Chapter 16　二端口网络
Two-port Networks

目标：

a. 理解二端口网络特性的描述方法；

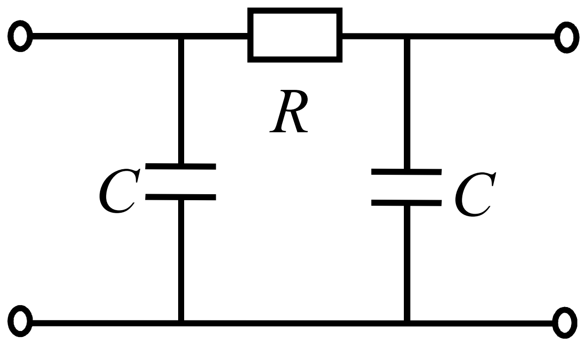
b. 计算、测量二端口网络的任何参数，并可相互转换；

c. 计算带负载二端口网络的端口电量；

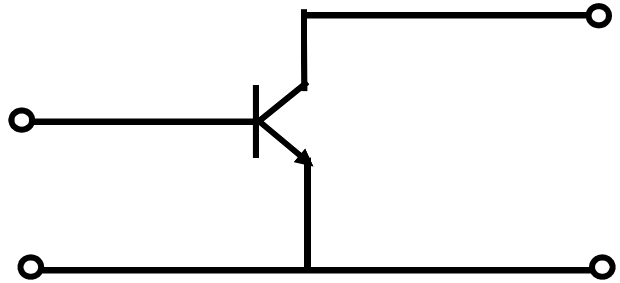
d. 分析相互连接的二端口网络。

16.1 概述

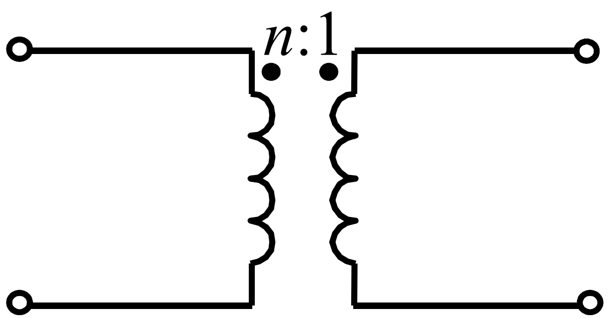
在工程实际中，研究信号及能量的传输和信号变换时，经常碰到如下形式的电路：二端口网络。



滤波器



三极管



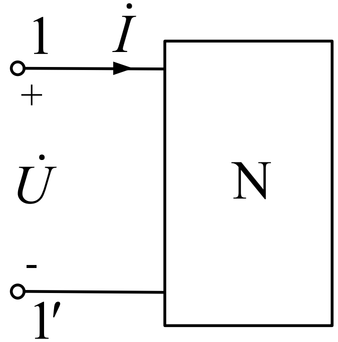
变压器

传输线

16.2　二端口网络的端口特性方程

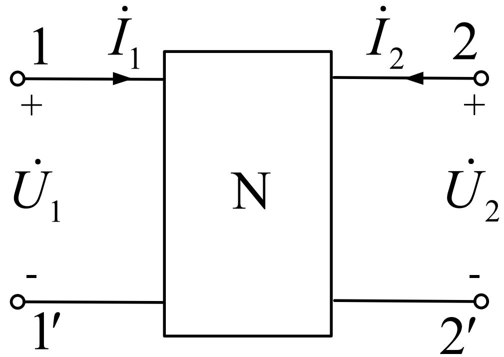
1 多端网络端口的定义

- 一端口 (port)



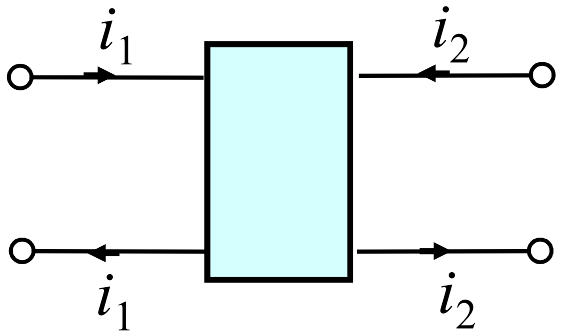
端口由一对端钮构成，且满足如下条件：从一个端钮流入的电流等于从另一个端钮流出的电流。

- 二端口（two-port)

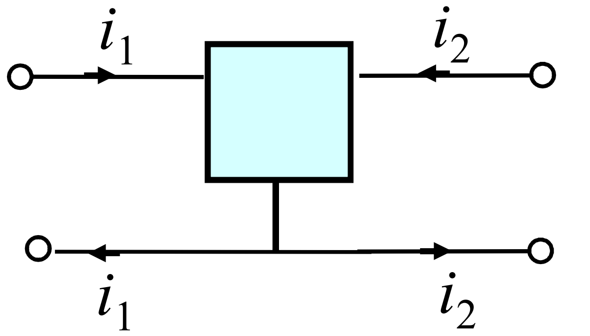


当一个电路与外部电路通过两个端口连接时称此电路为二端口网络。

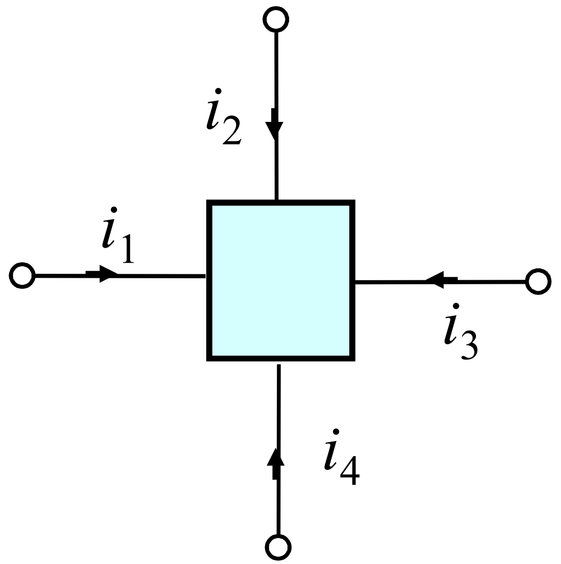
➢二端口网络与四端网络



二端口

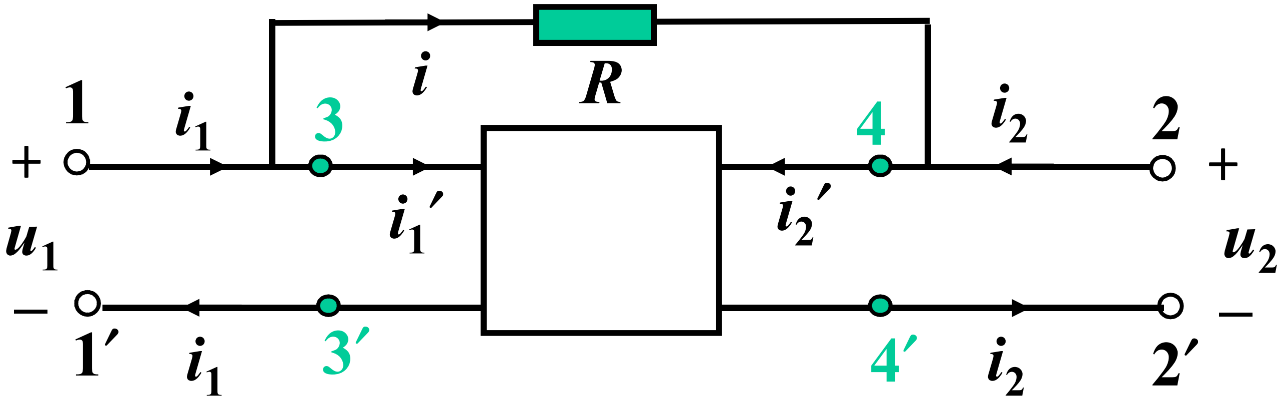


具有公共端的二端口



四端网络，非二端口网络

➤四端二端口的两个端口间若有外部连接，则会破坏原二端口的端口条件。



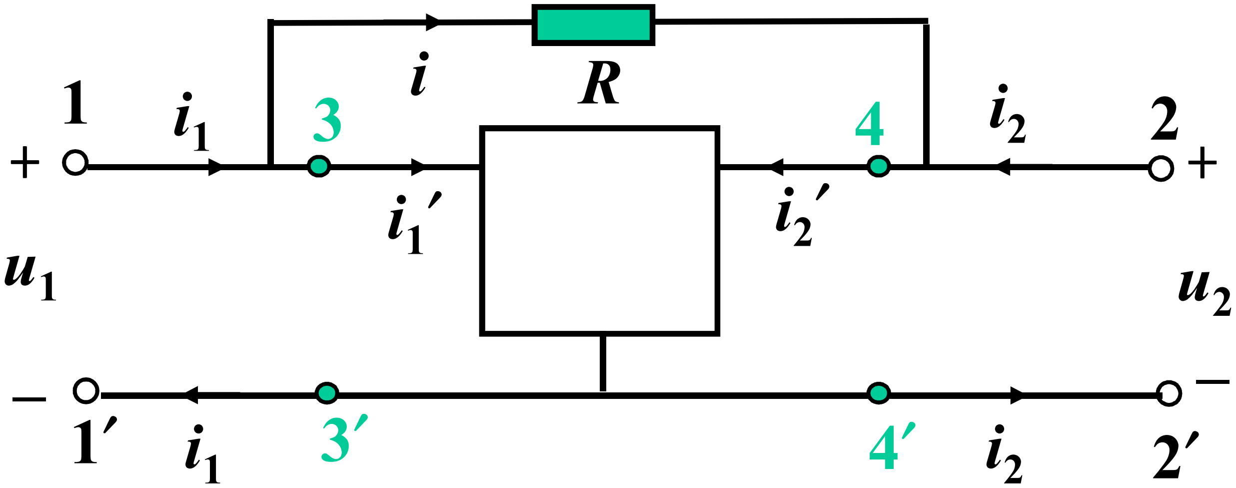
若1-1’ 2-2’是二端口网络

3-3’ 4-4’不是二端口网络，是四端网络

$$\left.\begin{aligned} i_1' &= i_1 - i \neq i_1 \\ i_2' &= i_2 + i \neq i_2 \end{aligned}\right\} \text{端口条件破坏}$$

思考：若3-3’ 4-4’是二端口网络，则1-1’ 2-2’ 是二端口吗?

➢三端二端口的两个端口间若有外部连接，则仍然是二端口，满足端口条件。



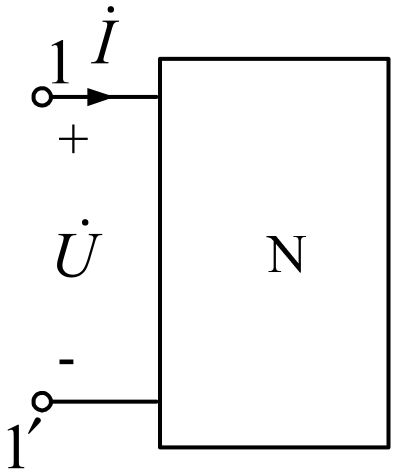
若1-1’ 2-2’是二端口网络

3-3’ 4-4’ 是二端口网络，满足端口条件

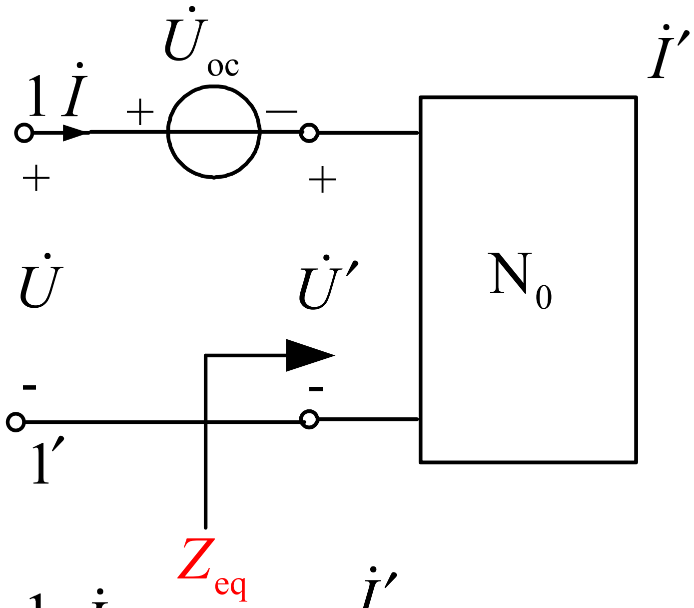
$$\left.\begin{aligned} i_1' &= i_1 - i = i_1 \\ i_2' &= i_2 + i = i_2 \end{aligned}\right\}$$ 端口条件成立

思考：若3-3’ 4-4’是二端口网络，则1-1’ 2-2’ 是二端口吗?

