

第9章

二阶电路的暂态分析

9.1 概述

9.2 零输入响应

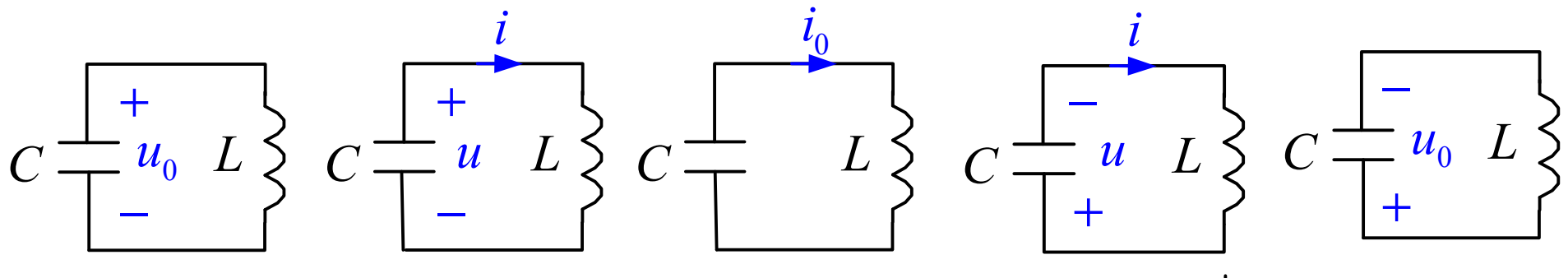
9.3 直流电源激励下的响应

9.4 一般二阶电路

9.1 概述

二阶电路及能量流动

能用二阶微分方程描述的电路称为二阶电路，有两个独立的储能元件。



初始时刻，
 $u_c(0_-) = u_0$
 $i_L(0_-) = 0$

电容放电
→
电流增长

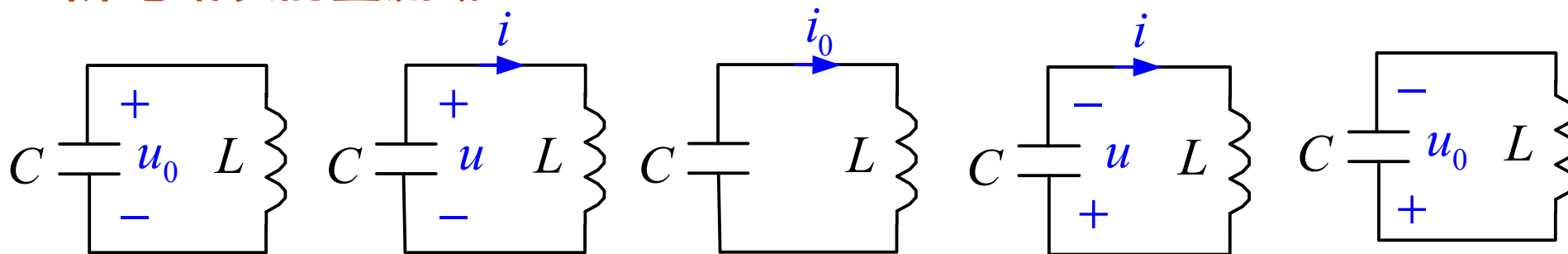
$\frac{di}{dt} = 0$
电容电压为0，
电流为最大值，
能量存入电感

电容充电
→
电流减小

u_0 与初始
时刻相同

9.1 概述

二阶电路及能量流动



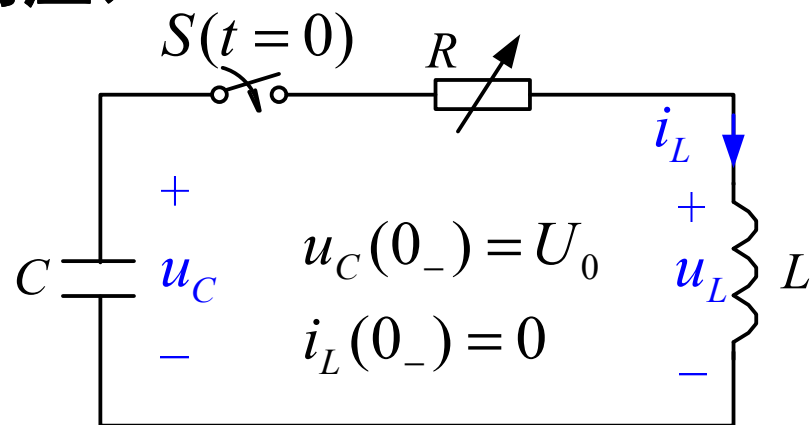
- 由电容和电感两种不同储能元件构成的电路中，储能能在电场和磁场之间往返转移。
- 电流和电压不断改变大小和极性，形成周而复始的振荡。这种由初始储能引起的振荡是等幅的。
- 如果电路中存在电阻，则振幅不可能相等，幅度将逐渐衰减而趋于0。
- 如果电阻过大, 储能能在初次转移时被电阻吸收，则不会产生振荡。

9.2 零输入响应（自然响应）

$$\left\{ \begin{array}{l} LC \frac{d^2 u_C}{dt^2} + RC \frac{du_C}{dt} + u_C = 0 \\ u_C(0_+) = u_C(0_-) = U_0 \\ \left. \frac{du_C}{dt} \right|_{0_+} = \frac{i_C(0_+)}{C} = \frac{i_L(0_+)}{C} = 0 \end{array} \right.$$

特征根

$$\begin{aligned} s_{1,2} &= \frac{-RC \pm \sqrt{(RC)^2 - 4LC}}{2LC} \\ &= -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \\ &= -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \end{aligned}$$



讨论

$$(1) \alpha > \omega_0 \rightarrow R > 2\sqrt{\frac{L}{C}}$$

过阻尼情况

$$(2) \alpha < \omega_0 \rightarrow R < 2\sqrt{\frac{L}{C}}$$

欠阻尼情况

$$(3) \alpha = \omega_0 \rightarrow R = 2\sqrt{\frac{L}{C}}$$

临界阻尼情况

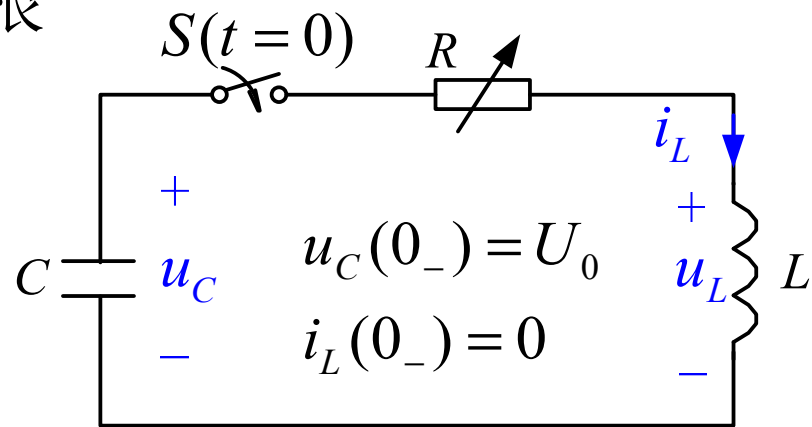
1. $R > 2\sqrt{\frac{L}{C}}$ s_1, s_2 不等的负实根

