OWNER OF THE FILE

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COURSE:- B.Sc(Hons.) MATHEMATICS

ROLL NO:- 2232139

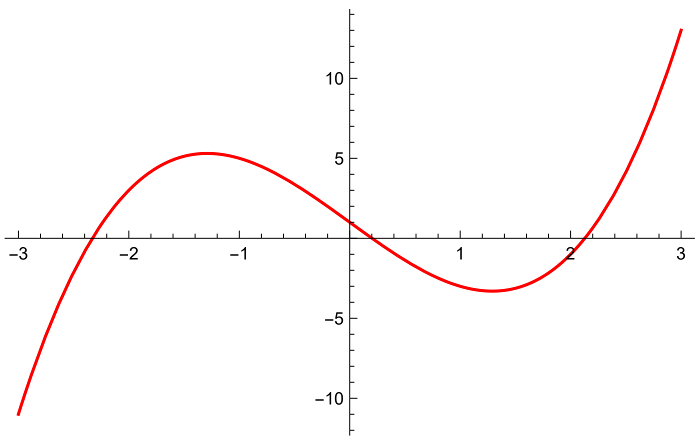
PRACTICAL 1 :- *BISECTION METHOD*

Ques.1 :- Perform 10 iterations of Bisection Method to find the root of the function f(x) = $x^3 - 5x + 1$ in the interval [0,1]

In[∘]:=

```
f[x_] := x^3 - 5 x + 1;
Plot[f[x], {x, -3, 3}, PlotStyle → Red]
a = 0;
b = 1;
n = 10;
If[f[a] * f[b] > 0, Print["We cannot continue with the Bisection Method."], i = 1;
 p = (a + b) / 2;
 OutputDetails = {{i, a, p, b, f[p]}};
 While[i < n, If[Sign[f[a]] == Sign[f[p]], a = p, b = p];
  p = (a + b) / 2;
  i++;
  OutputDetails = Append[OutputDetails, {i, a, p, b, f[p]}];]]
(*Combining the output details with the headings of the table*)
Print[NumberForm[
   N[TableForm[OutputDetails, TableHeadings → {None, {"i", "a_i", "p_i", "b_i", "f[p_i]"}}]],
   8]];(*Printing Table*)
Print["Root After ", n, " iterations ", NumberForm[N[p], 8]]
(*8 is used to give the result with 8 digits precision*)
```

Out[∘]=



i	a_i	p_i	b_i	$f[p_i]$
1.	0.	0.5	1.	-1.375
2.	0.	0.25	0.5	-0.234375
3.	0.	0.125	0.25	0.37695313
4.	0.125	0.1875	0.25	0.069091797
5.	0.1875	0.21875	0.25	-0.083282471
6.	0.1875	0.203125	0.21875	-0.0072441101
7.	0.1875	0.1953125	0.203125	0.030888081
8.	0.1953125	0.19921875	0.203125	0.011812866
9.	0.19921875	0.20117188	0.203125	0.0022820756
10.	0.20117188	0.20214844	0.203125	-0.0024815956

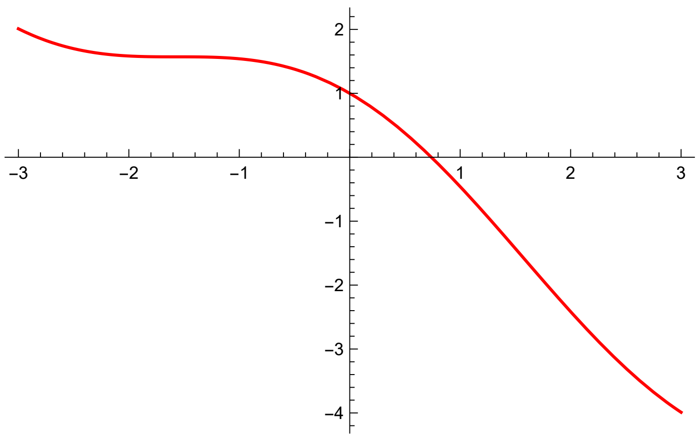
Root After 10 iterations 0.20214844

Ques.2 :- Perform 10 iterations of Bisection Method to find the root of the function f(x) = Cosx – x = 0 in the interval [0,1]

In[∘]:=

```
f[x_] := Cos[x] - x;
Plot[f[x], {x, -3, 3}, PlotStyle → Red]
a = 0;
b = 1;
n = 10;
If[f[a] * f[b] > 0, Print["We cannot continue with the Bisection Method."], i = 1;
 p = (a + b) / 2;
 OutputDetails = {{i, a, p, b, f[p]}};
 While[i < n, If[Sign[f[a]] == Sign[f[p]], a = p, b = p];
  p = (a + b) / 2;
  i++;
  OutputDetails = Append[OutputDetails, {i, a, p, b, f[p]}];]]
(*Combining the output details with the headings of the table*)
Print[NumberForm[
   N[TableForm[OutputDetails, TableHeadings → {None, {"i", "ai", "pi", "bi", "f[pi]"}}]],
   8]];(*Printing Table*)
Print["Root After ", n, " iterations ", NumberForm[N[p], 8]]
(*8 is used to give the result with 8 digits precision*)
```

Out[∘]=



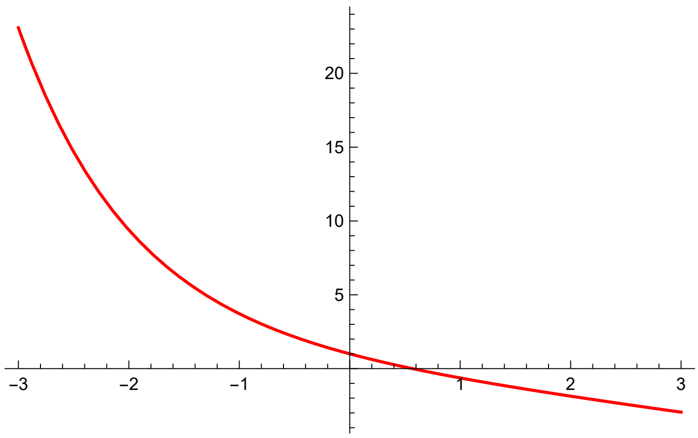
i	a_i	p_i	b_i	$f[p_i]$
1.	0.	0.5	1.	0.37758256
2.	0.5	0.75	1.	-0.018311131
3.	0.5	0.625	0.75	0.18596312
4.	0.625	0.6875	0.75	0.085334946
5.	0.6875	0.71875	0.75	0.033879372
6.	0.71875	0.734375	0.75	0.0078747255
7.	0.734375	0.7421875	0.75	-0.0051957117
8.	0.734375	0.73828125	0.7421875	0.0013451498
9.	0.73828125	0.74023438	0.7421875	-0.0019238728
10.	0.73828125	0.73925781	0.74023438	-0.00028900915

Root After 10 iterations 0.73925781

Ques.3 :- Perform 10 iterations of Bisection Method to find the root of the function f(x) = $e^x - x = 0$ in the interval [0,1]

In[◦]:=
```
f[x_] := Exp[-x] - x;
Plot[f[x], {x, -3, 3}, PlotStyle → Red]
a = 0;
b = 1;
n = 10;
If[f[a] * f[b] > 0, Print["We cannot continue with the Bisection Method."], i = 1;
 p = (a + b) / 2;
 OutputDetails = {{i, a, p, b, f[p]}};
 While[i < n, If[Sign[f[a]] == Sign[f[p]], a = p, b = p];
  p = (a + b) / 2;
  i++;
  OutputDetails = Append[OutputDetails, {i, a, p, b, f[p]}];]]
(*Combining the output details with the headings of the table*)
Print[NumberForm[
   N[TableForm[OutputDetails, TableHeadings → {None, {"i", "aᵢ", "pᵢ", "bᵢ", "f[pᵢ]"}}]],
   8]];(*Printing Table*)
Print["Root After ", n, " iterations ", NumberForm[N[p], 8]]
(*8 is used to give the result with 8 digits precision*)
```

Out[◦]=



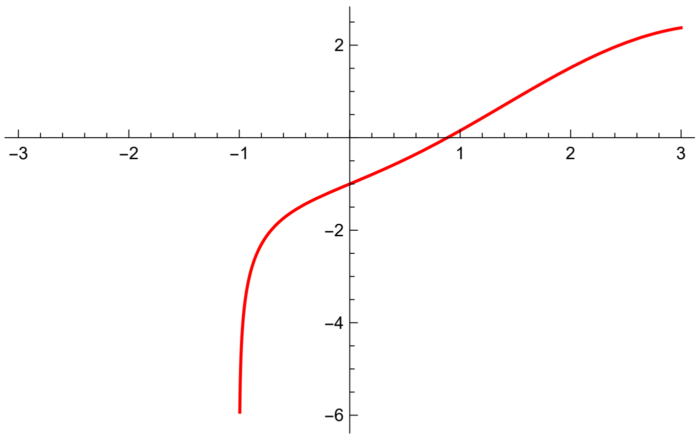
i	a_i	p_i	b_i	$f[p_i]$
1.	0.	0.5	1.	0.10653066
2.	0.5	0.75	1.	-0.27763345
3.	0.5	0.625	0.75	-0.089738571
4.	0.5	0.5625	0.625	0.0072828247
5.	0.5625	0.59375	0.625	-0.04149755
6.	0.5625	0.578125	0.59375	-0.017175839
7.	0.5625	0.5703125	0.578125	-0.0049637604
8.	0.5625	0.56640625	0.5703125	0.001155202
9.	0.56640625	0.56835938	0.5703125	-0.0019053596
10.	0.56640625	0.56738281	0.56835938	-0.00037534917

