Playing w/ RLC circuits (Part I)

11th January 2014

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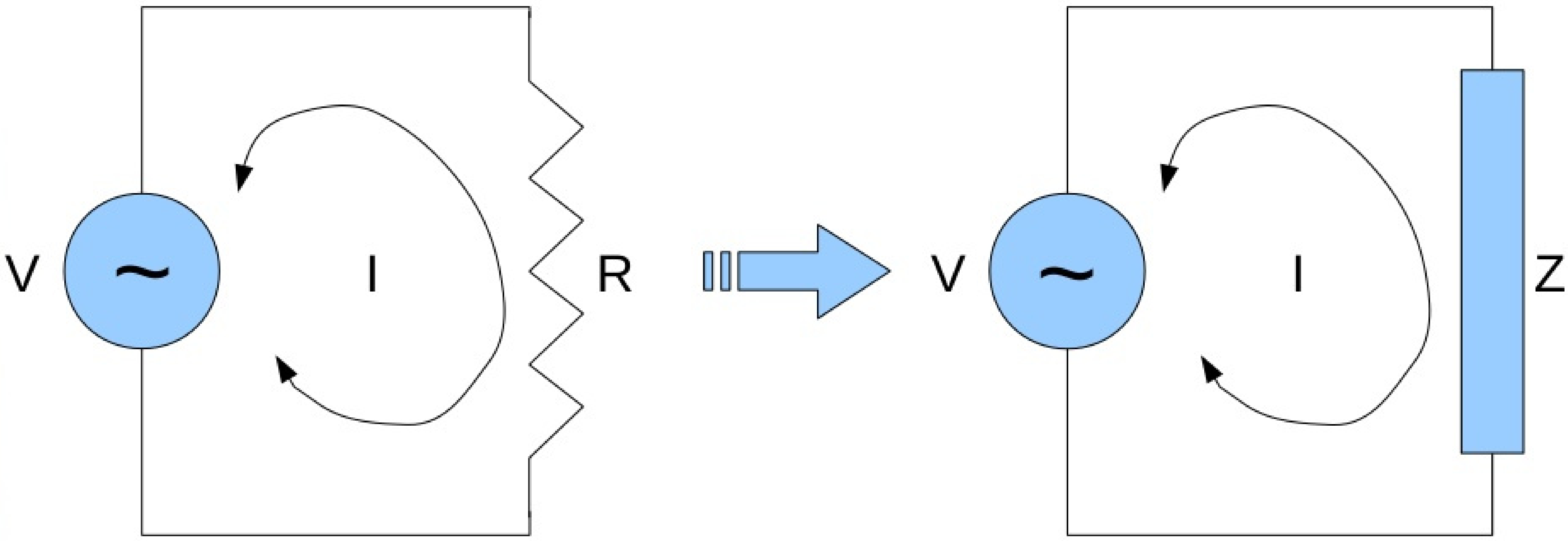
Agenda

- RLC Basics
- RC Visualization
- CR Oscilloscope
- Function Generator
- Frequency Filters

