of electron if its speed is 2×10^{8} m/s.

Soln:

$$\lambda = \frac{h}{p}$$

$$= \frac{h}{m.v}$$

where,

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{9.1 \times 10^{-31}}{\sqrt{1 - \left(\frac{2 \times 10^{8}}{3 \times 10^{8}}\right)^2}} = 1.22 \times 10^{-30} \, kg$$

then, $$\lambda = \frac{6.6 \times 10^{-34}}{1.2 \times 10^{-30} \times 2 \times 10^{8}}$$

$$= 2.75 \times 10^{-12} \, m$$

(if its speed is 20 m/s, we can neglect the term $\sqrt{1 - v^2/c^2}$. so, $m = 9.1 \times 10^{-31}$ kg)

$M = Mega = 10^6$
$\mu = 10^{-6}$

Page No.:
Date: / /

Q. Find de'Broglie wavelength of electron if its kinetic energy is 2 MeV.

Soln:

Given,

$E_K = 2 MeV$

$= 2 \times 10^6 \times 1.6 \times 10^{-19}$ J

$= 3.2 \times 10^{-13}$

$$\therefore \frac{E_K}{2m_0c^2} = \frac{2 \times 10^6 \times 1.6 \times 10^{-19}}{2 \times 9.1 \times 10^{-31} \times (3 \times 10^8)^2}$$

$$\frac{E_K}{2m_0c^2} = 1.95$$

we must use Relativistic expression.

$$\lambda = \frac{h}{\sqrt{2m_0E_K\left(1 + \frac{E_K}{2m_0c^2}\right)}}$$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 3.2 \times 10^{-13} (1 + 1.95)}}$$

$= 5.04 \times 10^{-13}$ m.

$(m_p \simeq m_n \simeq 1\,amu = 1.67 \times 10^{-27} kg)$

Page No.:
Date: / /

Q. A termal neutron is at temperature of 27°C then find its de'Broglie wavelength.

We know, $T = 27°C \Rightarrow 300K$

$$\lambda = \frac{h}{p}$$

$$= \frac{h}{\sqrt{2mE_k}}$$

$$= \frac{h}{\sqrt{2m.\frac{3}{2}kT}}$$

$$= \frac{h}{\sqrt{3mkT}}$$

$$= \frac{6.6 \times 10^{-34}\,Js}{\sqrt{3 \times 1.67 \times 10^{-27} \times 1.38 \times 10^{-23} \times 300}}$$

$\left(\because m_n = 1.67 \times 10^{-27}, \quad k = \text{Boltzmann constant} = 1.38 \times 10^{-23}\, JK^{-1}, \quad h = 6.6 \times 10^{-34}\, Js \right)$

$$= 1.48 \times 10^{-10}\, m$$

Q. Find de'Broglie wavelength of electron of kinetic energy (i) 20eV
(ii) 1MeV

(i) $E_k = 20eV$

$= 20 \times 1.6 \times 10^{-19} J = 3.2 \times 10^{-18} J.$

$$\therefore \frac{E_k}{2m_0c^2} = \frac{3.2 \times 10^{-18}}{2 \times 9.1 \times 10^{-31} \times (3 \times 10^8)^2}$$

$$\frac{E_k}{2m_0c^2} = 1.97 \times 10^{-5}$$

Page No.:
Date: / /

This is very small ~~is~~ in comparision to 1 & hence we can use non-relativistic expression.

i.e, $\lambda = \dfrac{h}{\sqrt{2m_0E_k}}$

$$= \frac{6.6\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times3.2\times10^{-18}}}$$

$$= 2.73\times10^{-10}\ m.$$

(ii) $E_k = 1\,MeV$

$= 1\times10^{6}\times1.6\times10^{-19}\ J = 1.6\times10^{-13}\ J.$

$$\therefore \frac{E_k}{2m_0c^2} = \frac{1.6\times10^{-13}}{2\times9.1\times10^{-31}\times(3\times10^{8})^{2}}$$

$$= 0.976$$

This is very close to 1 & hence we use relativistic expression.

$$\lambda = \frac{h}{\sqrt{2m_0E_k\left(1+\frac{E_k}{2m_0c^2}\right)}}$$

$$= \frac{6.6\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-13}\,(1+0.976)}}$$

$$= 8.7\times10^{-13}\ m$$

Page No.:
Date: / /

Q. If total energy of electron is 2 Mev then find its de'Broglie wavelength.

Here,

$$E = 2\,MeV$$
$$= 2\times10^{6}\times1.6\times10^{-19}\,J \Rightarrow 3.2\times10^{-19}\,J$$

Now,

$$E = E_K + \text{rest mass energy}$$
$$3.2\times10^{-19} = E_K + m_0c^2$$
$$E_K = 3.2\times10^{-19} - 9.1\times10^{-31}\times(3\times10^{8})^2$$
or, $E_K = \cancel{2}.38\times10^{-13}\,J$

$$\therefore \frac{E_K}{2m_0c^2} = \frac{2.38\times10^{-13}}{2\times9.1\times10^{-31}\times(3\times10^{8})^2}$$
$$= \cancel{4}\;1.45 > 1$$

We must use relativistic expression.

