

Tribhuvan University  
Institute of Science and Technology  
2079  
★

Bachelor Level / First Year/ First Semester/ Science  
Computer Science and Information Technology (PHY. 113)  
(Physics )  
**(NEW COURSE)**

Full Marks: 60  
Pass Marks: 24  
Time: 3 hours.

*Candidates are required to give their answers in their own words as far as practicable.*  
The questions are of equal value.

### Section A

#### Long Answer Questions:

**(2x10=20)**

Attempt any TWO questions

1. Explain the meaning of 'fabrication of integrated circuits'. Describe following processes involved in the fabrication of integrated circuits: epitaxial growth, oxidation, oxide removal and pattern definition, doping and interconnection of components. [10]
2. Explain the effect of external magnetic field on current carrying loops. Describe torque on a current-carrying rectangular loop of wire on a pivot rod when placed in a magnetic field. Give alternative way of increasing the torque on the coil. [10]
3. What do you mean by the wavefunction? Discuss its physical significance. Set up time-independent and time-dependent Schrodinger wave equation. What are the implications of this equation? Discuss. [10]

### Section B

**(8x5=40)**

#### Short Answer Questions:

Attempt any EIGHT questions:

4. Derive expression for electrical conductivity of semiconductor in terms of impurity ionization energy. [5]
5. Describe behavior of mobile negative charges in the Hall effect experiment. [5]
6. Set up differential equation for an oscillation of a spring using Hooke's and Newton's second law. [5]
7. What are (a) the energy, (b) the momentum, and (c) the wavelength of the photon that is emitted when a hydrogen atom undergoes a transition from the state  $n = 4$  to  $n = 2$ ? [5]
8. An oscillating block of mass 250 g takes 0.15 sec to move between the endpoints of the motion, which are 40 cm apart. Find (a) frequency and (b) amplitude of the motion, and (c) force constant of the spring. [5]

9 A potential difference of 100 V is established between the two plates one being the high potential plate (say A). A proton of charge  $q = 1.6 \times 10^{-19}$  C is released from plate B, the other plate. What will be the velocity of the proton when it reaches plate A? The mass of the proton is  $1.67 \times 10^{-27}$  kg. [5]

10 An  $\alpha$ -particle is emitted from a radioactive nuclei with an energy of 6.8 MeV. Calculate its wavelength and compare it with the size of the emitting nucleus that has a radius of  $8 \times 10^{-15}$  m. [5]

11 (a) Calculate the Fermi energy aluminum that have density  $2.73 \text{ g/cm}^3$  and molecular weight 26.98 g/mole. (b) If the experimental value of Fermi energy ( $E_F$ ) is 11.8 eV, what is the effective mass of electron in aluminum? Aluminum is trivalent. [5]

12 The output of a digital circuit ( $y$ ) is given by this expression:

$$y = (\overline{AB} + \overline{BA})(\overline{A} + \overline{B} + C)$$

Where A, B and C represent inputs. Draw a circuit of above equation using OR, AND and NOT gate and hence find its truth table. [5]

$$N_A = 6.023 \times 10^{23}$$

$$K_B = ?$$

$$1.038 \times 10$$

In the fabrication of an IC, parts of several circuit components are often formed simultaneously. This is illustrated with the example of fabrication of a DTL NOR gate as discussed step by step below.

1. An n-type epilayer is grown on the p-type substrate

## Q.N;1 1st Part

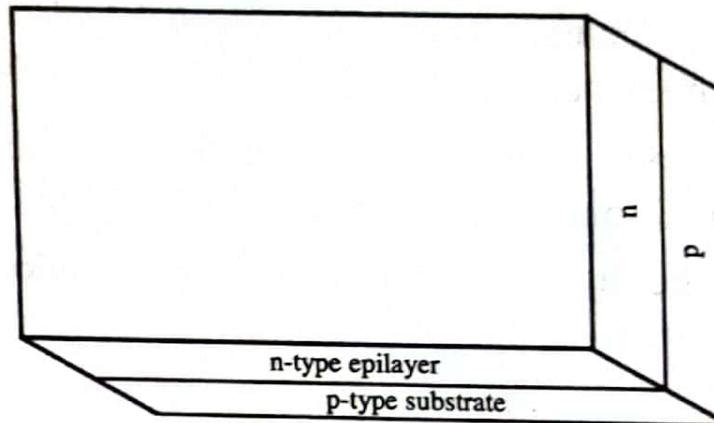


Figure 1

2. Coat with  $\text{SiO}_2$  by oxidation and then with a photo-resist.

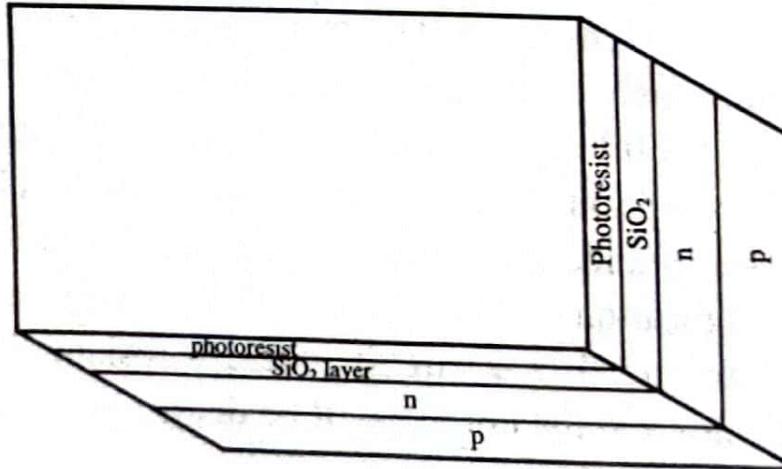


Figure 2

3. The wafer is exposed to uv radiation through the mask as shown in figure (3).

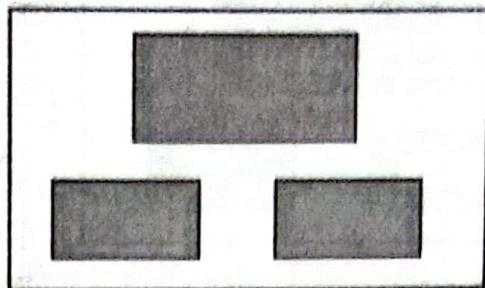


Figure 3

4. The photoresist in the region not covered by the mask is removed by uv radiation and the oxide is removed by etching. The islands of photo resist and oxide corresponding to mask region are as shown in figure (4).

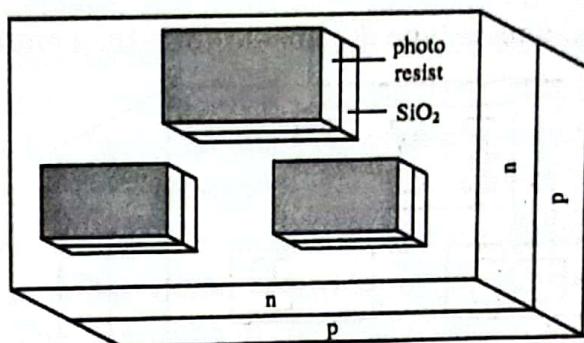


Figure 4

5. Dope the acceptors impurities on n-type epilayer so that the region except masked with  $\text{SiO}_2$  and photo resist becomes P-type and the masked region remain n-type. Remove photo resist with photoresist remover and  $\text{SiO}_2$  by etching.

Now the three n-type islands are formed on a p-type substrate. The first island (i) serve to construct the transistor, another (ii) the diodes, and the third (iii) the three resistors.

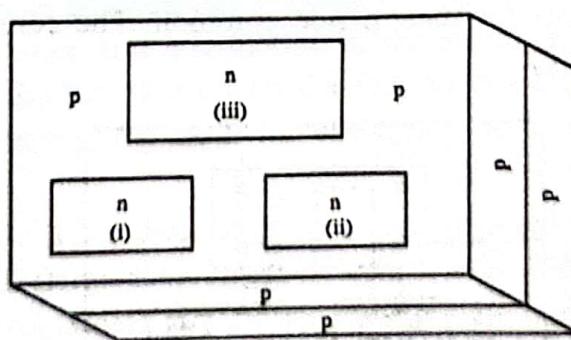
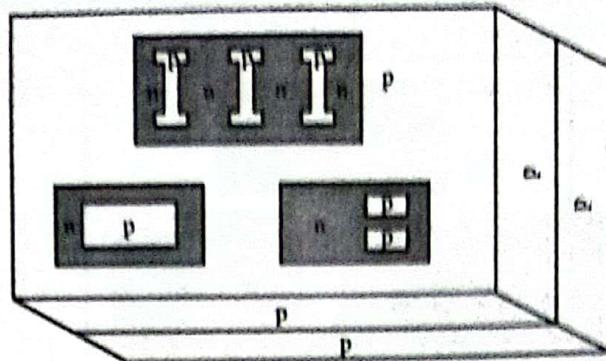


Figure 5

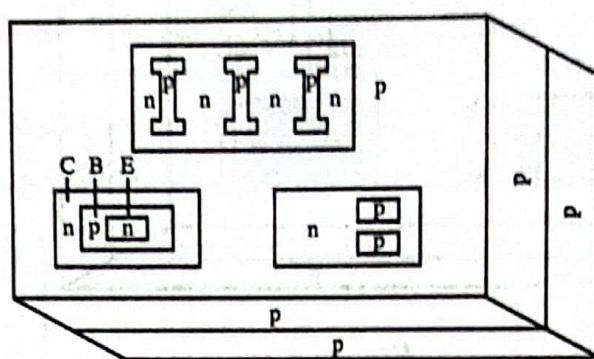
6. Now three n-type islands are doped with acceptor impurities as shown in figure (6). (after photolithography). In this case, the p-type dopant is not allowed to penetrate all the way through n-type layers. The  $\text{SiO}_2$  layer used in photolithography is now removed by etching.



