More on monads

Haskell and Cryptocurrencies

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Goals

- Revisit monads
- A few new monads
- Combining monads

Monads

The class hierarchy

```haskell
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

```haskell
class Functor f => Applicative f where
  pure  :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b
```

```haskell
class Applicative m => Monad m where
  (>>=) :: m a -> (a -> m b) -> m b
```

Laws

Functor:

```
fmap id x      = x
fmap (f . g) x = (fmap f . fmap g) x
```

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fmap id x      = x
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```

Applicative:

```
pure id <*> b                = b
pure f <*> pure x            = pure (f x)
a <*> pure x                 = pure ($ x) <*> a
pure (.) <*> a <*> b <*> c = a <*> (b <*> c)
```

Laws (contd.)

Monad:

```
pure x >>= f = f x
a >>= pure   = a
(a >>= f) >>= g = a >>= (\ x -> f x >>= g)
```

Instances

```haskell
instance Monad Maybe
instance Monad []
instance Monad (Either e)
instance Monad (State s)
instance Monad IO
instance Monad STM
instance Monad Gen
instance Monad (Parser t)
```

Extended interfaces

E.g. for `State`:

```
get :: State s s
put :: s -> State s ()
```

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E.g. for `State`:

```
get :: State s s
put :: s -> State s ()
```

Or for `Either`:

```
throwError :: e -> Either e a
catchError ::
  Either e a -> (e -> Either e a) -> Either e a
```

A few new monads

Reminder: Identity

Defined in `Data.Functor.Identity`:

```haskell
newtype Identity a = Identity {runIdentity :: a}
```

An interesting special case.

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Defined in `Data.Functor.Identity`:

```haskell
newtype Identity a = Identity {runIdentity :: a}
```

An interesting special case.

```haskell
instance Applicative Identity where
  pure = Identity
  (<*>) = ap

instance Monad Identity where
  (Identity x) >>= f = f x
```

Will become useful later today.

Reader

Defined[1] in `Control.Monad.Trans.Reader`:

```
newtype Reader r a = Reader {runReader :: r -> a}
```

For distributing (read-only) state.

[1] in a slightly different way, see later

Reader

Defined[1] in `Control.Monad.Trans.Reader`:

```
newtype Reader r a = Reader {runReader :: r -> a}
```

For distributing (read-only) state.

```
instance Applicative (Reader r) where
  pure x = Reader (\ _ -> x)
  (<*>) = ap
instance Monad (Reader r) where
  m >>= g =
    Reader (\ r -> runReader (g (runReader m r)) r)
```

[1] in a slightly different way, see later

Extended interface for reader

Accessing the state:

```
ask :: Reader r r
ask = Reader (\ r -> r)
```

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Accessing the state:

```
ask :: Reader r r
ask = Reader (\ r -> r)
```

Locally modifying the state:

```
local :: (r -> r) -> Reader r a -> Reader r a
local f m = Reader (\ r -> runReader m (f r))
```

A simpler version of reader

```haskell
newtype Reader r a = Reader {runReader :: r -> a}
```

```
Reader r a ≈ r -> a

Reader r   ≈ (->) r
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instance Monad ((->) r) where
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Reader r a ≈ r -> a

Reader r   ≈ (->) r
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```haskell
instance Monad ((->) r) where
  f >>= g = \ r -> g (f r) r
```

Can we overload `ask` and `local`?

An interface for readers

We want `ask` and `local` to have at least the following types:

```
ask   :: Reader r r
local :: (r -> r) -> Reader r a -> Reader r a
```

```
ask   :: r -> r
local :: (r -> r) -> (r -> a) -> (r -> a)
```

First attempt

```haskell
class Monad m => MonadReader m where
  ask   :: m r
  local :: (r -> r) -> m a -> m a
```

This is insufficient. Can you see why?

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```haskell
instance MonadReader ((->) r) where

  ask   = \ r -> r                -- type error

  local = \ f m r -> m (f r)   -- type error
```

The type variable `r` in the class is different from the `r` in the instance.

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```haskell
class Monad m => MonadReader m where
  ask   :: m r
  local :: (r -> r) -> m a -> m a
```

This is insufficient. Can you see why?