Introduction

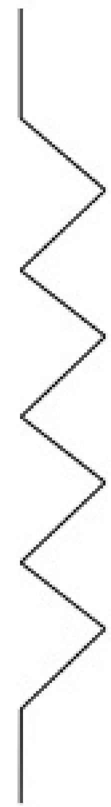


V = I * R

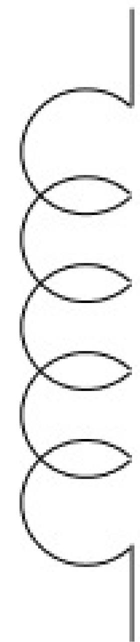
V = I * Z

Impedance (Z) Fundamentals

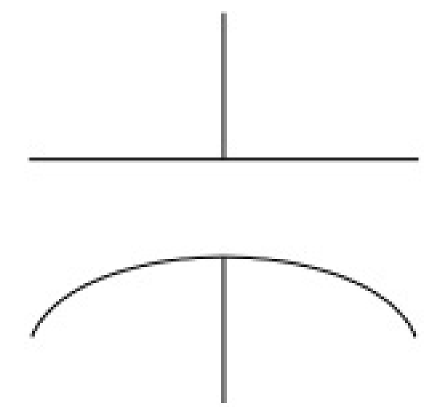
$\omega = 2\pi f$



$R \Rightarrow R$



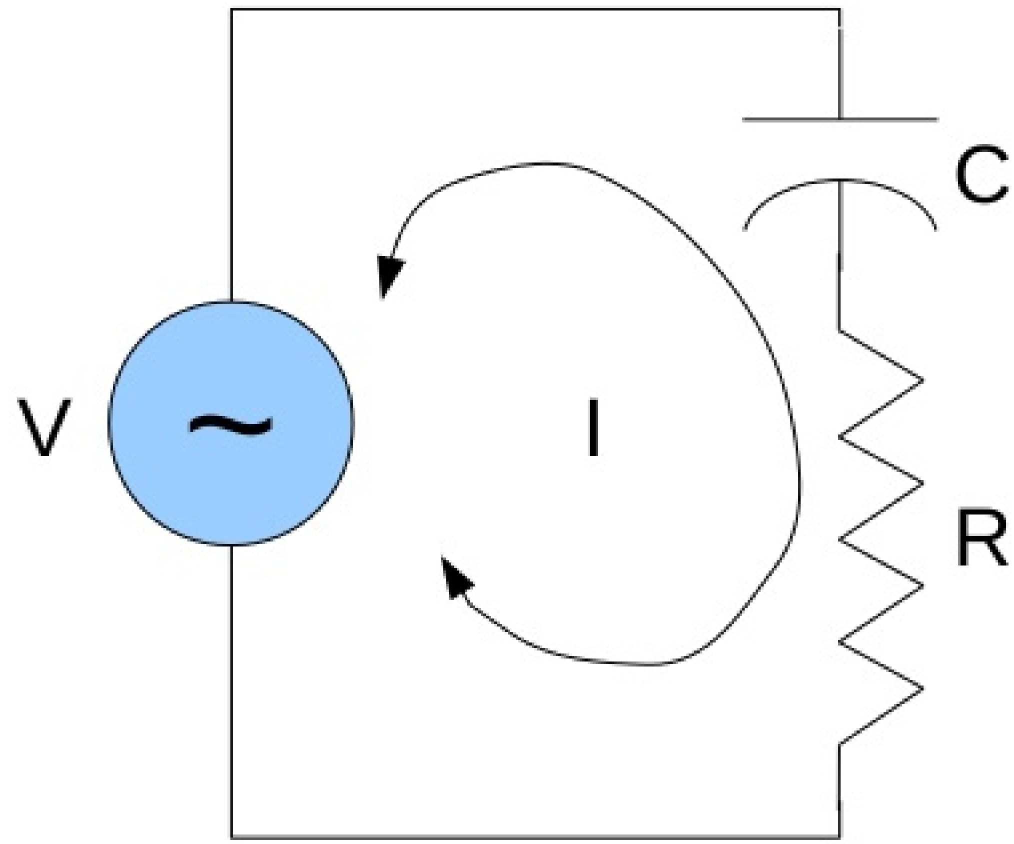
$L \Rightarrow j\omega L$



$C \Rightarrow \dfrac{1}{j\omega C} = \dfrac{-j}{\omega C}$

Goal: Observe these phase shifts

Basic RC Circuit