*Figure 6*

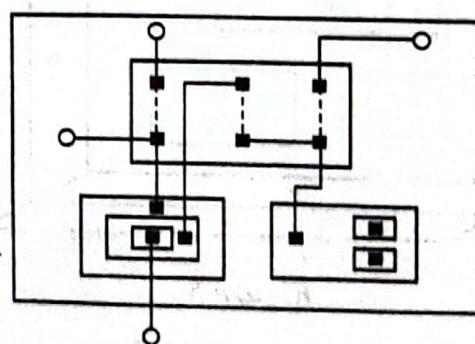
7. After the completion of step 6, there are three resistors in the upper island, the collector and base of npn transistor in the lower left island and two diodes with their n-side common in the lower right island.

Now the first island is doped with n-type dopants to form the emitter of npn transistor.



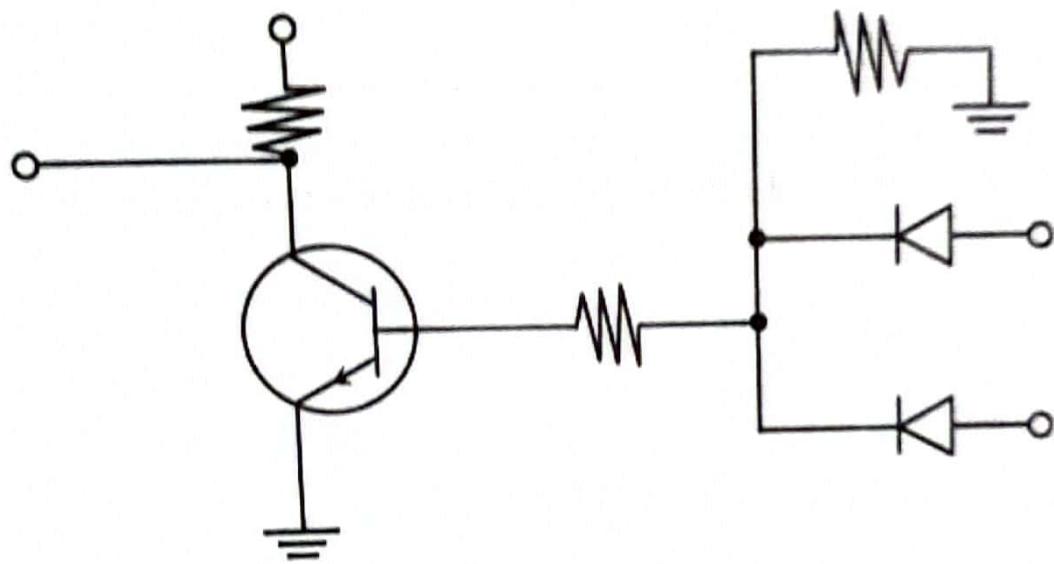
*Figure 7*

8. All the circuit components are now ready and they must now be interconnected. An aluminum evaporation (after masking the unwanted area by photolithography) is performed to make metal contact for interconnection. There are six metal contacts for three resistors, three metal contact for three components of npn transistor and two metal contacts for two diodes with one metal contact for their n-side common. The complete and interconnected circuit is as shown in figure (8).



*Figure 8*

9. The figure (8) can be drawn using electronic symbols as shown in figure (9) which is the required DTL NOR gate.



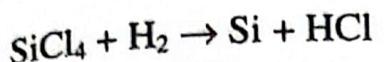
*Figure 9*

Epitaxy refers to the deposition of a crystalline over layer on a crystalline substrate. The over layer is called an epitaxial film or epitaxial layer. If the over layer either forms a random orientation with respect to the substrate or does not form an ordered over layer, it is termed as non epitaxial growth. The term epitaxy comes from Greek roots 'epi' meaning 'above' and 'taxis' meaning 'an ordered manner'.

## 2nd Part

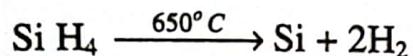
- a. In vapor phase epitaxy (VPE) atoms of Si from a vapor are deposited on the substrate in a layer that has the same crystal structure and orientation as the substrate. Thus the substrate serves as the seed crystal into which the epitaxial layer grows. This first layer in turn serves as the substrate for the second layer and so on.

A schematic diagram for vapor phase epitaxy is as shown in figure. Single crystal wafers of Si are placed in a heated chamber called the reactor. Gaseous compound of silicon ( $\text{SiCl}_4$ ) together with the appropriate reactant gas are introduced into the reactor. The temperature of the reactor is adjusted to produce the reaction that will liberate the silicon by decomposition of the compound. Thus for example at  $1250^\circ\text{C}$  the following reaction occurs.



Some of the Si atoms released in the reaction are deposited on the substrates, thereby forming epitaxial layers. If the chemicals used for the reaction are of high purity, the epitaxial layer of Si will be highly pure.

The Silicon VPE may also use pyrolytic (Pyrolytic: Chemical change because of heat) decomposition of silane.



This reaction is not reversible and takes place at lower temperature.

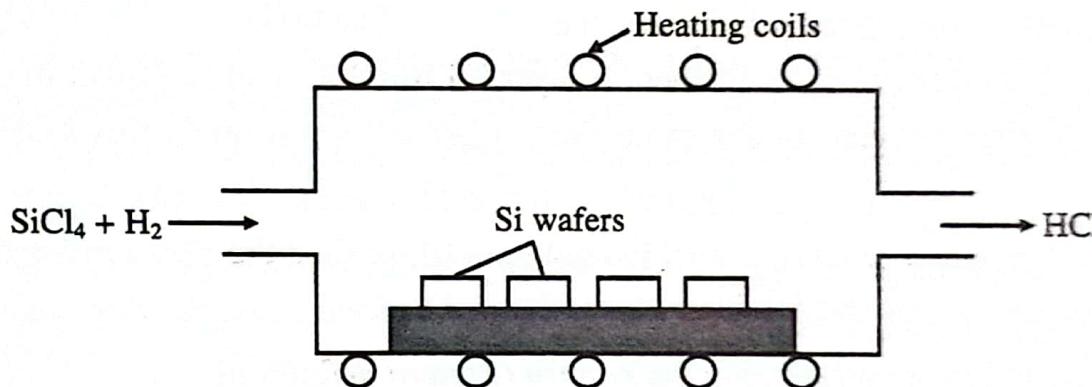


Figure: Vapor phase epitaxy.

- b. In liquid phase epitaxy (LPE), the substrate is dissolved into the melt of the material to be deposited. This happens at temperature well below the melting point of the substrate material. The substrate acts as a seed for material crystallizing directly from the melt.
- c. Solid phase epitaxy (SPE) is usually done by first depositing a film of amorphous material on a crystalline substrate. The substrate is then heated to crystallize the film.
- d. In molecular beam epitaxy (MBE), a source material is heated to produce an evaporated beam of particles. These particles travel through a very high vacuum to the substrate where they condense. Substrate temperature during this process ranges from 400 - 900°C.

## 2. Doping

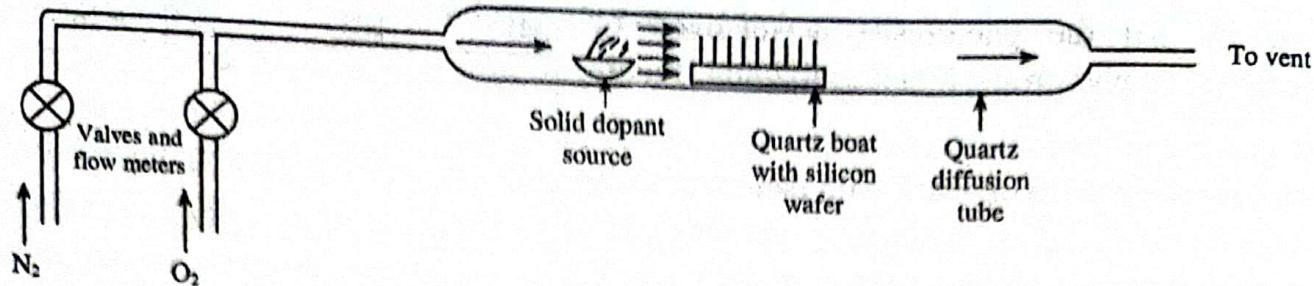
An integrated circuit has informal name chip. The formation of circuit components in a chip is achieved by the selective introduction of donor and acceptor impurities into the Si wafer to create localized n-type and p-type regions. The two most commonly used techniques for doping are diffusion and ion implantation.

