Balanced Binary Search Trees

Contents

Balanced Binary Search Trees

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Unbalanced	$O(n)$	$O(n)$	$O(n) + \beta$
Balanced	$O(\log n)$	$O(\log n) + \alpha$	$O(\log n) + \beta$

AVL (Adelson-Velskii and Landis) Tree

- Keep the tree perfectly balanced by comparing heights of the subtrees.

Red-Black Tree

- Keep the tree balanced by matchings colors of the nodes.

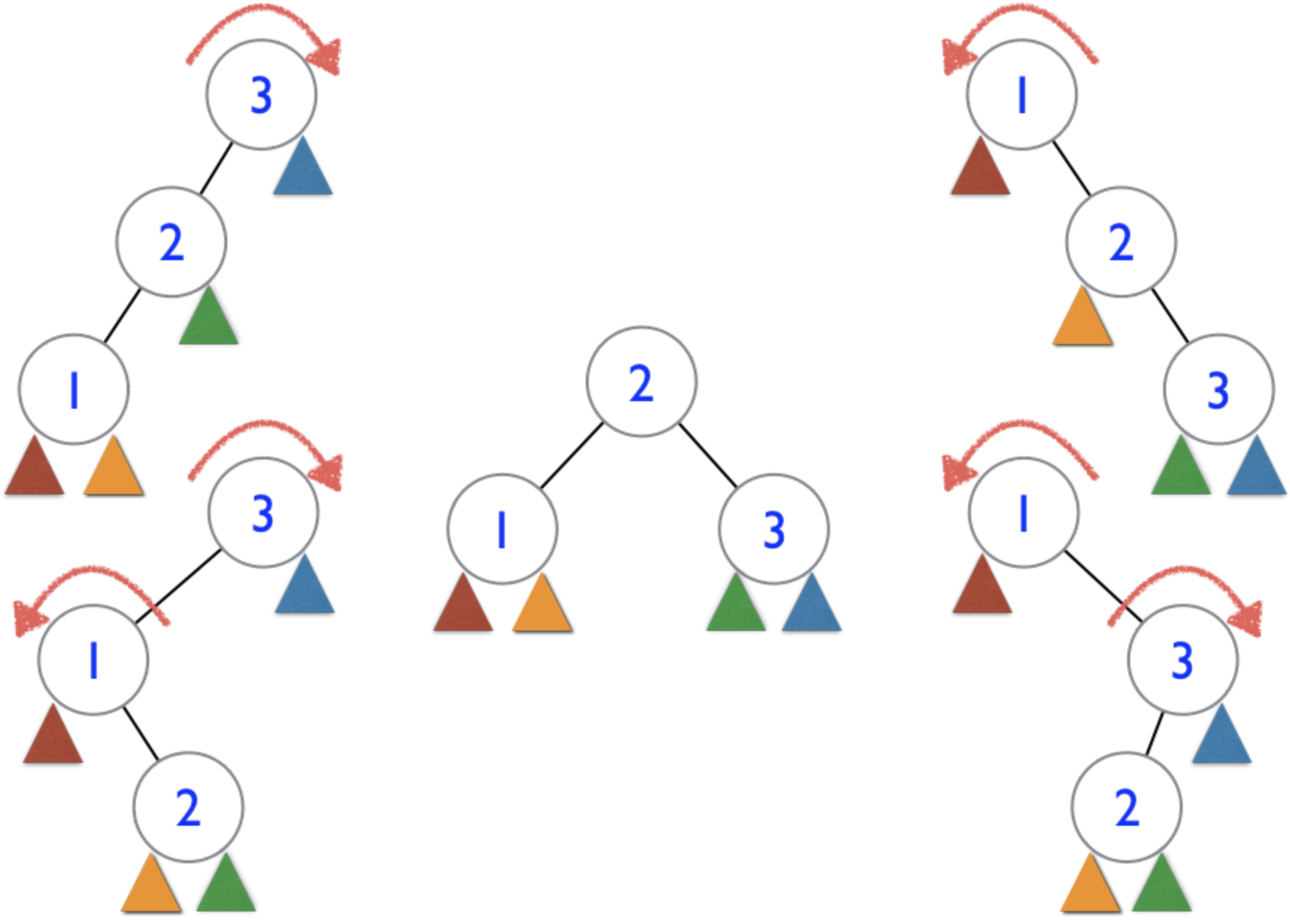
Abstract Balanced Binary Search Tree

Source: `AbstractBalancedBinarySearchTree.java`.

```
public abstract class AbstractBalancedBinarySearchTree
                      <T extends Comparable<T>, N extends AbstractBinaryNode<T, N>>
                      extends AbstractBinarySearchTree<T, N> {
    @Override
    public N add(T key) {
        N node = super.add(key);
        balance(node);
        return node;
    }

    @Override
    public N remove(T key) {
        N node = findNode(root, key);

        if (node != null) {
            N lowest = node.hasBothChildren() ? removeHibbard(node) : removeSelf(node);
            if (lowest != null && lowest != node) balance(lowest);
        }

        return node;
    }

    /** Preserves the balance of the specific node and its ancestors. */
    protected abstract void balance(N node);
```

- Abstract method: `balance()`.

Rotations



```
protected void rotateLeft(N node) {
    N child = node.getRightChild();

    node.setRightChild(child.getLeftChild());