$$Z = R - \frac{j}{\omega C} = A\, e^{-j\phi}$$

$$A = |Z| = \frac{\sqrt{\omega^2 R^2 C^2 + 1}}{\omega C}$$

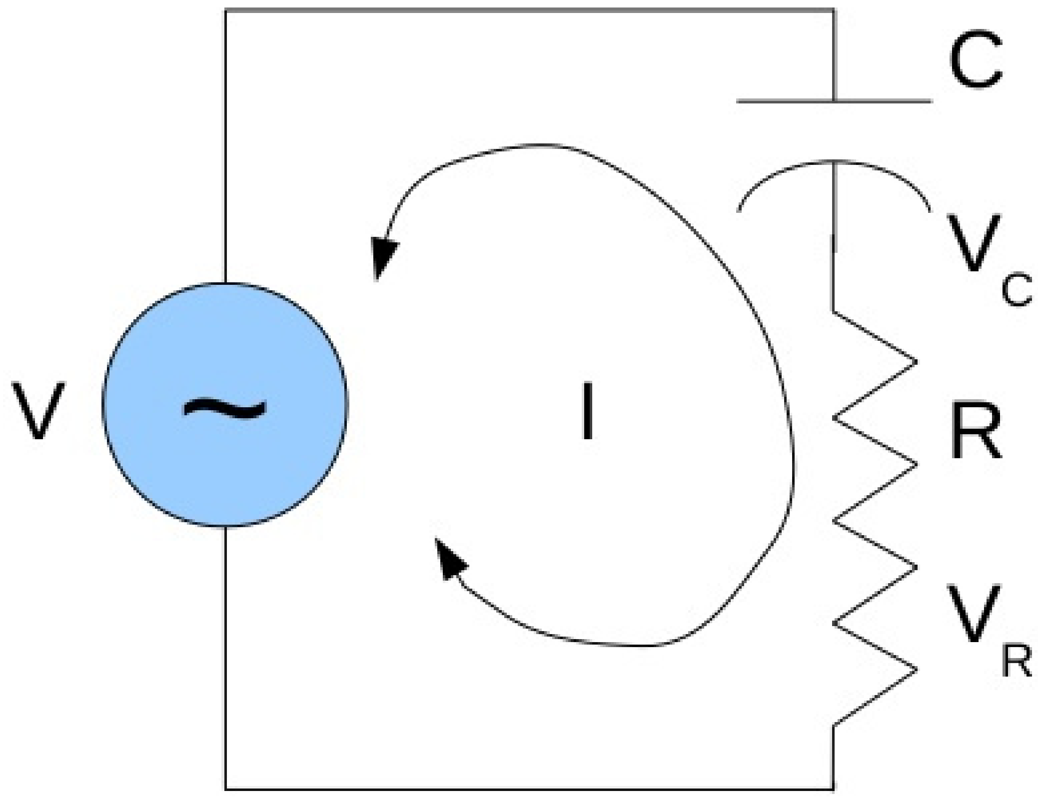
$$\tan(\phi) = arg\, Z = \frac{1}{\omega R C}$$

$$V = I * Z, Z = R - \frac{j}{\omega C}$$

$$I = \frac{V}{Z} = \frac{V}{A} e^{j\phi} = \frac{V}{A} < \phi >$$

RC Continued

$$A=\frac{\sqrt{\omega^2 R^2 C^2+1}}{\omega C}, \tan(\phi)=\frac{1}{\omega R C}, \quad I=\frac{V}{A}<\phi>$$



