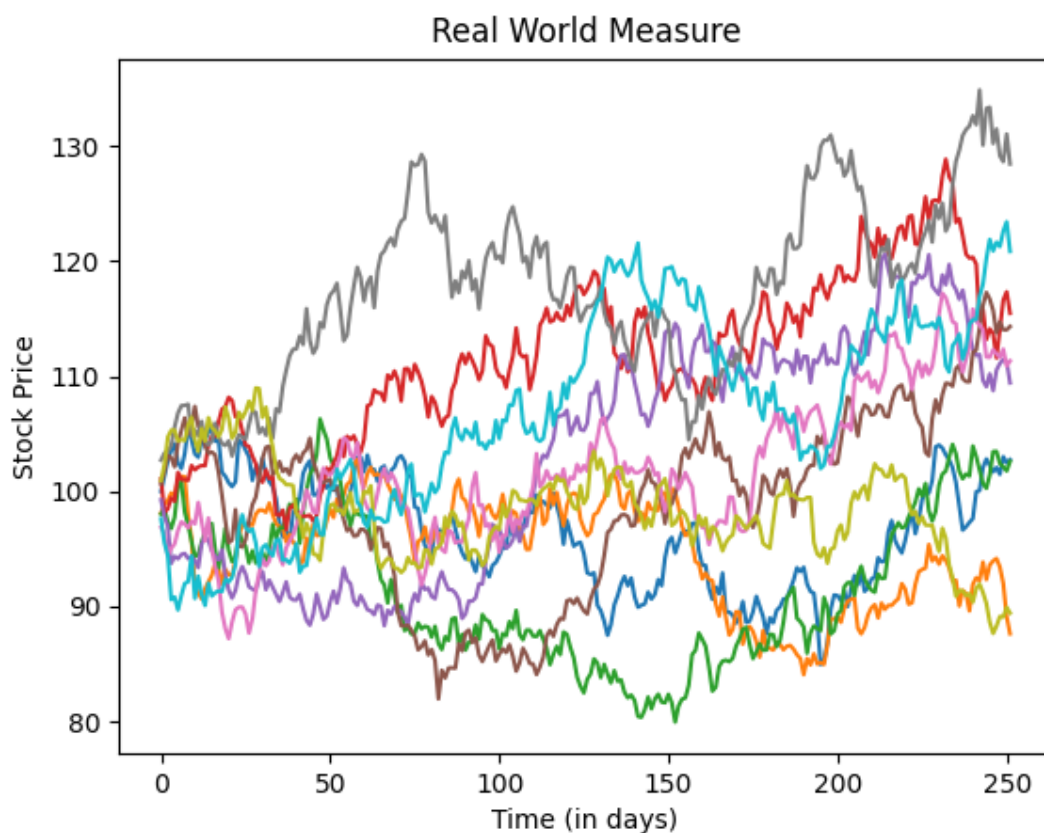# MA-374 Lab-10

Name- Dev Sandip Shah

Roll No.- 200123074

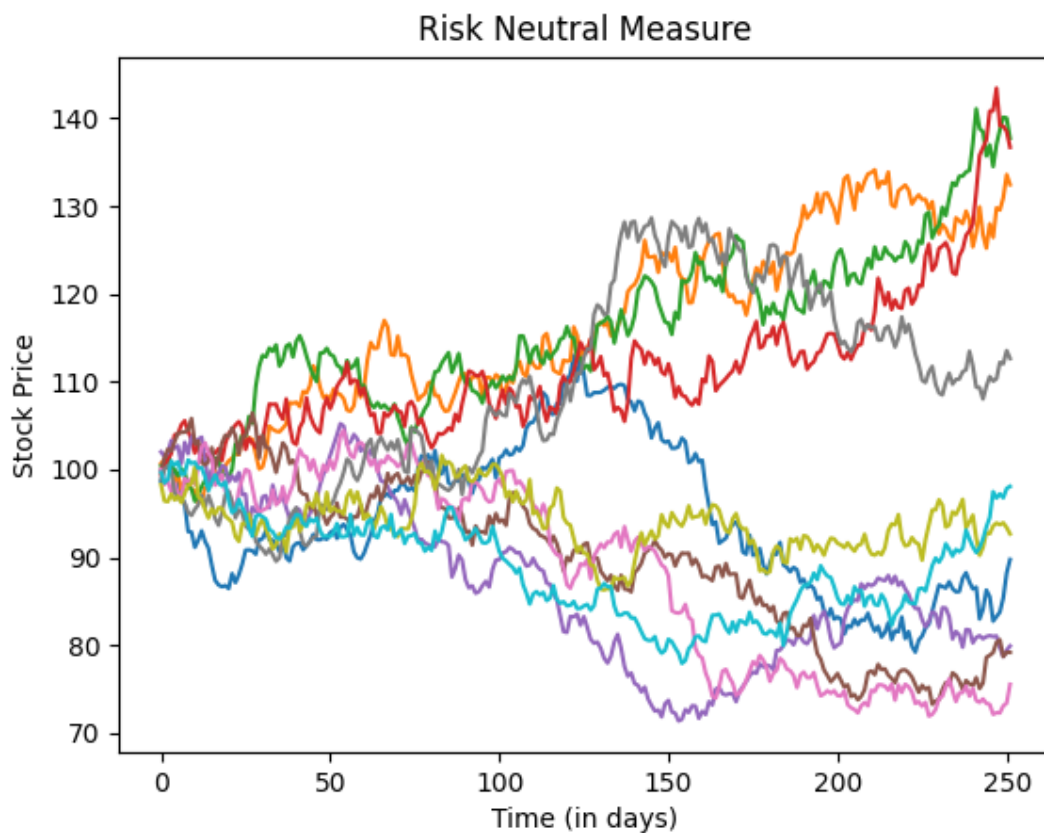## Question 1:

i) 10 different paths of the asset price making use of GBM in the real world is

Real World Measure

The evolution of the asset price in the real world is governed by the following differential equation:

$$dS = \mu S dt + \sigma S dW(t)$$

ii) 10 different paths of the asset price making use of GBM in the risk-neutral world is:

The evolution of the asset price in the risk-neutral world is governed by the following differential equation:

$$dS = \mu S dt + \sigma S d\widehat{W}(t)$$

where, $\widehat{W}$ is Brownian motion under risk-neutral probability.