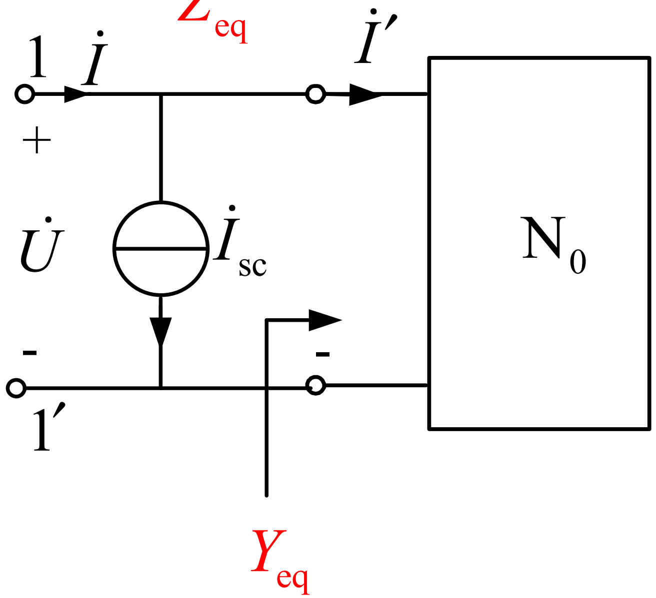
2 含源一端口网络



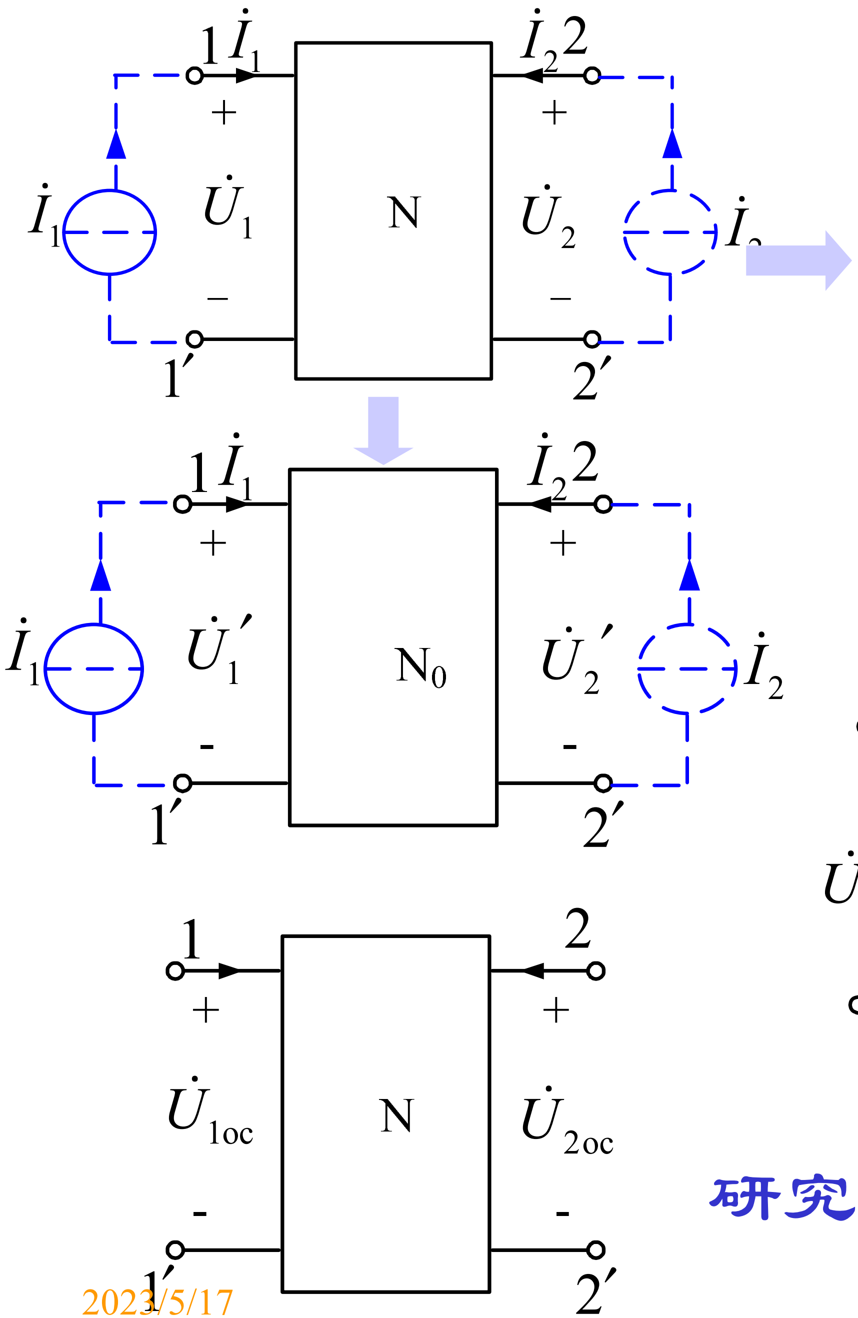
$$\dot U = \dot U_{oc} + Z_{eq}\dot I$$

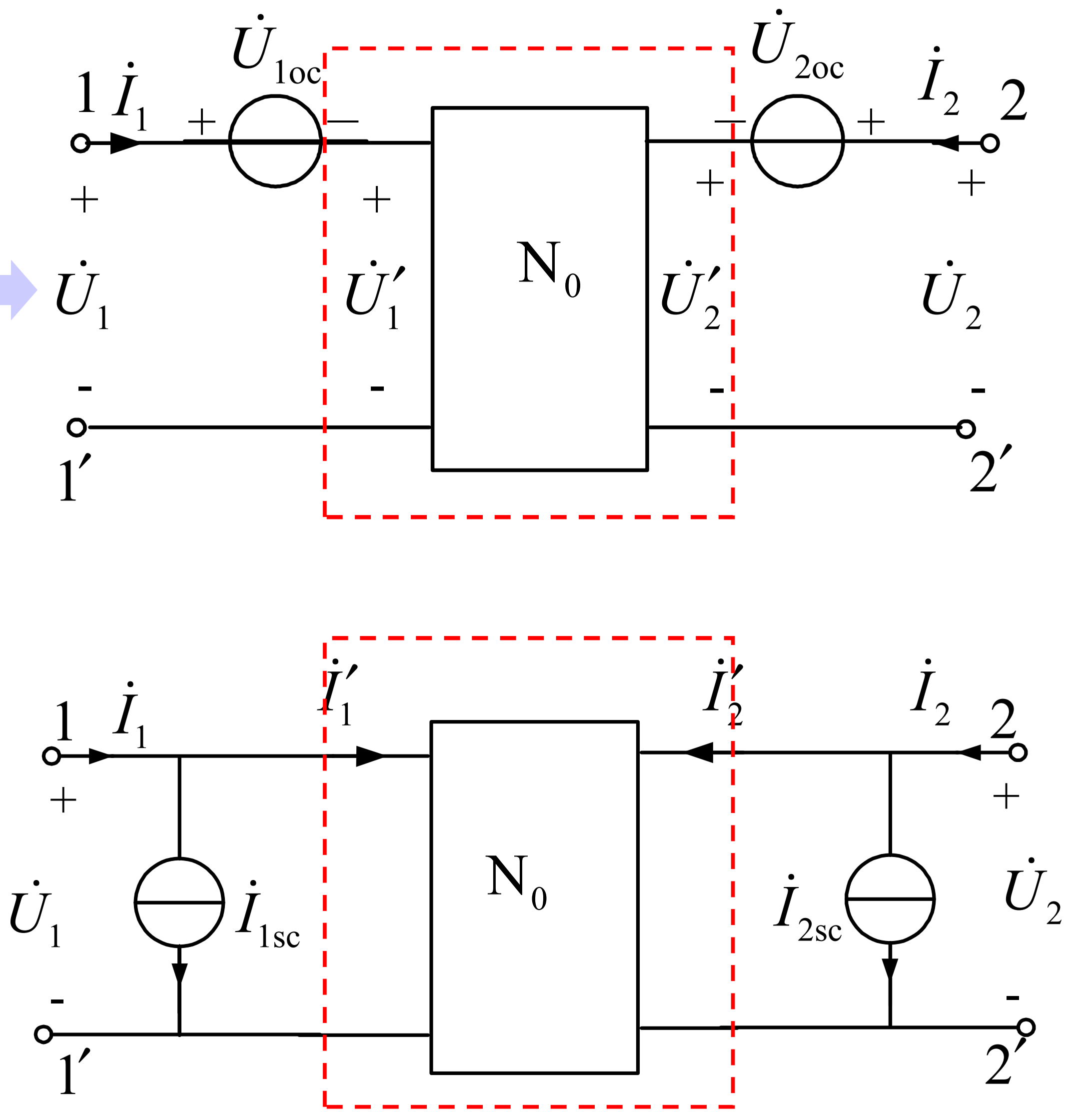


$$\dot I = Y_{eq}\dot U + \dot I_{sc}$$



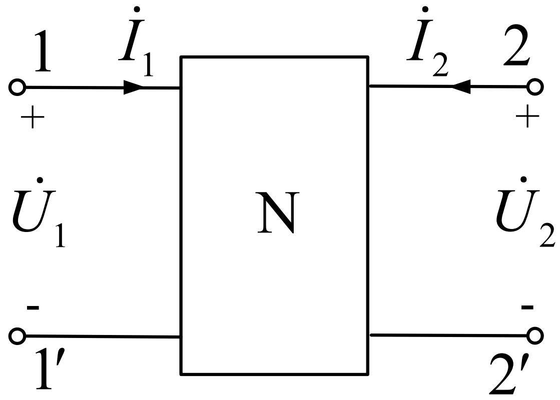
3 含源二端口网络





研究不含源二端口网络的端口特性

4 线性松弛二端口网络的端口特性方程

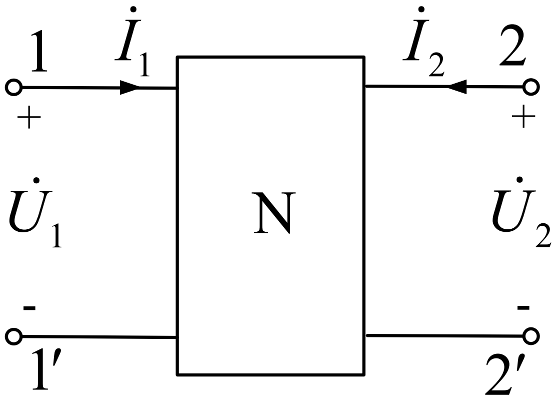


- 二端口网络的4个端口变量中，只有2个独立变量。他们构成2个端口特性方程

- 任选2个端口变量作为自变量，另外2个作为函数，即可得到二端口网络的一组特性方程；

- 可用六组参数描述二端口网络，六组不同的方程均可表示端口特性。

4 线性松弛二端口网络的端口特性方程



$$\begin{cases} \dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 \\ \dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2 \end{cases}$$

阻抗参数方程

$$\begin{cases} \dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2 \end{cases}$$

导纳参数方程

$$\begin{cases} \dot{U}_1 = h_{11}\dot{I}_1 + h_{12}\dot{U}_2 \\ \dot{I}_2 = h_{21}\dot{I}_1 + h_{22}\dot{U}_2 \end{cases} \qquad \begin{cases} \dot{I}_1 = g_{11}\dot{U}_1 + g_{12}\dot{I}_2 \\ \dot{U}_2 = g_{21}\dot{U}_1 + g_{22}\dot{I}_2 \end{cases}$$

混和参数方程

$$\begin{cases} \dot{U}_1 = A\dot{U}_2 + B(-\dot{I}_2) \\ \dot{I}_1 = C\dot{U}_2 + D(-\dot{I}_2) \end{cases} \qquad \begin{cases} \dot{U}_2 = A'\dot{U}_1 + B'(-\dot{I}_1) \\ \dot{I}_2 = C'\dot{U}_1 + D'(-\dot{I}_1) \end{cases}$$

传输参数方程

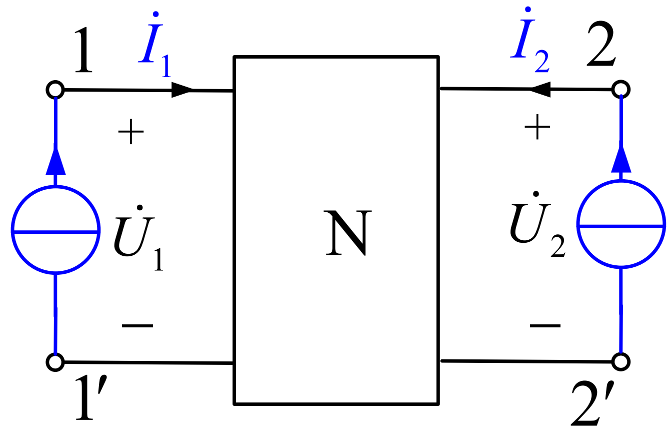
4 线性松弛二端口网络的端口特性方程

名称	自变量	方程
阻抗参数方程	$\dot{I}_1$、$\dot{I}_2$	$\left.\begin{array}{l}\dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 \\ \dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2\end{array}\right\}$
导纳参数方程	$\dot{U}_1$、$\dot{U}_2$	$\left.\begin{array}{l}\dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2\end{array}\right\}$
混合参数方程	$\dot{I}_1$、$\dot{U}_2$	$\left.\begin{array}{l}\dot{U}_1 = h_{11}\dot{I}_1 + h_{12}\dot{U}_2 \\ \dot{I}_2 = h_{21}\dot{I}_1 + h_{22}\dot{U}_2\end{array}\right\}$
逆混合参数方程	$\dot{U}_1$、$\dot{I}_2$	$\left.\begin{array}{l}\dot{I}_1 = g_{11}\dot{U}_1 + g_{12}\dot{I}_2 \\ \dot{U}_2 = g_{21}\dot{U}_1 + g_{22}\dot{I}_2\end{array}\right\}$
正向传输参数方程	$\dot{U}_2$、$\dot{I}_2$	$\left.\begin{array}{l}\dot{U}_1 = A\dot{U}_2 + B(-\dot{I}_2) \\ \dot{I}_1 = C\dot{U}_2 + D(-\dot{I}_2)\end{array}\right\}$
反向传输参数方程	$\dot{U}_1$、$\dot{I}_1$	$\left.\begin{array}{l}\dot{U}_2 = A'\dot{U}_1 + B'(-\dot{I}_1) \\ \dot{I}_2 = C'\dot{U}_1 + D'(-\dot{I}_1)\end{array}\right\}$

16.3 二端口网络的参数

16.3.1阻抗参数

1 阻抗参数方程（Z）



将$\dot{I}_1$、$\dot{I}_2$视为激励源，$\dot{U}_1$、$\dot{U}_2$是在$\dot{I}_1$、$\dot{I}_2$共同激励下的响应。由叠加定理得

$$\boxed{\begin{cases}\dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 \\ \dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2\end{cases}}$$

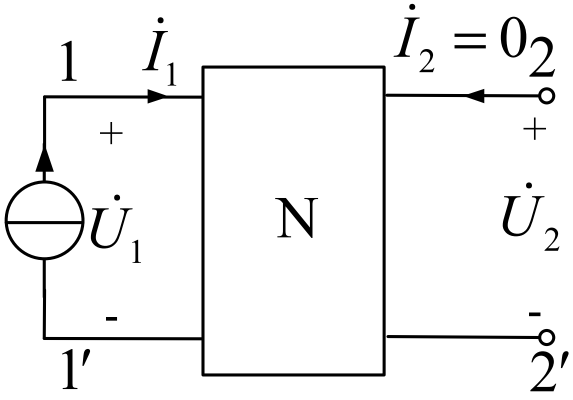
上述方程即为Z参数方程，写成矩阵形式为:

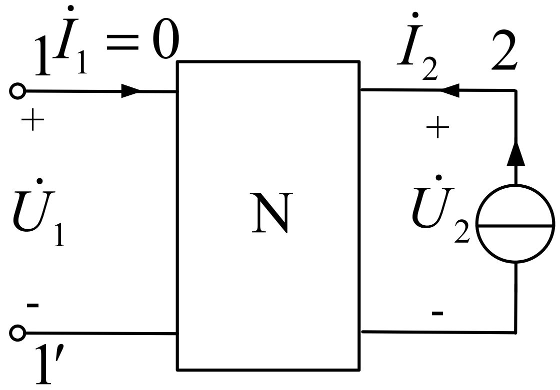
$$\begin{bmatrix}\dot{U}_1 \\ \dot{U}_2\end{bmatrix} = \begin{bmatrix}Z_{11} & Z_{12} \\ Z_{21} & Z_{22}\end{bmatrix}\begin{bmatrix}\dot{I}_1 \\ \dot{I}_2\end{bmatrix} \qquad [Z] = \begin{bmatrix}Z_{11} & Z_{12} \\ Z_{21} & Z_{22}\end{bmatrix}$$

称为Z参数矩阵

2. Z 参数计算与测量方法

$$\begin{cases} \dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 \\ \dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2 \end{cases}$$





由Z参数方程可得：

$$Z_{11} = \left.\frac{\dot{U}_1}{\dot{I}_1}\right|_{\dot{I}_2=0}$$

$$Z_{21} = \left.\frac{\dot{U}_2}{\dot{I}_1}\right|_{\dot{I}_2=0}$$

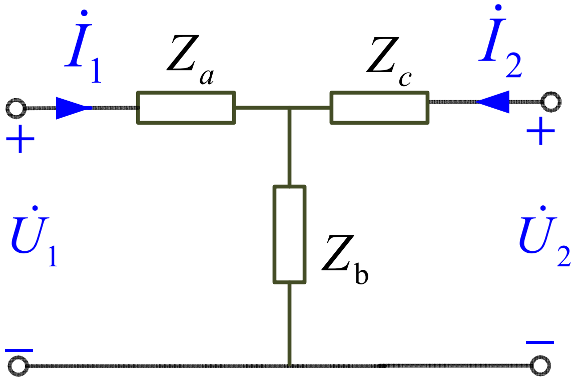
由Z参数方程可得：

$$Z_{12} = \left.\frac{\dot{U}_1}{\dot{I}_2}\right|_{\dot{I}_1=0}$$

$$Z_{22} = \left.\frac{\dot{U}_2}{\dot{I}_2}\right|_{\dot{I}_1=0}$$

3. 互易二端口：N为互易网络时，由互易定理： $Z_{12} = Z_{21}$

【例1】



$$\begin{cases} \dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 \\ \dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2 \end{cases}$$

