Tribhuvan University
Institute of Science and Technology
2078

GUPTA TUTORIAL

Bachelor Level / First Year/ First Semester/ Science
Computer Science and Information Technology (PHY. 113)
(Physics)
(NEW COURSE)

Full Marks: 60
Pass Marks: 24
Time: 3 hours.

Candidates are required to give their answers in their own words as for as practicable.
The questions are of equal value.

Section A

Long Answer Questions:
Attempt any TWO questions **(2x10=20)**

1. Discuss electrical conductivity of semieonductors. Derive expression for conductivity in terms of impurity ionization energy. Give a plot to discuss 'Theoretical temperature dependence of the electrical conductivity of an impurity semiconductor'. (10)

2. Describe processes involved in the fabrication of integrated circuits include epitaxial growth, oxidation, oxide removal and pattern definition, doping (impurities in the Si), and interconnection of components. (10)

3. Find expression for force on a current-carrying wire in a magnetic field to find the force experienced by a single charge. (10)

Section B

Short Answer Questions:
Attempt any EIGHT questions: **(8x5=40)**

4. Give spectrum of Hydrogen atom and discuss its lines. (5)

5. Give a brief account of Kronig-Penny model. (5)

6. Describe the term 'space quantization'. (5)

7. A given spring stretches 0.1 m when a force of 20 N pulls on it. A 2-kg block attached to it on a frictionless surface is pulled to the right 0.2 m and released.(a) What is the frequency of oscillation of the block? (b) What are the velocity and acceleration when $x = 0.12$ m, on the block's first passing this point? (5)

8. A proton is moving with a velocity $v = (3 \times 10^5 i + 7 \times 10^5 k)$ m/sec in a region where there is a magnetic field $B = 0.4 j$ T. Find the force experienced by the proton. (5)

12.12

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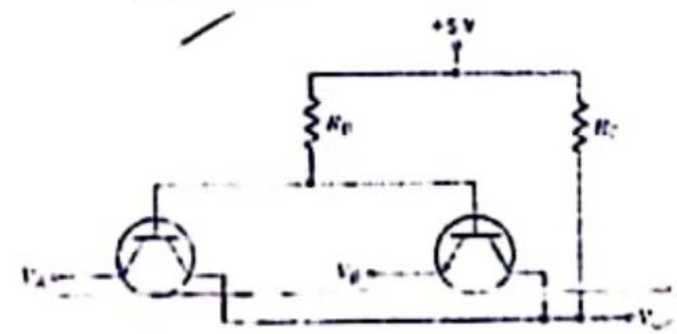
9. In neutron spectroscopy a beam of mono-energetic neutrons is obtained by reflecting reactor neutrons from a beryllium crystal. If the separation between the atomic planes of the beryllium crystal is 0.732 Å, what is the angle between the incident neutron beam and the atomic planes that will yield a monochromatic beam of neutrons of wavelength 0.1 Å? (5)

10. (a) How many atomic states are there in hydrogen with $n = 3$? (b) How are they distributed among the sub shells? Label each state with the appropriate set of quantum numbers n, l, m_l, m_s. (c) Show that the number of states in a shell, that is, states having the same n, is given by $2n^2$. (Hint: $1 + 2 + 3 + \ldots\ldots + n = n(n + 1)/2$). (5)

11. The density of aluminum is 2.70 g/cm^3 and its molecular weight is 26.98 g/mole. (a) Calculate the Fermi energy. (b) If the experimental value of E_F is 12 eV, what is the electron effective mass in alu-minum? [Aluminum is trivalent]. (5)

12. Analyze the circuit shown in the figure below. Determine the logic function performed by the circuit by making and justifying the appropriate truth table. (5)



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7. A given spring stretches 0.1m when a force of 20N pulls on it. A 2-kg block attached to it on a frictionless surface is pulled to the right 0.2m and released.
(a) What is the frequency of oscillation of the block?
(b) What are the velocity and acceleration when $x = 0.12m$. on the block's first passing this point?

Sol

Given,
The spring stretches $(x) = 0.1m$
Force $(F) = 20N$
Mass of block $(m) = 2kg$.
Distance toward right $(x) = 0.2m$

(a) Frequency of oscillation $(f) = ?$
(b) Velocity and acceleration when $x = 0.12m$ on the block's first passing $= ?$

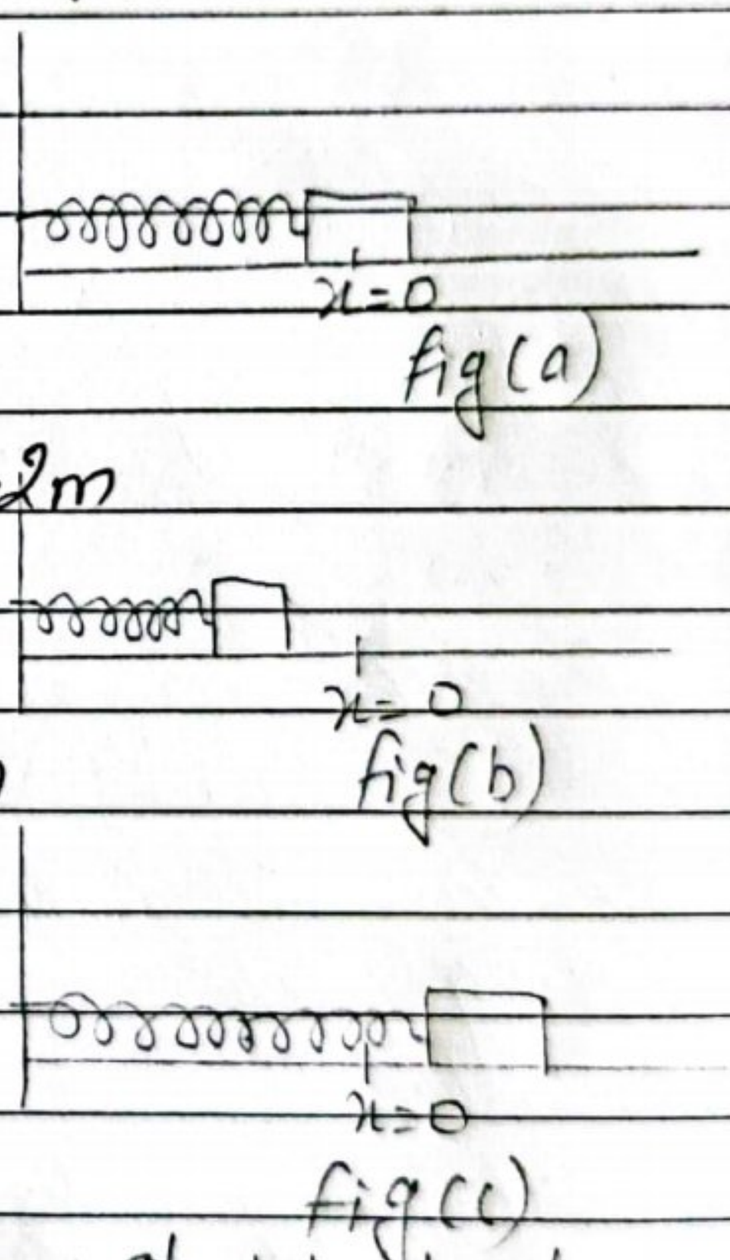

fig: Stretched string at different positions

we have,

$F = kx$ for spring where k = spring constant

or, $20 = k \times 0.1$

$\therefore k = \frac{20}{0.1} = 200 N/m$

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(a) Frequency (f) $= \frac{1}{2\pi}\sqrt{\frac{K}{m}}$

$= \frac{1}{2\pi}\sqrt{\frac{200}{2}}$

$= \frac{1}{2\pi}\sqrt{100}$

$= \frac{1}{2\pi} \times 10$

$\therefore f = 1.6 Hz$ Ans

(b) Velocity when $x = 0.12m$ passing this point,

$V = \omega\sqrt{r^2 - x^2}$

$= 2\pi f\sqrt{r^2 - x^2}$

$= 2\pi \times (1.6)\sqrt{(0.2)^2 - (0.12)^2}$

$\therefore V = 1.6 m/sec$ Ans

Again,
acceleration when $x = 0.12m$ passing this point,

$a = \omega^2 x$

$= (2\pi f)^2 \times x$

$= (2\pi \times 1.6)^2 \times 0.12$

$\therefore a = 12.12 m/sec^2$ Ans

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8. A proton is moving with a velocity $v = (3\times10^5 i + 7\times10^5 k)$ m/sec in a region where there is a magnetic field $\vec{B} = 0.4j$ T. Find the force experienced by the proton.

Sol

Given,
Velocity of proton $(\vec{v}) = (3\times10^5\vec{i} + 7\times10^5\vec{k})$ m/sec
Magnetic field $(\vec{B}) = 0.4\vec{j}$ T.
Mass of proton $(m_p) = 1.67\times10^{-27}$ kg
charge of proton $(q) = 1.6\times10^{-19}$ C
Force experienced by the proton $(\vec{F}) = ?$
we have,
Force experienced by proton = Lorentz force $= q(\vec{v}\times\vec{B})$

$$= 1.6\times10^{-19}[(3\times10^5\vec{i} + 7\times10^5\vec{k})\times 0.4\vec{j}]$$

$$= 1.6\times10^{-19}[1.2\times10^5\vec{k} - 2.8\times10^5\vec{i}]$$

$$= 1.6\times10^{-19}[1.2\vec{k} - 2.8\vec{i}]\times10^5$$

$$\vec{F} = (1.92\vec{k} - 4.48\vec{i})\times10^{-14}\,N$$ Ans

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9. In neutron spectroscopy a beam of monoenergetic neutrons is obtained by reflecting reactor neutrons from a beryllium crystal. If the separation between the atomic planes of the beryllium crystal is 0.732A°, What is the angle between the incident neutron beam and the atomic planes that will yield a monochromatic beam of neutrons of wavelength 0.1A°?

Sol Given,

separation between the atomic planes (d) = 0.732A° = 0.732×10^{-10} m

Wavelength (λ) = 0.1A° = 0.1×10^{-10} m

Angle (θ) = ?

we know,

$$2d\sin\theta = n\lambda$$

or, $$\sin\theta = \frac{n\lambda}{2d}$$

or, $$\theta = \sin^{-1}\left(\frac{n\lambda}{2d}\right)$$

or, $$\theta = \sin^{-1}\left(\frac{1 \times 0.1 \times 10^{-10}}{2 \times 0.732 \times 10^{-10}}\right)$$

$\therefore$ $\theta = 3.92^\circ$ Ans

$\therefore$ The angle between the incident neutron beam and the atomic planes is 3.92° Ans

10(a) How many atomic states are there in hydrogen with $n=3$?

