Union (set theory)

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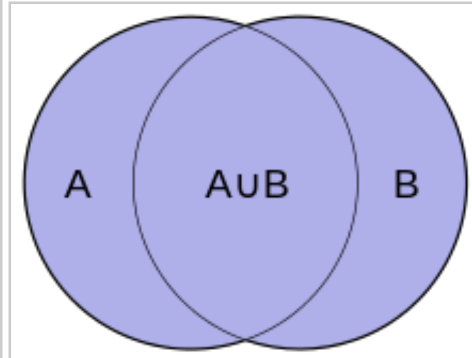
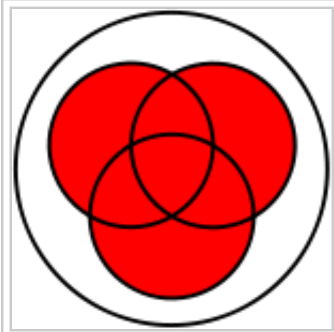
In set theory, the **union** (denoted by ∪) of a collection of sets is the set of all distinct elements in the collection.[1] It is one of the fundamental operations through which sets can be combined and related to each other.



Union of two sets:
$A \cup B$



Union of three sets:
$A \cup B \cup C$

Contents

Union of two sets

The union of two sets *A* and *B* is the set of elements which are in *A*, in *B*, or in both *A* and *B*. In symbols,

$$A \cup B = \{x : x \in A \text{ or } x \in B\}.$$

For example, if $A = \{1, 3, 5, 7\}$ and $B = \{1, 2, 4, 6\}$ then $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$. A more elaborate example (involving two infinite sets) is:

$A = \{x \text{ is an even integer larger than } 1\}$
$B = \{x \text{ is an odd integer larger than } 1\}$
$A \cup B = \{2, 3, 4, 5, 6, \dots\}$

Sets cannot have duplicate elements, so the union of the sets $\{1, 2, 3\}$ and $\{2, 3, 4\}$ is $\{1, 2, 3, 4\}$. Multiple occurrences of identical elements have no effect on the cardinality of a set or its contents.

The number 9 is *not* contained in the union of the set of prime numbers $\{2, 3, 5, 7, 11, \dots\}$ and the set of even numbers $\{2, 4, 6, 8, 10, \dots\}$, because 9 is neither prime nor even.

Algebraic properties

Binary union is an associative operation; that is,

$$A \cup (B \cup C) = (A \cup B) \cup C.$$

The operations can be performed in any order, and the parentheses may be omitted without ambiguity (i.e., either of the above can be expressed equivalently as $A \cup B \cup C$). Similarly, union is commutative, so the sets can be written in any order.

The empty set is an identity element for the operation of union. That is, $A \cup \varnothing = A$, for any set A.

These facts follow from analogous facts about logical disjunction.

Finite unions

One can take the union of several sets simultaneously. For example, the union of three sets A, B, and C contains all elements of A, all elements of B, and all elements of C, and nothing else. Thus, x is an element of $A \cup B \cup C$ if and only if x is in at least one of A, B, and C.

In mathematics a **finite union** means any union carried out on a finite number of sets: it doesn't imply that the union set is a finite set.

Arbitrary unions

The most general notion is the union of an arbitrary collection of sets, sometimes called an *infinitary union*. If $\mathbf{M}$ is a set whose elements are themselves sets, then x is an element of the union of $\mathbf{M}$ if and only if there is at least one element A of $\mathbf{M}$ such that x is an element of A. In symbols:

$$x \in \bigcup \mathbf{M} \iff \exists A \in \mathbf{M},\ x \in A.$$

That this union of $\mathbf{M}$ is a set no matter how large a set $\mathbf{M}$ itself might be, is the content of the axiom of union in axiomatic set theory.

This idea subsumes the preceding sections, in that (for example) $A \cup B \cup C$ is the union of the collection $\{A,B,C\}$. Also, if $\mathbf{M}$ is the empty collection, then the union of $\mathbf{M}$ is the empty set. The analogy between finite unions and logical disjunction extends to one between arbitrary unions and existential quantification.

Notations

The notation for the general concept can vary considerably. For a finite union of sets $S_1, S_2, S_3, \ldots, S_n$ one often writes $S_1 \cup S_2 \cup S_3 \cup \ldots \cup S_n$. Various common notations for arbitrary unions include $\bigcup \mathbf{M}$, $\bigcup_{A \in \mathbf{M}} A$, and $\bigcup_{i \in I} A_i$, the last of which refers to the union of the collection $\{A_i : i \in I\}$ where I is an index set and A_i is a set for every $i \in I$. In the case that the index set I is the set of natural numbers, one uses a notation $\bigcup_{i=1}^{\infty} A_i$ analogous to that of the infinite series. When formatting is difficult, this can also be written "$A_1 \cup A_2 \cup A_3 \cup \cdots$". (This last example, a union of countably many sets, is very common in analysis; for an example see the article on σ-algebras.)

Whenever the symbol "∪" is placed before other symbols instead of between them, it is of a larger size.

Union and intersection

Since sets with unions and intersections form a Boolean algebra, Intersection distributes over union:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C);$$

and union distributes over intersection:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

Within a given universal set, union can be written in terms of the operations of intersection and complement as

$$A \cup B = \left(A^C \cap B^C\right)^C$$

where the superscript ^{C} denotes the complement with respect to the universal set.

Arbitrary union and intersection also satisfy the law

$$\bigcup_{i \in I} \left(\bigcap_{j \in J} A_{i,j} \right) \subseteq \bigcap_{j \in J} \left(\bigcup_{i \in I} A_{i,j} \right).$$

See also

- Alternation (formal language theory), the union of sets of strings
- Cardinality
- Complement (set theory)
- Disjoint union
- Intersection (set theory)
- Iterated binary operation
- Naive set theory
- Symmetric difference

Notes

1. Weisstein, Eric W. "Union" (http://mathworld.wolfram.com/Union.html). Wolfram's Mathworld. Retrieved 2009-07-14.

External links

- Weisstein, Eric W., "Union" (http://mathworld.wolfram.com/Union.html), *MathWorld*.
- Hazewinkel, Michiel, ed. (2001), "Union of sets" (http://www.encyclopediaofmath.org/index.php?title=p/u095390), *Encyclopedia of Mathematics*, Springer, ISBN 978-1-55608-010-4
- Infinite Union and Intersection at ProvenMath (http://www.apronus.com/provenmath/sum.htm) De Morgan's laws formally proven from the axioms of set theory.

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