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Task # 1

$\omega_n = 72 \qquad \zeta = 0.5 \qquad J = 1 \qquad a = 1$

$$TF_{\text{Small F.B}} = \frac{\frac{K}{Js+a}}{1+\frac{KK_f}{Js+a}} = \frac{K}{Js+a+KK_f}$$

$$TF_{\text{cascade}} = \frac{K}{Js+a+KK_f} \times \frac{1}{s} = \frac{K}{Js^2+(a+KK_f)s}$$

$$TF = \frac{\frac{K}{Js^2+(a+KK_f)s}}{1+\frac{K}{Js^2+(a+KK_f)s}} = \frac{K}{Js^2+(a+KK_f)s+K}$$

$$\frac{K}{Js^2+(a+KK_f)s+K} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$K = \omega_n^2 = 72^2 = 5184$$

$$a+KK_f = 2\zeta\omega_n$$

$$1+5184K_f = 2\times 0.5\times 72 \longrightarrow K_f = \frac{71}{5184}$$

Task 2

$$G(s) = \frac{5184}{s^2+72s+8836} \times \frac{0.21}{s^2} = \frac{1088.64}{s^4+72s^3+5184s^2}$$

$$T.F = \frac{G}{1+GH} = \frac{\frac{1088.64}{s^4+72s^3+5184s^2}}{1+\frac{1088.64}{s^4+72s^3+5184s^2}} = \frac{1088.64}{s^4+72s^3+5184s^2+1088.64}$$

Task 10

$w_n = 7.2 \quad \zeta = 0.5$

$$t_r = \frac{\pi - \cos^{-1}\zeta}{w_n\sqrt{1-\zeta^2}} = \frac{\pi - \cos^{-1} 0.5}{72\sqrt{1-0.5^2}} = 0.0336 \text{ sec}$$

$$t_p = \frac{\pi}{w_n\sqrt{1-\zeta^2}} = \frac{\pi}{72\sqrt{1-0.5^2}} = 0.0504 \text{ sec}$$

$$t_s(2\%) = \frac{4}{\zeta w_n} = \frac{4}{0.5\times 72} = \frac{1}{9} \text{ sec} \qquad t_s(5\%) = \frac{3}{0.5\times 72} = \frac{1}{12} \text{ sec}$$

$$M_p = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} = e^{\frac{-\pi\times 0.5}{\sqrt{1-0.5^2}}} = 0.163 = 16.3\%$$

③ $e_{ss\ ramp} = \frac{1}{K_v}$

$$K_v = \lim_{s\to 0} s\, G(s)H(s) = \lim_{s\to 0} s \times \frac{1088.64}{s^4 + 72s^3 + 5184s^2 + 1088.64} = \infty$$

$$e_{ss_{ramp}} = \frac{1}{\infty} = 0$$

④ Use PD controller $\quad G_c(s) = K_p + K_d s$

Char eqn $\quad 1 + G_c(s)\, G(s)H(s) = 0$

$$1 + (K_p + K_d s) \times \frac{1088.64}{s^4 + 72s^3 + 5184s^2} = 0$$

$$s^4 + 72s^3 + 5184s^2 + 1088.64 K_d s + 1088.64 K_p = 0$$

$$a + b + 2\zeta w_n = 72$$
$$ab + (a+b)2\zeta w_n + w_n^2 = 5184$$
$$2\zeta w_n ab + (a+b) w_n^2 = 1088.64 K_d$$
$$ab w_n^2 = 1088.64 K_p$$

use conditions

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$0.2 = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$\zeta = 0.456$$

$$t_s\ (5\%) = \frac{3}{\zeta w_n}$$

$$2.5 = \frac{3}{0.456\, w_n}$$

$$w_n = 2.63$$

Subs above (first 2 equ.):

$$a + b + 2 \times 0.456 \times 2.63 = 72$$

$$a + b + 2.4 = 72$$

$$a + b = 69.6$$

$$ab + (69.6)(2.4) + (2.63)^2 = 5184$$

$$ab + 173.9569 = 5184$$

$$ab = 5010.0431$$

In last 2 equ.:

$$2.4 \times 5010.0431 + 69.6 \times (2.63)^2 = 1088.64 K_d$$

$$K_d = 11.49$$

$$5010.0431\ (2.63)^2 = 1088.64\, K_p$$

$$K_p = 31.83$$

After tuning

$K_p,\ K_v = \infty$ $\qquad e_{ramp} = 0$

$e_{step} = 0$

$$K_a = \lim_{s \to 0} s^2 (31.83 + 18.6s) \times \frac{1088.64}{s^2(s^2 + 72s + 5184)}$$

$$K_a = 6.6843 \qquad e_{ss} = \frac{1}{K_a} = \frac{1}{6.6843} = 0.1496 = 14.96\%$$

Task 17

Lead :

$$G = \frac{1088.64}{s^2(s^2+72s+5184)} \quad , \zeta = 0.456 \;\&\; \omega_n = 2.63$$

closed loop poles : $-\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} = -1.19928 \pm j2.3406$

$$G(s) \quad at \quad s = -1.19928 \pm j2.3406 = 0.03 \angle 123.91^\circ$$

$$\Phi = 180 - 123.91 = 56.09^\circ$$

$$G_{lead} = \frac{K_c(s+1.35)}{(s+5.15)}$$

$$|G \times G_{lead}| = 1$$

$$\left| \frac{1088.64}{s^2(s^2+72s+5184)} \times \frac{K_c(s+1.35)}{(s+5.15)} \right| = 1$$

@ $s = -1.19928 \pm j2.3406$

$$1088.64\, K_c\, (1.449 \times 10^5) = 1$$

$$K_c = \frac{1}{1088.64\,(1.449\times10^5)} = 63.4$$

$$G_{lead} = \frac{63.4(s+1.35)}{s+5.15}$$

$$K_a^{old} = \lim_{s\to 0} s^2 G_{lead} GH = s^2 \times \frac{1088.64}{s^2(s^2+72s+5184)} \times \frac{63.4(s+1.35)}{(s+5.15)} = 3.49$$

$$e_{ss} = \frac{1}{K_a^{old}} = \frac{1}{3.49} = 0.2865 = 28.65\%$$

Lag : (Decrease error to be within 1%)

$$K_a^{new} = 100$$

$$\beta = \frac{K_a^{new}}{K_a^{old}} = \frac{100}{3.49} = 28.653 \simeq 29$$

Assume $\frac{1}{T} = 0.029$

$$\frac{1/T}{\beta} = \frac{0.029}{29} = 0.001$$

$$G_{lag} = \frac{(s+0.029)}{s+0.001}$$