Polynomials

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Outline

Meaning of a Polynomial

A **polynomial** is a mathematical expression that consists of variables, coefficients, and exponents combined using **addition (+)**, **subtraction (-)**, and **multiplication ($\times$)**, but not division by a variable. Each term in a polynomial has a variable raised to a **whole number power**. **Examples:**

$$2x + 3, \quad 4x^2 + 3x + 7, \quad 5a^3 - 2a^2 + a - 9$$

Not polynomials:

$$\frac{1}{x} + 3x, \quad 2x^{1/2} + 5$$

Components of a Polynomial

1. **Term:** Each part of the polynomial separated by $+$ or $-$.
   Example: In $3x^2 + 2x + 1$, the terms are $3x^2, 2x, 1$.
2. **Coefficient:** The numerical part of each term.
   Example: In $5x^2$, the coefficient is 5.
3. **Variable:** The letter that represents an unknown number.
   Example: In $5x^2$, the variable is x.
4. **Exponent (or Power):** The number that shows how many times the variable is multiplied by itself.
   Example: In x^3, the exponent is 3.

Degree of a Polynomial

The **degree** of a polynomial is the highest power of the variable in the expression.

$4x^3 + 2x + 1 \Rightarrow \text{Degree} = 3$

$7x^2 - 5x + 4 \Rightarrow \text{Degree} = 2$

$5x + 9 \Rightarrow \text{Degree} = 1$

$8 \Rightarrow \text{Degree} = 0$ (constant polynomial)

Evaluation of a Polynomial

The value of $P(x)$ at $x = a$ is denoted by $P(a)$ and is obtain by substituting a for x in the polynomial.

Example 1: If $P(x) = 2x^2 + 3x + 1$, find $P(2)$.

Solution:$P(2) = 2(2)^2 + 3(2) + 1 = 15$

Example 2: If $P(x) = 2x^3 + 5x^2 - 9x - 18$, find $P(-1)$.

Solution:$P(-1) = 2(-1)^3 + 5(-1)^2 - 9(-1) - 18$

$P(-1) = 2(-1) + 5(1) - 9(-1) - 18 = -2 + 5 + 9 - 18 = -6$

Try $P(1) = -20$ **and** $P(0) = -18$

Division in polynomials

Division in polynomials is the process of determining how many times one polynomial (the divisor) is contained in another (the dividend), and what remains after the division. It is similar to long division of numbers, except we work with algebraic terms.

$$f(x) = g(x)q(x) + r(x)$$

where:

- $f(x) =$ Dividend
- $g(x) =$ Divisor
- $q(x) =$ Quotient
- $r(x) =$ Remainder

Division in polynomials Cont'd

The process of dividing a polynomial say $f(x) = x^3 - 7x^2 + 3x - 1$ by a linear expression $x - 2$ is given by a long division as shown below.

$$
\begin{array}{r l}
 & x^2 - 5x - 7 \\
\cline{2-2}
x - 2 \big) & x^3 - 7x^2 + 3x - 1 \\
 & x^3 - 2x^2 \\
\cline{2-2}
 & \quad -5x^2 + 3x \\
 & \quad -5x^2 + 10x \\
\cline{2-2}
 & \qquad -7x - 1 \\
 & \qquad -7x + 14 \\
\cline{2-2}
 & \qquad\quad -15
\end{array}
$$

Division in polynomials Cont'd

This means that when $f(x)$ is divided by $x-2$, the remainder is -15 and the quotient is x^2-5x-7.
Therefore, the algebraic terms

$$f(x) = g(x)q(x) + r(x)$$

will be

$$f(x) = (x-2)(x^2-5x-7) + (-15)$$

Remainder Theorem: If $f(x)$ is divided by $(x-a)$, the remainder is $f(a)$.
Example 1: Find the remainder when $f(x) = x^3 - 7x^2 + 3x - 1$ is divided by a linear expression $x - 2$.
Solution: All we need to do is to evaluate $f(2)$.
$f(2) = (2)^3 - 7(2)^2 + 3(2) - 1$
$f(2) = 8 - 7(4) + 3(2) - 1 = 8 - 28 + 6 - 1 = -15$
Try: Find the remainder when $f(x) = (x+3)(x-2)(x+2)$ is divided by $x + 1$
Hint: All you need to do is evaluate f(-1). Answer is -6

Theorems in polynomials:Factor Theorem

Factor Theorem: If $f(x)$ is divided by $(x-a)$ and the remainder $f(a)=0$, then $(x-a)$ is a factor of $f(x)$.
Example 1: Factorize $f(x)=x^2-5x+6$
Solution: $f(x)=(x-2)(x-3)$
This means $(x-2)$ and $(x-3)$ are factors to $f(x)=x^2-5x+6$.
So, according to factor theorem: $f(2)=0$ and $f(3)=0$**(you can check)**

Example 2: Show that $x + 1$ is a factor of $f(x) = 2x^3 + 3x^2 - 5x - 6$

Solution: All you need to do according to factor theorem is to check if $f(-1) = 0$

$f(-1) = 2(-1)^3 + 3(-1)^2 - 5(-1) - 6$

$f(-1) = 2(-1) + 3(1) - 5(-1) - 6$

$f(-1) = -2 + 3 + 5 - 6 = 0$

Hence, $x + 1$ is a factor of $f(x) = 2x^3 + 3x^2 - 5x - 6$

Example 3: Factorize $f(x) = x^3 - 2x^2 - 5x + 6$

Solution: We will do a process called inspection. Because it is a third degree polynomial, it means $f(x) = (x \pm a)(x \pm b)(x \pm c)$.

Let assume the first factor $(x \pm a)$ is either $(x + 1)$ or $(x - 1)$

If $(x + 1)$ is a factor, then we check if $f(-1) = 0$. $f(-1)$ is not equal to zero. So $(x + 1)$ is not a factor

If $(x - 1)$ is a factor, then we check if $f(1) = 0$. $f(1)$ is equal to zero. So $(x - 1)$ is one of the factors.

Example 3 Cont'd: We will then use long division method to find the quotient.

$$
\begin{array}{r l}
 & x^2 - x - 6 \\
\cline{2-2}
x-1 \big) & x^3 - 2x^2 - 5x + 6 \\
 & x^3 - x^2 \\
\cline{2-2}
 & \quad -x^2 - 5x \\
 & \quad -x^2 + x \\
\cline{2-2}
 & \qquad\qquad -6x + 6 \\
 & \qquad\qquad -6x + 6 \\
\cline{2-2}
 & \qquad\qquad\qquad 0
\end{array}
$$

Example 3 Cont'd:
Therefore, $x^3 - 2x^2 - 5x + 6 = (x - 1)(x^2 - x - 6)$
Factorizing the quadratic equation $(x^2 - x - 6)$, we get
$x^2 - x - 6 = (x + 2)(x - 3)$. Hence,
$x^3 - 2x^2 - 5x + 6 = (x - 1)(x + 2)(x - 3)$.
That implies that $(x - 1), (x + 2)$ and $(x - 3)$ are factors of
$x^3 - 2x^2 - 5x + 6$ because $f(1) = 0$, $f(-2) = 0$ and $f(3) = 0$ according to factor theorem.

Thank You