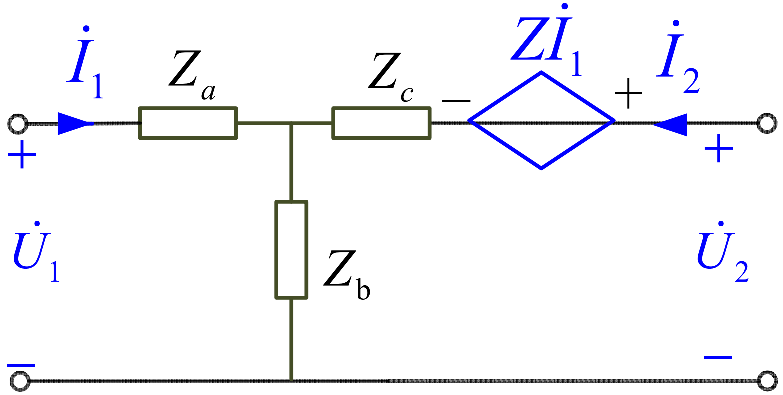
$$Z_{11} = \frac{\dot{U}_1}{\dot{I}_1}\bigg|_{\dot{I}_2=0} = Z_a + Z_b$$

$$Z_{21} = \frac{\dot{U}_2}{\dot{I}_1}\bigg|_{\dot{I}_2=0} = Z_b$$

$$Z_{12} = \frac{\dot{U}_1}{\dot{I}_2}\bigg|_{\dot{I}_1=0} = Z_b$$

$$Z_{22} = \frac{\dot{U}_2}{\dot{I}_2}\bigg|_{\dot{I}_1=0} = Z_b + Z_c$$

【例2】



$$Z_{11} = \frac{\dot{U}_1}{\dot{I}_1}\bigg|_{\dot{I}_2=0} = Z_a + Z_b$$

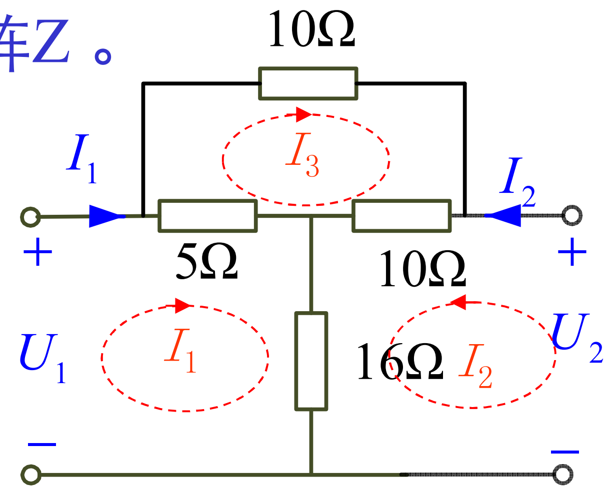
$$Z_{21} = \frac{\dot{U}_2}{\dot{I}_1}\bigg|_{\dot{I}_2=0} = Z_b + Z$$

$$Z_{12} = \frac{\dot{U}_1}{\dot{I}_2}\bigg|_{\dot{I}_1=0} = Z_b$$

$$Z_{22} = \frac{\dot{U}_2}{\dot{I}_2}\bigg|_{\dot{I}_1=0} = Z_b + Z_c$$

$$Z_{12} \neq Z_{21}$$

【例3】：求图示二端口网络的阻抗参数矩阵Z 。



解题思路：用网络方程法计算参数。以电流为自变量，列端口电压的方程，并整理系数即可得Z参数。

网孔电流方程为：

$$
\begin{aligned}
U_1 &= (5+16)I_1 + 16I_2 - 5I_3 \\
16I_1 + (10+16)I_2 + 10I_3 &= U_2 \\
-5I_1 + 10I_2 + (10+10+5)I_3 &= 0
\end{aligned}
$$

联立求解，消去 I_3：

$$
\begin{aligned}
U_1 &= 20I_1 + 18I_2 \\
U_2 &= 18I_1 + 22I_2
\end{aligned}
$$

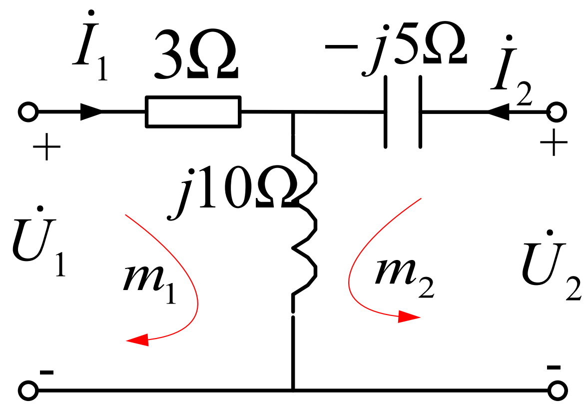
Z参数矩阵方程为：

$$
\boldsymbol{Z} = \begin{bmatrix} 20 & 18 \\ 18 & 22 \end{bmatrix} \Omega
$$

思考题: 求其他参数矩阵?

【练习】计算Z参数（p90，例16-3-1)

$$\boxed{\begin{cases}\dot U_1 = Z_{11}\dot I_1 + Z_{12}\dot I_2 \\ \dot U_2 = Z_{21}\dot I_1 + Z_{22}\dot I_2\end{cases}}$$



$$Z_{11} = \frac{\dot U_1}{\dot I_1}\Big|_{\dot I_2=0} = (3+\mathrm{j}10)\,\Omega \qquad Z_{21} = \frac{\dot U_2}{\dot I_1}\Big|_{\dot I_2=0} = \mathrm{j}10\,\Omega$$

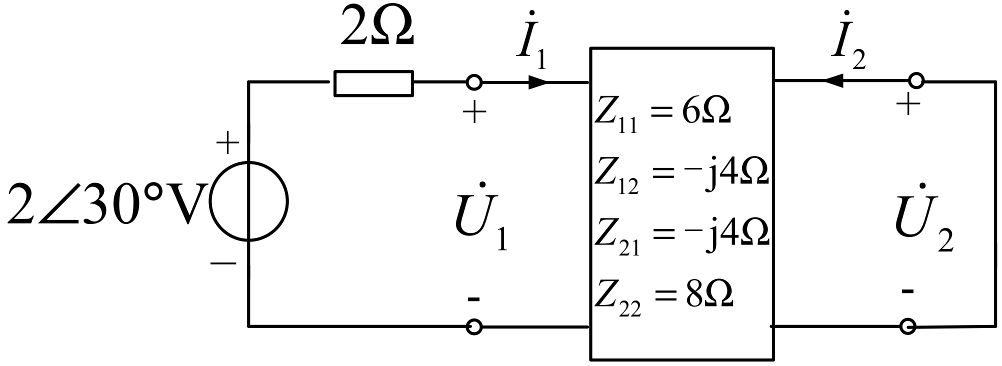
$$Z_{22} = \frac{\dot U_2}{\dot I_2}\Big|_{\dot I_1=0} = -\mathrm{j}5+\mathrm{j}10 = \mathrm{j}5\,\Omega \qquad Z_{12} = \frac{\dot U_1}{\dot I_2}\Big|_{\dot I_1=0} = \mathrm{j}10\,\Omega$$

方法2：网络方程

$$\begin{cases}(3+\mathrm{j}10)\dot I_1 + \mathrm{j}10\dot I_2 = \dot U_1 \\ \mathrm{j}10\dot I_1 + (\mathrm{j}10-\mathrm{j}5)\dot I_2 = \dot U_2\end{cases}$$

$$Z = \begin{bmatrix}3+\mathrm{j}10 & \mathrm{j}10 \\ \mathrm{j}10 & \mathrm{j}5\end{bmatrix}$$

例4.计算图中所示电流$\dot{I}_1$、$\dot{I}_2$



解：端接支路特性方程（VCR）

$$\begin{cases}\dot{U}_1 = 2\angle 30^0 - 2\dot{I}_1 \\ \dot{U}_2 = 0\end{cases}$$

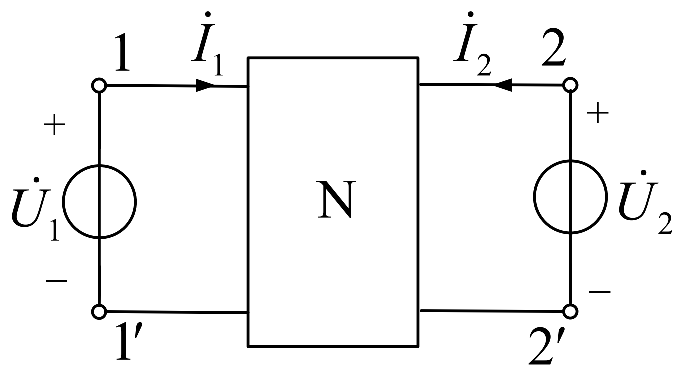
二端口网络的参数方程为

$$\begin{cases}\dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 = 6\dot{I}_1 - j4\dot{I}_2 \\ \dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2 = -j4\dot{I}_1 + 8\dot{I}_2\end{cases}$$

可以求出

$$\dot{I}_1 = 0.2\angle 30°\,\text{A}，\quad \dot{I}_2 = 0.1\angle 120°\,\text{A}$$

16.3.2 导纳参数



1 导纳参数方程（Y）

将两个端口各施加一电压源，则端口电流可视为这些电压源的叠加作用产生。

即:

$$\boxed{\begin{cases} \dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2 \end{cases}}$$

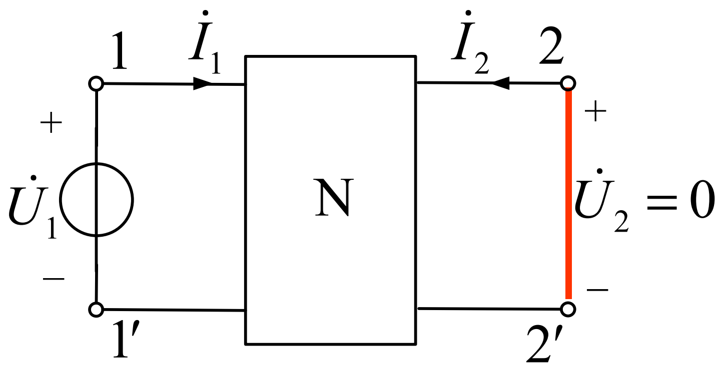
上述方程即为Y参数方程，写成矩阵形式为:

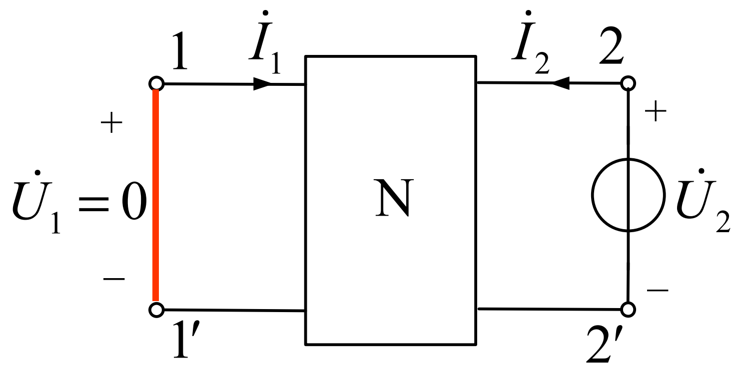
$$\begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix} = \boldsymbol{Y} \begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix}$$

称为Y 参数矩阵.

2. Y参数的计算和测定

$$\boxed{\begin{cases}\dot I_1 = Y_{11}\dot U_1 + Y_{12}\dot U_2 \\ \dot I_2 = Y_{21}\dot U_1 + Y_{22}\dot U_2\end{cases}}$$





由*Y*参数方程可得：

$$Y_{11} = \left.\frac{\dot I_1}{\dot U_1}\right|_{\dot U_2=0}$$

$$Y_{21} = \left.\frac{\dot I_2}{\dot U_1}\right|_{\dot U_2=0}$$

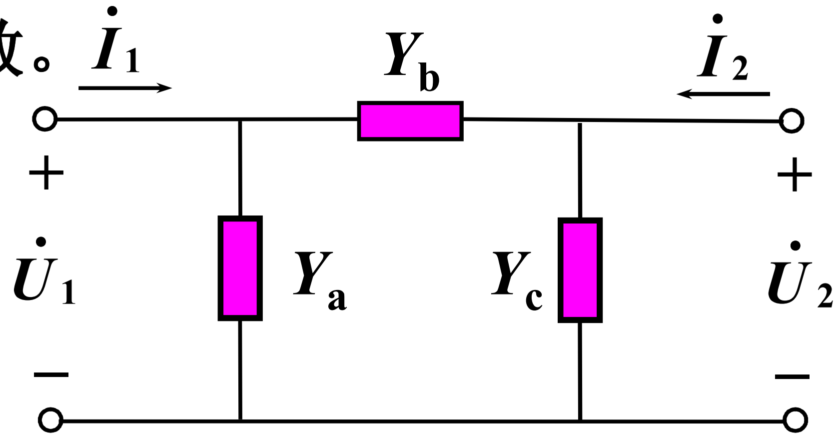
由*Y*参数方程可得：

$$Y_{12} = \left.\frac{\dot I_1}{\dot U_2}\right|_{\dot U_1=0}$$

$$Y_{22} = \left.\frac{\dot I_2}{\dot U_2}\right|_{\dot U_1=0}$$

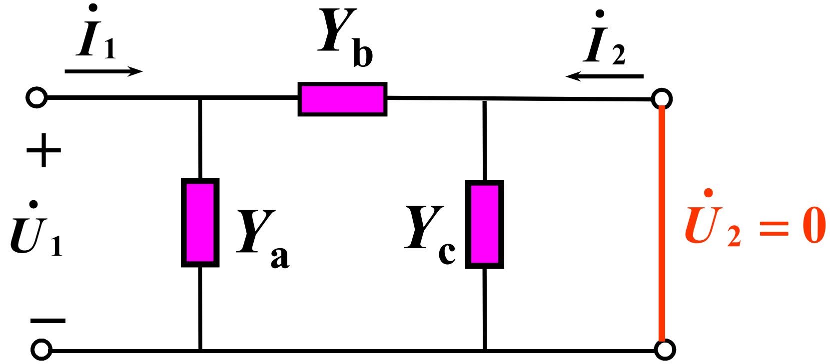
3. 互易二端口：N为互易网络时，由互易定理： $Y_{12} = Y_{21}$