$$u_C = k_1 e^{s_1 t} + k_2 e^{s_2 t}$$

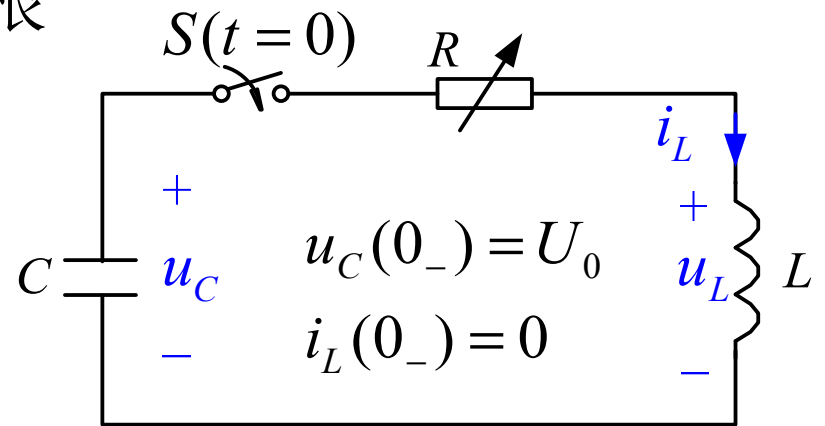
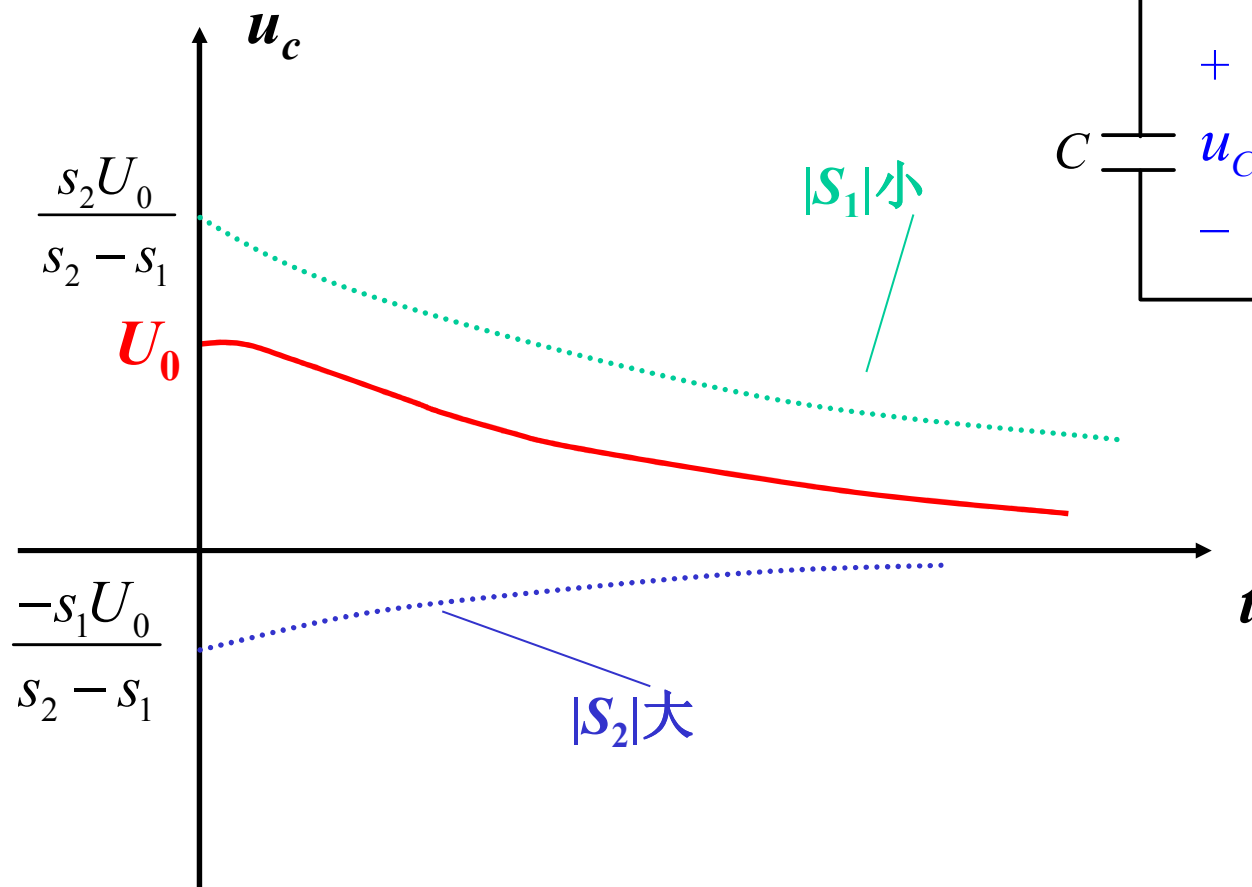
$$\begin{cases} u_C(0^+) = U_0 \rightarrow k_1 + k_2 = U_0 \\ \frac{du_C}{dt}(0^+) = 0 \rightarrow s_1 k_1 + s_2 k_2 = 0 \end{cases}$$

$$\therefore k_1 = \frac{s_2}{s_2 - s_1} U_0 \quad k_2 = \frac{-s_1}{s_2 - s_1} U_0$$

$$u_C = \frac{U_0}{s_2 - s_1} (s_2 e^{s_1 t} - s_1 e^{s_2 t})$$

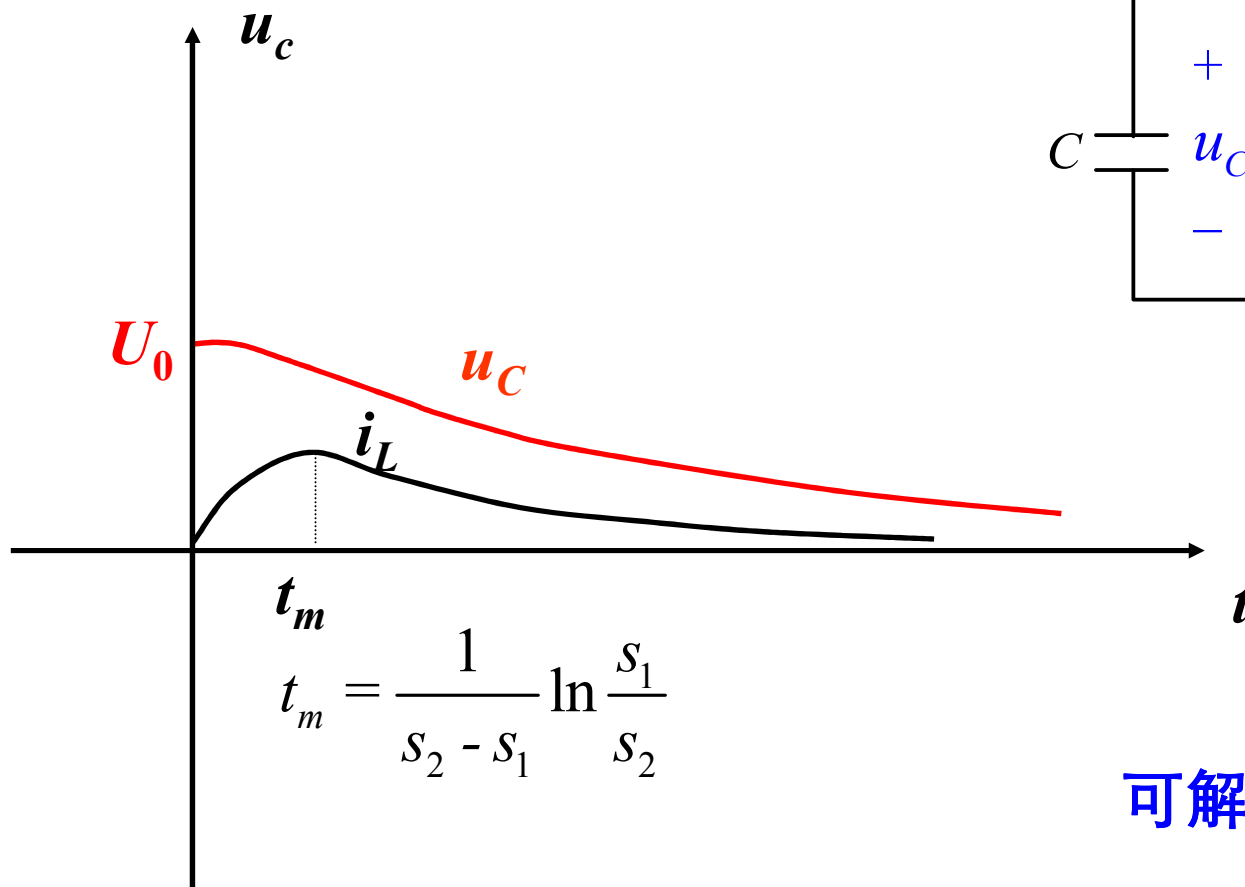
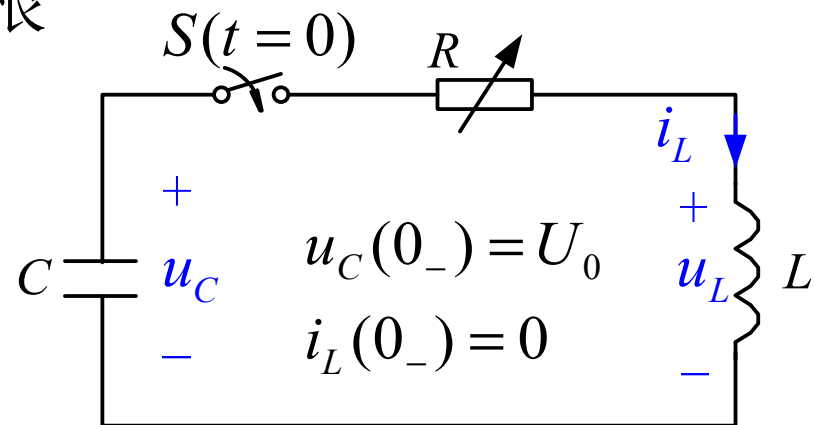


1. $R > 2\sqrt{\frac{L}{C}}$ s_1, s_2 不等的负实根



$$u_c = \frac{U_0}{s_2 - s_1} (s_2 e^{s_1 t} - s_1 e^{s_2 t}) \quad \text{设 } |s_1| < |s_2|$$