The prices of a six-month fixed-strike Asian option with various strike prices are

```
PS C:\Users\Dev Sandip Shah\IITG\Sem6\FE Lab\Lab-10> python Q1.py
For K =  90
Asian Call Option price:  13.488681292319177
Variance in Asian Call Option price:  157.404877611654

Asian Put Option price:  0.6488453945431237
Variance in Asian Put Option price:  3.4201368936462866


For K =  105
Asian Call Option price:  3.970333622230726
Variance in Asian Call Option price:  43.69617051867698

Asian Put Option price:  5.451834503903502
Variance in Asian Put Option price:  47.98496219747079


For K =  110
Asian Call Option price:  1.009094782643702
Variance in Asian Call Option price:  7.426643227521371

Asian Put Option price:  10.459366934139858
Variance in Asian Put Option price:  79.47283929080434
```
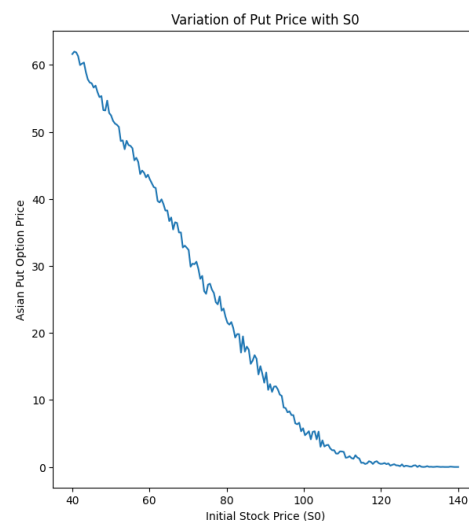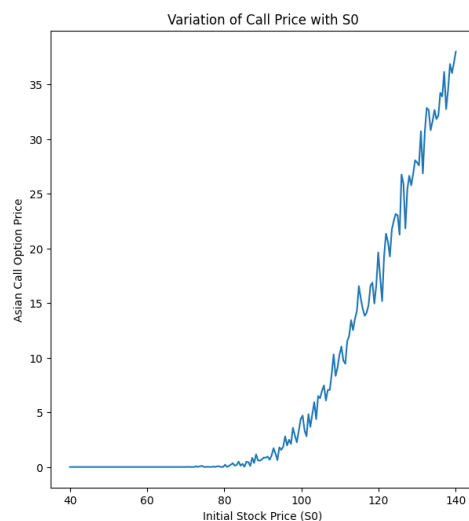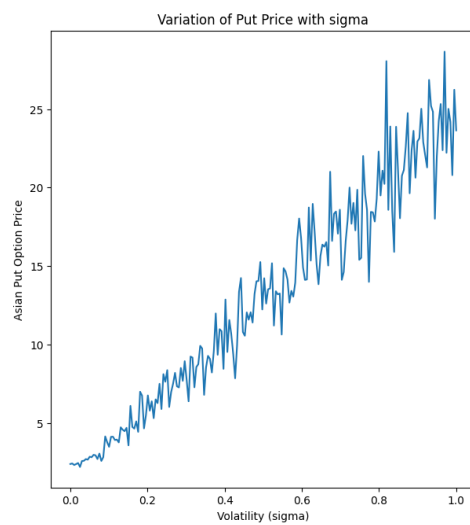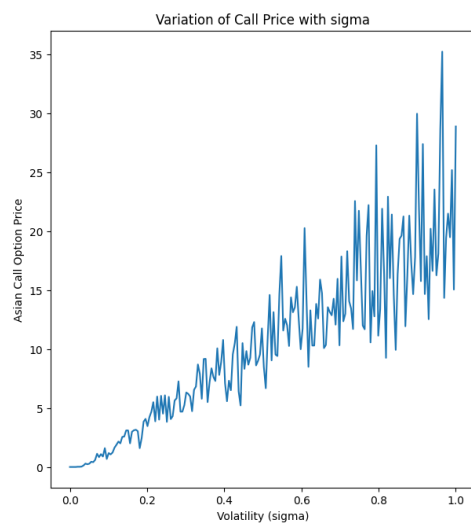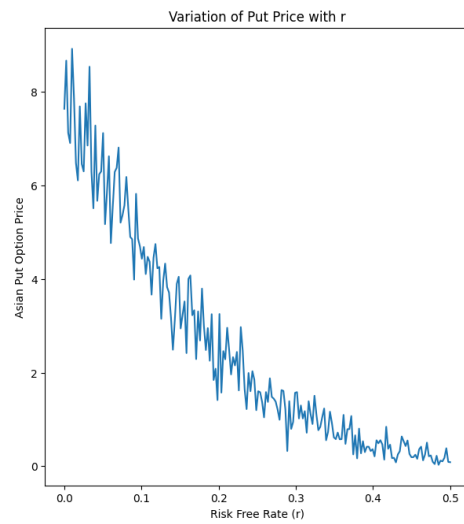
Sensitivity Analysis-