例1. 求 Y 参数。



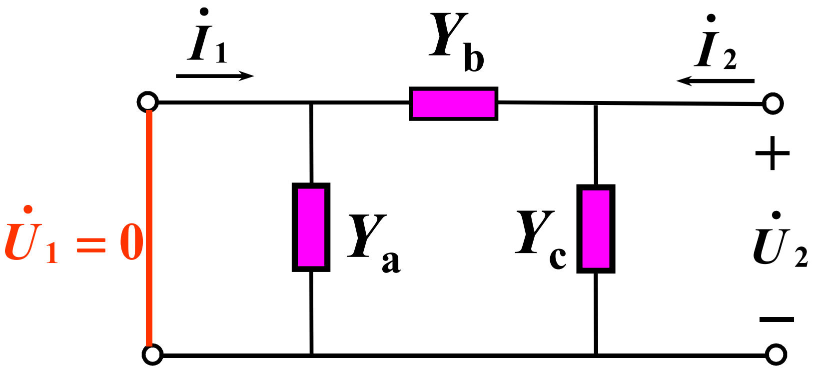
$$\begin{cases} \dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2 \end{cases}$$

解：



$$Y_{11} = \left.\frac{\dot{I}_1}{\dot{U}_1}\right|_{\dot{U}_2=0} = Y_a + Y_b$$

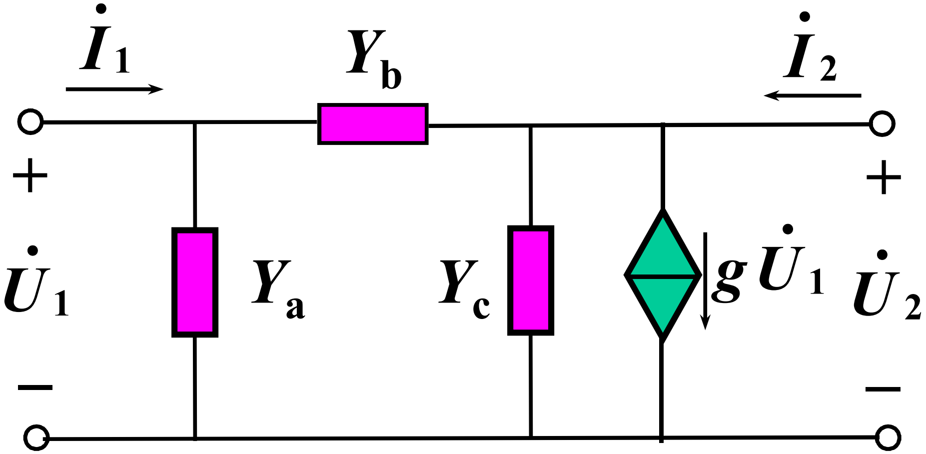
$$Y_{21} = \left.\frac{\dot{I}_2}{\dot{U}_1}\right|_{\dot{U}_2=0} = -Y_b$$



$$Y_{12} = \left.\frac{\dot{I}_1}{\dot{U}_2}\right|_{\dot{U}_1=0} = -Y_b$$

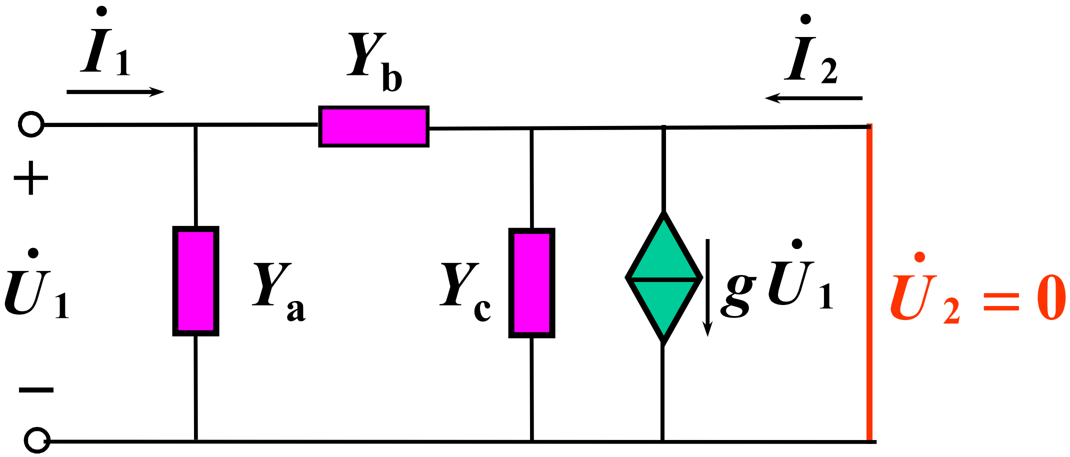
$$Y_{22} = \left.\frac{\dot{I}_2}{\dot{U}_2}\right|_{\dot{U}_1=0} = Y_b + Y_c$$

例2.



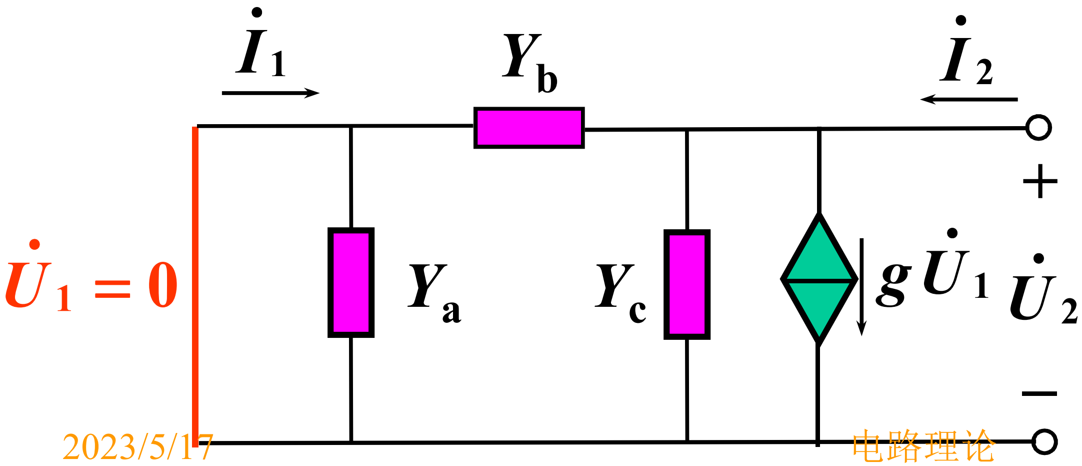
$$\begin{cases} \dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2 \end{cases}$$

解:



$$Y_{11} = \frac{\dot{I}_1}{\dot{U}_1}\bigg|_{\dot{U}_2=0} = Y_a + Y_b$$

$$Y_{21} = \frac{\dot{I}_2}{\dot{U}_1}\bigg|_{\dot{U}_2=0} = -Y_b + g$$



$$Y_{12} = \frac{\dot{I}_1}{\dot{U}_2}\bigg|_{\dot{U}_1=0} = -Y_b$$

$$Y_{22} = \frac{\dot{I}_2}{\dot{U}_2}\bigg|_{\dot{U}_1=0} = Y_b + Y_c$$

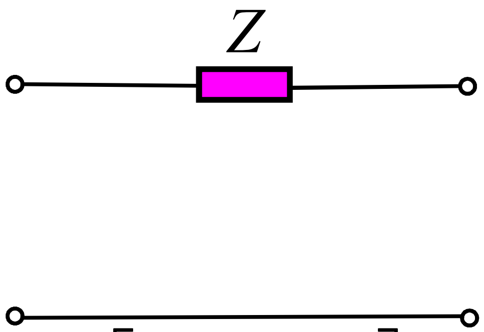
4. 对称二端口

除 $Y_{12}=Y_{21}$外，若 $Y_{11}=Y_{22}$，称为对称二端口。

对称二端口是指电路结构左右对称。两个端口电气特性上对称，结构不对称的二端口，其电气特性可能是对称的，这样的二端口也是对称二端口。

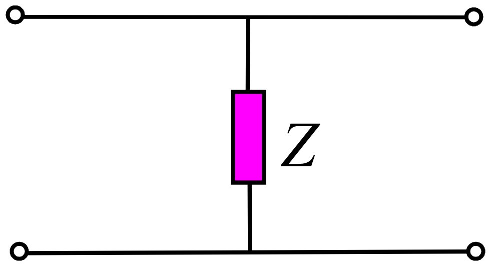
5. Z 参数矩阵与Y 参数矩阵互为逆矩阵。即：$Y=Z^{-1}$ $Z=Y^{-1}$

6. 并非所有的二端口均有Z,Y 参数。



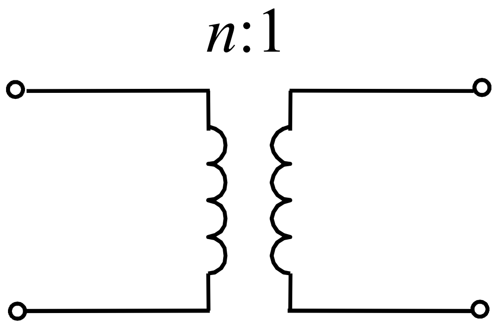
$$[Y]=\begin{bmatrix}\frac{1}{Z} & -\frac{1}{Z}\\ -\frac{1}{Z} & \frac{1}{Z}\end{bmatrix}$$

Z不存在



$$[Z]=\begin{bmatrix}Z & Z\\ Z & Z\end{bmatrix}$$

Y不存在



Z,Y 均不存在

16.3.3 混合参数H

***H* 参数和G参数称为混合参数， *H* 参数常用于晶体管等效电路。**

1．混合 参数方程

➢ *H* 参数

$$\begin{cases}\dot{U}_1 = h_{11}\dot{I}_1 + h_{12}\dot{U}_2 \\ \dot{I}_2 = h_{21}\dot{I}_1 + h_{22}\dot{U}_2\end{cases}$$

➢G参数

$$\begin{cases}\dot{I}_1 = g_{11}\dot{U}_1 + g_{12}\dot{I}_2 \\ \dot{U}_2 = g_{21}\dot{U}_1 + g_{22}\dot{I}_2\end{cases}$$

$$\begin{bmatrix}\dot{I}_1 \\ \dot{U}_2\end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22}\end{bmatrix}\begin{bmatrix}\dot{U}_1 \\ \dot{I}_2\end{bmatrix} = G\begin{bmatrix}\dot{U}_1 \\ \dot{I}_2\end{bmatrix}$$

矩阵形式:

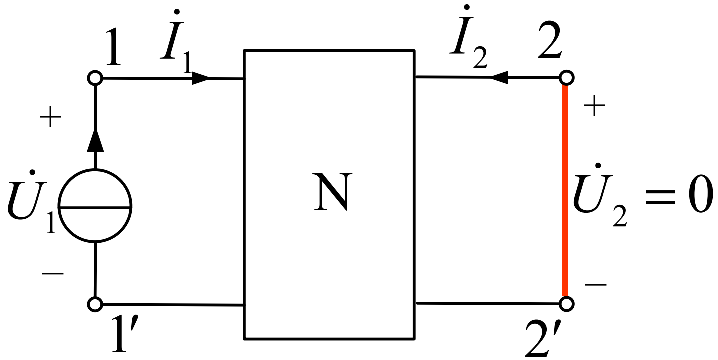
$$\begin{bmatrix}\dot{U}_1 \\ \dot{I}_2\end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22}\end{bmatrix}\begin{bmatrix}\dot{I}_1 \\ \dot{U}_2\end{bmatrix} = H\begin{bmatrix}\dot{I}_1 \\ \dot{U}_2\end{bmatrix}$$

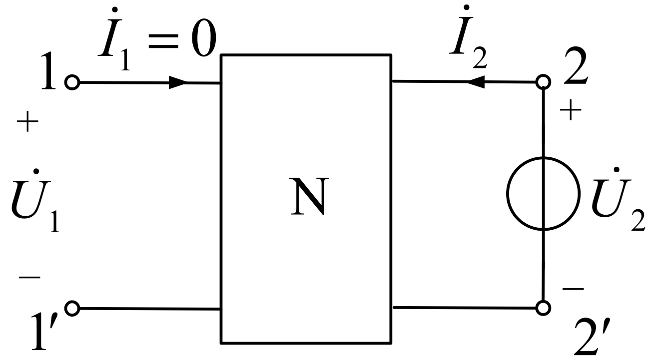
$$\begin{bmatrix}\dot{U}_1 \\ \dot{I}_2\end{bmatrix} = G^{-1}\begin{bmatrix}\dot{I}_1 \\ \dot{U}_2\end{bmatrix} = H\begin{bmatrix}\dot{I}_1 \\ \dot{U}_2\end{bmatrix}$$

G和H存在逆矩阵时，它们互为逆矩阵：$G=H^{-1}$或$H=G^{-1}$

2. *H* 参数的计算与测定

$$\begin{cases} \dot{U}_1 = h_{11}\dot{I}_1 + h_{12}\dot{U}_2 \\ \dot{I}_2 = h_{21}\dot{I}_1 + h_{22}\dot{U}_2 \end{cases}$$





$$h_{11} = \frac{\dot{\mathbf{U}}_1}{\dot{\mathbf{I}}_1}\bigg|_{\dot{\mathbf{U}}_2=0} \qquad h_{12} = \frac{\dot{\mathbf{U}}_1}{\dot{\mathbf{U}}_2}\bigg|_{\dot{\mathbf{I}}_1=0}$$

$$h_{21} = \frac{\dot{\mathbf{I}}_2}{\dot{\mathbf{I}}_1}\bigg|_{\dot{\mathbf{U}}_2=0} \qquad h_{22} = \frac{\dot{\mathbf{I}}_2}{\dot{\mathbf{U}}_2}\bigg|_{\dot{\mathbf{I}}_1=0}$$

3. 互易二端口：N为互易网络时，由互易定理：

$$h_{12} = -h_{21}$$

【例1】.