Root After 10 iterations 0.56738281

Ques.4:- Perform 10 iterations of Bisection Method to find the root of the function f(x) = ln(1+x)-cosx=0 in the interval [0,1]

In[]:=
```
f[x_] := Log[1 + x] - Cos[x];
Plot[f[x], {x, -3, 3}, PlotStyle → Red]
a = 0;
b = 1;
n = 10;
If[f[a] * f[b] > 0, Print["We cannot continue with the Bisection Method."], i = 1;
 p = (a + b) / 2;
 OutputDetails = {{i, a, p, b, f[p]}};
 While[i < n, If[Sign[f[a]] == Sign[f[p]], a = p, b = p];
  p = (a + b) / 2;
  i++;
  OutputDetails = Append[OutputDetails, {i, a, p, b, f[p]}];]]
(*Combining the output details with the headings of the table*)
Print[NumberForm[
   N[TableForm[OutputDetails, TableHeadings → {None, {"i", "ai", "pi", "bi", "f[pi]"}}]],
   8]];(*Printing Table*)
Print["Root After ", n, " iterations ", NumberForm[N[p], 8]]
(*8 is used to give the result with 8 digits precision*)
```

Out[]=



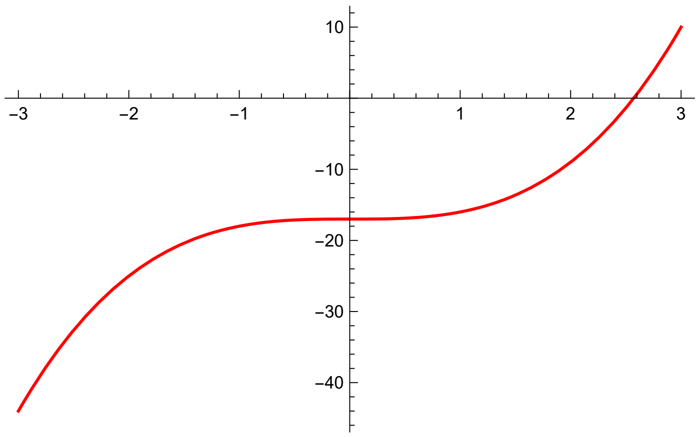
i	a_i	p_i	b_i	$f[p_i]$
1.	0.	0.5	1.	-0.47211745
2.	0.5	0.75	1.	-0.17207308
3.	0.75	0.875	1.	-0.012388199
4.	0.875	0.9375	1.	0.069593407
5.	0.875	0.90625	0.9375	0.028435895
6.	0.875	0.890625	0.90625	0.0079812284
7.	0.875	0.8828125	0.890625	-0.0022142544
8.	0.8828125	0.88671875	0.890625	0.0028808088
9.	0.8828125	0.88476563	0.88671875	0.00033260588
10.	0.8828125	0.88378906	0.88476563	-0.00094099231

Root After 10 iterations 0.88378906

Ques.5:- Perform 10 iterations of Bisection Method to find the root of the function cube root of 17 in the interval [0,1]

In[∘]:=
```
f[x_] := x^3 - 17;
Plot[f[x], {x, -3, 3}, PlotStyle → Red]
a = 2;
b = 3;
n = 10;
If[f[a] * f[b] > 0, Print["We cannot continue with the Bisection Method."], i = 1;
 p = (a + b) / 2;
 OutputDetails = {{i, a, p, b, f[p]}};
 While[i < n, If[Sign[f[a]] == Sign[f[p]], a = p, b = p];
  p = (a + b) / 2;
  i++;
  OutputDetails = Append[OutputDetails, {i, a, p, b, f[p]}];]]
(*Combining the output details with the headings of the table*)
Print[NumberForm[
   N[TableForm[OutputDetails, TableHeadings → {None, {"i", "ai", "pi", "bi", "f[pi]"}}]],
   8]];(*Printing Table*)
Print["Root After ", n, " iterations ", NumberForm[N[p], 8]]
(*8 is used to give the result with 8 digits precision*)
```

Out[∘]=



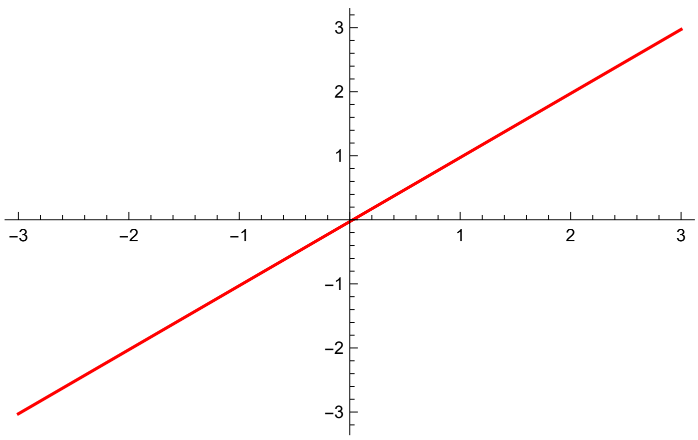
i	a_i	p_i	b_i	$f[p_i]$
1.	2.	2.5	3.	-1.375
2.	2.5	2.75	3.	3.796875
3.	2.5	2.625	2.75	1.0878906
4.	2.5	2.5625	2.625	-0.17358398
5.	2.5625	2.59375	2.625	0.44955444
6.	2.5625	2.578125	2.59375	0.13609695
7.	2.5625	2.5703125	2.578125	-0.019214153
8.	2.5703125	2.5742188	2.578125	0.058323562
9.	2.5703125	2.5722656	2.5742188	0.019525267
10.	2.5703125	2.5712891	2.5722656	0.00014820043

Root After 10 iterations 2.5712891

Ques.6:- Perform 10 iterations of Bisection Method to find the root of the function approximate value of $\frac{1}{37}$ in the interval [0,1]

In[∘]:=
```
f[x_] := x - 1/37;
Plot[f[x], {x, -3, 3}, PlotStyle → Red]
a = -1;
b = 1;
n = 10;
If[f[a] * f[b] > 0, Print["We cannot continue with the Bisection Method."], i = 1;
 p = (a + b) / 2;
 OutputDetails = {{i, a, p, b, f[p]}};
 While[i < n, If[Sign[f[a]] == Sign[f[p]], a = p, b = p];
  p = (a + b) / 2;
  i++;
  OutputDetails = Append[OutputDetails, {i, a, p, b, f[p]}];]]
(*Combining the output details with the headings of the table*)
Print[NumberForm[
   N[TableForm[OutputDetails, TableHeadings → {None, {"i", "aᵢ", "pᵢ", "bᵢ", "f[pᵢ]"}}]],
   8]];(*Printing Table*)
Print["Root After ", n, " iterations ", NumberForm[N[p], 8]]
(*8 is used to give the result with 8 digits precision*)
```

Out[∘]=



i	a_i	p_i	b_i	$f[p_i]$
1.	-1.	0.	1.	-0.027027027
2.	0.	0.5	1.	0.47297297
3.	0.	0.25	0.5	0.22297297
4.	0.	0.125	0.25	0.097972973
5.	0.	0.0625	0.125	0.035472973
6.	0.	0.03125	0.0625	0.004222973
7.	0.	0.015625	0.03125	-0.011402027
8.	0.015625	0.0234375	0.03125	-0.003589527
9.	0.0234375	0.02734375	0.03125	0.00031672297
10.	0.0234375	0.025390625	0.02734375	-0.001636402

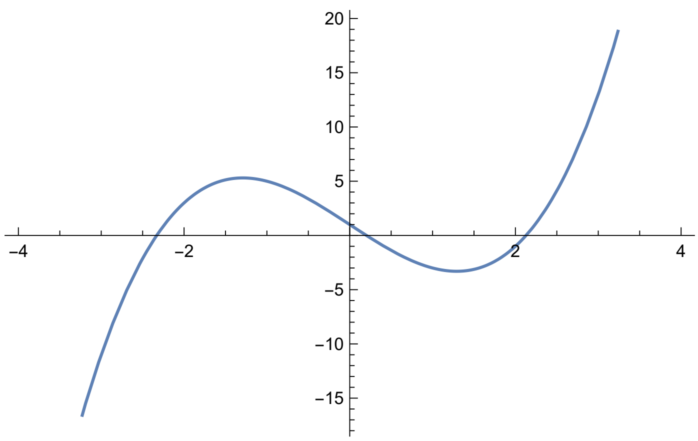
```
Root After 10 iterations 0.025390625
```

***PRACTICAL 2** :- NEWTON RAPHSON METHOD*

Ques. 1: Find the root of the function f(x)= $x^3 - 5x + 1$. Perform 5 iterations of Newton Raphson Method to find the root of the function starting with the initial approximation 0.5.

