Question Bank - 1

Discrete Mathematics

1. a) True

   b) False

2. a) WFF

   b) Not WFF

3. a. WFF

   b. Not WFF

4. a. True

   b. False

5. a. True

   b. False

6. a. False

   b. True

7. a. True

   b. True

8. p: It snows today

q: I will ski tomorrow.

converse : $q \to p$

I will ski tomorrow, only if it snows today

Contrapositive : $\sim q \to \sim p$

I will not ski tomorrow, if it doesn't snows today.

Inverse : $\sim p \to \sim q$

If it doesn't snows today, then i will not ski tomorrow

9. p: There is going to be a quiz.

q: I come to class

Converse : $q \to p$

If I come to class then there will be a quiz

contrapositive : $\sim q \to \sim p$

If I do not come to class then there will not be a quiz.

Inverse: $\sim p \to \sim q$

If there is not going to be a quiz, then I don't come to class

10. p: A positive integer is a prime.

q: Has no divisor other than 1 and itself.

converse: $q \to p$

A positive is a prime if it has no divisor other than 1 and itself.

contrapositive: $\sim q \to \sim p$

If a positive integer has divisor other than 1 and itsel

then it is not prime.

Inverse: $\sim p \to \sim q$

If a positive integer is not prime then it has a divisor other than 1 or itself.

11. a. True

b. False

12. a. True

b. False

13. a. True

b. True

14 a. True

b. True

15. a True

b. False

19. $(\neg p \wedge r) \wedge (\neg r \rightarrow (p \wedge q))$

20. opt. c. is not equivalent

21.

r	p	q	$r \rightarrow p$	$(r \rightarrow p) \rightarrow q$
0	0	0	0	
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

21

r	p	q	$r \to p$	$(r \to p) \to q$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	F
F	F	T	T	T
F	F	F	T	F

21

p	q	r	$r \to p$	$(r \to p) \to q$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	F
T	F	F	T	F
F	T	T	F	T
F	T	F	T	T
F	F	T	F	T
F	F	F	T	F

Opt d: Always true when q is true.

22. opt. d. II and III only

23. c and d.

24. $(\neg(\neg p \vee q) \vee \sim(\sim r \vee \sim s))$

$(p \wedge \sim q) \vee (r \wedge s)$

A/Q, $r \wedge s$ is False

$p \wedge \sim q$

Here only II is not q

$\therefore$ II satisfies the statement

25. option d.

26. I don't have even 1 perfect friend.

$\neg \exists x (F(x) \wedge P(x))$

27. $\exists x$: glitters$(x) \wedge \neg$ gold(x)

28. a.

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim q \to \sim p$	$(p \to q) \leftrightarrow (\neg q \to \sim p)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

b.

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

29. a.

p	q	$p \vee q$	$p \oplus q$	$(p \vee q) \to (p \oplus q)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

b.

p	q	$p \oplus q$	$\sim q$	$p \oplus \sim q$	$(p \oplus q) \to (p \oplus \sim q)$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	T	F	F	F
F	F	F	T	T	T

30. a. Jason is not rich or not happy.

b. Carlos will neither bicycle nor run tomorrow

31. a. Shakila neither walks nor takes the bus to class.

b. Ibrahim is ~~neither~~ smart ~~nor~~ not hard working.

24. a.

p	q	$p \wedge q$	$(p \wedge q) \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

$(p \wedge q) \to p$

$\sim(p \wedge q) \vee p$

$(\sim p \vee \sim q) \vee p$

$(\sim p \vee p) \vee (\sim q \vee p)$

$T \vee (\sim q \vee p)$

T

b.

p	q	$p \vee q$	$p \to p \vee q$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

$\sim p \vee (p \vee q)$

$(\sim p \vee p) \vee (p \vee q)$

$T \vee (\sim p \vee q)$

T

c.

p	q	~p	p→q	~p→(p→q)
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

$\sim p \to (p \to q)$

$\sim(\sim p) \vee (\sim p \vee q)$ Logical equivalence

$p \vee (\sim p \vee q)$ Negation law

$(p \vee \sim p) \vee (p \vee q)$ Distributive laws

$T \vee (p \vee q)$

T

26.

p	q	r	(p→q)	(q→r)	(p→r)	(p→q)∧(q→r)	(p→q)∧(q→r)→(p→r)
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Since last column is T for all condition so tautology

27. a.

p	q	p↔q	~(p↔q)	~q	p↔~q
T	T	T	F	F	F
T	F	F	T	T	T
F	T	F	T	F	T
F	F	T	F	T	F

∵ Truth values of ~(p↔q) and p↔~q is same

∴ They are logically equivalent.

28.

p	q	r	p→q	q→r	(p→r)∧(q→r)	pvq	(pvq)→r
T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	F
T	F	T	F	T	F	T	T
T	F	F	F	T	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	F
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	F

28.

p	q	r	p→q	q→r	(p→q)∧(q→r)	(pvq)	(pvq)→r
T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	F
T	F	T	F	T	F	T	T
T	F	F	F	T	T	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	F
F	F	T	T	T	T	F	T
F	F	F	T	T	T	F	T

28.

p	r	q	$p \to r$	$q \to r$	$(p \to q) \land (q \to r)$	$p \lor q$	$(p \lor q) \to r$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	F	F	T	F
T	F	F	F	T	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	F	T
F	F	T	T	F	F	T	F
F	F	F	T	T	T	F	T

29.

p	q	r	$p \to q$	$(p \to q) \to r$	$q \to r$	$p \to (q \to r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	F	T	T

∴ Not logically equivalent.

