RC Computations

by

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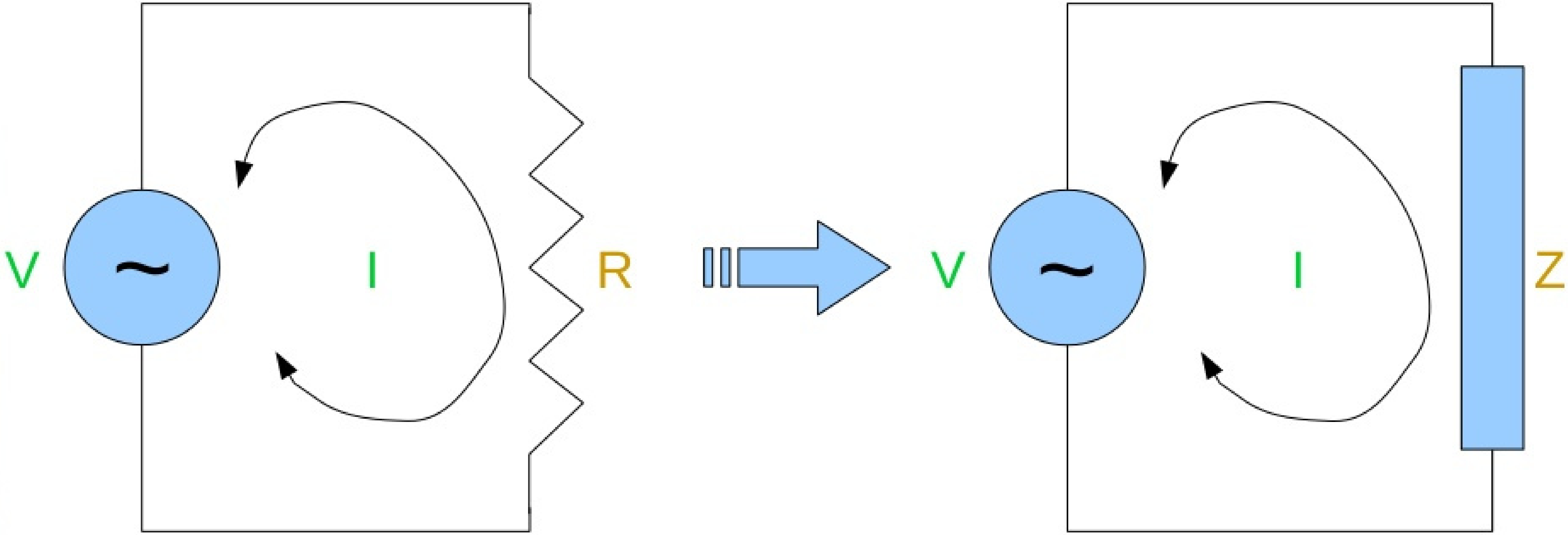
Introduction