1. $R > 2\sqrt{\frac{L}{C}}$ s_1, s_2 不等的负实根



$$t_m = \frac{1}{s_2 - s_1} \ln \frac{s_1}{s_2}$$

$$t=0_+ \quad i_L=0, t=\infty \quad i_L=0$$

$t = t_m$ 时 i 最大

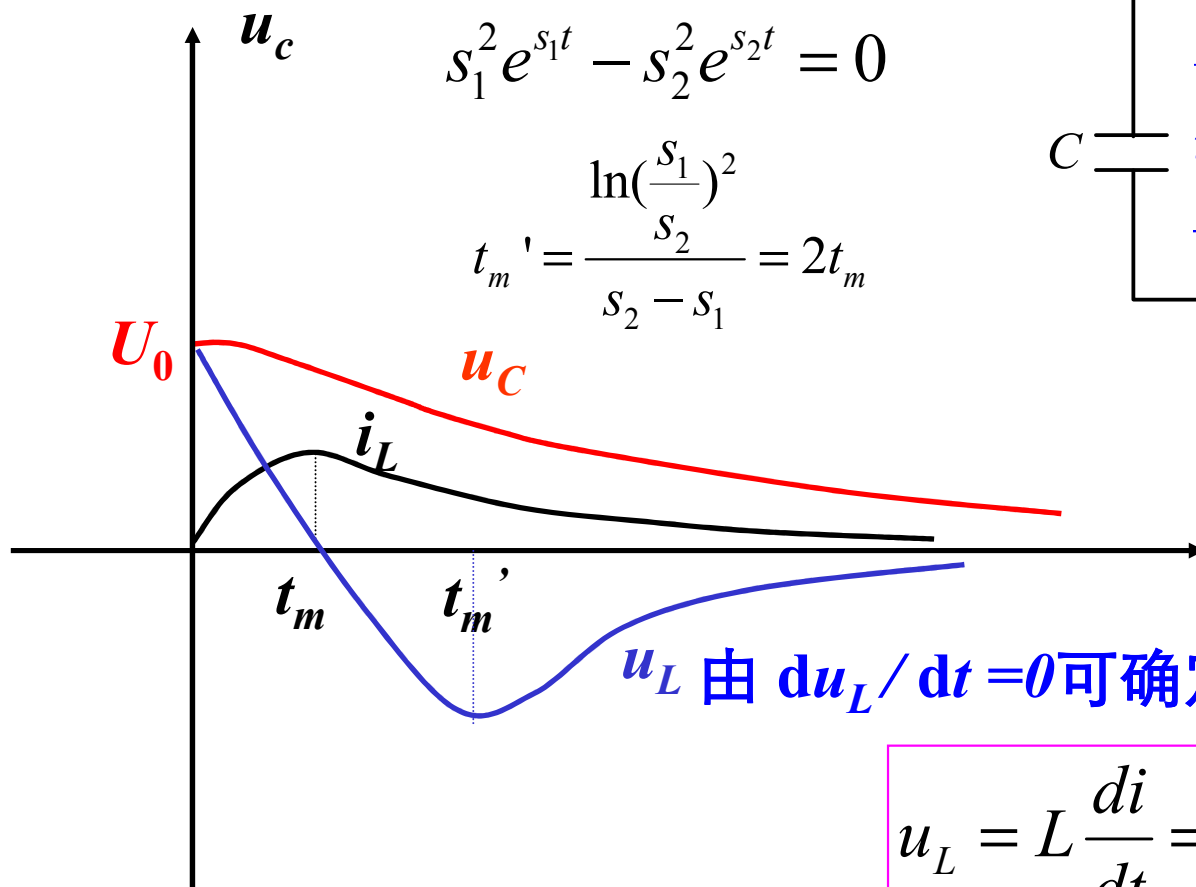
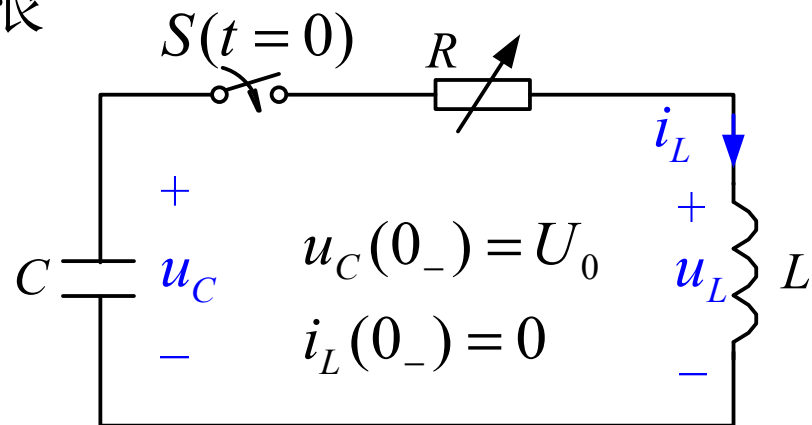
$$\text{令 } \frac{di_L}{dt} = 0$$

可解得 i_L 达到极值的时间 t_m

$$u_C = \frac{U_0}{s_2 - s_1} (s_2 e^{s_1 t} - s_1 e^{s_2 t})$$

$$i_L = -C \frac{du_C}{dt} = \frac{CU_0 s_1 s_2}{(s_1 - s_2)} (e^{s_1 t} - e^{s_2 t})$$

1. $R > 2\sqrt{\frac{L}{C}}$ s_1, s_2 不等的负实根



$$u_L(0+) = U_0 \quad u_L(\infty) = 0$$

$0 < t < t_m$ i 增加, $u_L > 0$

$t > t_m$, i 减小, $u_L < 0$

u_L 由 $du_L/dt = 0$ 可确定 u_L 为极小值的时间 t_m'

$$u_L = L \frac{di}{dt} = \frac{-U_0}{(s_2 - s_1)} (s_1 e^{s_1 t} - s_2 e^{s_2 t})$$

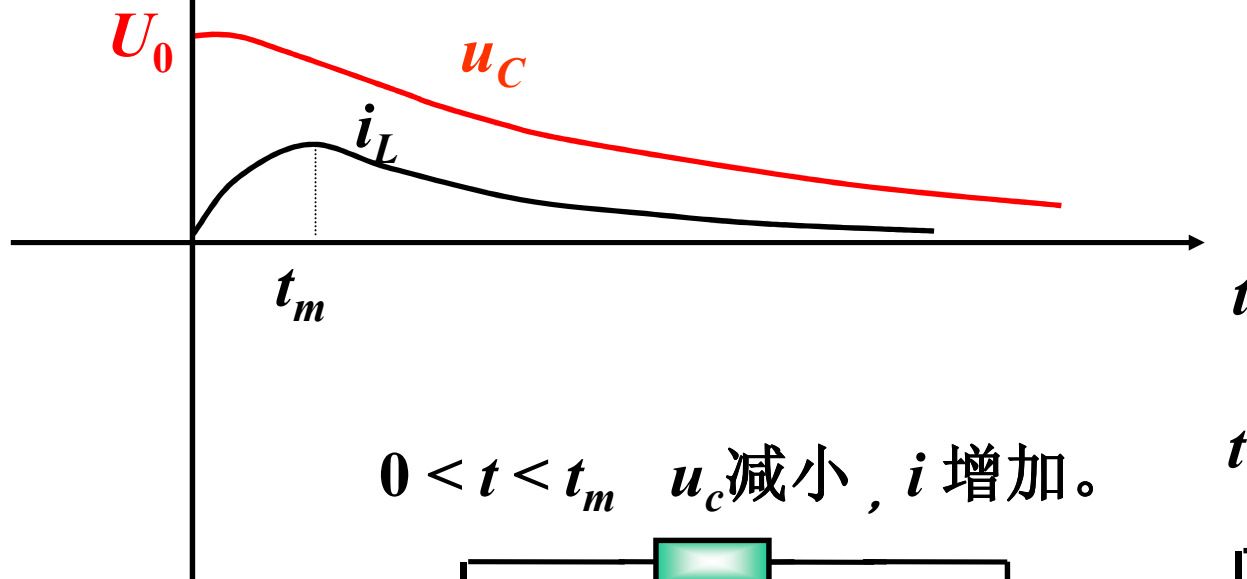
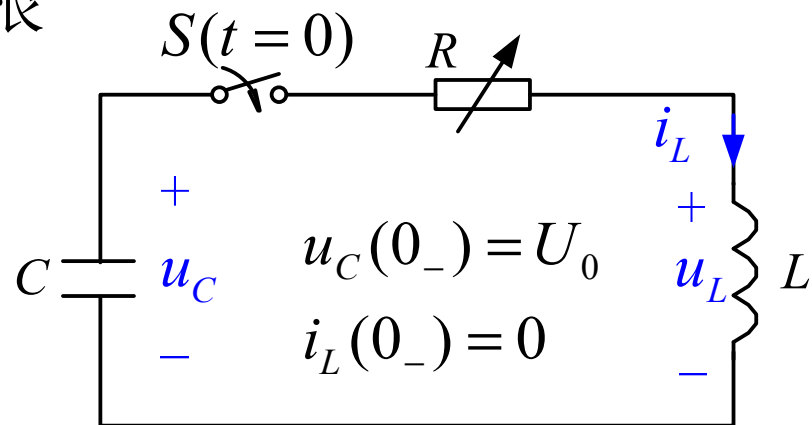
$$u_C = \frac{U_0}{s_2 - s_1} (s_2 e^{s_1 t} - s_1 e^{s_2 t})$$

$$i_L = -C \frac{du_C}{dt} = \frac{CU_0 s_1 s_2}{(s_2 - s_1)} (e^{s_1 t} - e^{s_2 t})$$