### a. Diffusion

When Si is heated to temperatures in the range of 1000°C, some of the semiconductor atoms move out of their lattice sites, leaving behind empty lattice sites that can migrate through the sample. If the heating is done in an atmosphere of either acceptors or donors, these impurity atoms move into the vacant lattice sites formed at high temperature. The diffusion of the dopant impurities can be stopped by cooling down the wafer. Because the diffusion of impurities is time and temperature dependent, the depth of the diffusion layer can be controlled by varying these two parameters. The  $\text{SiO}_2$  pattern, formed by photolithography acts as a mask that permits the diffusion of the impurities only in specific regions of the wafer.

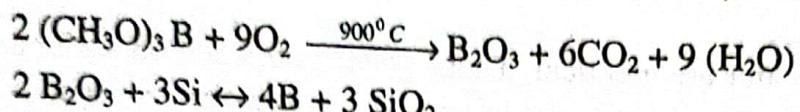
The open furnace tube system using solid, liquid and gaseous dopant sources is the most common diffusion technology used in IC fabrication. The wafers are loaded vertically into a quartz boat and put into the furnace where the wafers are heated to high temperature. In general, diffusion systems are similar to oxidation furnace.

- (i) **Solid Source:** In this system, the dopant source is in solid form as shown in figure (1). The carrier gases  $\text{N}_2$  or  $\text{O}_2$  picks up the vapour from the dopant source and transport it to the furnace tube, where the dopant atoms are deposited on the surface of wafer.

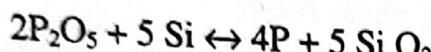


*Figure (1): Open furnace tube diffusion system: Solid impurity source*

The common solid source of Boron is Trimethyl Borate (TMB)

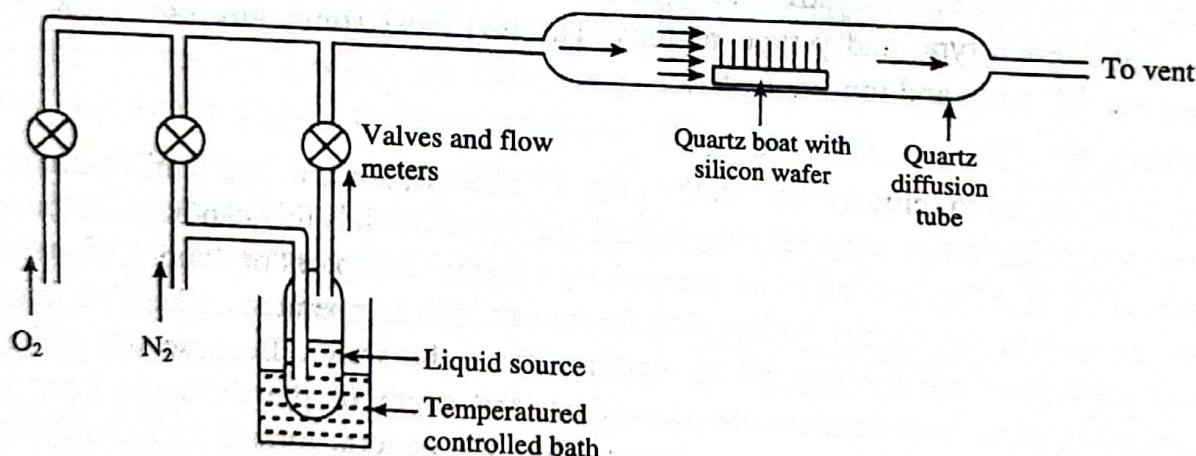


The common solid source of Phosphorous is Phosphorous Pentoxide.



## (ii) Liquid Source

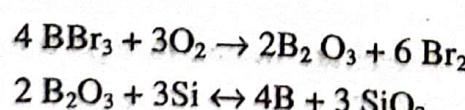
In this system the dopant source is in liquid form as shown in figure (2). The carrier gas passes through a bubbler where it picks up the vapour of the liquid source. The carrier gas carries the vapour into the furnace tube where it reacts with the surface of the silicon wafer.



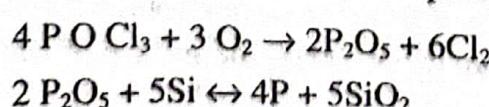
*Figure (2): Open furnace tube diffusion system: Liquid impurity source*

The most common liquid source of Boron is Boron Tribromide.

The reaction is



The common liquid source of 'P' is Phosphorous Oxychloride.



### (iii) Gas Source

In gas sources, the dopants are directly supplied to the furnace tube. The common gas sources are extremely toxic, an additional system is required to ensure that all the source gas is removed from the system before wafer entry or removal.

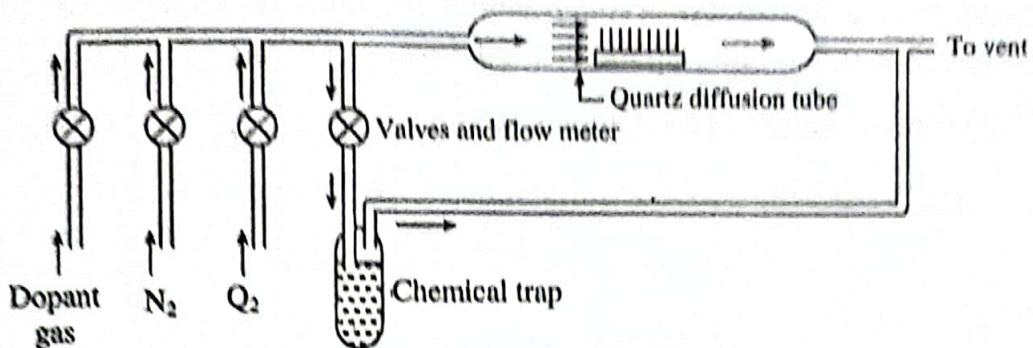
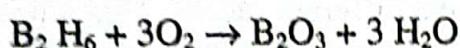
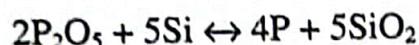
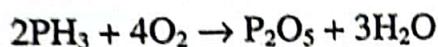


Figure (3): Open furnace tube diffusion system: Gas impurity source

The primary gaseous source of Boron is Diborane. The reaction is,



Phosphene is used as gaseous source for Phosphorous.



For Antimony, the solid source is Sb<sub>2</sub>O<sub>3</sub> and Sb<sub>2</sub>O<sub>4</sub> at the temperature of 900°C. The liquid source for Sb is Sb<sub>3</sub>Cl<sub>5</sub> in a bubbler.

For Arsenic, the solid source is As<sub>2</sub>O<sub>3</sub> and the gas source is AsH<sub>3</sub>.

### b. Ion Implantation

Ion implantation is an engineering process by which ions of a material are accelerated in an electric field and impacted into a solid. This process is used to change the physical, chemical or electrical properties of solid. Ion implantation is used in semiconductor device fabrication and in metal finishing as well as in various applications of material science research.

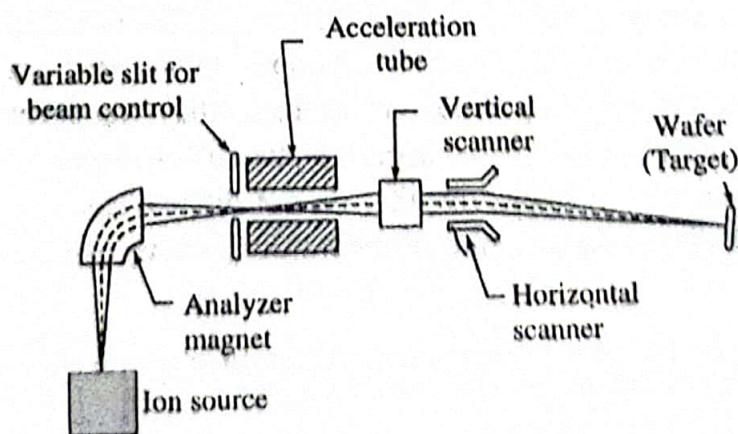


Figure 1: Ion implanter

An ion implantation equipment consists of an ion source where plasma of desired impurity are produced, an accelerator (the accelerating voltage may be from 20 kV to as much as 250 kV) where the ions are accelerated to a high energy and a target chamber where the ions impinge on a target which is the material to be implanted. An analyzer magnet bends the ion beam through a right angle to select the desired impurity ion. Scanning system consist of a vertical scanner and a horizontal scanner which provides necessary deflection to give a uniform implantation and to build up the desired dose.

The centrifugal force is balanced by magnetic force.

$$Bqv = \frac{mv^2}{r}$$

$$B = \frac{mv}{qr}$$

$$r = \frac{mv}{Bq} \quad \dots(1)$$

This means the impurities having different mass have different radius. This allows us to pass desired impurity species by providing slit on the path of beam.

And, the electrostatic energy provides necessary kinetic energy.

$$qV = \frac{1}{2} mv^2$$

$$v = \sqrt{\frac{2qV}{m}}$$

$$B = \frac{mv}{qr} = \frac{m}{qr} \sqrt{\frac{2qV}{m}} = \sqrt{\frac{2qV}{m} \times \frac{m^2}{q^2 r^2}}$$

$$B = \sqrt{\frac{2mV}{qr^2}} \quad \dots(2)$$

So the magnitude of magnetic field can be adjusted for a required ion of mass 'm'.

The target chamber is maintained at relativity low temperature during the implantation which prevents undesired spreading of impurities by diffusion. It is very important in VLSI (very large scale integration) fabrication.