(b) How are they distributed among the sub shells? Label each state with the appropriate set of quantum number n, l, m_l, m_s.

(c) Show that the number of states in a shell, that is, states having the same n, is given by $2n^2$ [Hint $1+2+3+\dots+n = \frac{n(n+1)}{2}$]

Sol^n

(a) Principle quantum number $(n)=3$

No. of atomic state $= \frac{n(n+1)}{2} = \frac{3(3+1)}{2} = 6$

(b)

S.N	State	n	l	m_l	m_s	No. of states in shell
1	For 1s state	1	0	0	1/2	1
2	For 2s state	2	0	0	1/2	1
3	For 2p state	2	1	0, ±1	1/2	3
4	For 3s state	3	0	0	1/2	1
5	For 3p state	3	1	0, ±1	1/2	3
6	For 3d state	3	2	0, ±1, ±2	1/2	5

No. of state in shell
s=1
p=3
d=5

(c) Now, number of states in shell having quantum number $1s^2, 2s^2, 2p^6, 3s^2 3p^6 = 18$

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11. The density of aluminum is $2.70 g/cm^3$ and its molecular weight is 26.98 g/mole.
(a) Calculate the Fermi energy.
(b) If the experimental value of E_F is 12eV, what is the electron effective mass in aluminum? [Aluminum is trivalent]

Sol: Given,
Density of aluminum (ρ) = $2.70 g/cm^3$
Molecular weight of aluminum (m) = 26.98 g/mole
(a) Fermi energy (E_F) = ?
(b) Fermi energy (E_F) = 12eV = $12 \times 1.6 \times 10^{-19} J$
Effective mass (m_e^*) = ?

we know that,
Number of free electron per unit volume (N) = $\frac{v \rho N_A}{m}$

where,
v = valency of atom
ρ = density , N_A = Avogardro's constant

then,
$$N = \frac{3 \times 2.70 \times (6.02 \times 10^{23})}{26.98}$$
$$N = 1.807 \times 10^{23} \text{ electrons}/cm^3$$
$$\therefore N = 1.807 \times 10^{29} \text{ electrons}/m^3$$

Now, we have,
$$E_F = \frac{\hbar^2}{2m_e}(3\pi^2 N)^{2/3}$$

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$$E_F = \frac{\left(\frac{6.626 \times 10^{-33}}{2\pi}\right)^2 \times (3 \times \pi^2 \times 1.807 \times 10^{29})^{2/3}}{2 \times 9.1 \times 10^{-31}}$$

$$E_F = 1.867 \times 10^{-18} J$$

$$E_F = \frac{1.867 \times 10^{-18}}{1.6 \times 10^{-19}} eV$$

$$\therefore E_F = 11.66 eV$$

(b) Again, For m_e^*

$$E_F' = \frac{\hbar^2}{2m_e} (3N\pi^2)^{2/3}$$

$$m_e^* = \frac{\hbar^2}{2E_F'} (3N\pi^2)^{2/3}$$

$$m_e^* = \frac{\left(\frac{6.626 \times 10^{-33}}{2\pi}\right)^2 \times (3 \times \pi^2 \times 1.807 \times 10^{29})^{2/3}}{(2 \times 12 \times 1.6 \times 10^{-19})}$$

$$m_e^* = \frac{8.847 \times 10^{-31}}{(9.1 \times 10^{-31})} kg$$

$$\therefore m_e^* \simeq 0.97 m_e$$

Hence, required Fermi energy and electron effective mass in Aluminum are 11.66 eV and 0.97 m_e respectively.

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12. Analyze the circuit shown in the figure below Determine the logic function performed by the circuit by making and justifying the appropriate truth table

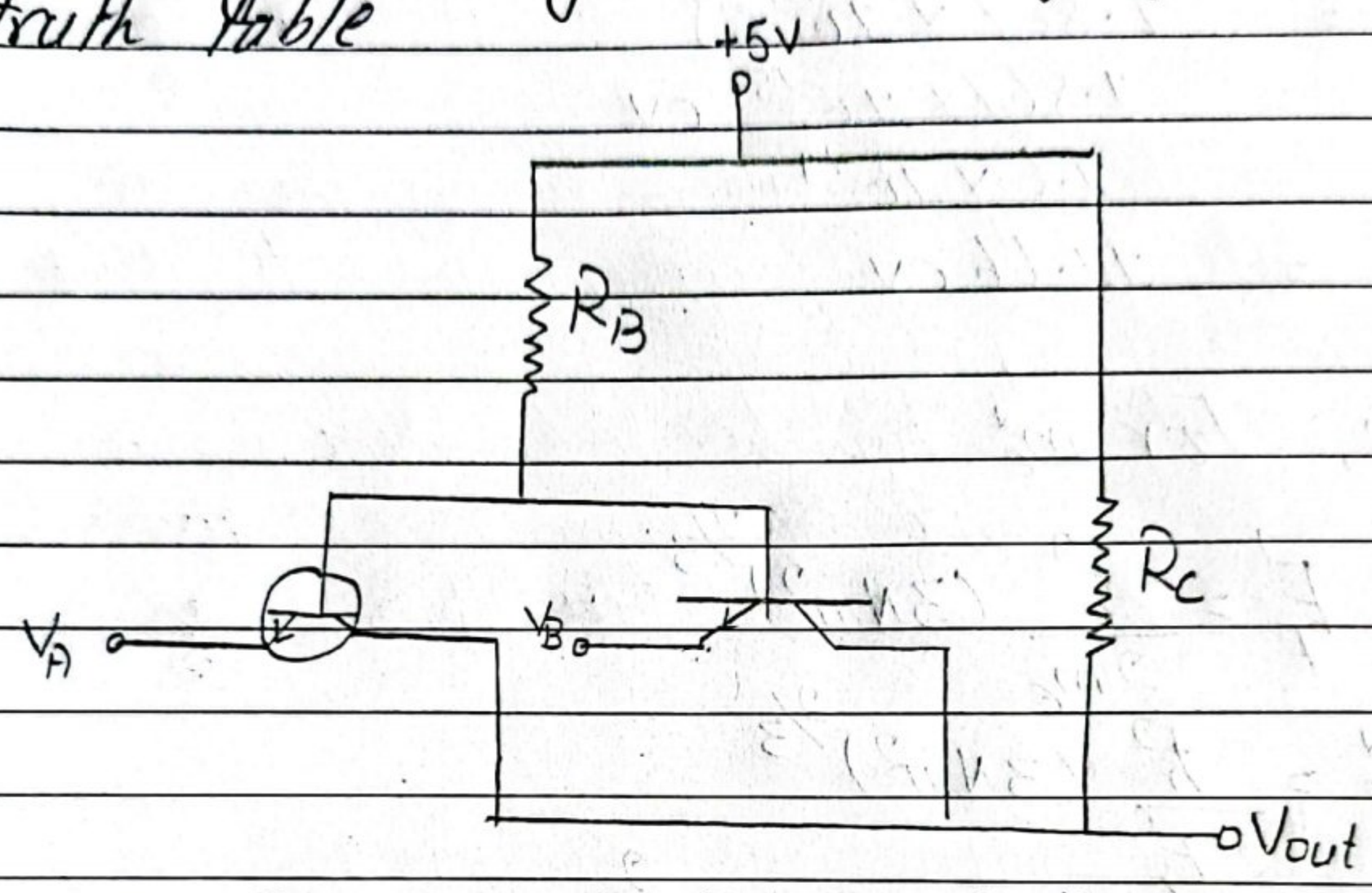

Fig:- Circuit diagram of logic function

Sol: The logic function performed by the given circuit:
Output $Y = V_A \cdot V_B$ i.e AND operation

Truth table

Inputs		Output
V_A(V)	V_B(V)	$Y = V_A \cdot V_B$ (V)
0	0	0
0	5	0
5	0	0
5	5	5

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Electrical Conductivity of Semiconductors

Q.N;1

When an electric field is applied to the semiconductor valance holes drift in the direction of applied field where as conduction electron drift in the direction opposite to the applied field. The drifting of both electrons in conduction band and holes in valence band contribute to current in one direction. Therefore the total current density in semiconductor can be written as:

$$J = n\,e\,v_{de} + p\,e\,v_{dh}$$

Where, n = electrons concentration in conduction band

p = holes concentration in valence band

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v_{de} = drift velocity of electron

v_{dh} = drift velocity of holes.

Since, $\mu = \frac{v_d}{E} \Rightarrow v_d = \mu E$

$\therefore J = n e \mu_e E + pe \mu_h E$

$\sigma E = (n e \mu_e + pe \mu_h) E$

$\sigma = n e\mu_e + p e \mu_h$

Where μ_e and μ_h are electrons and hole mobilities

$\mu_e = \frac{e \tau_e}{m_e^*}$, $\mu_h = \frac{e \tau_h}{m_h^*}$; τ_e and τ_h represents the mean time between the collisions with ions for electrons and holes respectively

m_e^* and m_h^* are the effective masses of electrons and holes respectively

i. Pure or Intrinsic Semiconductor

For an intrinsic semiconductor in equilibrium the density of holes is the valence band is equal to density of electrons in condition band.

i.e. $n = p = n_i$

Hence, $\sigma = en_i (\mu_e + \mu_h)$

Now,

$$n_i = 2\left(\frac{2\pi KT}{h^2}\right)^{\frac{3}{2}} (m_e^* m_h^*) e^{-\frac{E_g}{KT}}$$

