AB: Potenzen, Wurzeln, Logarithmen

a) $2^3 \cdot 2^2 = 2^{3+2} = 2^5 = 32$

b) $(5^2)^3 = 5^{2\cdot 3} = 5^6 = 15.625$

c) $\frac{3^4}{2^4} = \left(\frac{3}{2}\right)^4 = \frac{81}{16}$

d) $3^3 \div 3 = \frac{3^3}{3} = \frac{3^3}{3^1} = 3^3 \cdot 3^{-1} = 3^{3+(-1)} = 3^{3-1} = 3^2 = 9$

e) $7^{-2} = \frac{1}{7^2} = \frac{1}{49}$

f) $2^3 \cdot 3^3 = (2\cdot 3)^3 = 6^3 = 216$

g) $3 \cdot 7^2 + 2 \cdot 7^2 = (3+2)\cdot 7^2 = 5 \cdot 7^2 = 5 \cdot 49 = 245$

h) $4 + 28 + 49 = 2^2 + 2\cdot 2\cdot 7 + 7^2 = (2+7)^2 = 9^2 = 81$

a) $\sqrt[3]{8} = 2$

b) $\sqrt[2]{3} \cdot \sqrt[2]{27} = \sqrt[2]{3 \cdot 27} = \sqrt[2]{81} = 9$

c) $\sqrt[3]{27} + 2 \cdot \sqrt[3]{27} = (1+2)\cdot\sqrt[3]{27} = 3 \cdot \sqrt[3]{27} = 3 \cdot 3 = 9$

d) $\left(\sqrt{\frac{9}{2}} - \sqrt{2}\right)^2 = \left(\frac{\sqrt{9}}{\sqrt{2}} - \frac{\sqrt{2}\,\sqrt{2}}{\sqrt{2}}\right)^2 = \left(\frac{\sqrt{9}}{\sqrt{2}} - \frac{2}{\sqrt{2}}\right)^2$
$= \left(\frac{\sqrt{9}-2}{\sqrt{2}}\right)^2 = \left(\frac{3-2}{\sqrt{2}}\right)^2 = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1^2}{(\sqrt{2})^2}$
$= \frac{1}{2}$

e) $\left(\sqrt{\frac{9}{2}} \cdot \sqrt{2}\right)^2 = \left(\frac{\sqrt{9}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{1}\right)^2 = \left(\frac{\sqrt{9}}{1}\right)^2 = 9$

f) $\sqrt[3]{-27} = -3$

g) $\sqrt[7]{2^{21}} = 2^{21/7} = 2^3 = 8$

a) $\ln(e) = \log_e(e) = 1$

b) $\log_{16}(8) = \log_{2^4}(2^3) = \frac{\log_2(3)}{\log_2(4)} = \frac{3}{4}$

c) $\lg(3000) - \lg(3) =$

$\log_{10}\left(\frac{3000}{3}\right) = \log_{10}(1000) = \log_{10}(10^3) = 3$

d) $\lg(4) + 2 \cdot \lg(5) =$

$\log_{10}(4) + \log_{10}(5^2) = \log_{10}(4 \cdot 5^2) = \log_{10}(100) = \log_{10}(10^2) = 2$

e) $2^x = 16$ Später

f) $\ln(\sqrt{e}) = \log_e(e^{1/2}) = 1/2$

g) $\mathrm{ld}(32) = \log_2(32) = \log_2(2^5) = 5$

a) $a^n \cdot a^m =$

b) $= a^{n-m}$

c) $(a^n)^m =$

d) $= (p+q) \cdot a^n$

e) $= (a \cdot b)^n$

f) $a^n \div b^n =$

g) $= \frac{1}{a^n}$

h) $a^{\frac{n}{m}} =$

a) $\sqrt[n]{a} \cdot \sqrt[m]{a} =$

b) $= \sqrt[m\cdot n]{a^{m-n}}$

c) $\sqrt[n]{\sqrt[m]{a}} =$

d) $p \cdot \boxed{\sqrt[n]{a}} + q \cdot \boxed{\sqrt[n]{a}} = (p+q)\cdot\boxed{\sqrt[n]{a}}$

e) $\sqrt{a} =$

f) $= \sqrt[n]{a \cdot b}$

g) $\sqrt[n]{a} \div \sqrt[n]{b} =$

h) $(\sqrt[n]{a})^m = \quad =$

i) $\sqrt[n\cdot p]{a^{n\cdot q}} =$

j) $\sqrt[n]{a^n} =$

a) $\log_a(1) =$

b) $\log_a(a) =$

c) $\log_a(a^x) =$

d) $= w \cdot \log_a(u)$

e) $\lg(x) =$

f) $\mathrm{ld}(x) =$

g) $\log_a(u \cdot v) =$

h) $= \log_a(u) - \log_a(v)$

i) $\log_{\frac{a}{b}}(c) =$

j) $\ln(x) =$

k) $\log_a(v) = \frac{\log_x(u)}{\log_x(a)}$

Im Cheat-Sheet

$\log_a(u) = \frac{\log_x(a)}{\log_x(u)}$

a und u tauschen!