$$\lambda = \frac{h}{\sqrt{2m_0E_K\left(1+\frac{E_K}{2m_0c^2}\right)}}$$

$$= \frac{6.6\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times2.38\times10^{-13}(1+1.45)}}$$

$$= 6.41\times10^{-13}\,m.$$

* K.E of gas

$$K.E = \left(\frac{1}{2}KT\right) \times f$$

where, f = degree of freedom

$f = 3$ (for monoatomic gas ; He, Ne)

$f = 5$ (for diatomic gas ; O_2, H_2, N_2)

$f = 6$ (for triatomic gas/polyatomic gas; CO_2)

Q. Find de'Broglie wavelength of oxygen molecule at 27°C.

Given,

$T = 27°C = 300K$

$$\lambda = \frac{h}{p}$$

$$= \frac{h}{\sqrt{2mE_K}}$$

$$= \frac{h}{\sqrt{2m\left(\frac{1}{2}KT\right)\times f}}$$

$$\lambda = \frac{h}{\sqrt{5mKT}} \quad [\because f = 5 \text{ for } O_2]$$

Here,

$K = 1.38 \times 10^{-23} JK^{-1}$, $h = 6.6 \times 10^{-34}$ JS

$m = ?$

1 mole of O_2 = 32g

6.023×10^{23} molecule of O_2 = 32g

Page No.:
Date: / /

1 molecule of $O_2 = \frac{32g}{6.023 \times 10^{23}}$ g

$\therefore m = \frac{32 \times 10^{-3}}{6.023 \times 10^{23}}$ kg.

$= 5.313$ kg.

Now,

$\lambda = \frac{h}{\sqrt{5mkT}}$

$= \frac{6.62 \times 10^{-34}}{\sqrt{5 \times 5.313 \times 1.38 \times 10^{-24} \times 300}}$

$= 6.31 \times 10^{-24}$ m.

Q. Calculate mass of a single molecule of CO_2 and find de'Broglie wavelength at 20°C?

6.023×10^{23} molecule of $CO_2 = 44g$.

1 molecule of $CO_2 = \frac{44}{6.023 \times 10^{23}}$ g

$= \cancel{\frac{44 \times 10^{-3}}{6.023 \times 10^{23}}}$ kg

and mass (m) $= \frac{44 \times 10^{-3}}{6.023 \times 10^{23}}$ kg.

Now,

$\lambda = \frac{h}{\sqrt{5mkT}}$

$= \frac{6.62 \times 10^{-34}}{\sqrt{5 \times \frac{44 \times 10^{-3}}{6.023 \times 10^{23}} \times 1.38 \times 10^{-24} \times 293}}$

CamScanner

$= 5.45 \times 10^{-11} m.$

(phase-angle in radian)

* Phase velocity and group velocity

Phase velocity/wave velocity: It is defined as a speed of constant phase or speed of a wave.
consider a wave, associated with moving particle be $\psi = a \sin(\omega t - kx)$.
where,

a = amplitude.

$\omega = 2\pi f$

$k = \frac{2\pi}{\lambda}$

Since phase is a constant velocity.

i.e, $\omega t - kx$ = constant.

differentiating w.r.to 't' we get.

$$\omega - k\left(\frac{dx}{dt}\right) = 0$$

$$k \cdot \frac{dx}{dt} = \omega$$

$$\boxed{v = \frac{dx}{dt} = \frac{\omega}{k}} \quad \text{——— (1)}$$

where v = phase velocity or wave velocity.
eqn (1) can also be written as,

$$v = \frac{2\pi f}{2\pi/\lambda}$$

$$\boxed{v = f \times \lambda} \quad \text{——— (2)}$$

Eqns (1) and (2) are reqd expression for phase

Page No.:
Date: / /

velocity / wave velocity.

from eqn ①

$$v = \frac{\omega}{k}$$

$$= \frac{\hbar\omega}{\hbar k}$$

$$= \frac{(hf)}{(h/\lambda)}$$

$$= \frac{E}{p}$$

$\because \hbar = h/2\pi$, $\hbar\omega = \frac{h}{2\pi} \times 2\pi f$

$= hf$

$\because \hbar k = \frac{h}{2\pi} \times \frac{2\pi}{\lambda} = \frac{h}{\lambda}$

[$\because E = hf$ and $p = h/\lambda$]

$$\therefore \boxed{v = \frac{E}{p}} \quad \text{——— } (3)$$

where, $p = mu$ = momentum.

For relativistic particle,

$$v = \frac{E}{p} = \frac{\cancel{m}c^2}{\cancel{m}u}$$

$$\therefore \text{phase velocity}(v) = \left(\frac{c^2}{u}\right) > c$$

(u = velocity of particle)

This shows that phase velocity is greater than speed of light but according to Einstein's special theorey of relativity, Speed of light is the upper limit of speed and no particle can have speed equal to the speed of light. Clearly, phase velocity of single de'Broglue wave is physically meaningless.

CamScanner

Group velocity : According to Schrodinger every moving particle is always associated with an infinite number waves having slightly different frequency and wavelength. These waves are so selected that, they interfare constructively at a point/in a region where probability of finding of particle is maximum and they interfare ~~destru~~ distructively in a region where probability of finding of particle is minimum. These selected waves are called wave group and ~~con~~ their construction is called wave packet.

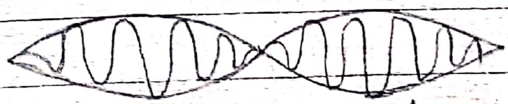


fig: wave packet.