Ion implantation has many advantages over diffusion. The process is performed at room temperature. This permits the implantation of doped layers without disturbing previously implanted layers. Because the impurities are ionized, they represent a current that can be measured very accurately. This permits accurate control of the impurity concentration. Ion implantation can be used with impurities that do not diffuse easily in Si. The recent use of arsenic as a dopant in mos (metal-oxide semiconductor) devices is due to the advent of ion implantation.

Ion implantation usually follows a Gaussian distribution as given by

$$N(x) = N_p \exp \left[ -\frac{(x - R_p)^2}{2\Delta R_p^2} \right] \quad \dots(3)$$

Where,  $N(x)$  is the impurity concentration

$N_p$  is the peak concentration

$R_p$  is the projected range and  $\Delta R_p$  is the standard deviation called *straggle* (straggle: to spread out from others in a disorganized way)

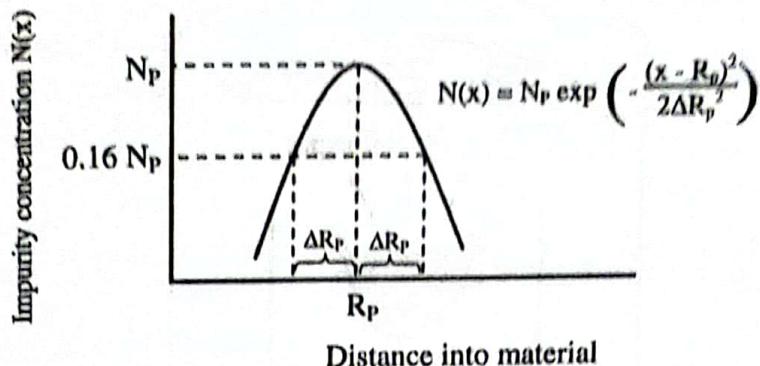


Figure 2: Gaussian distribution resulting from ion implantation

Figure (3) shows the result of varying the accelerating voltage several times during the implantation process. Each of the Gaussian distributions (dashed lines) corresponds to a different accelerating potential. The overall distribution (solid line) is obtained by summing up the individual Gaussian distributions, and the sum, as can be seen from the graph, is relatively flat.

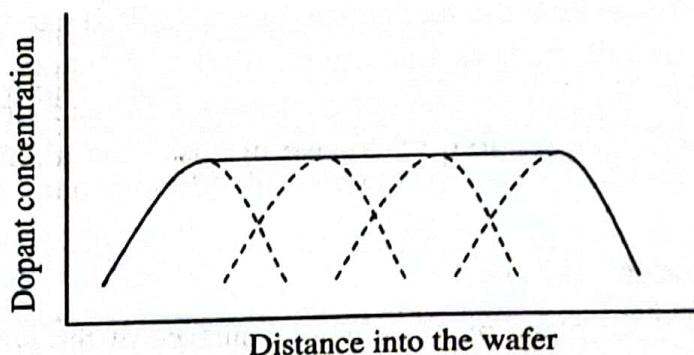


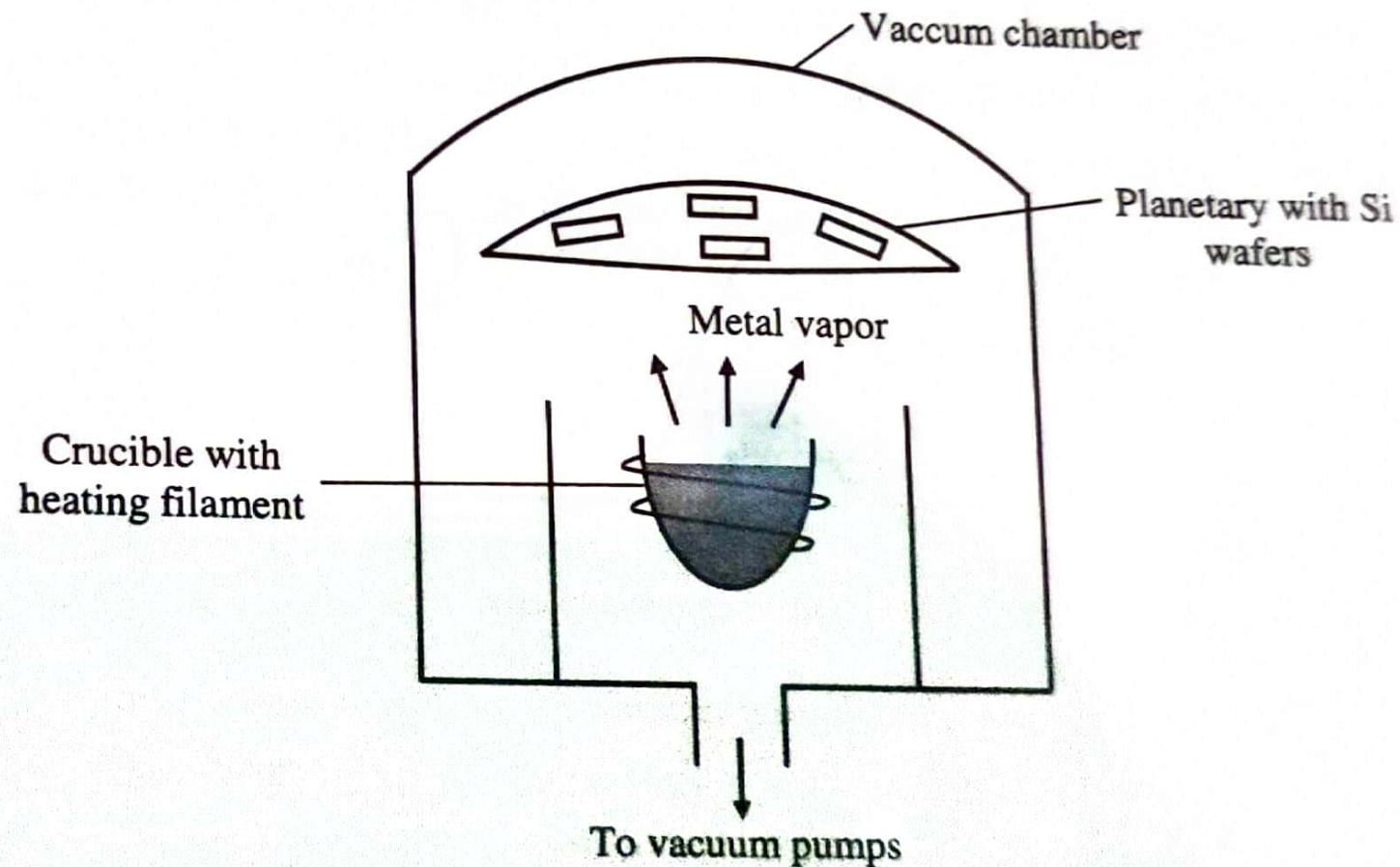
Figure 3: Profile of impurities obtained by varying the accelerating voltage four times during the implantation process.

### 3. Connection of components in chip (metallization)

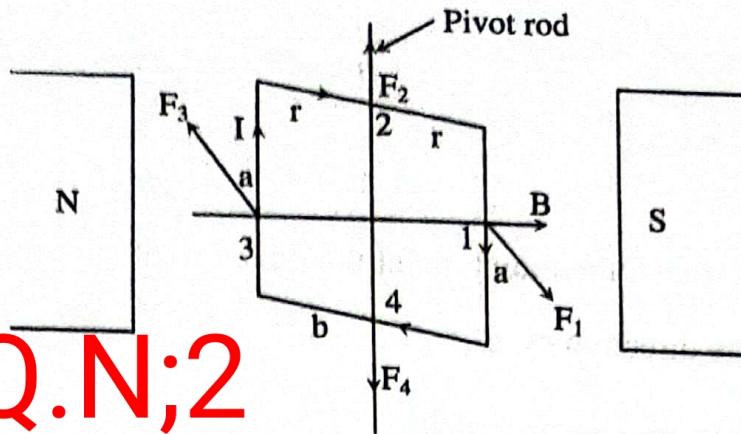
An IC chip consists of many superimposed doped layers. To complete the circuit, the electronic components within a layer as well as the layers themselves must be electrically connected. This can be done either by forming heavily doped regions of silicon or by metal electrodes (metallization). The metallization is performed by metallic thin film evaporation. A schematic of the setup used for the evaporation of thin metallic film is shown in figure. The metal (usually aluminum) is placed in a crucible. The silicon wafers are placed above the crucible in a device called the planetary.

Vapor of the metal is produced by heating the crucible with a heater coil wrapped around it. As the metal vapor hits the cooled, masked wafer, it condenses on it thus forming a thin metallic layer in a desired pattern that connects the different section of IC chip. The whole

evaporation process is performed in a vacuum to avoid contamination of the metal vapor with the oxygen in air.



*Figure: Schematic representation of a vacuum chamber used for metallization.*



**Q.N;2**

Figure: Torque on current-carrying rectangular loop of wire on a pivot rod when placed in a magnetic field.

