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COURSE : B.Sc. (Hons.) Mathematics/Year3/Complex Analysis
SECTION : A

PRACTICAL 1 : Make a geometric plot to show that the n^{th} roots of unity are equally spaced points that lie on the unit circle $C_1(0) = \{ z : |z| = 1\}$ and form the vertices of a regular polygon with n sides, for n = 4, 5, 6, 7, 8.

n = 3

In[•]:=
```
s = N[Solve[z^3 == 1, z]]
z1 = s[[1, 1, 2]]
z2 = s[[2, 1, 2]]
z3 = s[[3, 1, 2]]
a = Graphics[{PointSize[0.03], Red, Point[{Re[z1], Im[z1]}],
    Blue, Point[{Re[z2], Im[z2]}], Yellow, Point[{Re[z3], Im[z3]}]}];
b = PolarPlot[1, {h, 0, 2 π}, PolarAxes → True,
   PolarGridLines → True, PlotStyle → Thickness[0.01]];
c = Graphics[{Text[Style["z1", Bold], {1.2, 0.2}],
    Text[Style["z2", Bold], {-0.8, -1}], Text[Style["z3", Bold], {-0.8, 1}]}];
e = Graphics[{Purple, Thick, Line[{{0, 0}, {Re[z1], Im[z1]}}],
    Line[{{0, 0}, {Re[z2], Im[z2]}}], Line[{{0, 0}, {Re[z3], Im[z3]}}]}];
Show[a, b, c, e]
If[Arg[z2/z1] == Arg[z3/z2] == Arg[z1/z3], Print["All Points are equally spaced."],
 Print["All Points are not equally spaced."]]
f = Graphics[{Purple, Thick, Line[{{Re[z1], Im[z1]}, {Re[z2], Im[z2]}}],
    Line[{{Re[z2], Im[z2]}, {Re[z3], Im[z3]}}],
    Line[{{Re[z3], Im[z3]}, {Re[z1], Im[z1]}}]}];
Show[a, b, c, f]
If[Abs[z1 - z2] == Abs[z2 - z3] == Abs[z3 - z1],
 Print["The Points form the vertices of a regular polygon."],
 Print["Polygon formed is not regular."]]
```

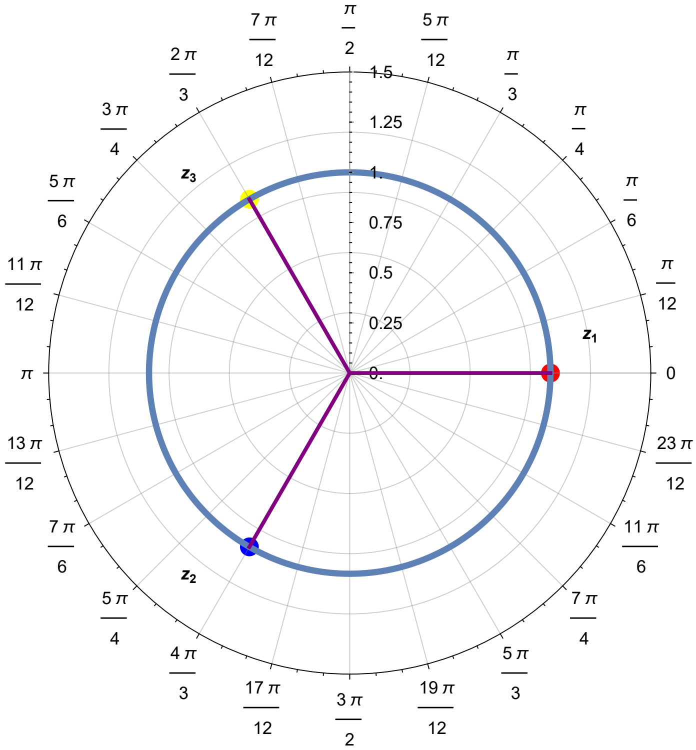
Out[•]=
```
{{z → 1.}, {z → -0.5 - 0.866025 i}, {z → -0.5 + 0.866025 i}}
```

Out[•]=
```
1.
```

Out[•]=
```
-0.5 - 0.866025 i
```

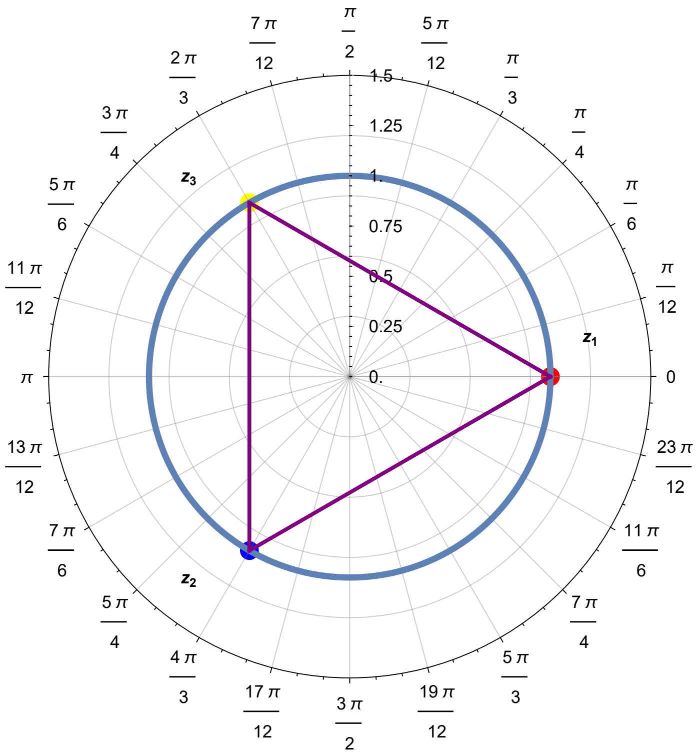
Out[•]=
```
-0.5 + 0.866025 i
```

Out[•]=



All Points are equally spaced.

Out[•]=



The Points form the vertices of a regular polygon.

n = 4

In[*]:=
```
s = N[Solve[z^4 == 1, z]]
z1 = s[[1, 1, 2]]
z2 = s[[2, 1, 2]]
z3 = s[[3, 1, 2]]
z4 = s[[4, 1, 2]]
a = Graphics[
   {PointSize[0.03], Red, Point[{Re[z1], Im[z1]}], Blue, Point[{Re[z2], Im[z2]}],
    Yellow, Point[{Re[z3], Im[z3]}], Purple, Point[{Re[z4], Im[z4]}]}];
b = PolarPlot[1, {h, 0, 2 π}, PolarAxes → True,
   PolarGridLines → True, PlotStyle → Thickness[0.01]];
c = Graphics[{Text[Style["z1", Bold], {-1.2, 0}], Text[Style["z2", Bold], {0, -1.2}],
    Text[Style["z4", Bold], {1.2, 0}], Text[Style["z3", Bold], {0, 1.2}]}];
e = Graphics[
   {Purple, Thick, Line[{{0, 0}, {Re[z1], Im[z1]}}], Line[{{0, 0}, {Re[z2], Im[z2]}}],
    Line[{{0, 0}, {Re[z3], Im[z3]}}], Line[{{0, 0}, {Re[z4], Im[z4]}}]}];
Show[a, b, c, e]
If[Arg[z3/z4] == Arg[z1/z3] == Arg[z2/z1] == Arg[z4/z2], Print["All Points are equally spaced."],
 Print["All Points are not equally spaced."]]
f = Graphics[{Purple, Thick, Line[{{Re[z4], Im[z4]}, {Re[z3], Im[z3]}}], Line[
     {{Re[z3], Im[z3]}, {Re[z1], Im[z1]}}], Line[{{Re[z1], Im[z1]}, {Re[z2], Im[z2]}}],
    Line[{{Re[z2], Im[z2]}, {Re[z4], Im[z4]}}]}];
Show[a, b, c, f]
If[Abs[z4 - z3] == Abs[z3 - z1] == Abs[z1 - z2] == Abs[z2 - z4],
 Print["The Points form the vertices of a regular polygon."],
 Print["Polygon formed is not regular."]]
```

Out[*]=
```
{{z → -1.}, {z → 0. - 1. i}, {z → 0. + 1. i}, {z → 1.}}
```

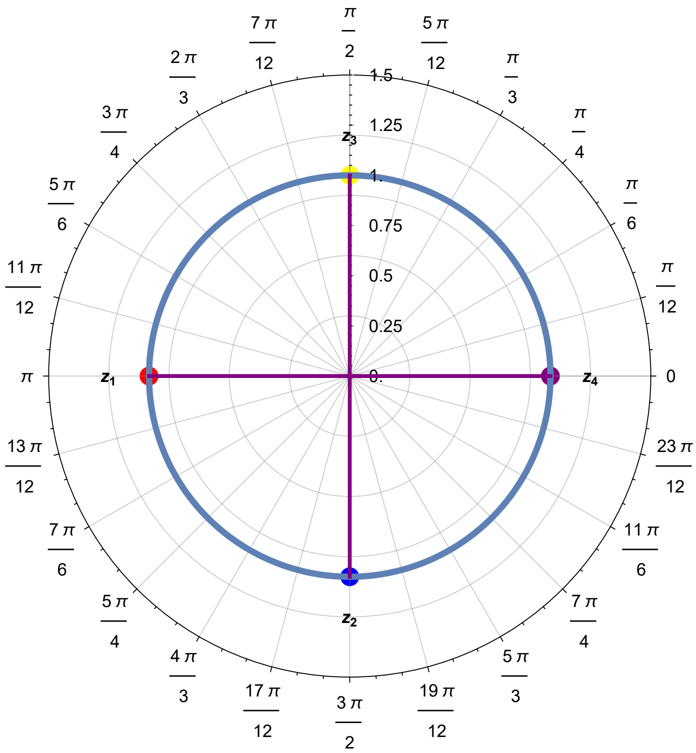
Out[*]=
```
-1.
```

Out[*]=
```
0. - 1. i
```

Out[*]=
```
0. + 1. i
```

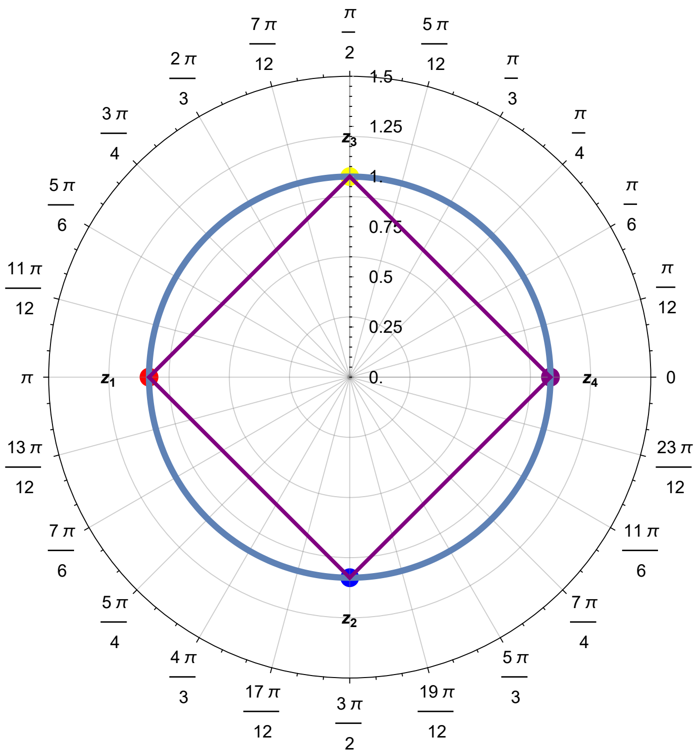
Out[*]=
```
1.
```

Out[ ]=



All Points are equally spaced.

Out[ ]=



The Points form the vertices of a regular polygon.

$n = 5$

In[∘]:=
```
s = N[Solve[z^5 == 1, z]]
z1 = s[[1, 1, 2]]
z2 = s[[2, 1, 2]]
z3 = s[[3, 1, 2]]
z4 = s[[4, 1, 2]]
z5 = s[[5, 1, 2]]
a = Graphics[{PointSize[0.03], Red, Point[{Re[z1], Im[z1]}],
    Blue, Point[{Re[z2], Im[z2]}], Yellow, Point[{Re[z3], Im[z3]}],
    Green, Point[{Re[z4], Im[z4]}], Magenta, Point[{Re[z5], Im[z5]}]}];
b = PolarPlot[1, {h, 0, 2 π}, PolarAxes → True,
   PolarGridLines → True, PlotStyle → Thickness[0.01]];
c = Graphics[{Text[Style["z1", Bold], {1.2, 0}],
    Text[Style["z2", Bold], {-1, -0.6}], Text[Style["z3", Bold], {0.5, 1}],
    Text[Style["z4", Bold], {0.5, -1}], Text[Style["z5", Bold], {-1, 0.6}]}];
e = Graphics[{Purple, Thick, Line[{{0, 0}, {Re[z1], Im[z1]}}],
    Line[{{0, 0}, {Re[z2], Im[z2]}}], Line[{{0, 0}, {Re[z3], Im[z3]}}],
    Line[{{0, 0}, {Re[z4], Im[z4]}}], Line[{{0, 0}, {Re[z5], Im[z5]}}]}];
Show[a, b, c, e]
If[Arg[z3/z1] == Arg[z5/z3] == Arg[z2/z5] == Arg[z4/z2] == Arg[z1/z4],
 Print["All Points are equally spaced."],
 Print["All Points are not equally spaced."]]
f = Graphics[{Purple, Thick, Line[{{Re[z1], Im[z1]}, {Re[z3], Im[z3]}}],
    Line[{{Re[z3], Im[z3]}, {Re[z5], Im[z5]}}], Line[
     {{Re[z5], Im[z5]}, {Re[z2], Im[z2]}}], Line[{{Re[z2], Im[z2]}, {Re[z4], Im[z4]}}],
    Line[{{Re[z4], Im[z4]}, {Re[z1], Im[z1]}}]}];
Show[a, b, c, f]
If[Abs[z1 - z3] == Abs[z3 - z5] == Abs[z5 - z2] == Abs[z2 - z4] == Abs[z4 - z1],
 Print["The Points form the vertices of a regular polygon."],
 Print["Polygon formed is not regular."]]
```

Out[∘]=

{{z → 1.}, {z → -0.809017 - 0.587785 i}, {z → 0.309017 + 0.951057 i},
 {z → 0.309017 - 0.951057 i}, {z → -0.809017 + 0.587785 i}}

Out[∘]=

1.

Out[∘]=

-0.809017 - 0.587785 i

Out[∘]=

0.309017 + 0.951057 i

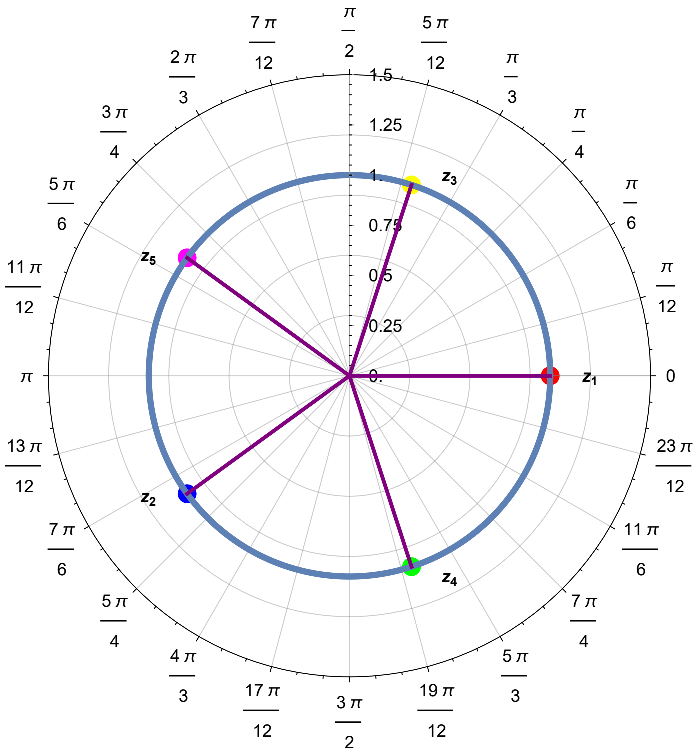
Out[∘]=

0.309017 - 0.951057 i

Out[∘]=

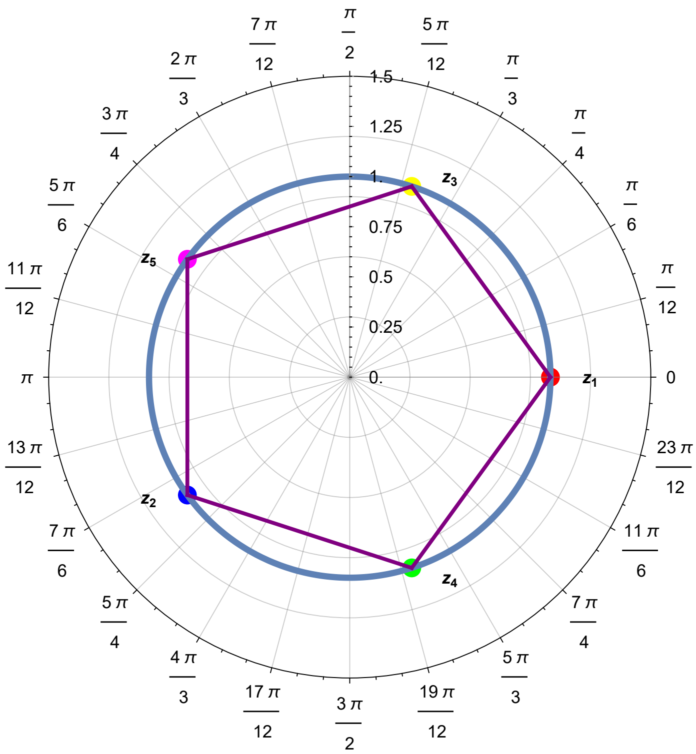
-0.809017 + 0.587785 i

Out[ ]=




```
All Points are equally spaced.
```

Out[ ]=




```
The Points form the vertices of a regular polygon.
```

n = 6

In[ ]:=
```
s = N[Solve[z^6 == 1, z]]
z1 = s[[1, 1, 2]]
z2 = s[[2, 1, 2]]
z3 = s[[3, 1, 2]]
z4 = s[[4, 1, 2]]
z5 = s[[5, 1, 2]]
z6 = s[[6, 1, 2]]
a = Graphics[
   {PointSize[0.03], Red, Point[{Re[z1], Im[z1]}], Blue, Point[{Re[z2], Im[z2]}],
    Yellow, Point[{Re[z3], Im[z3]}], Green, Point[{Re[z4], Im[z4]}],
    Magenta, Point[{Re[z5], Im[z5]}], Purple, Point[{Re[z6], Im[z6]}]}];
b = PolarPlot[1, {h, 0, 2 π}, PolarAxes → True,
   PolarGridLines → True, PlotStyle → Thickness[0.01]];
c = Graphics[{Text[Style["z1", Bold], {-1.2, 0.2}], Text[Style["z2", Bold], {1.2, 0.2}],
    Text[Style["z3", Bold], {-0.8, -1}], Text[Style["z4", Bold], {0.8, 1}],
    Text[Style["z5", Bold], {0.8, -1}], Text[Style["z6", Bold], {-0.8, 1}]}];
e = Graphics[
   {Purple, Thick, Line[{{0, 0}, {Re[z1], Im[z1]}}], Line[{{0, 0}, {Re[z2], Im[z2]}}],
    Line[{{0, 0}, {Re[z3], Im[z3]}}], Line[{{0, 0}, {Re[z4], Im[z4]}}],
    Line[{{0, 0}, {Re[z5], Im[z5]}}], Line[{{0, 0}, {Re[z6], Im[z6]}}]}];
Show[a, b, c, e]
If[Arg[z4/z2] == Arg[z6/z4] == Arg[z1/z6] == Arg[z3/z1] == Arg[z5/z3] == Arg[z2/z5],
 Print["All Points are equally spaced."],
 Print["All Points are not equally spaced."]]
f = Graphics[{Purple, Thick, Line[{{Re[z2], Im[z2]}, {Re[z4], Im[z4]}}],
    Line[{{Re[z4], Im[z4]}, {Re[z6], Im[z6]}}],
    Line[{{Re[z6], Im[z6]}, {Re[z1], Im[z1]}}], Line[
     {{Re[z1], Im[z1]}, {Re[z3], Im[z3]}}], Line[{{Re[z3], Im[z3]}, {Re[z5], Im[z5]}}],
    Line[{{Re[z5], Im[z5]}, {Re[z2], Im[z2]}}]}];
Show[a, b, c, f]
If[Abs[z2 - z4] == Abs[z4 - z6] == Abs[z6 - z1] == Abs[z1 - z3] == Abs[z3 - z5] == Abs[z5 - z2],
 Print["The Points form the vertices of a regular polygon."],
 Print["Polygon formed is not regular."]]
```

Out[ ]=
$\{\{z \to -1.\}, \{z \to 1.\}, \{z \to -0.5 - 0.866025\, i\}, \{z \to 0.5 + 0.866025\, i\}, \{z \to 0.5 - 0.866025\, i\}, \{z \to -0.5 + 0.866025\, i\}\}$

Out[ ]=
$-1.$

Out[ ]=
$1.$

Out[ ]=
$-0.5 - 0.866025\, i$

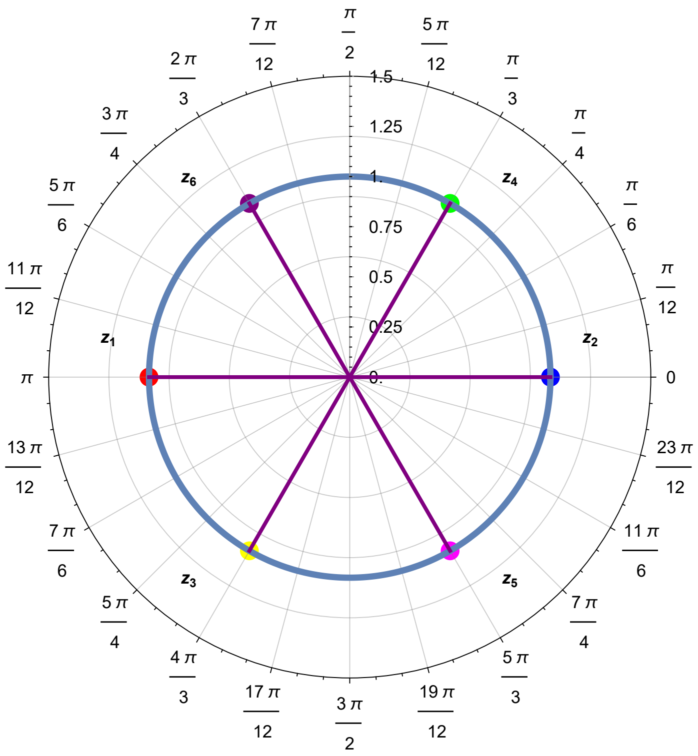
Out[ ]=
$0.5 + 0.866025\, i$

Out[ ]=
$0.5 - 0.866025\, i$

Out[•]=

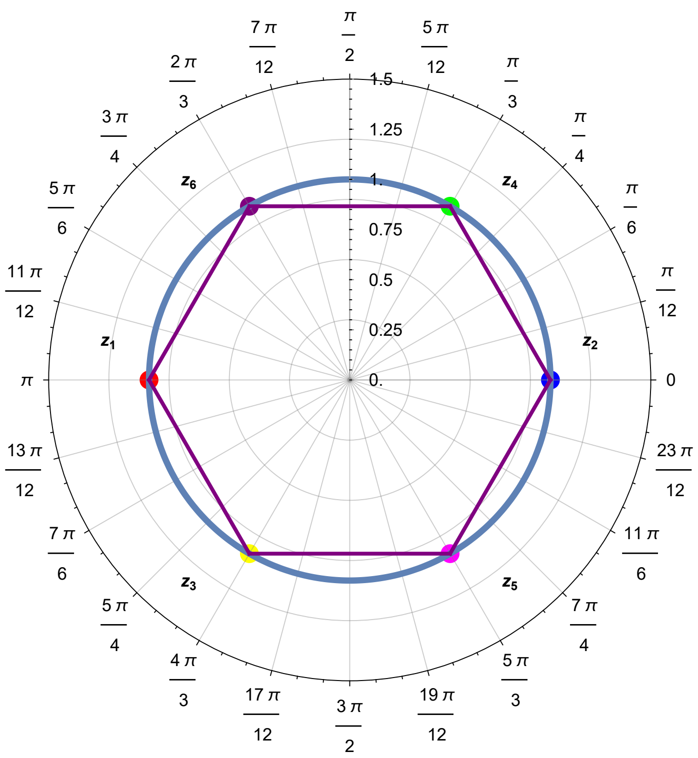
$-0.5 + 0.866025\, i$

Out[•]=



All Points are equally spaced.