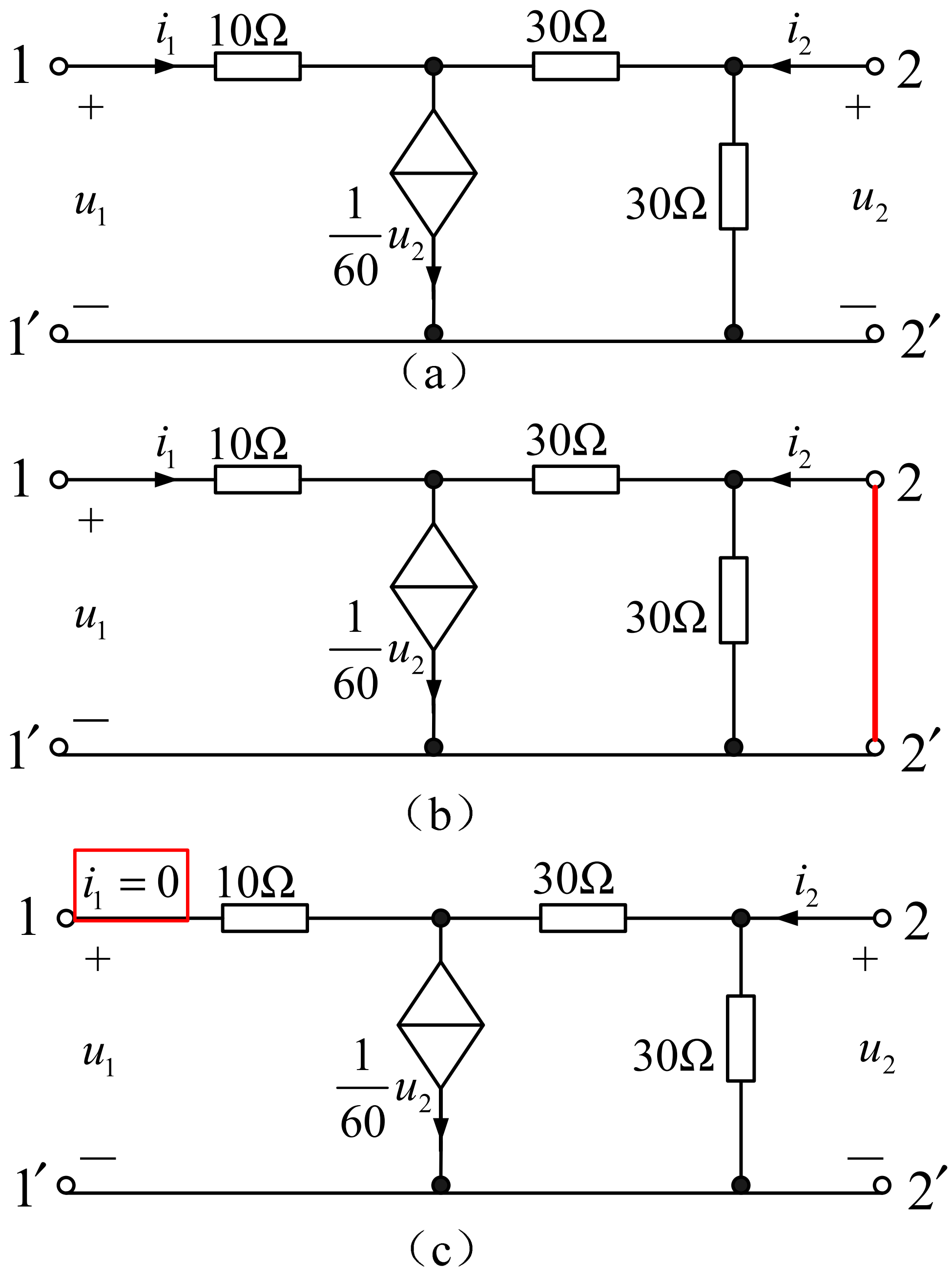
$$\begin{cases} \dot{U}_1 = h_{11}\dot{I}_1 + h_{12}\dot{U}_2 \\ \dot{I}_2 = h_{21}\dot{I}_1 + h_{22}\dot{U}_2 \end{cases}$$

$$h_{11} = \frac{u_1}{i_1}\bigg|_{u_2=0} = 10+30 = 40\Omega$$

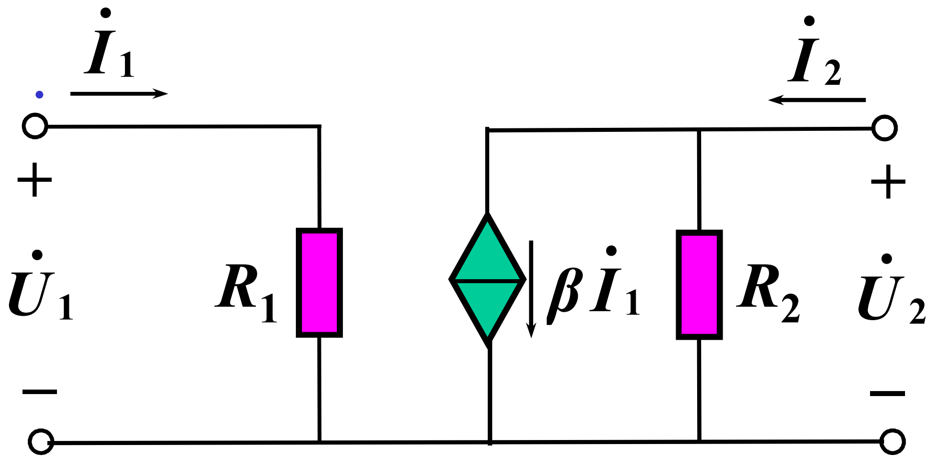
$$h_{21} = \frac{i_2}{i_1}\bigg|_{u_2=0} = \frac{-i_1}{i_1} = -1$$

$$h_{22} = \frac{i_2}{u_2}\bigg|_{i_1=0} = \frac{\frac{1}{60}u_2 + \frac{1}{30}u_2}{u_2} = \frac{1}{20}\text{S}$$

$$h_{12} = \frac{u_1}{u_2}\bigg|_{i_1=0} = \frac{u_2 - \frac{1}{60}u_2 \times 30}{u_2} = \frac{1}{2}$$



【课下练习】



$$\boxed{\begin{cases} \dot{U}_1 = h_{11}\dot{I}_1 + h_{12}\dot{U}_2 \\ \dot{I}_2 = h_{21}\dot{I}_1 + h_{22}\dot{U}_2 \end{cases}}$$

$$h_{11} = \frac{\dot{\mathbf{U}}_1}{\dot{\mathbf{I}}_1}\Big|_{\dot{\mathbf{U}}_2=0} = R_1 \qquad h_{12} = \frac{\dot{\mathbf{U}}_1}{\dot{\mathbf{U}}_2}\Big|_{\dot{\mathbf{I}}_1=0} = 0$$

$$h_{21} = \frac{\dot{\mathbf{I}}_2}{\dot{\mathbf{I}}_1}\Big|_{\dot{\mathbf{U}}_2=0} = \beta \qquad h_{22} = \frac{\dot{\mathbf{I}}_2}{\dot{\mathbf{U}}_2}\Big|_{\dot{\mathbf{I}}_1=0} = \frac{1}{R_2}$$

16.3.4 传输参数T

1. *T*参数和方程

$$\begin{cases}\dot{U}_1 = A\dot{U}_2 + B(-\dot{I}_2) \\ \dot{I}_1 = C\dot{U}_2 + D(-\dot{I}_2)\end{cases}$$

$$\begin{bmatrix}\dot{U}_1 \\ \dot{I}_1\end{bmatrix} = \begin{bmatrix}A & B \\ C & D\end{bmatrix}\begin{bmatrix}\dot{U}_2 \\ -\dot{I}_2\end{bmatrix} = \boldsymbol{T}\begin{bmatrix}\dot{U}_2 \\ -\dot{I}_2\end{bmatrix}$$

2. *T'*参数和方程

$$\begin{cases}\dot{U}_2 = A'\dot{U}_1 + B'(-\dot{I}_1) \\ \dot{I}_2 = C'\dot{U}_1 + D'(-\dot{I}_1)\end{cases}$$

$$\begin{bmatrix}\dot{U}_2 \\ \dot{I}_2\end{bmatrix} = \begin{bmatrix}A' & B' \\ C' & D'\end{bmatrix}\begin{bmatrix}\dot{U}_1 \\ -\dot{I}_1\end{bmatrix} = \boldsymbol{T}'\begin{bmatrix}\dot{U}_1 \\ -\dot{I}_1\end{bmatrix}$$

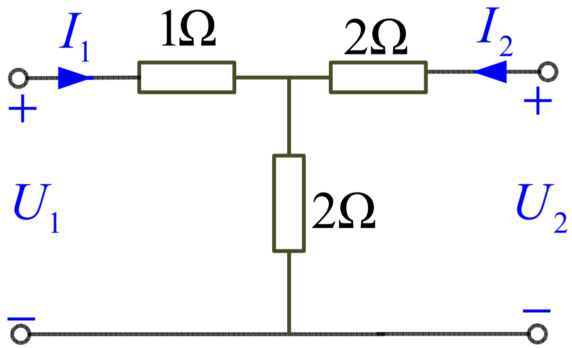
3. *T* 参数的计算或测定

$$\boldsymbol{A} = \frac{\dot{U}_1}{\dot{U}_2}\bigg|_{\dot{I}_2=0} \qquad \boldsymbol{B} = \frac{\dot{U}_1}{-\dot{I}_2}\bigg|_{\dot{U}_2=0}$$

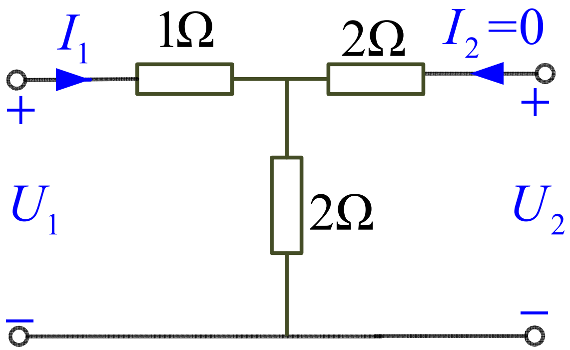
$$\boldsymbol{C} = \frac{\dot{I}_1}{\dot{U}_2}\bigg|_{\dot{I}_2=0} \qquad \boldsymbol{D} = \frac{\dot{I}_1}{-\dot{I}_2}\bigg|_{\dot{U}_2=0}$$

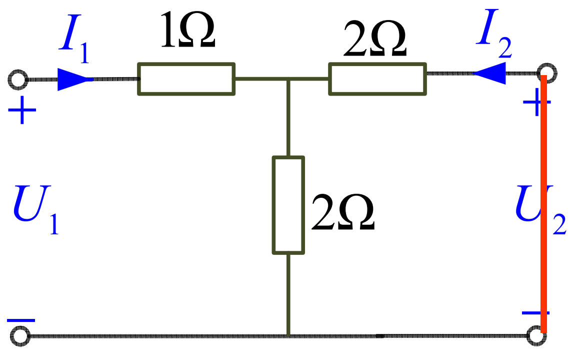
$$\begin{bmatrix}A & -B \\ C & -D\end{bmatrix}\begin{bmatrix}A' & -B' \\ C' & -D'\end{bmatrix} = \begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$$

【例1】求T参数



$$\begin{cases} \dot{U}_1 = A\dot{U}_2 + B(-\dot{I}_2) \\ \dot{I}_1 = C\dot{U}_2 + D(-\dot{I}_2) \end{cases}$$





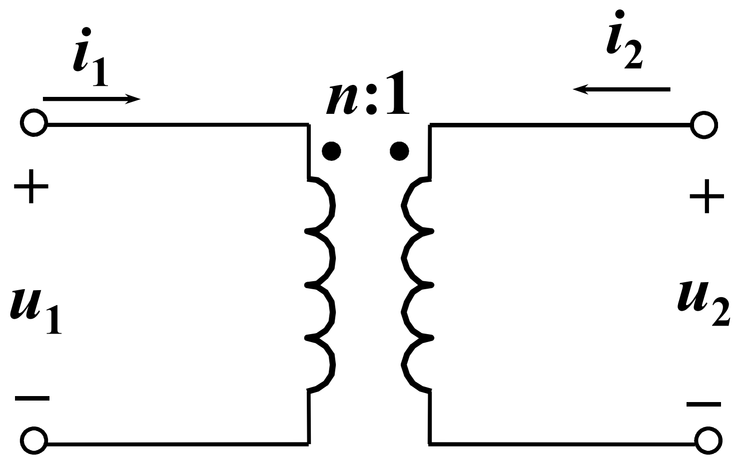
$$A = \frac{U_1}{U_2}\bigg|_{I_2=0} = \frac{1+2}{2} = 1.5$$

$$B = \frac{U_1}{-I_2}\bigg|_{U_2=0} = \frac{I_1[1+(2//2)]}{0.5I_1} = 4\,\Omega$$

$$C = \frac{I_1}{U_2}\bigg|_{I_2=0} = 0.5\,S$$

$$D = \frac{I_1}{-I_2}\bigg|_{U_2=0} = \frac{I_1}{0.5I_1} = 2$$

【练习】.



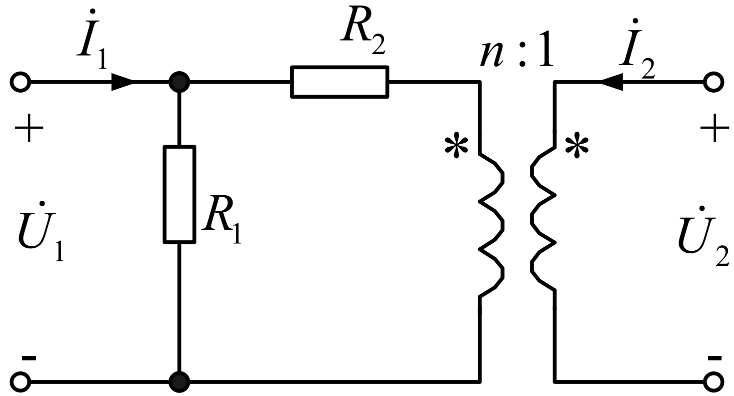
$$\begin{cases} u_1 = nu_2 \\ i_1 = -\dfrac{1}{n} i_2 \end{cases}$$

即 $$\begin{bmatrix} u_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} n & 0 \\ 0 & \dfrac{1}{n} \end{bmatrix} \begin{bmatrix} u_2 \\ -i_2 \end{bmatrix}$$

则 $$[T] = \begin{bmatrix} n & 0 \\ 0 & \dfrac{1}{n} \end{bmatrix}$$

【例2】求T参数

$$\begin{cases}\dot{U}_1 = A\dot{U}_2 + B(-\dot{I}_2) \\ \dot{I}_1 = C\dot{U}_2 + D(-\dot{I}_2)\end{cases}$$

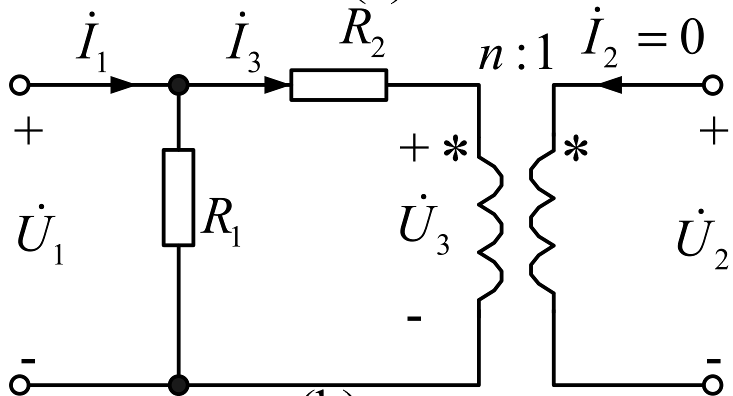


(a)

$$\dot{I}_3 = 0$$

$$\dot{U}_3 = \dot{U}_1 = n\dot{U}_2$$

$$A = \frac{\dot{U}_1}{\dot{U}_2}\bigg|_{\dot{I}_2=0} = n \quad C = \frac{\dot{I}_1}{\dot{U}_2}\bigg|_{\dot{I}_2=0} = \frac{\dfrac{\dot{U}_1}{R_1}}{\dfrac{1}{n}\dot{U}_1} = \frac{n}{R_1}$$