1. $R > 2\sqrt{\frac{L}{C}}$ s_1, s_2 不等的负实根

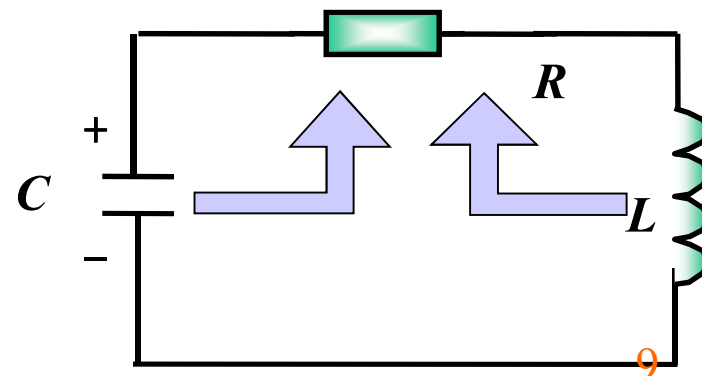
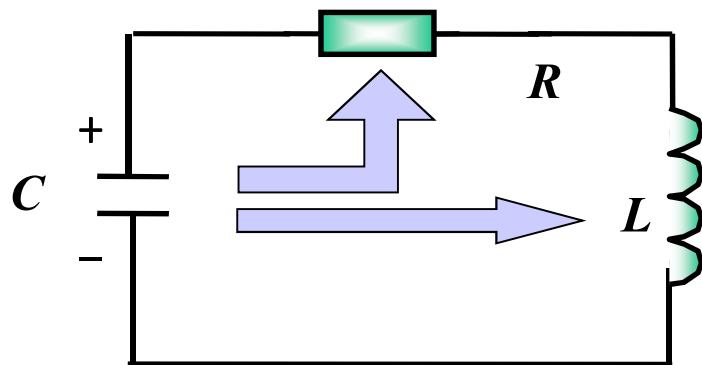
u_c 能量转换关系

过阻尼情况，非振荡放电



$0 < t < t_m$ u_c 减小, i 增加。

$t > t_m$ u_c 减小, i 减小。



2. $R < 2\sqrt{\frac{L}{C}}$ 特征根为一对共轭复根

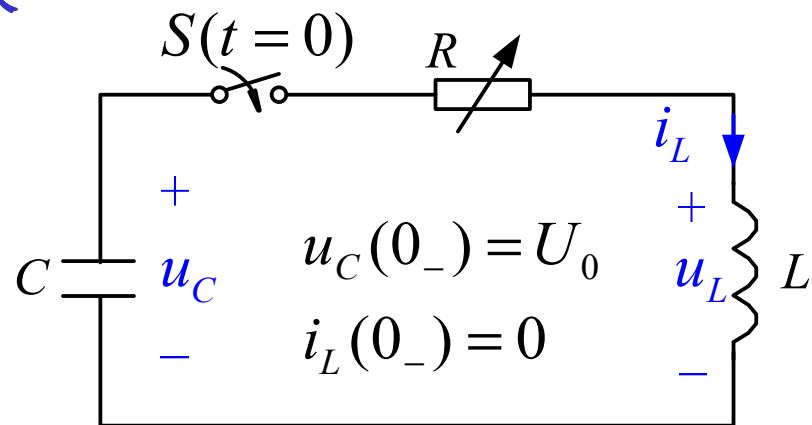
$$s_{1,2} = -\alpha \pm j\sqrt{\omega_0^2 - \alpha^2} = -\alpha \pm j\omega_d$$

$$u_C = ke^{-\alpha t} \sin(\omega_d t + \theta)$$

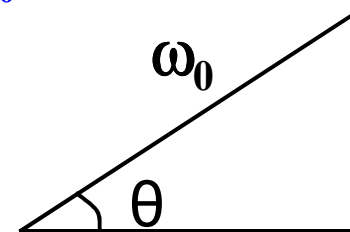
由初始条件

$$\begin{cases} u_C(0_+) = U_0 \rightarrow k \sin \theta = U_0 \\ \frac{du_C}{dt}(0_+) = 0 \rightarrow -\alpha \sin \theta + \omega_d \cos \theta = 0 \end{cases}$$

$$\rightarrow k = \frac{\omega_0 U_0}{\omega_d}, \quad \theta = \tan^{-1} \frac{\omega_d}{\alpha}$$



ω_0 无阻尼振荡角频率



ω_d 阻尼振荡角频率

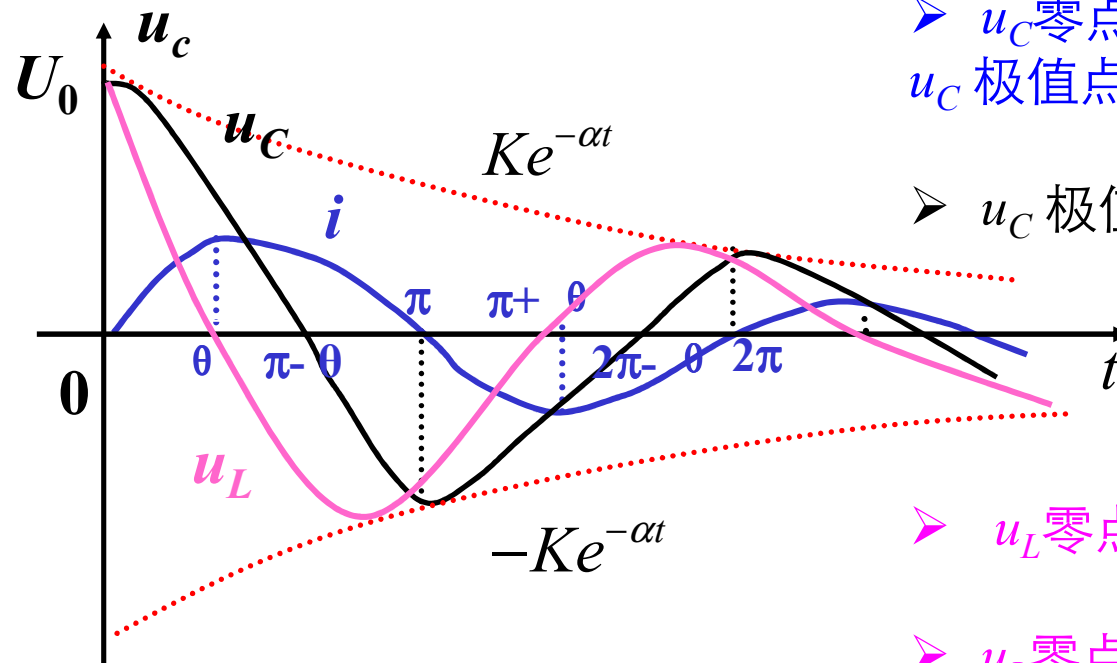
α 衰减因子

$$u_C = \frac{U_0 \omega_0}{\omega_d} e^{-\alpha t} \sin(\omega_d t + \theta) = K e^{-\alpha t} \sin(\omega_d t + \theta)$$

$$i_L = -C \frac{du_C}{dt} = \frac{U_0}{\omega_d L} e^{-\alpha t} \sin \omega_d t$$

$$u_L = L \frac{di}{dt} = -\frac{U_0 \omega_0}{\omega_d} e^{-\alpha t} \sin(\omega_d t - \theta)$$

2. $R < 2\sqrt{\frac{L}{C}}$ 特征根为一对共轭复根



➤ u_C 零点: $\omega_d t = \pi - \theta, (2\pi - \theta \dots n\pi - \theta,)$
 u_C 极值点: $\omega_d t = 0, (\pi, 2\pi \dots n\pi,)$