Where E_g is the energy gap = $E_c - E_v$

$$\therefore \quad \sigma = 2e (\mu_e + \mu_h) \left(\frac{2\pi KT}{h^2}\right)^{\frac{3}{2}} (m_e^* m_h^*) e^{-\frac{E_g}{KT}}$$

or $$\ln \sigma = -\frac{E_g}{2K}\frac{1}{T} + \frac{3}{2} \ln T + C$$

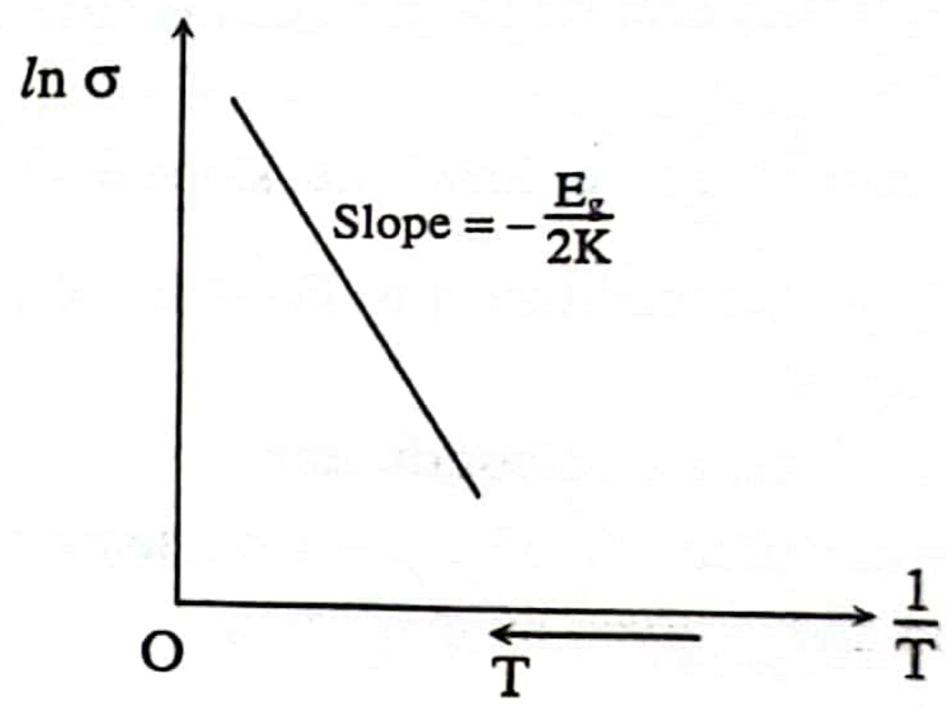


Figure 1: Temperature dependence of electrical conductivity of an Intrinsic semiconductor

where, $C = \text{constant} = \ln 2e + \ln T (\mu_e + \mu_h) + \frac{3}{2} \ln \frac{2\pi K}{h^2} + \frac{3}{4} (m_e^* m_h^*)$

It is also assumed that electron and hole mobilities are independent of temperature. The main variation in $\ln \sigma$ is due to the term containing $\frac{1}{T}$. When a graph is plotted between $\ln\sigma$ and $\frac{1}{T}$ gives a straight line as shown in figure (1).

The slope of the straight line gives the estimation of the value of band gap.

ii. n – type semiconductor

The conductivity of n – type semiconductor is mainly due to the electrons in the conduction band and is given by

$$\sigma = ne\,\mu_e$$

Now,

$$n_e = \frac{N_d}{1 + e^{\frac{E_f - E_d}{KT}}}$$

Therefore,

$$\sigma = e\mu_e N_d \frac{1}{e^{\frac{E_f - E_d}{KT}}}$$

or $\quad ln\,\sigma = -\,ln\left[e^{\frac{E_f - E_d}{KT}} + 1\right] + C_1$

where C_1 is a constant given by $C_1 = ln\, e\mu_e N_d$, N_d is the number of donor atoms per unit volume.

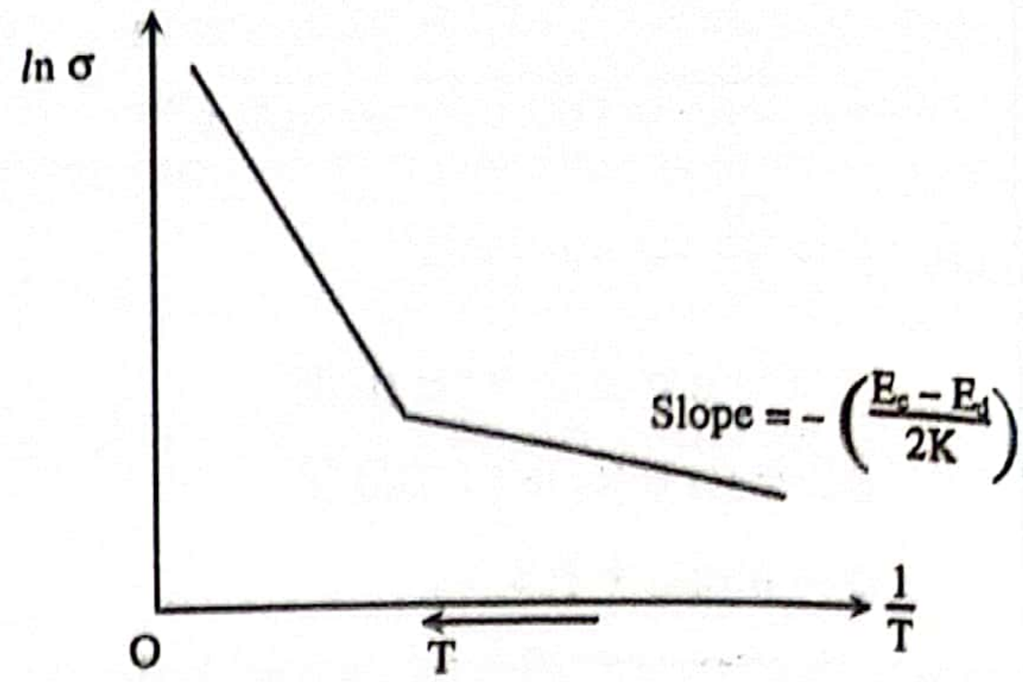


Figure 2: Dependence of electric conducitivity with temperature for an impurity semiconductor

When a graph is plotted between $ln\sigma$ and $\frac{1}{T}$ a straight line for high T and a flatter line for smaller T as shown in figure (2).

At low temperature ($E_f > E_d$), $ln\,\sigma$ vary linearly with $\frac{1}{T}$ and the slope will be $-\frac{(E_f - E_d)}{2K}$

for, $E_f = \frac{E_c + E_d}{2}$ at low T i.e. slope $= -\left(\frac{E_c - E_d}{2K}\right)$

At high temperature (as $E_d - E_f >> KT$) intrinsic conductivity begins to dominate the expression.

iii. p – type semiconductor

The conductivity of a p – type semiconductor is primarily due to the holes in the valence band and is given by $\sigma = n\,p\,\mu_h$

Now, $p = \frac{N_a}{e^{\frac{E_a - E_f}{KT}} + 1}$

$$\therefore \quad \sigma = e\,\mu_h N_a \frac{1}{e^{\frac{E_a - E_f}{KT}} + 1}$$

or $\quad ln\,\sigma = -\,ln\left[e^{\frac{E_a - E_f}{KT}} + 1\right] + C_2$

Where C_2 is a constant $= ln\, e\mu_h N_a$, N_a is the number of acceptor atoms per unit volume at acceptor level.

If we plot $ln\,\sigma$ against $\frac{1}{T}$, $ln\,\sigma$ behaves similar to that of an n – type semiconductor.

. Fabrication of a Simple IC (Fabrication of DTL NOR gate)

In the fabrication of an IC, parts of several circuit components are often formed simultaneously. This is illustrated with the example of fabrication of a DTL NOR gate as discussed step by step below.