```haskell
instance MonadReader ((->) r') where
  ask :: r' -> r
  ask   = \ r -> r                -- type error
  local :: (r -> r) -> (r' -> a) -> (r' -> a)
  local = \ f m r -> m (f r)   -- type error
```

The type variable `r` in the class is different from the `r'` in the instance.

Solution 1: move to higher kinds

```
class MonadReader (m :: * -> * -> *) where
  ask   :: m r r
  local :: (r -> r) -> m r a -> m r a
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  ask   = \ r -> r
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Solution 1: move to higher kinds

```haskell
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  ask   :: m r r
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```

```haskell
instance MonadReader (->) where
  ask   = \ r -> r
  local = \ f m r -> m (f r)
```

```haskell
instance MonadReader Reader where
  ask         = Reader (\ r -> r)
  local f m = Reader (\ r -> runReader m (f r))
```

Solution 1: move to higher kinds

```haskell
class MonadReader (m :: * -> * -> *) where
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instance MonadReader (->) where
  ask   = \ r -> r
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```

```haskell
instance MonadReader Reader where
  ask         = Reader (\ r -> r)
  local f m = Reader (\ r -> runReader m (f r))
```

We can no longer easily impose a superclass constraint.

Solution 2: multi-parameter type classes

```haskell
class Monad m => MonadReader r m where
  ask   :: m r
  local :: (r -> r) -> m a -> m a
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```

Solution 2: multi-parameter type classes

```
class Monad m => MonadReader r m where
  ask   :: m r
  local :: (r -> r) -> m a -> m a
```

```
instance MonadReader r ((->) r) where
  ask   = \ r -> r
  local = \ f m r -> m (f r)
```

```
instance MonadReader r (Reader r) where
  ask        = Reader (\ r -> r)
  local f m = Reader (\ r -> runReader m (f r))
```

Language extensions

This solution requires the `MultiParamTypeClasses` extensions.

In general, code that uses multi-parameter type classes will also require the `FlexibleInstances` and `FlexibleContexts` extensions.

Solution 2 is somewhat more flexible

```haskell
type Env = Map String Int

newtype EnvErr a =
  EnvErr {runEnvErr :: Env -> Either String a}
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```haskell
instance Applicative EnvErr where
  pure x = EnvErr $ \ env -> pure x
  (<*>) = ap
```

```haskell
instance Monad EnvErr where
  EnvErr f >>= g = EnvErr $ \ env ->
    f env >>= \ a -> runEnvErr (g a) env
```

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```

```haskell
instance Applicative EnvErr where
  pure x = EnvErr $ \ env -> pure x
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```

```haskell
instance Monad EnvErr where
  EnvErr f >>= g = EnvErr $ \ env ->
    f env >>= \ a -> runEnvErr (g a) env
```

```haskell
instance MonadReader Env EnvErr where
  ask = EnvErr $ \ env -> pure env
  local f m = EnvErr $ \ env -> runEnvErr m (f env)
```

Solution 2 also creates new problems

Let's try to define the following abstraction:

```
withDouble m = local (* 2) m  -- type error
```

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Let's try to define the following abstraction:

```
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```

Yields a type error similar to this:

```
Could not deduce (Num r0)
from the context: (Num r, MonadReader r m)
  bound by the inferred type for ‘withDouble’:
      (Num r, MonadReader r m) => m a -> m a
The type variable ‘r0’ is ambiguous
```

Recall: "ambiguous" type variable usually mean there's not sufficient contextual info to resolve the overloading.

Ambiguity

We can annotate the type of the state locally:

```
withDouble m = local (* (2 :: Int)) m
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But we cannot just provide a type signature:

```
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withDouble m = local (* 2) m  -- type error
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```
withDouble m = local (* (2 :: Int)) m
```

But we cannot just provide a type signature:

```
withDouble :: MonadReader m Int => m a -> m a
withDouble m = local (* 2) m  -- type error
```

And we cannot easily leave the state type polymorphic:

```
withDouble :: forall r m a .
  (MonadReader r m, Num r) => m a -> m a
withDouble m = local (* (2 :: r)) m  -- type error
```

This causes an error despite scoped type variables.