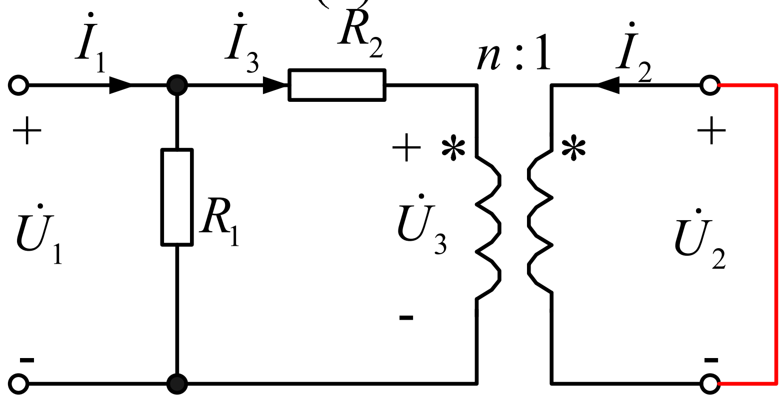


(b)

$$\dot{U}_3 = 0 \quad \dot{I}_3 = \frac{\dot{U}_1}{R_2} \quad \dot{I}_1 = \frac{\dot{U}_1}{R_1//R_2}$$

$$\dot{I}_2 = -n\dot{I}_3 = -\frac{n}{R_2}\dot{U}_1$$

$$D = \frac{\dot{I}_1}{-\dot{I}_2}\bigg|_{\dot{U}_2=0} = \frac{(\dfrac{1}{R_1} + \dfrac{1}{R_2})\dot{U}_1}{\dfrac{n}{R_2}\dot{U}_1} = \frac{R_1 R_2}{nR_1}$$

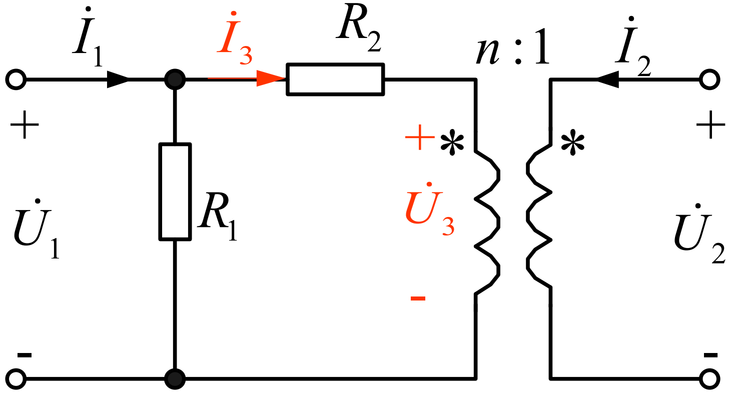


(c)

$$B = \frac{\dot{U}_1}{-\dot{I}_2}\bigg|_{\dot{U}_2=0} = \frac{\dot{U}_1}{\dfrac{n}{R_2}\dot{U}_1} = \frac{R_2}{n}$$

【例2】求T参数 $\begin{cases}\dot{U}_1 = A\dot{U}_2 + B(-\dot{I}_2) \\ \dot{I}_1 = C\dot{U}_2 + D(-\dot{I}_2)\end{cases}$

Method2:



(a)

$$\dot{I}_3 = -\frac{1}{n}\dot{I}_2$$

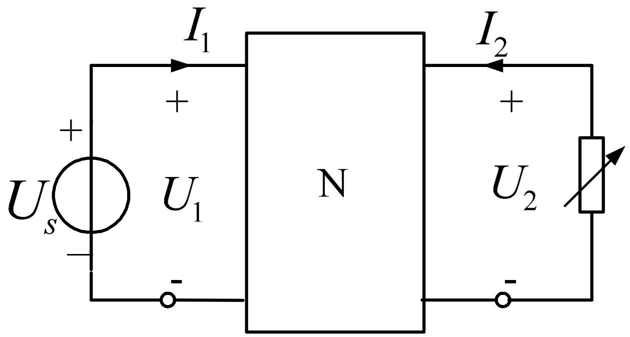
$$\dot{U}_3 = n\dot{U}_2$$

$$\dot{U}_1 = \dot{I}_3 R_2 + \dot{U}_3 = n\dot{U}_2 - \frac{R_2}{n}\dot{I}_2$$

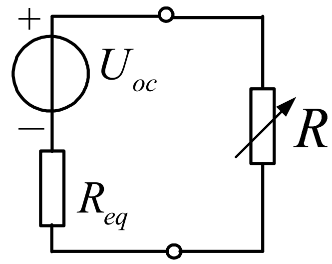
$$\dot{I}_1 = \frac{\dot{U}_1}{R_1} + \dot{I}_3 = \frac{1}{R_1}(n\dot{U}_2 - \frac{R_2}{n}\dot{I}_2) - \frac{1}{n}\dot{I}_2$$

$$\begin{cases}\dot{U}_1 = n\dot{U}_2 + \dfrac{R_2}{n}(-\dot{I}_2) \\ \dot{I}_1 = \dfrac{n}{R_1}\dot{U}_2 + \left(\dfrac{R_2}{nR_1} + \dfrac{1}{n}\right)(-\dot{I}_2)\end{cases}$$

例3：已知：U_S=9V，无源双口网络的传输矩阵 $\boldsymbol{T}=\begin{bmatrix}2.5 & 6\\ 0.5 & 1.6\end{bmatrix}$，求当R为何值时，R吸收的功率为最大？



(a)



(b)

$$\begin{cases}\dot{U}_1 = A\dot{U}_2 + B(-\dot{I}_2)\\ \dot{I}_1 = C\dot{U}_2 + D(-\dot{I}_2)\end{cases}$$

解：

求开路电压：$$U_{oc}=U_2\big|_{I_2=0}=\frac{U_1}{A}=\frac{9}{2.5}=3.6\,\text{V}$$

求短路电流：$$I_{sc}=-I_2\big|_{U_2=0}=\frac{U_1}{B}=\frac{9}{6}=1.5\,\text{A}$$

则等效电阻：$$R_{eq}=\frac{3.6}{1.5}=2.4\Omega$$

则R=2.4 Ω时,获得最大功率：$$P=\left(\frac{U_{oc}}{R+R_{eq}}\right)^2 R=1.35\text{W}$$

思考: 直接由T参数求等效电阻?

16.3.5 参数间的互换关系

1　各参数间的互换关系：将一种参数方程进行自变量和变量转换，将其变成另外一种参数方程，从而获得两种参数之间的互换关系。

- 双口网络可以用六组参数来表征根据具体情况，可以选用一种合适的参数表示：
- Z参数和Y参数常用于理论的探讨和基本定律得推导中；
- H参数广泛用于低频晶体管电路的分析问题中；
- 有些电路只存在某几种参数

如果知道双口网络的任一参数矩阵，通过对变量的运算，可以求得任何其他的参数矩阵，只要这一矩阵是存在的。

参考书中表 161-3-1 二端口网络六种参数的互换

【例4】： 二端口网络的Y参数矩阵为 $Y=\begin{bmatrix} \frac{1}{15} & -\frac{1}{30} \\ -\frac{1}{30} & \frac{1}{15} \end{bmatrix}$ ，求该网络的 Z参数矩阵。

解题思路：列写Y参数矩阵方程，整理方程，经过系数比较就可以获得相应的Z参数矩阵。

Y参数矩阵方程为：
$$\begin{cases} \dot{I}_1 = \frac{1}{15}\dot{U}_1 - \frac{1}{30}\dot{U}_2 \\ \dot{I}_2 = -\frac{1}{30}\dot{U}_1 + \frac{1}{15}\dot{U}_2 \end{cases}$$

整理矩阵方程：
$$\begin{cases} \dot{U}_1 = 20\dot{I}_1 + 10\dot{I}_2 \\ \dot{U}_2 = 10\dot{I}_1 + 20\dot{I}_2 \end{cases}$$

Z参数矩阵方程为：
$$\boldsymbol{Z} = \begin{bmatrix} 20 & 10 \\ 10 & 20 \end{bmatrix}\Omega$$

思考题: 求T参数矩阵?

16.3.5 参数间的互换关系

Z parameters ➡ T parameters:

$$\left.\begin{aligned}\dot U_1 &= Z_{11}\dot I_1 + Z_{12}\dot I_2\\ \dot U_2 &= Z_{21}\dot I_1 + Z_{22}\dot I_2\end{aligned}\right\}$$

$$\dot I_1 = \underbrace{\boxed{\frac{1}{Z_{21}}}}_{C}\dot U_2 + \underbrace{\boxed{\frac{Z_{22}}{Z_{21}}}}_{D}(-\dot I_2)$$

$$\dot U_1 = \underbrace{\boxed{\frac{Z_{11}}{Z_{21}}}}_{A}\dot U_2 + \underbrace{\boxed{\frac{Z_{11}Z_{22}-Z_{12}Z_{21}}{Z_{21}}}}_{B}(-\dot I_2)$$

$$AD-BC=\frac{Z_{11}Z_{22}}{Z_{21}^2}-\frac{Z_{11}Z_{22}-Z_{12}Z_{21}}{Z_{21}^2}=\frac{Z_{12}}{Z_{21}}=1$$

$$\frac{\dot U_2\big|_{\dot I_2=0}}{\dot I_1}=\frac{\dot U_1\big|_{\dot I_1=0}}{\dot I_2}$$

$$Z_{21}=Z_{12}$$

互易定理 1

$$\frac{\dot I_2\big|_{\dot U_2=0}}{\dot U_1}=\frac{\dot I_1\big|_{\dot U_1=0}}{\dot U_2}$$

$$Y_{21}=Y_{12}$$

互易定理 2

$$\frac{\dot U_2\big|_{\dot I_2=0}}{\dot U_1}=\frac{-\dot I_1\big|_{\dot U_1=0}}{\dot I_2}$$

$$g_{12}=-g_{21}$$

$$h_{12}=-h_{21}$$

互易定理 3

16.3.5 参数间的互换关系

表16-1 互易及对称二端口网络的参数特点

	互易网络二端口	对称二端口网络	
Z	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$	$Z_{12} = Z_{21}$
Y	$Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$	$Y_{12} = Y_{21}$
H/G	$h_{12} = -h_{21}$	$h_{11}h_{22} - h_{12}h_{21} = 1$	$h_{12} = -h_{21}$
T/T’	$AD - BC = 1$	$A = D$	$AD - BC = 1$

16.4 二端口网络的等效电路模型

两个二端口网络等效:

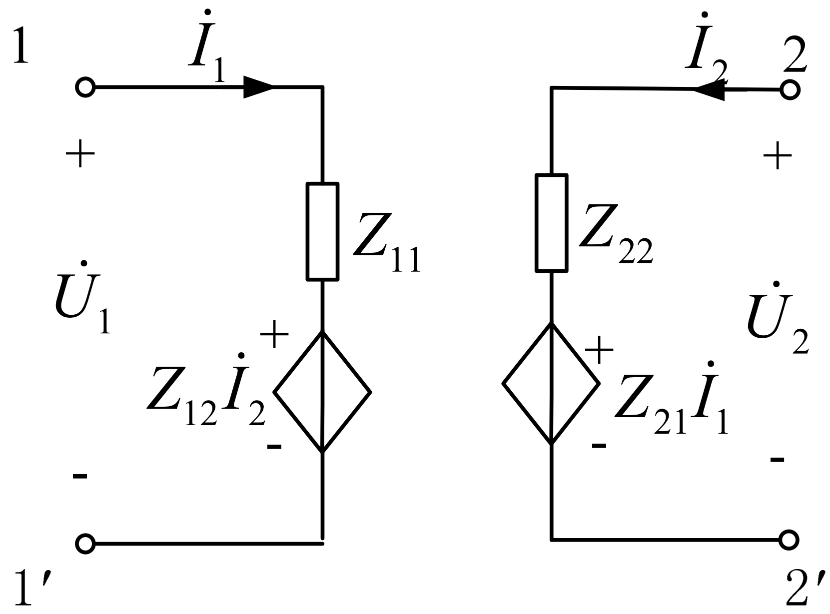
- 是指对外电路而言，端口的电压，电流关系相同。
- 求等效电路即根据给定的参数方程画出电路。

1由Z参数方程表示的等效模型

方法1：直接由参数方程得到等效电路。

$$\begin{cases} \dot{U}_1 = Z_{11}\dot{I}_1 + Z_{12}\dot{I}_2 \\ \dot{U}_2 = Z_{21}\dot{I}_1 + Z_{22}\dot{I}_2 \end{cases}$$

等效电路为:



1由Z参数方程表示的等效模型

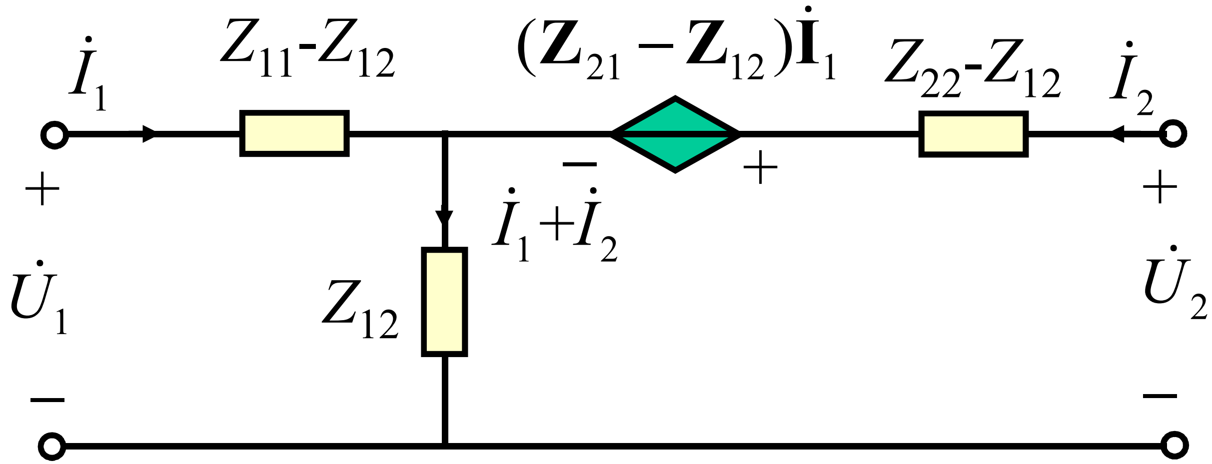
$$\begin{cases}\dot U_1 = Z_{11}\dot I_1 + Z_{12}\dot I_2 \\ \dot U_2 = Z_{21}\dot I_1 + Z_{22}\dot I_2\end{cases}$$

方法2：采用等效变换的方法。

$$\dot{\mathbf{U}}_1 = \mathbf{Z}_{11}\dot{\mathbf{I}}_1 + \mathbf{Z}_{12}\dot{\mathbf{I}}_2 + \mathbf{Z}_{12}\dot{\mathbf{I}}_1 - \mathbf{Z}_{12}\dot{\mathbf{I}}_1 = (\mathbf{Z}_{11} - \mathbf{Z}_{12})\dot{\mathbf{I}}_1 + \mathbf{Z}_{12}(\dot{\mathbf{I}}_1 + \dot{\mathbf{I}}_2)$$

$$\dot{\mathbf{U}}_2 = \mathbf{Z}_{21}\dot{\mathbf{I}}_1 + \mathbf{Z}_{22}\dot{\mathbf{I}}_2 + \mathbf{Z}_{12}\dot{\mathbf{I}}_1 - \mathbf{Z}_{12}\dot{\mathbf{I}}_1 + \mathbf{Z}_{12}\dot{\mathbf{I}}_2 - \mathbf{Z}_{12}\dot{\mathbf{I}}_2$$