In[ ]:=
```
Clear[x, f, a, b, m, n, i]
f[x_] := x^3 - 5 x + 1;
Plot[f[x], {x, -4, 4}]
```

Out[ ]=



In[ ]:=
```
ClearAll;
```

In[]:=
```
k = 0.5;(*k = p0*)
n = 5;
If[f'[k] == 0,
 Print["We cannot continue with the Newton Raphson Mehtod"],
 i = 1;
 p = k - f[k]/f'[k]; (*k = pn-1*)
 OutputDetails = {{i, k}};
 While[i < n, p = k - f[k]/f'[k]; k = p; i++;
  OutputDetails = Append[OutputDetails, {i, k}];]]
(*Combining the output details with the headings of the table*)
Print[
  NumberForm[N[TableForm[OutputDetails, TableHeadings → {None, {"i", "ki"}}]], 8]];
(*Printing Table*)
Print["Root after ", n, " iterations ", NumberForm[N[k], 8]]
(*8 is used to give the result with 8 digits precision*)
```

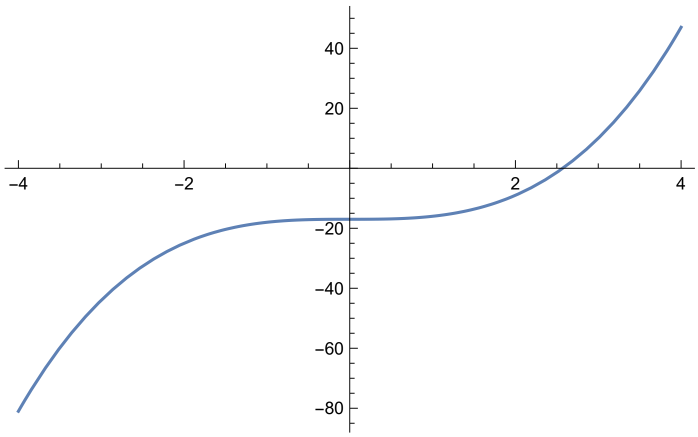
i	k_i
1.	0.5
2.	0.17647059
3.	0.20156807
4.	0.20163968
5.	0.20163968

Root after 5 iterations 0.20163968

Ques 2: Find the root of the function f(x)= x^3 – 17. Perform 5 iterations of Newton Raphson Method to find the root of the function starting with the initial approximation 2.

In[]:=
```
Clear[x, f, a, b, m, n, i]
f[x_] := x^3 - 17;
Plot[f[x], {x, -4, 4}]
```

Out[]=



In[]:=
```
ClearAll;
```

In[]:=
```
f[x_] := x^3 - 17
```

In[ ]:=
```
k = 2; (*k = p0*)
n = 5;
If[f'[k] == 0,
 Print["We cannot continue with the Newton Raphson Mehtod"],
 i = 1;
 p = k - f[k]/f'[k]; (*k = pn-1*)
 OutputDetails = {{i, k}};
 While[i < n, p = k - f[k]/f'[k]; k = p; i++;
  OutputDetails = Append[OutputDetails, {i, k}];]]
(*Combining the output details with the headings of the table*)
Print[
  NumberForm[N[TableForm[OutputDetails, TableHeadings → {None, {"i", "ki"}}]], 8]];
(*Printing Table*)
Print["Root after ", n, " iterations ", NumberForm[N[k], 8]]
(*8 is used to give the result with 8 digits precision*)
```

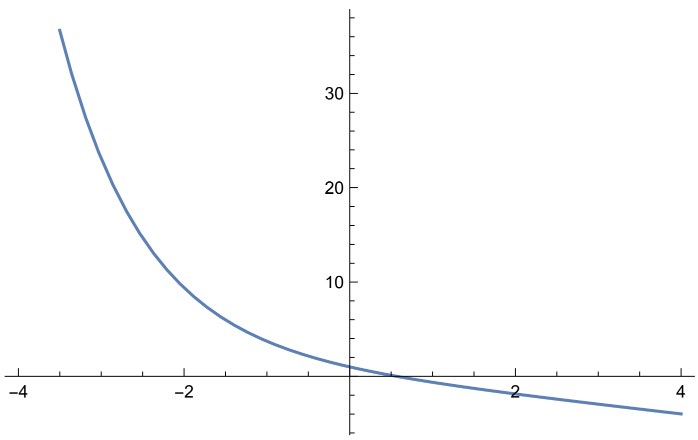
i	k_i
1.	2.
2.	2.75
3.	2.5826446
4.	2.5713315
5.	2.5712816

Root after 5 iterations 2.5712816

Ques 3: Find the root of the function $f(x) = e^{-x} - x$. Perform 5 iterations of Newton Raphson Method to find the root of the function starting with the initial approximation 0.5.

In[ ]:=
```
Clear[x, f, a, b, m, n, i]
f[x_] := Exp[-x] - x;
Plot[f[x], {x, -4, 4}]
```

Out[ ]=



In[*]:=
```
k = 0.5; (*k = p0*)
n = 5;
If[f'[k] == 0,
 Print["We cannot continue with the Newton Raphson Mehtod"],
 i = 1;
 p = k - f[k]/f'[k]; (*k = pn-1*)
 OutputDetails = {{i, k}};
 While[i < n, p = k - f[k]/f'[k]; k = p; i++;
  OutputDetails = Append[OutputDetails, {i, k}];]]
(*Combining the output details with the headings of the table*)
Print[
  NumberForm[N[TableForm[OutputDetails, TableHeadings → {None, {"i", "ki"}}]], 8]];
(*Printing Table*)
Print["Root after ", n, " iterations ", NumberForm[N[k], 8]]
(*8 is used to give the result with 8 digits precision*)
```

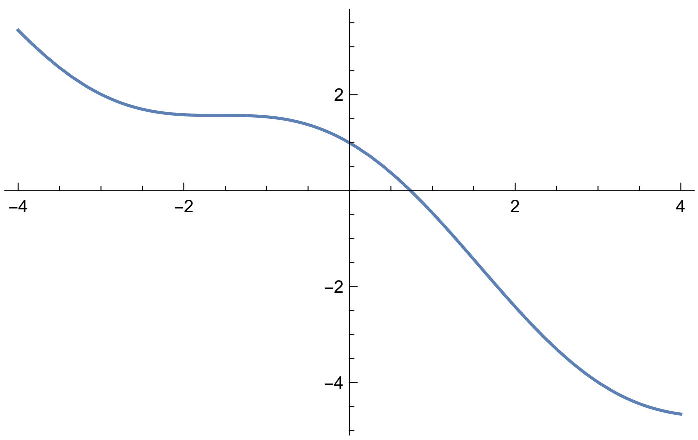
i	k_i
1.	0.5
2.	0.566311
3.	0.56714317
4.	0.56714329
5.	0.56714329

Root after 5 iterations 0.56714329

Ques 4: Find the root of the function $f(x) = \cos x - x$. Perform 5 iterations of Newton Raphson Method to find the root of the function starting with the initial approximation 0.75.

In[*]:=
```
Clear[x, f, a, b, m, n, i]
f[x_] := Cos[x] - x;
Plot[f[x], {x, -4, 4}]
```

Out[*]=



In[•]:=
```
k = 0.75; (*k = p0*)
n = 5;
If[f'[k] == 0,
 Print["We cannot continue with the Newton Raphson Mehtod"],
 i = 1;
 p = k - f[k]/f'[k]; (*k = pn-1*)
 OutputDetails = {{i, k}};
 While[i < n, p = k - f[k]/f'[k]; k = p; i++;
  OutputDetails = Append[OutputDetails, {i, k}];]]
(*Combining the output details with the headings of the table*)
Print[
  NumberForm[N[TableForm[OutputDetails, TableHeadings → {None, {"i", "ki"}}]], 8]];
(*Printing Table*)
Print["Root after ", n, " iterations ", NumberForm[N[k], 8]]
(*8 is used to give the result with 8 digits precision*)
```

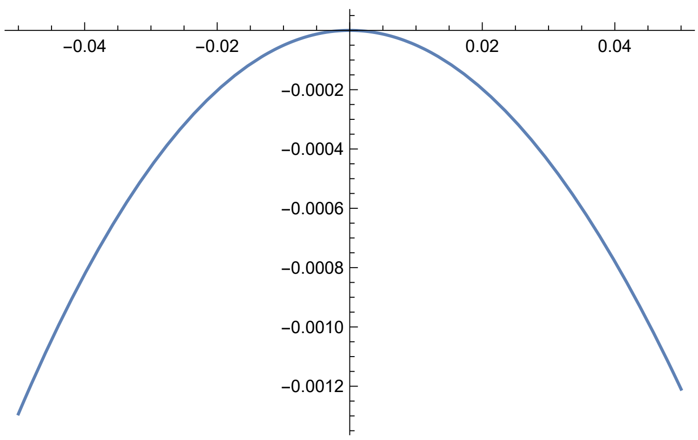
i	k_i
1.	0.75
2.	0.73911114
3.	0.73908513
4.	0.73908513
5.	0.73908513

Root after 5 iterations 0.73908513

Ques 4: Find the root of the function $f(x) = \ln(1+x) - x$. Perform 5 iterations of Newton Raphson Method to find the root of the function starting with the initial approximation 0.5.

In[•]:=
```
Clear[x, f, a, b, m, n, i]
f[x_] := Log[1 + x] - x;
Plot[f[x], {x, -0.05, 0.05}]
```

Out[•]=



In[◦]:=
```
k = 0.5; (*k = p0*)
n = 10;
If[f'[k] == 0,
 Print["We cannot continue with the Newton Raphson Mehtod"],
 i = 1;
 p = k - f[k]/f'[k]; (*k = pn-1*)
 OutputDetails = {{i, k}};
 While[i < n, p = k - f[k]/f'[k]; k = p; i++;
  OutputDetails = Append[OutputDetails, {i, k}];]]
(*Combining the output details with the headings of the table*)
Print[
  NumberForm[N[TableForm[OutputDetails, TableHeadings → {None, {"i", "ki"}}]], 8]];
(*Printing Table*)
Print["Root after ", n, " iterations ", NumberForm[N[k], 8]]
(*8 is used to give the result with 8 digits precision*)
```

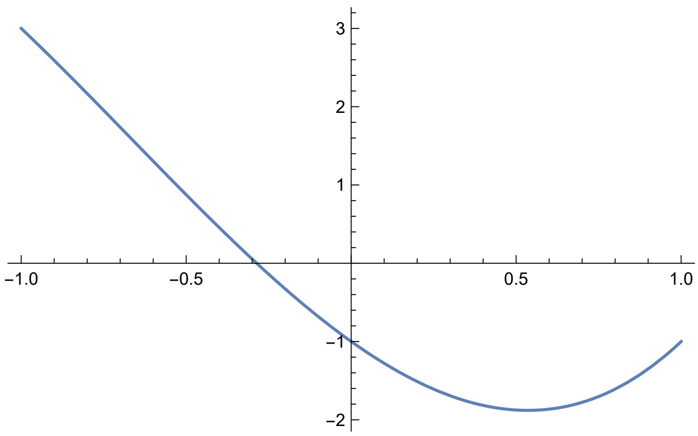
i	k_i
1.	0.5
2.	0.21639532
3.	0.10114167
4.	0.048947214
5.	0.024083797
6.	0.011946374
7.	0.0059495423
8.	0.0029688891
9.	0.0014829777
10.	0.00074112258

Root after 10 iterations 0.00074112258

Ques 5: Find the root of the function $f(x) = x^3 + 2x^2 - 3x - 1$. Perform 5 iterations of Newton Raphson Method to find the root of the function starting with the initial approximation 0.75.