Ambiguous types

```
withDouble :: forall r m a .
  (MonadReader r m, Num r) => m a -> m a
withDouble m = local (* (2 :: r)) m  -- type error
```

Our example is much like

```
readShow ::
  forall a . (Show a, Read a) => String -> String
readShow x = show (read x :: a)  -- type error
```

Both have ambiguous types. There are type variables appearing in the class context that do not occur in the type itself. There is no way for type inference to ever resolve the constraint from contextual information.

Is there true ambiguity?

```
withDouble :: forall r m a .
  (MonadReader r m, Num r) => m a -> m a
withDouble m = local (* (2 :: r)) m  -- type error
```

In our case, there should not really be any ambiguity, because the type `r` should be computable from `m`.

Consider `m` to be one of our `MonadReader` examples so far:

```
Reader r   -- state type must be r
(->) r     -- state type must be r
EnvErr     -- state type must be Env
```

In all three cases, the state type is determined by `m`. But GHC does not know that.

Solution 3a: use a type family

Requires the `TypeFamilies` extension:

```
type family EnvType (m :: * -> *) :: *

class Monad m => MonadReader m where
  ask   :: m (EnvType m)
  local :: (EnvType m -> EnvType m) -> m a -> m a
```

A type family is an open, type-level function.

The type family `EnvType` maps types of kind `* -> *` to types of kind `*`.

Whenever we provide an instance declaration for `MonadReader`, we can (and must) extend the type family as well.

Solution 3a (contd.)

```haskell
type instance EnvType (Reader r) = r

instance MonadReader (Reader r) where
  ask       = Reader (\ r -> r)
  local f m = Reader (\ r -> runReader m (f r))
```

Solution 3a (contd.)

```haskell
type instance EnvType (Reader r) = r

instance MonadReader (Reader r) where
  ask       = Reader (\ r -> r)
  local f m = Reader (\ r -> runReader m (f r))
```

```haskell
type instance EnvType EnvErr = Env

instance MonadReader EnvErr where
  ask = EnvErr $ \ env -> Right env
  local f m = EnvErr $ \ env -> runEnvErr m (f env)
```

Solution 3a (contd.)

The problem with `withDouble` disappears now:

```
withDouble ::
  (MonadReader m, Num (EnvType m)) => m a -> m a
withDouble m = local (* 2) m
```

The type no longer mentions `r`, and `EnvType m` is clearly computable from `m`.

Solution 3b: use a functional dependency

Requires the `FunctionalDependencies` extension:

```
class Monad m => MonadReader r m | m -> r where
  ask   :: m r
  local :: (r -> r) -> m a -> m a
```

A functional dependency restricts the instances we can define for the class. For example, declaring

```
instance MonadReader Env EnvErr
instance MonadReader Int EnvErr
```

would be rejected as violating the functional dependency.

In turn, GHC now treats `r` as determined by `m`.

Solution 3b (contd.)

Again, the problem with `withDouble` disappears:

```
withDouble ::
  (MonadReader r m, Num r) => m a -> m a
withDouble m = local (* 2) m
```

While the type looks as before, `r` is now treated as determined by `m`, so the type is no longer considered to be ambiguous.

Comparison

- Both solutions are viable.
- Both functional dependencies and type families are widely applicable for a large number of programming problems that go far beyond this particular scenario.
- While in general, type families are often preferred over functional dependencies (their implementation in GHC is somewhat more principled), historically, for the use in the `MonadReader` class and similar classes, functional dependencies still dominate.
- Both implementations exist on Hackage: `mtl` implements the version with functional dependencies, and e.g. `monads-tf` implements a version with type families.

Other similar interfaces

```haskell
class Monad m => MonadError e m | m -> e where
  throwError :: e -> m a
  catchError :: m a -> (e -> m a) -> m a
```

```haskell
class Monad m => MonadState s m | m -> s where
  get :: m s
  put :: s -> m ()
```

```haskell
class Monad m => MonadIO m where
  liftIO :: IO a -> m a
```

Combining monads

Combined monads

`Reader Env` and `Either String`:

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newtype EnvErr a =
  EnvErr {runEnvErr :: Env -> Either String a}
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`Reader Env` and `Either String`:

```
newtype EnvErr a =
  EnvErr {runEnvErr :: Env -> Either String a}
```

`State s` and `Either String` (from the exercises):

```
newtype ErrorState s a =
  ErrorState
    {runErrorState :: s -> Either String (a, s)}
```

Combined monads

`Reader Env` and `Either String`:

```
newtype EnvErr a =
  EnvErr {runEnvErr :: Env -> Either String a}
```

`State s` and `Either String` (from the exercises):

```
newtype ErrorState s a =
  ErrorState
    {runErrorState :: s -> Either String (a, s)}
```

`State [t]` and `[]` (from the lecture on parsers):

```
newtype Parser t a =
  MkParser {runParser :: [t] -> [(a, [t])]}
```

Combining systematically

Rather than defining all these combinations by hand, can we just combine the individual monads into larger ones?