Out[•]=



The Points form the vertices of a regular polygon.

n = 7

```
In[∘]:= s = N[Solve[z^7 == 1, z]]
z1 = s[[1, 1, 2]]
z2 = s[[2, 1, 2]]
z3 = s[[3, 1, 2]]
z4 = s[[4, 1, 2]]
z5 = s[[5, 1, 2]]
z6 = s[[6, 1, 2]]
z7 = s[[7, 1, 2]]
a = Graphics[{PointSize[0.03], Red, Point[{Re[z1], Im[z1]}],
    Blue, Point[{Re[z2], Im[z2]}], Yellow, Point[{Re[z3], Im[z3]}],
    Green, Point[{Re[z4], Im[z4]}], Magenta, Point[{Re[z5], Im[z5]}],
    Purple, Point[{Re[z6], Im[z6]}], Brown, Point[{Re[z7], Im[z7]}]}];
b = PolarPlot[1, {h, 0, 2 π}, PolarAxes → True,
   PolarGridLines → True, PlotStyle → Thickness[0.01]];
c = Graphics[{Text[Style["z1", Bold], {1.2, 0}],
    Text[Style["z2", Bold], {-1.2, -0.6}], Text[Style["z3", Bold], {0.9, 1}],
    Text[Style["z4", Bold], {-0.4, -1.2}], Text[Style["z5", Bold], {-0.4, 1.2}],
    Text[Style["z6", Bold], {0.9, -1}], Text[Style["z7", Bold], {-1.2, 0.6}]}];
e = Graphics[{Purple, Thick, Line[{{0, 0}, {Re[z1], Im[z1]}}],
    Line[{{0, 0}, {Re[z2], Im[z2]}}], Line[{{0, 0}, {Re[z3], Im[z3]}}],
    Line[{{0, 0}, {Re[z4], Im[z4]}}], Line[{{0, 0}, {Re[z5], Im[z5]}}],
    Line[{{0, 0}, {Re[z6], Im[z6]}}], Line[{{0, 0}, {Re[z7], Im[z7]}}]}];
Show[a, b, c, e]
If[Arg[z3/z1] == Arg[z5/z3] == Arg[z7/z5] == Arg[z2/z7] == Arg[z4/z2] == Arg[z6/z4] == Arg[z1/z6],
 Print["All Points are equally spaced."],
 Print["All Points are not equally spaced."]]
f = Graphics[{Purple, Thick, Line[{{Re[z1], Im[z1]}, {Re[z3], Im[z3]}}], Line[
     {{Re[z3], Im[z3]}, {Re[z5], Im[z5]}}], Line[{{Re[z5], Im[z5]}, {Re[z7], Im[z7]}}],
    Line[{{Re[z7], Im[z7]}, {Re[z2], Im[z2]}}], Line[
     {{Re[z2], Im[z2]}, {Re[z4], Im[z4]}}], Line[{{Re[z4], Im[z4]}, {Re[z6], Im[z6]}}],
    Line[{{Re[z6], Im[z6]}, {Re[z1], Im[z1]}}]}];
Show[a, b, c, f]
If[Abs[z1 - z3] == Abs[z3 - z5] == Abs[z5 - z7] == Abs[z7 - z2] == Abs[z2 - z4] == Abs[z4 - z6] ==
  Abs[z6 - z1], Print["The Points form the vertices of a regular polygon."],
 Print["Polygon formed is not regular."]]
```

```
Out[∘]=
{{z → 1.}, {z → -0.900969 - 0.433884 i}, {z → 0.62349 + 0.781831 i},
 {z → -0.222521 - 0.974928 i}, {z → -0.222521 + 0.974928 i},
 {z → 0.62349 - 0.781831 i}, {z → -0.900969 + 0.433884 i}}

Out[∘]=
1.

Out[∘]=
-0.900969 - 0.433884 i

Out[∘]=
0.62349 + 0.781831 i

Out[∘]=
-0.222521 - 0.974928 i
```

Out[*]=

$-0.222521 + 0.974928\,i$

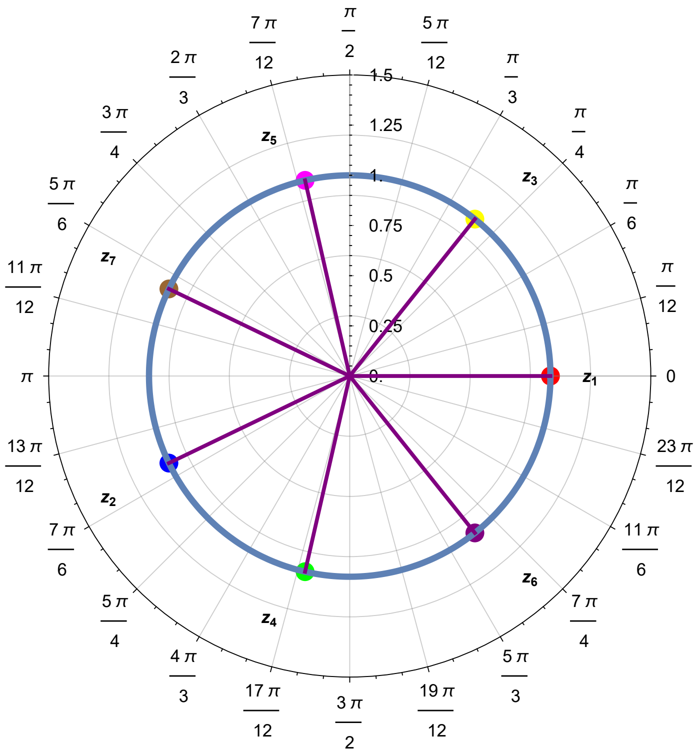
Out[*]=

$0.62349 - 0.781831\,i$

Out[*]=

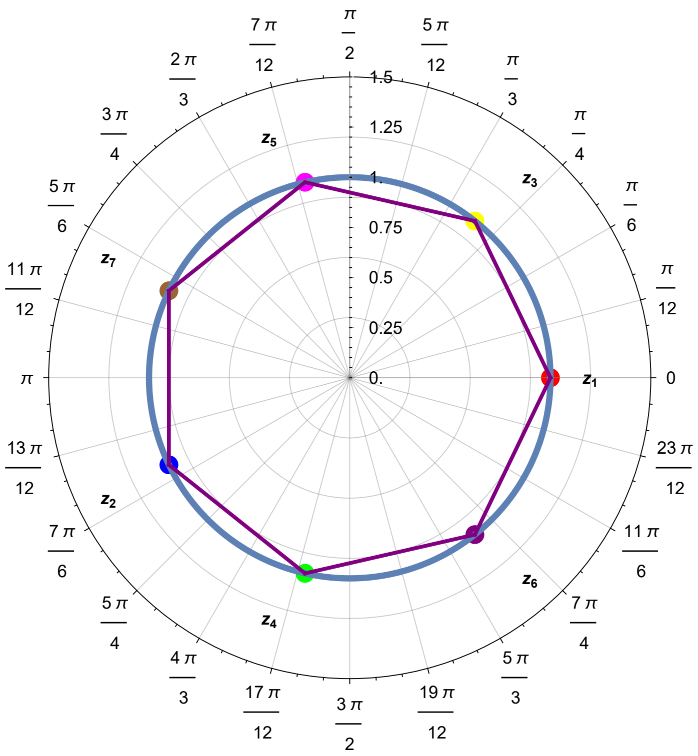
$-0.900969 + 0.433884\,i$

Out[*]=



All Points are not equally spaced.

Out[•]=



The Points form the vertices of a regular polygon.

n = 8

In[ ]:=
```
s = N[Solve[z^8 == 1, z]]
z1 = s[[1, 1, 2]]
z2 = s[[2, 1, 2]]
z3 = s[[3, 1, 2]]
z4 = s[[4, 1, 2]]
z5 = s[[5, 1, 2]]
z6 = s[[6, 1, 2]]
z7 = s[[7, 1, 2]]
z8 = s[[8, 1, 2]]
a = Graphics[
   {PointSize[0.03], Red, Point[{Re[z1], Im[z1]}], Blue, Point[{Re[z2], Im[z2]}],
    Yellow, Point[{Re[z3], Im[z3]}], Green, Point[{Re[z4], Im[z4]}],
    Magenta, Point[{Re[z5], Im[z5]}], Purple, Point[{Re[z6], Im[z6]}],
    Brown, Point[{Re[z7], Im[z7]}], Orange, Point[{Re[z8], Im[z8]}]}];
b = PolarPlot[1, {h, 0, 2 π}, PolarAxes → True,
   PolarGridLines → True, PlotStyle → Thickness[0.01]];
c = Graphics[{Text[Style["z1", Bold], {-1.2, 0}], Text[Style["z2", Bold], {0, -1.2}],
    Text[Style["z3", Bold], {0, 1.2}], Text[Style["z4", Bold], {1.2, 0}],
    Text[Style["z5", Bold], {-1, -1}], Text[Style["z6", Bold], {1, 1}],
    Text[Style["z7", Bold], {1, -1}], Text[Style["z8", Bold], {-1, 1}]}];
e = Graphics[
   {Purple, Thick, Line[{{0, 0}, {Re[z1], Im[z1]}}], Line[{{0, 0}, {Re[z2], Im[z2]}}],
    Line[{{0, 0}, {Re[z3], Im[z3]}}], Line[{{0, 0}, {Re[z4], Im[z4]}}],
    Line[{{0, 0}, {Re[z5], Im[z5]}}], Line[{{0, 0}, {Re[z6], Im[z6]}}],
    Line[{{0, 0}, {Re[z7], Im[z7]}}], Line[{{0, 0}, {Re[z8], Im[z8]}}]}];
Show[a, b, c, e]
If[Arg[z3/z6] == Arg[z8/z3] == Arg[z1/z8] == Arg[z5/z1] == Arg[z2/z5] == Arg[z7/z2] ==
   Arg[z4/z7] == Arg[z6/z4] == Arg[z3/z6], Print["All Points are equally spaced."],
  Print["All Points are not equally spaced."]]
f = Graphics[{Purple, Thick, Line[{{Re[z6], Im[z6]}, {Re[z3], Im[z3]}}],
    Line[{{Re[z3], Im[z3]}, {Re[z8], Im[z8]}}],
    Line[{{Re[z8], Im[z8]}, {Re[z1], Im[z1]}}], Line[
     {{Re[z1], Im[z1]}, {Re[z5], Im[z5]}}], Line[{{Re[z5], Im[z5]}, {Re[z2], Im[z2]}}],
    Line[{{Re[z2], Im[z2]}, {Re[z7], Im[z7]}}], Line[
     {{Re[z7], Im[z7]}, {Re[z4], Im[z4]}}], Line[{{Re[z4], Im[z4]}, {Re[z6], Im[z6]}}],
    Line[{{Re[z6], Im[z6]}, {Re[z3], Im[z3]}}]}];
Show[a, b, c, f]
If[Abs[z6 - z3] == Abs[z3 - z8] == Abs[z8 - z1] == Abs[z1 - z5] ==
   Abs[z5 - z2] == Abs[z2 - z7] == Abs[z7 - z4] == Abs[z4 - z6] == Abs[z6 - z3],
  Print["The Points form the vertices of a regular polygon."],
  Print["Polygon formed is not regular."]]
```

Out[ ]=

{{z → -1.}, {z → 0. - 1. i}, {z → 0. + 1. i}, {z → 1.}, {z → -0.707107 - 0.707107 i},
 {z → 0.707107 + 0.707107 i}, {z → 0.707107 - 0.707107 i}, {z → -0.707107 + 0.707107 i}}

Out[ ]=

-1.

Out[ ]=

0. − 1. i

Out[ ]=

0. + 1. i

Out[ ]=

1.

Out[ ]=

−0.707107 − 0.707107 i

Out[ ]=

0.707107 + 0.707107 i

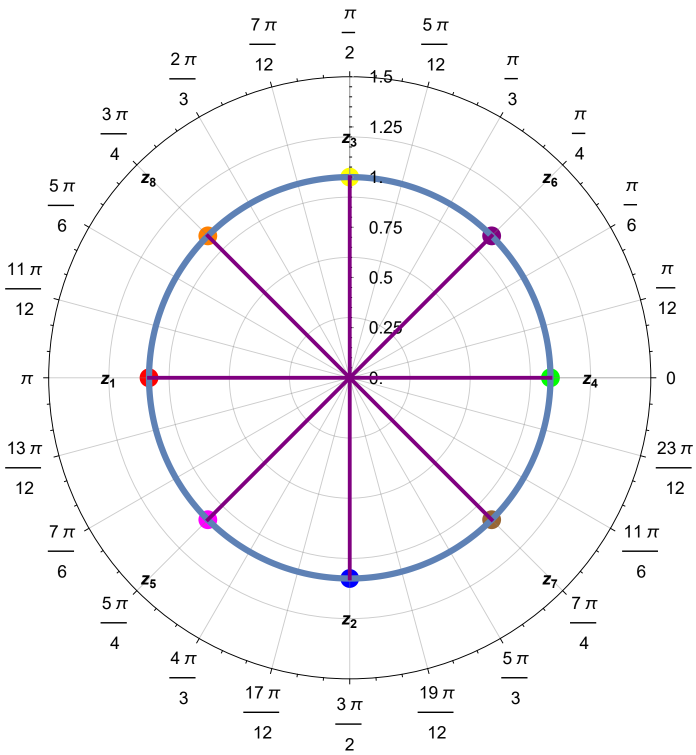
Out[ ]=

0.707107 − 0.707107 i

Out[ ]=

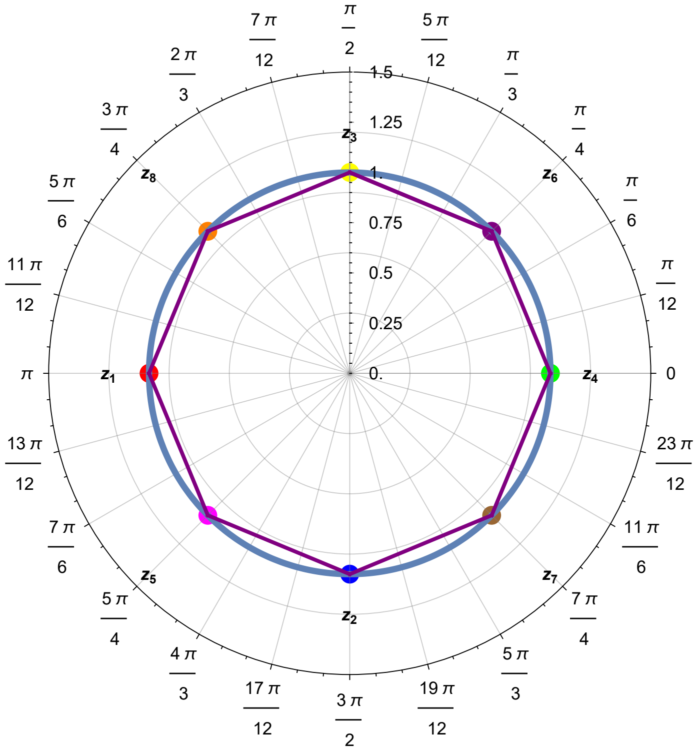
−0.707107 + 0.707107 i

Out[ ]=



All Points are equally spaced.

Out[∘]=



The Points form the vertices of a regular polygon.

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COLLEGE ROLL NO.: 2232139
UNIVERSITY ROLL NO.: 22036563034
COURSE : B.Sc. (Hons.) Mathematics/Year3/Complex Analysis
SECTION : A

PRACTICAL 2 : Find all the solutions of the equation z^3=8i and represent these geometrically.

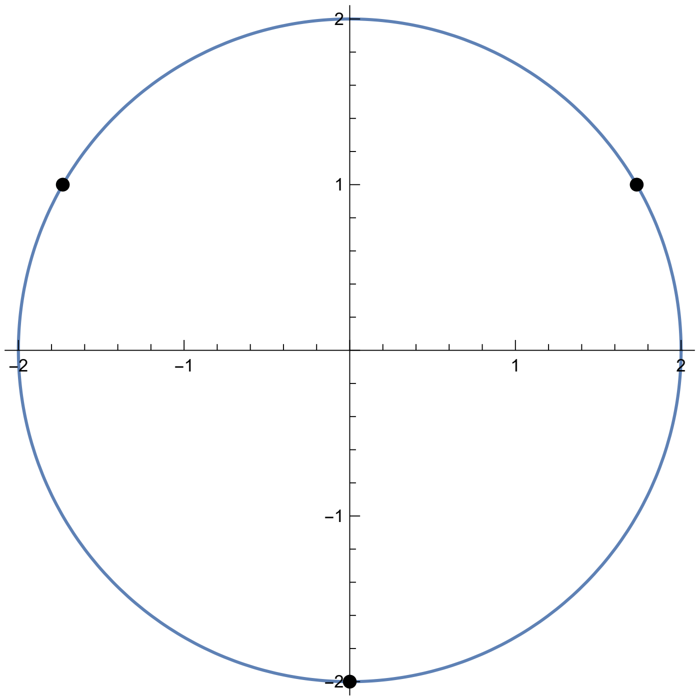
(a) Solve the equation $z^3 = 8i$

In[∘]:=
```
sol = N[Solve[z^3 == 8 I, z]]
```
Out[∘]=
```
{{z → 0. - 2. i}, {z → 1.73205 + 1. i}, {z → -1.73205 + 1. i}}
```
In[∘]:=
```
z1 = sol[[1, 1, 2]]
z2 = sol[[2, 1, 2]]
z3 = sol[[3, 1, 2]]
```
Out[∘]=
```
0. - 2. i
```
Out[∘]=
```
1.73205 + 1. i
```
Out[∘]=
```
-1.73205 + 1. i
```

(b) Show that the roots lie on a circle with radius 2, centered at origin

In[ ]:=
```
a = Graphics[{PointSize[0.02],
    Point[{Re[z1], Im[z1]}],
    Point[{Re[z2], Im[z2]}],
    Point[{Re[z3], Im[z3]}]
   }];
b = PolarPlot[2, {τ, 0, 2 π}];
Show[b, a]
```

Out[ ]=



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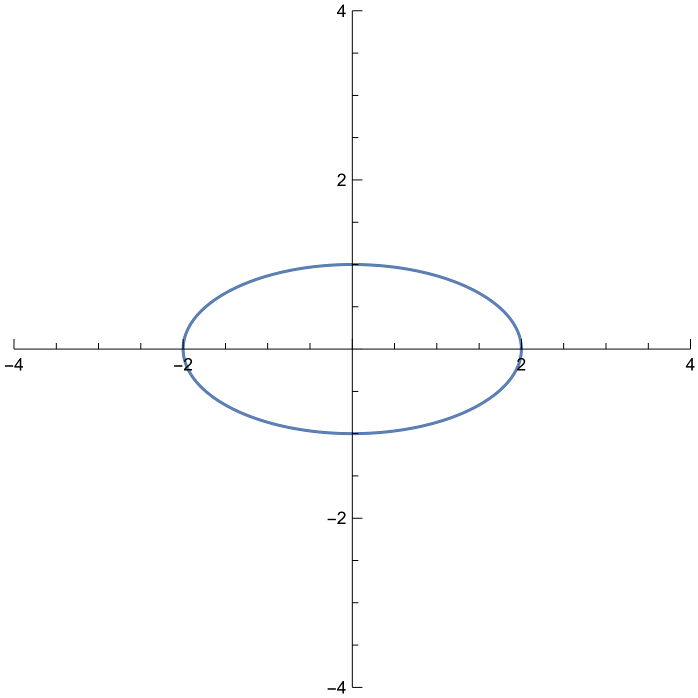
PRACTICAL 3 : Write the parametric equations and make a parametric plot for an ellipse centered at the origin with the horizontal major axis of 4 units and vertical minor axis of 2 units

(a) Write parametric equations and make a parametric plot for an ellipse centered at the origin with horizontal major axis of 4 units and vertical minor axis of 2 units.
(b) Show the effect of rotation of this ellipse by an angle of $\frac{\pi}{6}$ radians and shifting of the centre from (0,0) to (2,1), by making a parametric plot.

In[ ]:=
```
(*To plot a parametric equations, we may use the ParametricPlot[] command*)
```

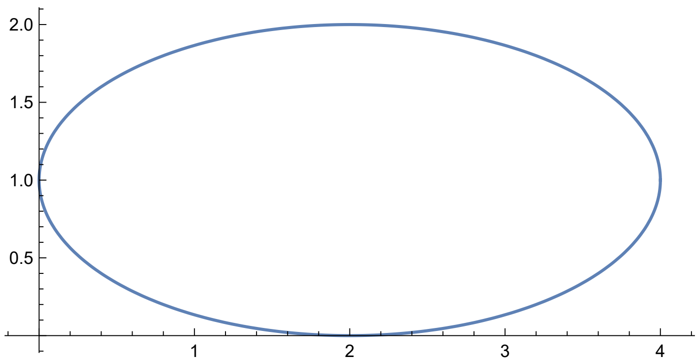
In[ ]:=
```
(*the equation of the ellipse is x^2/2^2 + y^2/1 =1*)
a = ParametricPlot[{2 * Cos[t], Sin[t]}, {t, 0, 2 π}, PlotRange → 4]
```

Out[ ]=



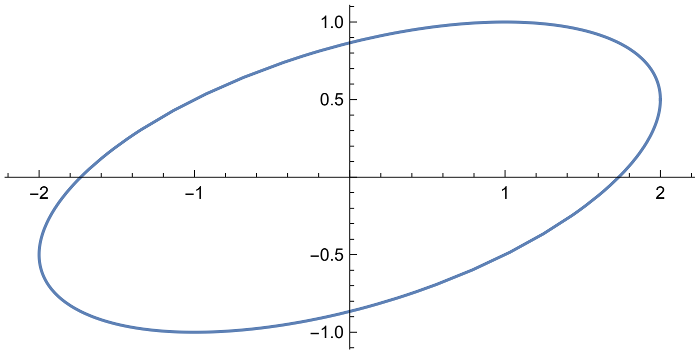
In[ ]:=
```
(*Translate the center to the point (2,1)*)
b = ParametricPlot[{2 * Cos[t] + 2, Sin[t] + 1}, {t, 0, 2 π}]
```

Out[ ]=



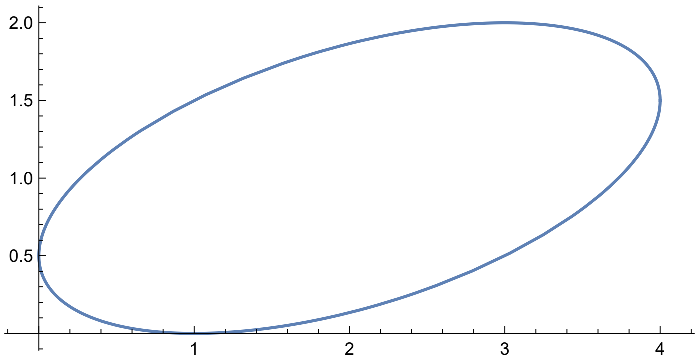
In[•]:=
```
(*Rotate the ellipse about the origin by an angle of π/6*)
c = ParametricPlot[{2 * Cos[t], Sin[t + π/6]}, {t, 0, 2 π}]
```

Out[•]=



In[•]:=
```
(*Rotate and translate*)
d = ParametricPlot[{2 * Cos[t] + 2, Sin[t + π/6] + 1}, {t, 0, 2 π}]
```

Out[•]=

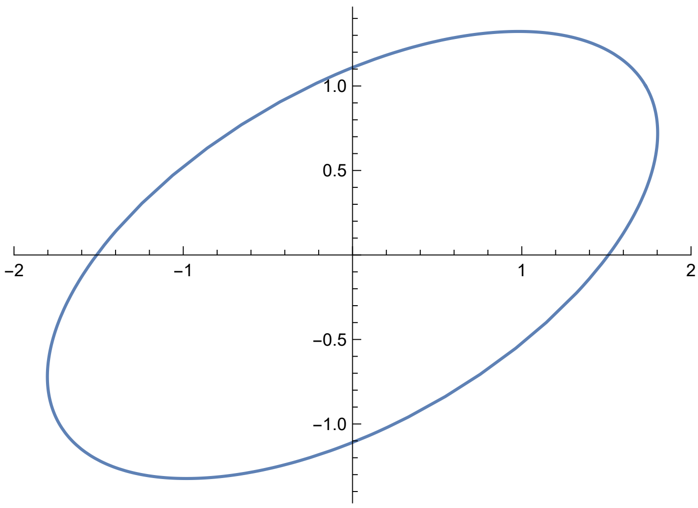


Using ParametricPlot command

In[•]:=
```
ClearAll;
```

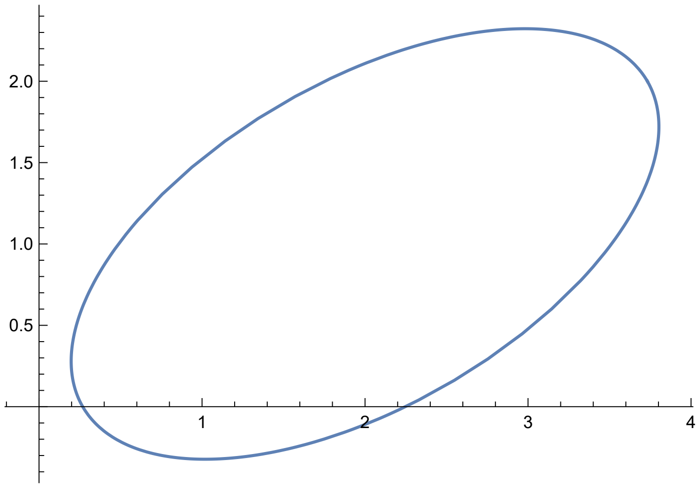
In[•]:=
```
e = ParametricPlot[RotationTransform[π/6][{2 * Cos[t], Sin[t]}], {t, 0, 2 π}]
```