➤ u_C 极值点为 i_L 零点。

➤ u_L 零点为 i_L 极值点。

➤ u_L 零点: $\omega_d t = \theta, \pi + \theta, \dots n\pi + \theta$

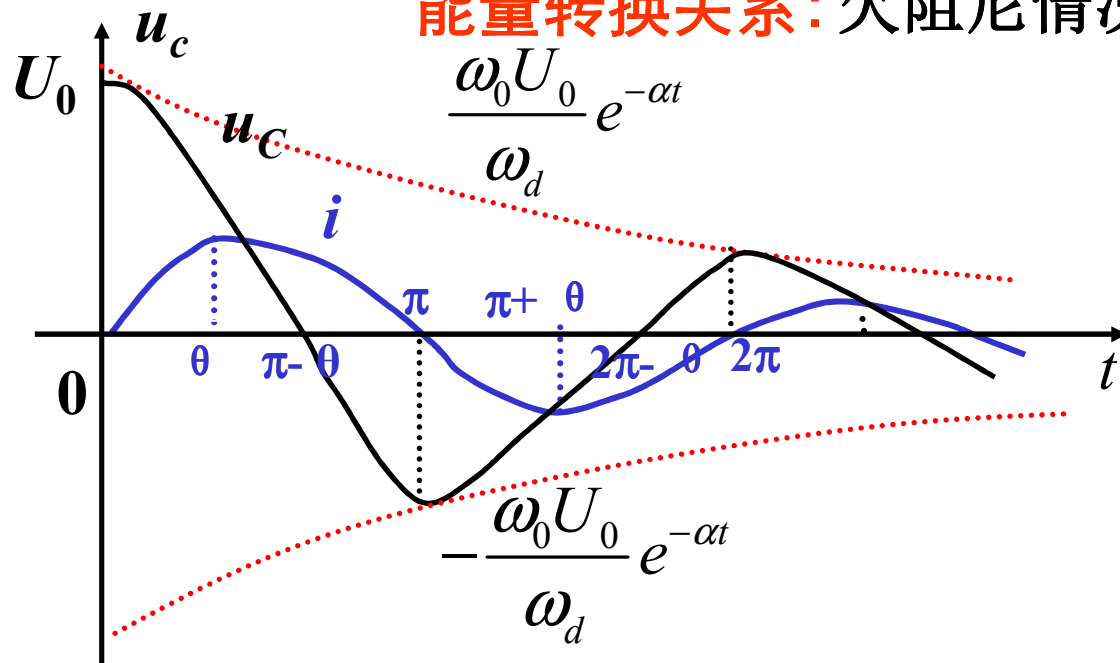
$$u_C = \frac{U_0 \omega_0}{\omega_d} e^{-\alpha t} \sin(\omega_d t + \theta) = Ke^{-\alpha t} \sin(\omega_d t + \theta)$$

$$i_L = -C \frac{du_C}{dt} = \frac{U_0}{\omega_d L} e^{-\alpha t} \sin \omega_d t$$

$$u_L = L \frac{di}{dt} = -\frac{U_0 \omega_0}{\omega_d} e^{-\alpha t} \sin(\omega_d t - \theta)$$

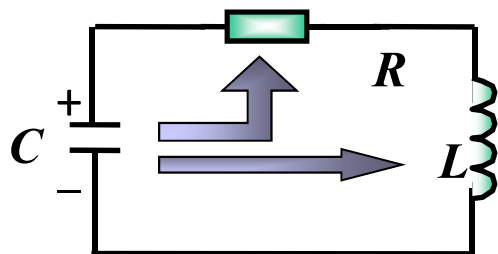
2. $R < 2\sqrt{\frac{L}{C}}$ 特征根为一对共轭复根

能量转换关系：欠阻尼情况，衰减振荡



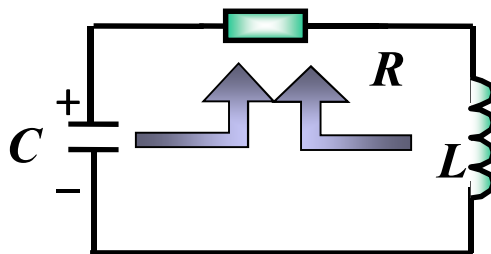
$$0 < \omega_d t < \theta$$

u_C 减小, i 增大



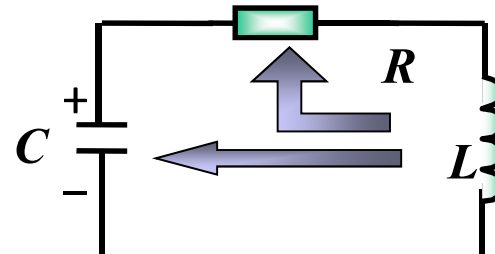
$$\theta < \omega_d t < \pi - \theta$$

u_C 减小, i 减小



$$\pi - \theta < \omega_d t < \pi$$

$|u_C|$ 增大, i 减小



2. $R < 2\sqrt{\frac{L}{C}}$ 特征根为一对共轭复根

特例 $R = 0 \Rightarrow \alpha = 0$

$$s_{1,2} = \pm j\omega_0$$

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}, \quad \theta = \frac{\pi}{2}$$

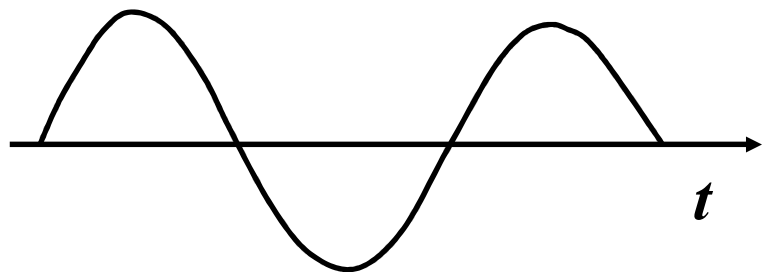
$$u_C = U_0 \sin(\omega_0 t + \frac{\pi}{2}) = U_0 \cos \omega_0 t = u_L$$

$$i = -U_0 \omega_0 C \sin \omega_0 t$$

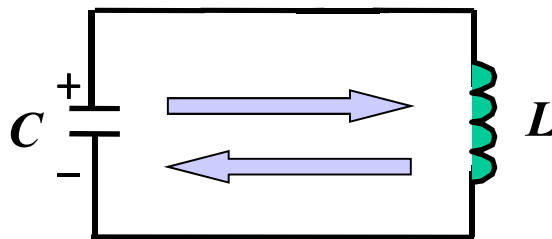
$$u_C = \frac{U_0 \omega_0}{\omega_d} e^{-\alpha t} \sin(\omega_d t + \theta)$$

$$i_L = -C \frac{du_C}{dt} = \frac{U_0}{\omega_d L} e^{-\alpha t} \sin \omega_d t$$

$$u_L = -\frac{U_0 \omega_0}{\omega_d} e^{-\alpha t} \sin(\omega_d t - \theta)$$



等幅振荡：无阻尼



3. $R = 2\sqrt{\frac{L}{C}}$ s_1, s_2 相等的实根

$$s_{1,2} = -\alpha \quad u_C = (k_1 + k_2 t)e^{-\alpha t}$$

由初始条件 $\begin{cases} u_C(0_+) = U_0 \rightarrow k_1 = U_0 \\ \frac{du_C}{dt}(0_+) = 0 \rightarrow k_1(-\alpha) + k_2 = 0 \end{cases} \quad \begin{cases} k_1 = U_0 \\ k_2 = U_0 \alpha \end{cases}$

$$\begin{aligned} u_C &= U_0(1 + \alpha t)e^{-\alpha t} \\ i &= -C \frac{du_C}{dt} = \frac{U_0}{L} t e^{-\alpha t} \\ u_L &= L \frac{di}{dt} = U_0(1 - \alpha t)e^{-\alpha t} \end{aligned}$$

- 非振荡放电，临界阻尼情况
- 能量转换过程与阻尼类似

二阶电路响应变化规律:

$$R > 2\sqrt{\frac{L}{C}} \quad \text{过阻尼, 非振荡放电} \quad u_c = k_1 e^{s_1 t} + k_2 e^{s_2 t}$$

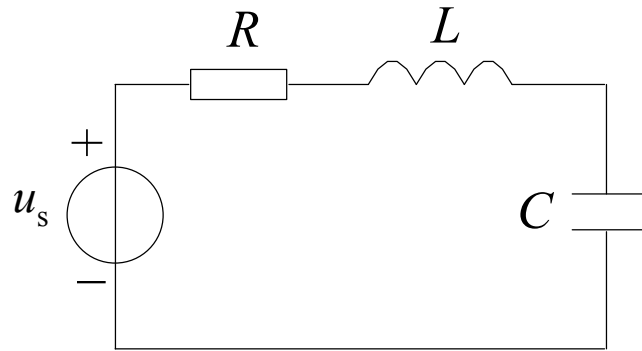
$$R = 2\sqrt{\frac{L}{C}} \quad \text{临界阻尼, 非振荡放电} \quad u_c = e^{-\alpha t} (k_1 + k_2 t)$$

$$R < 2\sqrt{\frac{L}{C}} \quad \text{欠阻尼, 振荡放电} \quad u_c = k e^{-\alpha t} \sin(\omega t + \theta)$$

