Factoring a calculus expression

An expression that occurs in calculus is given. Factor completely. Write the expression as a single quotient in which only positive exponents and/or radicals appear.

1. $\left(8-x^2\right)^{1/2}+x\cdot\frac{1}{8}\left(8-x^2\right)^{-1/2}\left(-8x\right)$

2. $\dfrac{5\left(x-1\right)^5\left(7x+4\right)^6-5\left(7x+4\right)^5\left(7\right)\left(x-1\right)^6}{\left[\left(7x+4\right)^6\right]^2}$

3. $\left(30x^4\right)\left(x^2+8\right)^{2/3}+\left(6x^5\right)\left(\frac{2}{3}\right)\left(x^2+8\right)^{-1/3}\left(2x\right)$

4. $\left(x^5+2\right)^3\left(\frac{1}{2}\right)\left(5x+1\right)^{-1/2}\left(5\right)-\left(5x+1\right)^{1/2}\left(3\right)\left(x^5+2\right)^2\left(5x^4\right)$

Solving equations

Find **all** complex solutions to the following equations.

5. $x^{8/3}=81x^{2/3}$

6. $27x^5-8x^2=0$

7. $8x^3-12x^2=3-2x$

8. $x^{-4}-36=5x^{-2}$

Polynomial and Rational Inequalities

Solve each inequality. Justify your answer using a SIGN CHART and state your answer in interval notation.

9. $7x^2+5x\le-9\left(7x+5\right)$

10. $2x^3+9x^2<32x+144$

11. $32x\ge2x^3$

12. $\dfrac{8}{x-9}\ge3$

13. $\dfrac{7}{1-x}\le\dfrac{2}{3-x}$

14. $\dfrac{2x^2-5x-12}{x^2-10x+24}\le0$

Answers

1. $\dfrac{-2(x-2)(x+2)}{\left(8-x^2\right)^{1/2}}$

2. $\dfrac{55(x-1)^5}{(7x+4)^7}$

3. $\dfrac{2x^4\left(19x^2+120\right)}{\left(x^2+8\right)^{1/3}}$

4. $\dfrac{5\left(x^5+2\right)^2\left(-29x^5-6x^4+2\right)}{2(5x+1)^{1/2}}$

5. $\{0, \pm 9\}$

6. $\left\{0, \dfrac{2}{3}, \dfrac{-1 \pm i\sqrt{3}}{3}\right\}$

7. $\left\{\dfrac{3}{2}, \pm\dfrac{1}{2}i\right\}$

8. $\left\{\pm\dfrac{1}{3}, \pm\dfrac{1}{2}i\right\}$

9. $\left[-9, -\dfrac{5}{7}\right]$

10. $\left(-\infty, -\dfrac{9}{2}\right) \cup (-4, 4)$

11. $(-\infty, -4] \cup [0, 4]$

12. $\left(9, \dfrac{35}{3}\right]$

13. $(-\infty, 1) \cup \left(3, \dfrac{19}{5}\right]$

14. $\left[-\dfrac{3}{2}, 4\right) \cup (4, 6)$