Out[•]=



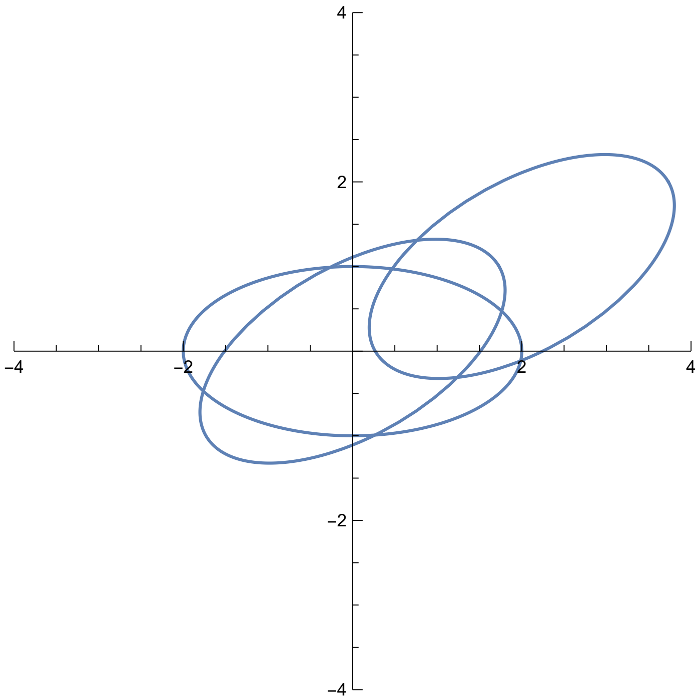
In[◦]:=
```
f = ParametricPlot[TranslationTransform[{2, 1}][
    RotationTransform[π/6][{2 * Cos[t], Sin[t]}]], {t, 0, 2 π}]
```

Out[◦]=



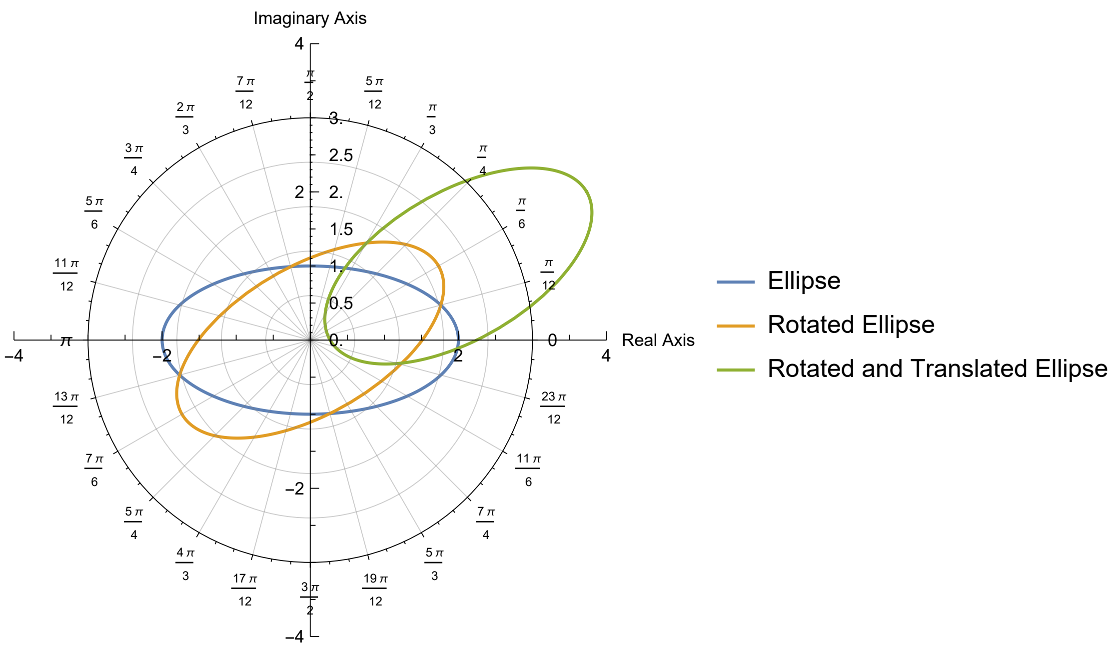
In[◦]:=
```
Show[a, e, f]
```

Out[◦]=



In[ ]:=
```
ClearAll;
a = ParametricPlot[
   {{2 Cos[t], Sin[t]},
    RotationTransform[π/6][{2 Cos[t], Sin[t]}],
    TranslationTransform[{2, 1}][RotationTransform[π/6][{2 Cos[t], Sin[t]}]]},
   {t, 0, 2 π},
   PlotLegends → {"Ellipse", "Rotated Ellipse", "Rotated and Translated Ellipse"},
   AxesLabel → {"Real Axis", "Imaginary Axis"}, PlotRange → 4
  ];
b = PolarPlot[2, {θ, 0, 2 π},
   PolarGridLines → True, PlotStyle → Opacity[0], PolarAxes → True];
Show[a, b]
```

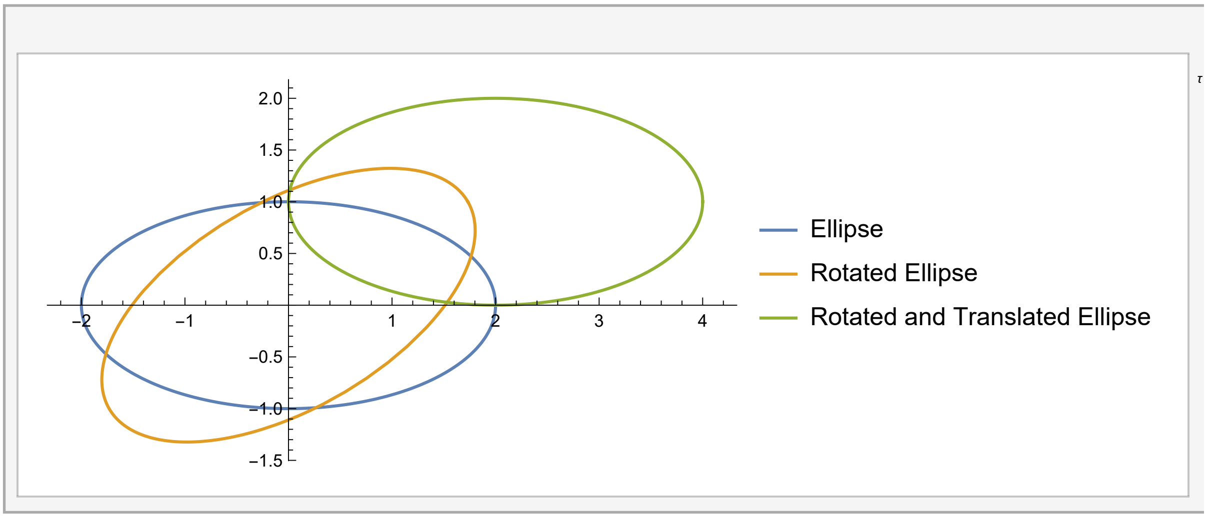
Out[ ]=



In[ ]:=

In[∘]:=
```
Manipulate[
 ParametricPlot[
  {{2 Cos[t], Sin[t]},
   RotationTransform[π/6][{2 Cos[t], Sin[t]}],
   TranslationTransform[{2, 1}][RotationTransform[τ][{2 Cos[t], Sin[t]}]]},
  {t, 0, 2 π},
  PlotLegends → {"Ellipse", "Rotated Ellipse", "Rotated and Translated Ellipse"}
 ], {τ, 0, 2 π}
]
```

Out[∘]=



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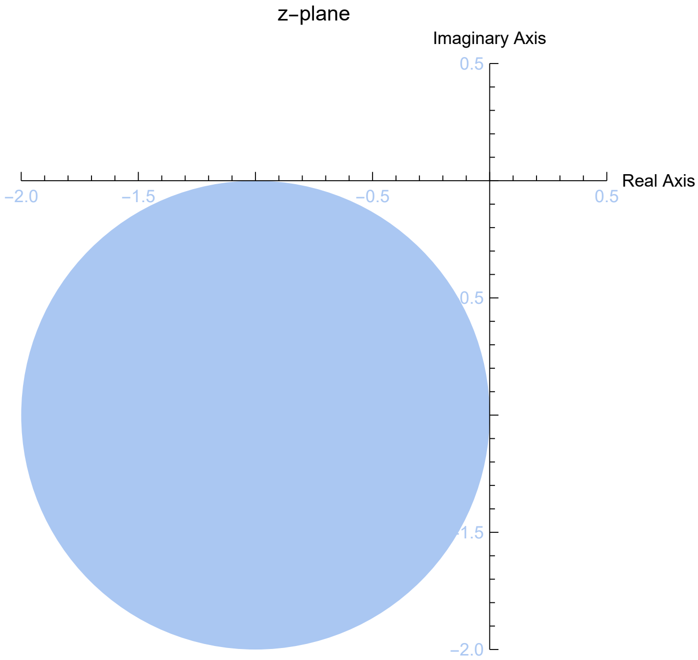
PRACTICAL 4 : Show that the image of the open disk D_1(-1,-i)= {z : |z+1+i|<1} under the linear transformation w = f(z) = (3-4i) z + 6 + 2i is the open disk:
D_5(-1+3i) = {w: |w+1-3i|<5}.

In[ ]:=
```
a = Region[Disk[{-1, -1}, 1], Axes → True, AxesLabel → {"Real Axis", "Imaginary Axis"},
  PlotLabel → "z-plane", PlotRange → {{-2, 0.5}, {-2, 0.5}}]
```

Out[ ]=



In[ ]:=
```
z = x + ⅈy
```

Out[ ]=

$x + i y$

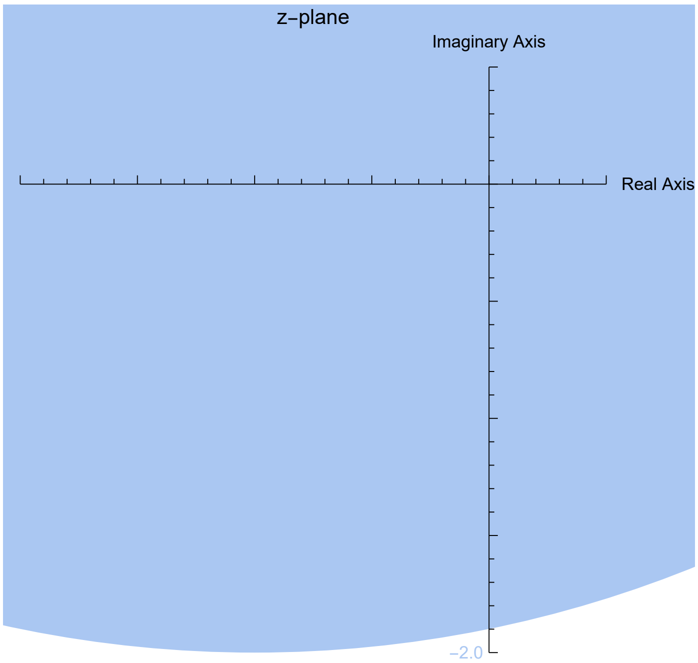
In[ ]:=
```
w = ComplexExpand[(3 - 4 ⅈ) z + (6 + 2 ⅈ)]
```

Out[ ]=

$6 + 3 x + i (2 - 4 x - 4 i y) + 3 i y$

In[∘]:=
```
b = Region[
  TransformedRegion[a, Function[p, {6 + 3 p[[1]] + 4 p[[2]], 2 - 4 p[[1]] + 3 p[[2]]}]],
  PlotLabel → "w-plane", PlotRange → {{-7, 5}, {-3, 9}}
 ]
```

Out[∘]=



In[∘]:=
```
RegionCentroid[b]
```

Out[∘]=

```
{-1., 3.}
```

In[∘]:=

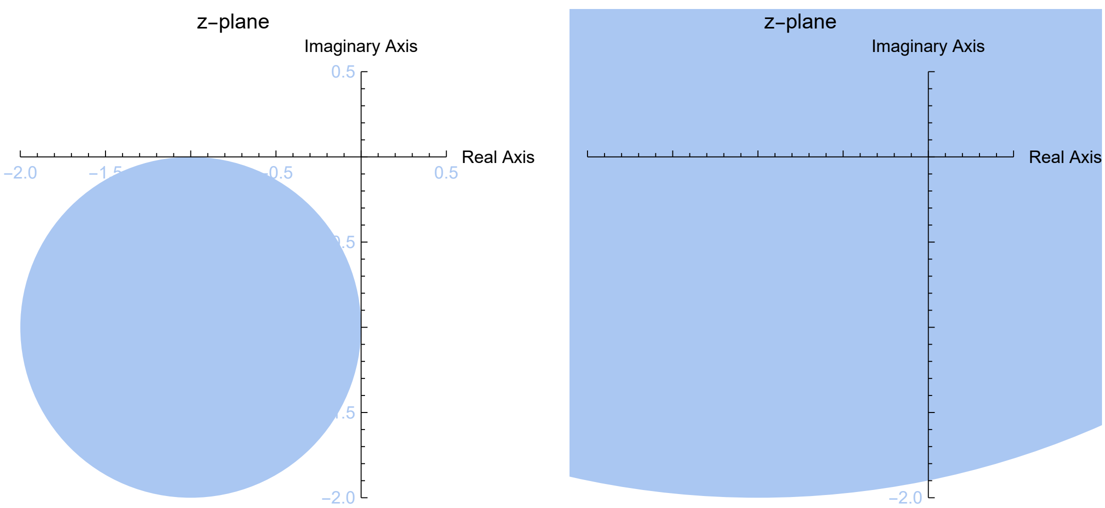
$$\text{radius} = \sqrt{\frac{\text{Area}[b]}{\pi}}$$

Out[∘]=

```
5.
```

In[∘]:=
```
GraphicsRow[{a, b}]
```

Out[∘]=



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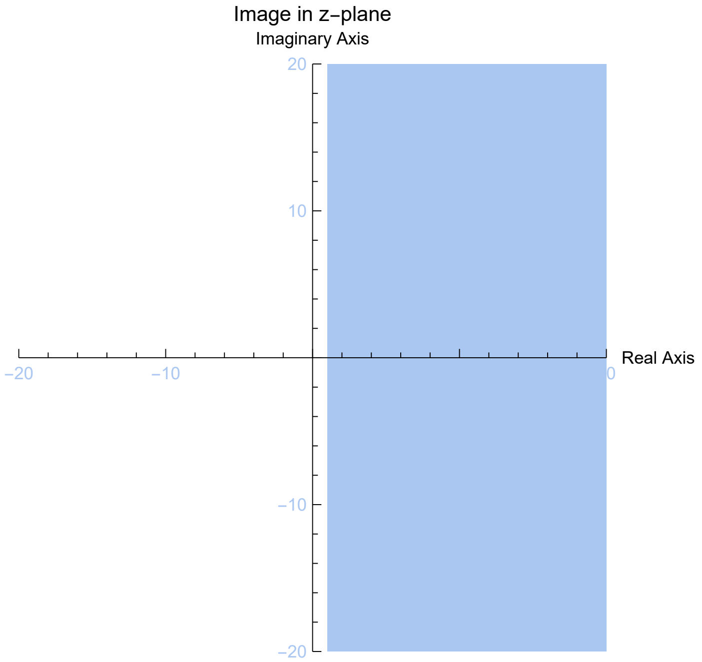
PRACTICAL 5 : Show that the image of the half plane Re[z]>1 under the linear transformation ω=f(z)=(-1+i)z+(-2+3i) is the half plane {w:v>u+7} where u=Re[ω] and v=Im[ω].

In[∘]:=
```
a = Region[
  HalfPlane[{{1, -5}, {1, 9}}, {1, 0}], Axes → True, PlotRange → {{-20, 20}, {-20, 20}},
  AxesLabel → {"Real Axis", "Imaginary Axis"},
  PlotLabel → "Image in z-plane"
 ]
```
Out[∘]=



In[∘]:=
```
Area[a]
```
Out[∘]=

∞

In[∘]:=
```
z = x + y * ⅈ
```
Out[∘]=

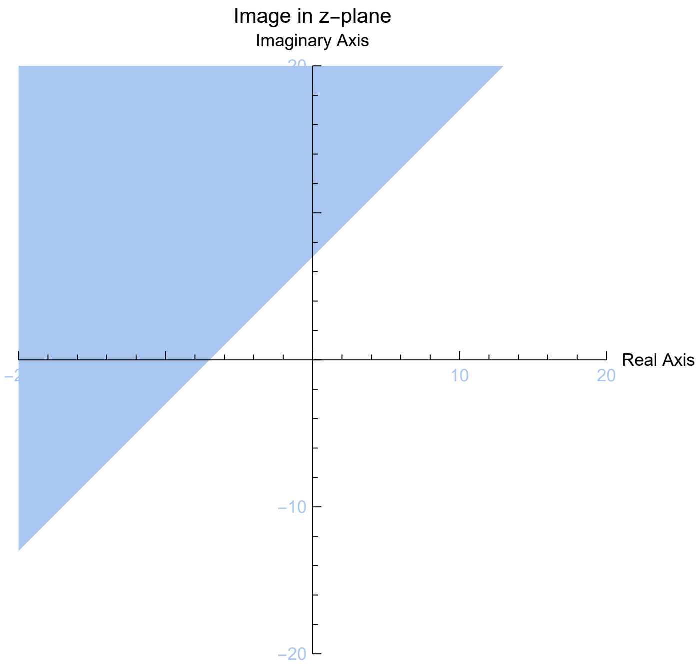
$x + i\, y$

In[∘]:=
```
ω = ComplexExpand[(-1 + ⅈ) * z + (-2 + 3 ⅈ)]
```
Out[∘]=

$-2 - x + i\,(3 + x - y) - y$

In[ ]:=
```
b = Region[
  TransformedRegion[a, Function[p, {-2 - p[[1]] - p[[2]], 3 + p[[1]] - p[[2]]}]],
  Axes → True, PlotRange → {{-20, 20}, {-20, 20}},
  AxesLabel → {"Real Axis", "Imaginary Axis"},
  PlotLabel → "Image in ω-plane"
 ]
```

Out[ ]=



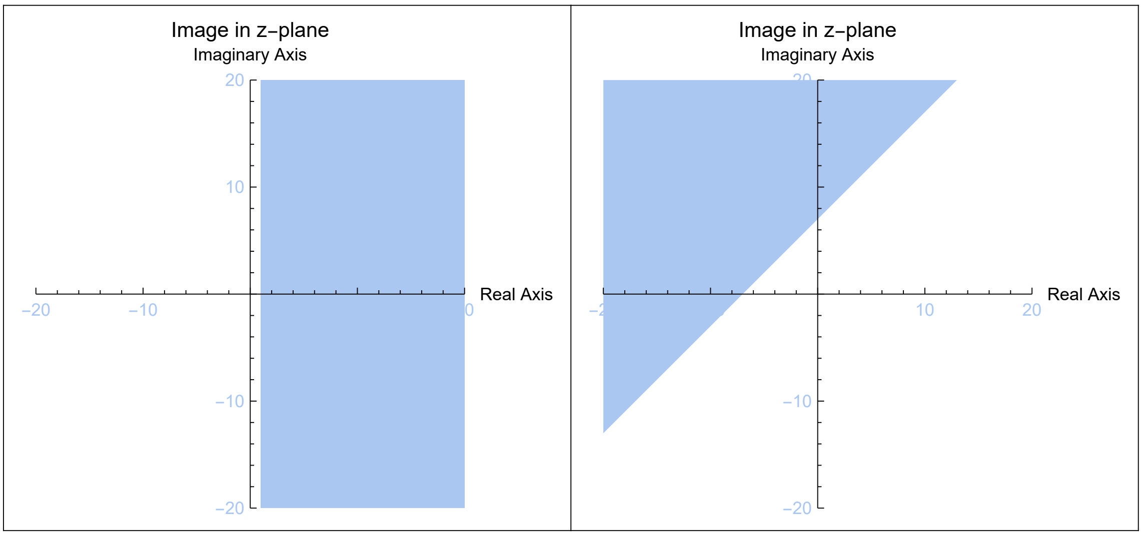
In[ ]:=
```
Area[b]
```

Out[ ]=

∞

In[ ]:=
```
GraphicsRow[{a, b}, Frame → All]
```

Out[ ]=



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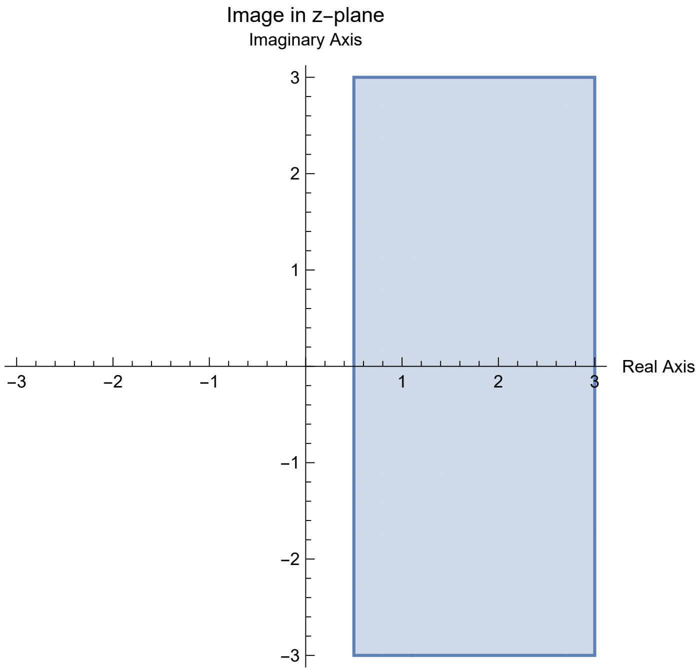
PRACTICAL 6 : Show that image of the half plane {z: Re[z] >= 1/2 } under the linear transformation $\omega = f(z) = \frac{1}{2}$ is the disk $\{\omega: |\omega - 1| < 1\}$

In[ ]:=
```
z = x + ⅈ * y
```
Out[ ]=
$x + i\, y$

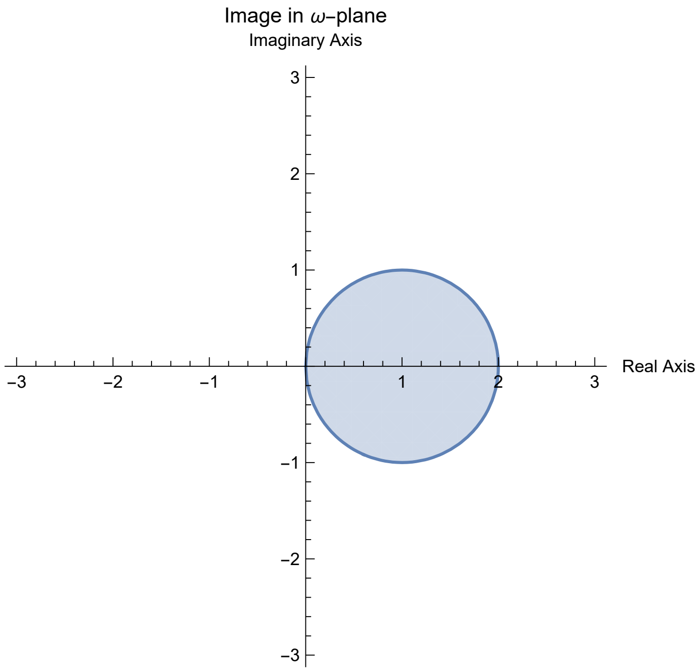
In[ ]:=
```
a = RegionPlot[
  Re[z] ≥ 1/2, {x, -3, 3}, {y, -3, 3}, Axes → True, Frame → False,
  AxesLabel → {"Real Axis", "Imaginary Axis"}, PlotLabel → "Image in z-plane"
 ]
```
Out[ ]=



In[ ]:=
```
f[z_] := 1/z
```
In[ ]:=
```
ω = ComplexExpand[f[z]]
```
Out[ ]=
$$\frac{x}{x^2+y^2} - \frac{i\, y}{x^2+y^2}$$