Q.N;2

1. An n-type epilayer is grown on the p-type substrate

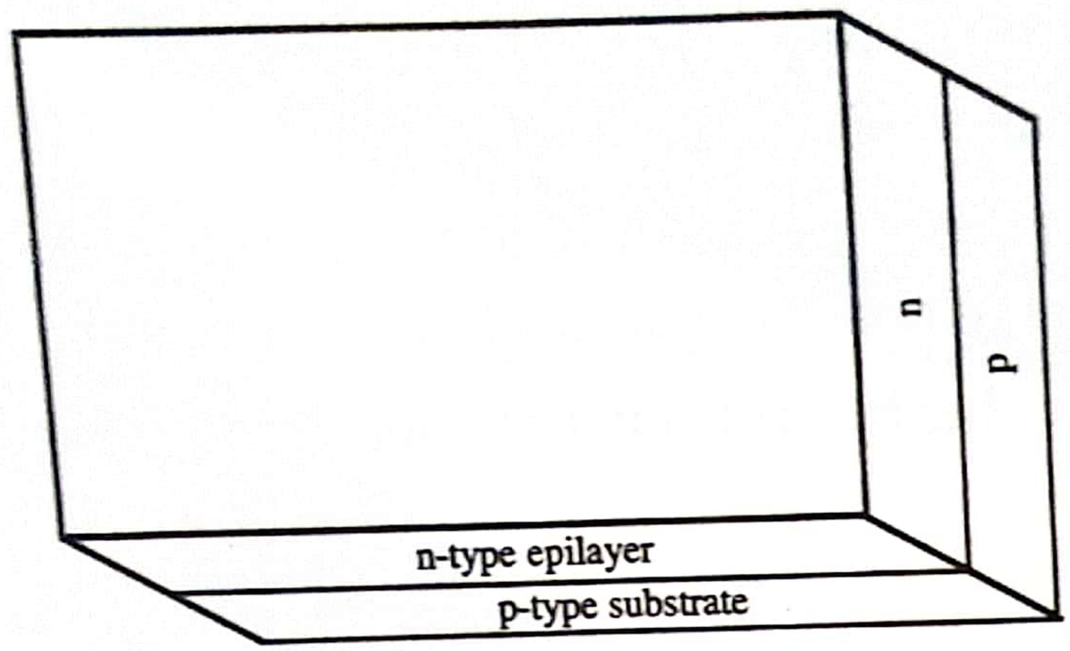


Figure 1

2. Coat with SiO_2 by oxidation and then with a photo-resist.

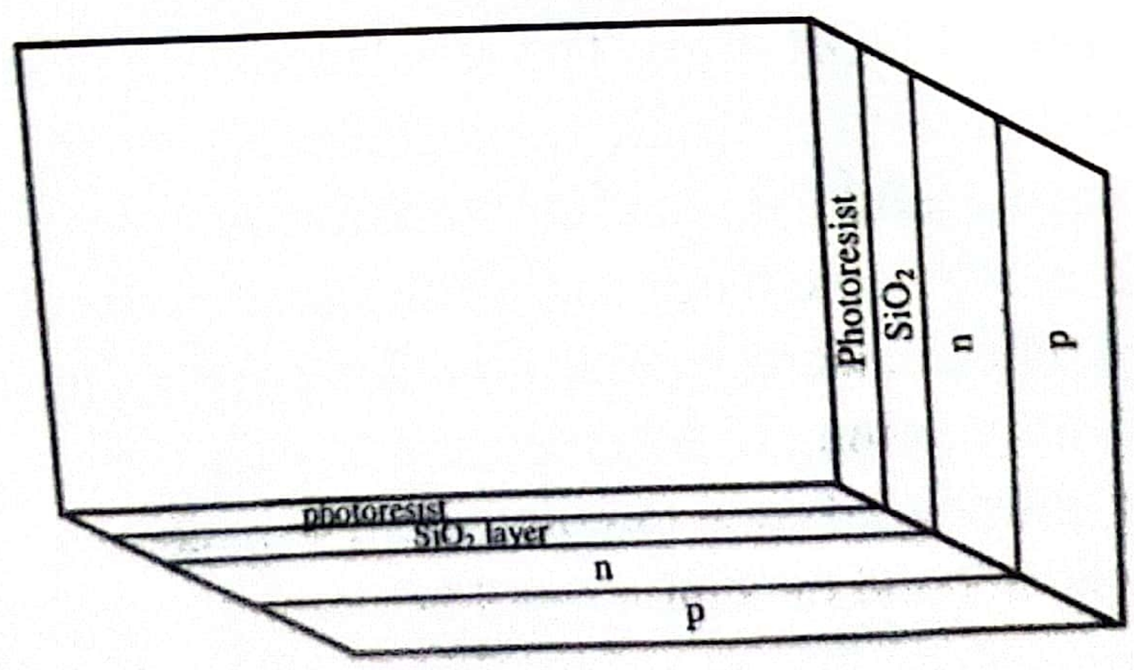


Figure 2

3. The wafer is exposed to uv radiation through the mask as shown in figure (3).

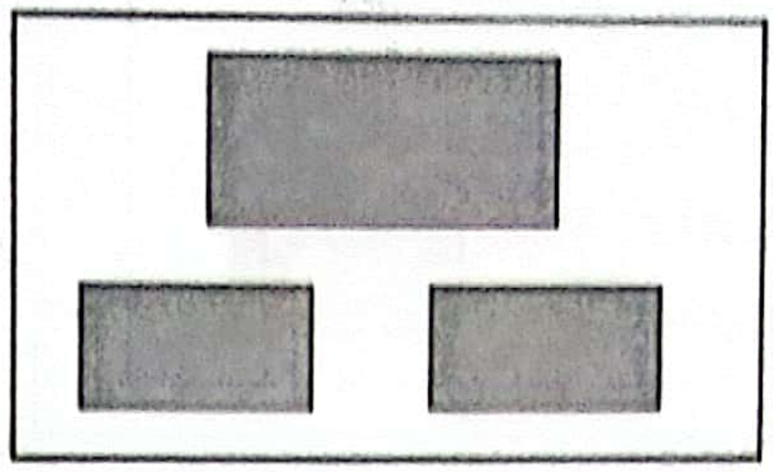


Figure 3

4. The photoresist in the region not covered by the mask is removed by uv radiation and the oxide is removed by etching. The islands of photo resist and oxide corresponding to mask region are-as shown in figure (4).

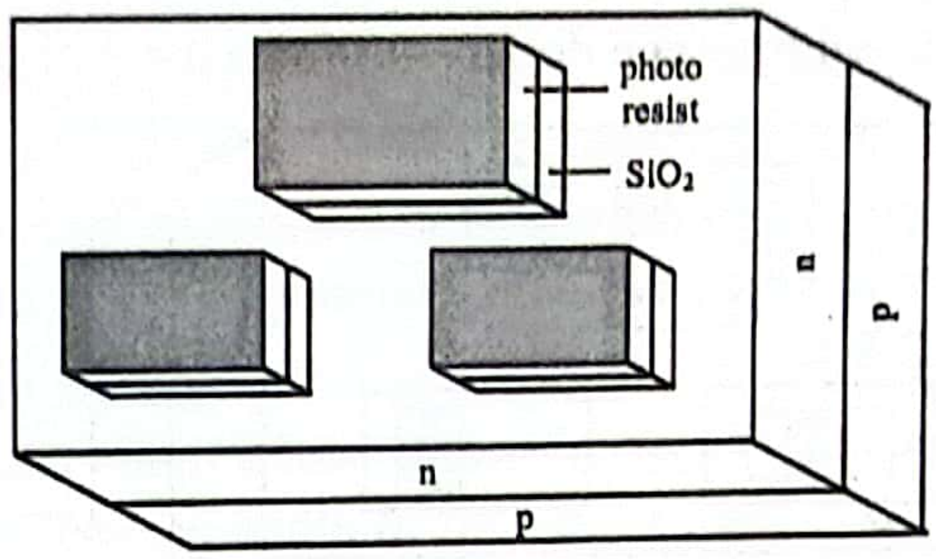


Figure 4

5. Dope the acceptors impurities on n-type epilayer so that the region except masked with SiO_2 and photo resist becomes P-type and the masked region remain n-type. Remove photo resist with photoresist remover and SiO_2 by etching.

Now the three n-type islands are formed on a p-type substate. The first island (i) serve to construct the transistor, another (ii) the diodes, and the third (iii) the three resistors.

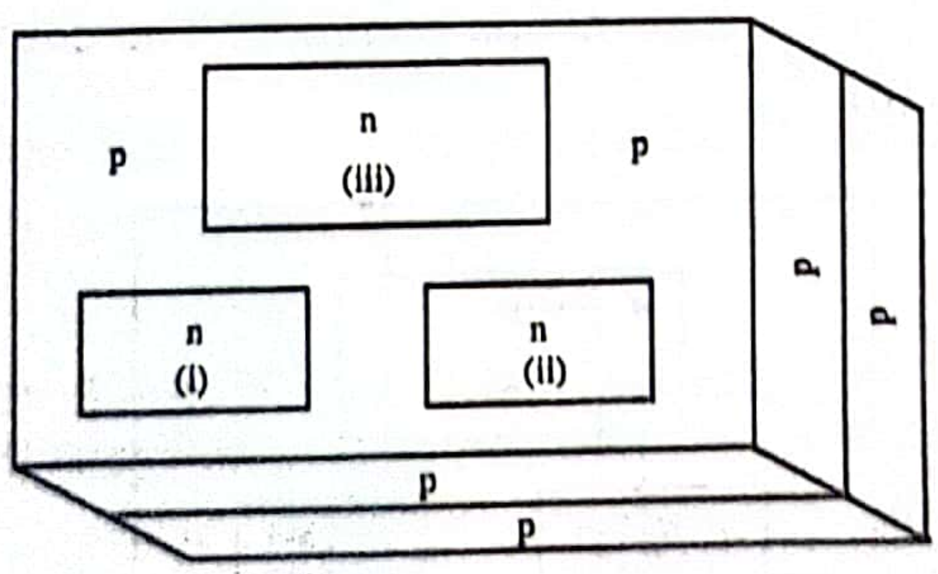


Figure 5

6. Now three n-type islands are doped with acceptor impurities as shown in figure (6). (after photolithography). In this case, the p-type dopant is not allowed to penetrate all the way through n-type layers. The SiO_2 layer used in photolithography is now removed by etching.

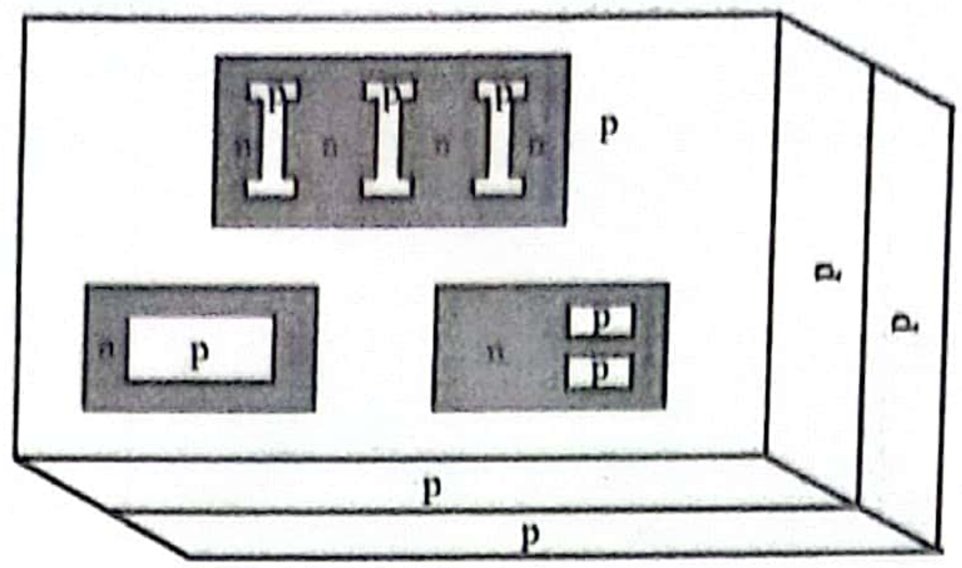


Figure 6

7. After the completion of step 6, there are three resistors in the upper island, the collector and base of npn transistor in the lower left island and two diodes with their n-side common in the lower right island.

Now the first island is doped with n-type dopants to form the emitter of npn transistor.

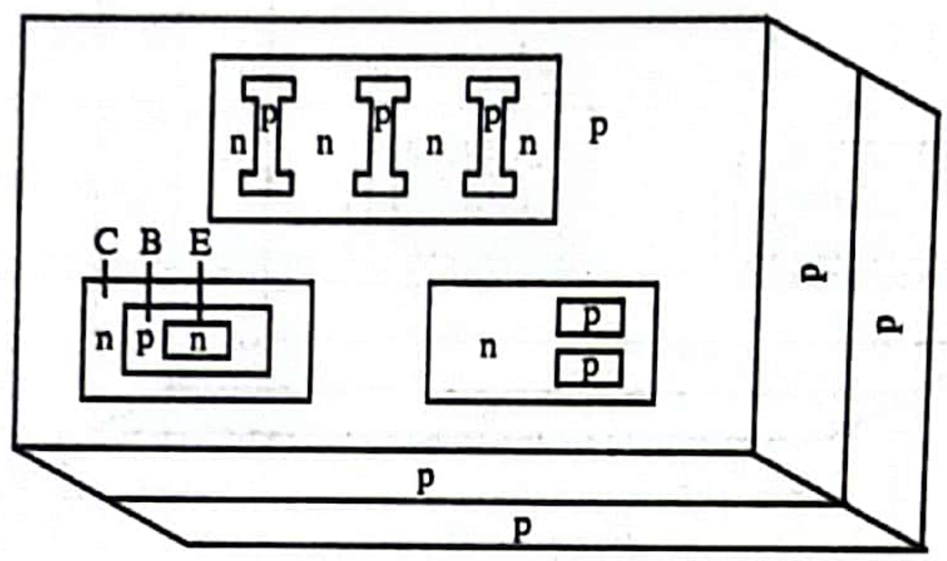


Figure 7

8. All the circuit components are now ready and they must now be interconnected. An aluminum evaporation (after masking the unwanted area by photolithography) is performed to made metal contact for interconnection. There are six metal contacts for three resistors, three metal contact for three components of npn transistor and two metal contacts for two diodes with one metal contact for their n-side common. The complete and interconnected circuit is as shown in figure (8).