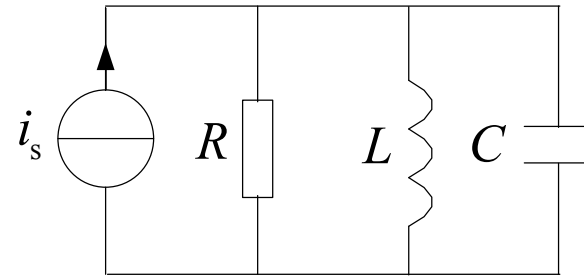
$$\text{由} \begin{cases} u_c(0^+) \\ \frac{du_c}{dt}(0^+) \end{cases} \quad \text{定积分常数}$$

可推广应用于一般二阶电路

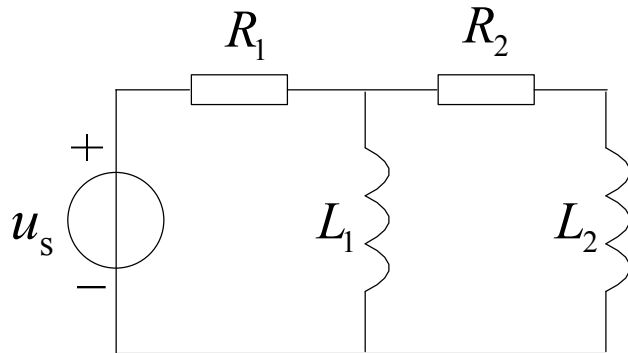
二阶电路



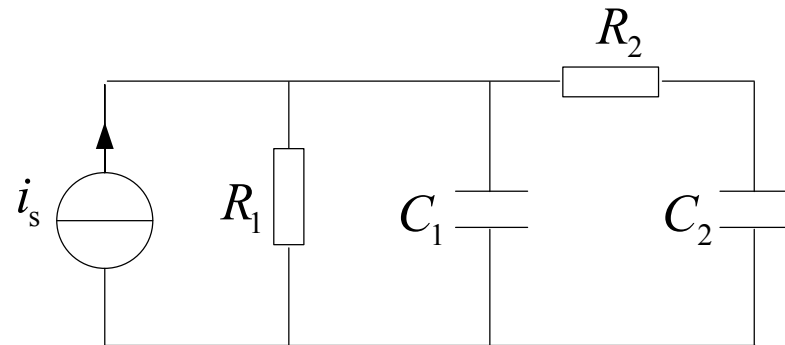
RLC串联电路



RLC并联电路



一般二阶电路



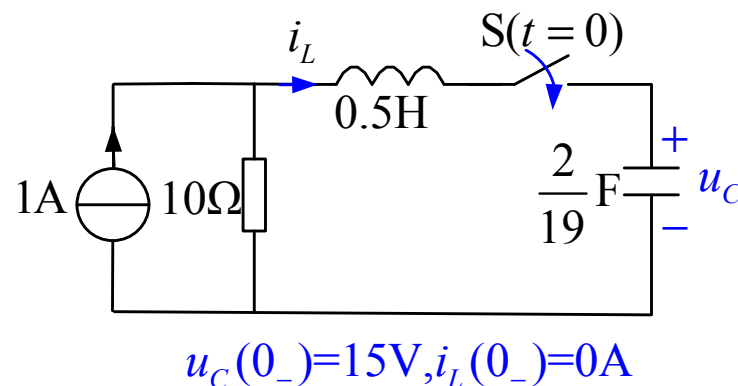
一般二阶电路

9.3 直流电源激励下的响应

全响应——二阶电路响应计算

$$u_C(0_+) = u_C(0_-) = 15\text{V} \quad i_L(0_+) = i_L(0_-) = 0$$

$$\left. \frac{du_C}{dt} \right|_{0_+} = \frac{i_C(0_+)}{C} = \frac{i_L(0_+)}{C} = 0$$



$$\text{KVL: } 0.5 \frac{d}{dt} \left(\frac{2}{19} \frac{du_C}{dt} \right) + 10 \left(\frac{2}{19} \frac{du_C}{dt} - 1 \right) + u_C = 0$$

$$\frac{d^2 u_C}{dt^2} + 20 \frac{du_C}{dt} + 19 u_C = 190 \quad s_{1,2} = -10 \pm \sqrt{100 - 19} = \begin{cases} -1 \\ -19 \end{cases}$$

$$u_C = k_1 e^{-t} + k_2 e^{-19t} + 10$$

$$u_C = \frac{95}{18} e^{-t} - \frac{5}{18} e^{-19t} + 10$$

$$k_1 = \frac{95}{18} \quad k_2 = -\frac{5}{18}$$

$$i_L = C \frac{du_C}{dt} = -\frac{95}{18} e^{-t} + \frac{95}{18} e^{-19t}$$

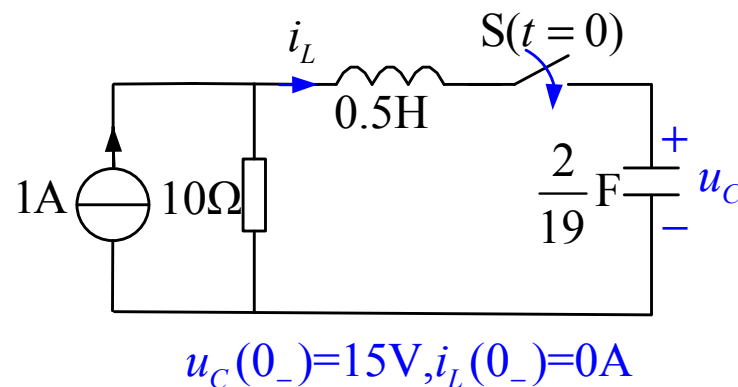
两种分析思路——
—列写方程

9.3 直流电源激励下的响应

全响应——二阶电路响应计算

$$u_C(0_+) = u_C(0_-) = 15V \quad i_L(0_+) = i_L(0_-) = 0$$

$$\left. \frac{du_C}{dt} \right|_{0_+} = \frac{i_C(0_+)}{C} = \frac{i_L(0_+)}{C} = 0$$



$$\alpha = \frac{R}{2L} = 10 \quad \omega_0^2 = \frac{1}{LC} = 19$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -10 \pm \sqrt{100 - 19} = \begin{cases} -1 \\ -19 \end{cases}$$

$$u_{Ch} = k_1 e^{-t} + k_1 e^{-19t}$$

$$u_C = \frac{95}{18} e^{-t} - \frac{5}{18} e^{-19t} + 10$$

$$u_{Cp} = u_C(\infty) = 10$$

$$u_C = k_1 e^{-t} + k_1 e^{-19t} + 10$$

两种分析思路——套用结论

9.3 直流电源激励下的响应

零状态响应

求所示电路中电流 i 的零状态响应。

解：1 列写微分方程 由KVL

$$-u_1 + 2i_1 + 6 \int i_1 dt + \frac{di}{dt} + 2i = 0$$

$$u_1 = 2(2 - i)$$

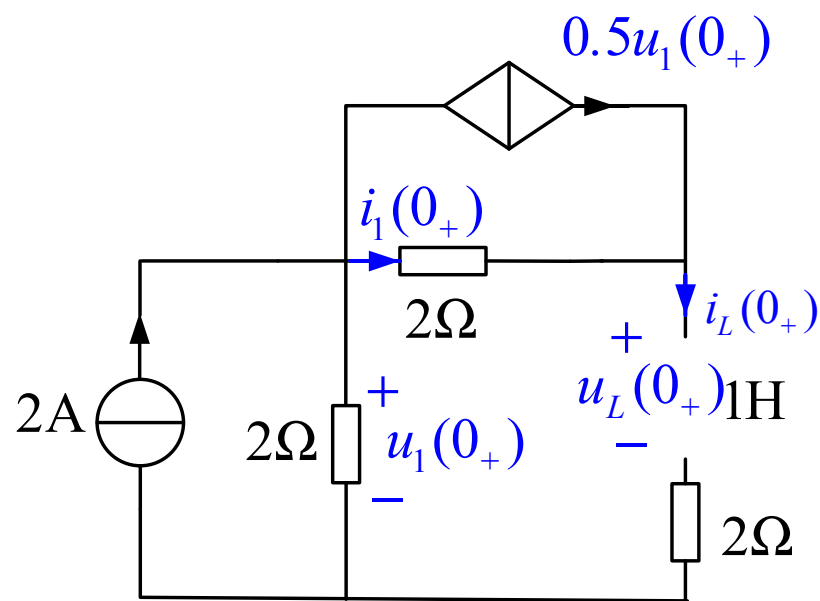
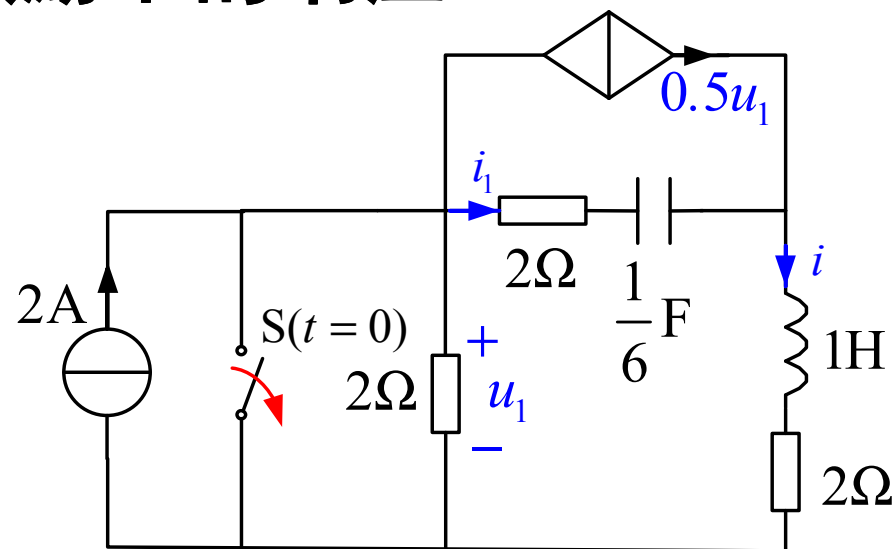
$$i_1 = i - 0.5u_1 = 2i - 2$$