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Rather than defining all these combinations by hand, can we just combine the individual monads into larger ones?

Unfortunately, the answer is not a simple yes.

Excursion: composition of functors

Defined in `Data.Functor.Compose`:

```haskell
newtype Compose f g a =
  Compose {getCompose :: f (g a)}
```

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Defined in `Data.Functor.Compose`:

```
newtype Compose f g a =
  Compose {getCompose :: f (g a)}
```

If `f` and `g` are functors, then `Compose f g` is a functor:

```
instance (Functor f, Functor g)
   => Functor (Compose f g) where
  fmap f (Compose x) = Compose (fmap (fmap f) x)
```

Examples

```
GHCi> fmap (+ 1) (Compose (Just [3, 4]))
Compose {getCompose = Just [4, 5]}
```

```
GHCi> getCompose
        (fmap (+ 1)
          (Compose (flip replicate 3))) 7
[4, 4, 4, 4, 4, 4, 4]
```

Composition of applicative functors

Applicative functors are closed under composition as well:

```
instance (Applicative f, Applicative g)
   => Applicative (Compose f g) where
  pure x = Compose (pure (pure x))
  Compose f <*> Compose x =
    Compose (pure (<*>) <*> f <*> x)
```

Example

```
GHCi> getCompose
  pure (+)
   <*> Compose [Just 1, Just 10]
   <*> Compose [Nothing, Just 100]
[Nothing, Just 101, Nothing, Just 110]
```

And for monads?

Unfortunately, we cannot make `Compose` an instance of `Monad`.

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Unfortunately, we cannot make `Compose` an instance of `Monad`.

The story for monads is more complicated and less general:

- For some monads, we can define monad transformers.
- A monad transformer takes an existing monad to a new monad.
- The order in which we apply monad transformers matters.
- Not all monads have associated transformers.

Monad transformers

The reader transformer

Defined in `Control.Monad.Trans.Reader` in package `transformers`:

```
newtype ReaderT r m a =
  ReaderT {runReaderT :: r -> m a}
```

The `ReaderT` type is parameterized over the "underlying" monad – compare to the "old" type:

```
newtype Reader r a = Reader {runReader :: r -> a}
```

Note that the kind of `ReaderT` is

```
* -> (* -> *) -> (* -> *)
```

A monad transformer

The `ReaderT` type indeed takes monads to monads:

```
instance Monad m => Monad (ReaderT r m) where
  m >>= g =
    ReaderT $ \ r ->
      runReaderT m r >>= \ a -> runReaderT (g a) r
```

Compare to the old `Reader` instance:

```
instance Monad (Reader r) where
  m >>= g =
    Reader (\ r -> runReader (g (runReader m r)) r)
```

Functors and applicatives

We still need to handle the superclasses – we do not use the “standard” instances here because we can do with weaker constraints on `m`:

```
instance Functor m => Functor (ReaderT r m) where
  fmap f m =
    ReaderT (\ r -> fmap f (runReaderT m r))
```

```
instance Applicative m
   => Applicative (ReaderT r m) where
  pure x = ReaderT (\ _ -> pure x)
  f <*> x =
    ReaderT $ \ r ->
      runReaderT f r <*> runReaderT x r
```

We obtain a reader

```
instance Monad m
   => MonadReader r (ReaderT r m) where
  ask = ReaderT (\ r -> pure r)
  local f m =
    ReaderT $ \ env -> runReaderT m (f env)
```

(More or less the same instance we used for `EnvErr`.)