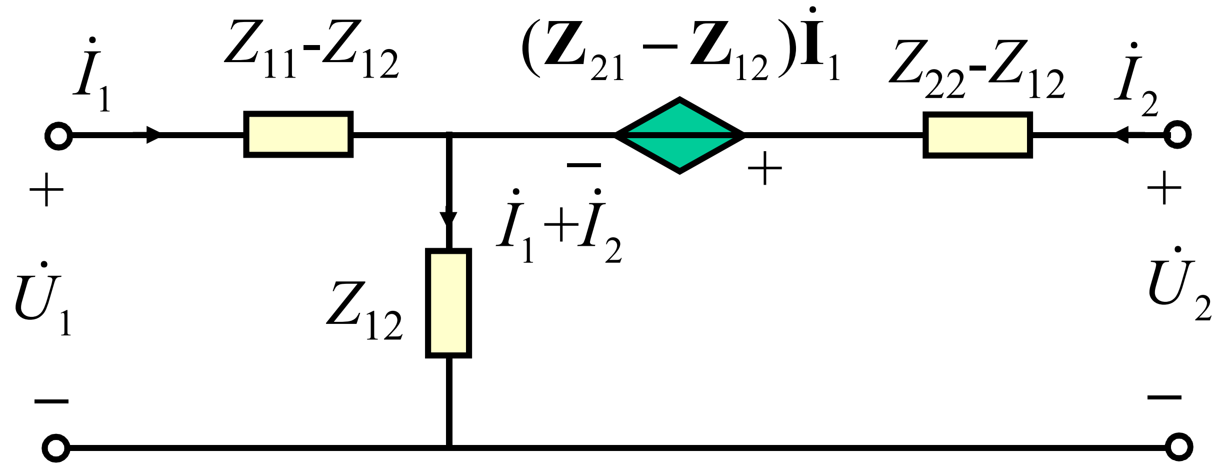
$$= (Z_{22} - Z_{12})\dot I_2 + Z_{12}(\dot I_1 + \dot I_2) + (Z_{21} - Z_{12})\dot I_1$$



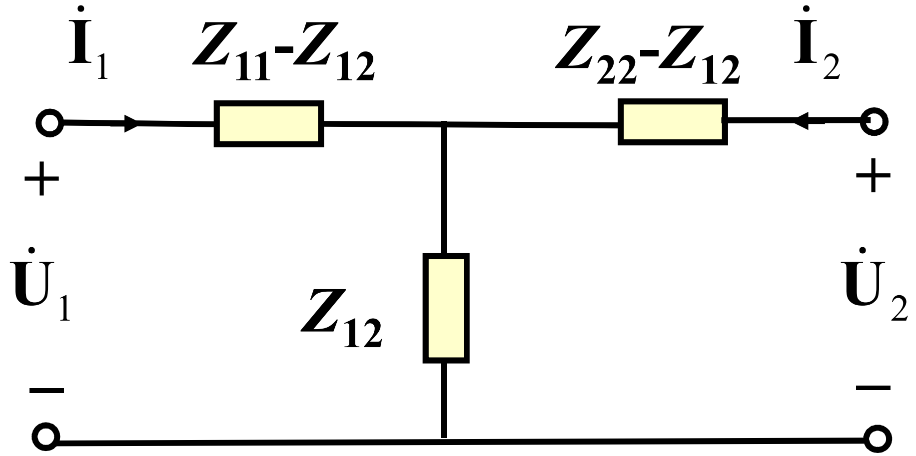
同一个参数方程，可以画出结构不同的等效电路。

等效电路不唯一。

1由Z参数方程表示的等效模型



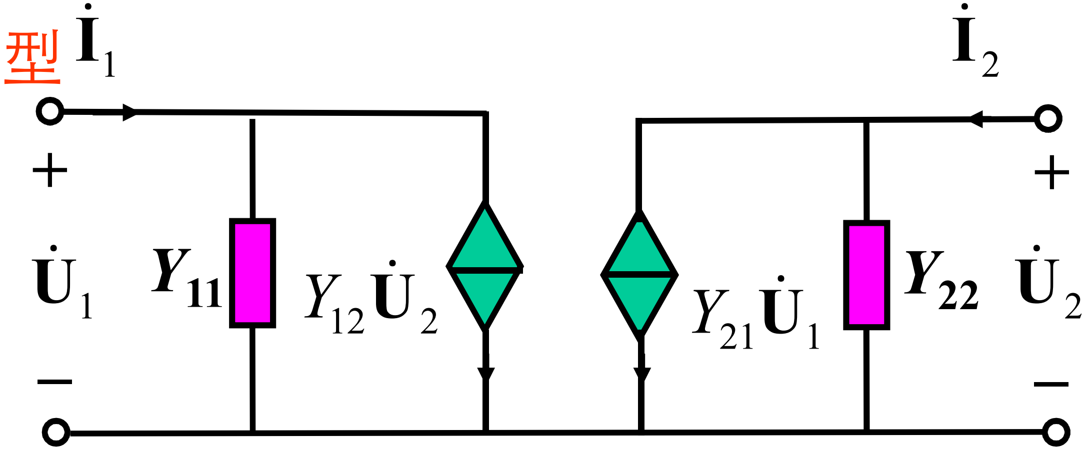
互易网络 $Z_{12}=Z_{21}$



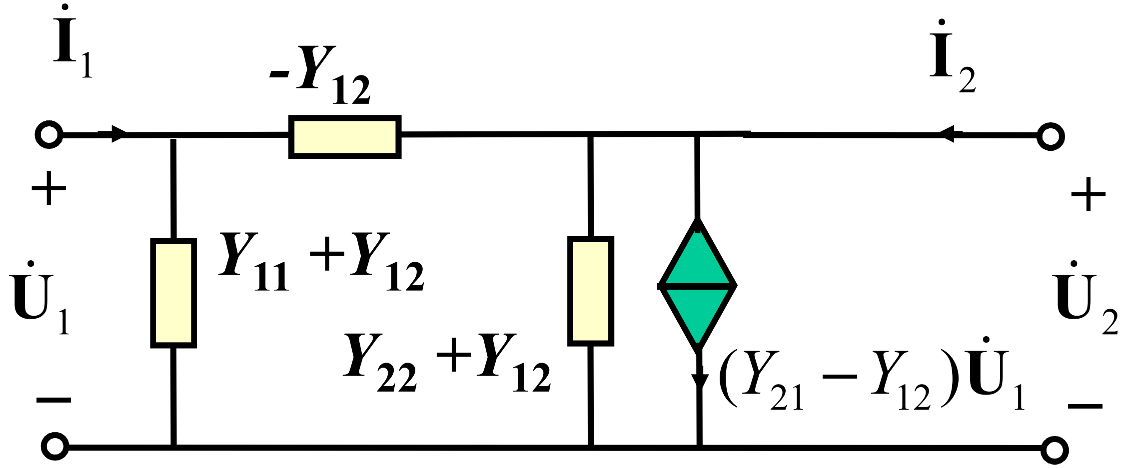
任何二端口网络都有T形等效模型。

2 由Y参数方程表示的等效模型

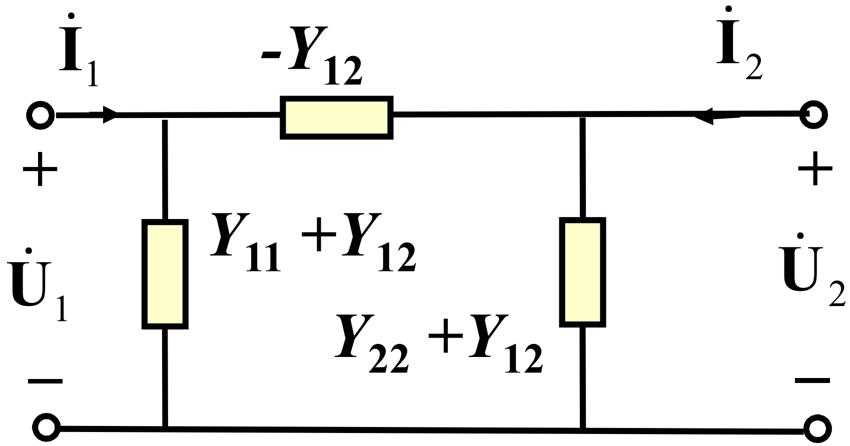
$$\begin{cases} \dot{I}_1 = Y_{11}\dot{U}_1 + Y_{12}\dot{U}_2 \\ \dot{I}_2 = Y_{21}\dot{U}_1 + Y_{22}\dot{U}_2 \end{cases}$$



另一种形式

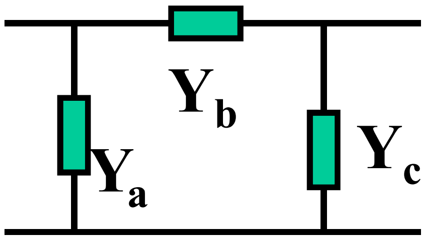


互易网络 $Y_{12}=Y_{21}$

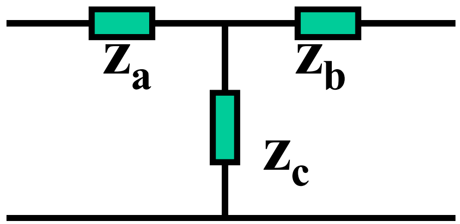


任何二端口网络都有π形等效模型。

3　互易二端口的等效电路



π型等效电路

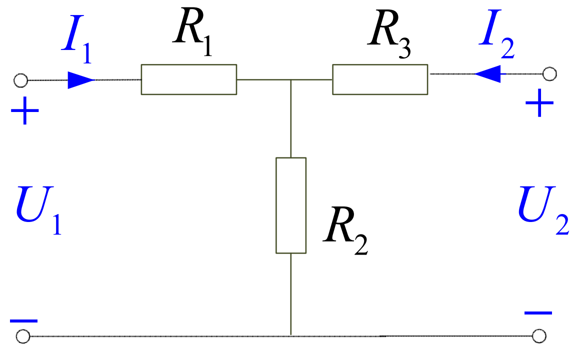


T型等效电路

例：电路如图所示，已知互易二端口网络的正向传输参数为

$$T=\begin{bmatrix}1.5 & 2.5\\ 0.5 & 1.5\end{bmatrix}$$ ，求该网络的T形等效电路。

解： $1.5\times1.5-0.5\times2.5=1$



$$\begin{bmatrix}\dot U_1\\ \dot I_1\end{bmatrix}=\begin{bmatrix}1.5 & 2.5\\ 0.5 & 1.5\end{bmatrix}\begin{bmatrix}\dot U_2\\ -\dot I_2\end{bmatrix}$$

$$A=\frac{\dot U_1}{\dot U_2}\bigg|_{\dot I_2=0}=\frac{R_1+R_2}{R_2}$$

$$B=\frac{\dot U_1}{-\dot I_2}\bigg|_{\dot U_2=0}=\frac{\dot U_1}{\dfrac{\dot U_1}{R_1+R_2//R_3}\times\dfrac{R_2}{R_2+R_3}}\bigg|_{\dot U_2=0}$$

$$C=\frac{\dot I_1}{\dot U_2}\bigg|_{\dot I_2=0}=\frac{1}{R_2}$$

$$=\frac{R_1(R_2+R_3)+R_2R_3}{R_2}$$

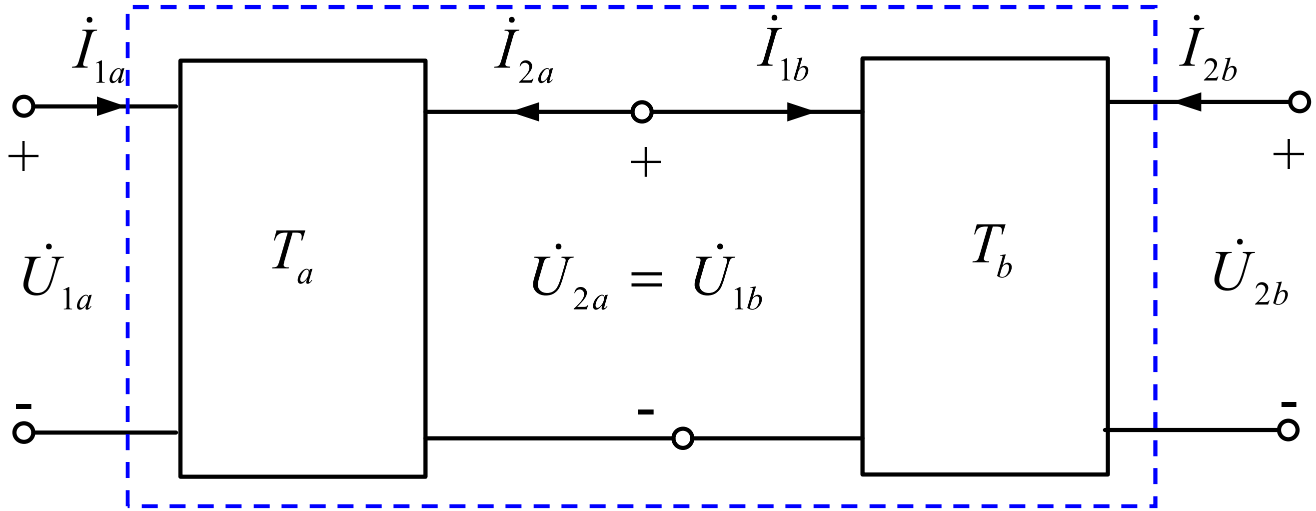
比较系数得$R_1=1\Omega$， $R_2=2\Omega$， $R_3=1\Omega$

16.5 二端口网络的相互连接

- 二端口网络可以在端口处相互联结构成复杂网络。
- 可以是串联、并联和级联。

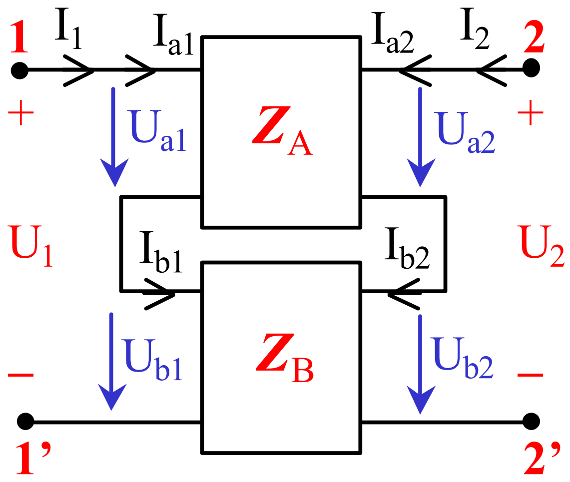
16.5.1级联　Cascade connection

- 一个网络的输出是另一个网络的输入端。



$$\begin{bmatrix} \dot{U}_{1a} \\ \dot{I}_{1a} \end{bmatrix} = \boldsymbol{T}_a \begin{bmatrix} \dot{U}_{2a} \\ -\dot{I}_{2a} \end{bmatrix} = \boldsymbol{T}_a \begin{bmatrix} \dot{U}_{1b} \\ \dot{I}_{1b} \end{bmatrix} = \boldsymbol{T}_a \times \boldsymbol{T}_b \begin{bmatrix} \dot{U}_{2b} \\ -\dot{I}_{2b} \end{bmatrix} \qquad \boldsymbol{T} = \boldsymbol{T}_a \times \boldsymbol{T}_b$$