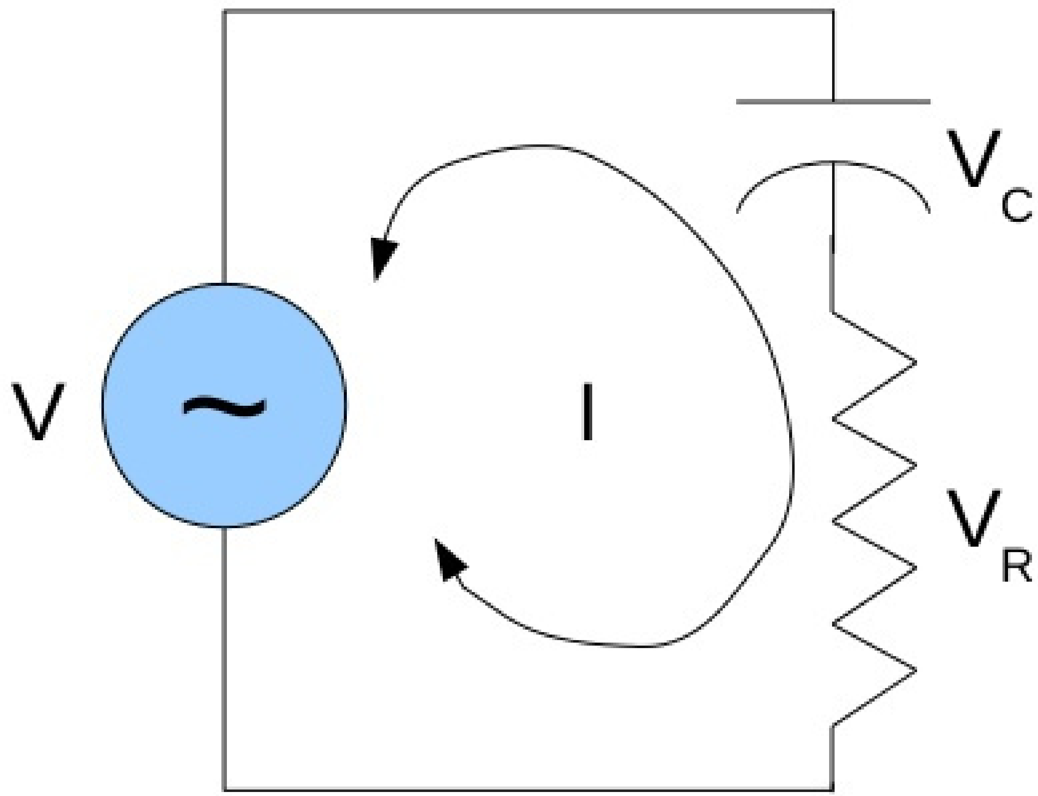
$$V_R=I*R=\frac{V*R}{A}<\phi>$$

$$V=V_m*\sin(\omega t)$$

$$V_R=\left(V_m*\frac{R}{A}\right)*\sin(\omega t+\phi)$$

$$\frac{R}{A}=\frac{\omega R C}{\sqrt{(\omega R C)^2+1}}=\alpha$$

RC Conclusion



$$V = V_m * \sin(\omega t)$$

$$V_R = (V_m * \alpha) * \sin(\omega t + \phi)$$

$$\alpha = \frac{\omega R C}{\sqrt{(\omega R C)^2 + 1}}$$

$$\tan(\phi) = \frac{1}{\omega R C}$$

RC Predictions

$$\alpha = \frac{\omega R C}{\sqrt{(\omega R C)^2 + 1}}, \tan(\phi) = \frac{1}{\omega R C}, R = 220\,\Omega$$

f (Hz)	1000		2000	
C (μF)	α	Φ (deg)	α	Φ (deg)
100	1.00	0.4	1.00	0.2
10	1.00	4.1	1.00	2.1
1	0.81	35.9	0.94	19.9
0.1	0.14	82.1	0.27	74.5
0.01	0.01	89.2	0.03	88.4

Oscilloscope

- Build your own
- Total Cost: ~₹20,000
- Parts
  - Computer (~₹20,000)
  - OSS: xoscope (₹0)
  - Audio Cable (~₹200)
- Net Additional Cost: ~₹200
- Limitations
  - Only for audio range (100Hz-20kHz)
  - DC is filtered off – shows AC with 0 average

RC Predictions

$$\alpha = \frac{\omega R C}{\sqrt{(\omega R C)^2 + 1}}, \tan(\phi) = \frac{1}{\omega R C}, R = 220\,\Omega$$

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Function Generator

- Build your own
- Electronics-way: Use electronic components
  - 555, Opamp, ...
  - Good got higher frequencies
- Programming way: Use micro-controller
  - Like AVR, ...
  - Excellent for low frequencies

FG Programming

- Requirements for the uC
  - Programmable Timer/Counter
  - Digital to Analog Convertor (DAC), Or
  - Pulse Width Modulator (PWM)
- Concept of FG
  - Run a frequency-divider timer/counter
  - Every count, generate the function output
  - Repeat for the frequency

Sine Wave Example

sin(0°)	0.00000	sin(180°)
sin(18°)	0.30902	sin(162°)
sin(36°)	0.58779	sin(144°)
sin(54°)	0.80902	sin(126°)
sin(72°)	0.95106	sin(108°)
sin(90°)	1.00000	

	-0.00000	sin(360°)
sin(198°)	-0.30902	sin(342°)
sin(216°)	-0.58779	sin(324°)
sin(234°)	-0.80902	sin(306°)
sin(252°)	-0.95106	sin(288°)
sin(270°)	-1.00000	

Add +1 to make all positive.
Multiply by 5 / 2 to scale between 0 & 5.

sin(0°)	2.50000	sin(180°)
sin(18°)	3.27254	sin(162°)
sin(36°)	3.96946	sin(144°)
sin(54°)	4.52254	sin(126°)
sin(72°)	4.87764	sin(108°)
sin(90°)	5.00000	

	2.50000	sin(360°)
sin(198°)	1.72746	sin(342°)
sin(216°)	1.03054	sin(324°)
sin(234°)	0.47746	sin(306°)
sin(252°)	0.12236	sin(288°)
sin(270°)	0.00000	