Consider a rectangular loop of wire lengths 'a' and 'b' connected to a pivot rod as shown in figure. For the sides 1 and 3 the magnitude of forces are same because the angle between 'a' and 'B' is  $90^\circ$  and both wires have the same length 'a', that is,

$$F_1 = F_3 = I (\vec{a} \times \vec{B}) = I a B \sin\theta = I a B \sin 90^\circ = I a B \quad \dots\dots(1)$$

From the definition of cross-product we see that  $F_1$  is out of page toward reader whereas  $F_3$  is into the page.

Similarly, the magnitude of the forces on sides 2 and 4 are equal.

$$F_2 = F_4$$

The direction of  $F_2$  is upward, while that of  $F_4$  is downward. We conclude that there is no net force in any direction. However there exists a torque that tends to rotate the loop about the pivot rod. The torque is given by

$$\tau = r \times F$$

Applied to the present situation,  $F_2$  and  $F_4$  cancel each other.  $F_1$  and  $F_3$  exert a torque on the loop. The net torque is the sum of individual torques caused by  $F_1$  and  $F_3$ , but because they are equal, the total torque is

$$\begin{aligned}\tau &= 2 r \times F \\ &= 2 r F \sin\theta\end{aligned}$$

where  $F$  stands for either  $F_1$  or  $F_3$  and  $r = b/2$

Therefore,  $\tau = 2 (b/2) I a B \sin\theta = I a b B \sin\theta$

$$\tau = I A B \sin\theta \quad \dots\dots(2)$$

Where  $A = ab$  is the area of loop.

Although equation (2) has been derived for a rectangular loop of wire, it is valid for any other geometric configuration.

From equation (2) we conclude that,

$\tau_{\max} = IAB$ , when  $\theta = 90^\circ$ , i.e. when  $A$  and  $B$  are perpendicular

$\tau_{\min} = 0$ , when  $\theta = 0$ , i.e. when  $A$  and  $B$  are parallel.

## **Wave Function and It's Significance**

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According to Max Born, in the case of quantum mechanical particle, the guiding wave is represented by a function,  $\psi(r, t)$ , called a wave function. The method of quantum mechanics consist in first finding the wave function associated with a particle or system of particles. All the information about the physical properties of the system can be obtained from it.

**Q.N;3(1st Part)**

The physical significances of the wave function are

- i. At some instant  $t$ , a measurement can be done to locate the particle associated with the wave function  $\psi$ .
- ii. The wave function  $\psi$  operated with Schrödinger wave equation describes the motion of particle associated with  $\psi$  as done by Newton's second law in classical mechanics.
- iii. The probability  $P(r, t) dV$  that the particle will be found with in a small volume  $dV$  is equal to  $|\psi|^2 dV$ ,

$$\text{that is, } P(r, t) dV = |\psi|^2 dV = \psi \psi^* dV \quad \dots (1)$$

where  $\psi^*$  is the complex conjugate of  $\psi$ .

Since the total probability of finding the particle in the entire space is unity.

$$\int_{-\infty}^{\infty} |\psi|^2 dV = 1 \quad \dots (2)$$

The wave function satisfying this condition is called normalized wave function. Every acceptable wave function can be normalized by multiplying it with an appropriate constant called normalizing constant. A wave function can be normalized by using equation (2), therefore it is also called the normalizing condition.

For a free particle, the wave function is given by,

$$\psi = A e^{i(kx - \omega t)} \quad \dots (3)$$

$$\text{Since, } P = \frac{h}{\lambda} = \frac{h}{2\pi} \frac{2\pi}{\lambda} = \hbar k \Rightarrow k = \frac{P}{\hbar}$$

$$\text{and } E = h f = \frac{h}{2\pi} \cdot 2\pi f = \hbar \omega \Rightarrow \omega = \frac{E}{\hbar}$$

$$\psi = A e^{i((Px/\hbar) - (Et/\hbar))}$$

$$\psi = A e^{-\frac{i}{\hbar}(Et - Px)} \quad \dots, (4)$$

$$\text{Here } |\psi|^2 = \psi^* \psi = A^* e^{\frac{i}{\hbar}(Et - Px)} \cdot A e^{-\frac{i}{\hbar}(Et - Px)} \\ = A^* A$$

Which is a real quantity. Because an imaginary probability is not defined mathematically, the probability of finding a particle at any point in space, that is  $|\psi|^2 dV$ , and  $|\psi|^2$  must be both real and positive.

## The Time Dependent Schrödinger Equation

The basis of the modern theory of quantum mechanics was developed in 1925 by Erwin Schrödinger.

If a particle has well-defined momentum and energy. We can use a sinusoidal traveling wave, that is either,

$$\psi = A \sin(kx - \omega t)$$

or  $\psi = A \cos(kx - \omega t)$

or A linear combination of both. If we want to describe a free particle which is partially localized, we could use a wave packet. If a particle is not free and that is acted on by a force, its momentum and energy will not be constant. Such a particle cannot be described by a function with  $\omega$  and  $k$  because these are changing. de-Broglie hypothesis does not tell what type of wave one can associate to describe such particles. The Schrödinger theory tells us how to obtain the wave function  $\psi(x, t)$  associated with a particle acted upon by a force by giving the potential energy associated with force.

Schrödinger developed a differential equation whose solutions yield the possible wave-functions that can be associated with a particle in a given physical situation. This equation, known as the Schrödinger equation, tells us how the wave function changes as a result of the force acting on the particle.

The wave function associated with the motion of free particle is

$$\psi = Ae^{-i\frac{Et}{\hbar}} (Et - Px)$$

Differentiating with respect to  $x$ ,  $\frac{\partial \psi}{\partial x} = \left(\frac{-i}{\hbar}\right) (-P) \psi = \frac{iP}{\hbar} \psi$

Again differentiating with respect to  $x$ ,

$$\frac{\partial^2 \psi}{\partial x^2} = \left(\frac{iP}{\hbar}\right)^2 \psi = -\frac{P^2}{\hbar^2} \psi$$

$$\Rightarrow P^2 \psi = -\hbar^2 \frac{\partial^2 \psi}{\partial x^2} \quad \dots(1)$$

Now, differentiating  $\psi$  with respect to  $t$ ,

$$\frac{\partial \psi}{\partial t} = \frac{-iE}{\hbar} \psi \Rightarrow E\psi = \frac{-\hbar}{i} \frac{\partial \psi}{\partial t} = \frac{i^2 \hbar}{i} \frac{\partial \psi}{\partial t}$$

$$\text{Therefore, } E\psi = i \hbar \frac{\partial \psi}{\partial t} \quad \dots(2)$$

## 2nd Part

The total energy of a particle is given by,  $E = K.E. + P.E.$

$$E = \frac{P^2}{2m} + V$$

Multiplying both sides by  $\psi$ ,

$$E\psi = \frac{P^2\psi}{2m} + V\psi$$

Using equations (1) and (2)

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi \quad \dots(3)$$

This is the time dependent Schrödinger wave equation

Equation (3) can be re-written as

$$\frac{-\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi = i\hbar \frac{\partial \psi}{\partial t}$$

The Hamiltonian operator is given by,  $\hat{H} = \left[ \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V \right]$

Therefore time dependent Schrödinger wave equation takes the form

$$\hat{H}\psi = i\hbar \frac{\partial \psi}{\partial t} \quad \dots(4)$$

If the potential energy  $V$  is known, this equation can be solved and the solution will yield the possible wave-functions that we can associate with the particle. The Schrödinger equation is to quantum mechanics what Newton's Second Law is to classical physics.

Here we have derived time dependent SWE by using the wave-function of free particle. When the particle is not free, the relation cannot be proved, but we postulate that it still holds and experiment bears this out. The Schrödinger equation cannot be derived from first principles; it is a first principle, which cannot be mathematically derived, just as Newton's laws of motion are not derivable. The justification lies in the fact that its predictions agree with the experiment.

**Note:** In mathematics, when a function depends on more than one independent variable, the derivative of the function with respect to one of them, while treating the other variables as constants, is called the partial derivative with respect to that variable.

### Time Independent Schrödinger Equation

Schrödinger wave equation describes the motion of quantum mechanical particle as Newton's second law in classical mechanics. It has been observed that in many situations, potential ( $V$ ) acting on the particle does not depend upon time and varies only with its position only. For such conditions time independent form of Schrödinger's equation is applicable.

The wave function associated with the motion of free particle is given by

$$\psi = A e^{-i(Et - px)} = A e^{-i\hbar(Et - px)}$$

Differentiating with respect to x.

$$\frac{d\psi}{dx} = \left(-\frac{i}{\hbar}\right) (-P) A e^{-i/\hbar(Et - Px)}$$
$$= \frac{iP}{\hbar} \psi$$

Again differentiating with respect to x.

$$\frac{d^2\psi}{dx^2} = \left(\frac{iP}{\hbar}\right)^2 \psi$$

$$\frac{d^2\psi}{dx^2} = \frac{-P^2}{\hbar^2} \psi$$

$$\Rightarrow P^2\psi = -\hbar^2 \frac{d^2\psi}{dx^2} \quad \dots(1)$$

The total energy of a particle is given by,  $E = KE + PE$

$$E = \frac{1}{2}mv^2 + V = \frac{1}{2} \frac{(mv)^2}{m} + V = \frac{P^2}{2m} + V$$

$$E = \frac{P^2}{2m} + V$$

Multiplying both sides by  $\psi$

$$\left(\frac{P^2}{2m} + V\right)\psi = E\psi$$

$$\frac{P^2\psi}{2m} + V\psi = E\psi$$

Using equation (1)

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V\psi = E\psi \quad \dots(2)$$

$$\frac{d^2\psi}{dx^2} + \frac{2m(E-V)}{\hbar^2} \psi = 0 \quad \dots(3)$$

This is the *time independent Schrödinger wave equation*.