    if (node.hasParent())
        node.getParent().replaceChild(node, child);
    else
        setRoot(child);

    child.setLeftChild(node);
}

protected void rotateRight(N node) {
    N child = node.getLeftChild();

    node.setLeftChild(child.getRightChild());

    if (node.hasParent())
        node.getParent().replaceChild(node, child);
    else
        setRoot(child);

    child.setRightChild(node);
}
```

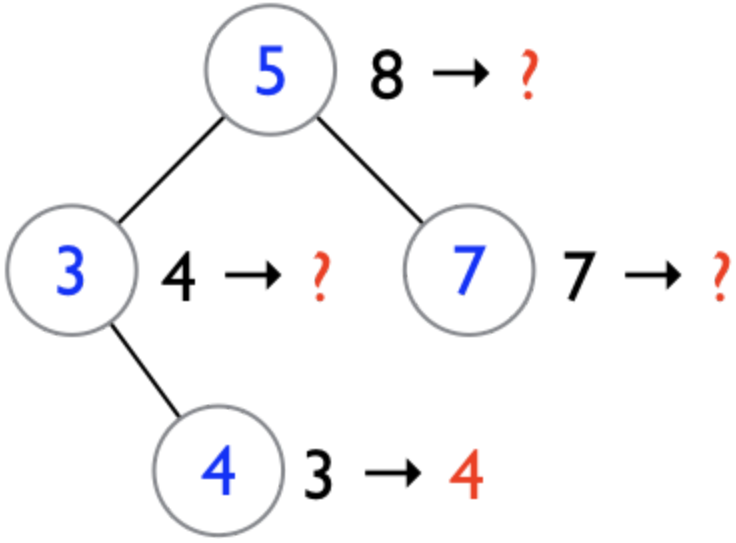
AVL Node

Source: `AVLNode.java`.

```
public class AVLNode<T extends Comparable<T>>
             extends AbstractBinaryNode<T, AVLNode<T>> {

    private int height;

    public AVLNode(T key) {
        super(key);
        height = 1;
    }

    @Override
    public void setLeftChild(AVLNode<T> node) {
        super.setLeftChild(node);
        resetHeights();
    }

    @Override
    public void setRightChild(AVLNode<T> node) {
        super.setRightChild(node);
        resetHeights();
    }
```

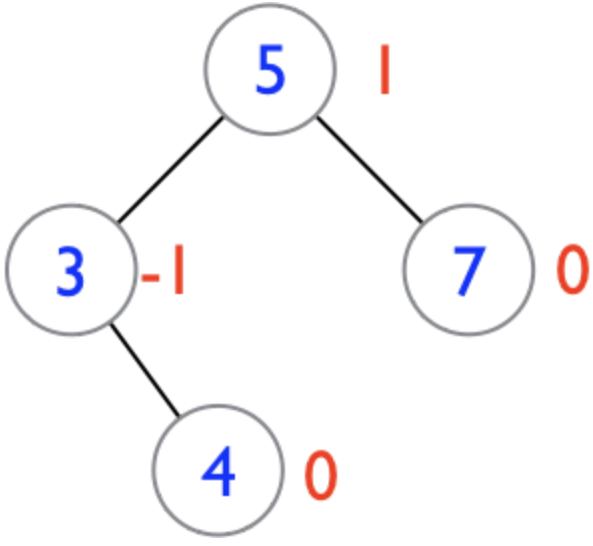
Reset Heights

```
public void resetHeights() { resetHeightsAux(this); }

private void resetHeightsAux(AVLNode<T> node) {
    if (node != null) {
        int lh = node.hasLeftChild() ? node.getLeftChild().getHeight() : 0;
        int rh = node.hasRightChild() ? node.getRightChild().getHeight() : 0;
        int height = Math.max(lh, rh) + 1;

        if (height != node.getHeight()) {
            node.setHeight(height);
            resetHeightsAux(node.getParent());
        }
    }
}
```