$$\frac{d^2 i}{dt^2} + 8 \frac{di}{dt} + 12i = 12$$

2 求初值及稳态值

$$\begin{cases} i(0_+) = i(0_-) = 0 \\ \left. \frac{di}{dt} \right|_{0_+} = \frac{1}{L} u_L(0_+) = 8V \end{cases}$$

$$u_L(0_+) = 0.5u_1 \times 2 + u_1 = 8V$$



0_+ 等效电路

9.3 直流电源激励下的响应

零状态响应

求所示电路中电流 i 的零状态响应。

解：1 列写微分方程 由KVL

$$-u_1 + 2i_1 + 6 \int i_1 dt + \frac{di}{dt} + 2i = 0$$

$$u_1 = 2(2 - i)$$

$$i_1 = i - 0.5u_1 = 2i - 2$$

$$\frac{d^2 i}{dt^2} + 8 \frac{di}{dt} + 12i = 12$$

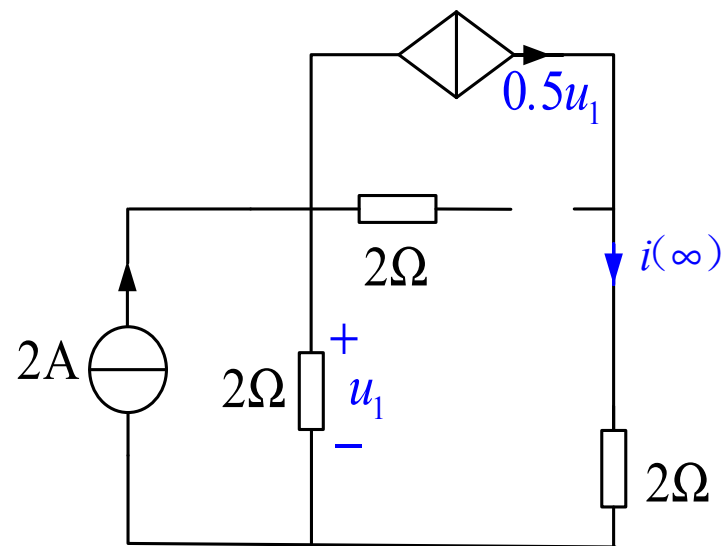
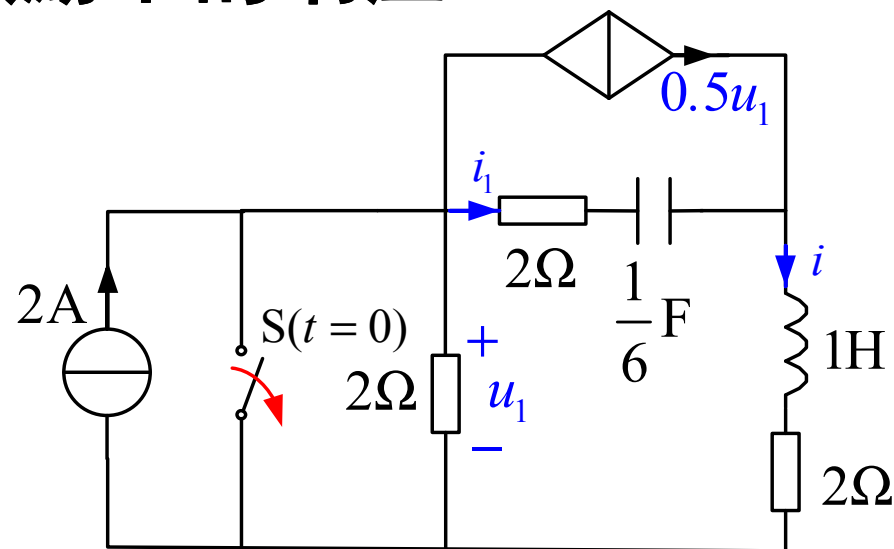
2 求初值及稳态值

$$\begin{cases} i(0_+) = i(0_-) = 0 \end{cases}$$

$$\left. \frac{di}{dt} \right|_{0_+} = \frac{1}{L} u_L(0_+) = 8V$$

$$u_L(0_+) = 0.5u_1 \times 2 + u_1 = 8V$$

$$i(\infty) = 1A$$



∞等效电路

9.3 直流电源激励下的响应

零状态响应

求所示电路中电流 i 的零状态响应。

解：1 列写微分方程 由KVL

$$\frac{d^2 i}{dt^2} + 8 \frac{di}{dt} + 12i = 12$$

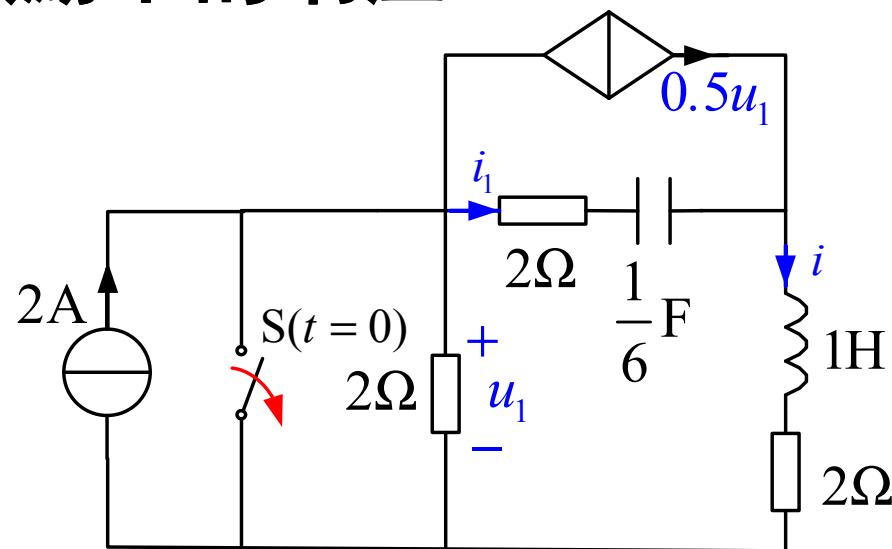
2 求初值及稳态值

$$\begin{cases} i(0_+) = i(0_-) = 0 \\ \left. \frac{di}{dt} \right|_{0_+} = 8V \end{cases}$$

3 求特征根

$$s^2 + 8s + 12 = 0$$

$$s_1 = -2, \quad s_2 = -6$$



4 求解微分方程

$$i = 1 + k_1 e^{-2t} + k_2 e^{-6t}$$

$$\begin{cases} 0 = 1 + k_1 + k_2 \\ 8 = -2k_1 - 6k_2 \end{cases} \rightarrow \begin{cases} k_1 = 0.5 \\ k_2 = -1.5 \end{cases}$$

$$i(t) = 1 + 0.5e^{-2t} - 1.5e^{-6t} \text{ A} \quad t \geq 0$$

9.4 一般二阶电路

已知： $i_L(0_-)=2\text{A}$, $u_C(0_-)=0$, 求： $i_L(t)$

解：1 列微分方程，由KCL得：

$$i_R = i_L + i_C$$

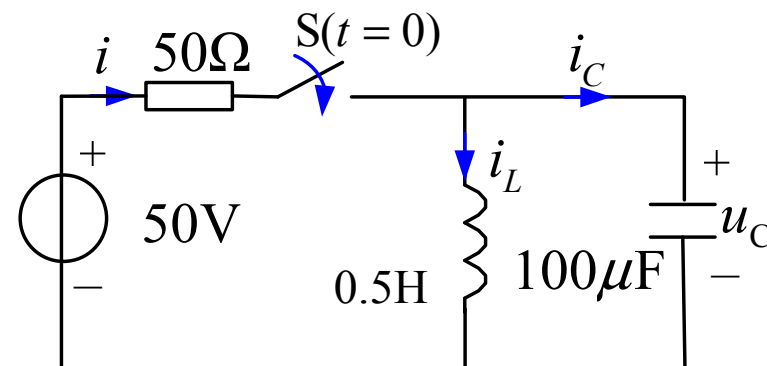
$$\frac{50 - L \frac{di_L}{dt}}{R} = i_L + C \frac{du_C}{dt} \quad u_C = u_L = L \frac{di_L}{dt}$$
$$\frac{d^2 i_L}{dt^2} + 200 \frac{di_L}{dt} + 2 \times 10^4 i_L = 2 \times 10^4$$