In three dimensions it can be expressed as

$$\nabla^2 \psi + \frac{2m(E-V)}{\hbar^2} \psi = 0 \quad \dots(4)$$

Where,  $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$  is called Laplacian operator.

The equation (2) is

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + V\psi = E\psi$$

$$\left(-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V\right)\psi = E\psi$$

The Hamiltonian operator is given by,  $\hat{H} = \left[ \frac{-\hbar^2}{2m} \frac{d^2}{dx^2} + V \right]$

Therefore time independent Schrödinger wave equation takes the form,

$$\hat{H}\psi = E\psi \quad \dots(5)$$

Electrical conductivity of intrinsic semiconductor due to electrons and holes

Q.N;4

$$\begin{aligned} J_e &= \sigma_e E \\ J_h &= \sigma_h E \end{aligned} \quad \text{--- (1)}$$

where  $J_e$  and  $J_h$  are current density of electrons and holes. Current density including drift velocity is given by.

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$$J_e = n(-e) v_{de} \quad \text{--- (2)}$$

$$J_e = nev \quad \text{--- (3)}$$

$$J_e = ne \mu_e E \quad \text{--- (3)}$$

Similarly for holes,

$$J_h = P e \mu_h E \quad \text{--- (4)}$$

from eq<sup>2</sup> (1) (3) and (4)

$$\sigma_e = ne \mu_e \quad \text{--- (5)}$$

$$\sigma_h = Pe \mu_h$$

Total electrical conductivity

$$\sigma = \sigma_e + \sigma_h$$

$$\sigma = ne \mu_e + Pe \mu_h$$

$$\sigma = e \ell (n \mu_e + P \mu_h) \quad \text{--- (6)}$$

In Intrinsic Semiconductor  $n = p = n_i$

but

$$n_i = \sqrt{N_c N_v} \propto e^{-\frac{E_g}{2k_B T}}$$

where,

$$N_v = 2 \left( \frac{m_e k_B T}{2\pi\hbar^2} \right)^{3/2}, N_c = 2 \left( \frac{m_h k_B T}{2\pi\hbar^2} \right)^{3/2}$$

so,

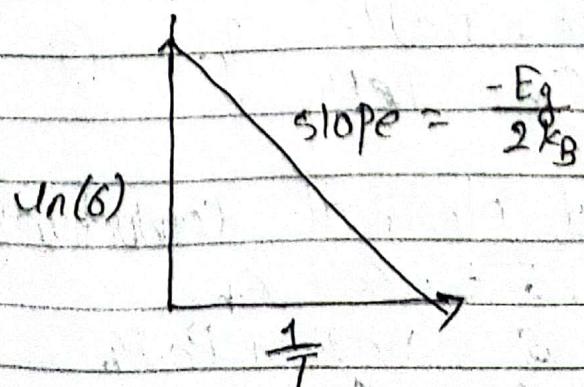
$$\sigma = e n_i (\mu_e + \mu_h)$$

$$\text{or, } \sigma = e (\mu_e + \mu_h) \sqrt{N_c N_v} e^{-\frac{E_g}{2k_B T}}$$

$$\therefore \sigma = \sigma_0 e^{-\frac{E_g}{2k_B T}}$$

where,  $\sigma_0 = \sqrt{N_c N_v}$  let  $(\mu_e + \mu_h)$  is constant and independent of temperature.

Graph between  $\ln(\sigma)$  and  $\frac{1}{T}$  is given in figure which shows that conductivity is exponentially dependent.

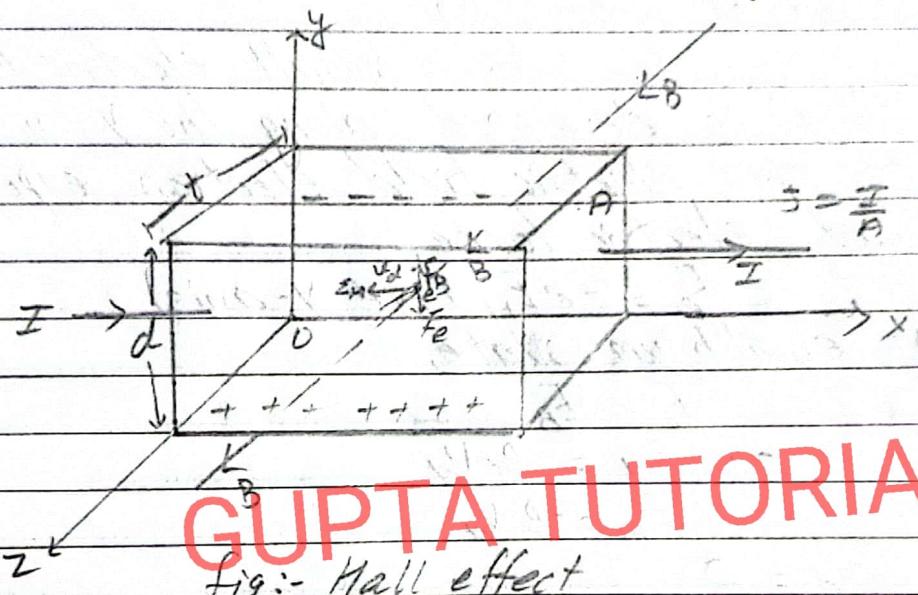


~~Hall Effect~~

20/7/9

## Q.N;5

When magnetic field is applied to the current carrying conductor the voltage is developed across the conductor. The direction of voltage is perpendicular to the direction of both magnetic field and current. This phenomenon is called Hall effect and voltage developed is called Hall voltage.



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Fig:- Hall effect

### Explanation:-

Let us consider, a conductor in the form of rectangular strip of thickness 't' having cross sectional area 'A'. If current 'I' is flowing in the conductor along x-axis, then current density ( $J = \frac{I}{A}$ ) is also along x-axis

so that drift velocity of electron,

$$v_d = \frac{J}{ne} \quad \dots \textcircled{1}$$

Now, magnetic field intensity  $B$  is applied in the conductor along z-axis. Hence, magnetic force experienced

by electron

$$F_B = B e V_d$$

acting upward (+ve Y-axis) due to this force, electrons are collected at the top of conductor resulting -ve charge. According to conservation of charge, +ve charge is developed at the bottom of conductor.

As a result electric field "E\_H" (Hall effect) is developed along the Y-axis (+ve to -ve). Due to this electric field electron experienced downward electric force,

$$F_e = -e E_H \text{ (-ve Y-axis)}$$

At equilibrium state,

$$F_B = F_e$$

$$-e \cdot E_H = B e V_d$$

$$E_H = -B \cdot V_d$$

Put  $V_d = \frac{I}{n e}$

$$E_H = -\frac{J B}{n e}$$

$I$  = constant for given conductor known as Hall coefficient ( $R_H$ )

$$E_H = -R_H \cdot J B$$

$$R_H = -\frac{E_H}{J B}$$

If "d" be the width of conductor then

$$\text{Hall voltage } V_H = E_H \cdot d$$

$$V_H = -JB \cdot d$$

$n e$

$$\text{Put } J = \frac{I}{A} = \frac{I}{t \times d}$$

$$\Rightarrow \boxed{V_H = -B I \text{ net}}$$

$$-\frac{1}{n e} = \text{constant} = R_H$$

$$R_H = \frac{E_H}{J B}$$

### Mobility ( $\mu$ ) :-

When electric field is applied in the conductor, then electrons move in the direction opposite to applied electric field with an average velocity known as drift velocity. Hence, mobility of electron is defined as the drift velocity per unit applied electric field. It is denoted by  $\mu$  and given by

$$\mu = \frac{v_d}{E} \quad \dots \textcircled{1}$$

We know,

$$J = n e v_d$$

$$v_d = \frac{I}{n e}$$

$$\text{and } \boxed{J = \sigma E}$$

where " $\sigma$ " is conductivity

$$v_d = \sigma F$$

Then eqn 1 becomes

$$\boxed{\mu = \frac{\sigma}{n e}}$$

Put  $\frac{1}{ne} = R_H$

$$V = R_H \cdot \sigma$$

Put  $\sigma = \frac{I}{A}$

where 'σ' is resistivity

$$V = \frac{R_H}{\sigma}$$

Hall resistance ( $R_H$ ) :-

If 'V<sub>H</sub>' be the hall voltage and 'I' be the current flowing in the conductor then hall resistance 'R' is given by

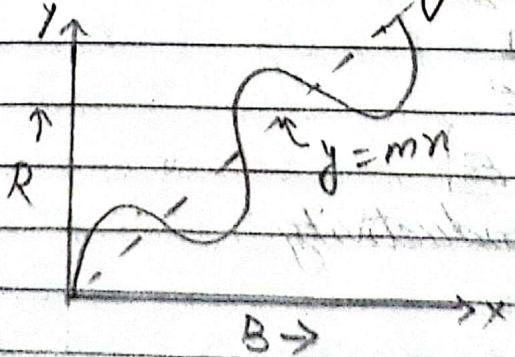
$$R = V_H / I$$

Put  $V_H = BI$   
net

$$R = \left( \frac{1}{\text{net}} \right) B$$

which is in form of  $y = mx$

Hence, graph of 'B' versus R is straight line passing through origin as shown in figure below by dotted line



But, experimentally it is found that graph is non Linear as shown by solid line known as quantum Hall effect.

