

Max Bethune-Waddell and Kenneth J. Chau

Applied Electromagnetics Laboratory, School of Engineering, The University of British Columbia Okanagan, Kelowna BC, Canada

## Introduction

There are different models for the electromagnetic momentum, stress, and force density inside a material (see Table 1). Some experiments have taken steps towards resolving their validity.

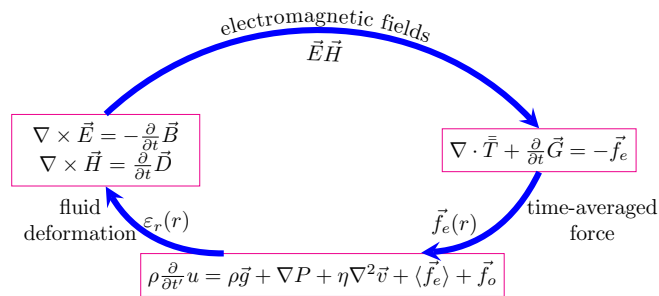
- The 1973 experiment by Ashkin and Dziedzic [1] showed that pulses directed onto an air-water interface caused it to bulge towards air (the lower index region) regardless of the illumination direction.
- A similar experiment by Casner et al. [2] showed that a beam directed onto a fluid-fluid interface caused it to bulge downwards, also towards the lower index region.

These observation suggest the Minkowski form but this has not been firmly established [3, 4].

We study radiation pressure on a fluid interface using a coupled electromagnetic and fluid dynamic simulator that solves Maxwell's equations, electromagnetic momentum continuity, and the Navier Stokes equation. Force densities of Abraham, Minkowski, and Einstein-Laub all predict fluid deformations consistent with experiment. Force densities of Amperian and Chu however, predict polarization-dependent deformation inconsistent with experiments.

## Methodology

Maxwell's equations are solved using FDTD. The force density is calculated from momentum continuity given the stress tensor  $\bar{T}$  and momentum density  $\vec{G}$  from the postulate sets listed in Table 1. Fluid deformation is modelled using the Navier Stokes equation.

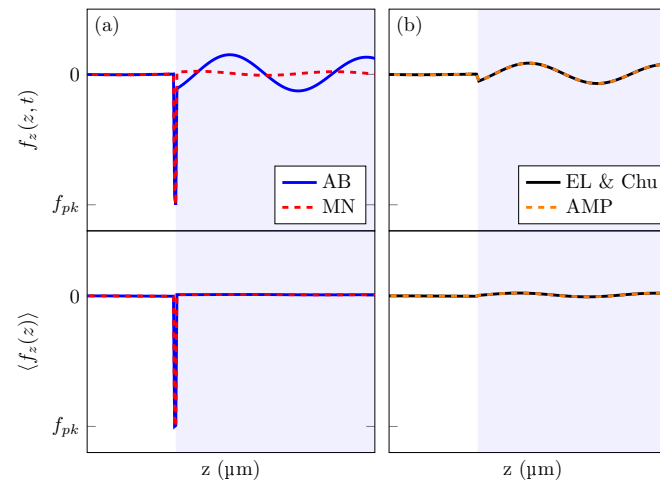


**Figure 1.** Simulation framework used to study radiation pressure in fluid media. Because  $t \ll t'$ , the time-averaged (as opposed to instantaneous) force density is used to calculate fluid deformation.

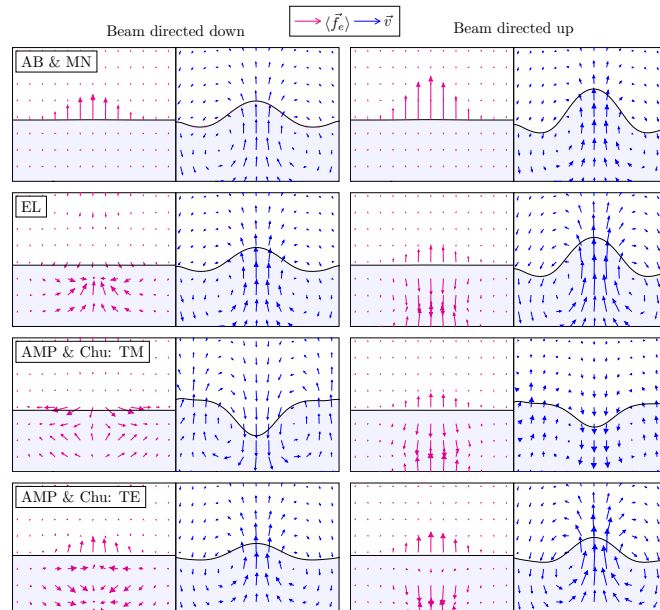
Form	$\vec{G}$	$\vec{S}$	$\bar{T}$
Minkowski	$\vec{D} \times \vec{B}$	$c^2 \vec{D} \times \vec{B}$	$(\vec{D} \cdot \vec{E} + \vec{B} \cdot \vec{H})\bar{I}/2 - \vec{D}\vec{E} - \vec{B}\vec{H}$
Abraham	$\vec{E} \times \vec{H}/c^2$	$\vec{E} \times \vec{H}$	$(\vec{D} \cdot \vec{E} + \vec{B} \cdot \vec{H})\bar{I}/2 - \vec{D}\vec{E} - \vec{B}\vec{H}$
Einstein-Laub	$\vec{E} \times \vec{H}/c^2$	$\vec{E} \times \vec{H}$	$(\epsilon_0 \vec{E}^2 + \mu_0 \vec{H}^2)\bar{I}/2 - \vec{D}\vec{E} - \vec{B}\vec{H}$
Amperian	$\epsilon_0 \vec{E} \times \vec{B}$	$\vec{E} \times \vec{B}/\mu_0$	$(\epsilon_0 \vec{E}^2 + \mu_0^{-1} \vec{B}^2)\bar{I}/2 - \epsilon_0 \vec{E}\vec{E} - \mu_0^{-1} \vec{B}\vec{B}$
Chu	$\vec{E} \times \vec{H}/c^2$	$\vec{E} \times \vec{H}$	$(\epsilon_0 \vec{E}^2 + \mu_0 \vec{H}^2)\bar{I}/2 - \epsilon_0 \vec{E}\vec{E} - \mu_0 \vec{H}\vec{H}$