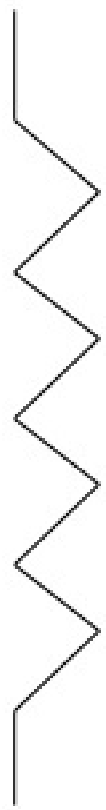


V = I * R

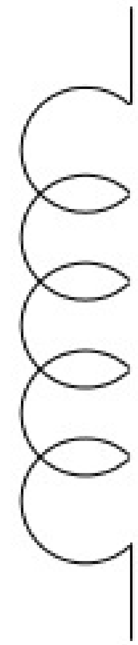
V = I * Z

Impedance (Z) Fundamentals

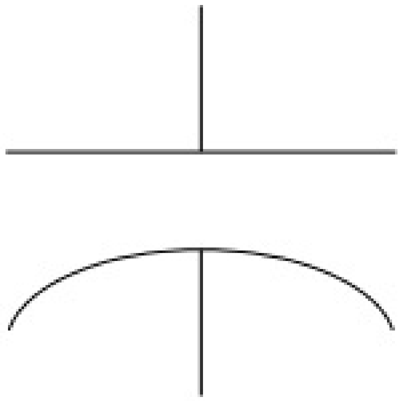
$$\boxed{\omega = 2\pi f}$$



$$R \Rightarrow R$$

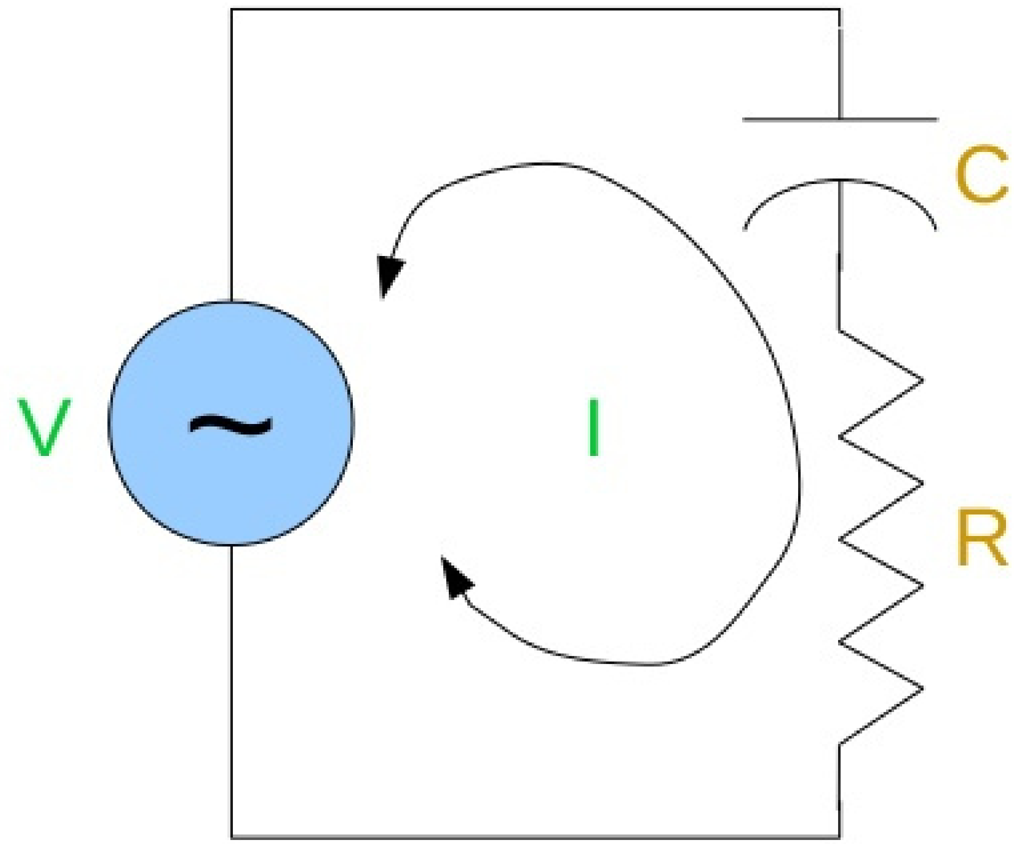


$$L \Rightarrow j\omega L$$



$$C \Rightarrow \frac{1}{j\omega C} = \frac{-j}{\omega C}$$

Basic RC Circuit



$$Z = R - \frac{j}{\omega C} = A e^{-j\varphi}$$