In[◦]:=
```
Clear[x, f, a, b, m, n, i]
f[x_] := x^3 + 2 x^2 - 3 x - 1;
Plot[f[x], {x, -1, 1}]
```

Out[◦]=



In[∘]:=
```
k = -0.25; (*k = p0*)
n = 5;
If[f'[k] == 0,
 Print["We cannot continue with the Newton Raphson Mehtod"],
 i = 1;
 p = k - f[k]/f'[k]; (*k = pn-1*)
 OutputDetails = {{i, k}};
 While[i < n, p = k - f[k]/f'[k]; k = p; i++;
  OutputDetails = Append[OutputDetails, {i, k}];]]
(*Combining the output details with the headings of the table*)
Print[
  NumberForm[N[TableForm[OutputDetails, TableHeadings → {None, {"i", "ki"}}]], 8]];
(*Printing Table*)
Print["Root after ", n, " iterations ", NumberForm[N[k], 8]]
(*8 is used to give the result with 8 digits precision*)
```

i	k_i
1.	-0.25
2.	-0.28688525
3.	-0.28646212
4.	-0.28646207
5.	-0.28646207

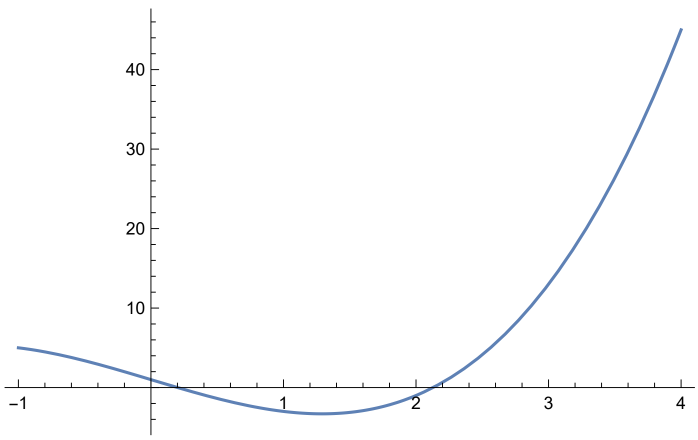
Root after 5 iterations -0.28646207

PRACTICAL 3:- SECANT METHOD

Ques. 1: Find the root of the function $f(x) = x^3 - 5x + 1$. Perform 6 iterations of Secant Method to find the root of the function starting with the initial approximation 0 and 1

In[∘]:=
```
Clear[x, f, a, b, m, n, i]
f[x_] := x^3 - 5 x + 1;
Plot[f[x], {x, -1, 4}]
```

Out[∘]=



In[ ]:=
```
k = 0 ; m = 1; (*k = p_o and m = p_1*)
n = 6;
i = 1;
p = m - f[m] (m - k)/(f[m] - f[k]); (*k = p_(n-1) and m = p_n*)
OutputDetails = {{i, p, f[p]}};
While[i < n, p = m - f[m] (m - k)/(f[m] - f[k]);
 k = m;
 m = p;
 i++;
 OutputDetails = Append[OutputDetails, {i, p, f[p]}];]
(*Combining the output details with the headings of the table*)
Print[NumberForm[
   N[TableForm[OutputDetails, TableHeadings → {None, {"i", "p_i", "f[p_i]"}}]], 8]];
(*Printing Table*)
Print["Root After ", n, " iterations ", NumberForm[N[p], 8]]
(*8 is used to give the result with 8 digits precision*)
```

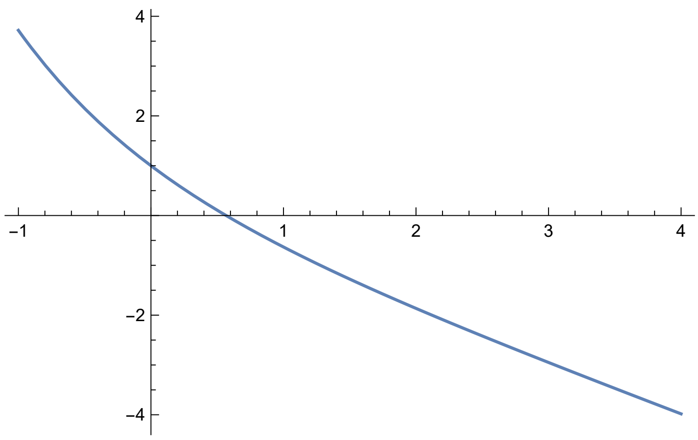
i	p_i	$f[p_i]$
1.	0.25	-0.234375
2.	0.25	-0.234375
3.	0.18644068	0.074277312
4.	0.20173626	-0.00047111617
5.	0.20163985	-8.642293×10^{-7}
6.	0.20163968	1.0352523×10^{-11}

Root After 6 iterations 0.20163968

Ques. 2: Find the root of the function $f(x) = e^{-x} - x$. Perform 6 iterations of Secant Method to find the root of the function starting with the initial approximation 0 and 1

In[ ]:=
```
Clear[x, f, a, b, m, n, i]
f[x_] := Exp[-x] - x;
Plot[f[x], {x, -1, 4}]
```

Out[ ]=



In[ ]:=
```
k = 0 ; m = 1; (*k = p0 and m = p1*)
n = 6;
i = 1;
p = m - f[m] (m - k)/(f[m] - f[k]); (*k = pn-1 and m = pn*)
OutputDetails = {{i, p, f[p]}};
While[i < n, p = m - f[m] (m - k)/(f[m] - f[k]);
 k = m;
 m = p;
 i++;
 OutputDetails = Append[OutputDetails, {i, p, f[p]}];]
(*Combining the output details with the headings of the table*)
Print[NumberForm[
   N[TableForm[OutputDetails, TableHeadings → {None, {"i", "pi", "f[pi]"}}]], 8]];
(*Printing Table*)
Print["Root After ", n, " iterations ", NumberForm[N[p], 8]]
(*8 is used to give the result with 8 digits precision*)
```

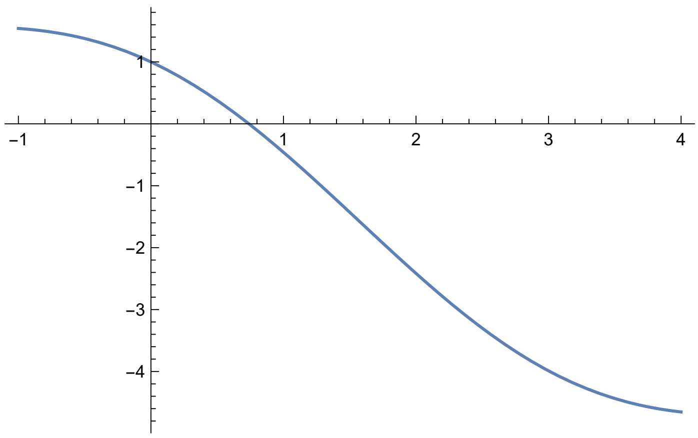
i	p_i	$f[p_i]$
1.	0.61269984	-0.070813948
2.	0.61269984	-0.070813948
3.	0.56383839	0.0051823545
4.	0.56717036	-0.000042419242
5.	0.56714331	-2.5380167×10^{-8}
6.	0.56714329	1.2423092×10^{-13}

Root After 6 iterations 0.56714329

Ques. 3: Find the root of the function $f(x) = \text{Cos}x - x$. Perform 6 iterations of Secant Method to find the root of the function starting with the initial approximation 0 and 1

In[ ]:=
```
Clear[x, f, a, b, m, n, i]
f[x_] := Cos[x] - x;
Plot[f[x], {x, -1, 4}]
```

Out[ ]=



In[ ]:=
```
k = 0 ; m = 1; (*k = po and m = p1*)
n = 6;
i = 1;
p = m - f[m] (m - k)/(f[m] - f[k]); (*k = pn-1 and m = pn*)
OutputDetails = {{i, p, f[p]}};
While[i < n, p = m - f[m] (m - k)/(f[m] - f[k]);
 k = m;
 m = p;
 i++;
 OutputDetails = Append[OutputDetails, {i, p, f[p]}];]
(*Combining the output details with the headings of the table*)
Print[NumberForm[
   N[TableForm[OutputDetails, TableHeadings → {None, {"i", "pi", "f[pi]"}}]], 8]];
(*Printing Table*)
Print["Root After ", n, " iterations ", NumberForm[N[p], 8]]
(*8 is used to give the result with 8 digits precision*)
```