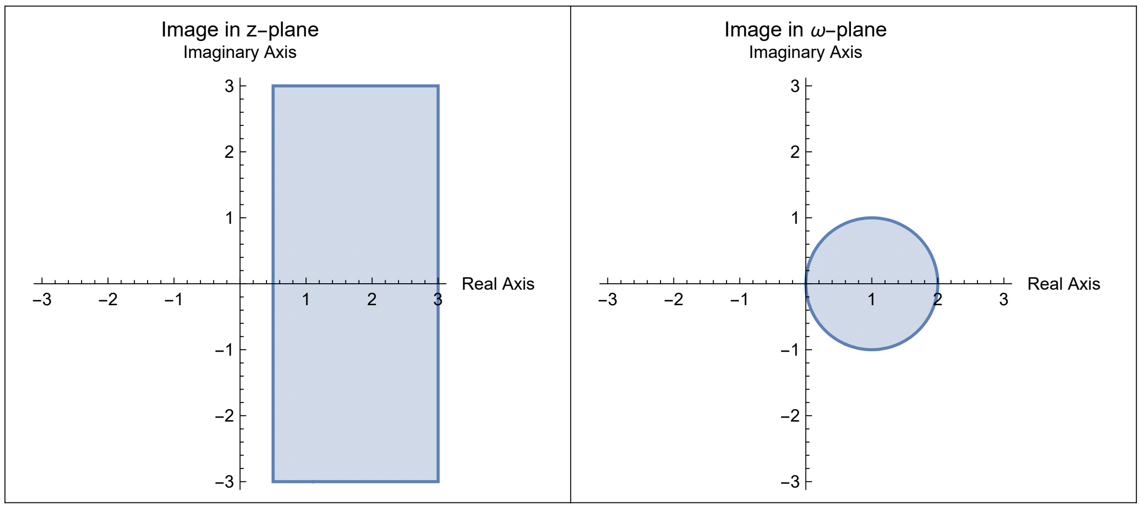
In[ ]:=
```
b = RegionPlot[
  Re[1/z] ≥ 1/2, {x, -3, 3}, {y, -3, 3}, Axes → True, Frame → False,
  AxesLabel → {"Real Axis", "Imaginary Axis"}, PlotLabel → "Image in ω-plane"
 ]
```

Out[ ]=



In[ ]:=
```
GraphicsRow[{a, b}, Frame → All]
```

Out[ ]=



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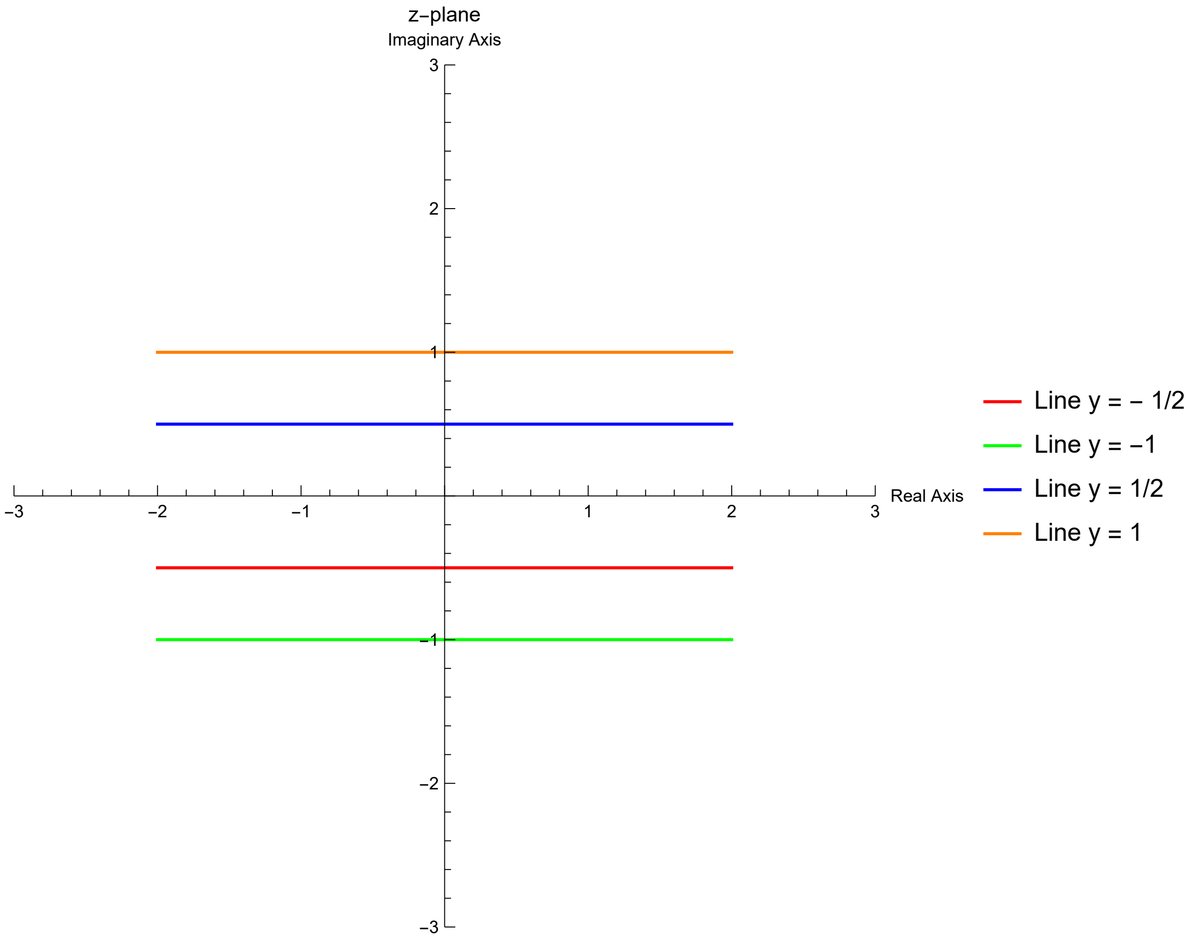
PRACTICAL 7 : Plot the lines y = a, where a = -1/2, -1, 1/2, 1; the lines x = a where a = -1/2, -1, 1/2, 1;

and the corresponding grid. Also find the image of this grid under the mapping f(z) = 1/z .

In[ ]:=
```
f[z_] := 1 / z
```

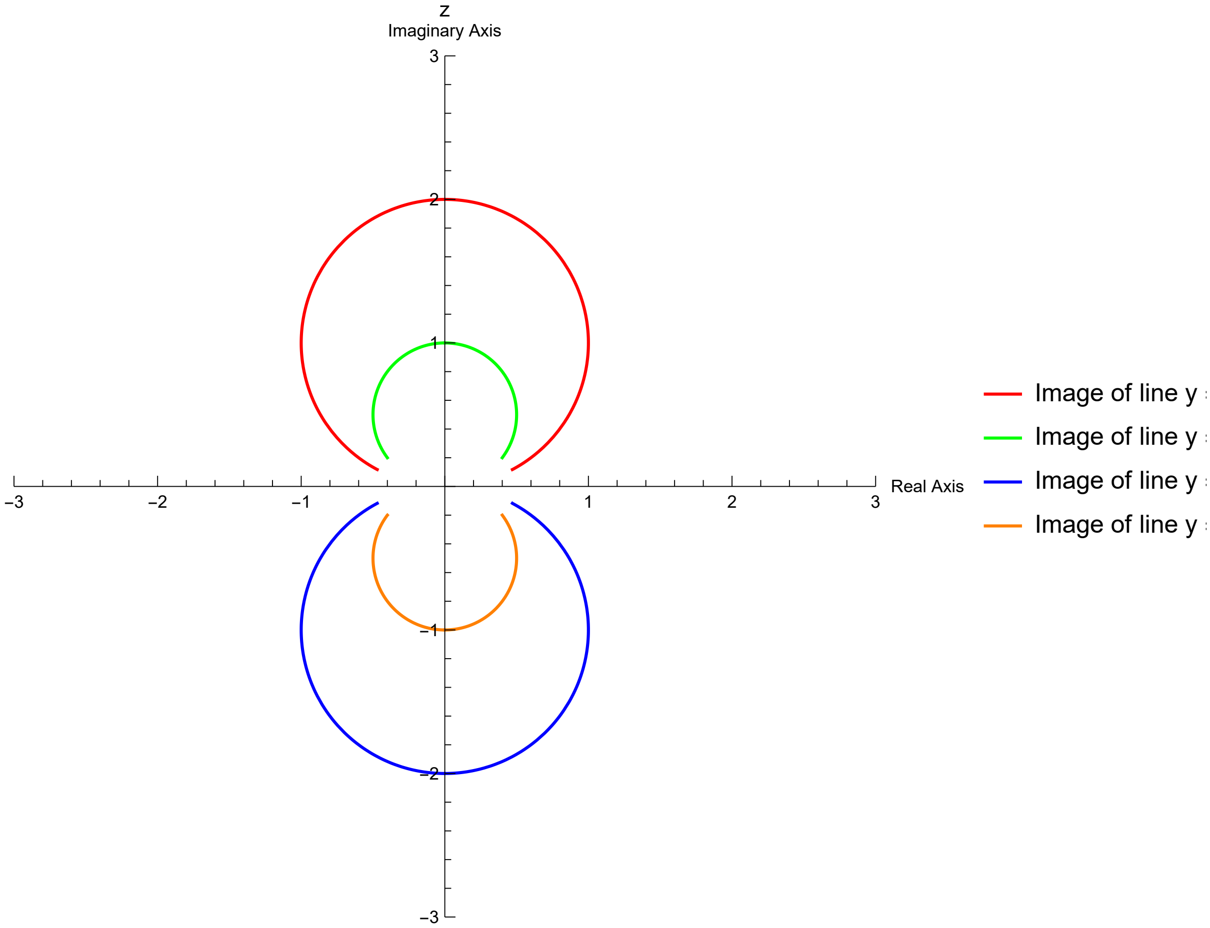
In[ ]:=
```
a1 = ParametricPlot[
  Evaluate[Table[{Re[t + ⅈ a], Im[t + ⅈ a]}, {a, {- 1 / 2 , -1, 1 / 2 , 1}}]],
  {t, -2, 2}, PlotStyle → {Red, Green, Blue, Orange}, Axes → True, Frame → False,
  AxesLabel → {"Real Axis", "Imaginary Axis"}, PlotLabel → "z-plane",
  PlotLegends → {"Line y = - 1/2 ", "Line y = -1", "Line y = 1/2 ", "Line y = 1"},
  PlotRange → 3, ImageSize → {500, 500}]
```

Out[ ]=



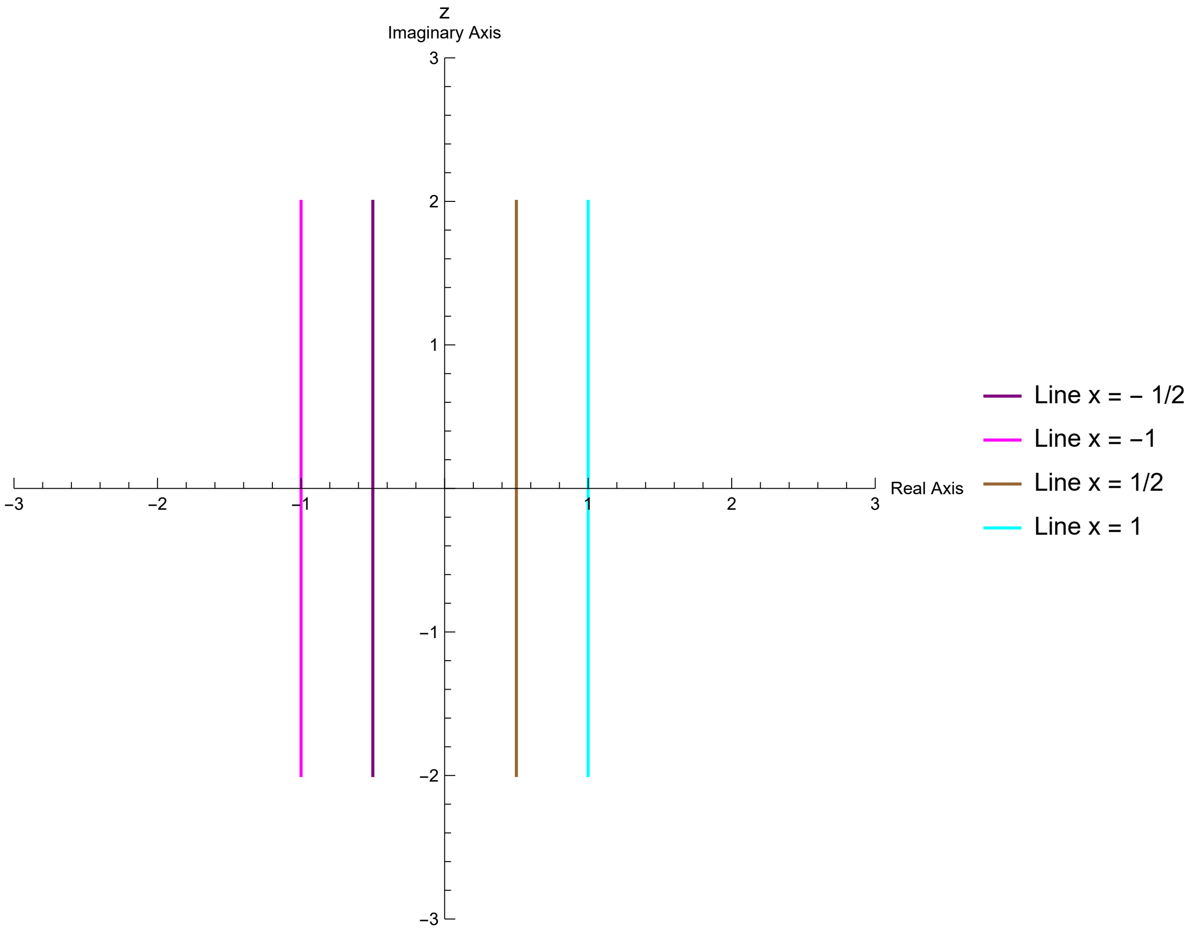
In[∘]:=
```
b1 = ParametricPlot[
  Evaluate[Table[{Re[f[t + ⅈ a]], Im[f[t + ⅈ a]]}, {a, {- 1 / 2 , -1, 1 / 2 , 1}}]],
  {t, -2, 2}, PlotStyle → {Red, Green, Blue, Orange}, Axes → True, Frame → False,
  AxesLabel → {"Real Axis", "Imaginary Axis"}, PlotLabel → "z", PlotLegends →
   {"Image of line y = - 1/2 ", "Image of line y = -1", "Image of line y = 1/2 ",
    "Image of line y = 1"}, PlotRange → 3, ImageSize → {500, 500}]
```

Out[∘]=



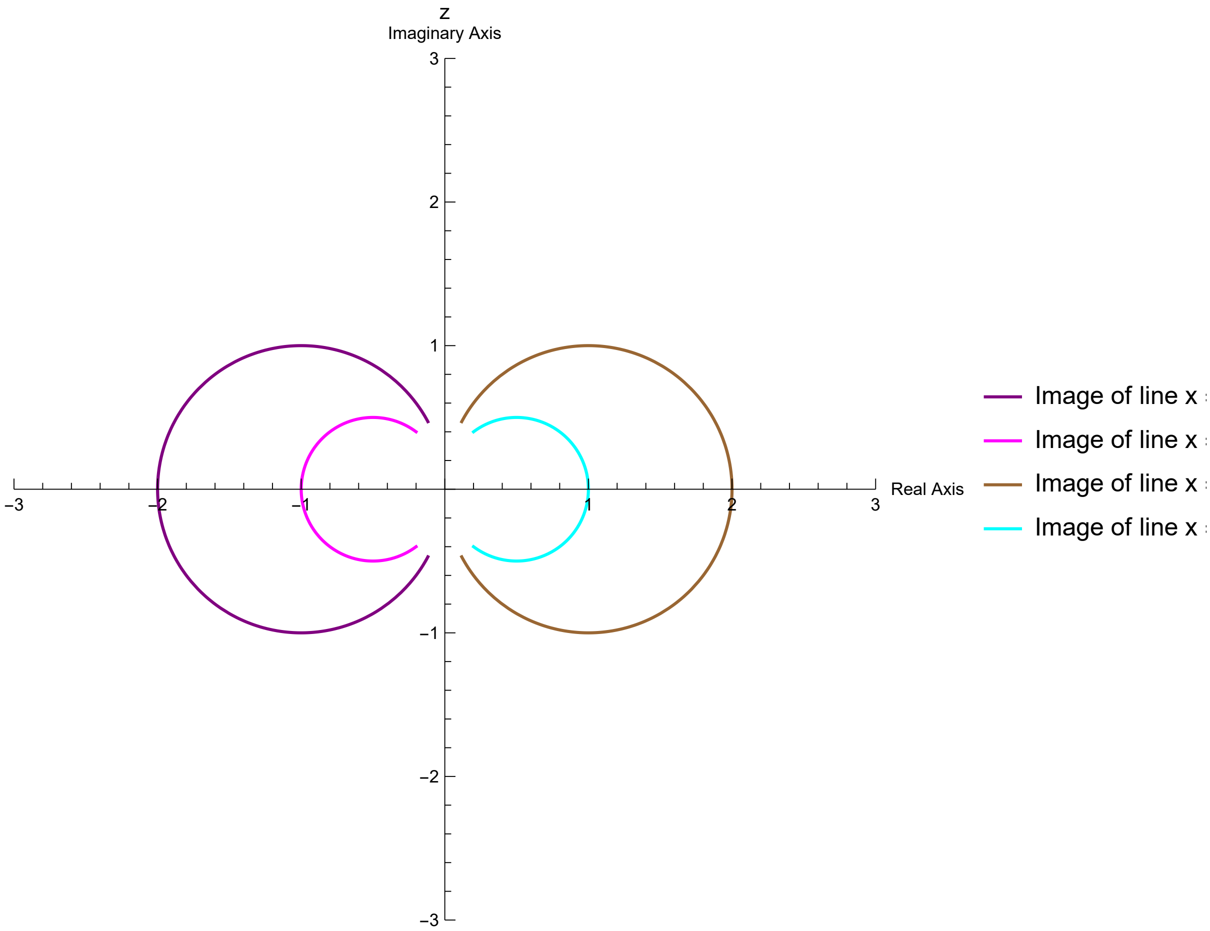
In[∘]:=
```
a2 = ParametricPlot[
  Evaluate[Table[{Re[a + ⅈ t], Im[a + ⅈ t]}, {a, {- 1 / 2 , -1, 1 / 2 , 1}}]],
  {t, -2, 2}, PlotStyle → {Purple, Magenta, Brown, Cyan}, Axes → True,
  Frame → False, AxesLabel → {"Real Axis", "Imaginary Axis"}, PlotLabel → "z",
  PlotLegends → {"Line x = - 1/2 ", "Line x = -1", "Line x = 1/2 ", "Line x = 1"},
  PlotRange → 3, ImageSize → {500, 500}]
```

Out[∘]=

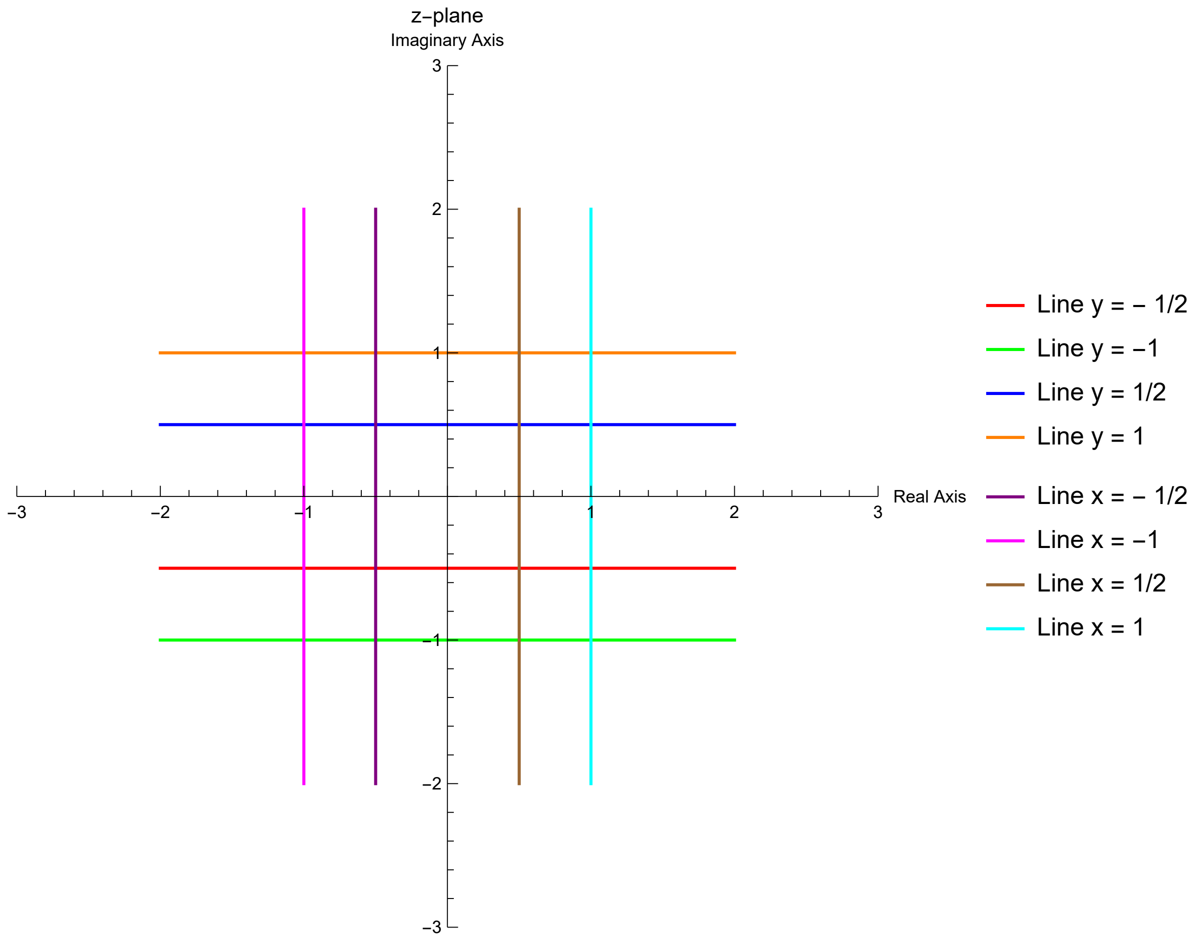


In[*]:=
```
b2 = ParametricPlot[
  Evaluate[Table[{Re[f[a + ⅈ t]], Im[f[a + ⅈ t]]}, {a, {- 1 / 2 , -1, 1 / 2 , 1}}]],
  {t, -2, 2}, PlotStyle → {Purple, Magenta, Brown, Cyan}, Axes → True,
  Frame → False, AxesLabel → {"Real Axis", "Imaginary Axis"},
  PlotLabel → "z", PlotLegends → {"Image of line x = - 1/2 ",
    "Image of line x = -1", "Image of line x = 1/2 ", "Image of line x = 1"},
  PlotRange → 3, ImageSize → {500, 500}]
```

Out[*]=



In[∘]:=
```
{Show[a1, a2], Show[b1, b2]}
```
Out[∘]=

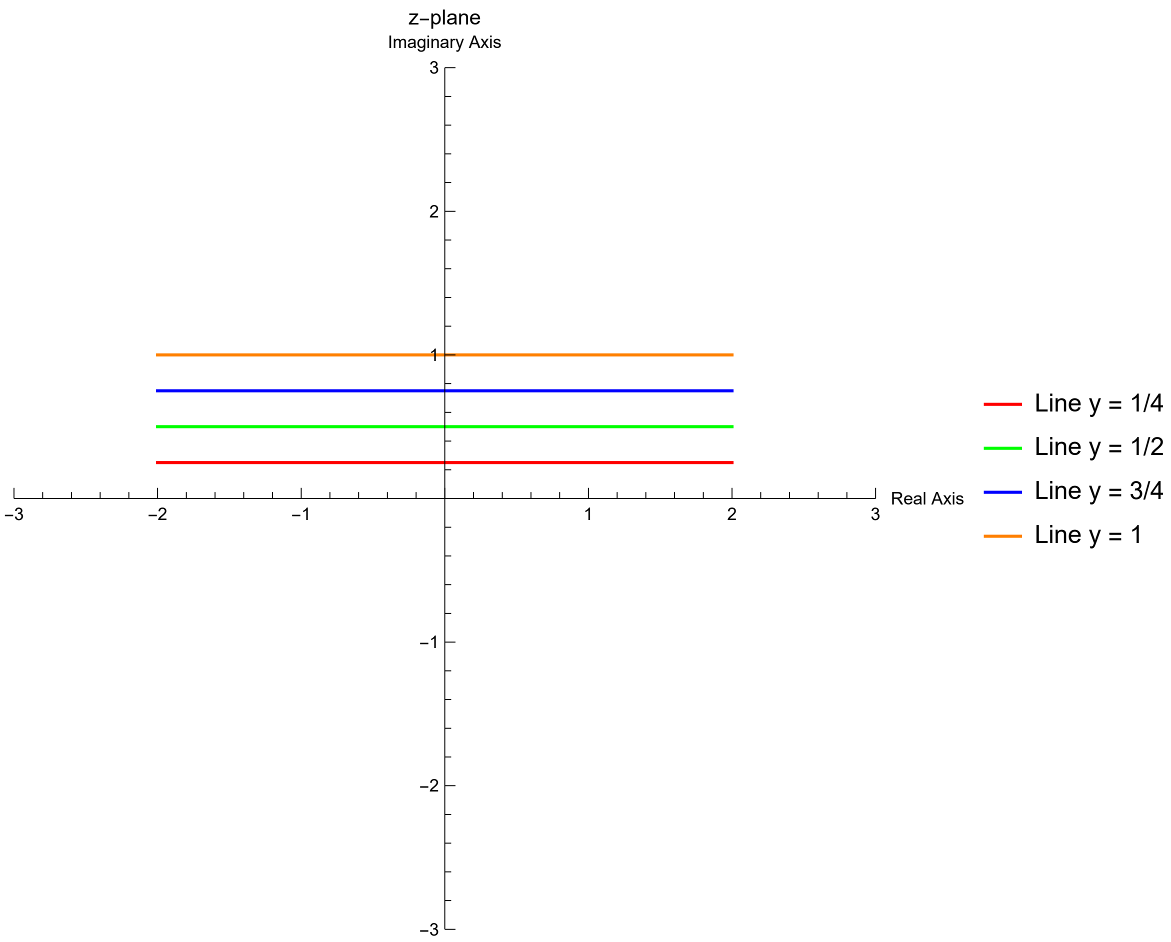
{ 

Question 2: Plot the lines y = a, where a = 1/4 , 1/2 , 3/4 , 1; the lines x = a, where a = 1/4 , 1/2 , 3/4 , 1 and the corresponding grid. Also find the image of this grid under the mapping f(z) = z^2.