Balance Factor

```
public int getBalanceFactor() {
    if (hasBothChildren())
        return left_child.getHeight() - right_child.getHeight();
    else if (hasLeftChild())
        return left_child.getHeight();
    else if (hasRightChild())
        return -right_child.getHeight();
    else
        return 0;
}
```



AVL Tree

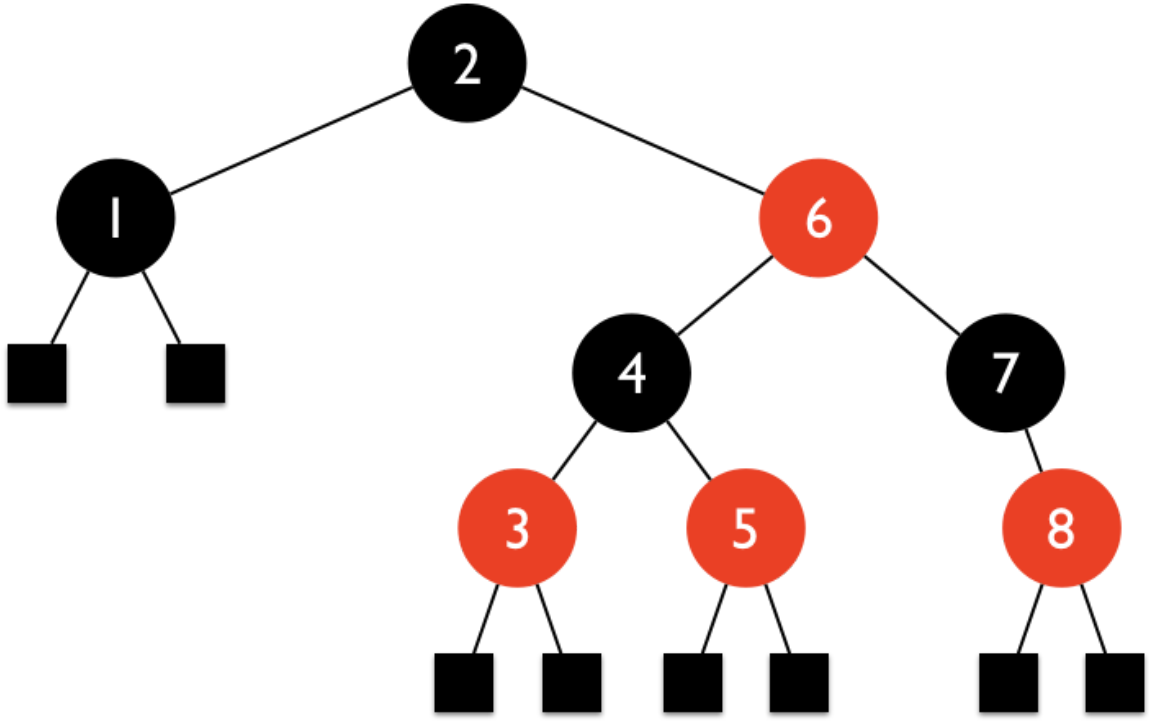
Source: `AVLTree.java`.

```
public class AVLTree<T extends Comparable<T>>
             extends AbstractBalancedBinarySearchTree<T, AVLNode<T>> {
    @Override
    public AVLNode<T> createNode(T key) {
        return new AVLNode<T>(key);
    }

    @Override
    protected void rotateLeft(AVLNode<T> node) {
        super.rotateLeft(node);
        node.resetHeights();
    }

    @Override
    protected void rotateRight(AVLNode<T> node) {
        super.rotateRight(node);
        node.resetHeights();
    }
```

```java
@Override
protected void balance(AVLNode<T> node) {
    if (node == null) return;
    int bf = node.getBalanceFactor();

    if (bf == 2) {
        AVLNode<T> child = node.getLeftChild();

        if (child.getBalanceFactor() == -1)  // case 1
            rotateLeft(child);

        rotateRight(node);  // case 2
    } else if (bf == -2) {
        AVLNode<T> child = node.getRightChild();

        if (child.getBalanceFactor() == 1)  // case 3
            rotateRight(child);

        rotateLeft(node);  // case 4
    } else
        balance(node.getParent());
}
```