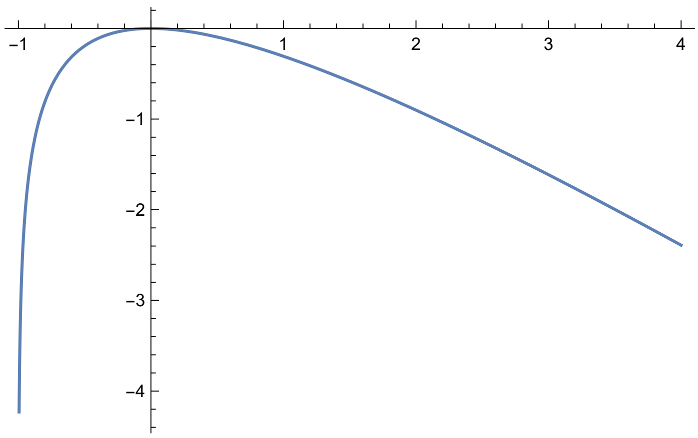
i	p_i	$f[p_i]$
1.	0.68507336	0.089299276
2.	0.68507336	0.089299276
3.	0.736299	0.004660039
4.	0.73911936	-0.000057285991
5.	0.73908511	3.5292623×10^{-8}
6.	0.73908513	2.6678659×10^{-13}

Root After 6 iterations 0.73908513

Ques. 4: Find the root of the function $f(x) = \ln(1 + x) - x$. Perform 6 iterations of Secant Method to find the root of the function starting with the initial approximation 0.1 and 1

In[ ]:=
```
Clear[x, f, a, b, m, n, i]
f[x_] := Log[1 + x] - x;
Plot[f[x], {x, -1, 4}]
```

Out[ ]=



In[ ]:=
```
k = 0.1; m = 1;(*k = p0 and m = p1*)
n = 6;
i = 1;
p = m - f[m] (m - k)/(f[m] - f[k]);(*k = pn-1 and m = pn*)
OutputDetails = {{i, p, f[p]}};
While[i < n, p = m - f[m] (m - k)/(f[m] - f[k]);
 k = m;
 m = p;
 i++;
 OutputDetails = Append[OutputDetails, {i, p, f[p]}];]
(*Combining the output details with the headings of the table*)
Print[NumberForm[
   N[TableForm[OutputDetails, TableHeadings → {None, {"i", "pi", "f[pi]"}}]], 8]];
(*Printing Table*)
Print["Root After ", n, " iterations ", NumberForm[N[p], 8]]
(*8 is used to give the result with 8 digits precision*)
```

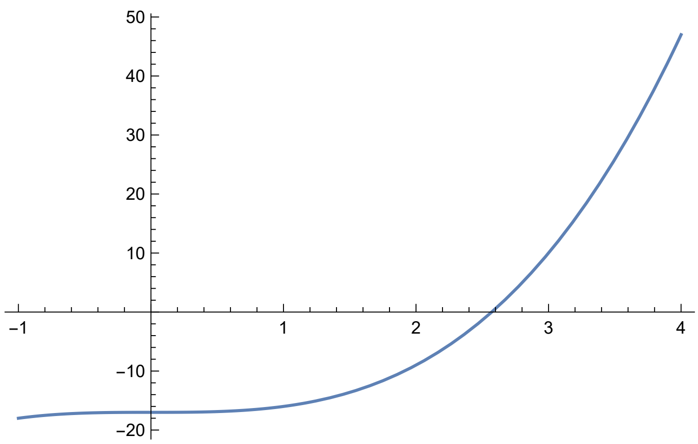
i	p_i	$f[p_i]$
1.	0.086031254	-0.0035012539
2.	0.086031254	-0.0035012539
3.	0.075482317	-0.002713089
4.	0.039169855	-0.00074767693
5.	0.025355963	-0.00031612973
6.	0.015236603	-0.00011491126

Root After 6 iterations 0.015236603

Ques. 5: Find the root of the function $f(x) = x^3 - 17$. Perform 6 iterations of Secant Method to find the root of the function starting with the initial approximation 0 and 1

In[ ]:=
```
Clear[x, f, a, b, m, n, i]
f[x_] := x^3 - 17;
Plot[f[x], {x, -1, 4}]
```

Out[ ]=



In[•]:=
```
k = 0; m = 1; (*k = p0 and m = p1*)
n = 6;
i = 1;
p = m - f[m] (m - k)/(f[m] - f[k]); (*k = pn-1 and m = pn*)
OutputDetails = {{i, p, f[p]}};
While[i < n, p = m - f[m] (m - k)/(f[m] - f[k]);
 k = m;
 m = p;
 i++;
 OutputDetails = Append[OutputDetails, {i, p, f[p]}];]
(*Combining the output details with the headings of the table*)
Print[NumberForm[
   N[TableForm[OutputDetails, TableHeadings → {None, {"i", "pi", "f[pi]"}}]], 8]];
(*Printing Table*)
Print["Root After ", n, " iterations ", NumberForm[N[p], 8]]
(*8 is used to give the result with 8 digits precision*)
```

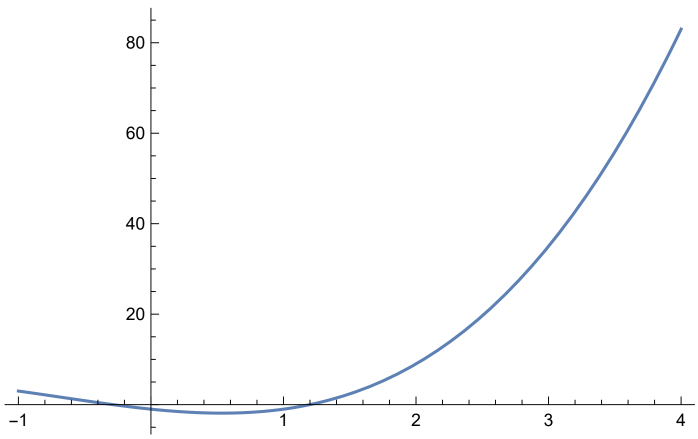
i	p_i	$f[p_i]$
1.	17.	4896.
2.	17.	4896.
3.	1.0521173	-15.835358
4.	1.1035319	-15.656138
5.	5.5949642	158.14265
6.	1.508129	-13.569831

Root After 6 iterations 1.508129

Ques. 6: Find the root of the function $f(x) = x^3 + 2x^2 - 3x - 1$. Perform 6 iterations of Secant Method to find the root of the function starting with the initial approximation 0.1 and 1

In[•]:=
```
Clear[x, f, a, b, m, n, i]
f[x_] := x^3 + 2 x^2 - 3 x - 1;
Plot[f[x], {x, -1, 4}]
```

Out[•]=



```
In[]:= k = 0.1; m = 1;(*k = p0 and m = p1*)
n = 6;
i = 1;
p = m - f[m] (m - k)/(f[m] - f[k]);(*k = pn-1 and m = pn*)
OutputDetails = {{i, p, f[p]}};
While[i < n, p = m - f[m] (m - k)/(f[m] - f[k]);
 k = m;
 m = p;
 i++;
 OutputDetails = Append[OutputDetails, {i, p, f[p]}];]
(*Combining the output details with the headings of the table*)
Print[NumberForm[
   N[TableForm[OutputDetails, TableHeadings → {None, {"i", "pi", "f[pi]"}}]], 8]];
(*Printing Table*)
Print["Root After ", n, " iterations ", NumberForm[N[p], 8]]
(*8 is used to give the result with 8 digits precision*)
```

i	p_i	$f[p_i]$
1.	4.2258065	97.499547
2.	4.2258065	97.499547
3.	1.0327495	-0.86360442
4.	1.0607837	-0.73816727
5.	1.2257585	0.16937497
6.	1.1949692	-0.022646757

Root After 6 iterations 1.1949692

Practical 4 - LU Decomposition

Ques 1:- Find the LU Decomposition of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{pmatrix}$

In[∘]:=
```
Clear [A, L, U, i, j]
A = {{1, 2, 3}, {1, 4, 9}, {1, 8, 27}}
{m, p, c} = LUDecomposition[A]
MatrixForm[m]
L = m SparseArray[{i_, j_} /; j < i → 1, {3, 3}] + IdentityMatrix[3];
U = m SparseArray[{i_, j_} /; j ≥ i → 1, {3, 3}];
MatrixForm /@ {A, L, U, L.U}
A == L.U
```

Out[∘]=

{{1, 2, 3}, {1, 4, 9}, {1, 8, 27}}

Out[∘]=

{{{1, 2, 3}, {1, 2, 6}, {1, 3, 6}}, {1, 2, 3}, 0}

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 6 \\ 1 & 3 & 6 \end{pmatrix}$$

Out[∘]=

$$\left\{\begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 6 \\ 0 & 0 & 6 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 9 \\ 1 & 8 & 27 \end{pmatrix}\right\}$$

Out[∘]=

True

Ques 2: Find the LU decomposition of the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and solve the system Ax = $[4\ 6]^T$

In[ ]:=
```
Clear[A, L, U, i, j]
A = {{1, 2}, {3, 4}}
{m, p, c} = LUDecomposition[A]
L = m SparseArray[{i_, j_} /; j < i → 1, {2, 2}] + IdentityMatrix[2];
U = m SparseArray[{i_, j_} /; j ≥ i → 1, {2, 2}];
MatrixForm /@ {A, L, U, L.U}
A == L.U
X = {{x1}, {x2}};
B = {{4}, {6}}
Y = {{y1}, {y2}};
Ysol = Solve[L.Y == B, {y1, y2}]
Xsol = Solve[U.X == Y, {x1, x2}]
Xsol /. Ysol
```

Out[ ]=

{{1, 2}, {3, 4}}

Out[ ]=

{{{1, 2}, {3, -2}}, {1, 2}, 0}

Out[ ]=

$\left\{\begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}, \begin{pmatrix}1 & 0\\3 & 1\end{pmatrix}, \begin{pmatrix}1 & 2\\0 & -2\end{pmatrix}, \begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}\right\}$