**Q.N;6**

Differential equation of the oscillation of horizontal spring by application of Hook's Law and Newton's Law: Write its solution. Also find acceleration, time period and velocity.

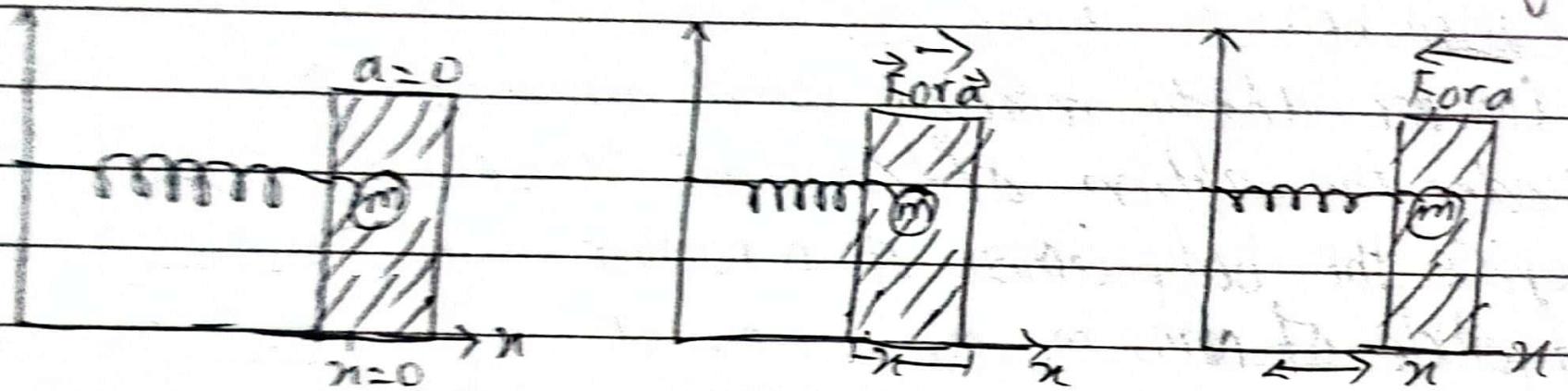


Fig: motion of horizontal spring along n-axis

Let  $m$  be the mass of the particle which is attached by horizontal spring. At first it is rest ( $a=0$ ) after application of force it stretched then Force  $F = kn \dots \text{--- (1)}$  which is given by Hooke's Law, where  $n$  is the stretching length. When we apply force in one direction then there is reaction force in another direction with the help of Newton's third law of motion so Force becomes:

$$F = -kn \dots \text{--- (2)}$$

where  $K$  is Force constant or spring constant

$$F = ma$$

$$F = m \frac{d^2n}{dt^2} \quad \text{(double derivative of velocity)}$$

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From eq? (1) and (3)

$$m \frac{d^2n}{dt^2} = -kn$$

$$\boxed{m \frac{d^2n}{dt^2} + kn = 0} \quad \text{--- (4)}$$

which is the eq? of horizontal spring. This eq? is same as the eq? of S.H.M so, it is oscillatory motion.

Solution of this equation is

$$n = A \sin(\omega t + \phi)$$

where  $\phi$  is the phase angle,  $(\omega t + \phi)$  is phase factor and  $A$  is Amplitude.

using relation,

$$ma = -kn$$

$$a = \frac{-k}{m} n$$

$$\therefore \boxed{\text{acceleration (a)} = -\frac{k}{m} n}$$

Again, From acceleration of oscillatory motion

$$a = -\omega^2 x$$

$$\frac{-k \times x}{m} = -\omega^2 x$$

$$\omega = \sqrt{\frac{k}{m}}$$

$\therefore \text{Angular velocity } (\omega) = \sqrt{\frac{k}{m}}$

velocity  $\sqrt{\frac{k}{m}} \times r$  ( $v = \omega r$ )

Time period,

$T = \frac{2\pi}{\omega}$  *The time taken by particle to complete a cycle of oscillation is called time period 'T'.*

$$\text{or } T = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

$$\therefore T = 2\pi \sqrt{\frac{m}{k}}$$

Frequency,

$F = \frac{1}{T}$  *The number of complete oscillations made by an oscillating particle in one second is called frequency.*

$$\text{or, } F = \frac{1}{2\pi \sqrt{\frac{m}{k}}}$$

$$\therefore F = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

7. What are (a) the energy, (b) the momentum and  
 (c) the wavelength of the photon that is emitted  
 when a hydrogen atom undergoes a transition  
 from the state  $n=4$  to  $n=2$ .

So?

Given,

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energy state of Hydrogen atom are  $n_1=2, n_2=4$

Energy ( $E$ ) = ?

Momentum ( $p$ ) = ?

Wavelength ( $\lambda$ ) = ?

we know that

$$\frac{1}{\lambda} = R \left[ \frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

where,  $R$  = Rydberg constant i.e  $R = 1.09 \times 10^7 \text{ m}^{-1}$

Then,

$$\frac{1}{\lambda} = 1.09 \times 10^7 \left[ \frac{1}{2^2} - \frac{1}{4^2} \right]$$

$$\text{or, } \frac{1}{\lambda} = 1.09 \times 10^7 \left[ \frac{4-1}{16} \right]$$

$$\text{or, } \frac{1}{\lambda} = 2043750$$

$$\text{or, } \lambda = \frac{1}{2043750}$$

$$\therefore \lambda = 4.89 \times 10^{-7} \text{ m } \text{ Air}$$

Again,

$$\begin{aligned}\text{Energy } (E) &= \frac{hc}{\lambda} \\&= \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{4.89 \times 10^{-7}} \\&= 4.06 \times 10^{-19} \text{ J} \\&= \frac{4.06 \times 10^{-19}}{1.06 \times 10^{-19}} \text{ eV} \\&= 3.83 \text{ eV Ans}\end{aligned}$$

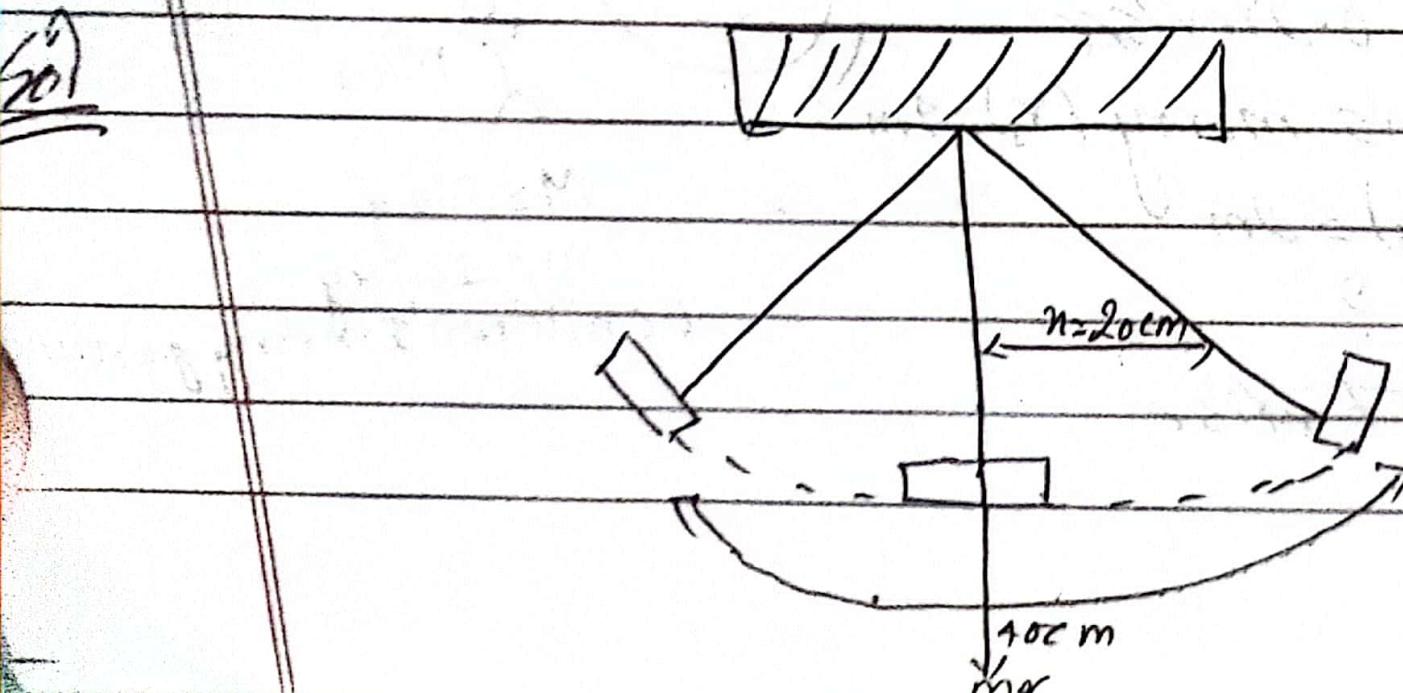
$$\begin{aligned}\text{Momentum } (P) &= \frac{h}{\lambda} \\&= \frac{6.62 \times 10^{-34}}{4.89 \times 10^{-7}} \\&= 1.353 \times 10^{-27} \text{ kg/ms Ans}\end{aligned}$$

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An oscillating block of mass 250kg takes 0.15sec to move between the end points of the motion, which are 40cm apart

**Q.N;8**

- (a) What is the frequency of motion?
- (b) What is the amplitude of motion?
- (c) What is force constant of the spring.



