6-2 设控制系统的开环频率特性为

$$G(j\omega)H(j\omega)=\frac{40}{j\omega(1+0.0625j\omega)(1+0.25j\omega)}$$

① 绘出系统的 Bode 图，并确定系统的相角裕度和幅值裕度以及系统的稳定性；

② 如引入传递函数 $G_c(s)=\frac{0.05(s+0.25)}{(s+0.0125)}$ 的相位滞后校正装置，试绘出校正后系统的 Bode 图，并确定校正后系统的相角裕度和幅值裕度。

① 转折频率 $\omega_1=\frac{1}{0.25}=4$　　$\omega_2=\frac{1}{0.0625}=16$

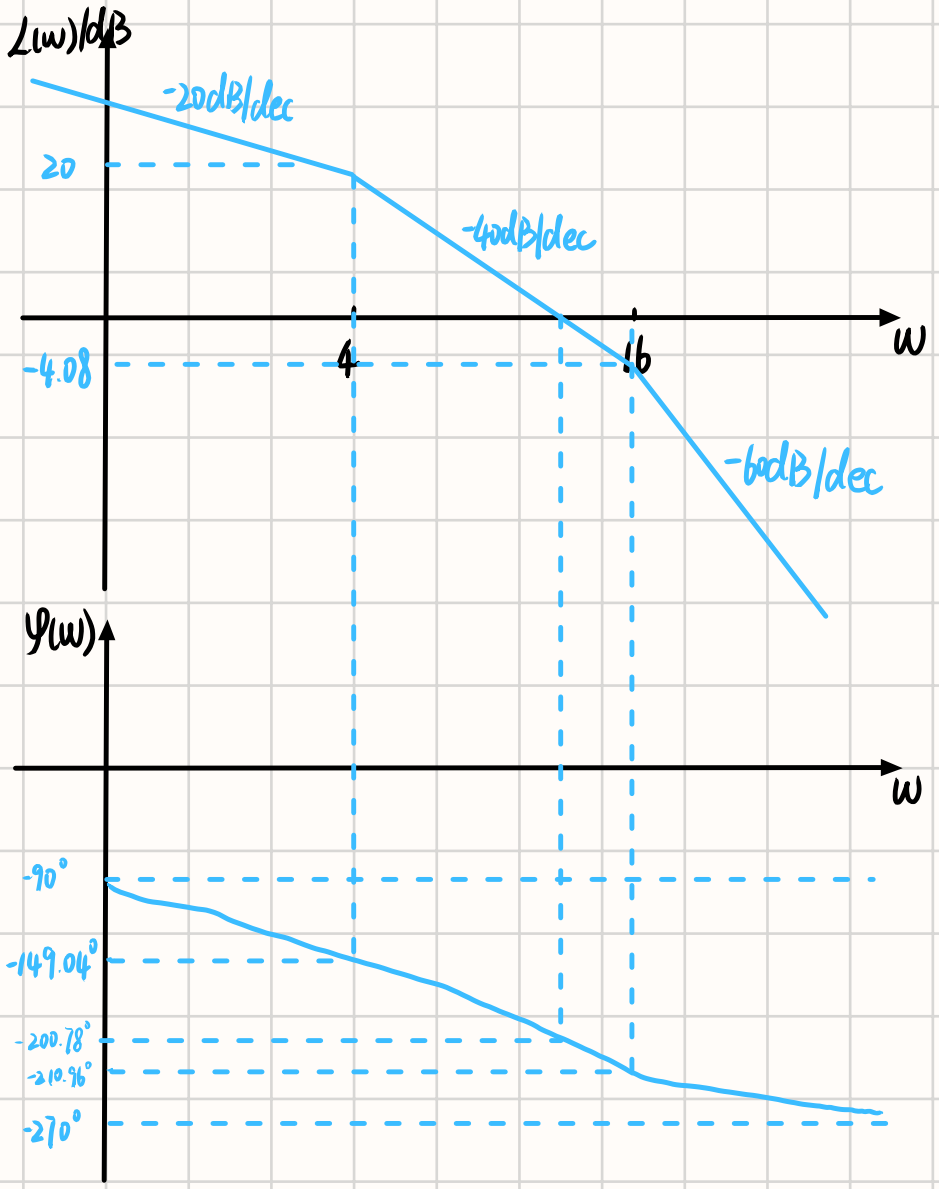
$L(\omega_1)=20\lg\frac{40}{4}=20\,dB$　　$L(\omega_2)=20\lg\frac{40}{16\times\frac{1}{4}\times16}=-4.08\,dB$