33. a) $p \land \sim q \land \sim r$

$\Downarrow$ DUAL

$p \lor \sim q \lor \sim r$

b) $(p \land q \land r) \lor s$

$\Downarrow$ DUAL

$(p \lor q \lor r) \land s$

c) $(p \lor F) \land (q \lor T)$

$\Downarrow$ DUAL

$(p \land T) \lor (q \land F)$

34. $(p \vee q) \rightarrow \neg r$

$\Rightarrow \neg(p \vee q) \vee \neg r$

$\Rightarrow (\neg p \wedge \neg q) \vee \neg r$

35. $(p \vee r) \wedge (p \leftrightarrow q)$

p	r	q	$(\sim p \rightarrow r) \wedge (p \leftrightarrow q)$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	F

DNF of $(\sim p \rightarrow r) \wedge (p \leftrightarrow q) \Rightarrow (p \wedge r \wedge q) \vee (p \wedge \sim r \wedge q) \vee (\sim p \wedge r \wedge \sim q)$

38.

p	q	r	$(p \land q) \lor (\sim p \land q \land r)$
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	F

$(\sim p \lor q \lor \sim r) \land (\sim p \lor q \lor r) \land (p \lor \sim q \lor r) \land (p \lor q \lor \sim r) \land (p \lor q \lor r)$

40. a. $\neg \exists x (P(x))$

b. $\neg \forall x (P(x))$

c. $\forall x (F(x) \to P(x))$

d. $\exists x (F(x) \land P(x))$

e. $\forall x F(x) \land \forall x P(x)$

f. $\neg \forall x F(x) \land (\exists x \neg P(x))$

41. p : Socrates is human

q : Socrates is mortal

$p \rightarrow q$

p

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q

This argument is valid because it is constructed by modus ponens which is a valid argument form. Thus, can conclude that a conclusion is true when the premises are true.

42. p : It is sunny this afternoon.

r : We will go swimming

q : It is colder than yesterday

s : We will take a canoe trip.

t : We will be home by sunset.

Hypothesis : $\sim p \wedge q$, $r \rightarrow p$, $\sim r \rightarrow s$, $s \rightarrow t$

Conclusion : t

$\sim p \wedge q$	Premise
$\sim p$	Simplification
$r \rightarrow p$	Premise
$\sim r$	Modus tollens
$\sim r \rightarrow s$	Premise
s	Modus ponens
$s \rightarrow t$	Premise
t	Modus Ponens

Rule of Inference	Name
p $p \to q$ $\therefore q$	Modus Ponens
$\sim q$ $p \to q$ $\therefore \sim p$	Modus Tollens
$p \to q$ $q \to r$ $\therefore p \to r$	Hypothetical syllogism
$\sim p$ $p \vee q$ $\therefore q$	Disjunctive Syllogism
p $\therefore (p \vee q)$	Addition
$(p \wedge q) \to r$ $\therefore p \to (q \to r)$	Exportation
$p \vee q$ $\sim p \vee r$ $\therefore q \vee r$	Resolution

43. p: You invest in the stock market

q: You get rich.

r: You are happy

$p \to q$ Premise

$q \to r$ Premise

$\therefore p \to r$ Hypothetical Syllogism

By the rule of inference, the argument is valid

44. p: Randy works hard

q: He is dull boy

r: ~~He is dull~~ He will ~~not~~ get the job

$p \to q$ Premise

q Modus ponen

$q \to \sim r$ Premise

$\sim r$ Modus ponens

Thus, by the rule of inference, we can conclude that the conclusion is true when the premises are true.

45. C- represents class

x - represents "is a student in this class"

W- know how to write of programs in JAVA

J- get a high paying job

Then the premises are: $C(\text{Danish})$, $W(\text{Danish})$, $\forall x(W(x) \rightarrow J(x))$

Conclusion: someone in this class can get a high-paying Job.

Proof: $\forall x(W(x) \rightarrow J(x))$, $C(\text{Danish})$, $W(\text{Danish})$ premises

From 1st premise $W \rightarrow J$ ~~Premise~~ By Universal

J Modus ponens

Now, from ~~2nd~~ premise and the above result

$C \wedge J$ Conjunction

$\exists x(C(x) \wedge J(x))$ By existential generalization

Conclusion is valid

16. a. Statement is not valid, as it asserts converse of the first statement, $q \rightarrow p$ which is not equivalent to the first statement, $p \rightarrow q$.

b. Statement is valid as it states the equivalent in the form of contrapositive of first statement.

c. Statement is not valid. It uses the fallacy of denying the hypothesis.