In[∘]:=
```
f[z_] := z^2
```

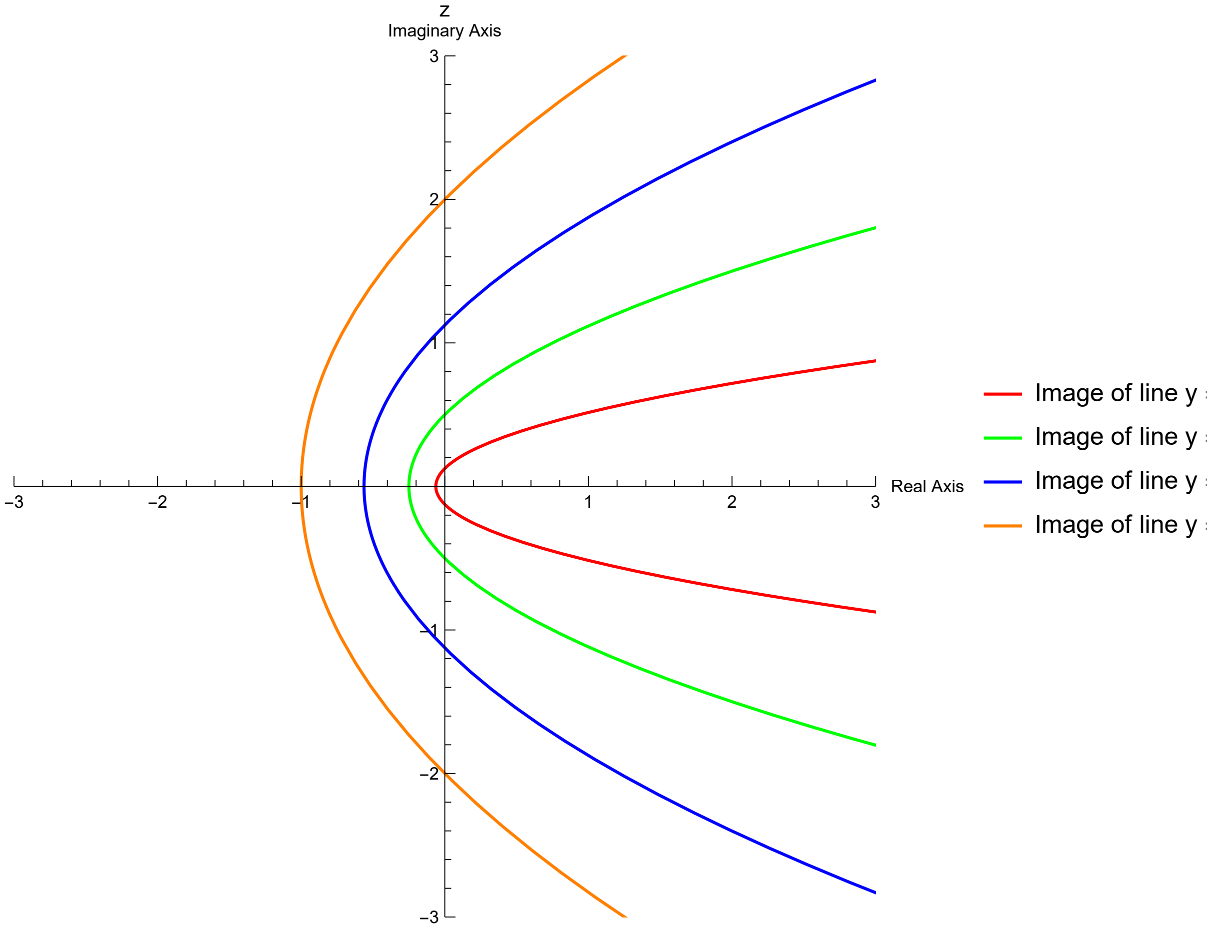
In[ ]:=
```
a1 = ParametricPlot[
  Evaluate[Table[{Re[t + ⅈ a], Im[t + ⅈ a]}, {a, {1 / 4, 1 / 2, 3 / 4, 1}}]],
  {t, -2, 2}, PlotStyle → {Red, Green, Blue, Orange}, Axes → True, Frame → False,
  AxesLabel → {"Real Axis", "Imaginary Axis"}, PlotLabel → "z-plane",
  PlotLegends → {"Line y = 1/4 ", "Line y = 1/2", "Line y = 3/4 ", "Line y = 1"},
  PlotRange → 3, ImageSize → {500, 500}]
```

Out[ ]=



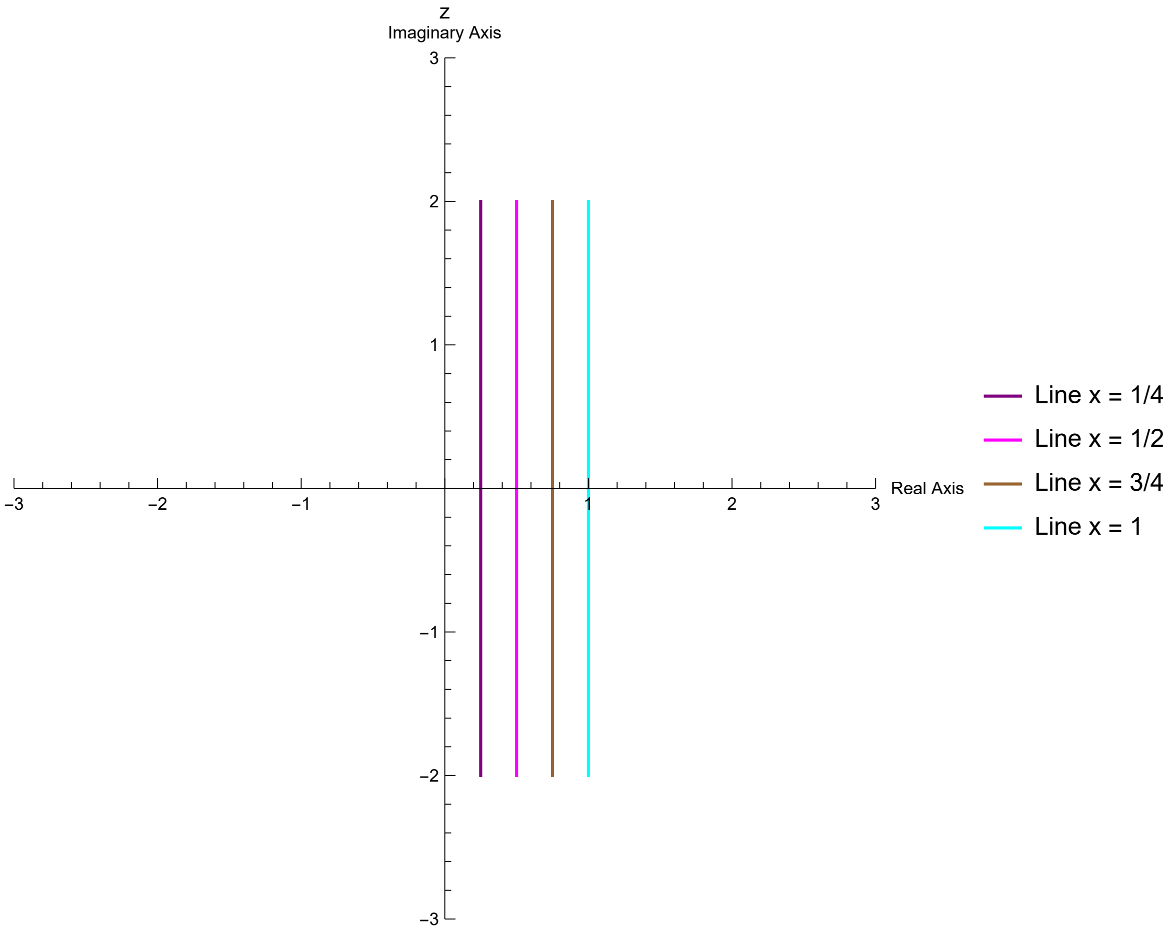
In[∘]:=
```
b1 = ParametricPlot[
  Evaluate[Table[{Re[f[t + ⅈ a]], Im[f[t + ⅈ a]]}, {a, {1 / 4, 1 / 2, 3 / 4, 1}}]],
  {t, -2, 2}, PlotStyle → {Red, Green, Blue, Orange}, Axes → True, Frame → False,
  AxesLabel → {"Real Axis", "Imaginary Axis"}, PlotLabel → "z", PlotLegends →
   {"Image of line y = 1/4 ", "Image of line y = 1/2", "Image of line y = 3/4 ",
    "Image of line y = 1"}, PlotRange → 3, ImageSize → {500, 500}]
```

Out[∘]=



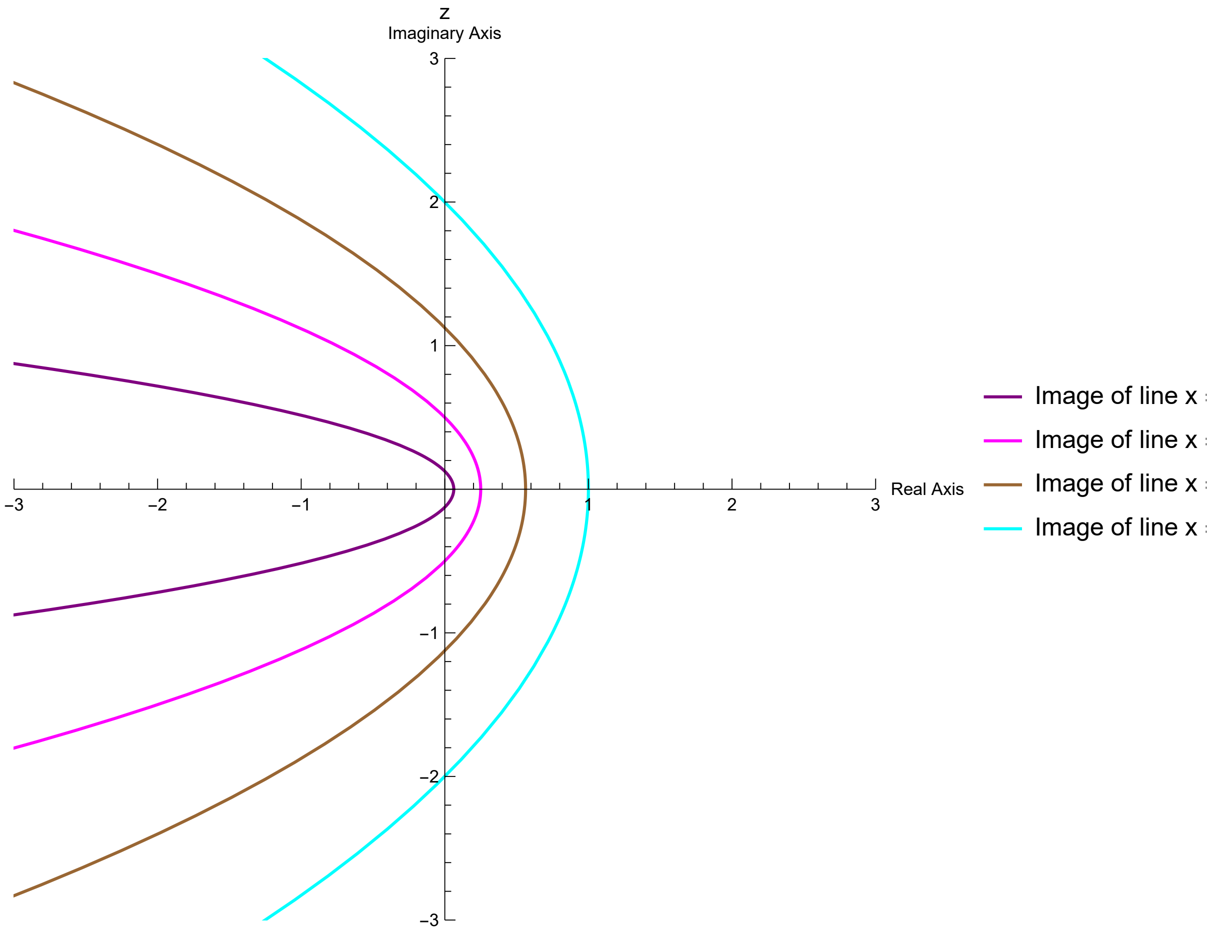
In[∘]:=
```
a2 = ParametricPlot[
  Evaluate[Table[{Re[a + ⅈ t], Im[a + ⅈ t]}, {a, {1 / 4, 1 / 2, 3 / 4, 1}}]],
  {t, -2, 2}, PlotStyle → {Purple, Magenta, Brown, Cyan}, Axes → True,
  Frame → False, AxesLabel → {"Real Axis", "Imaginary Axis"}, PlotLabel → "z",
  PlotLegends → {"Line x = 1/4 ", "Line x = 1/2", "Line x = 3/4 ", "Line x = 1"},
  PlotRange → 3, ImageSize → {500, 500}]
```

Out[∘]=

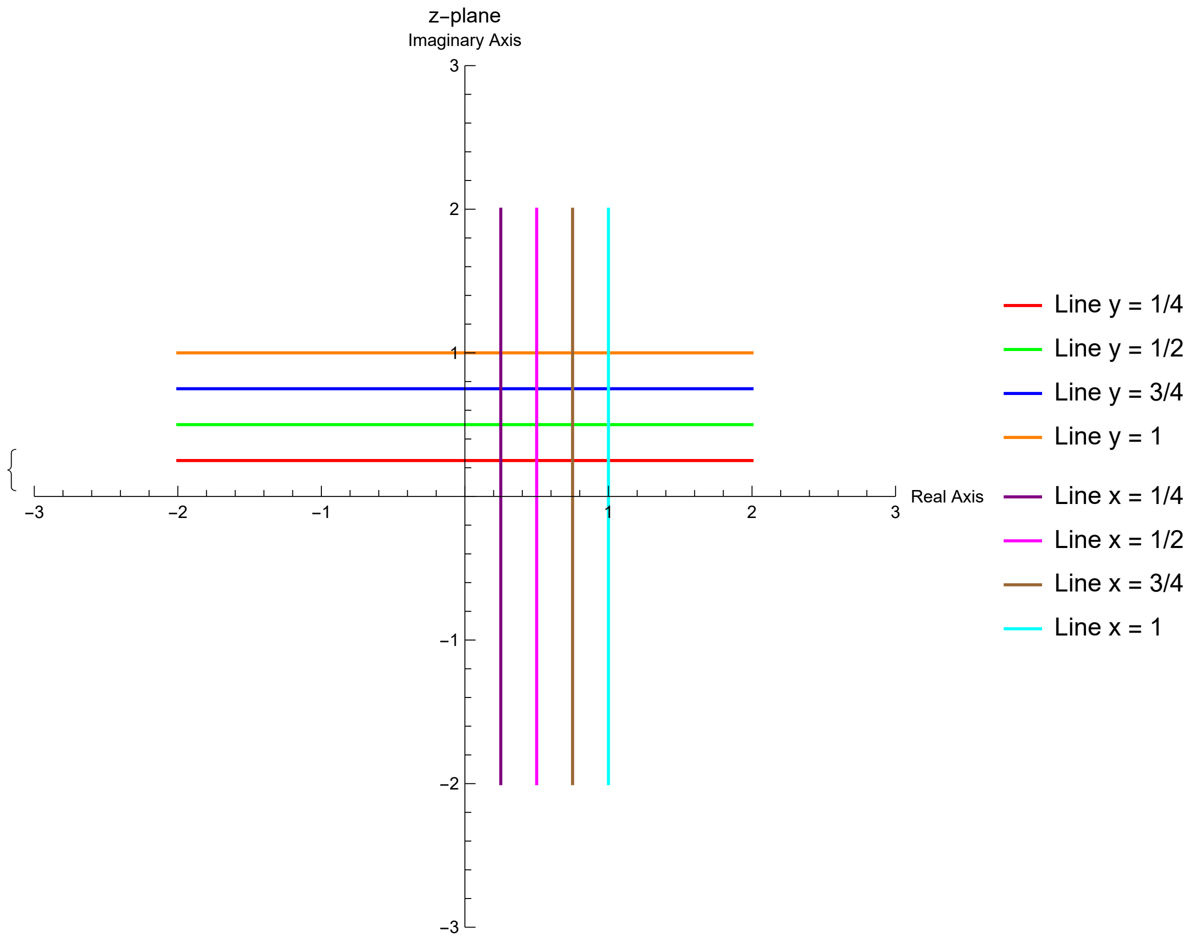


In[ ]:=
```
b2 = ParametricPlot[
  Evaluate[Table[{Re[f[a + ⅈ t]], Im[f[a + ⅈ t]]}, {a, {1 / 4, 1 / 2, 3 / 4, 1}}]],
  {t, -2, 2}, PlotStyle → {Purple, Magenta, Brown, Cyan}, Axes → True,
  Frame → False, AxesLabel → {"Real Axis", "Imaginary Axis"},
  PlotLabel → "z", PlotLegends → {"Image of line x = 1/4 ",
    "Image of line x = 1/2", "Image of line x = 3/4 ", "Image of line x = 1"},
  PlotRange → 3, ImageSize → {500, 500}]
```

Out[ ]=



In[∘]:=
```
{Show[a1, a2], Show[b1, b2]}
```
Out[∘]=



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PRACTICAL 8 : Find a parametrization of a line segment joining the two specified end points. Also plot the line segment

1.1 : z0 = -1 + i , z1 = 2 - i

In[∘]:=
```
(* Initial and Terminal points *)
z0 = -1 + i; z1 = 2 - i;
(* Real and Imaginary parts of given points *)
x0 = Re[z0];
y0 = Im[z0];
x1 = Re[z1];
y1 = Im[z1];
```

In[∘]:=
```
z[t_] = (x0 + (x1 - x0) t) + i (y0 + (y1 - y0) t);
```

In[∘]:=
```
dot0 = Graphics[{PointSize[0.03], Red, Point[{Re[z0], Im[z0]}]}];
dot1 = Graphics[{PointSize[0.03], Green, Point[{Re[z1], Im[z1]}]}];
```

In[∘]:=
```
line =
  ParametricPlot[{Re[z[t]], Im[z[t]]}, {t, 0, 1}, PlotRange → {{-2, 3}, {-2, 2}},
Ticks → {Range[-2, 3, 1 / 2], Range[-2, 2, 1 / 2]},
   PlotStyle → Blue, AxesLabel → {"Real Axis", "Imaginary Axis"}];
```

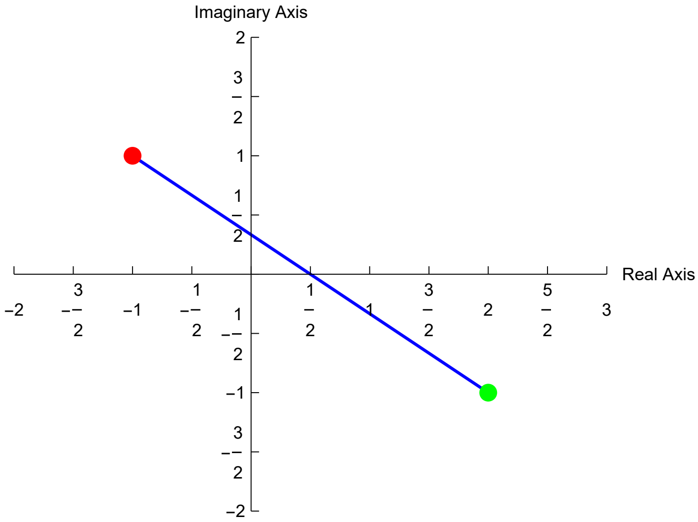
In[∘]:=
```
Print["Equation of the line segment joining the points ", z0, " and ", z1, " is "];
Print[" z[t] = ", z[t], " where 0 ≤ t ≤ 1"];
Print["The required plot is "];
Show[line, dot0, dot1]
```

Equation of the line segment joining the points $-1+i$ and $2-i$ is

$z[t] = -1 + i\,(1-2t) + 3t$ where $0 \le t \le 1$

The required plot is

Out[∘]=



1.2 : $z0 = -1 + 3\,i,\ z1 = 2 + i$

In[∘]:=
```
(* Initial and Terminal points *)
z0 = -1 + 3 i; z1 = 2 + i;
(* Real and Imaginary parts of given points *)
x0 = Re[z0];
y0 = Im[z0];
x1 = Re[z1];
y1 = Im[z1];
```

In[∘]:=
```
z[t_] = (x0 + (x1 - x0) t) + i (y0 + (y1 - y0) t);
```

In[∘]:=
```
dot0 = Graphics[{PointSize[0.03], Red, Point[{Re[z0], Im[z0]}]}];
dot1 = Graphics[{PointSize[0.03], Green, Point[{Re[z1], Im[z1]}]}];
```

In[∘]:=
```
line =
  ParametricPlot[{Re[z[t]], Im[z[t]]}, {t, 0, 1}, PlotRange → {{-2, 3}, {-2, 2}},
Ticks → {Range[-2, 3, 1 / 2], Range[-2, 2, 1 / 2]},
   PlotStyle → Blue, AxesLabel → {"Real Axis", "Imaginary Axis"}];
```

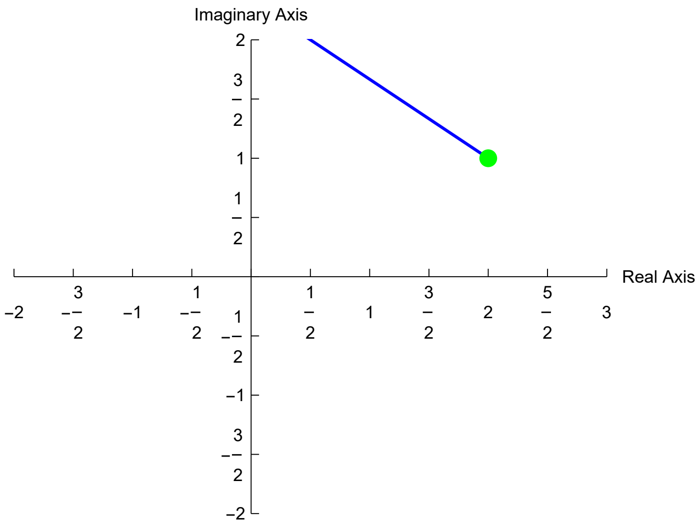
In[ ]:=
```
Print["Equation of the line segment joining the points ", z0, " and ", z1, " is "];
Print[" z[t] = ", z[t], " where 0 ≤ t ≤ 1"];
Print["The required plot is "];
Show[line, dot0, dot1]
```

Equation of the line segment joining the points $-1+3\,i$ and $2+i$ is

z[t] = $-1+i\,(3-2\,t)+3\,t$ where 0 ≤ t ≤ 1

The required plot is

Out[ ]=



1.3 : z0 = -2 - i , z1 = 1 + i

In[ ]:=
```
(* Initial and Terminal points *)
z0 = -2 - i; z1 = 1 + i;
(* Real and Imaginary parts of given points *)
x0 = Re[z0];
y0 = Im[z0];
x1 = Re[z1];
y1 = Im[z1];
```

In[ ]:=
```
Clear[z]
z[t_] = (x0 + (x1 - x0) t) + i (y0 + (y1 - y0) t);
```

In[ ]:=
```
dot0 = Graphics[{PointSize[0.03], Red, Point[{Re[z0], Im[z0]}]}];
dot1 = Graphics[{PointSize[0.03], Green, Point[{Re[z1], Im[z1]}]}];
```

In[ ]:=
```
line =
  ParametricPlot[{Re[z[t]], Im[z[t]]}, {t, 0, 1}, PlotRange → {{-2, 3}, {-2, 2}},
Ticks → {Range[-2, 3, 1 / 2], Range[-2, 2, 1 / 2]},
   PlotStyle → Blue, AxesLabel → {"Real Axis", "Imaginary Axis"}];
```

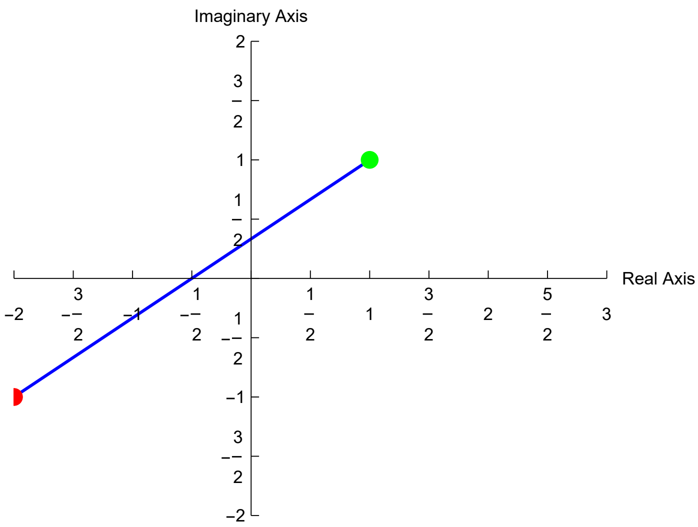
In[ ]:=
```
Print["Equation of the line segment joining the points ", z0, " and ", z1, " is "];
Print[" z[t] = ", z[t], " where 0 ≤ t ≤ 1"];
Print["The required plot is "];
Show[line, dot0, dot1]
```

Equation of the line segment joining the points $-2 - i$ and $1 + i$ is

$z[t] = -2 + 3t + i(-1 + 2t)$ where $0 \leq t \leq 1$

The required plot is

Out[∘]=



Question 2: Find a parametrization of a polygonal path C. Also make a plot of it.