16.5.2串联（Series Connection ）



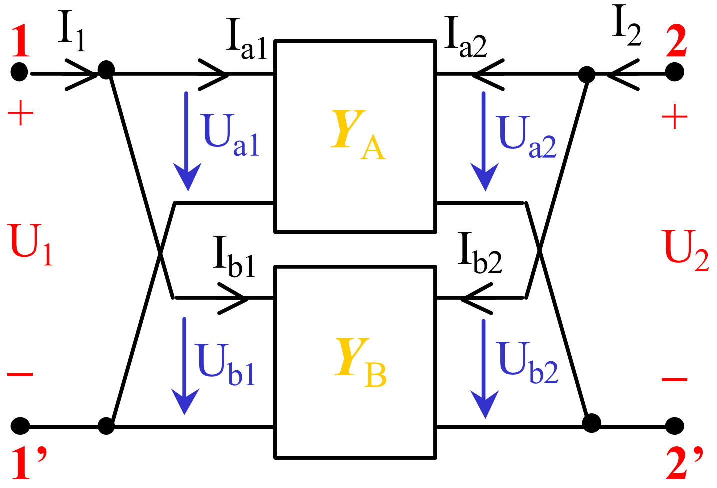
$$I_{a1} = I_{b1} = I_1 \,, \quad I_{a2} = I_{b2} = I_2$$

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} U_{a1} \\ U_{a2} \end{bmatrix} + \begin{bmatrix} U_{b1} \\ U_{b2} \end{bmatrix} = \boldsymbol{Z}_A \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + \boldsymbol{Z}_B \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$= (\boldsymbol{Z}_A + \boldsymbol{Z}_B) \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \boldsymbol{Z} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\boldsymbol{Z} = \boldsymbol{Z}_A + \boldsymbol{Z}_B$$

16.5.3并联（Parallel Connection）



$$U_{a1}=U_{b1}=U_1\ ,\quad U_{a2}=U_{b2}=U_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}=\begin{bmatrix} I_{a1} \\ I_{a2} \end{bmatrix}+\begin{bmatrix} I_{b1} \\ I_{b2} \end{bmatrix}=\boldsymbol{Y}_A\begin{bmatrix} U_1 \\ U_2 \end{bmatrix}+\boldsymbol{Y}_B\begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

$$=(\boldsymbol{Y}_A+\boldsymbol{Y}_B)\begin{bmatrix} U_1 \\ U_2 \end{bmatrix}=\boldsymbol{Y}\begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

$$\boldsymbol{Y}=\boldsymbol{Y}_A+\boldsymbol{Y}_B$$

【例1】一电阻二端口N，其传输参数矩阵为 $\boldsymbol{T}=\begin{bmatrix} 2 & 8\Omega \\ 0.5\mathrm{S} & 2.5 \end{bmatrix}$，

(1)求其T型等效电路

(2)若端口1接U_S=6V、R_1=2Ω的串联支路，端口2接电阻R（1），求R=? 时可使其上获得最大功率，并求此最大功率值。

(3)若端口1接电压源$u_s=6+10\sin t$ V与电阻 R_1=2Ω的串联支路，端口2接L=1H与C=1F的串联支路（图2），求电容C上电压的有效值。

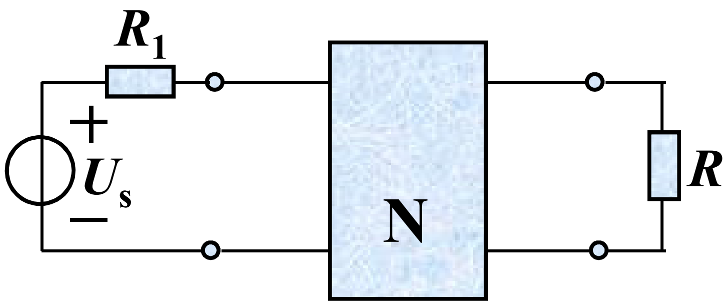


图1

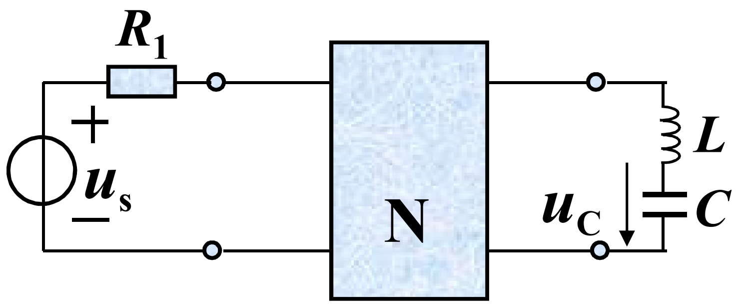
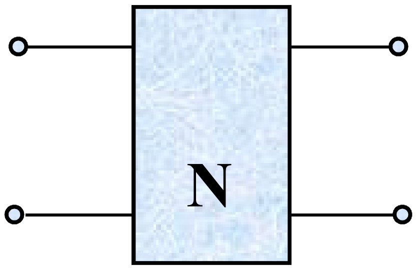


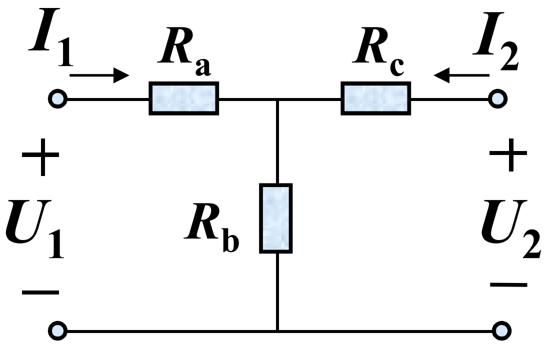
图2

(1) 求T型等效电路



$$T=\begin{bmatrix}2 & 8\Omega\\ 0.5\,\mathrm{S} & 2.5\end{bmatrix}$$

$$\begin{cases}U_1=AU_2-BI_2\\ I_1=CU_2-DI_2\end{cases}$$



$$\begin{cases}A=\left.\dfrac{U_1}{U_2}\right|_{I_2=0}=\dfrac{R_a+R_b}{R_b}=2\\ C=\left.\dfrac{I_1}{U_2}\right|_{I_2=0}=\dfrac{1}{R_b}=0.5\\ D=\left.\dfrac{I_1}{-I_2}\right|_{U_2=0}=\dfrac{R_c+R_b}{R_b}=2.5\end{cases}$$

解之得：

$$\begin{cases}R_a=2\Omega\\ R_b=2\Omega\\ R_c=3\Omega\end{cases}$$

(2)若端口1接U_S=6V、R_1=2Ω的串联支路，端口2接电阻R（图1），求R=？时可使其上获得最大功率，并求此最大功率值。

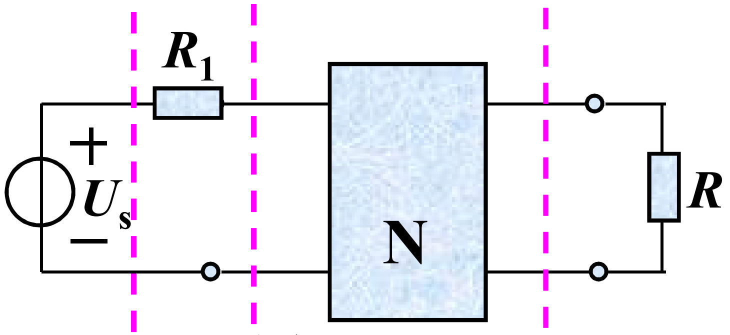


图1

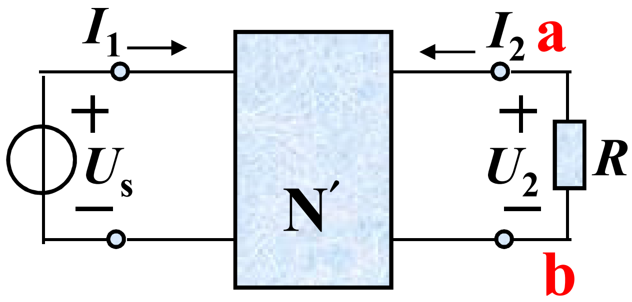
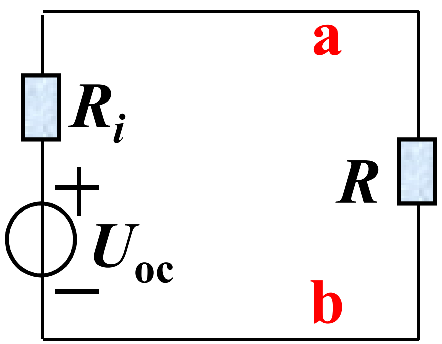


图1

$$T=\begin{bmatrix} 2 & 8\Omega \\ 0.5\,\mathrm{S} & 2.5 \end{bmatrix} \qquad T'=\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 8 \\ 0.5 & 2.5 \end{bmatrix}=\begin{bmatrix} 3 & 13\Omega \\ 0.5\mathrm{S} & 2.5 \end{bmatrix} \qquad \begin{cases} U_S=3U_2-13I_2 \\ I_1=0.5U_2-2.5I_2 \end{cases}$$

计算a、b以左电路的戴维南等效电路:



$$U_{OC}=U_2\Big|_{I_2=0}=\frac{U_s}{A}=\frac{6}{3}=2\,V \qquad R_i=\frac{U_2}{I_2}\Big|_{U_S=0}=\frac{13}{3}=4.33\Omega$$

R 等于R_i时其上功率最大，此时最大功率为:

$$P_{\max}=\frac{U_{oc}^2}{4R_i}=\frac{4}{4\times4.33}=0.231W$$

(3)若端口1接电压源u_s=6+10sin$t$ V与电阻R_1=2Ω的串联支路，端口2接L=1H与C=1F的串联支路（图2），求电容C上电压的有效值。

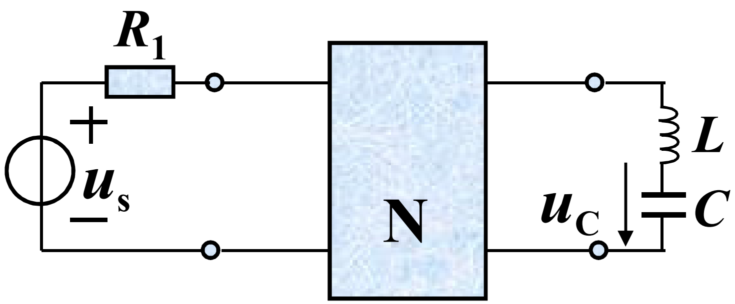
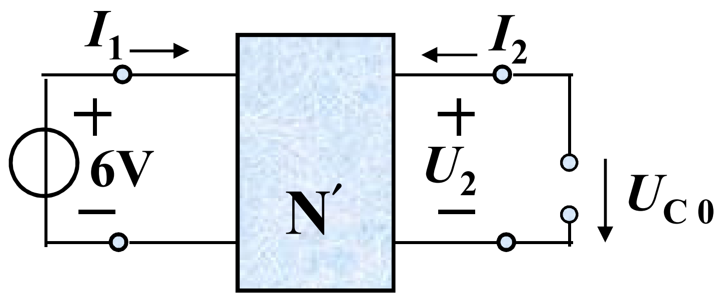


图2

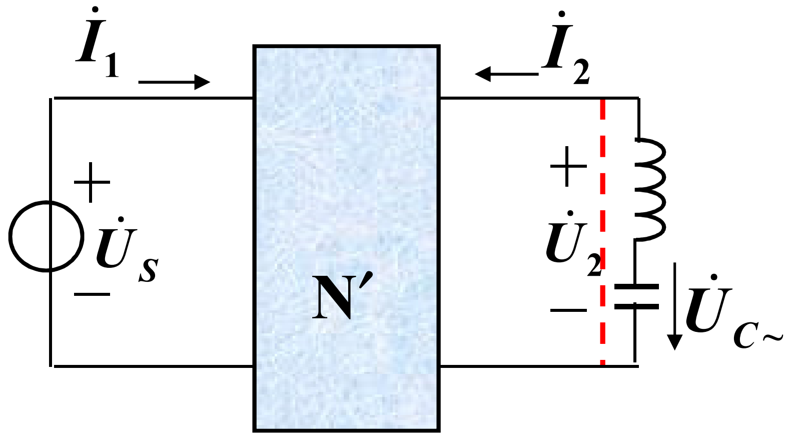


(1) 6V电压源单独作用，L短路、C开路

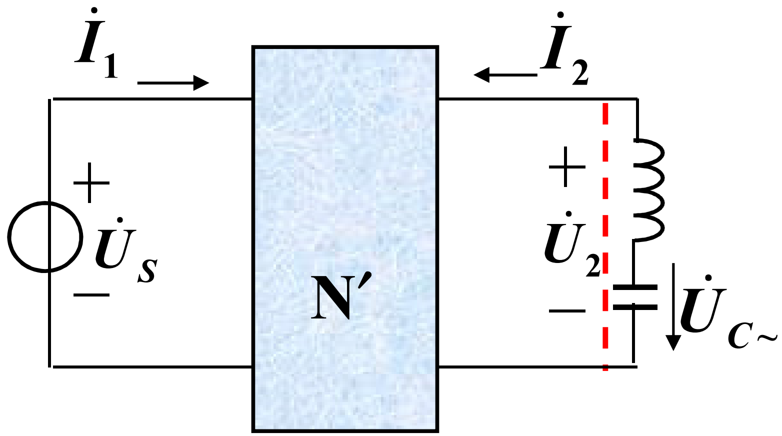
$$\begin{cases} 6 = 3U_2 - 13I_2 \\ I_1 = 0.5U_2 - 2.5I_2 \end{cases} \qquad U_{c0} = U_2\big|_{I_2=0} = 2V$$

(2) 正弦电源单独作用，L、C发生串联谐振相当于短（U_2=0）

$$\begin{cases} \dot{U}_S = 3\dot{U}_2 - 13\dot{I}_2 \\ \dot{I}_1 = 0.5\dot{U}_2 - 2.5\dot{I}_2 \end{cases} \qquad \begin{aligned} \dot{U}_S &= 10/\sqrt{2}\angle 0^\circ\ V \\ \dot{U}_2 &= 0 \end{aligned}$$



$$\begin{cases} \dot{U}_S = 3\dot{U}_2 - 13\dot{I}_2 \\ \dot{I}_1 = 0.5\dot{U}_2 - 2.5\dot{I}_2 \end{cases} \quad \dot{U}_S = {}^{10}\!\!\diagup\!_{\sqrt{2}}\angle 0^\circ\ V \quad \dot{U}_2 = 0$$



$$\dot{I}_2 = \frac{\dot{U}_S}{-13} = -\frac{7.07\angle 0^\circ}{13} = 0.544\angle 180^\circ\ A$$

$$\dot{U}_C = -j\frac{1}{\omega C}(-\dot{I}_2) = 0.544\angle 90^\circ\ V \qquad u_c(t) = 0.544\sqrt{2}\sin(t + 90^\circ)\ V$$

$$u_c = U_{C0} + u_c(t) = 2 + 0.544\sqrt{2}\sin(t + 90^\circ)\ \mathrm{V}$$

有效值 $U_C = \sqrt{2^2 + 0.544^2} = \sqrt{4.296} = 2.073\ A$

计划学时：5学时；课后学习15学时

作业：

16-2/含源二端口端口方程
16-11,16-16,16-26 /参数
16-30/级联
16-36，16-38/综合应用