47. b

$\overset{(T)}{p} \to (\overset{(T)}{q} \to \overset{(T)}{r}) \to T$

$\overset{(T)}{q} \to (\overset{(F)}{r} \to \overset{(F)}{s}) \to T$

$\overset{(T)}{p} \wedge \overset{(T)}{q} \wedge \overset{(T)}{\sim s} \to T$

$\because$ r can't be both T and F

$\therefore$ System is inconsistent

18 $P(x)$: x is an integer

$R(x)$: x is a rational no.

$S(x)$: x is a power of 2

We need to show $\forall x\, [P(x) \to R(x)]$, $(\exists x)\, [P(x) \wedge S(x)] \to (\exists x)\, [R(x) \wedge S(x)]$

$\exists x\, [P(x) \wedge S(x)]$ Premise

$P(a) \wedge S(a)$

$P(a)$

$S(a)$

$(\forall x)\, [P(x) \to R(x)]$ Premise

$P(a) \to R(a)$

$R(a)$ Modus Ponens

$R(a) \wedge S(a)$

$(\exists x)\, [R(x) \wedge S(x)]$

$\therefore$ The argument is valid.

51. Proof by Contrapositive

The given statement is "If $3n+2$ is odd, n is odd".

The contrapositive of the given statement is "If n is even, then $3n+2$ is even".

Suppose n is even, if n is even, then we can write n as:

$n = 2k$, $k \in \mathbb{Z}$

$3n+2 = 6k+2$

The term $6k$ is always even and 2 is odd. Accordingly $6k+2$ is even. Thus $3n+2$ is even.

This implies that ~~that the statement~~ "if n is even then $3n+2$ is even" is true. Therefore, the contrapositive "If $3n+2$ is odd, then n is odd" is also true. Hence, proved.

53. Proof by contrapositive

The given statement is "If n^2 is odd, then n is odd

The contrapositive is "If n is even then n^2 is even

Suppose n is even, then we can write n as:

$n = 2k$, $k \in \mathbb{Z}$

$n^2 = 2(2k^2)$

let $r = 2k^2$

$n^2 = 2r$

$\because$ $2r$ is clearly an even no. , n^2 is even.

This implies, the statement, "if n is even, then n^2 is even" is true.
Therefore the contrapositive "if n^2 is odd, then n is odd" is also true.
Hence proved.

53. The given statement is "If n is odd, then n^2 is odd".

Proof: If n is odd, then $n = 2k+1$, for some integer K.

Thus $n^2 = (2k+1)^2 = 4k^2 + 1 + 4k = 2(2k^2 + 2k) + 1$.

Therefore n^2 is in the form $2j+1$ (j is the integer $2k^2+2k$), thus n^2 is odd.

This implies "If n is odd, then n^2 is odd". Hence, proved.

54. The given statement is "$\sqrt{2}$ is an irrational number".

Proof by contradiction.

The contradiction of given statement is "$\sqrt{2}$ is a rational no.

So, it can be expressed in the form p/q, where p, q are co-primes integers.

$\sqrt{2} = p/q$

Solving:

$\sqrt{2} = p/q$

On squaring both side we get

$2 = \left(p/q\right)^2$

$2q^2 = p^2$

$\frac{p^2}{2} = q^2$

So 2 divides p and p is a multiple of 2

$\Rightarrow p = 2m$

$p^2 = 4m^2$

$2q^2 = 4m^2$

$q^2 = 2m^2$

q^2 is a multiple of 2

$\Rightarrow$ q is a multiple of 2

Hence, p, q have a common factor 2. This contradicts our assumption that they are co-primes. Therefore p/q is not a rational no..

This implies "$\sqrt{2}$ is an irrational no." Hence, proved.

55. Proof by contradiction.

Suppose $3n+2$ is odd, Assume n is even.

If n is even we can write n as:

$n=2k$

$\therefore 3n+2 = 3(2k)+2$

$= 6k+2$

$= 2(3k+1)$

This $3n+2$ is in the form of $2r$ (where r is $3k+1$)

This implies $3n+2$ is even. But this is contradictory. Our assumption that "n is even" is incorrect. Hence "n is even" is false. This implies n must be odd. This proves that if $3n+2$ is odd. Then n is odd $\forall n \in \mathbb{Z}$.

Question Bank 2

6. No of Permutation $= 6! = 720$

11. $6P3 = 6 \times 5 \times 4 = 120$

12. $7! = 5040$

13. b) $52 \times 52 \times 52 = 140,608$

a) $= 52C_3 = \frac{52 \times 51 \times 50}{3 \times 2} = 22100$

14 $^4C_3 = 4$

15 $^6C_3 \times {}^5C_2 \times {}^8C_4 = \frac{6!}{3!3!} \times \frac{5!}{2! \times 3!} \times \frac{8!}{4! \times 4!} = 20 \times 10 \times 70 = 14000$ choices

16. Pairs are : 1:9, 2:8, 3:7, 4:6, 5:5 $\Rightarrow 5$

Pigeonholes are $= 5+1 = 6$

17 Here, $n = 12$ months is the pigeonhole

$k+1 = 3$

$\Rightarrow k = 2$

So $kn + 1$

$2 \times 12 + 1 = 25$