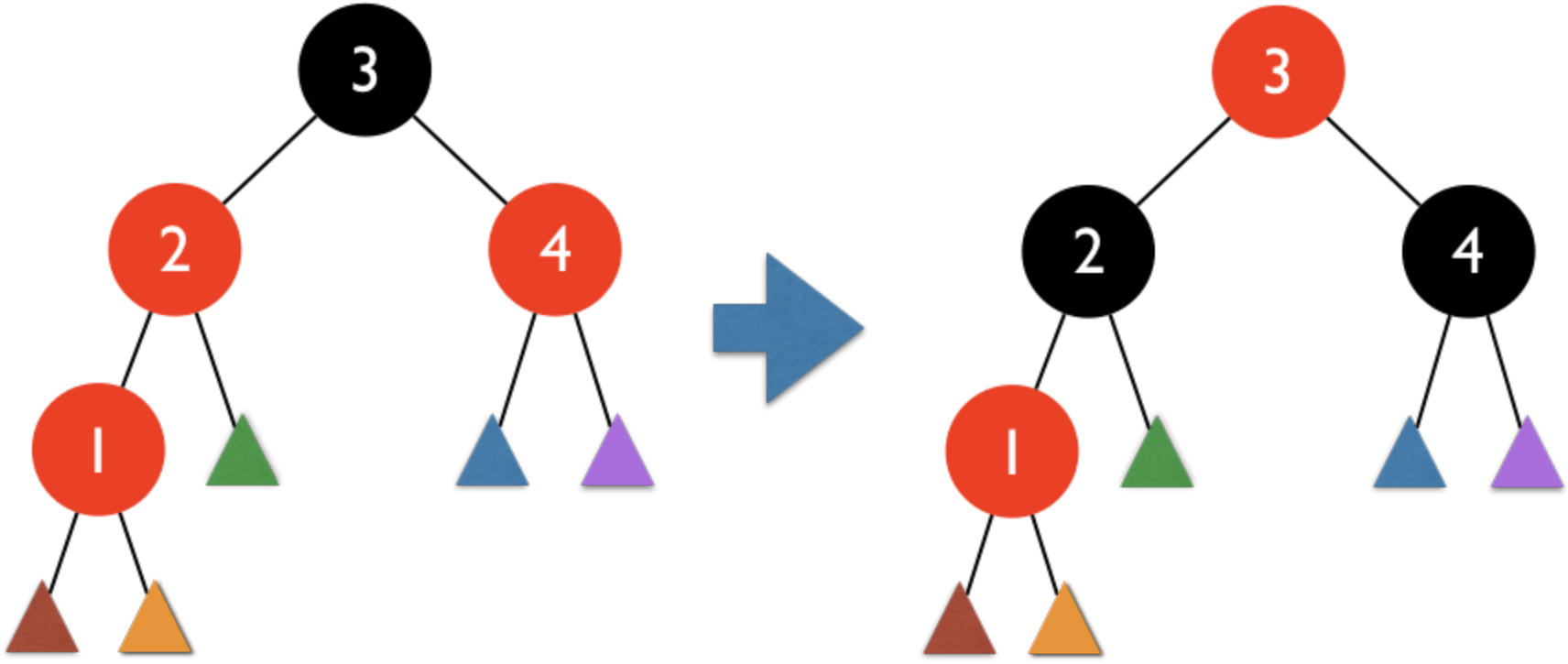
- What are the cases 1 - 4?