Other transformers

```haskell
newtype StateT s m a =
  StateT {runStateT :: s -> m (a, s)}
```

```haskell
newtype MaybeT m a =
  MaybeT {runMaybeT :: m (Maybe a)}
```

```haskell
newtype ExceptT e m a =
  ExceptT {runExceptT :: m (Either e a)}
```

Identity as a base case

```
StateT s Identity  ≈ State s
ReaderT r Identity ≈ Reader r
MaybeT Identity    ≈ Maybe
ExceptT e Identity ≈ Either e
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StateT s Identity  ≈ State s
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```

There is also an `IdentityT`, but it's rarely useful, as it adds no extra functionality.

Building combinations

```haskell
newtype EnvErr a =
  EnvErr {runEnvErr :: Env -> Either String a}
```

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```haskell
newtype EnvErr a =
  EnvErr {runEnvErr :: Env -> Either String a}
```

```haskell
type EnvErr =
  ReaderT Env (ExceptT String Identity)
```

Building combinations (contd.)

```haskell
newtype ErrorState s a =
  ErrorState
    {runErrorState :: s -> Either String (a, s)}
```

Building combinations (contd.)

```haskell
newtype ErrorState s a =
  ErrorState
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```

```haskell
type ErrorState s =
  StateT s (ExceptT String Identity)
```

Building combinations (contd.)

```
newtype Parser t a =
  MkParser {runParser :: [t] -> [(a, [t])]}
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Building combinations (contd.)

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newtype Parser t a =
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```haskell
type Parser t =
  StateT [t] []
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Building combinations (contd.)

```haskell
newtype Parser t a =
  MkParser {runParser :: [t] -> [(a, [t])]}
```

```haskell
type Parser t =
  StateT [t] []
```

Note: There is a `ListT` in the libraries, but it should be used with extreme care, as it often does not produce valid monads.

IO as a base case

We can also build monads on top of `IO`:

```
type Example =
  ReaderT Env (ExceptT Message IO)
```

IO as a base case

We can also build monads on top of `IO`:

```
type Example =
  ReaderT Env (ExceptT Message IO)
```

There is no transformer version of `IO`.

Are we done?

Not quite – consider again:

```
type Example =
  ReaderT Env (ExceptT Message IO)
```

- `Example` is an instance of `MonadReader Env`.
- Similarly, `ExceptT Message IO` is an instance of `MonadError Message`.
- But shouldn't `Example` also be an instance of `MonadError Message`?

Lifting `MonadError` through `ReaderT`

```
instance MonadError e m
   => MonadError e (ReaderT r m) where
  throwError e = ReaderT $ pure (throwError e)
  catchError m h =
    ReaderT $ \ r ->
      catchError
        (runReaderT m r)
        (\ e -> runReaderT (h e) r)
```

This instance requires the `UndecidableInstances` extension (GHC is *extremely* conservative without it in trying to guarantee the termination of instance search; the extension is more harmless than it sounds.)

Lifting `MonadState` through `ReaderT`

Unfortunately, we need lifting instances for *all combinations* of monad transformers:

```
instance MonadState s m
   => MonadState s (ReaderT r m) where
  get = ReaderT $ pure get
  put s = ReaderT $ pure (put s)
```

This makes monad transformers less modular and less extensible than would be desirable.

Lifting generically

Fortunately, most liftings are very simple:

```
class MonadTrans t where
  lift :: Monad m => m a -> t m a
```

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```haskell
class MonadTrans t where
  lift :: Monad m => m a -> t m a
```

Example instances:

```haskell
instance MonadTrans (ReaderT r) where
  lift m = ReaderT $ pure m
```

```haskell
instance MonadTrans (StateT s) where
  lift m = StateT $ \ s -> fmap (\ a -> (a, s)) m
```

```haskell
instance MonadTrans (ExceptT e) where
  lift m = ExceptT $ fmap pure m
```

Monad transformer laws

```
lift (pure x) = pure x
lift (m >>= f) = lift m >>= \ a -> lift (f a)
```

Lifting generically (contd.)

Some lifting instances get near trivial.

```
instance MonadState s m
   => MonadState s (ReaderT r m) where
  get   = lift get
  put s = lift (put s)
```

Others (e.g. the `catchError` definition) are less straight-forward.

Monad transformers summary

- Monad transformers provide a way to build monads out of smaller building blocks and to combine effects as needed.
- Unfortunately, the underlying theory is not extremely beautiful, leading to several restrictions and complications.
- A few approaches have been tried to address some of the shortcomings, but `mtl` remains dominant.