Given,

Block of mass ( $m$ ) = 250 kg

Time taken ( $T$ ) = 0.15 sec

① Frequency ( $f$ ) = ?

② Amplitude of motion ( $x_m$ ) = ?

③ Force constant ( $K$ ) = ?

We know,

$$\textcircled{1} \quad F = \frac{1}{T} = \frac{1}{0.15} = 6.66 \text{ Hz}$$

$$\textcircled{2} \quad x_m = \text{midpoint of total amplitude (50 cm)} \\ = \frac{50}{2} = 25 \text{ cm}$$

$$\textcircled{3} \quad T = 2\pi\sqrt{\frac{m}{K}}$$

$$\text{or, } \frac{T}{2\pi} = \sqrt{\frac{m}{K}}$$

S.B.S

$$\frac{T^2}{4\pi^2} = \frac{m}{K}$$

$$K = \frac{m \cdot \pi^2}{T^2} = \frac{250 \times 9 \times \pi \times \pi}{(0.15)^2}$$

$$= 433,626.22 \text{ N/m} \quad 43864.908 \text{ N/m}$$

A potential difference of 100V is established between the two parallel plates. A proton of charge  $q = 1.6 \times 10^{-19} C$  is released from the positive plate. What will be the velocity of the proton when it reaches negative plate? The mass of proton is  $1.67 \times 10^{-27} kg$

Given,

**Q.N;9**

$$\text{Potential difference } (V) = 100V$$

$$\text{charge of proton } (q) = 1.6 \times 10^{-19} C$$

$$\text{Mass of proton } (m) = 1.67 \times 10^{-27} kg$$

$$\text{velocity of proton } (v) = ?$$

we know,

$$\frac{1}{2}mv^2 = qV$$

$$\text{or, } v^2 = \frac{2qV}{m}$$

$$\text{or, } v = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 100}{1.67 \times 10^{-27}}}$$

$$\therefore v = 1.38 \times 10^5 m/s  
(138425.70)$$

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10. An  $\alpha$ -particle is emitted from a radioactive nuclei with an energy of 6.8 Mev. Calculate its wavelength and compare it with the size of the emitted emitting nucleus that has a radius of  $8 \times 10^{-15} \text{ m}$

Given.

$$\text{Mass } (M) = 6.8 \text{ Mev} \cdot 1.6 \times 10^{-27} \text{ kg}$$

$$\text{Mass of } \alpha\text{-particle} = 4 \times \text{mass of proton}$$

$$= 4 \times 1.6 \times 10^{-27} \text{ kg}$$

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$$\begin{aligned}\text{Energy } (E_k) &= 6.8 \text{ Mev} = 6.8 \times 10^6 \text{ eV} \\ &= 6.8 \times 10^6 \times 1.6 \times 10^{-19} \text{ J} \\ &= 1.088 \times 10^{-18} \text{ J}\end{aligned}$$

From De-broglie hypothesis

$$\begin{aligned}\lambda &= \frac{h}{\sqrt{2mE_k}} = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 4 \times 1.6 \times 10^{-27} \times 1.088 \times 10^{-18}}} \\ &= \frac{6.62 \times 10^{-34}}{\sqrt{1.39264 \times 10^{-44}}} \\ &= 5.609 \times 10^{-12} \text{ m}\end{aligned}$$

*Ans*

11. (a) Calculate the Fermi energy aluminum that have density  $2.73 \text{ g/cm}^3$  and molecular weight  $26.98 \text{ g/mole}$ . (b) If the experimental value of Fermi energy ( $E_F$ ) is  $11.8 \text{ eV}$ , what is the effective mass of electron in aluminum? Aluminum is trivalent?

SQ

Here,

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- (a) Density of aluminium ( $\rho$ ) =  $2.73 \text{ g/cm}^3$   
 Molecular weight ( $M_{\text{mo}}$ ) =  $26.98 \text{ g/mole}$   
 Fermi energy ( $E_F$ ) = ?  
 Effective mass ( $m^*$ ) = ?  
 Experimental value of fermi energy ( $E_F$ ) =  $11.8 \text{ eV}$   
 $= 11.8 \times 1.6 \times 10^{-19} \text{ J}$

$V_a$  = valency of electron = 3

$$\text{no. of free electrons per unit volume } (N) = \frac{V_a N_A \rho}{M_{\text{mo}}}$$

$$= \frac{3 \times 6.02 \times 10^{23} \times 2.73}{26.98}$$

$$= 1.82 \times 10^{23} \text{ electron/m}^3$$

$$\begin{aligned}
 E_F &= \frac{\hbar^2}{2m} (3\pi^2 N)^{2/3} = \frac{(1.05 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31}} (3\pi^2 \times 1.82 \times 10^{23})^{2/3} \\
 &= \frac{(1.1025 \times 10^{-68}) \times (3.073 \times 10^{16})}{(2 \times 9.1 \times 10^{-31})} \\
 &= \frac{1.861 \times 10^{-22} \text{ J}}{1.6 \times 10^{-19}} \\
 &= 1.163 \times 10^{-3} \text{ eV}
 \end{aligned}$$

Ans.

Again,

for  $m_e^*$

$$F = F \propto \frac{h^2}{2m_e^*} (3\pi^2 N)^{\frac{2}{3}}$$

$$m_e^* = \frac{h^2}{2F} (3\pi^2 N)^{\frac{2}{3}}$$

$$m_e^* = \frac{(1.05 \times 10^{-34})^2}{(2 + 11.8 \times 1.6 \times 10^{-19})} (3\pi^2 \times 1.82 \times 10^{23})^{\frac{2}{3}}$$

$$m_e^* = 8.974 \times 10^{-35}$$

$$\therefore m_e^* = 8.974 \times 10^{-35} m_e$$

$$m_e^* = 9.85 \times 10^{-5} m_e$$

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$$Y = (\overline{AB} + \overline{BA})(\overline{A} + \overline{B} + C)$$

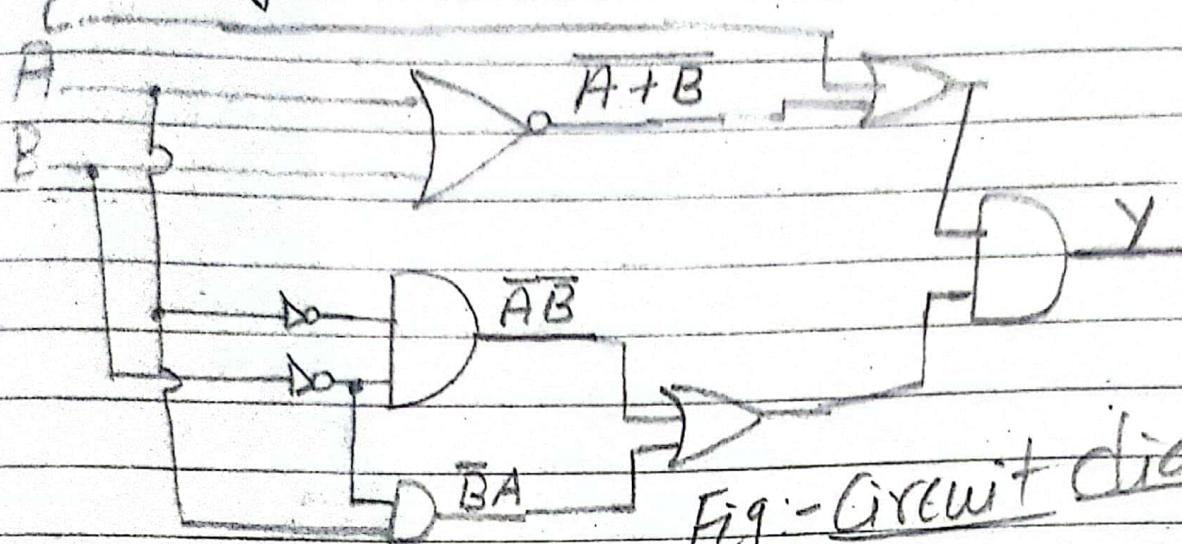


Fig - Circuit diagram

Input	Intermediated state						output		
A	B	C	$\overline{A}$	$\overline{B}$	$\overline{AB}$	$\overline{BA}$	$\overline{A} + \overline{B}$	$\overline{A} + \overline{B} + C$	Y
0	0	0	1	1	1	0	1	0	1
0	0	1	1	1	1	0	1	1	1
0	1	0	1	0	0	0	0	1	1
0	1	1	1	0	0	0	0	1	0
1	0	0	0	1	0	1	1	1	0
1	0	1	0	1	0	1	1	0	1
1	1	0	0	0	0	0	0	0	0
1	1	1	0	0	0	0	1	0	1

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