Red-Black Tree



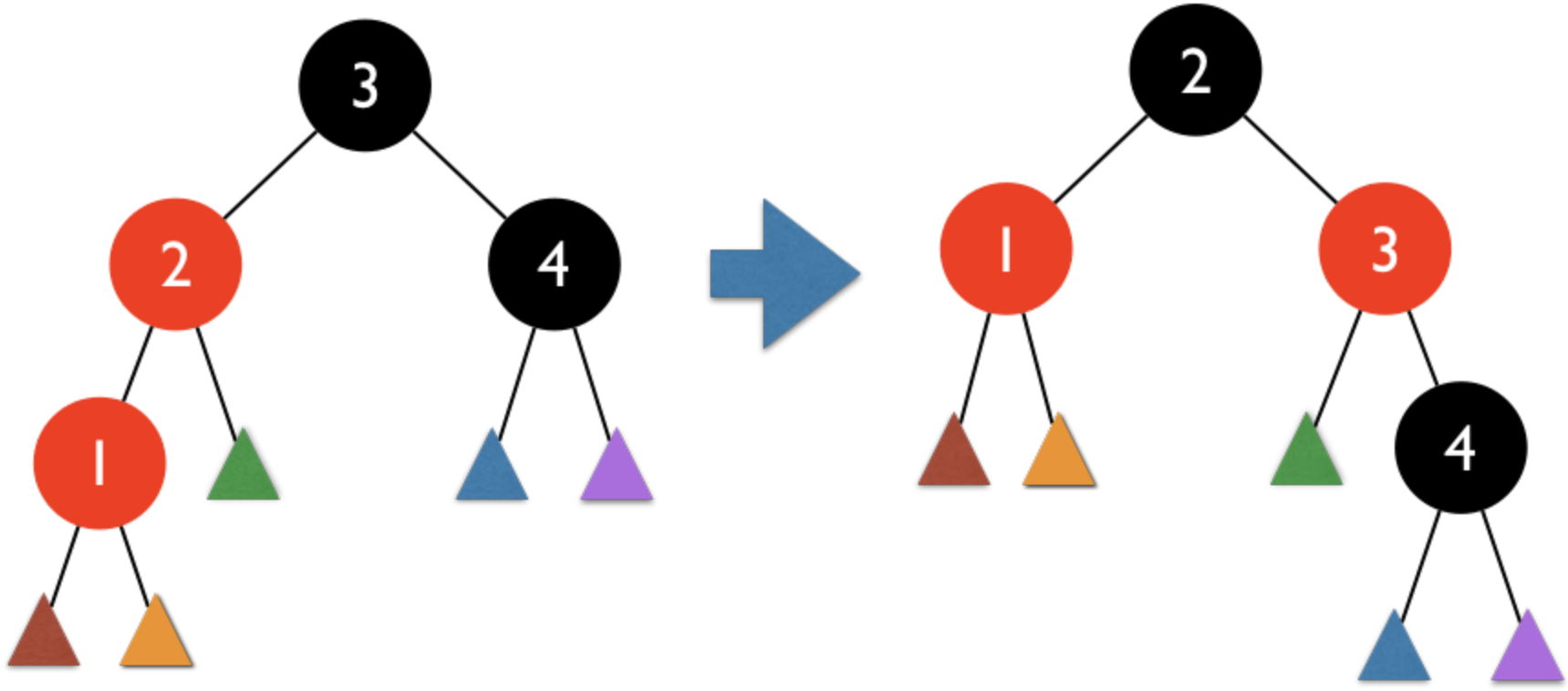
- Every node is either red or black.
- The root and all leaves (null) are black.
- Every red node must have two black child nodes.
- Every path from a node to any of its leaves must contain the same number of black nodes.