Variation of Call Price with K

Variation of Put Price with K

Variation of Call Price with r

Variation of Put Price with r

Variation of Call Price with sigma

Variation of Put Price with sigma
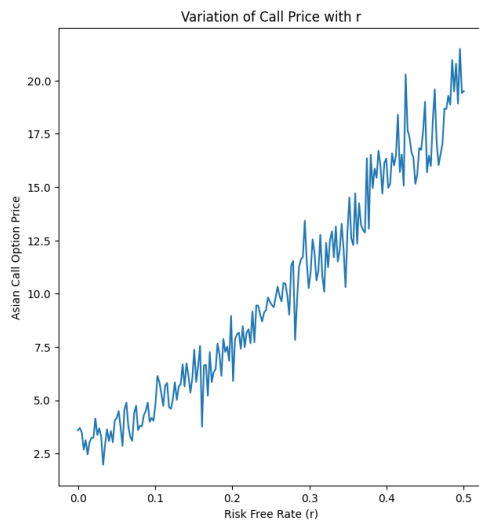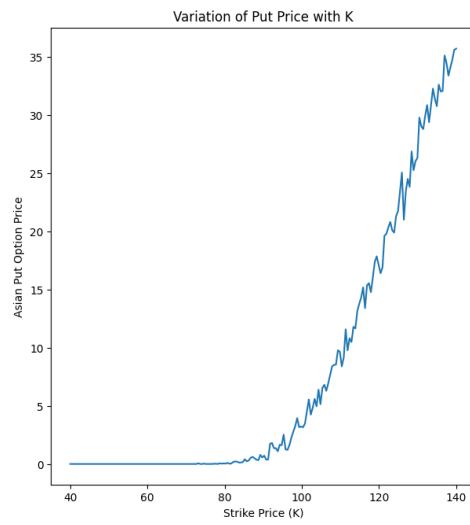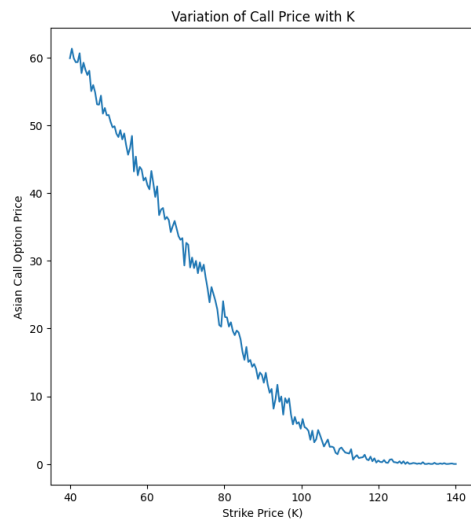
Observations:

1. The price of the call option increases while that of the put option decreases, with an increase in the initial asset price, S0.

2. The price of the call option decreases while that of the put option increases, with an increase in the strike prices, K.

3. The price of the call option increases while that of the put option decreases, with an increase in the risk-free interest, r

4. The price of both call and put options increases with an increase in volatility.

5. There appear to be some fluctuations in the plots, which we try to minimize using the variance reduction schemes in the next question.

Question 2:

The prices of a six-month fixed-strike Asian option with various strike prices, after performing variance reduction, are:

```
PS C:\Users\Dev Sandip Shah\IITG\Sem6\FE Lab\Lab-10> python Q2.py
For K =  90
Asian Call Option price:  13.551551129349685
Variance in Asian Call Option price:  99.9872791134783

Asian Put Option price:  1.0388729391220033
Variance in Asian Put Option price:  6.47527687837578


For K =  105
Asian Call Option price:  4.751138214060936
Variance in Asian Call Option price:  46.24595585335243

Asian Put Option price:  4.646533730744174
Variance in Asian Put Option price:  29.367568192857142


For K =  110
Asian Call Option price:  2.4483827340103783
Variance in Asian Call Option price:  24.833757971328026

Asian Put Option price:  8.929152460580422
Variance in Asian Put Option price:  67.9737257326362
```