**Table 1.** Five formulations of electrodynamics used here to model radiation pressure on fluid interfaces.

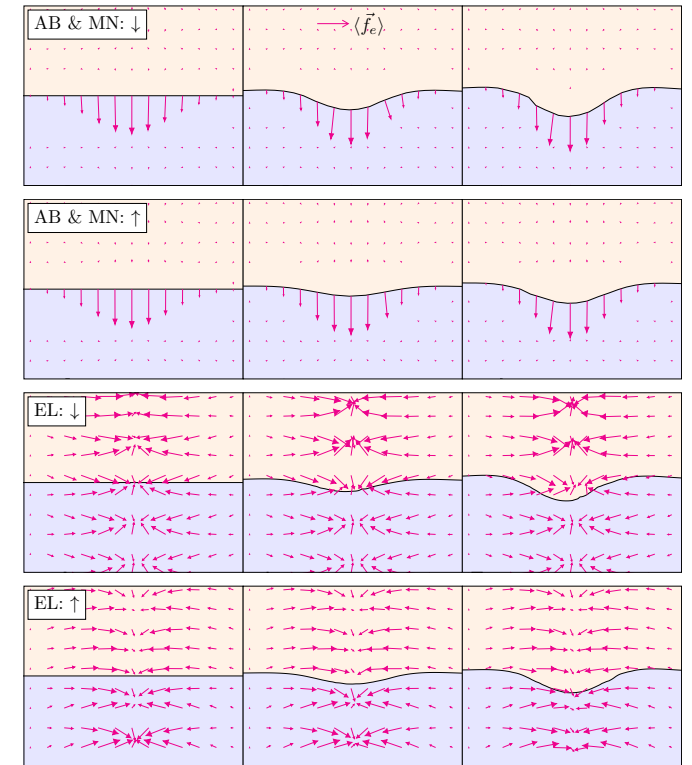
## Results



**Figure 3.** Degeneracy of the time-averaged force densities predicted by different formulation of electrodynamics. Here we compare the (top row) instantaneous and (bottom row) time-averaged force densities for the case of monochromatic wave of 500 nm wavelength incident from air into water.



**Figure 2.** Simulation of the Ashkin-Dziedzic experiment using five electrodynamic models for illumination from above and below. (magenta arrows) depict time-averaged force density and the (blue-arrows) depict the velocity field of the fluid.



**Figure 4.** Simulation of the Casner et al. experiment using the three models validated in Figure 2. Magenta arrows depict time-averaged force density.

## Conclusions

- Radiation pressure on fluid interfaces cannot distinguish between Abraham and Minkowski models.
- The Abraham, Minkowski, and Einstein-Laub models all correctly describe observations, but present different mechanisms for bulge formation. The first two rely interface forces, the latter relies on body forces.
- The Amperian and Chu models are inconsistent with experiments.

## References

- [1] Ashkin A and Dziedzic J M. Radiation Pressure on a Free Liquid Surface. *Phys. Rev. Lett.*, 30:139-142, 1973.
- [2] Casner A and Delville J-P. Laser-Induced Hydrodynamic Instability of Fluid Interfaces. *Phys. Rev. Lett.*, 90:144503, 2003.
- [3] Kemp B A. Resolution of the Abraham-Minkowski debate: Implications for the electromagnetic wave theory of light in matter. *J. Appl. Phys.*, 109:111101, 2011.
- [4] Astrath N G C, Malacarne L C, Baesso M L, Lukasiewicz G V B and Bialkowski S E. Unravelling the effects of radiation forces in water. *Nat. Commun.*, 5:023826, 2014.