$$A = |Z| = \frac{\sqrt{\omega^2 R^2 C^2 + 1}}{\omega C}$$

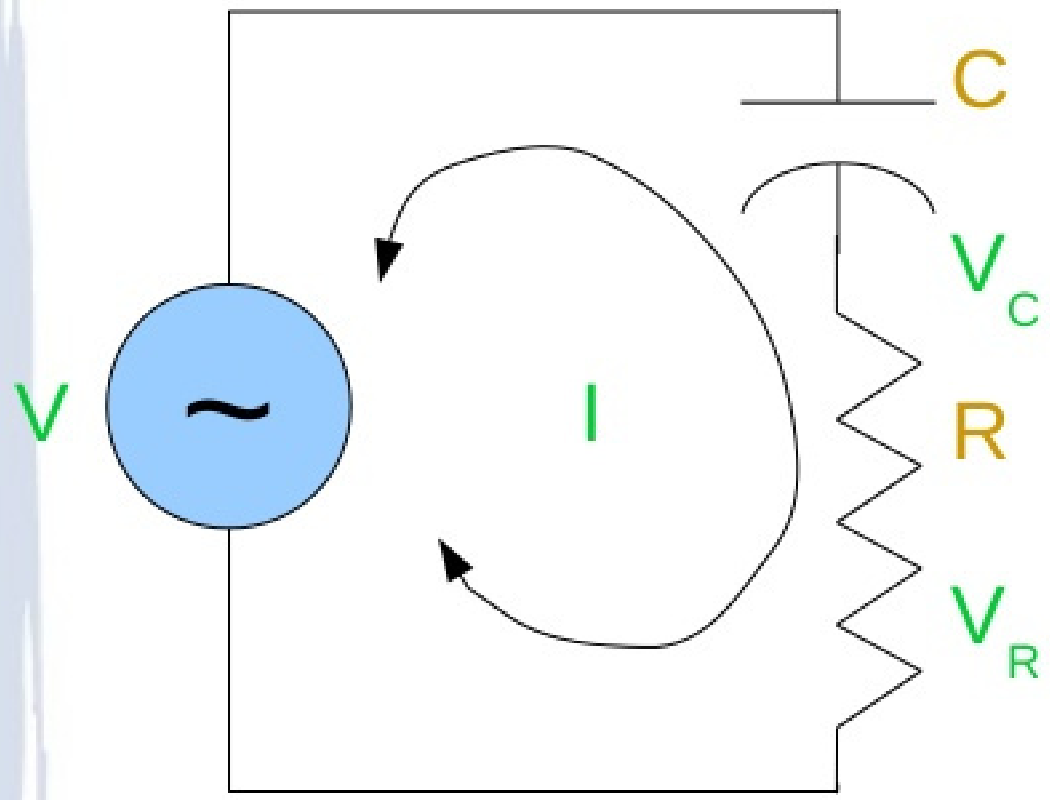
$$\tan(\varphi) = arg\, Z = \frac{1}{\omega R C}$$

$$\boxed{V = I * Z, Z = R - \frac{j}{\omega C}}$$

$$I = \frac{V}{Z} = \frac{V}{A} e^{j\varphi} = \frac{V}{A} < \varphi >$$

Voltage across the Resistor

$$A = \frac{\sqrt{\omega^2 R^2 C^2 + 1}}{\omega C}, \tan(\varphi) = \frac{1}{\omega R C}, \quad I = \frac{V}{A} < \varphi >$$