<u>Observations</u> –

The price of both call and put options obtained using both with and without variance reduction are comparable. The respective variances are compared in the following table:
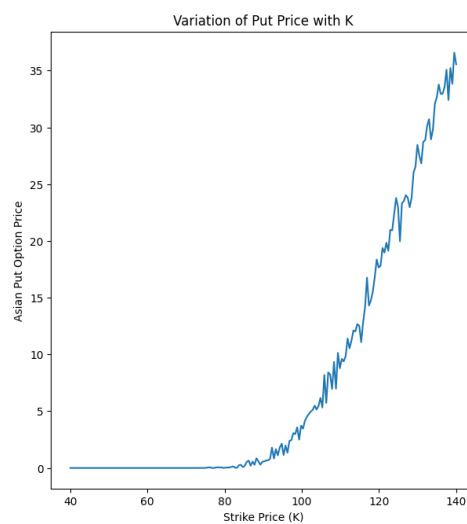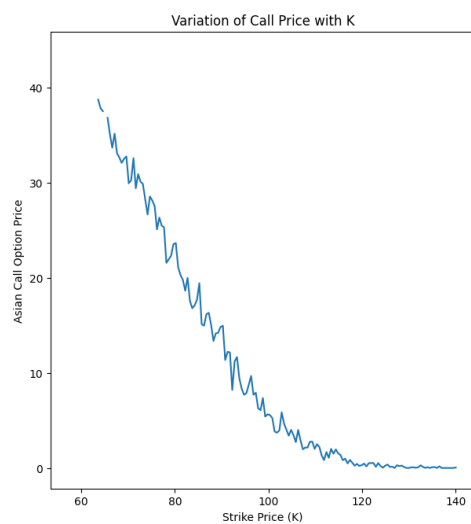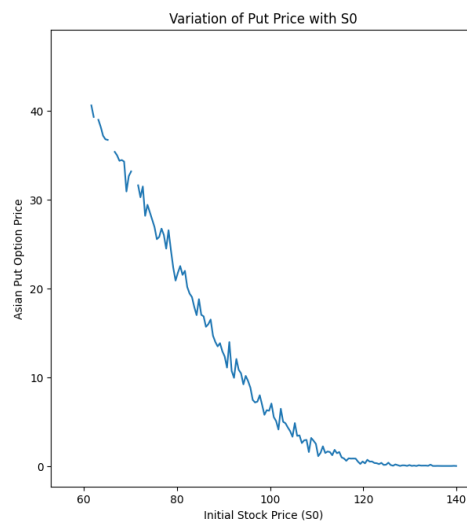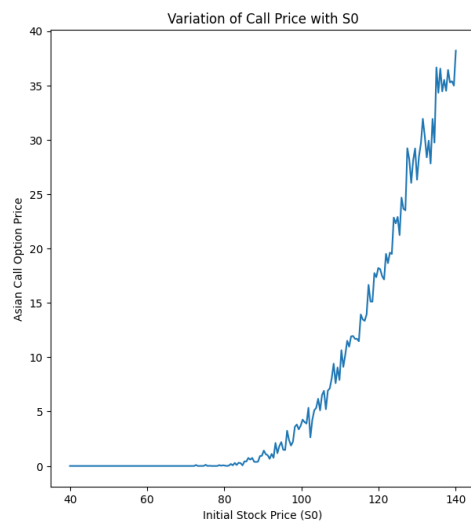
<u>For Call Option</u>:

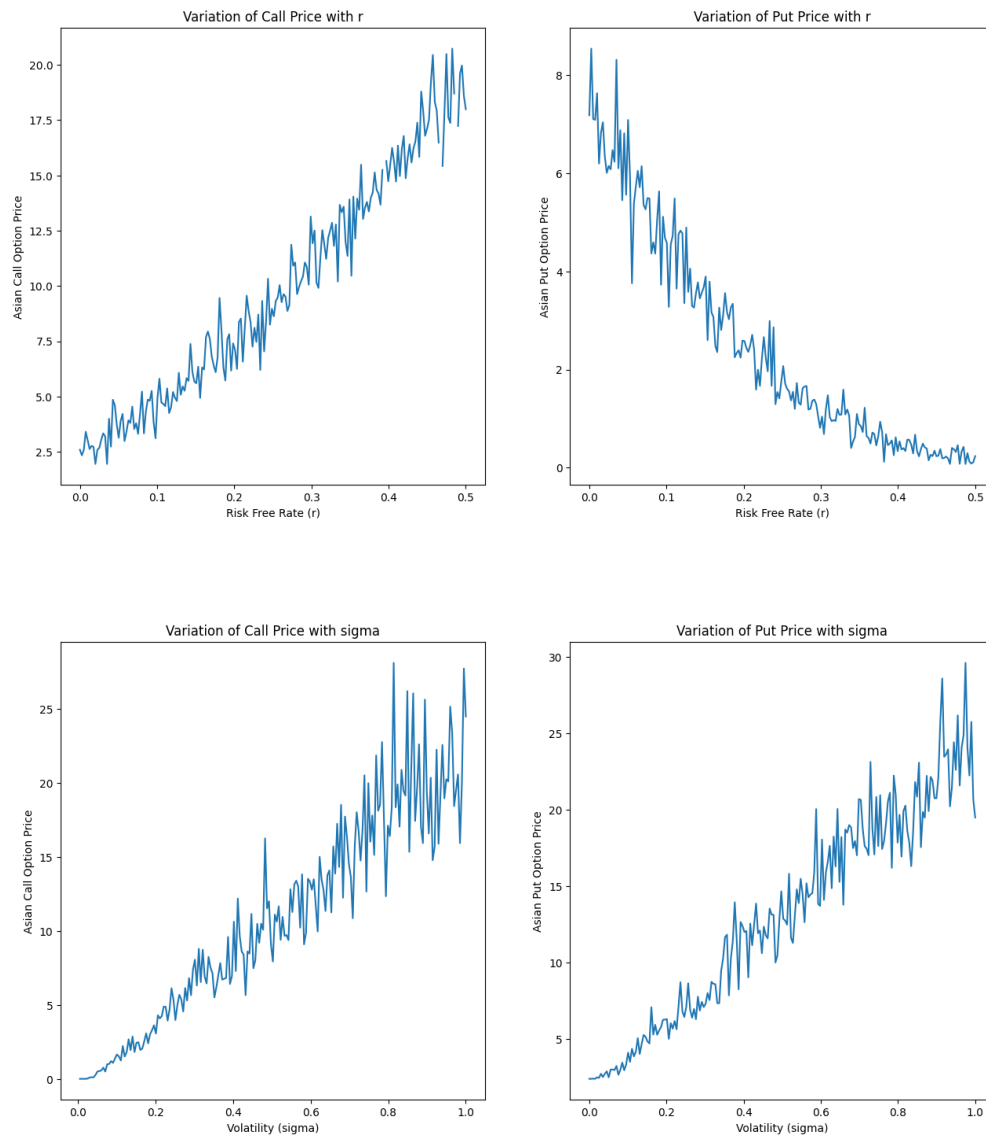| Sr No.	Strike Price	Variance (without Reduction)	Variance (with Reduction)
1.	90	120.3495560839533	80.03017841066529
2.	105	70.4942939544021	37.7690382996848
3.	110	33.91978507164524	13.72475570281891

<u>For Put Option</u>:

Sr No.	Strike Price	Variance (without Reduction)	Variance (with Reduction)
1.	90	4.075186729940523	3.275267181307292
2.	105	52.86539053443688	42.7343038123033
3.	110	69.92061191652613	58.3401369895063

So, we can clearly observe that the variance reduction is successful, and we have reduced the variance in calculating the option prices.

Sensitivity Analysis after performing Variance Reduction-

Variation of Call Price with r   Variation of Put Price with r

Variation of Call Price with sigma   Variation of Put Price with sigma

Observations:

1. Earlier, we quantitatively demonstrated that the variance reduction was achieved. The constructed plots even more support this claim.

2. On careful analysis, the plot fluctuations seem to be less than when variance reduction was not applied. So, the scheme achieves its goal.

3. The nature of the plots is consistent with our expectations, which is explained in the last question