Consider two waves

$$\psi_1 = a \sin(\omega t - kx) \quad \text{——} (1)$$

$$\psi_2 = a \sin\{(\omega + \Delta\omega)t - (k + \Delta k)x\} \quad \text{——} (2)$$

Using Principle of Super position, resultant wave is given by.

$$\psi = \psi_1 + \psi_2$$

$$\psi = a\left[\sin(\omega t - kx) + \sin\{(\omega + \Delta\omega)t - (k + \Delta k)x\}\right]$$

$$\psi = a\left[2 \times \sin\left(\frac{\omega t - kx + (\omega + \Delta\omega)t - (k + \Delta k)x}{2}\right) \cdot \cos\left(\frac{\omega t - kx - \omega t - \Delta\omega t + kx + \Delta kx}{2}\right)\right]$$

Page No.:
Date: / /

$$\left[\because \sin(C+D) = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\right]$$

or, $\psi = 2a\left\{\sin\left(\frac{2\omega t + \Delta\omega t - 2kx - \Delta kx}{2}\right)\right\}\left\{\cos\left(\frac{-\Delta\omega t + \Delta kx}{2}\right)\right\}$

or, $\psi = 2a\cos\left(\frac{\Delta kx - \Delta\omega t}{2}\right)\sin\left(\frac{(2\omega+\Delta\omega)t}{2} - \frac{(2k+\Delta k)x}{2}\right)$

or, $\psi = 2a\cos\left(\frac{\Delta kx}{2} - \frac{\Delta\omega t}{2}\right)\sin\left\{\left(\omega+\frac{\Delta\omega}{2}\right)t - \left(k+\frac{\Delta k}{2}\right)x\right\}$

or, $\psi = 2a\cos\left(\frac{\Delta kx}{2} - \frac{\Delta\omega t}{2}\right)\sin(\omega' t - k' x)$

(where, $\omega' = \omega + \frac{\Delta\omega}{2}$ and $k' = k + \frac{\Delta k}{2}$)

$$\psi = A\sin(\omega' t - k' x) \quad \text{——} \quad (3)$$

where,

$A = 2a\cos\left(\frac{\Delta kx}{2} - \frac{\Delta\omega t}{2}\right)$ (4) is amplitude of resultant wave and also known as modulated amplitude.

↳ Group velocity is defined as the velocity of centre of wave packet. In other words, it is also defined as the phase velocity of modulated amplitude.

Clearly, from eqn (4)

Phase of modulated amplitude $= \left(\frac{\Delta kx}{2} - \frac{\Delta\omega t}{2}\right)$

$\because$ phase = constant

$$\frac{\Delta kx}{2} - \frac{\Delta\omega t}{2} = \text{constant}$$

Page No.:
Date: / /

Differentiating w.r.to t, we get

$$\frac{\Delta k}{2}\left(\frac{dx}{dt}\right) - \frac{\Delta\omega}{2} = 0$$

or, $\frac{\Delta k}{2}\left(\frac{dx}{dt}\right) = \frac{\Delta\omega}{2}$

or, $\frac{dx}{dt} = \frac{\Delta\omega}{\Delta k}$

or, $\boxed{V_g = \frac{\Delta\omega}{\Delta k}}$ $\left[\because V = dx/dt\right]$

where, V_g = group velocity ~~of~~ superposition of two waves.

For large number of superposing waves, above expression can be written as:

$$\boxed{V_g = \frac{d\omega}{dk}} \quad \text{———} (5) \quad \left[\because \frac{d\omega}{dk} = \lim_{\Delta k \to 0} \frac{d\omega}{dk}\right]$$

from eqn (5),

$$V_g = \frac{d(\hbar\omega)}{d(\hbar k)}$$

or, $\boxed{V_g = \frac{dE}{dP}}$ ——— (6) $\left[\because \hbar\omega = E \ \& \ \hbar k = P\right]$

Eqn (5) and (6) are required expression for group velocity.

Page No.:
Date: / /

Q. Show that group velocity is equal to the particle velocity
↳ We know,

$$\text{group velocity } (V_g) = \frac{dE}{dP} \quad \text{--- (1)}$$

for free particle (Non-relativistic);

$$E = K.E + P.E \quad (\text{for free particle, } P.E = 0)$$

or, $E = \frac{1}{2} mv^2$

$$= \frac{(mv)^2}{2m}$$

$$\boxed{E = \frac{p^2}{2m}} \quad \text{--- (2)}$$

Using eqn (2) in eqn (1) we get:

$$V_g = \frac{d(p^2/2m)}{dp}$$

$$V_g = \frac{1}{2m} \cdot \frac{dp^2}{dp}$$

$$= \frac{1}{2m} \times 2p$$

$$= \frac{p}{m}$$

$$= \frac{mv}{m} = v$$

$$\therefore \boxed{V_g = V}$$

i.e. group velocity is equal to particle velocity.

For relativistic particle,

$$E^2 = p^2c^2 + m_0^2c^4$$

$$E = (p^2c^2 + m_0^2c^4)^{1/2}$$

Now,

$$\text{group velocity } (v_g) = \frac{dE}{dp}$$

$$= \frac{d}{dp}(p^2c^2 + m_0^2c^4)^{1/2}$$

$$= \frac{d(p^2c^2 + m_0^2c^4)^{1/2}}{d(p^2c^2 + m_0^2c^4)} \times \frac{d(p^2c^2 + m_0^2c^4)}{dp}$$

$$= \frac{1}{\cancel{2}}(p^2c^2 + m_0^2c^4)^{-1/2} \times \cancel{2}pc^2$$

$$= \frac{1}{\cancel{2}} \frac{pc^2}{(p^2c^2 + m_0^2c^4)^{1/2}}$$

$$v_g = \frac{pc^2}{E} = \frac{pc^2}{mc^2} = \frac{p}{m} = \frac{m \cdot v}{m} = v.$$

$$\therefore \boxed{v_g = v}$$

This means that group velocity is equal to the particle velocity.

