Problem:

* A unity Feed back control system has a feed forward transfer function :

$$G(s) = \frac{3}{s(s+2)}$$

- It's required to:

(1) Find the closed loop transfer function.

(2) Find the rise time, peak time, settling time, and maximum peak for a unit step input.

(3) Get the steady state error for a unit ramp input.

(4) Design a Controller in order to get a maximum peak of 10% , $t_s = 3$ sec.

(5) Design a Controller in order to get a settling time of 2sec. and steady state error of 10% due to a parabolic input.

* Solution:

(1) Unity Feedback $\Rightarrow H(s)=1$, $\because G(s)=\dfrac{3}{s(s+2)}$

$$\therefore T.F. = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{3}{s(s+2)}}{1+\frac{3}{s(s+2)}*1} = \boxed{\frac{3}{s^2+2s+3}} .$$

(2) The c/ch eq.: $1+G(s)H(s)=0$ (denominator of T.F.)

$\therefore s^2+2s+3=0$, Comparing with 2^{nd} order prototype

$\therefore s^2+2\eta\omega_n s+\omega_n^2=0$

$\therefore \omega_n=\sqrt{3}$ rad/sec , $\eta=\frac{1}{\sqrt{3}}$

$$\therefore t_r = \frac{\pi-\theta}{\omega} = \frac{\pi-\cos^{-1}\eta}{\omega_n\sqrt{1-\eta^2}} \Rightarrow \boxed{t_r = 1.545 \text{ sec}}$$

$$, t_p = \frac{\pi}{\omega} = \frac{\pi}{\omega_n\sqrt{1-\eta^2}} \Rightarrow \boxed{t_p = 2.221 \text{ sec}}$$

$$, t_s = \frac{4}{\eta\omega_n} \Rightarrow \boxed{t_s = 4 \text{ sec}}$$

$$, M_p = e^{\frac{-\eta\pi}{\sqrt{1-\eta^2}}} \Rightarrow \boxed{M_p = 0.1084 \equiv 10.84\%}$$

(3) $\because \left. e_{ss} \right|_{\text{ramp i/p}} = \frac{1}{k_v}$

$\because k_v = \lim_{s \to 0} s\,G(s)\,H(s) = \lim_{s \to 0} \frac{3\cancel{s}}{\cancel{s}(s+2)} = \frac{3}{2} = 1.5$

$\therefore \boxed{e_{ss} = \frac{2}{3} \equiv 66.66\%}$

(4) There is a condition on the Maximum peak (transient response) $\Rightarrow$ use PD Controller

$\therefore G_c(s) = k_p + k_d s$.

$\therefore$ The c/ch eq : $1 + G_c(s) \cdot G(s) \cdot H(s) = 0$

$\Rightarrow 1 + (k_p + k_d s) * \frac{3}{s(s+2)} * 1 = 0$

$\Rightarrow (s^2 + 2s) + 3(k_p + k_d s) = 0$

$\therefore s^2 + (2 + 3k_d)\,s + 3k_p = 0$

Comparing with the 2nd order proto type system:

$s^2 + 2\eta\omega_n s + \omega_n^2 = 0$

$\therefore \quad 2 + 3k_d = 2\eta\omega_n$

$, \quad 3k_p = \omega_n^2$

$\Rightarrow$ 2 equations, 4 unknowns: k_p, k_d, η, ω_n

$\Rightarrow$ Use the conditions:

- $M_p = e^{\frac{-\eta\pi}{\sqrt{1-\eta^2}}} = 10\% \quad \Rightarrow \quad \frac{-\eta\pi}{\sqrt{1-\eta^2}} = \ln(0.1) = -2.302$

$\therefore \quad \eta\pi = 2.302\sqrt{1-\eta^2} \quad \Rightarrow \quad \eta^2\pi^2 = (2.302)^2(1-\eta^2)$

$\therefore \quad [\pi^2 + (2.302)^2]\,\eta^2 = (2.302)^2 \quad \Rightarrow \quad \eta = 0.5911$

- $t_s = \frac{4}{\eta\omega_n} = 3 \Rightarrow \eta\omega_n = \frac{4}{3} \quad \Rightarrow \quad \omega_n = \frac{4}{3*\eta}$

$\therefore \quad \omega_n = \frac{4}{3*0.5911} \quad \Rightarrow \quad \omega_n = 2.2554 \text{ rad/sec}$

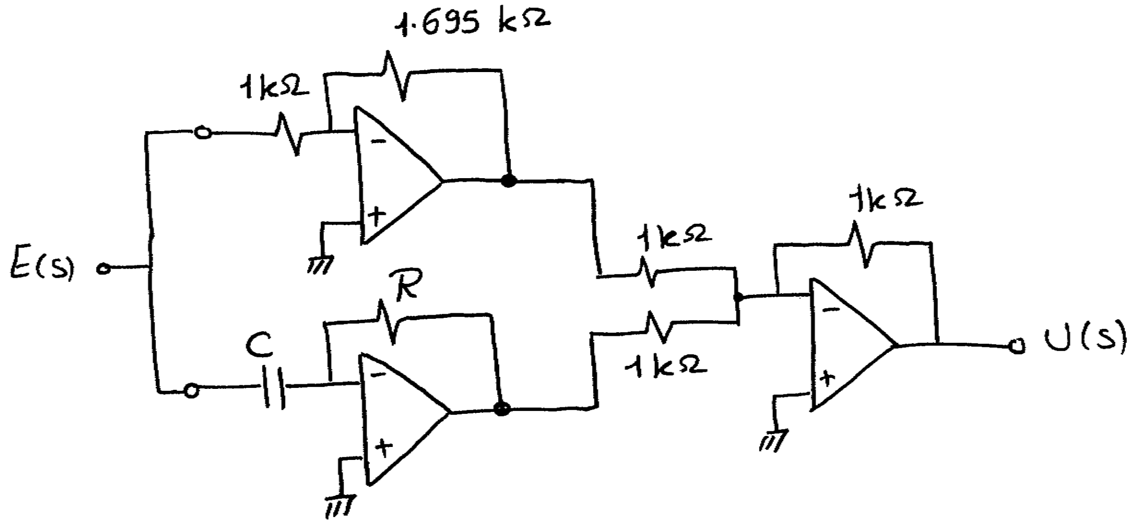
$\longrightarrow$ now substitute in the first 2 equations:

$\therefore \quad k_p = \frac{\omega_n^2}{3} \quad \Rightarrow \quad \boxed{k_p = 1.6957}$

$, \quad k_d = (2\eta\omega_n - 2)/3 \quad \Rightarrow \quad \boxed{k_d = 0.222}$

$\therefore$ the suitable controller is a PD controller with a transfer function: $\boxed{G_c(s) = 1.6957 + 0.222s}$

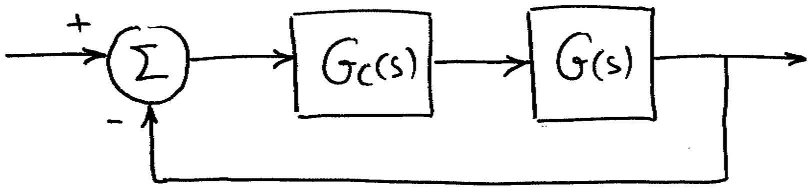
Designing:



note that: $k_d = CR = 0.222$

Choosing: $C = 100\ \mu f \Rightarrow R = 2.222\ k\Omega$

Then



(5) There are conditions on both transient and steady state responses $\Rightarrow$ use PID controller.

let $G_c(s) = k_p + k_d s + \frac{k_I}{s}$

$\therefore$ The c/ch eq is: $1 + G_c(s) * G(s) * H(s) = 0$

$$\therefore \quad 1 + \left(k_p + k_d s + \frac{k_I}{s}\right) * \frac{3}{s(s+2)} = 0$$

$$\Rightarrow \quad (s^2 + 2s) + 3\left(k_p + k_d s + \frac{k_I}{s}\right) = 0 \qquad (* s)$$

$$\therefore \quad (s^3 + 2s^2) + 3k_d s^2 + 3k_p s + 3k_I = 0$$

$$\therefore \quad s^3 + (2 + 3k_d) s^2 + 3k_p s + 3k_I = 0$$

Comparing with the 3rd order prototype c/ch eq:

$$(s+\alpha)(s^2 + 2\eta \omega_n s + \omega_n^2) = 0$$

$$\therefore \quad s^3 + (2\eta\omega_n + \alpha) s^2 + (\omega_n^2 + 2\eta\omega_n\alpha) s + \alpha\omega_n^2 = 0$$

$$\therefore \quad 2 + 3k_d = 2\eta\omega_n + \alpha \longrightarrow (1)$$

$$, \quad \omega_n^2 + 2\eta\omega_n\alpha = 3k_p \longrightarrow (2)$$

$$, \quad \alpha\omega_n^2 = 3k_I \longrightarrow (3)$$

$\Rightarrow$ 3 equations, 6 unknowns : $\eta, \omega_n, \alpha, k_p, k_d, k_I$

$\Rightarrow$ Using Conditions :

- $t_s = \frac{4}{\eta \omega_n} = 2 \text{ sec} \Rightarrow \boxed{\eta \omega_n = 2} \longrightarrow (4)$

- $e_{ss}\Big|_{\text{Parabolic i/p}} = \frac{1}{k_a}$, $k_a = \lim_{s \to 0} s^2 G_t(s) . H(s)$

$\therefore e_{ss} = \frac{1}{k_a} = 0.1 \Rightarrow k_a = 10$

$\therefore k_a = \lim_{s \to 0} s^2 * \frac{k_d s^2 + k_p s + k_I}{s} * \frac{3}{s(s+2)} * 1 = \frac{3 k_I}{2}$

$\therefore \frac{3 k_I}{2} = 10 \Rightarrow \boxed{k_I = 6.6667} \longrightarrow (5)$

now, we have 5 equations, 6 unknowns

$\Rightarrow$ assume $\alpha = 5 \eta \omega_n$ to have insignificant poles

$\therefore \boxed{\alpha = 10} \longrightarrow (6)$

now, solving these equations :

- from eq(1) : $2+3k_d = 2\eta\omega_n + \alpha$: using (4),(6)

$\therefore\ 2+3k_d = 2*2+10 \Rightarrow \boxed{k_d = 4}$

- from eq(3) : $\alpha\,\omega_n^2 = 3k_I \Rightarrow$: using (5),(6)

$\therefore\ 10\,\omega_n^2 = 20 \Rightarrow \omega_n = \sqrt{2}$ rad/sec. $\rightarrow$ (7)

- from eq (2) : $\omega_n^2 + 2\eta\omega_n\alpha = 3k_p$: using (4),(7),(6)

$\therefore\ 2+40 = 3k_p \Rightarrow \boxed{k_p = 14}$

$\therefore$ The suitable controller has the Transfer function:

$$\boxed{G_c(s) = 14 + 4s + \frac{6.6667}{s}}$$

* Designing :