2.1 : C = C1 + C2 + C3 from $-1 + i$ to $3 - i$

where C1 is a line from $-1 + i$ to -1,

C2 is a line from -1 to $1 + i$,

C3 is a line from $1 + i$ to $3 - i$.

In[∘]:=
```
(* Initial and Terminal points of the paths Ci *)
z0 = -1 + i; z1 = -1; z2 = 1 + i; z3 = 3 - i;
(* Real and Imaginary parts of given points *)
x0 = Re[z0];
y0 = Im[z0];
x1 = Re[z1];
y1 = Im[z1];
x2 = Re[z2];
y2 = Im[z2];
x3 = Re[z3];
y3 = Im[z3];
```

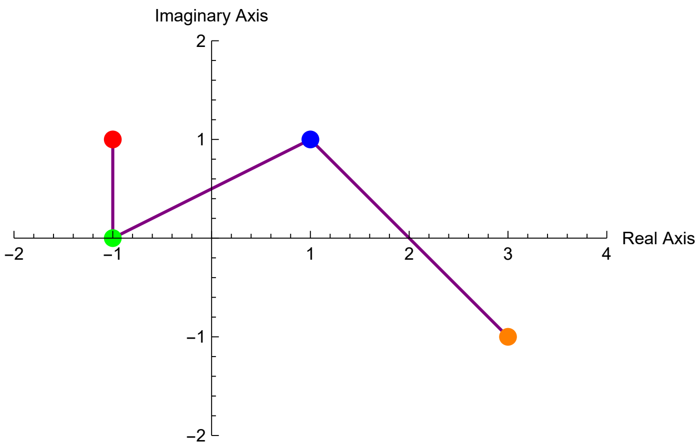
In[∘]:=
```
(* Equation of path C1: Line joining z0 to z1 *)
p1[t_] = (x0 + (x1 - x0) t) + i (y0 + (y1 - y0) t);
(* Equation of path C2: Line joining z1 to z2 *)
p2[t_] = (x1 + (x2 - x1) t) + i (y1 + (y2 - y1) t);
(* Equation of path C3: Line joining z2 to z3 *)
p3[t_] = (x2 + (x3 - x2) t) + i (y2 + (y3 - y2) t);
```

In[∘]:=
```
dot0 = Graphics[{PointSize[0.03], Red, Point[{Re[z0], Im[z0]}]}];
dot1 = Graphics[{PointSize[0.03], Green, Point[{Re[z1], Im[z1]}]}];
dot2 = Graphics[{PointSize[0.03], Blue, Point[{Re[z2], Im[z2]}]}];
dot3 = Graphics[{PointSize[0.03], Orange, Point[{Re[z3], Im[z3]}]}];
```

In[∘]:=
```
line = ParametricPlot[ {{Re[p1[t]], Im[p1[t]]},
{Re[p2[t]], Im[p2[t]]}, {Re[p3[t]], Im[p3[t]]}}, {t, 0, 1},
PlotRange → {{-2, 4}, {-2, 2}}, PlotStyle → {Purple},
AxesLabel → {"Real Axis", "Imaginary Axis"}];
```

In[∘]:=
```
Show[line, dot0, dot1, dot2, dot3]
```

Out[∘]=



Question 3: Find a parametrization of the given curve. Also make a plot of it.

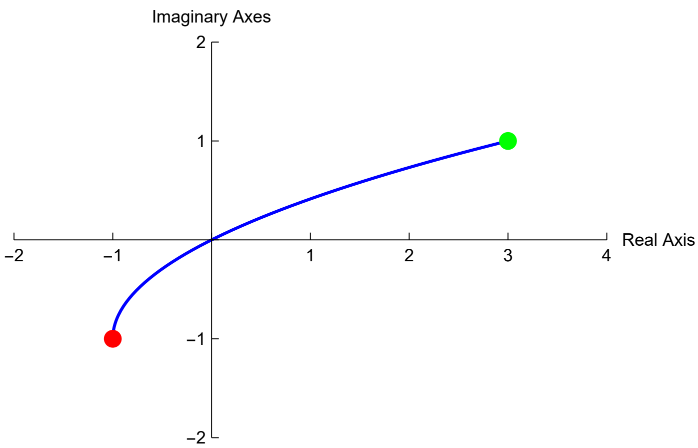
3.1 : C is the portion of the parabola x = y^2 + 2 y joining - 1 - *i* to 3 + *i*.

In[∘]:=
```
Clear[z]
z[t_] = (t^2 + 2 t) + i t;
g1 = ParametricPlot[{Re[z[t]], Im[z[t]]}, {t, -1, 1},
PlotRange → {{-2, 4}, {-2, 2}}, Ticks → {Range[-2, 4, 1],
Range[-2, 2, 1]}, AxesLabel → {"Real Axis", "Imaginary Axes"},
PlotStyle → Blue];

g2 = Graphics[{PointSize[0.03], Red, Point[{-1, -1}], Green, Point[{3, 1}]}];

Show[ g1, g2]
```

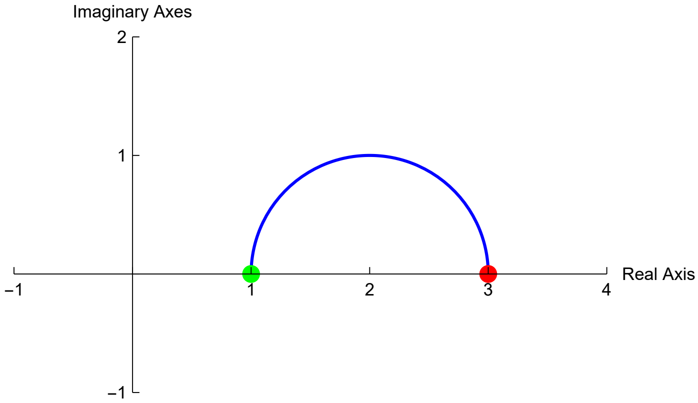
Out[∘]=



3.2 : C is the upper semicircle with radius 1 centered at point (2, 0) in the anticlockwise sense.

In[ ]:=
```
Clear[z]
z[t_] = (2 + Cos[t]) + i Sin[t];
g1 = ParametricPlot[{Re[z[t]], Im[z[t]]}, {t, 0, π},
PlotRange → {{-1, 4}, {-1, 2}}, Ticks → {Range[-2, 4, 1], Range[-2, 2, 1]},
AxesLabel → {"Real Axis", "Imaginary Axes"}, PlotStyle → Blue];
g2 = Graphics[{PointSize[0.03], Red, Point[{3, 0}], Green, Point[{1, 0}]}];
Show[ g1, g2]
```

Out[ ]=



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SECTION : A

PRACTICAL 9 : Plot the line segment joining the two specified end points. And find the integral of the given function over that specified line segment.

1. f (z) = z, z0 = -1 + 3 i, z1 = 2 + i

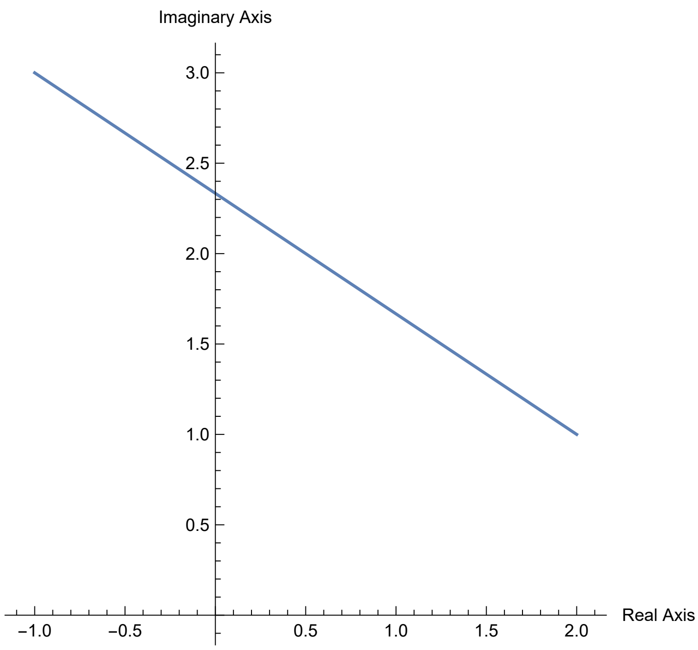
In[ ]:=
```
Integrate[z, {z, -1 + 3 i, 2 + i}]
```

Out[ ]=

$$\frac{11}{2} + 5\,i$$

In[ ]:=
```
ClearAll;
f[z_] = z ;
z0 = -1 + 3 ⅈ;
z1 = 2 + ⅈ;
x0 = Re[z0]; y0 = Im[z0];
x1 = Re[z1]; y1 = Im[z1];
L[t_] = x0 + (x1 - x0) t + ⅈ (y0 + (y1 - y0) t);
w[t_] = f[L[t]] * L'[t];
W1 = Integrate[w[t], {t, 0, 1} ];

ParametricPlot[{Re[L[t]], Im[L[t]]}, {t, 0, 1}, Axes → True,
 AxesOrigin → {0, 0}, AxesLabel → {"Real Axis", "Imaginary Axis"}]

Print["The value of the integration is"];
Print["∫ C ", f[z], " ⅆz = ", W1];
Print["where C: c[t] = ", L[t], ", for 0 ≤ t ≤ 1."];
```

Out[ ]=



The value of the integration is

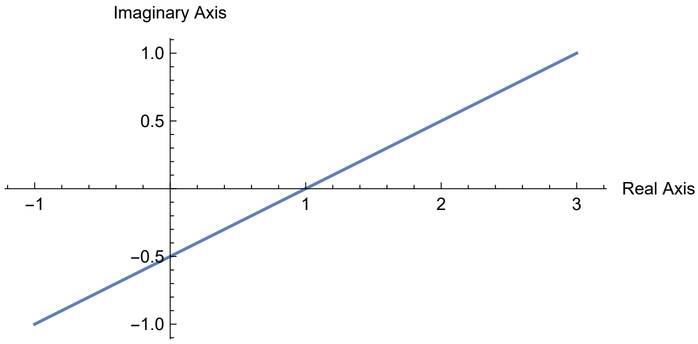
$\int C\ z\ dz = \frac{11}{2} + 5\,i$

where C: c[t] = −1 + ⅈ (3 − 2 t) + 3 t, for 0 ≤ t ≤ 1.

2. f (z) = z, z0 = -1 - ⅈ, z1 = 3 + ⅈ

In[∘]:=
```
f[z_] = z ; z0 = -1 - i;
z1 = 3 + i;

x0 = Re[z0]; y0 = Im[z0];
x1 = Re[z1]; y1 = Im[z1];

L[t_] = x0 + (x1 - x0) t + i (y0 + (y1 - y0) t);
w[t_] = ComplexExpand[f[L[t]] * L'[t]];
W1 = Integrate[w[t], {t, 0, 1}] ;

ParametricPlot[{Re[L[t]], Im[L[t]]}, {t, 0, 1}, Axes → True,
AxesOrigin → {0, 0}, AxesLabel → {"Real Axis", "Imaginary Axis"}]

Print["The value of the integration is"];
Print["∫ C ", f[z], " dz = ", W1];
Print["where C: c[t] = ", L[t], ", for 0 ≤ t ≤ 1."];
```

Out[∘]=



The value of the integration is

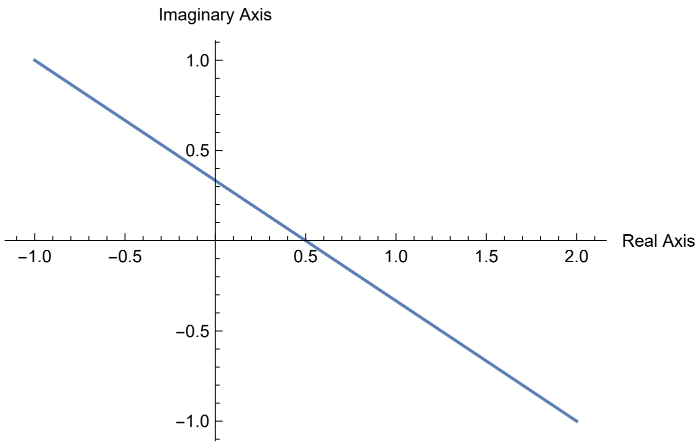
$\int$ C z dz = 4 + 2 i

where C: c[t] = −1 + 4 t + i (−1 + 2 t), for 0 ≤ t ≤ 1.

3: f (z) = sin z, z0 = -1 + *i*, z1 = 2 - *i*

In[•]:=
```
f[z_] = Sin[z] ;
z0 = -1 + i;
z1 = 2 - i;

x0 = Re[z0]; y0 = Im[z0];
x1 = Re[z1]; y1 = Im[z1];

L[t_] = x0 + (x1 - x0) t + i (y0 + (y1 - y0) t);
w[t_] = ComplexExpand[f[L[t]] * L'[t]];
W1 = Integrate[w[t], {t, 0, 1}] ;

ParametricPlot[{Re[L[t]], Im[L[t]]}, {t, 0, 1}, Axes → True,
AxesOrigin → {0, 0}, AxesLabel → {"Real Axis", "Imaginary Axis"}]

Print["The value of the integration is"];
Print[" ∫ C ", f[z], " ⅆz = ", W1];
Print["where C: c[t] = ", L[t], ", for 0 ≤ t ≤ 1."];
```

Out[•]=

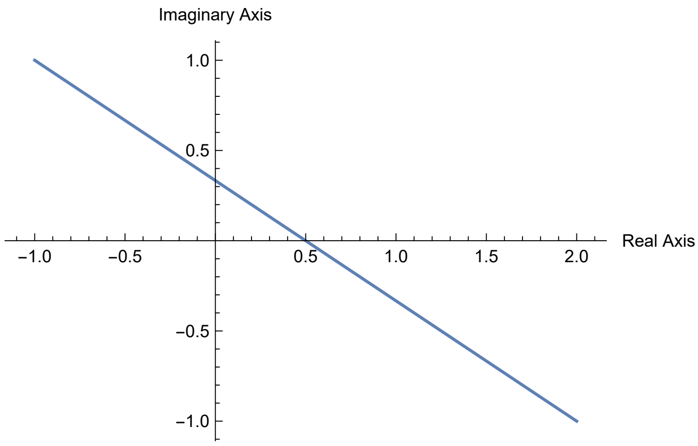



```
The value of the integration is
∫ C Sin[z] ⅆz = Cosh[1 + i] - Cosh[1 + 2 i]
where C: c[t] = -1 + i (1 - 2 t) + 3 t, for 0 ≤ t ≤ 1.
```

4 : f (z) = z, z0 = -1 +i, z1 = 2 - i

In[∘]:=
```
f[z_] = Conjugate[z];
z0 = -1 + ⅈ;
z1 = 2 - ⅈ;

x0 = Re[z0]; y0 = Im[z0];
x1 = Re[z1]; y1 = Im[z1];

L[t_] = x0 + (x1 - x0) t + ⅈ (y0 + (y1 - y0) t);
w[t_] = ComplexExpand[f[L[t]] * L'[t]];
W1 = Integrate[w[t], {t, 0, 1}];

ParametricPlot[{Re[L[t]], Im[L[t]]}, {t, 0, 1}, Axes → True,
AxesOrigin → {0, 0}, AxesLabel → {"Real Axis", "Imaginary Axis"}]

Print["The value of the integration is"];
Print["∫ C ", f[z] // TraditionalForm, " ⅆz = ", W1];
Print["where C: c[t] = ", L[t], ", for 0 ≤ t ≤ 1."];
```

Out[∘]=



The value of the integration is

$\int C\ z^{*}\ dz = \frac{3}{2} - i$

where C: c[t] = $-1 + i\,(1 - 2t) + 3t$, for 0 ≤ t ≤ 1.

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PRACTICAL 10 : Perform the following Line integrals.

1 : Perform the contour integration $\int_C \frac{1}{(z-2)}\, dz$

where C is the upper semicircle with radius 1 centered at point (2, 0) in the anticlockwise sense

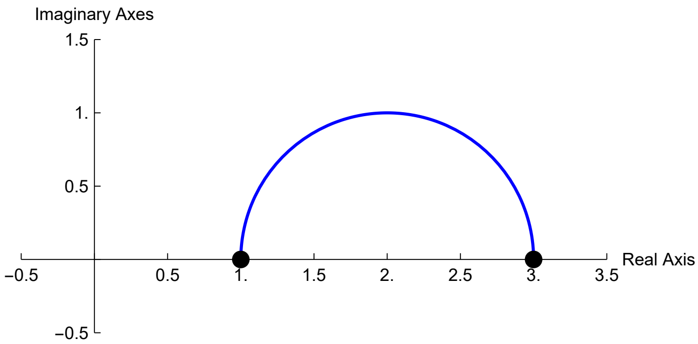
In[∘]:=
```
f[z_] = 1/(z - 2);
z[t_] = (2 + Cos[t]) + ⅈ Sin[t];
val = Integrate[f[z[t]] * z'[t], {t, 0, π}];
V[t_] = ComplexExpand[{Re[z[t]], Im[z[t]]}];

g1 = ParametricPlot[V[t], {t, 0, π}, PlotRange → {{-0.5, 3.5}, {-0.5, 1.5}},
Ticks → {Range[-0.5, 3.5, 0.5], Range[-0.5, 1.5, 0.5]},
AxesLabel → {"Real Axis", "Imaginary Axes"}, PlotStyle → Blue];

g2 = Graphics[{PointSize[0.03], Point[{3, 0}], Point[{1, 0}]}];

Show[g1, g2]
Print["The Value of the contour integration ", "∫_C",
  f[z], "ⅆz is ", "∫_0^π", "f[z[t]]z'[t]", "ⅆt ", "= ", val];
Print["where C: z[t] = ", z[t], ", for 0 ≤ t ≤ π."];
```

Out[∘]=



The Value of the contour integration $\int_C \frac{1}{-2+z} \, dz$ is $\int_0^{\pi} f[z[t]]\, z'[t] \, dt = i\pi$

where C: z[t] = 2 + Cos[t] + ⅈ Sin[t], for 0 ≤ t ≤ π.

2 : Perform the contour integration $\int_C \frac{2z}{(z^2+2)} \, dz$

where C is the circle $z - i = 1$ taken in anticlockwise sense.

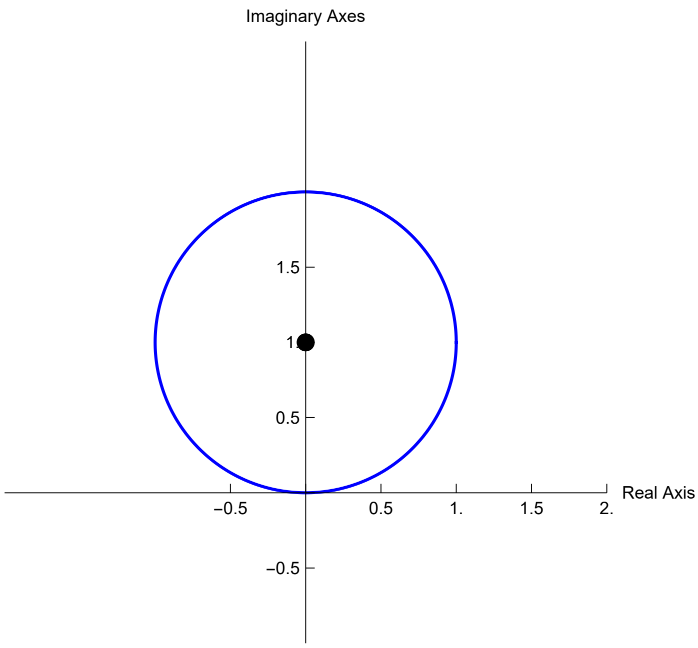
In[∘]:=
```
f[z_] = (2 z) / (z^2 + 2) ;
z[t_] = Cos[t] + ⅈ (1 + Sin[t]);
val = Integrate[ f[z[t]] * z '[t], {t, 0, 2 π}];
V[t_] = ComplexExpand[{Re[z[t]], Im[z[t]]}];

g1 = ParametricPlot[V[t], {t, 0, 2 π}, PlotRange → {{-2, 2}, {-1, 3}},
Ticks → {Range[-0.5, 3.5, 0.5], Range[-0.5, 1.5, 0.5]},
AxesLabel → {"Real Axis", "Imaginary Axes"}, PlotStyle → Blue];

g2 = Graphics[{PointSize[0.03], Point[{0, 1}]}];

Show[g1, g2]
Print["The Value of the contour integration ", "∫_C",
  f[z], "ⅆz is ", "∫_0^{2π}", "f[z[t]]z'[t]", "ⅆt ", "= ", val];
Print["where C: z[t] = ", z[t], ", for 0 ≤ t ≤ π."];
```

Out[∘]=



The Value of the contour integration $\int_C \frac{2z}{2+z^2} dz$ is $\int_0^{2\pi} f[z[t]]z'[t] dt = 2 i \pi$

where C: z[t] = Cos[t] + ⅈ (1 + Sin[t]), for 0 ≤ t ≤ π.

3 : Perform the contour integration $\int_C \frac{1}{z}\, dz$

where C is the circle z = 1 taken in clockwise sense.

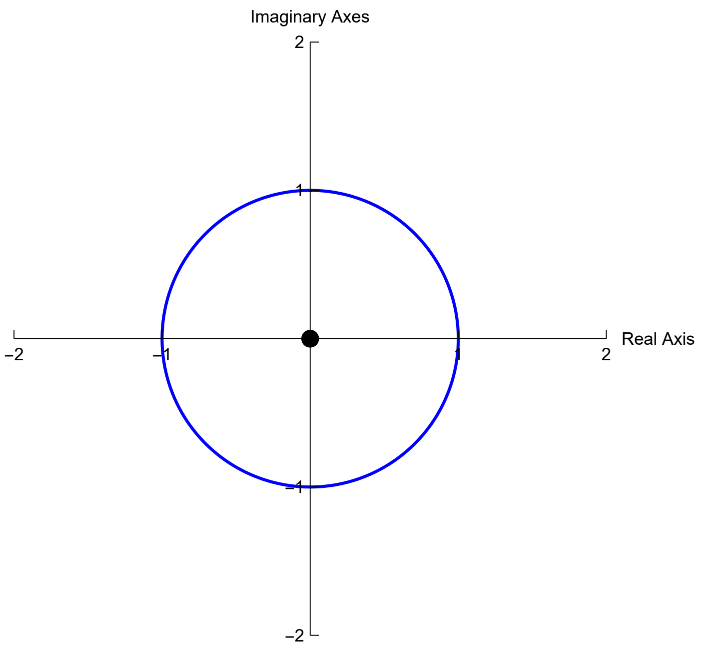
In[]:=
```
ClearAll;
f[z_] = 1 / z ;
z[t_] = Sin[t] + ⅈ Cos[t];
val = Integrate[ f[z[t]] * z'[t], {t, 0, 2 π}];
V[t_] = ComplexExpand[{Re[z[t]], Im[z[t]]}];

g1 = ParametricPlot[V[t], {t, 0, 2 π}, PlotRange → {{-2, 2}, {-2, 2}},
Ticks → {Range[-2, 2, 1], Range[-2, 2, 1]},
AxesLabel → {"Real Axis", "Imaginary Axes"}, PlotStyle → Blue];

g2 = Graphics[{PointSize[0.03], Point[{0, 0}]}];

Show[g1, g2]
Print["The Value of the contour integration ", "∫_C",
  f[z], "ⅆz is ", "∫_0^π ", "f[z[t]]z'[t]", "ⅆt ", "= ", val];
Print["where C: z[t] = ", z[t], ", for 0 ≤ t ≤ π."];
```

Out[]=



The Value of the contour integration $\int_C \frac{1}{z} \, dz$ is $\int_0^{\pi}$ f[z[t]]z'[t]ⅆt = $-2 \, i \, \pi$

where C: z[t] = ⅈ Cos[t] + Sin[t], for 0 ≤ t ≤ π.