Page No.:
Date: / /

Q. Why the concept of group velocity is necessary? Why phase velocity of single de-Broglie wave can't explain the physics?

→

* Relationship between phase velocity and group velocity

We have,

group velocity,

$$V_g = \frac{d\omega}{dk} \quad \text{--- } (1)$$

and,

phase velocity,

$$V_p = \frac{\omega}{k}$$

$$\Rightarrow \boxed{\omega = V_p k} \quad \text{--- } (2)$$

Using eq^n (2) in eq^n (1) we get,

$$V_g = \frac{d(V_p k)}{dk}$$

or, $$V_g = V_p \frac{dk}{dk} + \cancel{\frac{dk}{dk}}\, k \frac{dV_p}{dk}$$

or, $$\boxed{V_g = V_p + k\frac{dV_p}{dk}} \quad \text{--- } (3)$$

$$\because k = \frac{2\pi}{\lambda} \quad \text{--- } 4$$

Now,

$$\frac{dk}{d\lambda} = \frac{d(2\pi/\lambda)}{d\lambda}$$

$$= 2\pi \frac{d}{d\lambda}\lambda^{-1}$$

$$= 2\pi(-1)\lambda^{-2}$$

$$\frac{dk}{d\lambda} = -\frac{2\pi}{\lambda^2}$$

$$\boxed{dk = -\frac{2\pi}{\lambda^2}\, d\lambda} \quad \text{—— (5)}$$

Using eq^n (4) and (5) we get,

$$V_g = V_p + \left(\frac{2\pi}{\lambda}\right) \frac{d(V_p)}{(-2\pi/\lambda^2)\, d\lambda}$$

$$\boxed{V_g = V_p - \lambda \frac{dV_p}{\lambda}} \quad \text{—— (6)}$$

This is the req^d relationship between group velocity and phase velocity.

Q. An electron jumps from 3rd excited to 2nd second excited state the find wave length of radiation emitted in hydrogen atom.

Sol^n

Given,

$n_2 = 4$

$n_1 = 3$

Now,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\text{or, } \frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

or, $\frac{1}{\lambda} = R\left(\frac{1}{9} - \frac{1}{16}\right)$

or, $\frac{1}{\lambda} = R \times 0.05$

or, $\lambda = \frac{1}{R \times 0.05}$

or, $\lambda = \frac{1}{1.097 \times 10^{7} \times 0.05}$

or, $\lambda = 1.82 \times 10^{-6}$ m. #

Q. Find longest wavelength of Balmer series?

Given,

$n_1 = 2$

$n_2 = \infty$.

Now,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

or, $\frac{1}{\lambda} = 1.097 \times 10^{7}\left(\frac{1}{2^2} - \frac{1}{\infty}\right)$

or, $\lambda = \frac{1}{1.097 \times 10^{7}} = \frac{9.12 \times 10^{-8}}{2} = 9.12 \times 10^{-8}$.

$n_1 = 2$

$n_2 = 3$

Now,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$= 1.097 \times 10^{7}\left(\frac{1}{4} - \frac{1}{9}\right)$$

Page No.:
Date: / /

or, $\lambda = \frac{1}{15236111.111}$

$= 6.56 \times 10^{-7}$ m. ✗

Q. Physics student perform on experiment and claims that 13.6eV energy is sufficient to excite an electron of H-atom from ground state to infinity. Do you agree with this statement if yes then calculate wavelength of radiation incident on an atom?

→ We know,

$$E = -\frac{13.6eV}{n^2}$$

For n=1,

$$E_1 = -13.6eV.$$

For $n = \infty$,

$$E_2 = \frac{-13.6}{\infty} = 0$$

$\therefore E = E_2 - E_1$

$= 0 - (-13.6eV)$

$= 13.6eV.$

~~Also~~, Yes we agree with this statement and

Now,

$$E = \frac{hc}{\lambda}$$

$$\text{or, } \lambda = \frac{hc}{E}$$

CamScanner

Page No.:
Date: / /

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{(13.6 \times 1.\cancel{0}6 \times 10^{-19})}$$

$$= 9.13 \times 10^{-8} \text{ m.}$$

Page No.:
Date: / /

* Heisenberg Uncertainty Principle:

Statement: Canonically conjugate variables can't be measured accurately and simultaneously.

Canonically conjugate variables are:

1) Linear momentum and position
11) Angular momentum and angular position.
111) Energy and time.

Mathematically, for linear momentum & linear position,

$$\Delta p \cdot \Delta x \geq \hbar$$

OR,

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$$

where, Δx = uncertainity in the measurement of position.
Δp = uncertainity in the measurement of momentum (along x-direction)

Similarly, for angular momentum & angular position,

$$\Delta L \cdot \Delta \theta \geq \hbar$$

OR,

$$\Delta L \cdot \Delta \theta \geq \frac{\hbar}{2}$$

where,

$\Delta \theta$ = uncertainity in the measurement of angular position.