$L(\omega_c)=20\lg\frac{40}{\omega_c\times\frac{1}{4}\omega_c}=0 \Rightarrow \omega_c=4\sqrt{10}\approx12.65\ rad/s$

$\varphi(\omega_c)=-90°-\arctan(\frac{1}{4}\omega_c)-\arctan(\frac{1}{16}\omega_c)=-200.78°$

$\gamma=180°+\varphi(\omega_c)=-20.78°<0$　系统不稳定

$\varphi(\omega_1)=-149.04°$　　$\varphi(\omega_2)=-210.96°$



② $G_c(j\omega)=\frac{40(1+4j\omega)}{j\omega(1+80j\omega)(1+0.0625j\omega)(1+0.25j\omega)}$

转折频率　$\omega_1=\frac{1}{80}$　　$\omega_2=\frac{1}{4}$　　$\omega_3=4$　　$\omega_4=16$

$$L(\omega_1)=20\lg\frac{40}{\omega_1}=50.1\,dB \qquad L(\omega_b)=20\lg\frac{40\times4\omega_3}{80\omega_3^2}=-6\,dB$$

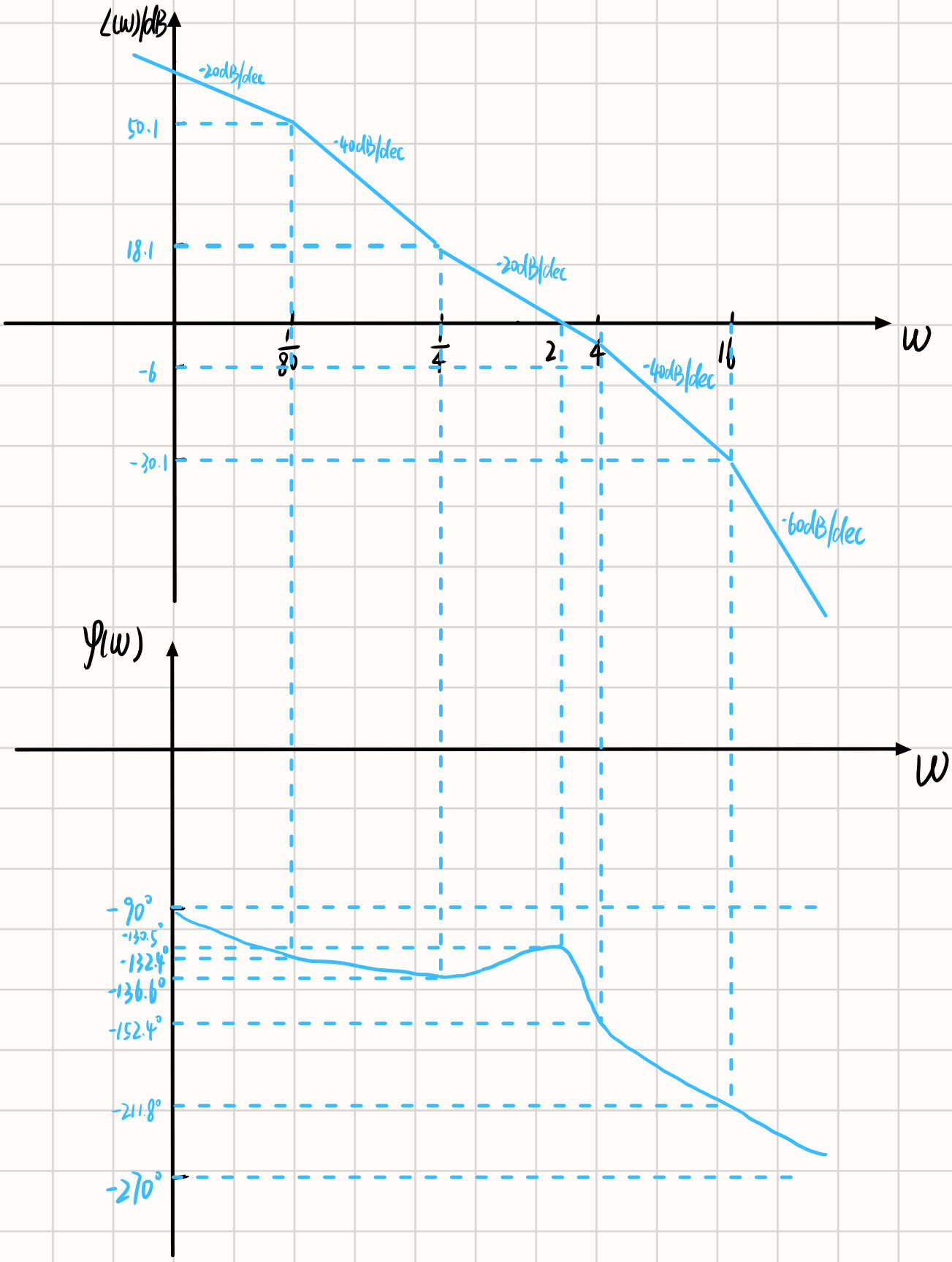
$$L(\omega_2)=20\lg\frac{40}{80\omega_2^2}=18.1\,dB \qquad L(\omega_4)=20\lg\frac{40\times4\omega_4}{80\times\frac{1}{4}\omega_4^3}=-30.1\,dB$$