Out[ ]=

True

Out[ ]=

{{4}, {6}}

Out[ ]=

$\{\{y_1 \to 4, y_2 \to -6\}\}$

Out[ ]=

$\left\{\left\{x_1 \to y_1 + y_2, x_2 \to -\frac{y_2}{2}\right\}\right\}$

Out[ ]=

$\{\{\{x_1 \to -2, x_2 \to 3\}\}\}$

Ques 3: Find the LU Decomposition of the matrix $\begin{pmatrix}2 & 7 & 5\\6 & 20 & 10\\4 & 3 & 0\end{pmatrix}$ and then solve the system Ax = $[0\ 4\ 1]^T$

```
In[•]:= Clear [A, L, U, i, j]
A = {{2, 7, 5}, {6, 20, 10}, {4, 3, 0}}
{m, p, c} = LUDecomposition[A]
MatrixForm[m]
L = m SparseArray[{i_, j_} /; j < i → 1, {3, 3}] + IdentityMatrix[3];
U = m SparseArray[{i_, j_} /; j ≥ i → 1, {3, 3}];
MatrixForm /@ {A, L, U, L.U}
A == L.U
X = {{x1}, {x2}, {x3}};
B = {{0}, {4}, {1}}
Y = {{y1}, {y2}, {y3}};
Ysol = Solve[L.Y == B, {y1, y2, y3}]
Xsol = Solve[U.X == Y, {x1, x2, x3}]
Xsol /. Ysol
```

Out[•]=

{{2, 7, 5}, {6, 20, 10}, {4, 3, 0}}

Out[•]=

{{{2, 7, 5}, {3, -1, -5}, {2, 11, 45}}, {1, 2, 3}, 0}

Out[•]//MatrixForm=

$$\begin{pmatrix} 2 & 7 & 5 \\ 3 & -1 & -5 \\ 2 & 11 & 45 \end{pmatrix}$$

Out[•]=

$$\left\{\begin{pmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 11 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 7 & 5 \\ 0 & -1 & -5 \\ 0 & 0 & 45 \end{pmatrix}, \begin{pmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{pmatrix}\right\}$$

Out[•]=

True

Out[•]=

{{0}, {4}, {1}}

Out[•]=

$$\{\{y_1 \to 0, y_2 \to 4, y_3 \to -43\}\}$$

Out[•]=

$$\left\{\left\{x_1 \to \frac{1}{6}(3 y_1 + 21 y_2 + 2 y_3), x_2 \to \frac{1}{9}(-9 y_2 - y_3), x_3 \to \frac{y_3}{45}\right\}\right\}$$

Out[•]=

$$\left\{\left\{\left\{x_1 \to -\frac{1}{3}, x_2 \to \frac{7}{9}, x_3 \to -\frac{43}{45}\right\}\right\}\right\}$$

Practical 5 - GAUSS JACOBI METHOD

Ques 1:- Solve the system of equation by performing 15 iterations of Gauss Jacobi Iterative Method with initial approximation $x_0 = [0, 0, 0]^T$

$5 x_1 + x_2 + 2 x_3 = 10$

$-3 x_1 + 9 x_2 + 4 x_3 = -14$

$x_1 + 2 x_2 - 7 x_3 = -33$

```
In[•]:= A = {{5, 1, 2}, {-3, 9, 4}, {1, 2, -7}};
b = {{10}, {-14}, {-33}}
x0 = {{0}, {0}, {0}}
max = 15
k = 0
Size = Dimensions[A]
m = Size[[1]]
n = Size[[2]]
xk = x0 (*x_k*)
If[m ≠ n, Print["Not a square matrix, so we cannot proceed"],
 OutputDetails = {xk};
 xk1 = Table[0, {m}]; (*x_{k+1}*)
 While[k < max,
  For[i = 1, i ≤ m, i++, xk1[[i]] = 1/A[[i, i]] (b[[i]] - Sum_{j=1}^{i-1} A[[i, j]] * xk[[j]] - Sum_{j=i+1}^{m} A[[i, j]] * xk[[j]]);];
  k++;
  OutputDetails = Append[OutputDetails, xk1]; xk = xk1;];
 ColumnHeading = Table[x[j], {j, 1, m}];]
Print[
 NumberForm[N[TableForm[OutputDetails, TableHeadings → {None, ColumnHeading}]], 8]]
```

```
Out[•]=
{{10}, {-14}, {-33}}
Out[•]=
{{0}, {0}, {0}}
Out[•]=
15
Out[•]=
0
Out[•]=
{3, 3}
Out[•]=
3
Out[•]=
3
Out[•]=
{{0}, {0}, {0}}
```

x[1.]	x[2.]	x[3.]
0.	0.	0.
2.	-1.5555556	4.7142857
0.42539683	-2.984127	4.5555556
0.77460317	-3.438448	3.922449
1.11871	-3.0406652	3.8425296
1.0711212	-2.8904432	4.00534
0.97595265	-2.9786663	4.0414621
0.9791484	-3.0264434	4.00266
1.0042247	-3.0081328	3.9894659
1.0058402	-2.99391	3.9982799
0.99947004	-2.9972888	4.0025743
0.99842803	-3.0013208	4.0006989
0.99998459	-3.0008346	3.9993981
1.0004077	-2.9997376	3.9997593
1.0000438	-2.9997571	4.0001332
0.99989814	-3.0000446	4.0000756

PRACTICAL - 6 GAUSS SEIDEL METHOD

Ques. 2: Solve the system of equations by performing 10 iterations of the Gauss Seidel Iterative method with initial approximation x_0= [0, 0, 0]^T

```
A = {{5, 1, 2}, {-3, 9, 4}, {1, 2, -7}};
b = {{10}, {-14}, {-33}}
x0 = {{0}, {0}, {0}}
max = 15
k = 0
Size = Dimensions[A]
m = Size[[1]]
n = Size[[2]]
xk = x0 (*x_k*)
If[m ≠ n, Print["Not a square matrix, so we cannot proceed"],
 OutputDetails = {xk};
 xk1 = Table[0, {m}]; (*x_(k+1)*)
 While[k < max, For[i = 1, i ≤ m, i++,
    xk1[[i]] = 1/A[[i, i]] (b[[i]] - Sum[A[[i, j]] * xk1[[j]], {j, 1, i-1}] - Sum[A[[i, j]] * xk[[j]], {j, i+1, m}]);];
   k++;
   OutputDetails = Append[OutputDetails, xk1]; xk = xk1;];
 ColumnHeading = Table[x[j], {j, 1, m}];]
Print[
 NumberForm[N[TableForm[OutputDetails, TableHeadings → {None, ColumnHeading}]], 8]]
```

Out[•]=
{{10}, {-14}, {-33}}

Out[•]=
{{0}, {0}, {0}}

Out[•]=
15

Out[ ]=
0

Out[ ]=
{3, 3}

Out[ ]=
3

Out[ ]=
3

Out[ ]=
{{0}, {0}, {0}}

x[1.]	x[2.]	x[3.]
0.	0.	0.
2.	-0.88888889	4.7460317
0.27936508	-3.5717813	3.7336861
1.2208818	-2.808011	4.0864086
0.92703877	-3.0627242	3.9716558
1.0238825	-2.9794417	4.0092856
0.99217411	-3.0067356	3.9969576
1.0025641	-2.9977931	4.0009968
0.99915989	-3.0007231	3.9996734
1.0002753	-2.9997631	4.000107
0.99990981	-3.0000776	3.9999649
1.0000295	-2.9999746	4.0000115
0.99999032	-3.0000083	3.9999962
1.0000032	-2.9999973	4.0000012
0.99999896	-3.0000009	3.9999996
1.0000003	-2.9999997	4.0000001

PRACTICAL 7 - LAGRANGE INTERPOLATION

Ques. 1: Find Lagrange Interpolating polynomial for the following set of points: x_0=0, x_1=1, x_2=3, $f(x_0)$=1, $f(x_1)$=3, $f(x_2)$=55. And hence approximate the value of the function at the point x=2 using the resulting polynomial. Also plot the Lagrange polynomial and the points (x_i, f_i) on the same axes.

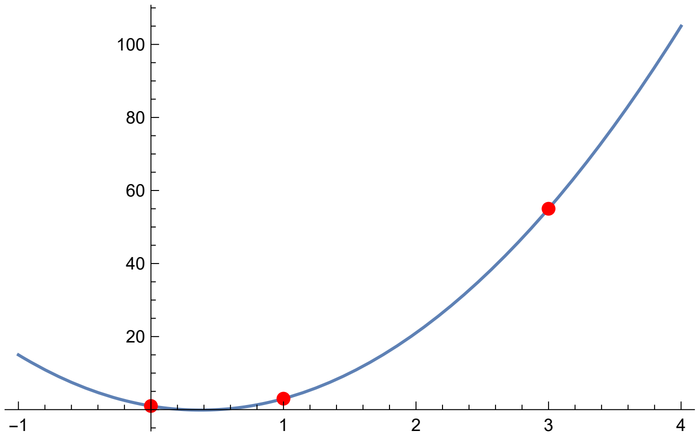
In[•]:=
```
xi = {0, 1, 3};
fi = {1, 3, 55};
n = Length[xi];
m = Length[fi];
If[n ≠ m, Print["List of points and function values are not of same size"],
 For[i = 1, i ≤ n, i++, L[i, x_] = (∏_{j=1}^{i-1} (x - xi[[j]])/(xi[[i]] - xi[[j]])) (∏_{j=i+1}^{n} (x - xi[[j]])/(xi[[i]] - xi[[j]]));];
 Polynomial[x_] = ∑_{k=1}^{n} L[k, x] * fi[[k]];]
Print["Langrange Polynomial = ", Simplify[Polynomial[x]]]
Polynomial[2]
A = Plot[Polynomial[x], {x, -1, 4}];
B = Graphics[{Red, PointSize[0.02], Point[{0, 1}], Point[{1, 3}], Point[{3, 55}]}];
Show[A, B]
```