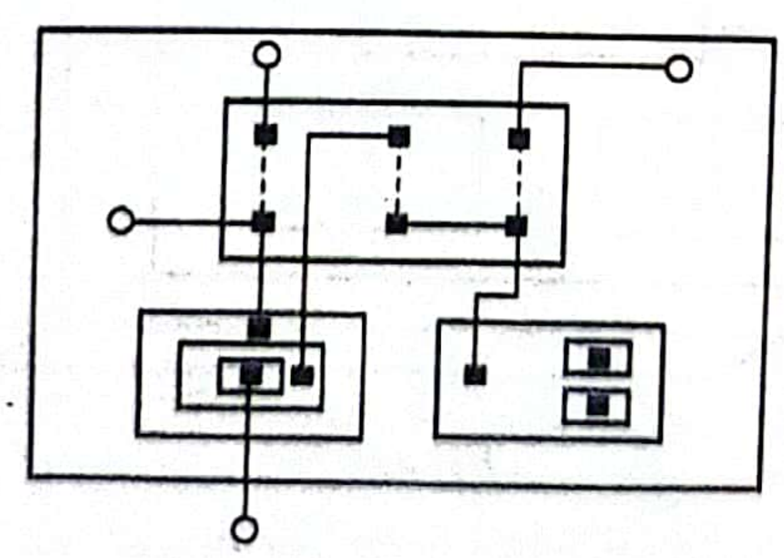


Figure 8

. Doping

An integrated circuit has informal name chip. The formation of circuit components in a chip s achieved by the selective introduction of donor and acceptor impurities into the Si wafer to create localized n-type and p-type regions. The two most commonly used techniques for doping are diffusion and ion implantation.

a. Diffusion

When Si is heated to temperatures in the range of 1000°C, some of the semiconductor atoms move out of their lattice sites, leaving behind empty lattice sites that can migrate through the sample. If the heating is done in an atmosphere of either acceptors or donors, these impurity atoms move into the vacant lattice sites formed at high temperature. The diffusion of the dopant impurities can be stopped by cooling down the wafer. Because the diffusion of impurities is time and temperature dependent, the depth of the diffusion layer can be controlled by varying these two parameters. The SiO_2 pattern, formed by photolithography acts as a mask that permits the diffusion of the impurities only in specific regions of the wafer.

The open furnace tube system using solid, liquid and gaseous dopant sources is the most common diffusion technology used in IC fabrication. The wafers are loaded vertically into a quartz boat and put into the furnace where the wafers are heated to high temperature. In general, diffusion systems are similar to oxidation furnace.

(i) **Solid Source:** In this system, the dopant source is in solid form as shown in figure (1). The carrier gases N_2 or O_2 picks up the vapour from the dopant source and transport it to the furnace tube, where the dopant atoms are deposited on the surface of wafer.

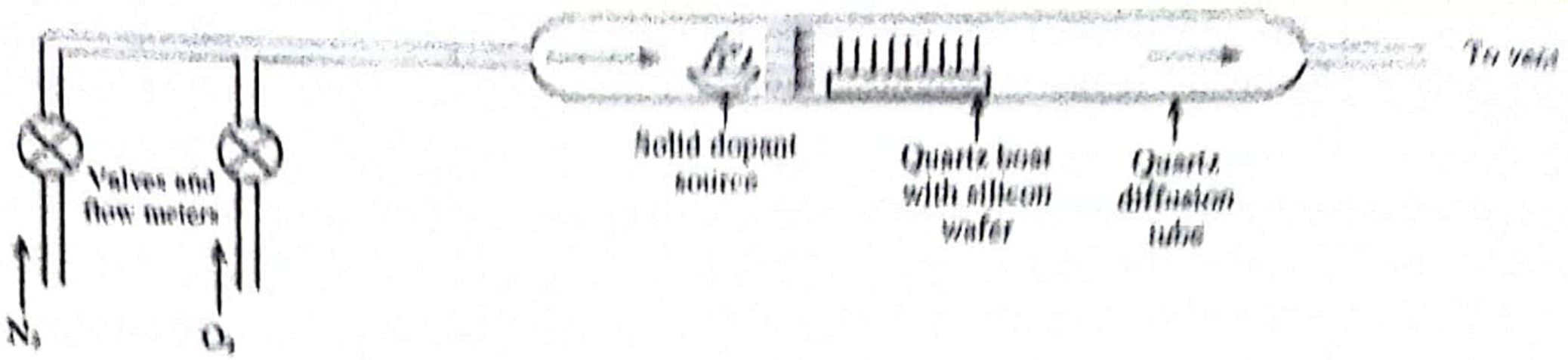


Figure (1): Open furnace tube diffusion system: Solid impurity source

The common solid source of Boron is Trimethyl Borate (TMB)

$$2\,(CH_3O)_3B + 9O_2 \xrightarrow{900^0 C} B_2O_3 + 6CO_2 + 9\,(H_2O)$$

$$2\,B_2O_3 + 3Si \leftrightarrow 4B + 3\,SiO_2$$

The common solid source of Phosphorous is Phosphorous Pentaoxide.

$$2P_2O_5 + 5\,Si \leftrightarrow 4P + 5\,Si\,O_2$$

(ii) Liquid Source

In this system the dopant source is in liquid form as shown in figure (2). The carrier gas passes through a bubbler where it picks up the vapour of the liquid source. The carrier gas carries the vapour into the furnace tube where it reacts with the surface of the silicon wafer.

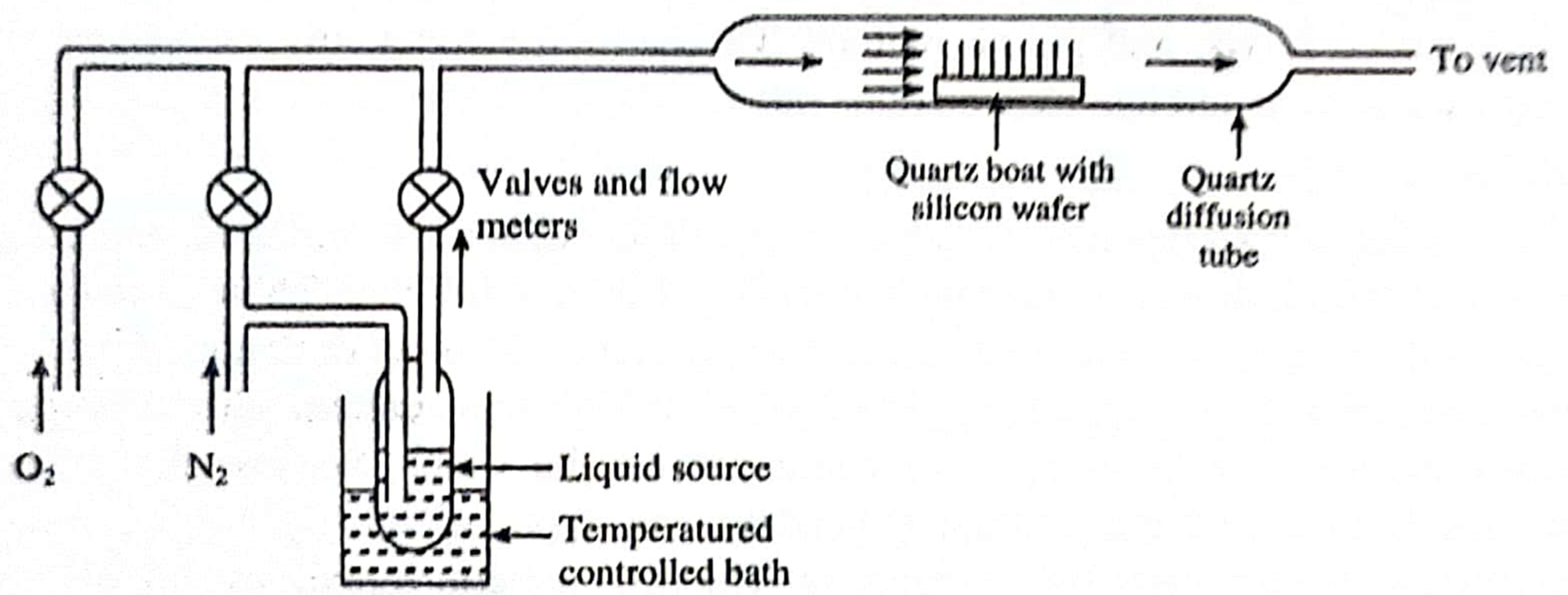


Figure (2): Open furnace tube diffusion system: Liquid impurity source

The most common liquid source of Boron is Boron Tribromide.

The reaction is

$$4\,BBr_3 + 3O_2 \rightarrow 2B_2\,O_3 + 6\,Br_2$$

$$2\,B_2O_3 + 3Si \leftrightarrow 4B + 3\,SiO_2$$

The common liquid source of 'P' is Phosphorous Oxychloride.

$$4\,P\,O\,Cl_3 + 3\,O_2 \rightarrow 2P_2O_5 + 6Cl_2$$

$$2\,P_2O_5 + 5Si \leftrightarrow 4P + 5SiO_2$$

. Epitaxy

Epitaxy refers to the deposition of a crystalline over layer on a crystalline substrate. The over layer is called an epitaxial film or epitaxial layer. If the over layer either forms a random orientation with respect to the substrate or does not form an ordered over layer, it is termed as non epitaxial growth. The term epitaxy comes from Greek roots 'epi' meaning 'above' and taxis meaning 'an ordered manner'.

a. In vapor phase epitaxy (VPE) atoms of Si from a vapor are deposited on the substrate in a layer that has the same crystal structure and orientation as the substrate. Thus the substrate serves as the seed crystal into which the epitaxial layer grows. This first layer in turn serves as the substrate for the second layer and so on.

A schematic diagram for vapor phase epitaxy is as shown in figure. Single crystal wafers of Si are placed in a heated chamber called the reacter. Gaseous compound of silicon ($SiCl_4$) together with the appropriate reactant gas are introduced into the reactor. The temperature of the reactor is adjusted to produce the reaction that will liberate the silicon by decomposition of the compound. Thus for example at 1250°C the following reaction occurs.

$SiCl_4 + H_2 \rightarrow Si + HCl$

Some of the Si atoms released in the reaction are deposited on the substrates, thereby forming epitaxial layers. If the chemicals used for the reaction are of high purity, the epitaxial layer of Si will be highly pure.

The Silicon VPE may also use pyrolytic (Pyrolytic: Chemical change because of heat) decomposition of silane.

$$Si\,H_4 \xrightarrow{650^\circ C} Si + 2H_2$$

This reaction is not reversible and takes place at lower temperature.