a) $\frac{(v^2-n^2)\cdot 2n}{v-n} = \frac{\cancel{(v-n)}\cdot(v+n)\cdot 2n}{\cancel{(v-n)}} = (v+n)\cdot 2n = 2vn + 2n^2$

"Summen kürzen nur die Dummen!*"

*außer man Kürzt richtig.

b) $(\sqrt{x}-\sqrt{2})\cdot(\sqrt{x}+\sqrt{2}) = (\sqrt{x})^2 - (\sqrt{2})^2 = x - 2$

sofern x „gutartig" ist

c) $7(a-b)^3 + 3(b-a)^3 = 7(a-b)^3 + 3(-a+b)^3$

$= 7(a-b)^3 + 3(1\cdot(-a+b))^3$

$= 7(a-b)^3 + 3((-1)\cdot(-1)\cdot(-a+b))^3$

$= 7(a-b)^3 + 3((-1)(a-b))^3$

$(x\cdot y)^3 = x^3\cdot y^3$

$= 7(a-b)^3 + 3\cdot\underbrace{(-1)^3}_{=-1}(a-b)^3$

$= 7(a-b)^3 - 3\cdot(a-b)^3$

$= (7-3)\cdot(a-b)^3 = 4(a-b)^3$

d) $\frac{\sqrt{x}\cdot\sqrt[3]{y}}{\sqrt[3]{x}\cdot\sqrt{y}} = \frac{x^{1/2}\cdot y^{1/3}}{x^{1/3}\cdot y^{1/2}} = x^{1/2}\cdot x^{-1/3}\cdot y^{-1/2}\cdot y^{1/3} = x^{\frac12-\frac13}\cdot y^{-\frac12+\frac13}$

$= x^{1/6}\cdot y^{-1/6} = \sqrt[6]{x}/\sqrt[6]{y} = \sqrt[6]{\frac{x}{y}}$

a) $2^x = 16 \Leftrightarrow 2^x = 2^4 \quad |\log_2(\,)$

$\Rightarrow \log_2(2^x) = \log_2(2^4) \Leftrightarrow x = \log_2(2^4) \Leftrightarrow x = 4$

÷ gemischter Bruch ..... viel besser ☺

b) $\left(\frac{2}{3}\right)^x = 3\frac{3}{8} \Leftrightarrow \left(\frac{2}{3}\right)^x = \frac{27}{8} \quad |\log_{2/3}(\,)$

$\Rightarrow \log_{2/3}\left(\left(\frac{2}{3}\right)^x\right) = \log_{2/3}\left(\frac{27}{8}\right)$

$\Leftrightarrow x = \log_{2/3}\left(\frac{3^3}{2^3}\right) = \log_{2/3}\left(\left(\frac{3}{2}\right)^3\right)$

$= -\log_{3/2}\left(\left(\frac{3}{2}\right)^3\right) = -3$

c) $\log_3\left(\frac{9x}{4x-3}\right) = 2 \quad |\,3^{(\,)}$

$\Rightarrow 3^{\log_3\left(\frac{9x}{4x-3}\right)} = 3^2 \Leftrightarrow \frac{9x}{4x-3} = 3^2 \quad |\cdot(4x-3) \Leftrightarrow 9x = 9\cdot(4x-3) \quad |\cdot\frac{1}{9} \Leftrightarrow x = 4x-3$

$|-x|+3 \Leftrightarrow 3x = 3 \quad |:3 \Leftrightarrow x = 1$

d) $\log_3(3x-5) = \log_3(2x+3) \quad |\,3^{(\,)}$

$\Rightarrow 3^{\log_3(3x-5)} = 3^{\log_3(2x+3)} \Leftrightarrow 3x-5 = 2x+3 \quad |-2x \quad |+5$

$$\Rightarrow 3^{\log_3(3x-5)} = 3^{\log_3(2x+3)} \Leftrightarrow 3x-5 = 2x+3 \quad | -2x \quad | +5$$

$$x = 8$$