4 : Perform the contour integration $\int_C z^3 + 2 z^2 + 1 \, dz$

where C is the contour given by x2 + y2 = 1 taken in the positive sense.

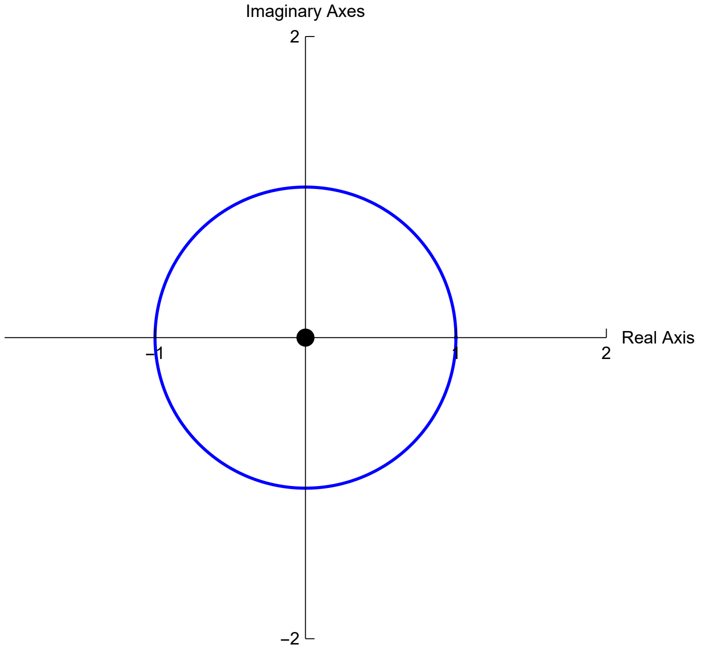
In[ ]:=
```
f[z_] = z^3 + 2 z^2 + 1;
z[t_] = Cos[t] + ⅈ Sin[t];
val = Integrate[f[z[t]] * z '[t], {t, 0, 2 π}];
V[t_] = ComplexExpand[{Re[z[t]], Im[z[t]]}];

g1 = ParametricPlot[V[t], {t, 0, 2 π}, PlotRange → {{-2, 2}, {-2, 2}},
Ticks → {Range[-1, 2, 1], Range[-2, 2, 2]},
AxesLabel → {"Real Axis", "Imaginary Axes"}, PlotStyle → Blue];

g2 = Graphics[{PointSize[0.03], Point[{0, 0}]}];

Show[g1, g2]
Print["The Value of the contour integration ", "∫_C",
  f[z], "ⅆz is ", "∫_0^(2 π) ", "f[z[t]]z'[t]", "ⅆt ", "= ", val];
Print["where C: z[t] = ", z[t], ", for 0 ≤ t ≤ π."];
```

Out[ ]=



The Value of the contour integration $\int_C 1+2z^2+z^3 dz$ is $\int_0^{2\pi}$ f[z[t]]z'[t]ⅆt = 0

where C: z[t] = Cos[t] + ⅈ Sin[t], for 0 ≤ t ≤ π.

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PRACTICAL 11 : Perform the following Line integrals.

1 : Perform the contour integration $\int$C z dz

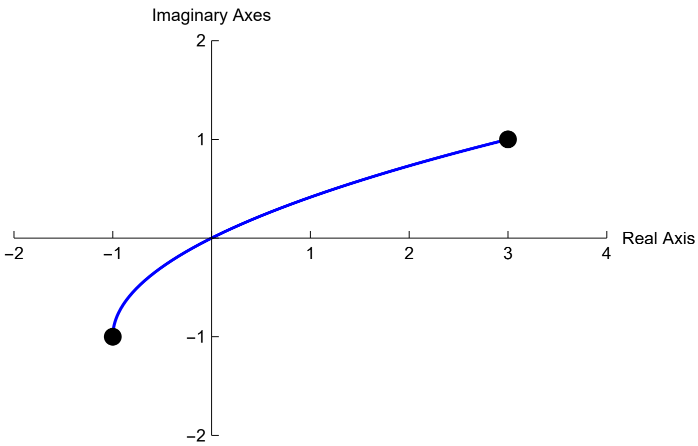
where C is the portion of the parabola x = y2 + 2 y joining - 1 - ⅈ to 3 + ⅈ.

In[ ]:=
```
ClearAll;
f[z_] = z;
z[t_] = (t^2 + 2 t) + ⅈ t;
val = Integrate[ f[z[t]] * z '[t], {t, -1, 1}];
V[t_] = ComplexExpand[{Re[z[t]], Im[z[t]]}];

g1 = ParametricPlot[V[t], {t, -1, 1}, PlotRange → {{-2, 4}, {-2, 2}},
Ticks → {Range[-2, 4, 1], Range[-2, 2, 1]},
AxesLabel → {"Real Axis", "Imaginary Axes"}, PlotStyle → Blue];

g2 = Graphics[{PointSize[0.03], Point[{-1, -1}], Point[{3, 1}]}];
Show[g1, g2]
Print["The Value of the contour integration ", "∫ C ",
  f[z], "ⅆz is ", "∫ -1 1 ", "f[z[t]]z'[t]", "ⅆt ", "= ", val];
Print["where C: z[t] = ", z[t], ", for -1 ≤ t ≤ 1"];
```

Out[ ]=



The Value of the contour integration ∫ C zⅆz is ∫ -1 1 f[z[t]]z'[t]ⅆt = 4 + 2 ⅈ

where C: z[t] = (2 + ⅈ) t + t^2, for -1 ≤ t ≤ 1

2 : Perform the contour integration ∫C (z2 - 2 z + 1) dz

where C is the contour given by x = y2 + 1 where - 2 ≤ y ≤ 2.

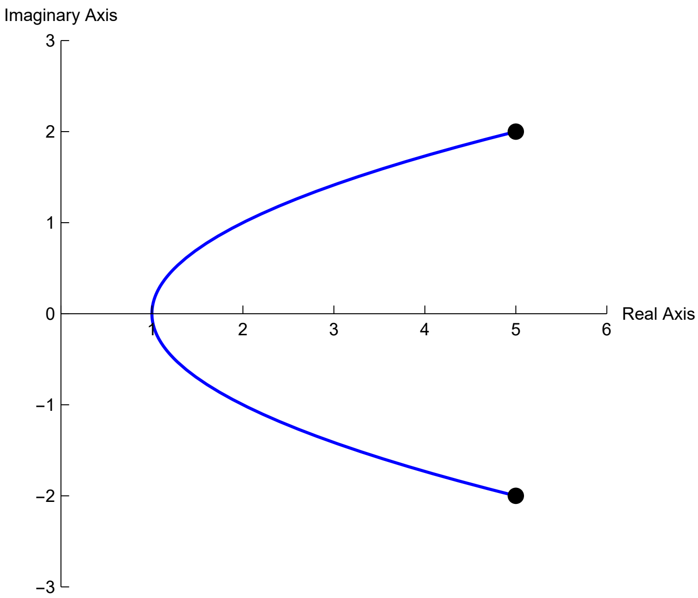
In[•]:=
```
f[z_] = z^2 + 2 * z + 1;
z[t_] = (t^2 + 1) + ⅈ t;
val = Integrate[ f[z[t]] * z'[t], {t, -2, 2}];
V[t_] = ComplexExpand[{Re[z[t]], Im[z[t]]}];

g1 = ParametricPlot[V[t], {t, -2, 2}, PlotRange → {{0, 6}, {-3, 3}},
Ticks → {Range[0, 6, 1], Range[-3, 3, 1]},
   AxesLabel → {"Real Axis", "Imaginary Axis"}, PlotStyle → Blue];

g2 = Graphics[{PointSize[0.03], Point[{5, -2}], Point[{5, 2}]}]; Show[g1, g2]

Print["The Value of the contour integration ", "∫ C ",
  f[z], " ⅆz is ", "∫ -2 2 ", "f[z[t]]z'[t]", "ⅆt ", "= ", val];
Print["where C: z[t] = ", z[t], ", for -2 ≤ t ≤ 2"];
```

Out[•]=



The Value of the contour integration $\int$ C $1+2z+z^2$ ⅆz is $\int$ −2 2 f[z[t]]z'[t]ⅆt = $\frac{416\,i}{3}$

where C: z[t] = $1+i\,t+t^2$, for $-2 \le t \le 2$

3 : Show that $\int$C1 z dz = $\int$C2 z dz

where C1 is the line segment from - 1 - ⅈ to 3 + ⅈ and

C2 is the portion of the parabola x = y2 + 2 y joining - 1 - ⅈ to 3 + ⅈ .

Also plot the contours C1 and C2 .

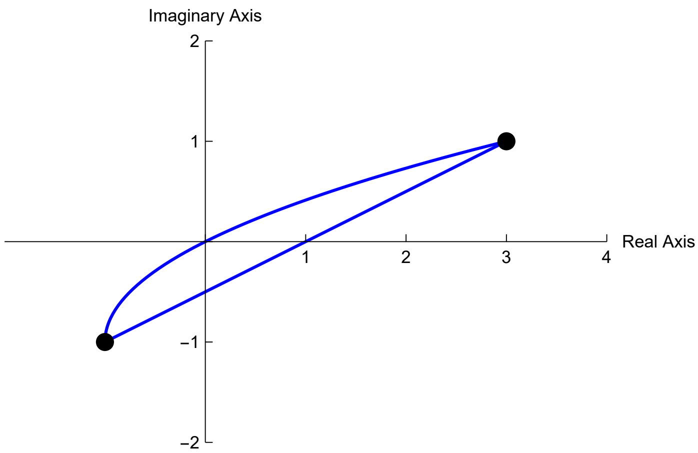
In[ ]:=
```
ClearAll;
f[z_] = z;
z0 = -1 - i;
z1 = 3 + i;
x0 = Re[z0]; y0 = Im[z0];
x1 = Re[z1]; y1 = Im[z1];
c1[t_] = x0 + (x1 - x0) t + i (y0 + (y1 - y0) t);
c2[t_] = (t^2 + 2 t) + i t;

Int1 = Integrate[f[c1[t]] * (c1'[t]), {t, 0, 1}];
Int2 = Integrate[f[c2[t]] * (c2'[t]), {t, -1, 1}];

V1[t_] = ComplexExpand[{Re[c1[t]], Im[c1[t]]}];
V2[t_] = ComplexExpand[{Re[c2[t]], Im[c2[t]]}];

g1 = ParametricPlot[V1[t], {t, 0, 1},
  PlotRange → {{-2, 4}, {-2, 2}}, Ticks → {Range[0, 6, 1], Range[-3, 3, 1]},
  AxesLabel → {"Real Axis", "Imaginary Axis"}, PlotStyle → Blue];
g2 = ParametricPlot[V2[t], {t, -1, 1}, PlotRange → {{-2, 4}, {-2, 2}},
  Ticks → {Range[0, 6, 1], Range[-3, 3, 1]},
  AxesLabel → {"Real Axis", "Imaginary Axis"}, PlotStyle → Blue];
g3 = Graphics[{PointSize[0.03], Point[{x0, y0}], Point[{x1, y1}]}];
Show[g1, g2, g3]
```

Out[ ]=



In[ ]:=
```
If[Int1 == Int2,
 Print["The integrals ∫ C1 z dz and ∫ C2 z dz are equal and equal to ", Int1, "."],
 Print["Integrals are not equal."]]
```

The integrals ∫ C1 z dz and ∫ C2 z dz are equal and equal to 4 + 2 i.

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SECTION : A

PRACTICAL 12 : Use ML-inequality to show that $\left| \int_C \frac{1}{z^2+1} \, dz \right| \leq \frac{1}{5}$,where C is the line segment form 2 to 2 + i. While solving, represent the distance from the point z to the points i and -i, respectively

In[∘]:=
```
f[z_] := 1/(z^2 + 1)
c[t_] := 2 + ⅈ t
k[t_] := ComplexExpand[f[c[t]]]
r[t_] := Refine[Re[k[t]], t ∈ Reals]
s[t_] := Refine[Im[k[t]], t ∈ Reals]
A[t_] := Simplify[Sqrt[s[t]^2 + r[t]^2]]
M = MaxValue[{A[t], 0 ≤ t ≤ 1}, t]
L = ∫_0^1 Abs[c'[t]] ⅆt
Print[
 "An upper bound to the absolute value of the integral |∫_C 1/(z^2 + 1) ⅆz| is found to be ",
 M * L, "."]
p = {2, RandomReal[{0, 1}]}
a = ParametricPlot[{Re[c[t]], Im[c[t]]},
   {t, 0, 1}, PlotRange → {{-1, 2.5}, {-1.5, 1.5}}];
b = Graphics[{Red, PointSize[0.03], Point[p], Green,
    DotDashed, Line[{p, {0, 1}}], Blue, DotDashed, Line[{p, {0, -1}}]}];
Show[a, b]
```

Out[∘]=

$\frac{1}{5}$

Out[∘]=

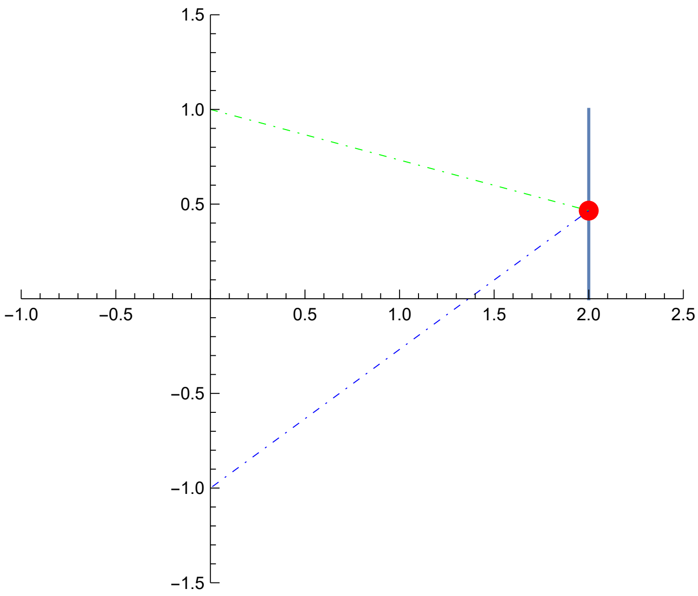
1

An upper bound to the absolute value of the integral $\left|\int_C \frac{1}{z^2+1}\,dz\right|$ is found to be $\frac{1}{5}$.

Out[∘]=

{2, 0.465359}

Out[∘]=



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SECTION : A

PRACTICAL 13 : Series Representation

Let c_n be complex numbers for n ∈ Integers. The doubly infinite series $\sum_{n=-\infty}^{\infty} c_n (z-\alpha)^n$ defined by

$$\sum_{n=-\infty}^{\infty} c_n (z-\alpha)^n = \sum_{n=1}^{\infty} c_{-n} (z-\alpha)^{-n} + \sum_{n=0}^{\infty} c_n (z-\alpha)^n$$

is called a Laurent Series about the point z = α.

7.1 Find the series representation for
f (z) = (cos (z) - 1) / z^4
that involves powers of z about z = 0.
Also plot the magnitude of the function and magnitude of its series expansion.

```
Normal[Series[f[z], {z, 0, 10}]]
```

Out[◦]=

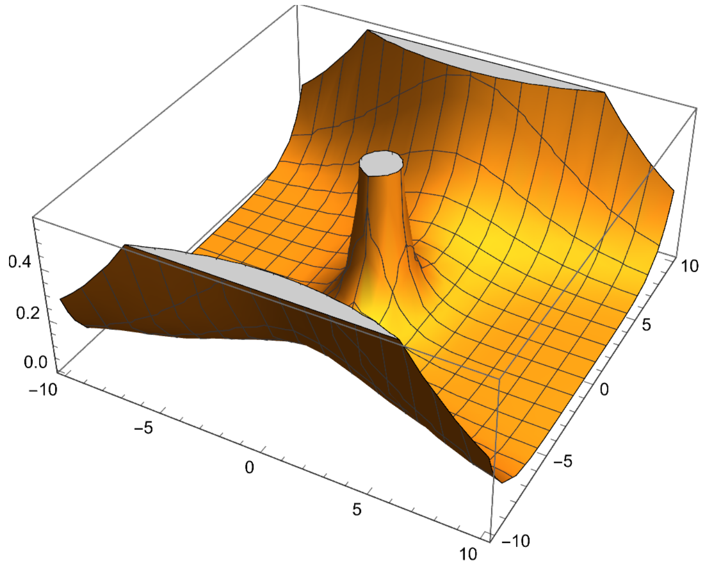
$$f[0] + z f'[0] + \frac{1}{2} z^2 f''[0] + \frac{1}{6} z^3 f^{(3)}[0] + \frac{1}{24} z^4 f^{(4)}[0] + \frac{1}{120} z^5 f^{(5)}[0] + \frac{1}{720} z^6 f^{(6)}[0] + \frac{z^7 f^{(7)}[0]}{5040} + \frac{z^8 f^{(8)}[0]}{40320} + \frac{z^9 f^{(9)}[0]}{362880} + \frac{z^{10} f^{(10)}[0]}{3628800}$$

```
(* Laurent Series Expansion *)
f[z_] = (Cos[z] - 1) / z^4;
L[z_] = Normal[Series[f[z], {z, 0, 10}]];
Print["The Laurent Series expansion for the function f(z) = ",
  f[z], ", is given by ", L[z], " + ... ."];
```

The Laurent Series expansion for the function f(z) = $\frac{-1 + \mathrm{Cos}[z]}{z^4}$, is given by $\frac{1}{24} - \frac{1}{2 z^2} - \frac{z^2}{720} + \frac{z^4}{40320} - \frac{z^6}{3628800} + \frac{z^8}{479001600} - \frac{z^{10}}{87178291200}$ + ...

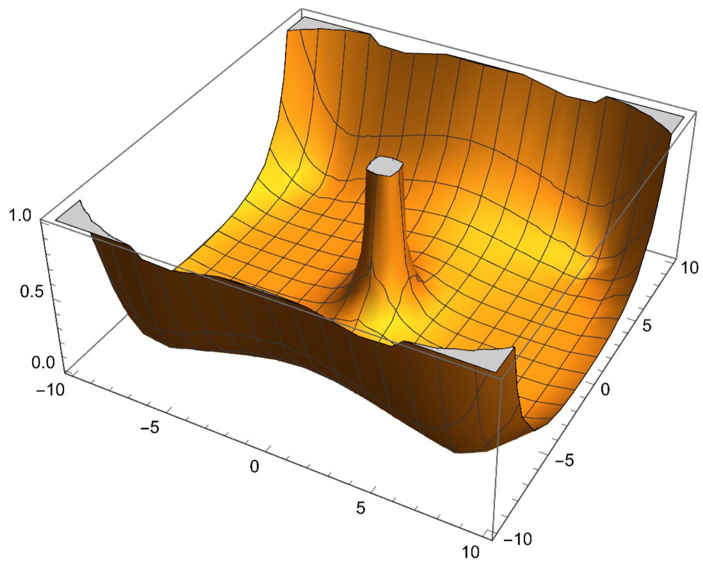
```
(*Magnitude of the function*)
Plot3D[Abs[f[x + ⅈ y]], {x, -10, 10}, {y, -10, 10}]
```

Out[∘]=




```
(*Magnitude of the Laurent Series*)
Plot3D[Abs[L[x + ⅈ y]], {x, -10, 10}, {y, -10, 10}]
```

Out[∘]=



7.2 : Find the Laurent series representation for
f(z) = (sin (z) - 1)/ z^4
that involves powers of z about z = 0.
Also plot the magnitude of the function and magnitude of its Taylor series expansion.

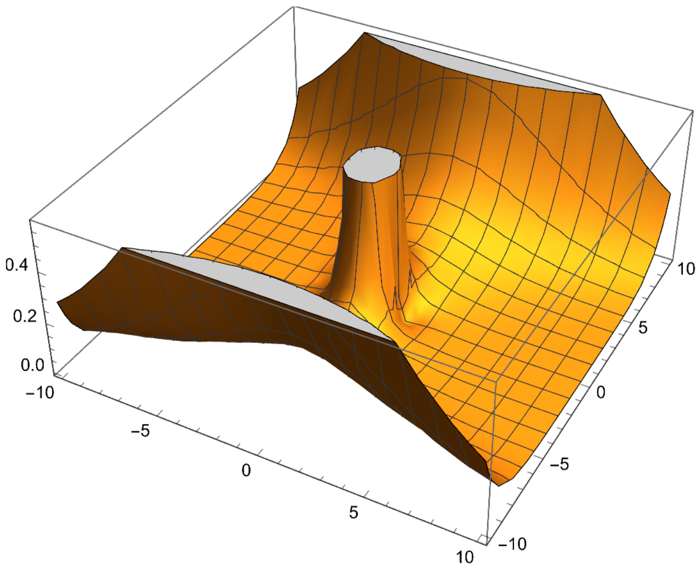
```
(* Laurent Series Expansion *)
f[z_] = (Sin[z] - 1) / z^4;
L[z_] = Normal[Series[f[z], {z, 0, 10}]];
Print["The Laurent Series expansion for the function f(z) = ",
  f[z], ", is given by ", L[z], " + ... ."];
```

The Laurent Series expansion for the function f(z) = $\frac{-1+\mathrm{Sin}[z]}{z^4}$

, is given by $-\frac{1}{z^4}+\frac{1}{z^3}-\frac{1}{6\,z}+\frac{z}{120}-\frac{z^3}{5040}+\frac{z^5}{362\,880}-\frac{z^7}{39\,916\,800}+\frac{z^9}{6\,227\,020\,800}$ + ...

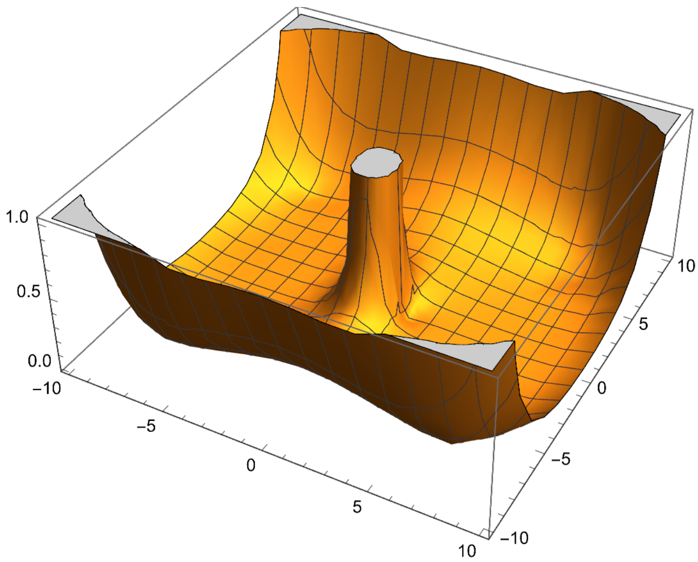
```
(*Magnitude of the function*)
Plot3D[Abs[f[x + i y]], {x, -10, 10}, {y, -10, 10}]
```

Out[•]=




```
(*Magnitude of the Laurent Series*)
Plot3D[Abs[L[x + i y]], {x, -10, 10}, {y, -10, 10}]
```

Out[•]=



7.3 : Find the Laurent series representation for f(z) = (Cos(z))/ z^4

that involves powers of z about z = 0.

Also plot the magnitude of the function and magnitude of its Taylor series expansion.