Red-Black Tree



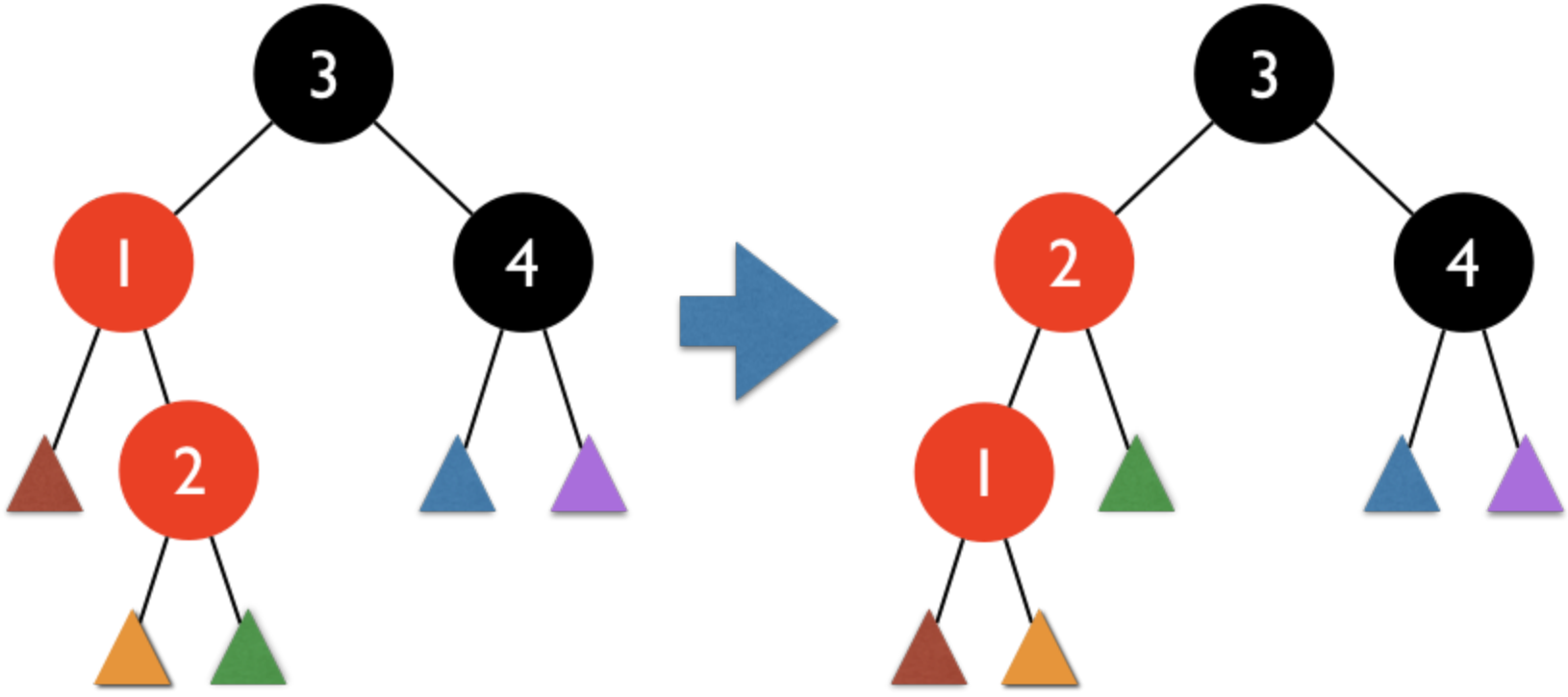
- Is the right tree a red-black tree?

Red-Black Tree



- Is the right tree a red-black tree?

Red-Black Tree



- Is the right tree a red-black tree?

Source: `RedBlackTree.java`.

```
public class RedBlackTree<T extends Comparable<T>>
             extends AbstractBalancedBinarySearchTree<T, RedBlackNode<T>> {
    public RedBlackNode<T> createNode(T key) {
        return new RedBlackNode<T>(key);
    }

    protected void balance(RedBlackNode<T> node) {
        if (isRoot(node))
            node.setToBlack();
        else if (node.getParent().isRed()) {
            RedBlackNode<T> uncle = node.getUncle();

            if (uncle != null && uncle.isRed())
                balanceWithRedUncle(node, uncle);
            else
                balanceWithBlackUncle(node);
        }
    }
```

```
private void balanceWithRedUncle(RedBlackNode<T> node, RedBlackNode<T> uncle) {
    node.getParent().setToBlack();
    uncle.setToBlack();
    RedBlackNode<T> grandParent = node.getGrandParent();
    grandParent.setToRed();
    balance(grandParent);
}
```

```
private void balanceWithBlackUncle(RedBlackNode<T> node) {
    RedBlackNode<T> grandParent = node.getGrandParent();

    if (grandParent != null) {
        RedBlackNode<T> parent = node.getParent();

        if (grandParent.isLeftChild(parent) && parent.isRightChild(node)) {  // case 1
            rotateLeft(parent);
            node = parent;
        }
        else if (grandParent.isRightChild(parent) && parent.isLeftChild(node)) {  // case 2
            rotateRight(parent);
            node = parent;
        }

        node.getParent().setToBlack();
        grandParent.setToRed();

        if (node.getParent().isLeftChild(node))        // case 3
            rotateRight(grandParent);
        else
            rotateLeft(grandParent);            // case 4
    }
}
```

- What are the cases 1 - 4?

References

- AVL Tree.
- Red-Black Tree.