ΔL = uncertainity in the measurement of angular momentum.

Page No.:
Date: / /

Also, for Energy and time.

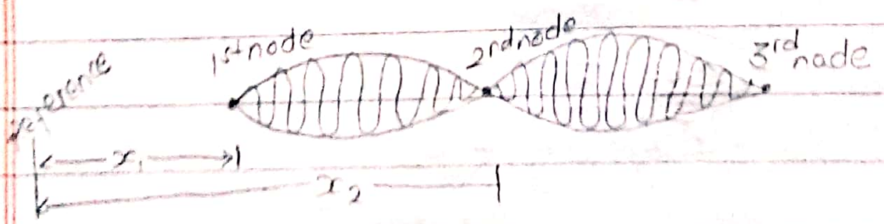
$$\Delta E \cdot \Delta t \geq \hbar$$

OR, $$\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$$

where, ΔE = uncertainity in the measurement of energy

Δt = uncertainity in the measurement of time.

→ Proof: According to Schrodinger; every moving particle is always associated with an infinite number of waves having slightly different frequency and wavelength. These waves are so selected that they interface constructively at a point/in a region where probability of finding ~~etc~~ of particle is maximum and they interface distructively in a region where probability of finding of particle is minimum. These waves are called wave group and their construction is wave packet.



consider two waves.

$$\psi_1 = a \sin(\omega t - kx) \quad — ①$$

$$\psi_2 = a \sin \{(\omega + \Delta\omega)t - (k + \Delta k)x\} — ②$$

Using principle of super position, resultant wave is

Page No.:
Date: / /

given by.

$$\psi = \psi_1 + \psi_2$$

$$\psi = a[\sin(\omega t - kx) + \sin(\omega + \Delta\omega)t - (k+\Delta k)x]$$

$$\psi = a\left[2 \times \sin\left(\frac{\omega t - kx + \omega + \Delta\omega - (k - \Delta k)x}{2}\right)\right]$$

$$\psi = a\left(\sin \frac{\omega t - kx + \omega + \Delta\omega - kx + \Delta kx}{2}\right).$$

$$\cos\left|\cos\left(\frac{\omega t - kx - \omega t - \Delta kx + \Delta kx}{2}\right)\right.$$

$$\left[\because \sin(c+\theta) = 2\sin\frac{c+\theta}{2}\cos\left(\frac{c+\theta}{2}\right)\right.$$

$$\text{or, } \psi = 2a\left\{\sin\left(\frac{2\omega t + \Delta\omega t - 2kx - \cancel{2}\Delta kx}{2}\right\}\cos\left(\frac{-\Delta\omega t + \Delta kx}{2}\right)\right]$$

$$\text{or, } \psi = 2a\cos\left(\frac{\Delta kx - \Delta\omega t}{2}\right)\sin\left(\frac{(2\omega + \Delta\omega)t}{2} - \frac{(2k + \Delta x)\cancel{x}}{2}\right.$$

$$\text{or, } \psi = 2a\cos\left(\frac{\Delta kx}{2} - \frac{\Delta\omega t}{2}\right)\sin(\omega' t - k' x)$$

where, $\omega' = \omega + \frac{\Delta\omega}{2}$

$$k' = k + \frac{\Delta k}{2}$$

$$\psi = A\sin(\omega' t - k' x) \quad \text{--- (3)}$$

where,

$$a = 2a\cos\left(\frac{\Delta kx}{2} - \frac{\Delta\omega t}{2}\right) \quad \text{--- (4)}$$

~~For nodes~~

CamScanner

For nodes,

$$A = 0$$

$$2a\cos\left(\frac{\Delta kx}{2} - \frac{\Delta\omega t}{2}\right) = 0 \quad \text{(using eqn (4))}$$

or, $\cos\left(\frac{\Delta kx}{2} - \frac{\Delta\omega t}{2}\right) = 0$

or, $\cos\left(\frac{\Delta kx}{2} - \frac{\Delta\omega t}{2}\right) = \cos\left(\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\right)$

or, $\left(\frac{\Delta kx}{2} - \frac{\Delta\omega t}{2}\right) = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$

$\left(\frac{\Delta kx - \Delta\omega t}{2}\right) = (2n+1)\frac{\pi}{2}$ where, $n = 0, 1, 2, 3, \ldots$

$\therefore \Delta kx - \Delta\omega t = (2n+1)\pi$

For 1st node at same time,

$$\Delta kx_1 - \Delta\omega t = (2\times 0 + 1)\pi$$

$$\Delta kx_1 - \Delta\omega t = \pi$$

For second node at same time,

$$\Delta kx_2 - \Delta\omega t = 3\pi \quad \text{--- (6)}$$

Now, subtracting eqn (5) from eqn (iv)

$$\Delta kx_2 - \Delta\omega t = 3\pi$$

$$\Delta kx_1 - \Delta\omega t = \pi$$

$$\Delta k(x_2 - x_1) = 3\pi - \pi$$

or, $\Delta k \cdot \Delta x = 2\pi$

or, $\frac{\Delta p}{\hbar} \cdot \Delta x = 2\pi$ $\left[\because p = \hbar k,\ \Delta p = \hbar\Delta k\right]$

or, $\Delta p \cdot \Delta x = 2\pi \cdot \hbar$

. radius of nucleus = 1×10^{-4} m

Page No.:
Date: / /

or, $\Delta p \cdot \Delta x > \hbar$ [$\because 2\pi h > \hbar$]

In general,

$$\Delta p \cdot \Delta x \geq \hbar$$ proved

* **Application of Heisenberg Uncertainity principle:**

① Non existence of election inside nucleus.