Sine Wave Example (Contd.)

sin(0°)	2.50000	sin(180°)
sin(18°)	3.27254	sin(162°)
sin(36°)	3.96946	sin(144°)
sin(54°)	4.52254	sin(126°)
sin(72°)	4.87764	sin(108°)
sin(90°)	5.00000	

	2.50000	sin(360°)
sin(198°)	1.72746	sin(342°)
sin(216°)	1.03054	sin(324°)
sin(234°)	0.47746	sin(306°)
sin(252°)	0.12236	sin(288°)
sin(270°)	0.00000	

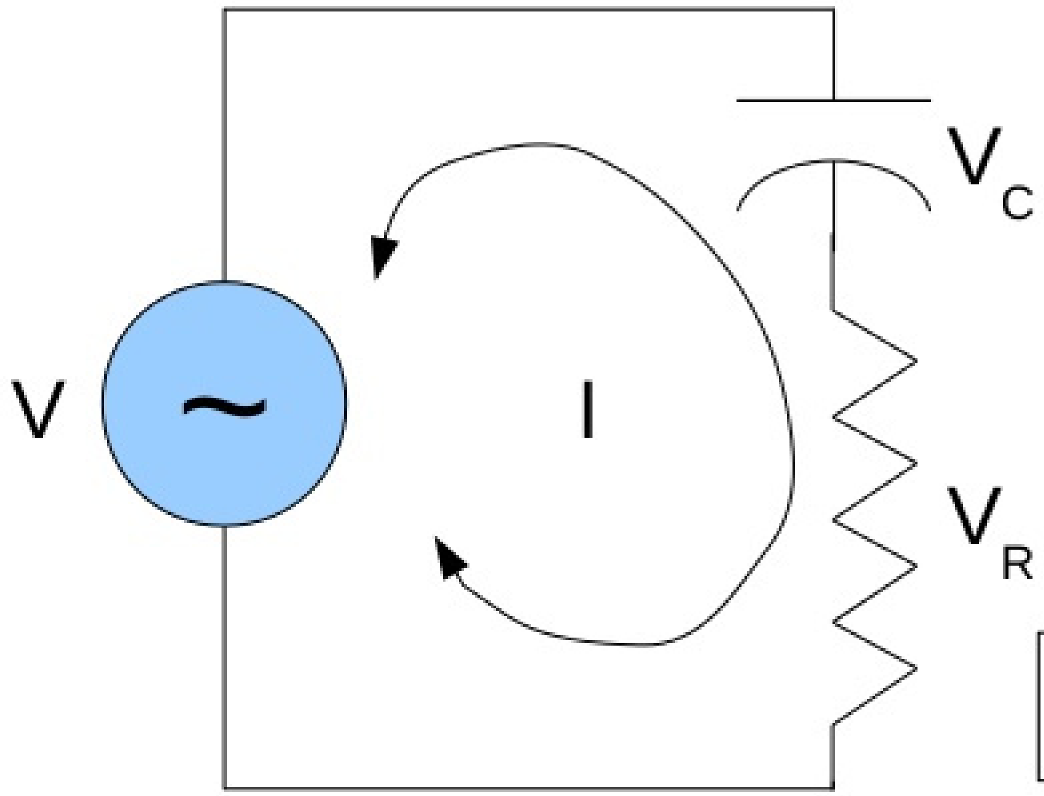
- We have 20 points
- For f = 1kHz => T = 1/1kHz = 1ms
- 20 points in 1ms = 1000us
- 1 point in 50us => Timer trigger every 50us

Frequency Filters

$$V = V_m * \sin(\omega t)$$

HPF => $$V_R = (V_m * \alpha) * \sin(\omega t + \phi)$$

LPF => $$V_C = (V_m * \sqrt{1-\alpha^2}) * \sin\left(\omega t + \left(\phi - \frac{\pi}{2}\right)\right)$$



$$\alpha = \frac{\omega R C}{\sqrt{(\omega R C)^2 + 1}} = \cos(\phi)$$

$$\tan(\phi) = \frac{1}{\omega R C}$$

$\boxed{\text{C=1µF, R=10Ω => f=16kHz}}$

Wrap Up

- Function Generator
- Hobbyist Oscilloscope
- RLC Basics
- RC Visualization
- RC Filters

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Any Queries