$$V_R = I * R = \frac{V * R}{A} < \varphi >$$

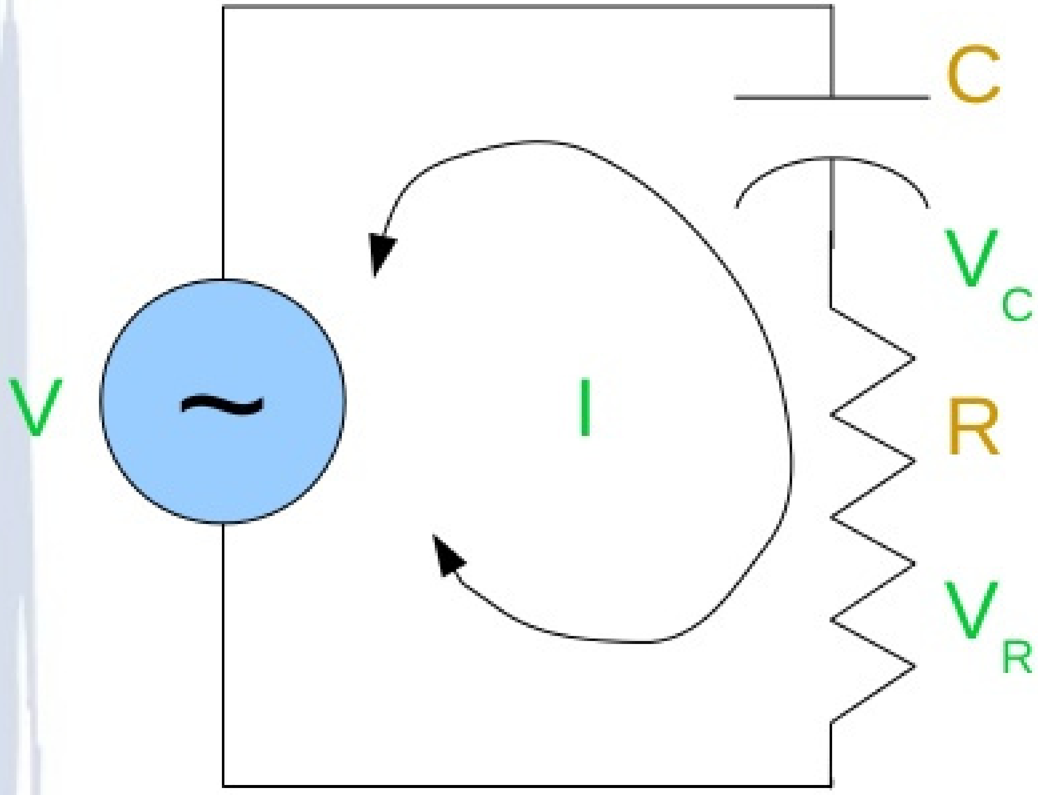
$$V = V_m * \sin(\omega t)$$

$$V_R = \left(V_m * \frac{R}{A}\right) * \sin(\omega t + \varphi)$$

$$\frac{R}{A} = \frac{\omega R C}{\sqrt{(\omega R C)^2 + 1}} = \alpha$$

Voltage across the Capacitor

$$A=\frac{\sqrt{\omega^2R^2C^2+1}}{\omega C},\tan(\varphi)=\frac{1}{\omega RC},\quad I=\frac{V}{A}<\varphi>$$