Suppose an electron is inside nucleus then max^m uncertainity in the measurement of electron is

$$(\Delta x)_{max} = 2r$$

where r = radius of nucleus
$\approx 10^{-14}$ m

$\therefore (\Delta x)_{max} = 2 \times 10^{-14}$ m.

From Heisenberg ~~p~~ uncertainity principle,

$$\Delta x \Delta p \geq \hbar$$

$$\Delta p \geq \frac{\hbar}{\Delta x}$$

$$(\Delta p)_{min} = \frac{\hbar}{(\Delta x)_{max}}$$

$$(\Delta p)_{min} = \frac{\hbar}{2 \times 10^{-14} m}$$

$$= \frac{6.62 \times 10^{-34}}{2\pi \times 2 \times 10^{-4}}$$ [$\because \hbar = h/2\pi$]

=

$MeV = 10^6 \times 1.6 \times 10^{-19}$
$MeV = 1.6 \times 10^{-13} J$

Page No.:
Date: / /

This is relativistic case and hence using.

$$E^2 = p^2c^2 + m_0^2c^4$$

or, $E = \sqrt{p^2c^2 + m_0^2c^4}$

or, $E_{min} = \sqrt{(\Delta P_{min})^2 c^2 + m_0^2 c^4}$

Using, $c = 3 \times 10^8 m/s$

$M_0 = 9.1 \times 10^{-31} kg$

∴ Experimental result shows that maximum energy of electron emitted in β-decay is 4 MeV. This means that electron doesn't exist inside nucleus.

② Energy and radius of electron in 1^{st} orbit of H-atom

We know that,

Energy of H-atom,

$$E = K.E + PE$$

$$= \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

$$= \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\frac{(+e)(-e)}{r}$$

$$= \frac{(mv)^2}{2m} - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

Page No.:
Date: / /

$$\therefore \boxed{E = \frac{p^2}{2m} - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}} \longrightarrow (1)$$

From Heisenberg Uncertainity principle, we have,

$$\Delta x \cdot \Delta p \geq \hbar \qquad (\because x = r)$$

$$\Delta p \geq \frac{\hbar}{\Delta r}$$

$$(\Delta P)_{min} = \frac{\hbar}{\Delta r} \qquad [\Delta P \rightarrow P,\ \Delta r \rightarrow r]$$

$$\therefore \boxed{P_{min} = \frac{\hbar}{r}} \longrightarrow (2)$$

From eq^{ns} (1) and (2)

$$E_{min} = \frac{p^2}{2m} - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

$$\boxed{E_{min} = \frac{\hbar^2}{2mr^2} - \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}} \longrightarrow (3)$$

for E_{min},

$$\frac{dE_{min}}{dr} = 0$$

$$\frac{d}{dr}\left(\frac{h^2 r^{-2}}{2m} - \frac{1}{4\pi\varepsilon_0} e^2 r^{-1}\right) = 0$$

or, $\frac{h^2}{\cancel{2}m} \times -\cancel{2} r^{-3} - \frac{e^2}{4\pi\varepsilon_0}(-1) \times r^{-2} = 0$

or, $-\frac{\cancel{2}h^2}{mr^3} + \frac{e^2}{4\pi\varepsilon_0 r^2} = 0$

CamScanner

or, $\frac{\cancel{-}\hbar^2}{mr^{\cancel{3}}} = \frac{\cancel{-}e^2}{4\pi\varepsilon_0 r^{\cancel{2}}}$

or, $\frac{\hbar^2}{mr} = \frac{e^2}{4\pi\varepsilon_0}$

or, $mr \times e^2 = \hbar^2\, 4\pi\varepsilon_0$

or, $r = \frac{4\pi\varepsilon_0\hbar^2}{m \times e^2}$

or, $r = \frac{\cancel{4}\cancel{\pi}\varepsilon_0}{me^2} \times \frac{h^2}{\cancel{4}\pi^{\cancel{2}}}$

$$\boxed{r = \frac{\varepsilon_0 h^2}{\pi m e^2}} \quad \text{——— (4)}$$

This is the radius of first orbit (Bohr's orbit) of H-atom.

Using eqn (4) in eqn (3)

$$E_{min} = \frac{\hbar^2}{2m}\left(\frac{\pi m e^2}{\varepsilon_0 h^2}\right)^2 - \frac{1}{4\pi\varepsilon_0} \times e^2 \times \frac{\pi m e^2}{\varepsilon_0 h^2}$$

$$E_{min} = \frac{\hbar^2}{2m}\,\frac{\pi^2 m^2 e^4}{\varepsilon_0^2 h^4} - \frac{m\, e^4 \pi}{4\pi\varepsilon_0^2 h^2}$$

or, $E_{min} = \frac{h^2}{4\pi^2\, 2m} \times \frac{\pi^2 m^2 e^4}{\varepsilon_0^2 h^4} - \frac{me^4}{4\varepsilon_0^2 h^2}$ $[\because \hbar = h/2\pi]$

or, $E_{min} = \frac{me^4}{8\varepsilon_0^2 h^2} - \frac{me^4}{4\varepsilon_0^2 h^2}$

$$= \frac{me^4}{4\varepsilon_0^2 h^2}\left(\frac{1}{2} - 1\right)$$

$$\boxed{E_{min} = -\frac{me^4}{8\varepsilon_0^2 h^2}} \quad \text{——— (5)}$$

Page No.:
Date: / /

∴ This is the energy of electron in 1st orbit of H-atom.