Langrange Polynomial = $1 - 6x + 8x^2$

Out[•]=

21

Out[•]=



Ques. 2: let f(x)=ln x. Find the Lagrange Interpolating polynomial for the following set of points: x_0=1, x_1=2, x_2=3, $f(x_0)$=ln 1, $f(x_1)$=ln 2, $f(x_2)$=ln 3. And hence approximate the value of the function at the point x=1.5 and x=2.4 using the resulting polynomial. Plot the function f(x) and the resulting Lagrange polynomial on the same axes over the range [1,3]. Next generate the plot of the difference between the Lagrange polynomial and the function f(x)= ln x

In[ ]:=
```
xi = {1, 2, 3};
fi = {N[Log[1]], N[Log[2]], N[Log[3]]};
n = Length[xi];
m = Length[fi];
If[n ≠ m, Print["List of points and function values are not of same size"],
 For[i = 1, i ≤ n, i++, L[i, x_] = (∏_{j=1}^{i-1} (x - xi[[j]])/(xi[[i]] - xi[[j]])) (∏_{j=i+1}^{n} (x - xi[[j]])/(xi[[i]] - xi[[j]]));];
 Polynomial[x_] = ∑_{k=1}^{n} L[k, x] * fi[[k]];]
Print["Langrange Polynomial = ", Simplify[Polynomial[x]]]
Polynomial[1.5]
Polynomial[2.4]
Plot[{Polynomial[x], Log[x]}, {x, 1, 3}, PlotLegends → "Expressions"]
Plot[Log[x] - Polynomial[x], {x, 1, 3}]
```

Langrange Polynomial = $-0.980829 + 1.12467\,x - 0.143841\,x^2$

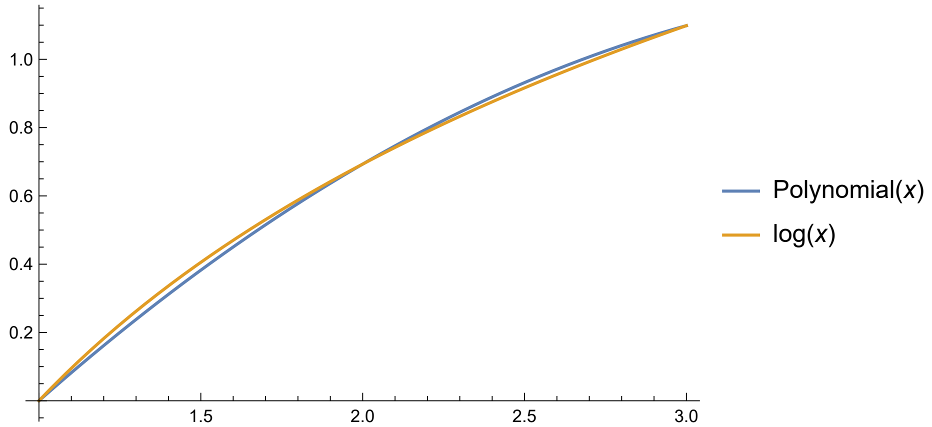
Out[ ]=

0.382534

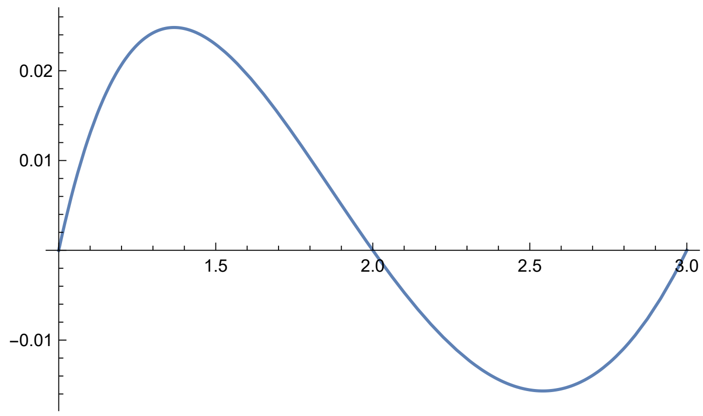
Out[ ]=

0.889855

Out[ ]=



Out[ ]=



Ques. 3: let f(x)=sin x. Find the Lagrange Interpolating polynomial for the following set of points: $x_0=0$, $x_1=\frac{\pi}{4}$, $x_2=\frac{\pi}{2}$, $f(x_0)=\sin 0$, $f(x_1)=\sin\frac{\pi}{4}$, $f(x_2)=\sin\frac{\pi}{2}$. And hence approximate the value of the

function at the point x=1.5 and x=2.4 using the resulting polynomial. Plot the function f(x) and the resulting Lagrange polynomial on the same axes over the range $[0, \frac{\pi}{2}]$. Next generate the plot of the difference between the Lagrange polynomial and the function f(x)= sin x

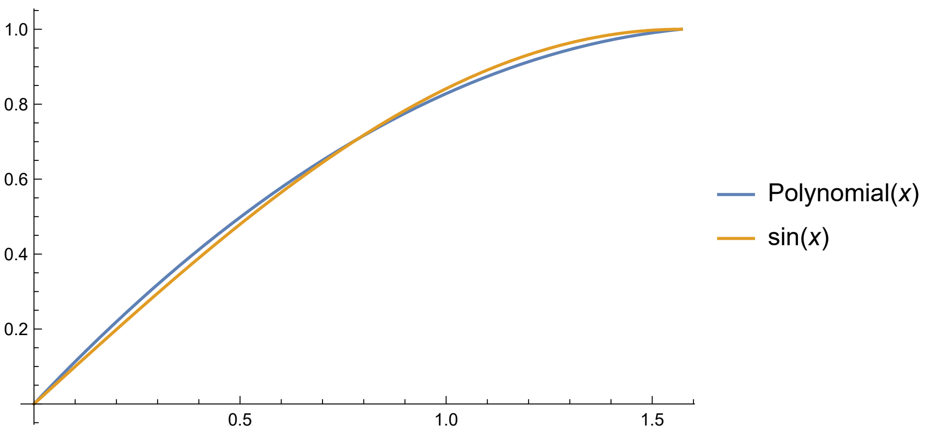
In[•]:=
```
ClearAll
xi = {0, π/4, π/2};
fi = {N[Sin[0]], N[Sin[π/4]], N[Sin[π/2]]};
n = Length[xi];
m = Length[fi];
If[n ≠ m, Print["List of points and function values are not of same size"],
  For[i = 1, i ≤ n, i++, L[i, x_] = (∏_{j=1}^{i-1} (x - xi⟦j⟧)/(xi⟦i⟧ - xi⟦j⟧)) (∏_{j=i+1}^{n} (x - xi⟦j⟧)/(xi⟦i⟧ - xi⟦j⟧));];
  Polynomial[x_] = ∑_{k=1}^{n} L[k, x] * fi⟦k⟧;]
Print["Langrange Polynomial = ", Simplify[Polynomial[x]]]
Polynomial[π/3]
Polynomial[π/6]
Plot[{Polynomial[x], Sin[x]}, {x, 0, π/2}, PlotLegends → "Expressions"]
Plot[Sin[x] - Polynomial[x], {x, 0, π/2}]
```

Out[•]=
```
ClearAll
Langrange Polynomial = 0. + 1.16401 x - 0.335749 x^2
```

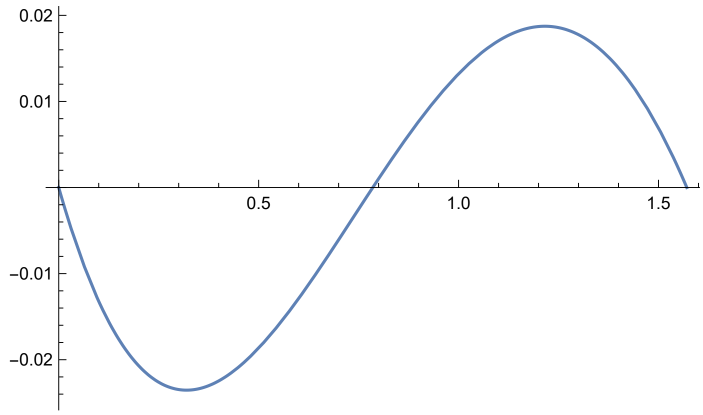
Out[•]=
```
0.850762
```

Out[•]=
```
0.517428
```

Out[•]=



Out[∘]=



Ques. 4: let $f(x) = e^x$. Find the Lagrange Interpolating polynomial for the following set of points: x_0=-1, x_1= 0, x_2= 1, $f(x_0)$= e^{-1}, $f(x_1)$= e^0, $f(x_2)$= e^1. And hence approximate the value of the function at the point x= $\sqrt{e}$ and $x = e^{-1/3}$ using the resulting polynomial. Plot the function f(x) and the resulting Lagrange polynomial on the same axes over the range [-1,1]. Next generate the plot of the difference between the Lagrange polynomial and the function f(x)= e^x

```
ClearAll
xi = {-1, 0, 1};
fi = {N[Exp[-1]], N[Exp[0]], N[Exp[]]};
n = Length[xi];
m = Length[fi];
If[n ≠ m, Print["List of points and function values are not of same size"],
 For[i = 1, i ≤ n, i++, L[i, x_] = (∏_{j=1}^{i-1} (x - xi[[j]])/(xi[[i]] - xi[[j]])) (∏_{j=i+1}^{n} (x - xi[[j]])/(xi[[i]] - xi[[j]]));];
 Polynomial[x_] = ∑_{k=1}^{n} L[k, x] * fi[[k]];]
Print["Langrange Polynomial = ", Simplify[Polynomial[x]]]
Polynomial[π/3]
Polynomial[π/6]
Plot[{Polynomial[x], Sin[x]}, {x, 0, π/2}, PlotLegends → "Expressions"]
Plot[Sin[x] - Polynomial[x], {x, 0, π/2}]
```

Practical 8 - Newton's Interpolation

Ques 1: Find newtons form of interpolating polynomial for the following set of points: x_0 =0 , x_1 =1, x_2=3, $f(x_0)$=1, $f(x_1)$=3, $f(x_2)$=55. And hence approximate the value of the function at the point x=2 using the resulting polynomial Also find the divided difference $f[x_1, x_2]$, $f[x_0, x_1]$ and $f[x_0, x_1, x_2]$.