$$L(\omega_c)=20\lg\frac{40\times4\omega_c}{80\omega_c^2}=0 \Rightarrow \omega_c=2\,rad/s$$

$$\varphi(\omega_c)=-90^\circ+\arctan(4\omega_c)-\arctan(80\omega_c)-\arctan(\tfrac{1}{4}\omega_c)-\arctan(\tfrac{1}{16}\omega_c)=-130.5^\circ$$

$$\gamma=180^\circ+\varphi(\omega_c)=49.5^\circ>0 \qquad \text{系统稳定}$$

$\varphi(\omega_1)=-132.4^\circ$ $\varphi(\omega_2)=-136.6^\circ$ $\varphi(\omega_3)=-152.4^\circ$ $\varphi(\omega_4)=-211.8^\circ$



6-8 设控制系统的开环频率特性为

$$G(j\omega)H(j\omega)=\frac{K}{(j\omega)^2(0.2j\omega+1)}$$

要使系统的相角裕度 $\gamma=35°$，系统的加速度误差系数 $K_a=10$，试用频率法设计串联超前校正装置。

$K=K_a=10$ $\omega_1=\frac{1}{0.2}=5$

$L(\omega_1)=20\lg\frac{10}{\omega_1^2}=-7.96\,dB$

$L(\omega_c)=20\lg\frac{10}{\omega_c^2}=0 \Rightarrow \omega_c=\sqrt{10}\ rad/s$

$\varphi(\omega_c)=-180°-\arctan(0.2\omega_c)=-212.3°$

$\gamma_{原}=180°+\varphi(\omega_c)=-32.3°$

$\varphi_m=\gamma-\gamma_{原}+\Delta\varphi=72.3°>60°$ 需采用两级串联校正

令 $\varphi_m=45°$ 则 $a=\frac{1+\sin\varphi_m}{1-\sin\varphi_m}=3+2\sqrt{2}\approx5.83$

$L(\omega_m)=-10\lg a\times2=-15.31\,dB$

$L(\omega_m)=20\lg\frac{10}{\omega_m^2\times0.2\omega_m}=-15.31\,dB \Rightarrow \omega_m=6.63\ rad/s$

由 $\omega_m=\frac{1}{T\sqrt{a}} \Rightarrow T=\frac{1}{\omega_m\sqrt{a}}=0.0625\,s$

$\frac{1}{T}=16\ rad/s$ $\frac{1}{aT}=0.274\ rad/s$

矫正后系统 $G(s)=G_c(s)G_0(s)=\frac{10(1+0.3645s)^2}{s^2(1+0.2s)(1+0.0625s)^2}$

$\varphi(\omega_m)=-180°+2\times\arctan(0.3645\omega_m)-\arctan(0.2\omega_m)-2\times\arctan(0.0625\omega_m)=-142.95°$