* Franck Hertz experiment:

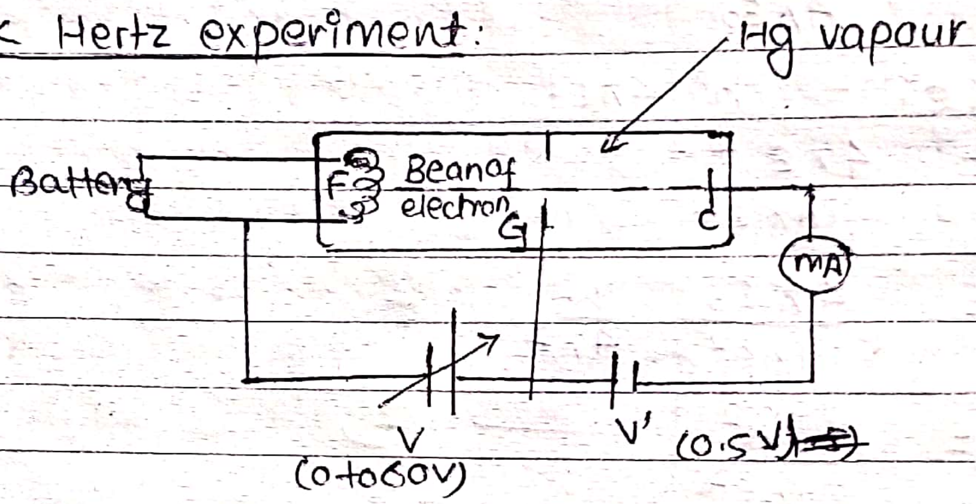
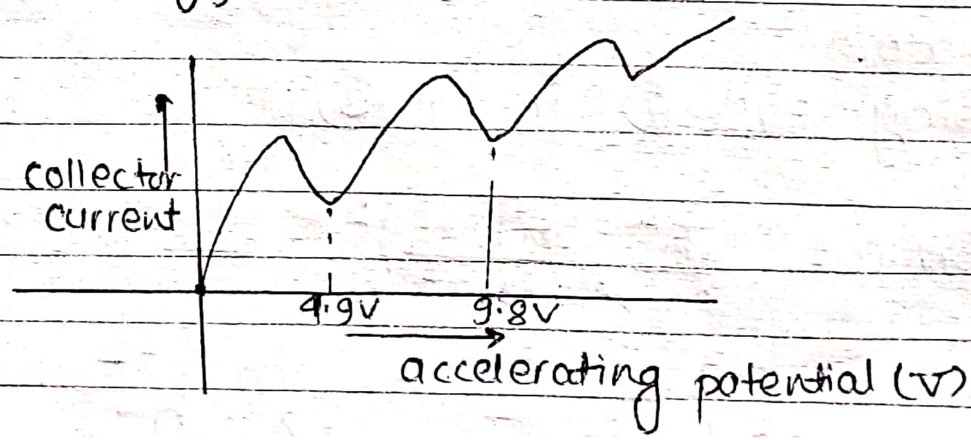


fig:- franck hertz experiment to explain discrete energy levels.



In this experiment, filament emits electrons when it is heated with the help of battery and these emitted electrons are accelerated with the help of variable battery of V volt. And the accelerated electrons undergo inelastic collision with mercury atoms present inside vaccum tube. Grid (G) is at positive potential with respect to collector (C) as shown in figure.

Page No.:
Date: / /

Experimental results shows that collector current is zero when $V < V'$ and current increases with increase in V for $V > V'$. But surprising result occur at 4.9V and 9.8V accelerating potential. Collector current sharply decreases at these potential and we can conclude that this is possible due to absorption of energy of 4.9eV and 9.8eV by mercury atoms.

This experimental result indicates that atom has discrete energy levels and supports bohr's atomic model.

Further, we observe that wavelength of radiation emitted during transition of electron of Hg-atom from 1st excited state to ground state is 2536 Å.

Then,

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{2536 \times 10^{-10}} = 4.9eV$$

* Limitation of Franck hertz experiment

This experimental cannot distinguish between excitation potential and ionization potential.

Black body

If a body absorbs all the radiation incident on it then it appear black and known as black body. Actually, black body absorbs all wavelength of incident radiation and according to Kirchoff's law of radiation, black body is a good emitter of radiation as it is good absorber of radiation.

Nobody is perfectly black body but for practical propose a copper empty vessel having small hole painted black to its inner surface is considered as black body.

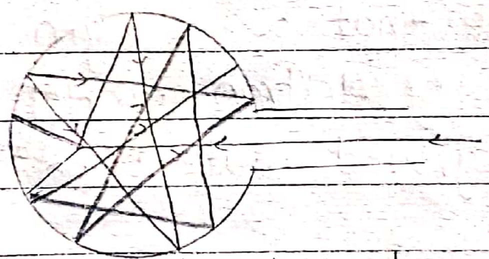


fig: black body.