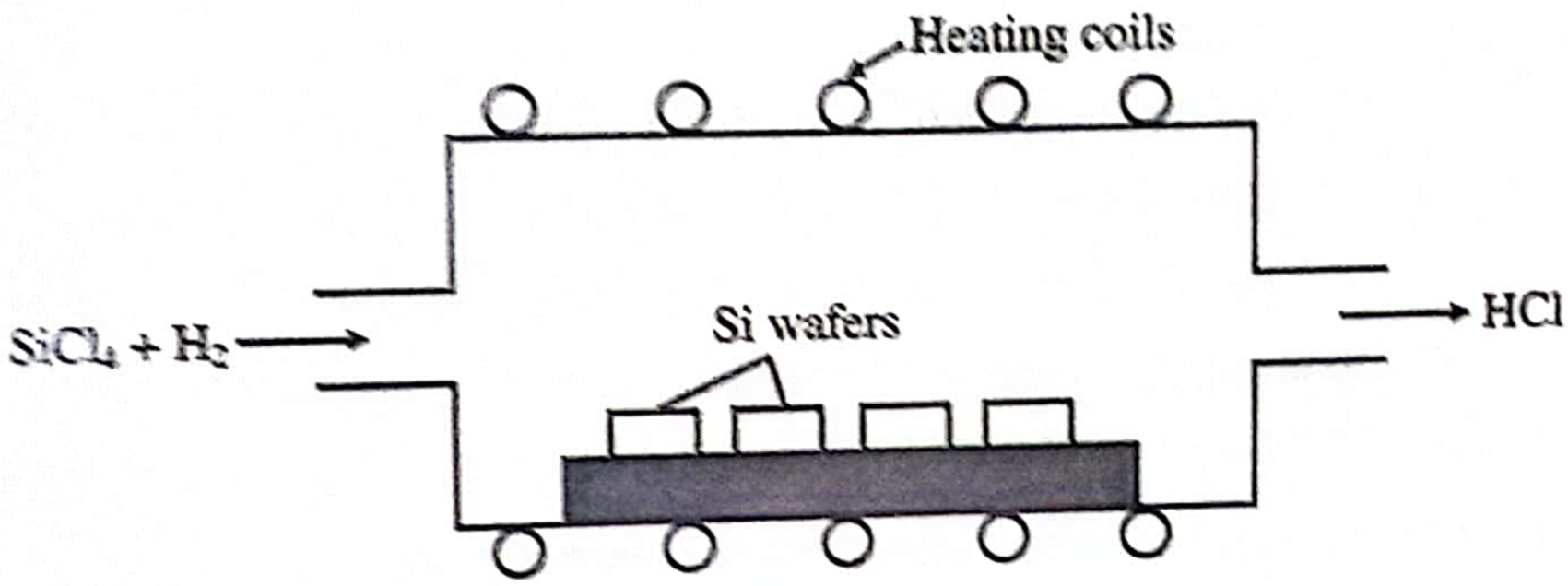


Figure: Vapor phase epitaxy.

b. In liquid phase epitaxy (LPE), the substrate is dissolved into the melt of the material to be deposited. This happens at temperature well below the melting point of the substrate material. The substrate acts as a seed for material crystallizing directly from the melt.

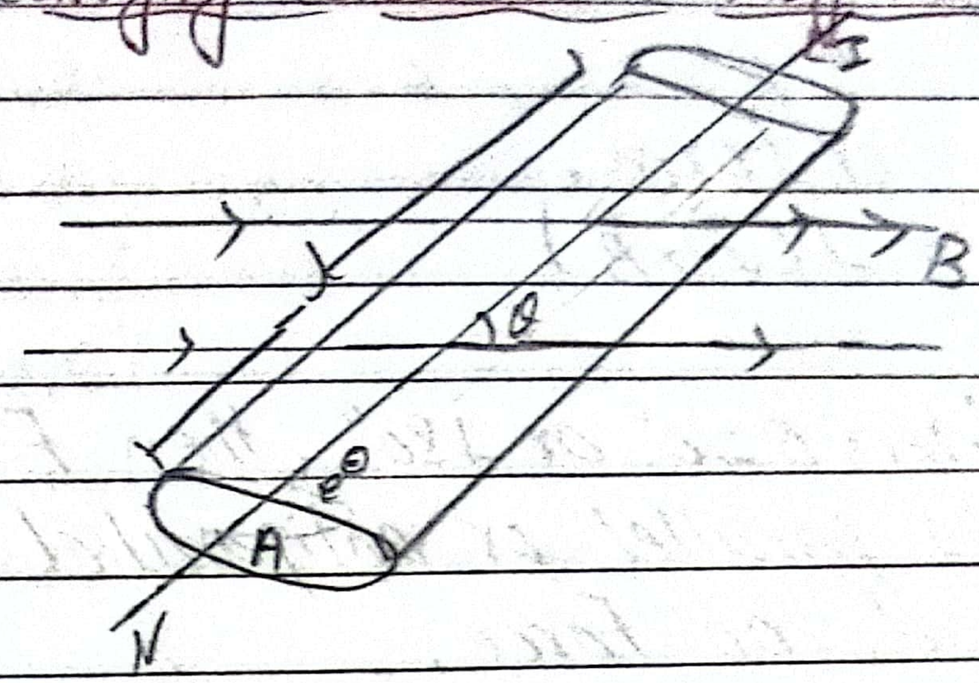
c. Solid phase epitaxy (SPE) is usually done by first depositing a film of amorphous material on a crystalline substrate. The substrate is then heated to crystallize the film.

d. In molecular beam epitaxy (MBE), a source material is heated to produce an evaporated beam of particles. These particles travel through a very high vaccum to the substrate where they condense. Substrate temperature during this process ranges from 400 - 900°C.

Force on current carrying conductors in magnetic field

⇒

Q.N;3



Consider a conductor of length 'l' and cross-sectional area 'A' placed in a magnetic field intensity (B) making an angle (θ) with it. Since, the conductor contains free electrons and these free electrons experienced force in magnetic field. Then,

the force experienced by single electron in the conductor by magnetic field is given by.

$$F_e = e(v \times B)$$

$$F_e = Bev\sin\theta \quad --- (1)$$

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And,

The total no. of electron (N) present in the conductor is

$N = nAl$

where n = no. of electrons per unit volume in the conductor

Also,

Total force experienced by conductor is the sum of Force experienced by all the electrons.

$F = N \cdot F_e$

$F = N \cdot Bev\sin\theta$

$F = nAl \cdot Bev\sin\theta$

$F = n(venA)l\sin\theta \quad (\because I = venA)$

$\boxed{F = BIl\sin\theta}$

And

In vector Form,

$F = I(\vec{l} \times \vec{B})$

Case-I :-

When $\theta = 0°$ or $180°$, then $F = 0$. It means when conductor is parallel or antiparallel to magnetic Field, it experienced no Force,

Case-II :-

when $\theta = 90°$, $F = BIl$. It means it experience maximum Force.

Spectral Lines Predicted by the Bohr Model

The energy levels of the Bohr atom are given by,

Q.N;4

$$E_{n_2} = \left(\frac{-me^4}{8\,\varepsilon_0^2\,h^2}\right)\frac{1}{n_2^2}$$

$$E_{n_1} = \left(\frac{-me^4}{8\,\varepsilon_0^2\,h^2}\right)\frac{1}{n_1^2}$$

Where E_{n_2} and E_{n_1} are the total energy of electron in the outer orbit of quantum number n_2 and total energy of electron in the inner orbit of quantum number n_1 respectively.

When the electron makes a transition from n_2^{th} orbit to the n_1^{th} orbit, the frequency f of the radiation emitted is given by

$$hf = E_{n_2} - E_{n_1}$$

$$= \left(\frac{-me^4}{8\,\varepsilon_0^2\,h^2}\right)\frac{1}{n_2^2} + \left(\frac{me^4}{8\,\varepsilon_0^2\,h^2}\right)\frac{1}{n_1^2}$$

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$$= \left(\frac{me^4}{8\,\varepsilon_0^2 h^2}\right)\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\therefore\ f = \frac{me^4}{8\,\varepsilon_0^2 h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Let λ be the wave length of radiation. Then $f = \frac{c}{\lambda}$; where c is the speed of light in vacuum.

$$\text{or,}\quad \frac{c}{\lambda} = \frac{me^4}{8\,\varepsilon_0^2 h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\text{or,}\quad \frac{1}{\lambda} = \frac{me^4}{8\,\varepsilon_0^2 c\, h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\therefore\quad \frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Where $R = \frac{me^4}{8\,\varepsilon_0^2 c\, h^3}$ is the Rydberg constant $= 1.097 \times 10^7\ m^{-1}$

This value is nearly the same as the experimental value for the hydrogen atom. The above equation is called Bohr's equation. The emission of all the spectral series of lines in the hydrogen spectrum are explained as follows:

1. Lyman series (in ultra-violet region, invisible)

   This series of lines is emitted by transition of the electron from outer orbits of quantum number, $n_2 = 2, 3, 4\ldots$ to the first orbit of quantum number $n_1 = 1$.

   The wavelength of this series of lines are given by, $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n_2^2}\right)$, where $n_2 = 2, 3, 4, \ldots$

2. Balmer Series (Visible): This series of lines is emitted by transitions of the electron from outer orbits of quantum number $n_2 = 3, 4, 5$ to the second orbit of quantum number $n_1 = 2$. Thus, the wavelength of this series of lines are given by

   $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_2^2}\right)$; where $n_2 = 3, 4, 5, \ldots$

   The lines are also called H_α, H_β, H_γ and H_δ lines for $n_2 = 3$, $n_2 = 4$, $n_2 = 5$ and $n_2 = 6$ respectively.

3. Paschen series (in infrared region, invisible)

   For this series $n_1 = 3$, and $n_2 = 4, 5, 6 \ldots$

4. Brackett series (in infrared region, invisible)

   For this series $n_1 = 4$, and $n_2 = 5, 6, 7, \ldots$

5. Pfund series (in far infrared region, invisible)

   For this series $n_1 = 5$, and $n_2 = 6, 7, 8, \ldots$

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The transitions responsible for some of the wavelength in each of the spectral series of hydrogen spectrum are shown in figure.