$\gamma'=180°+\varphi(\omega_m)=37.05°>35°$ 满足要求

6-10 设单位反馈控制系统的开环传递函数为

$$G(s)=\frac{K}{s(s+1)(0.2s+1)}$$

若使系统的相角裕度 $\gamma=45°$，速度误差系数 $K_v=8$，试设计串联滞后校正装置。

$K=K_v=8$ $\Delta\varphi=\gamma+10=55°$

$\varphi(\omega_c')=-180°+\Delta\varphi=-125°$ 即 $\varphi(\omega_c')=-90°-\arctan\omega_c'-\arctan(0.2\omega_c')=-125°$

$\arctan\frac{\omega_c'+0.2\omega_c'}{1-0.2\omega_c'^2}=35°$ $\Rightarrow \omega_c'=0.55$ rad/s

$L(\omega_c')=20\lg\frac{k}{\omega_c'}=-20\lg b$ $\Rightarrow b=0.06875$

$\frac{1}{bT}=\frac{\omega_c'}{10}=0.055$ $T=264.5s$ $bT=18.18s$

$G(s)=G_c(s)G_0(s)=\frac{8(1+18.18s)}{s(1+s)(1+0.2s)(1+264.5s)}$

$\varphi(\omega_c')=-90°+\arctan(18.18\omega_c')-\arctan\omega_c'-\arctan(0.2\omega_c')-\arctan(264.5\omega_c')\approx-130.4°$

$\gamma'=180°+\varphi(\omega_c')=49.6°>45°$ 满足要求

6-17 控制系统的开环传递函数为

$$G(s)=\frac{K}{s(1+0.1s)(1+0.2s)}$$

要使系统的相角裕度 $\gamma\geqslant40°$，单位斜坡输入时系统的稳态误差 $e_{ss}=0.01$，试用频率法设计串联滞后-超前校正网络。

$K=\frac{1}{e_{ss}}=100$ $\omega_1=5$ $\omega_2=10$

$L(\omega_1)=20\lg\frac{100}{5}=26.02dB$ $L(\omega_2)=20\lg\frac{100}{10\times0.2\times10}=13.98dB$

$L(\omega_c)=20\lg\frac{100}{\omega_c\times0.2\omega_c\times0.1\omega_c}=0$ $\Rightarrow \omega_c=17.1$ rad/s

$\varphi(\omega_c)=-90°-\arctan(0.2\omega_c)-\arctan(0.1\omega_c)=-223.4°$

$\gamma=180°+\varphi(\omega_c)=-43.4°$

超前校正：取 $\varphi_m=60°$ $a=\frac{1+\sin\varphi_m}{1-\sin\varphi_m}=13.93$

$L(\omega_m)=-10\lg a=-11.44dB$

$L(\omega_m)=20\lg\frac{100}{0.02\omega_m^3}=-11.44$ $\Rightarrow \omega_m=26.525$ rad/s

$T=\frac{1}{\omega_m\sqrt{a}}=0.01s$ $aT=0.1393s$

$$G(s)=\frac{100(1+0.1393s)}{s(1+0.2s)(1+0.1s)(1+0.01s)}$$

$$\varphi(\omega_m)=-90^\circ+\arctan(0.1393\omega_m)-\arctan(0.1\omega_m)-\arctan(0.2\omega_m)-\arctan(0.01\omega_m)$$
$$=-178.7^\circ$$

$$\gamma'=180^\circ+\varphi(\omega_m)=1.3^\circ$$

滞后校正：$\Delta\varphi=\gamma+5^\circ=45^\circ$

$$\varphi(\omega_c')=-180^\circ+\Delta\varphi=-135^\circ$$

$$\varphi(\omega_c')=-90^\circ+\arctan(0.1393\omega_c')-\arctan(0.1\omega_c')-\arctan(0.2\omega_c')-\arctan(0.01\omega_c')$$

$$\Rightarrow \omega_c'=5.04\ \text{rad/s}$$

$$L(\omega_c')=20\lg\frac{100}{\omega_c'\times0.2\omega_c'}=-20\lg b \quad\Rightarrow b=0.051$$

$$\frac{1}{bT}=\frac{\omega_c'}{10}=0.504 \quad\Rightarrow\quad T=38.9s \qquad bT=1.98s$$

$$G(s)=\frac{100(1+0.1393s)(1+1.98s)}{s(1+0.1s)(1+0.2s)(1+0.01s)(1+38.9s)}$$

$\varphi(\omega_c')=-135.22^\circ$　　$\gamma''=180^\circ+\varphi(\omega_c')=44.78^\circ>40^\circ$ 满足要求