* Stefan's law

statement: Energy radiated per unit area per unit time by a black body is directly proportional to the fourth power of ~~tem.~~ absolute temperature.

i.e.

$$\frac{\text{Energy}}{\text{Area} \times \text{time}} \propto T^4$$

$$E \propto T^4$$

$$\boxed{E = \sigma T^4}$$

where, $E = \frac{\text{energy}}{\text{Area} \times \text{time}}$

σ = stefan's constant.

T = temp^r in kelvin.

Stefan's law indicates that radiation emitted by a black body only depends upon the temperature of a black body and it is independent of nature of material of black body.

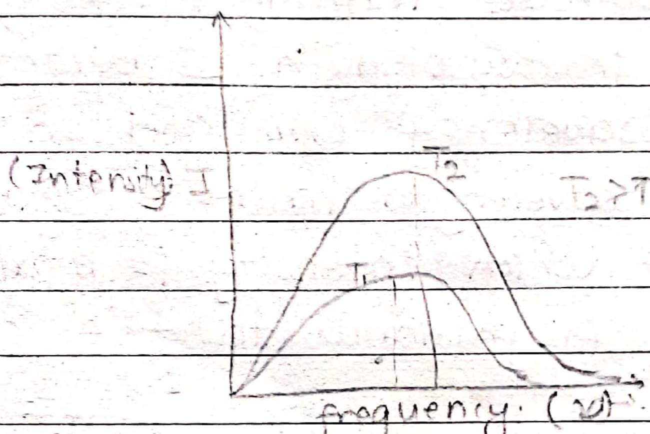


fig. graph between intensity & frequency of a black body at different tempr.

1) The spectrum is continuous with a board maximum.

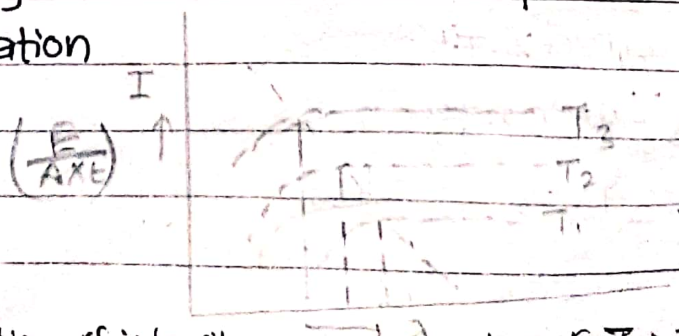
ii) The energy and intensity depends on fourth power of its temperature. ~~i.e~~

iii) This graph shows that at ~~higher temper~~ I/spectrum shifts towards right increasing the frequency as the temperature increases.

i.e, $\nu_{max} \propto T$

or

Energy distribution in a spectrum of black body radiation



Variation of intensity w.r.t. λ for tempr $T_3 > T_2 > T_1$

From graph, it is cleared that:

i) Non uniform distribution of intensity takes place w.r.to wavelength.

ii) For given temperature, initially intensity increases with increase in wavelength & becomes maximum for certain wavelength (λm) and on further increase in wavelength intensity decreases.

iii) Wavelength at which intensity is maximum (λm) is inversely proportional to the absolute temperature.

i.e, $\lambda m \propto \frac{1}{T}$ (Wien's displacement law)

iv) For given temperature, area enclosed by the curve gives energy radiated per unit area per unit time for range of wavelength.

v) ~~It is~~ Experimentally, it is found that area enclosed by curve is directly proportional to the fourth power of absolute temperature.

i.e, $E \propto T^4$ (stefan's law)

where,

$$E = \frac{\text{Energy}}{\text{Area} \times \text{time}}$$

$= \text{Intensity}$

Page No.:
Date: / /

→ According to classical concept, energy exchange between radiation & matter is continuous and average energy of classical oscillator is given by KT according to equipartition theorem.

where, K = Boltzmann constant.

$= 1.38 \times 10^{-23}$ J/K

and Number of modes of vibration per unit volume in the frequency range ν to $\nu + d\nu$ is $\left(\frac{8\pi\nu^2 d\nu}{c^3}\right)$

Now,

Energy density in the frequency range ν to $\nu + d\nu$ is; ~~$\left[\left(\frac{8\pi\nu\, d\nu}{c^3}\right)\right]$~~

$$E_\nu d\nu = \left[\left(\frac{8\pi\nu^2 d\nu}{c^3}\right)(KT)\right]$$

(Classical explanation known as Rayleigh Jeans law).

→ According to quantum physics (Planck's radiation law)

Energy exchange between radiation and matter is not continuous but it is of discrete values given by integral multiple of unit $h\nu$.

i.e,

$$\boxed{E = nh\nu}$$

where $n = 0, 1, 2, 3, \ldots$

Page No.:
Date: / /

Average energy of quantum oscillator is given by

$$\frac{h\nu}{e^{(h\nu/KT)}-1}$$

and number of modes of vibrations in the frequency range ν to $\nu+d\nu$ is

$$\left(\frac{8\pi\nu^2\,d\nu}{c^3}\right)$$

Now,

Energy density for frequency range ν to $\nu+d\nu$

$$E_\nu d\nu = \left(\frac{8\pi\,\nu^2\,d\nu}{c^3}\right)\left(\frac{h\nu}{e^{h\nu/KT}-1}\right)$$

This plank's radiation law holds good for ~~shoter~~ shorter and longer wavelength region but Rayleish Jeans law holds good for only longer wavelength region.