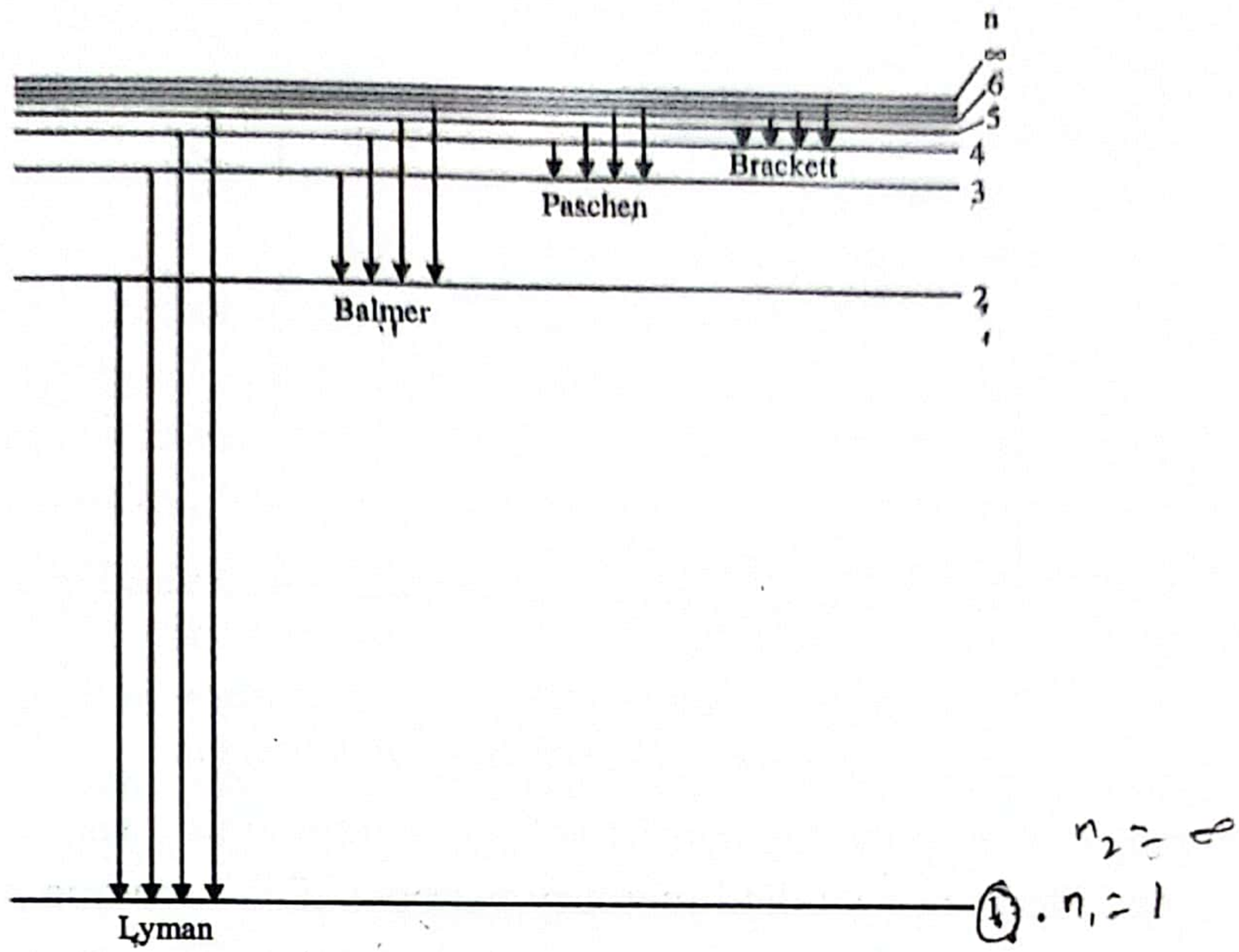

Figure: Energy levels and spectra of hydrogen

In the diagram, the energy of the electron $E_1 = -13.6$ eV in the ground state, and the values of energy $E_n = \frac{-13.6}{n^2}$ in the excited states for different values of n are plotted on a vertical scale, these energy levels are represented by horizontal lines.

As n becomes larger, the energy level becomes progressively closer. For $n = \infty$, $E_\infty = 0$. The energy in eV required to remove the electron from the ground state to infinity i.e. to the zero energy level is called ionization potential or the ionization energy for the hydrogen atom. Its value for the hydrogen atom is 13.6 eV.

The transitions of the electron, which give rise to the Lyman, Balmer, Paschen, Brackett and Pfund series of lines. As n increases the energy difference between successive energy level decreases. Therefore, the separation between the spectral lines of higher frequencies or smaller wavelengths in any series decreases.

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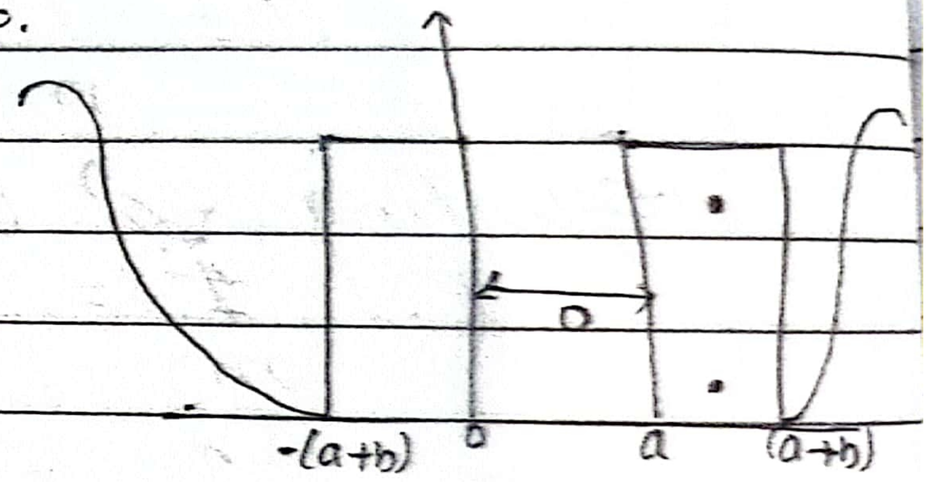
Kroning - Penny Model

It assume that the potential energy of an electron in a Linear array of positive nuclei has the Force of periodic array of square.

Q.N;5

The potential energy is given by

$$V(x) = 0 \quad \text{for} \quad 0 \le x \le a$$
$$= V_0 \quad \text{for} \quad -b \le x \le 0$$



The schrodinger wave equation is

$$-\frac{\hbar^2}{2m} * \frac{\delta \psi}{\delta x^2} + V(x)\,\psi(x) = E\psi(x) \quad \text{----} \quad (1)$$

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In region $0 \leq x \leq 0$

$$\frac{d^2\psi}{\Delta x^2} + \frac{2m\varepsilon}{\hbar^2}\psi_1(x) = 0$$

$$\frac{\Delta^2\psi}{\Delta x^2} + \alpha^2\psi_1(x) = 0 \quad \text{——— (2)}$$

In region $-b \leq x \leq 0$

$$\frac{\Delta^2\psi_2}{\Delta x^2} - \frac{2m}{\hbar^2}(V_0 - \varepsilon)\psi_2(x) = 0$$

$$\frac{\Delta^2\psi_2}{\Delta x^2} - \beta^2(\psi)(m) = 0 \quad \text{——— (3)}$$

Soln of eqn (2),

$$\psi_1(x) = Ae^{i\alpha x} + Be^{-i\alpha x} \quad \text{——— (4)}$$

Where,

$$\alpha^2 = \frac{2m\varepsilon}{\hbar^2} \quad ; \quad \beta = \frac{2m(V_0 - \varepsilon)}{\hbar}$$

Soln of eqn (3),

$$\psi_2(x) = Ce^{\beta x} + De^{-\beta x} \quad \text{——— (5)}$$

Since, the periodic potn the wave fxn is given by Bloch eqn,

$$\psi(x) = V_k(x)e^{ikx} \quad \text{——— (6)}$$

The soln of region $a \leq x \leq a+b$ is related with soln (5),

$$\psi_3(a \leq x \leq a+b) = \psi_2(-b \leq x \leq 0)$$

$$\psi_3(x) = (Ce^{\beta x} + De^{-\beta x})e^{ik(a+b)} \quad \text{——— (7)}$$

Applying boundary condn at $x = 0$,

let, $x = 0$

$$\psi_1(0) = \psi_2(0)$$

$$\psi_1'(0) = \psi_2'(0)$$

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From eqn (4) & (5),

$$A+B = (+D) \quad \text{———} \quad (8)$$

i.e $$(A-B) = \beta((-D) \quad \text{———} \quad (9)$$

Again,

at $x=0$, $\psi_1(a) = \psi_3(-b)$

$\psi_1'(a) = \psi_3'(-b)$

From eqn (4) & (7),

$$Ae^{i\alpha a} + Be^{-i\alpha a} = \left(Ce^{-\beta b} + De^{\beta b}\right)e^{ik(a+b)} \quad \text{———} \quad (10)$$

$$i\alpha\left(Ae^{i\alpha a} - Be^{-i\alpha a}\right) = \beta\left(-Ce^{-\beta b} + De^{\beta b}\right)e^{ik(a+b)} \quad \text{———} \quad (11)$$

The soln of these eqn is given by,

$$\begin{vmatrix} 1 & 1 & -1 & -1 \\ i\alpha & -1 & -\beta & \beta \\ e^{i\alpha a} & e^{-i\alpha a} & e^{-\beta b}\cdot e^{ik(a+b)} & -e^{\beta b}\cdot e^{ik(a+b)} \\ i\alpha e^{i\alpha a} & -i\alpha e^{i\alpha a} & -\beta e^{-\beta b}\cdot e^{ik(a+b)} & \beta e^{\beta b}\cdot e^{ik(a+b)} \end{vmatrix} = 0$$

By solving,

$$\frac{\beta^2-\alpha^2}{2\alpha\beta}\sinh\beta b\sin\alpha a + \cosh\beta b\cdot\cos\alpha a = \cos k(a+b)$$

Put, $\beta >>> \alpha$, & $b \to 0$

$$\frac{\beta^2}{2\alpha\beta}\beta b\sin\alpha a + 1\cdot\cos\alpha a = \cos ka \quad \text{———} \quad (12)$$

$$\frac{\beta^2 ab}{2\alpha a}\sin\alpha a + \cos\alpha a = \cos ka$$

Let, $P = \frac{\beta^2 ab}{2}$ so that,

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$$\frac{P}{\alpha a}\sin\alpha a + \cos\alpha a = \cos ka \quad \text{——— (13)}$$

The physical significance of P is that if P is increased, electron is bound strongly.

When $P \to 0$, potn barrier becomes weak, thus eqn (13) $P \to 0$

$$\cos\alpha a = \cos ka$$

$$\therefore \alpha = k$$

SBS,

$$\alpha^2 = k^2$$

$$k^2 = \frac{2mE}{\hbar^2}$$

$$\Rightarrow E = \frac{\hbar^2 k^2}{2m} \longrightarrow \text{This is the req expn}$$

Eqn (13) gives the condn which must be satisfied so that soln of wave eqn may exist. The value of $\cos ka$ lies "+1" to "-1".

Space Quantization: The Experiments

Q.N;6

1. The Zeeman Effect

Let an electron of mass m be rotating with speed v in a circular orbit of radius r around the nucleus of an atom. So it has a magnetic dipole moment associated with it, is called an orbital magnetic dipole moment μ_l.

i.e. $\mu_l = IA$ .... (1)

Where I is the conventional current in the direction opposite to v, A is the area of the loop.