$$V_C=I*\frac{-j}{\omega*C}=\frac{V}{A*\omega*C}<\varphi-\frac{\pi}{2}>$$

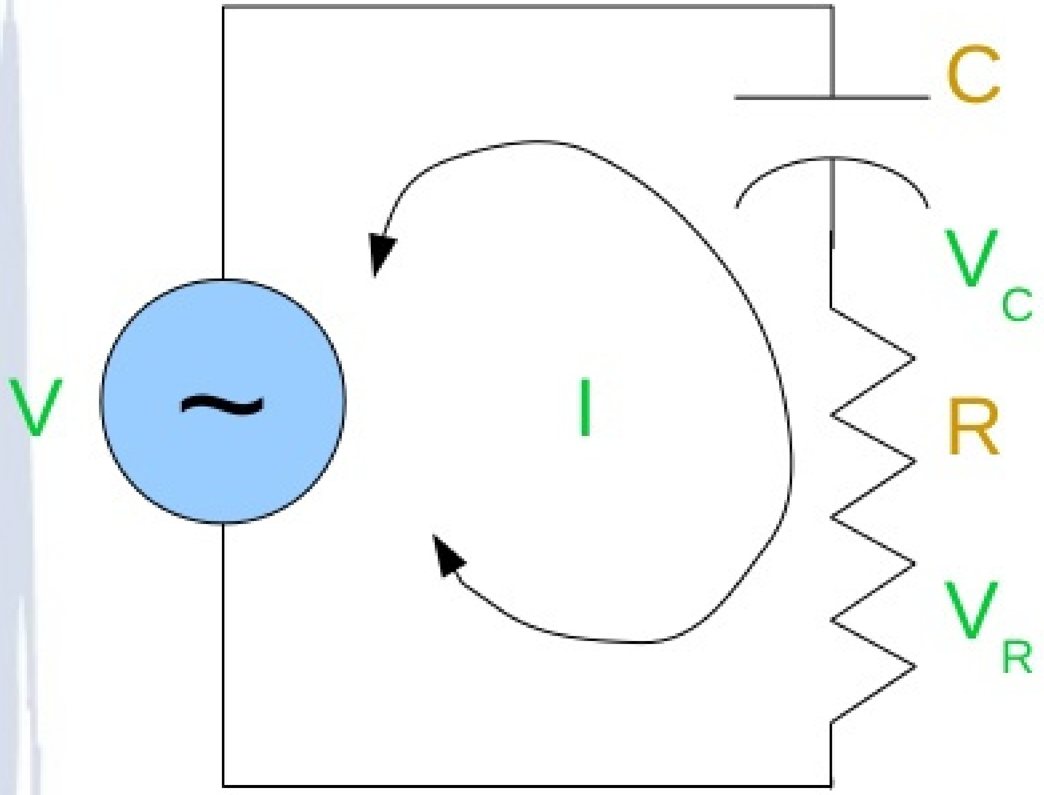
$$V=V_m*\sin(\omega t)$$

$$V_C=(\frac{V_m}{A*\omega*C})*\sin(\omega t+\varphi-\frac{\pi}{2})$$

$$\frac{1}{A*\omega*C}=\frac{1}{\sqrt{(\omega RC)^2+1}}=\beta$$

RC Conclusion

$$V = V_m * \sin(\omega t)$$



$$V_R = (V_m * \alpha) * \sin(\omega t + \varphi)$$

$$V_C = (V_m * \beta) * \sin\left(\omega t + \varphi - \frac{\pi}{2}\right)$$

$$\alpha = \frac{\omega R C}{\sqrt{(\omega R C)^2 + 1}}$$

$$\beta = \frac{1}{\sqrt{(\omega R C)^2 + 1}} = \sqrt{1 - \alpha^2}$$

$$\tan(\varphi) = \frac{1}{\omega R C}$$

RC Predictions

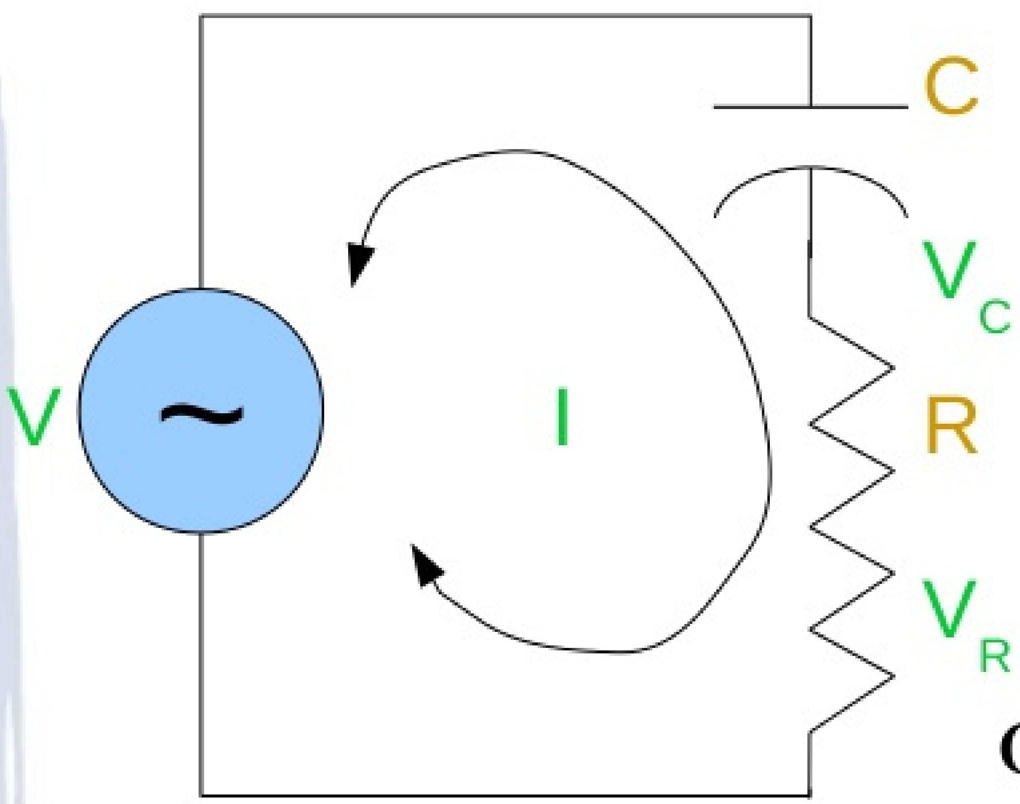
$$\alpha = \frac{\omega R C}{\sqrt{(\omega R C)^2 + 1}}, \tan(\varphi) = \frac{1}{\omega R C}, R = 220\,\Omega$$

f (Hz)	1000		2000	
C (μF)	α	Φ (deg)	α	Φ (deg)
100	1.00	0.4	1.00	0.2
10	1.00	4.1	1.00	2.1
1	0.81	35.9	0.94	19.9
0.1	0.14	82.1	0.27	74.5
0.01	0.01	89.2	0.03	88.4

Cut-Off Frequency of Filters

HPF => $V_R = (V_m * \alpha) * \sin(\omega t + \varphi)$

LPF => $V_C = (V_m * \sqrt{1-\alpha^2}) * \sin\left(\omega t + \left(\varphi - \frac{\pi}{2}\right)\right)$



$$\alpha = \frac{\omega RC}{\sqrt{(\omega RC)^2 + 1}} = \cos(\varphi)$$

$$\tan(\varphi) = \frac{1}{\omega RC}$$

$$\alpha = \frac{1}{\sqrt{(2)}} \Rightarrow \omega_c = \frac{1}{RC} \Rightarrow f_c = \frac{1}{2\pi RC}$$

$$V = V_m * \sin(\omega t)$$

Ex: C = 1μF, R = 10Ω => f_c = 16kHz