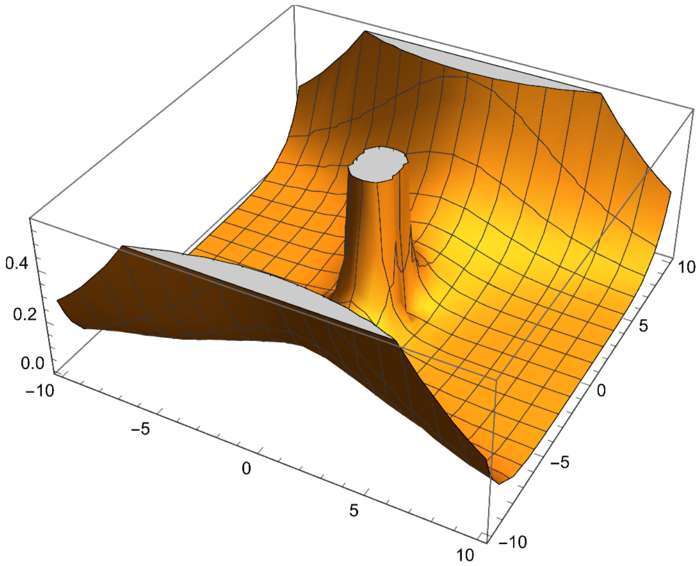
```
(* Laurent Series Expansion *)
f[z_] = (Cos[z]) / z^4;
L[z_] = Normal[Series[f[z], {z, 0, 10}]];
Print["The Laurent Series expansion for the function f(z) = ",
  f[z], ", is given by ", L[z], " + ... ."];
```

The Laurent Series expansion for the function $f(z) = \frac{\text{Cos}[z]}{z^4}$, is given by

$$\frac{1}{24} + \frac{1}{z^4} - \frac{1}{2\,z^2} - \frac{z^2}{720} + \frac{z^4}{40\,320} - \frac{z^6}{3\,628\,800} + \frac{z^8}{479\,001\,600} - \frac{z^{10}}{87\,178\,291\,200} + \ldots .$$

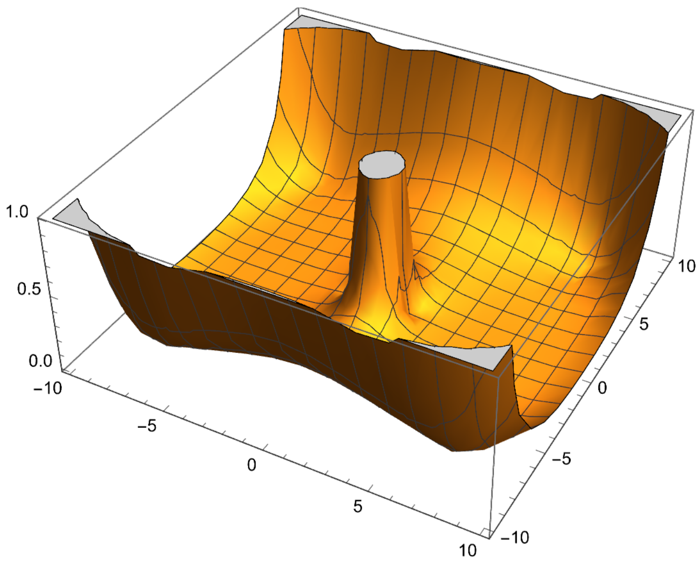
```
(*Magnitude of the function*)
Plot3D[Abs[f[x + i y]], {x, -10, 10}, {y, -10, 10}]
```

Out[ ]=




```
(*Magnitude of the Laurent Series*)
Plot3D[Abs[L[x + i y]], {x, -10, 10}, {y, -10, 10}]
```

Out[ ]=



7.4: Find the Laurent series representation for
f(z) = 1 / (2 + z + z^3)
that involves powers of z about z = 0.
Also plot the magnitude of the function and magnitude of its Taylor series expansion.

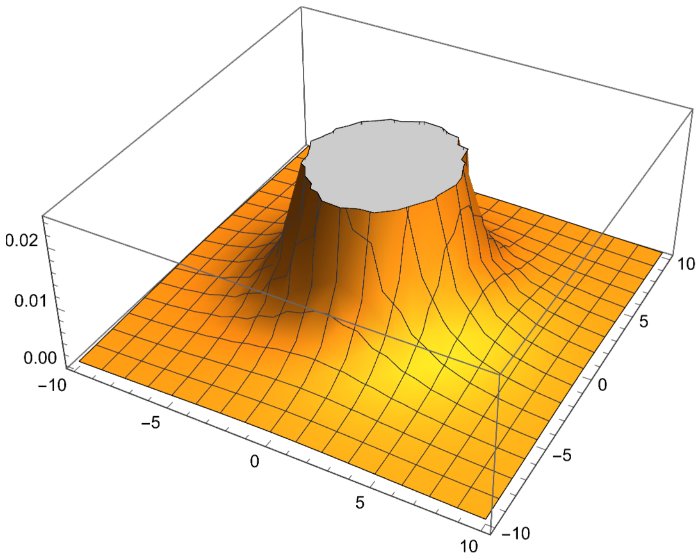
```
(* Laurent Series Expansion *)
f[z_] = 1 / (2 + z + z^3) ;
L[z_] = Normal[Series[f[z], {z, 0, 10}]];
Print["The Laurent Series expansion for the function f(z) = ",
  f[z], ", is given by ", L[z], " + ... ."];
```

The Laurent Series expansion for the function $f(z) = \frac{1}{2+z+z^3}$, is given by

$$\frac{1}{2} - \frac{z}{4} + \frac{z^2}{8} - \frac{5 z^3}{16} + \frac{9 z^4}{32} - \frac{13 z^5}{64} + \frac{33 z^6}{128} - \frac{69 z^7}{256} + \frac{121 z^8}{512} - \frac{253 z^9}{1024} + \frac{529 z^{10}}{2048} + \dots .$$

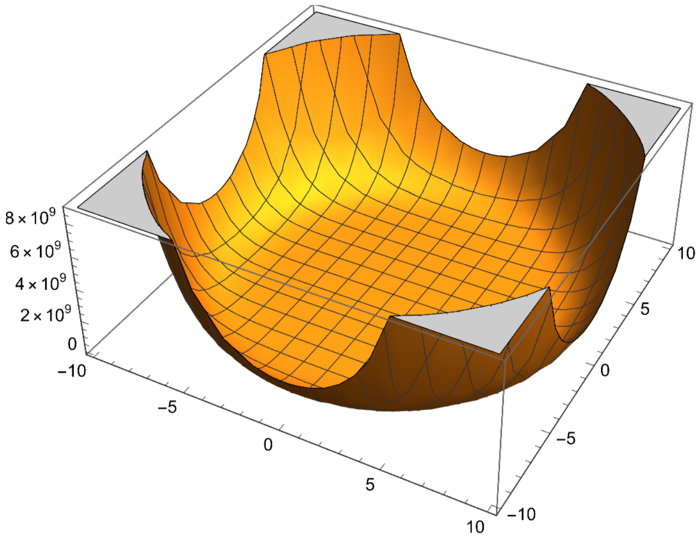
```
(*Magnitude of the function*)
Plot3D[Abs[f[x + i y]], {x, -10, 10}, {y, -10, 10}]
```

Out[•]=



```
(*Magnitude of the Laurent Series*)
Plot3D[Abs[L[x + i y]], {x, -10, 10}, {y, -10, 10}]
```

Out[•]=



7.5: Find the Laurent series representation for
f(z) = e ^ (-1/z^2)
that involves powers of z about z = 0.
Also plot the magnitude of the function and magnitude of its Taylor series expansion.

```
(* Laurent Series Expansion *)
f[z_] = e^(z);
T[z_] = Normal[Series[f[z], {z, 0, 10}]]
L[z_] = T[-1 / z^2];
Print["The Laurent Series expansion for the function f(z) = ",
  f[-1 / z^2], ", is given by ", L[z], " + ... ."];
```

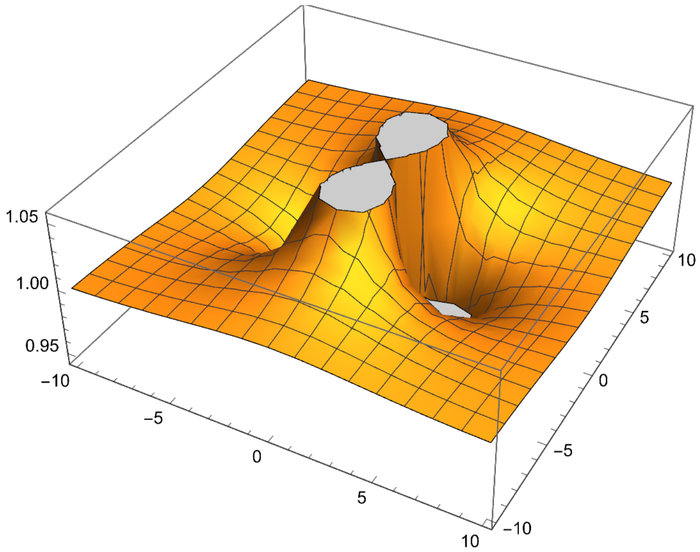
Out[•]=

$$1+z+\frac{z^2}{2}+\frac{z^3}{6}+\frac{z^4}{24}+\frac{z^5}{120}+\frac{z^6}{720}+\frac{z^7}{5040}+\frac{z^8}{40320}+\frac{z^9}{362880}+\frac{z^{10}}{3628800}$$

The Laurent Series expansion for the function f(z) = $e^{-\frac{1}{z^2}}$, is given by $1+\frac{1}{3628800\,z^{20}}-\frac{1}{362880\,z^{18}}+\frac{1}{40320\,z^{16}}-\frac{1}{5040\,z^{14}}+\frac{1}{720\,z^{12}}-\frac{1}{120\,z^{10}}+\frac{1}{24\,z^8}-\frac{1}{6\,z^6}+\frac{1}{2\,z^4}-\frac{1}{z^2}+\ldots$.

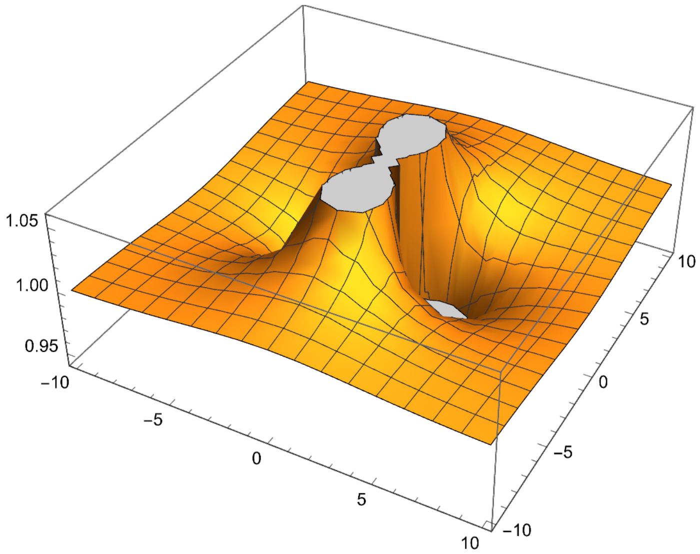
```
(*Magnitude of the function*)
Plot3D[Abs[f[-1 / (x + i y) ^2]], {x, -10, 10}, {y, -10, 10}]
```

Out[•]=




```
(*Magnitude of the Laurent Series*)
Plot3D[Abs[L[(x + i y)]], {x, -10, 10}, {y, -10, 10}]
```

Out[•]=



7.6: Find the Laurent series representation for
f(z) = 3 / (2 + z - z^3)
that involves powers of z about z = 0.
Also plot the magnitude of the function and magnitude of its Taylor series expansion.

```
Solve[2 + z - z^2 == 0, z]
```

Out[•]=

```
{{z → -1}, {z → 2}}
```

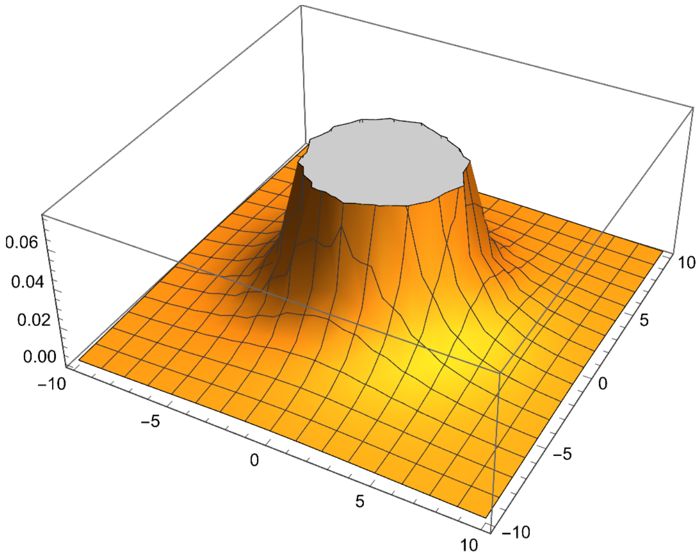
```
(*About the point z = 0*)
(* Laurent Series Expansion *)
f[z_] = 3 / (2 + z - z^3) ;
L1[z_] = Normal[Series[f[z], {z, 0, 10}]];
Print["The Laurent Series expansion for the function f(z) = ",
  f[z], ", is given by ", L1[z], " + ... ."];
```

The Laurent Series expansion for the function $f(z) = \frac{3}{2 + z - z^3}$, is given by

$$\frac{3}{2} - \frac{3z}{4} + \frac{3z^2}{8} + \frac{9z^3}{16} - \frac{21z^4}{32} + \frac{33z^5}{64} + \frac{3z^6}{128} - \frac{87z^7}{256} + \frac{219z^8}{512} - \frac{207z^9}{1024} - \frac{141z^{10}}{2048} + \ldots .$$

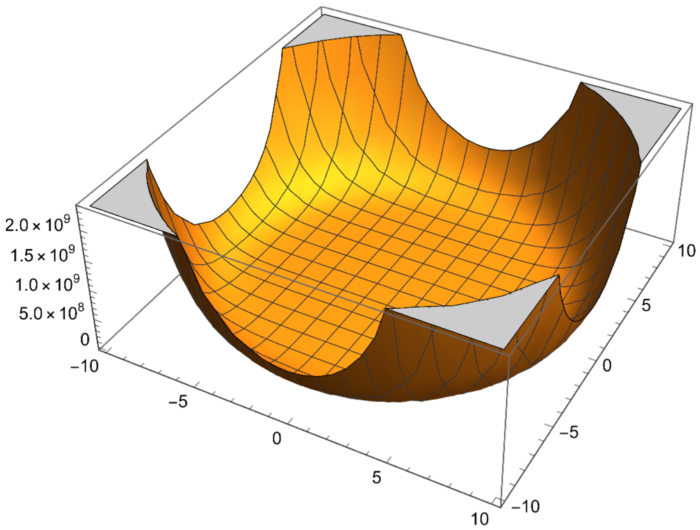
```
(*Magnitude of the function*)
Plot3D[Abs[f[x + i y]], {x, -10, 10}, {y, -10, 10}]
```

Out[=]=




```
(*Magnitude of the Laurent Series*)
Plot3D[Abs[L1[x + i y]], {x, -10, 10}, {y, -10, 10}]
```

Out[=]=



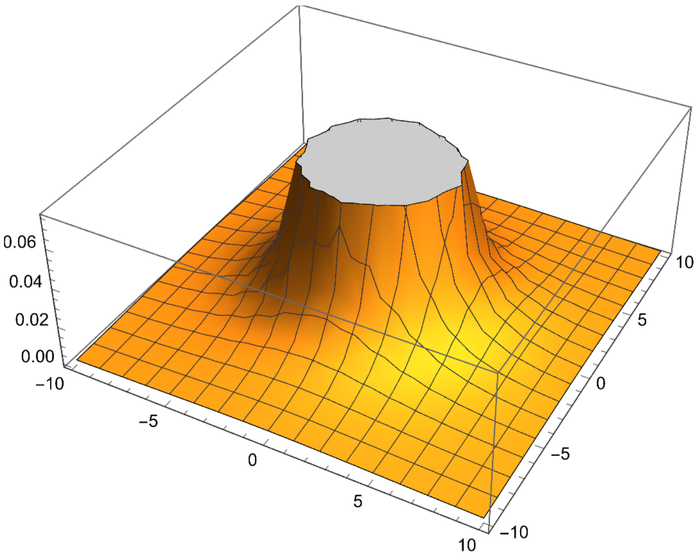
```
(* About the point z = -1 *)
(* Laurent Series Expansion *)
f[z_] = 3 / (2 + z - z^3);
L2[z_] = Normal[Series[f[z], {z, -1, 10}]];
Print["The Laurent Series expansion for the function f(z) = ",
  f[z], ", is given by ", L2[z], " + ... ."];
```

The Laurent Series expansion for the function f(z) = $\frac{3}{2+z-z^3}$

, is given by $\frac{3}{2}+\frac{3(1+z)}{2}-\frac{3}{4}(1+z)^2-\frac{9}{4}(1+z)^3-\frac{3}{8}(1+z)^4+\frac{21}{8}(1+z)^5+\frac{33}{16}(1+z)^6-\frac{33}{16}(1+z)^7-\frac{123}{32}(1+z)^8+\frac{9}{32}(1+z)^9+\frac{321}{64}(1+z)^{10}$ + ...

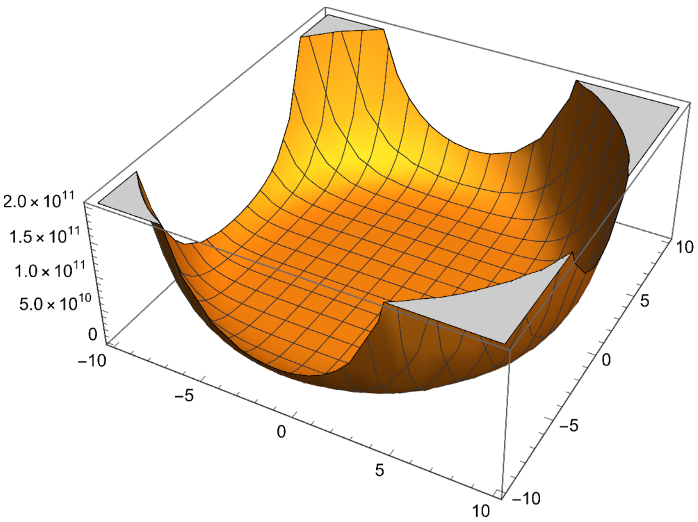
```
(*Magnitude of the function*)
Plot3D[Abs[f[x + i y]], {x, -10, 10}, {y, -10, 10}]
```

Out[ ]=




```
(*Magnitude of the Laurent Series*)
Plot3D[Abs[L2[x + i y]], {x, -10, 10}, {y, -10, 10}]
```

Out[ ]=



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PRACTICAL 14 : Locate the poles of f(z) = $\frac{1}{z^4+26z^2+5}$ and specify their order.

In[•]:=
```
f[z_] := 1/(5 z^4 + 26 z^2 + 5);
Solve[1/f[z] == 0, z]
```

Out[•]=
$$\left\{\left\{z \to -\frac{i}{\sqrt{5}}\right\}, \left\{z \to \frac{i}{\sqrt{5}}\right\}, \left\{z \to -i\sqrt{5}\right\}, \left\{z \to i\sqrt{5}\right\}\right\}$$

In[•]:=
```
Text["The function f has poles at z=-i/√5,z=i/√5,z=-i√5 and at z=i√5 of order 1."]
```

Out[•]=
The function f has poles at z=$-\frac{i}{\sqrt{5}}$,z=$\frac{i}{\sqrt{5}}$,z=$-i\sqrt{5}$ and at z=$i\sqrt{5}$ of order 1.

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PRACTICAL 15 : Locate the zero and poles of g(z) = $\frac{\pi\cot(\pi t)}{z^2}$ and determine their order. Also justify that Res(g,0)= $\frac{-\pi^2}{3}$.

In[•]:=
```
f[z_] := Pi Cot[Pi z]/z^2;
Solve[Cot[Pi z] == 0, z]
```

Out[•]=
$$\left\{\left\{z \to \boxed{\frac{\frac{\pi}{2} + \pi\, c_1}{\pi} \text{ if } c_1 \in \mathbb{Z}}\right\}\right\}$$

In[◦]:=
```
Text["Conclusion: The function f has zero at z=(π/2 + π n)/π (n ∈ Z) for order 1."]

Solve[1/f[z] == 0, z]
```

Out[◦]=

Conclusion: The function f has zero at z=$\frac{\frac{\pi}{2}+\pi n}{\pi}$(n ∈ Z) for order 1.

··· Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

Out[◦]=

{{z → 0}}

In[◦]:=
```
Text["Conclusion: The function f1 has pole at z=0 for order 2."]
```

Out[◦]=

Conclusion: The function f1 has pole at z=0 for order 2.

In[◦]:=
```
Residue[f[z], {z, 0}]
```

Out[◦]=

$$-\frac{\pi^2}{3}$$

In[◦]:=
```
SeriesCoefficient[f[z], {z, 0, -1}]
```

Out[◦]=

$$-\frac{\pi^2}{3}$$

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PRACTICAL 16 : A particular Contour Integral.

Ques: Perform the following Line Integrals.

(i) $\int_C \exp\left(\frac{2}{z}\right) dz$

(ii) $\int_C \frac{1}{z^4+z^3-2z^2} dz$

where C is the unit circle with center at z = 0 taken in the positive sense.

The curve C can be parametrized as

C : z (t) = x (t) + i y(t) , 0 ⩽ t ⩽ 2 π

where x(t) = Cos[t] and y(t) = Sin[t]

In[◦]:=
```
c[t_] := Cos[t] + i Sin[t];
```

(i) $\int_C \exp\left(\frac{2}{z}\right) dz$

In[ ]:=
```
f[z_] := e^(2/z)
val = ∫_0^(2π) f[c[t]] * c'[t] ⅆt;
Print["The Value of the contour integration", " ∫_C ", f[z], "ⅆz is", "= ", val];
Print["where C: z[t]=", c[t], ", for 0 ≤ t ≤ 2π"];
```

The Value of the contour integration $\int_C e^{2/z} dz$ is= 0 if Log[e] == 0

where C: z[t]=Cos[t] + ⅈ Sin[t], for 0 ≤ t ≤ 2π

(ii) $\int_C \frac{1}{z^4 + z^3 - 2 z^2} dz$

In[ ]:=
```
g[z_] := 1/(z^4 + z^3 - 2 z^2)
Solve[1/g[z] == 0, z]
```

Out[ ]=

{{z → -2}, {z → 0}, {z → 0}, {z → 1}}

In[ ]:=
```
Text["The function g has poles at z = 0 of order 2 and a pole at z = 1 of order 1."]
```

Out[ ]=

The function g has poles at z = 0 of order 2 and a pole at z = 1 of order 1.

In[ ]:=
```
Apart[g[z]]
```

Out[ ]=

$$\frac{1}{3(-1+z)} - \frac{1}{2 z^2} - \frac{1}{4 z} - \frac{1}{12(2+z)}$$

$$\frac{1}{z^4 + z^3 - 2 z^2} = -\frac{1}{2 z^2} - \frac{1}{4 z} - \frac{1}{12(2+z)} + \frac{1}{3(-1+z)}$$

So that

$$\int_C \frac{1}{z^4 + z^3 - 2 z^2} = -\int_C \frac{1}{2 z^2} - \int_C \frac{1}{4 z} - \int_C \frac{1}{12(2+z)} + \int_C \frac{1}{3(-1+z)}$$

Using

(i) Cauchy's Integral formula : $\int_C \frac{h[z]}{z-a} dz = 2 \pi i h[a]$

(ii) Derivative of an analytic function : $\int_C \frac{h[z]}{(z-a)^2} dz = 2 \pi i h'[a]$

where a is any point inside or on C.

In[•]:=
```
h1[z_] := 1/2
f1[z_] := h1[z]/z^2
h2[z_] := 1/4
f2[z_] := h2[z]/z
h3[z_] := 1/12
f3[z_] := h3[z]/(z + 2)
h4[z_] := 1/3
f4[z_] := h4[z]/(z - 1)
Val1 = 2 π i (h1'[z] /. z → 0);
Val2 = 2 π i (h2[z] /. z → 0);
Val3 = 0;
(*function is analytic inside and on C*)
Val4 = 2 π i (h4[z] /. z → 1);
V = -Val1 - Val2 - Val3 + Val4;
Print["The Value of the contour integration", " ∫C", f[z], "dz is", "= ", val];
Print["where C: z[t]=", c[t], ", for 0 ≤ t ≤ 2π"];
```

The Value of the contour integration $\int_C e^{2/z} dz$ is= 0 if Log[e] == 0

where C: z[t]=Cos[t] + i Sin[t], for 0 ≤ t ≤ 2π

(ii) $\int_C \frac{1}{z^4 + z^3 - 2 z^2} dz$

Using Method of Residues

In[•]:=
```
g[z_] := 1/(z^4 + z^3 - 2 z^2)
Solve[1/g[z] == 0, z]
```

Out[•]=

{{z → -2}, {z → 0}, {z → 0}, {z → 1}}

In[•]:=
```
a = Residue[g[z], {z, 0}]
```

Out[•]=

$-\frac{1}{4}$

In[•]:=
```
b = Residue[g[z], {z, 1}]
```

Out[•]=

$\frac{1}{3}$

In[ ]:= **Va1 = 2 ⅈ π (a + b)**

Out[ ]=

$$\frac{i\,\pi}{6}$$