2 求初值及稳态值

$$i_L(0_+) = 2\text{A} , \quad u_C(0_+) = 0 \quad (\text{已知})$$

$$\left. \frac{di_L}{dt} \right|_{0_+} = \frac{1}{L} u_L(0_+) = \frac{1}{L} u_C(0_+) = 0$$

$$i_L(\infty) = 1\text{A}$$



3 求特征根

$$s^2 + 200s + 20000 = 0$$

$$s_{1,2} = -100 \pm j100$$

9.4 一般二阶电路

已知： $i_L(0_-)=2\text{A}$, $u_C(0_-)=0$, 求： $i_L(t)$

解：1 列微分方程，由KCL得：

$$\frac{d^2 i_L}{dt^2} + 200 \frac{di_L}{dt} + 2 \times 10^4 i_L = 2 \times 10^4$$

2 求初值及稳态值

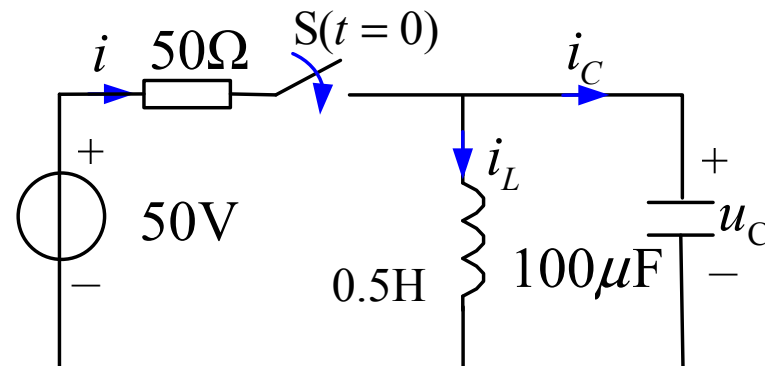
$$i_L(0_+)=2\text{A} , u_C(0_+)=0 \quad (\text{已知})$$

$$\left. \frac{di_L}{dt} \right|_{0_+} = \frac{1}{L} u_L(0_+) = \frac{1}{L} u_C(0_+) = 0$$

$$i_L(\infty)=1\text{A}$$

3 求特征根

$$s_{1,2} = -100 \pm j100$$



4 求解微分方程

$$i_L = 1 + k e^{-100t} \sin(100t + \theta)$$

$$\begin{cases} 1 + k \sin \theta = 2 \\ -100k \sin \theta + 100k \cos \theta = 0 \end{cases}$$

$$\rightarrow \begin{cases} k = \sqrt{2} \\ \theta = 45^\circ \end{cases}$$

$$\therefore i_L(t) = 1 + \sqrt{2} e^{-100t} \sin(100t + 45^\circ) \text{A} \quad t \geq 0$$

解二阶过渡过程包括以下几步：

- 换路后($t > 0^+$)电路列写微分方程
- 求特征根，由根的性质写出自由分量（积分常数待定）
- 求强制分量（稳态分量）
- 全解=自由分量+强制分量
- 将初值 $f(0^+)$ 和 $f'(0^+)$ 代入全解，定积分常数求响应
- 讨论物理过程，画出波形

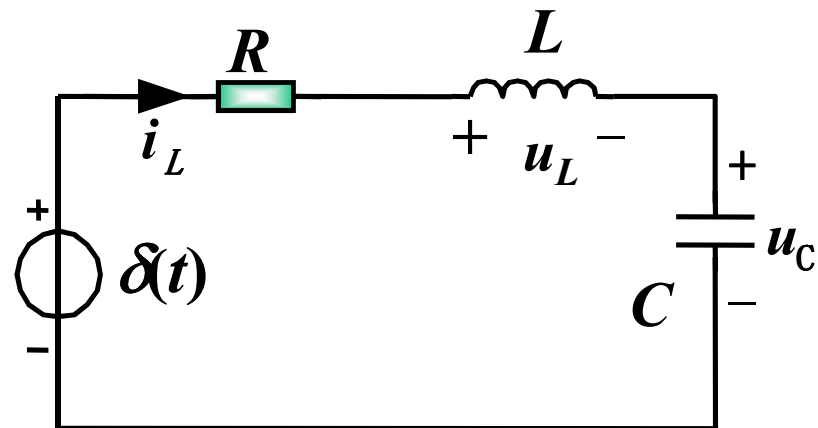
* (自学) 9.6 二阶电路的冲激响应

t 在 0^- 至 0^+ 间

$$u_L = \delta(t)$$

$$i_L(0^+) = i_L(0^-) + \frac{1}{L} \int_{0^-}^{0^+} u_L dt = \frac{1}{L}$$

$$u_C(0^+) = u_C(0^-) = 0$$



$$u_C(0^-) = 0, \quad i_L(0^-) = 0$$

$t > 0^+$ 为零输入响应

$$Ri_L + u_L + u_C = 0 \quad i_L = C \frac{du_C}{dt} \quad u_L = L \frac{di_L}{dt} = LC \frac{d^2 u_C}{dt^2}$$

$$LC \frac{d^2 u_C}{dt^2} + RC \frac{du_C}{dt} + u_C = 0$$

$$u_C(0^+) = u_C(0^-) = 0 \quad u_C'(0^+) = \frac{1}{C} i_C(0^+) = \frac{1}{LC}$$

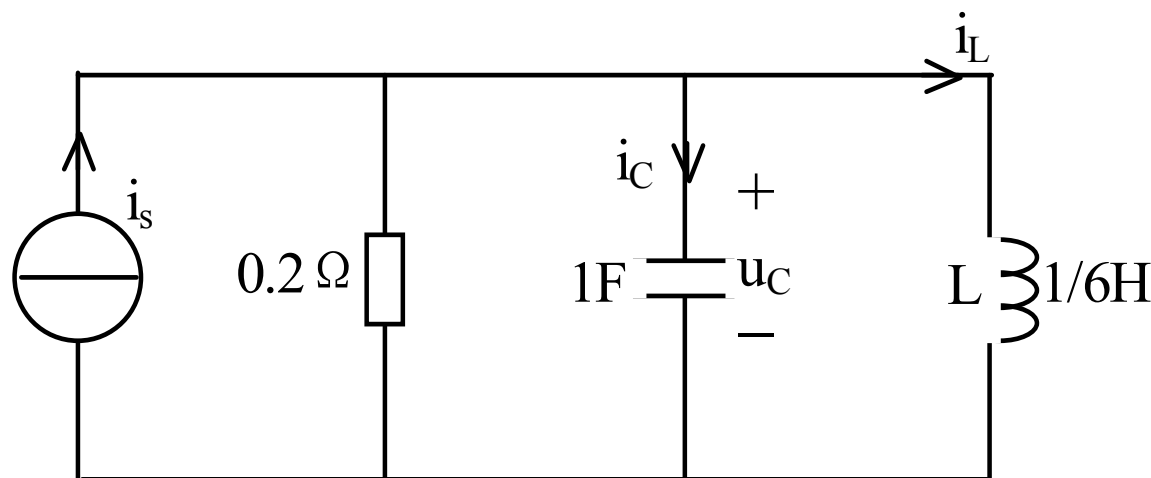
例: $i_s = \delta(t)$, 求 i_L 冲击响应.

解: 化为零输入响应

$$u_C(0_-) = i_L(0_-) = 0$$

$$t=0_- \text{ 时, } i_C(0) = \delta(t)$$

$$u_C(0_+) = \frac{1}{C} \int_{0_-}^{0_+} \delta(t) dt = 1V$$



列微分方程, 由KCL得:

$$i_R + i_C + i_L = 0 \quad i_R = \frac{u_L}{0.2} = \frac{5}{6} \frac{di_L}{dt} \quad i_C = C \frac{du_C}{dt} = \frac{1}{6} \frac{d^2 i_L}{dt^2}$$

$$\begin{cases} \frac{d^2 i_L}{dt^2} + 5 \frac{di_L}{dt} + 6i_L = 0 \\ i_L(0_+) = 0A \\ i_L'(0_+) = \frac{1}{L} u_C(0_+) = 6A \end{cases} \quad i_L = (6e^{-2t} - 6e^{-3t}) \varepsilon(t) A$$

计划学时：2学时；课后学习4学时

作业：

9-7 /二阶电路零输入响应

9-13 , 9-15/二阶电路在直流激励下的响应

9-19 /二阶电路全响应