The current is equivalent to the rate at which the electronic charge passes any given point on the orbit.

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$$\therefore \quad I = \frac{|e|}{T} = \frac{|e|}{\frac{2\pi r}{v}} = \frac{|e|\, v}{2\pi r} \qquad \ldots (2)$$

where T is the time period of rotation,

Then,

$$\mu_l = \frac{|e|\, v}{2\pi r} . A$$

$$= \frac{|e|\, v}{2\pi r} . \pi r^2$$

$$= \frac{|e|\, rv}{2} \qquad \ldots (3)$$

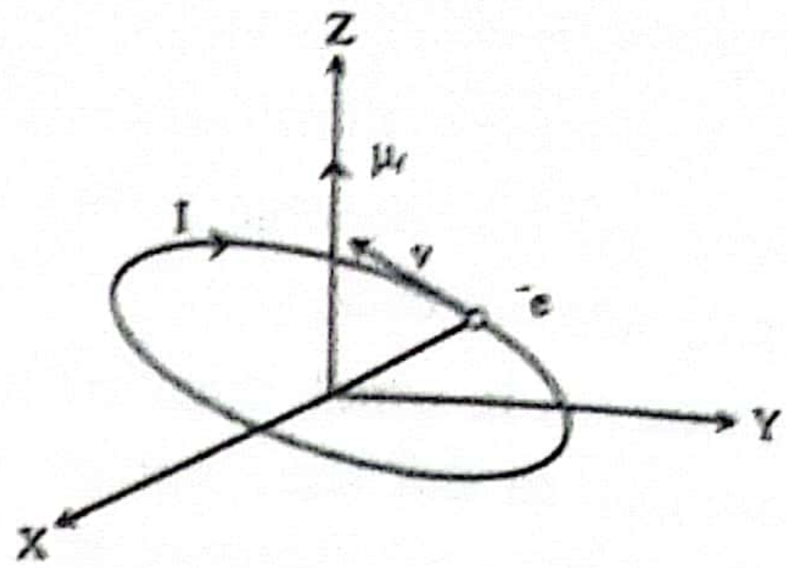

Figure 1: An electron revolving the nucleus

If L = mvr be the angular momentum then,

$$\mu_l = \frac{|e|\, rv}{2} = \frac{|e| mvr}{2m} = \frac{|e|}{2m} L$$

$$\mu_l = \frac{-|e|}{2m} L \qquad \ldots (4)$$

Negative sign is due to the direction of I is opposite to the direction of motion of electron and hence of L.

When the magnetic orbital dipole moment is placed in an external magnetic field B as shown in figure (2).

It will experience a torque

$$\tau = \mu_l \times B$$

$$= \mu_l B \sin\theta \qquad \ldots (5)$$

where θ is the angle between μ_l and B.

The torque is minimum when the dipole is perpendicular to the field and zero when it is parallel or antiparall to it.

The associated potential energy that depends on orientation of μ_l and B is given by

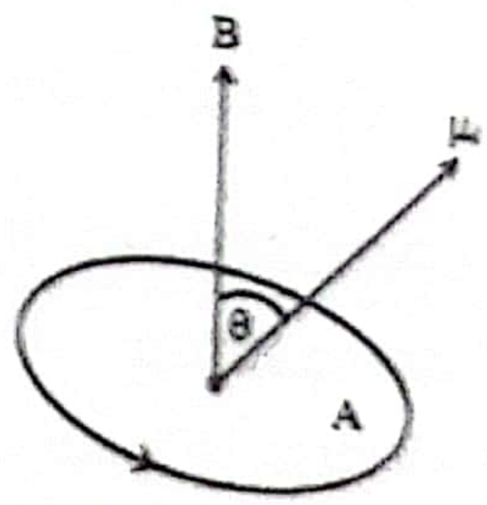

Figure 2: Orbital magnetic dipole moment in a magnetic field B

$$E_p = -\mu_l . B$$

$$= \frac{|e|}{2m} L.B$$

$$= \frac{|e|}{2m} BL \cos\theta$$

Since, $L_z = L \cos\theta$

or, $E_P = \frac{|e|}{2m} BL_z$ .... (6)

The total energy of the atom is the sum of energy resulting from the interaction of the electron with nucleus E_n and the energy resulting from the interaction of the magnetic dipole moment with the magnetic field.

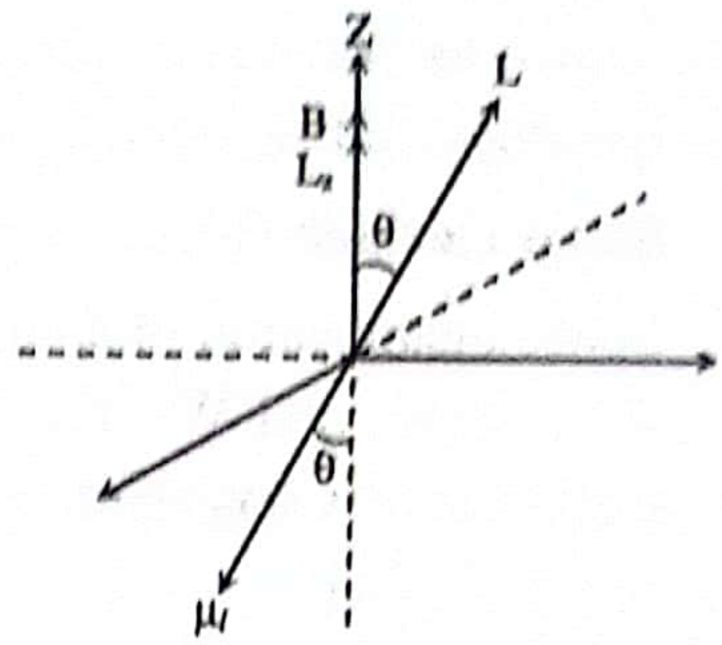

Figure 3: Relative orientation of L

i.e. $E_{total} = E_n + E_p$

$$= E_n + \frac{|e|}{2m} B\, L_z$$

Since $L_z = m_l\, \hbar$

Then,

$$E_{total} = E_n + \frac{|e|}{2m} \hbar B\, m_l \qquad \text{.... (7)}$$

As m_l takes the values 0, ± 1, ± 2, ..., ±l, the energy levels of quantum states with orbital quantum number l larger than 0 and thus, with several values of m_l, splits into several discrete sublevels as shown in figure (4).

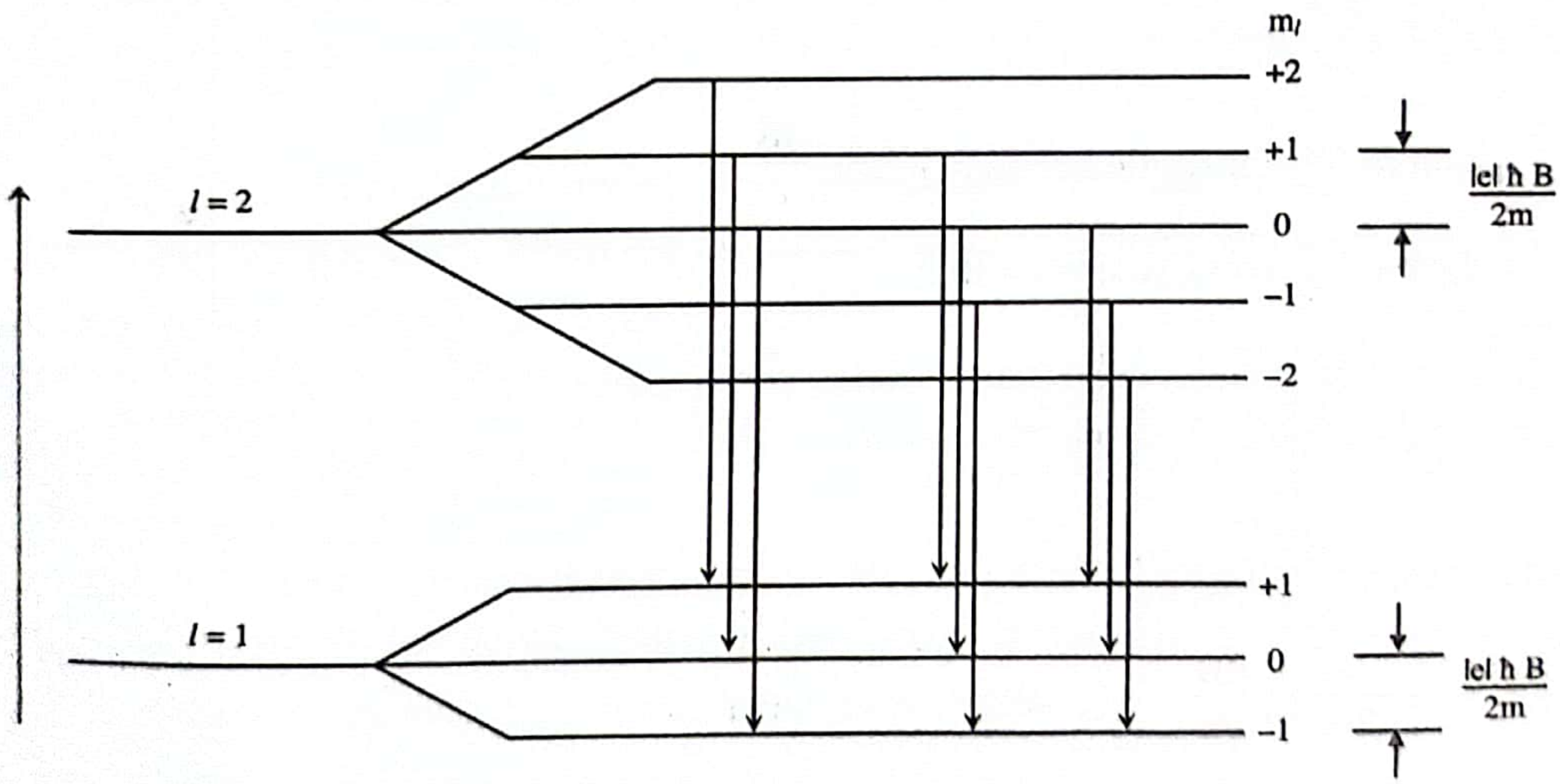

Figure 4: Energy level splitting in the Normal Zeeman Effect

In the transitions from higher n states, we may expect a greater number of lines. But the selection rule $\Delta m_l = 0, \pm 1$ restricts the number of possible lines. The nine transitions consistent with the selection rule give only three different energies because the energy differences between adjacent level is $\frac{|e|\hbar B}{2m}$ and is independent of L. This effect is known as Normal Zeeman Effect.