In[ ]:=
```
Polynomial = InterpolatingPolynomial[{{0, 1}, {1, 3}, {3, 55}}, x]
Simplify[Polynomial]
Polynomial /. x → 2
xi = {0, 1, 3};
fi = {1, 3, 55};
f[x0, x1] = (fi[[2]] - fi[[1]]) / (xi[[2]] - xi[[1]])
f[x1, x2] = (fi[[3]] - fi[[2]]) / (xi[[3]] - xi[[2]])
f[x0, x1, x2] = (f[x1, x2] - f[x0, x1]) / (xi[[3]] - xi[[1]])
```

Out[ ]= $1 + (2 + 8(-1 + x))x$

Out[ ]= $1 - 6x + 8x^2$

Out[ ]= 21

Out[ ]= 2

Out[ ]= 26

Out[ ]= 8

Ques. 2: Find Newton's form of Interpolating polynomial for the following set of points: x_0=-1, x_1=0, x_2=1, x_3=2, $f(x_0)$=5, $f(x_1)$=1, $f(x_2)$= 1, $f(x_3)$=11. And hence approximate the value of the function at the point x= 1.5 using the resulting polynomial. Also find the divided differences $f[x_2, x_3]$ and $f[x_1, x_2, x_3]$.

In[∘]:=
```
Polynomial = InterpolatingPolynomial[{{-1, 5}, {0, 1}, {1, 1}, {2, 11}}, x]
Simplify[Polynomial]
Polynomial /. x → 1.5
xi = {-1, 0, 1, 2};
fi = {5, 1, 1, 11};
f[x0, x1] = (fi[[2]] - fi[[1]]) / (xi[[2]] - xi[[1]])
f[x1, x2] = (fi[[3]] - fi[[2]]) / (xi[[3]] - xi[[2]])
f[x2, x3] = (fi[[4]] - fi[[3]]) / (xi[[4]] - xi[[3]])
f[x0, x1, x2] = (f[x1, x2] - f[x0, x1]) / (xi[[3]] - xi[[1]])
f[x1, x2, x3] = (f[x2, x3] - f[x1, x2]) / (xi[[4]] - xi[[2]])
f[x1, x2, x3] - f[x2, x3]
```

Out[∘]=
$5 + (1 + x)\ (-4 + x\ (1 + x))$

Out[∘]=
$1 - 3x + 2x^2 + x^3$

Out[∘]=
4.375

Out[∘]=
-4

Out[∘]=
0

Out[∘]=
10

Out[∘]=
2

Out[∘]=
5

Out[∘]=
-5

Practical 9 - Trapezoidal Rule

Ques.1 : Approximate the value of the integral $\int_1^2 \frac{1}{x}\, dx$ using trapezoidal rule.

```
In[]:= f[x_] := 1/x;
a = 1;
b = 2;
Exact = N[Integrate[f[x], {x, 1, 2}]]; (*exact value of integral*)
App = N[(b - a)/2 (f[a] + f[b])]; (*approximate value of integral*)
AE = Abs[Exact - App]; (*actual error*)
Print["Exact value of integral is:-  ", Exact]
Print["Approximate value of the integral is:-  ", App]
Print["Actual Error is:-  ", AE]
```

```
Exact value of integral is:-  0.693147
Approximate value of the integral is:-  0.75
Actual Error is:-  0.0568528
```

Ques.2 : Approximate the value of the integral $\int_0^1 e^{-x}\,dx$ using trapezoidal rule.

```
In[]:= f[x_] := Exp[-x];
a = 0;
b = 1;
Exact = N[Integrate[f[x], {x, 0, 1}]]; (*exact value of integral*)
App = N[(b - a)/2 (f[a] + f[b])]; (*approximate value of integral*)
AE = Abs[Exact - App]; (*actual error*)
Print["Exact value of integral is:-  ", Exact]
Print["Approximate value of the integral is:-  ", App]
Print["Actual Error is:-  ", AE]
```

```
Exact value of integral is:-  0.632121
Approximate value of the integral is:-  0.68394
Actual Error is:-  0.0518192
```

Ques.3 : Approximate the value of the integral $\int_0^1 \frac{1}{1+x^2}\,dx$ using trapezoidal rule.

```
In[]:= f[x_] := 1/(1 + x^2);
a = 0;
b = 1;
Exact = N[Integrate[f[x], {x, 0, 1}]]; (*exact value of integral*)
App = N[(b - a)/2 (f[a] + f[b])]; (*approximate value of integral*)
AE = Abs[Exact - App]; (*actual error*)
Print["Exact value of integral is:-  ", Exact]
Print["Approximate value of the integral is:-  ", App]
Print["Actual Error is:-  ", AE]
```

```
Exact value of integral is:-  0.785398
Approximate value of the integral is:-  0.75
Actual Error is:-  0.0353982
```

Ques.4 : Approximate the value of the integral $\int_0^1 \tan^{-1} x \, dx$ using trapezoidal rule.

```
In[•]:= f[x_] := ArcTan[x];
a = 0;
b = 1;
Exact = N[Integrate[f[x], {x, 0, 1}]]; (*exact value of integral*)
App = N[(b - a)/2 (f[a] + f[b])];(*approximate value of integral*)
AE = Abs[Exact - App];(*actual error*)
Print["Exact value of integral is:-  ", Exact]
Print["Approximate value of the integral is:-  ", App]
Print["Actual Error is:-  ", AE]
```

```
Exact value of integral is:-  0.438825
Approximate value of the integral is:-  0.392699
Actual Error is:-  0.0461255
```

Practical 10 - Simpson’s Rule

Ques.1 : Approximate the value of the integral $\int_1^2 \frac{1}{x} \, dx$ using simpson’s rule.

```
In[•]:= f[x_] := 1/x;
a = 1;
b = 2;
Exact = N[Integrate[f[x], {x, 1, 2}]]; (*exact value of integral*)
App = N[(b - a)/6 (f[a] + 4 f[(a + b)/2] + f[b])];(*approximate value of integral*)
AE = Abs[Exact - App];(*actual error*)
Print["Exact value of integral is:-  ", Exact]
Print["Approximate value of the integral is:-  ", App]
Print["Actual Error is:-  ", AE]
```

```
Exact value of integral is:-  0.693147
Approximate value of the integral is:-  0.694444
Actual Error is:-  0.00129726
```

Ques.2 : Approximate the value of the integral $\int_0^1 e^{-x} \, dx$ using simpson’s rule.

```
In[•]:= f[x_] := Exp[-x];
a = 0;
b = 1;
Exact = N[Integrate[f[x], {x, 0, 1}]]; (*exact value of integral*)
App = N[(b - a)/6 (f[a] + 4 f[(a + b)/2] + f[b])];(*approximate value of integral*)
AE = Abs[Exact - App];(*actual error*)
Print["Exact value of integral is:-  ", Exact]
Print["Approximate value of the integral is:-  ", App]
Print["Actual Error is:-  ", AE]
```

```
Exact value of integral is:-  0.632121

Approximate value of the integral is:-  0.632334

Actual Error is:-  0.000213121
```

Ques.3 : Approximate the value of the integral $\int_0^1 \frac{1}{1+x^2}\, dx$ using simpson's rule.

```
In[*]:= f[x_] := 1/(1 + x^2);
a = 0;
b = 1;
Exact = N[Integrate[f[x], {x, 0, 1}]]; (*exact value of integral*)
App = N[(b - a)/6 (f[a] + 4 f[(a + b)/2] + f[b])]; (*approximate value of integral*)
AE = Abs[Exact - App]; (*actual error*)
Print["Exact value of integral is:-  ", Exact]
Print["Approximate value of the integral is:-  ", App]
Print["Actual Error is:-  ", AE]
```

```
Exact value of integral is:-  0.785398

Approximate value of the integral is:-  0.783333

Actual Error is:-  0.00206483
```

Ques.4 : Approximate the value of the integral $\int_0^1 \tan^{-1} x \, dx$ using simpson's rule.

```
In[*]:= f[x_] := ArcTan[x];
a = 0;
b = 1;
Exact = N[Integrate[f[x], {x, 0, 1}]]; (*exact value of integral*)
App = N[(b - a)/6 (f[a] + 4 f[(a + b)/2] + f[b])]; (*approximate value of integral*)
AE = Abs[Exact - App]; (*actual error*)
Print["Exact value of integral is:-  ", Exact]
Print["Approximate value of the integral is:-  ", App]
Print["Actual Error is:-  ", AE]
```

```
Exact value of integral is:-  0.438825

Approximate value of the integral is:-  0.439998

Actual Error is:-  0.00117353
```

Practical 11 - Eular's Method

EULAR'S BLOCK

```
In[*]:= eular[f_, {x_, x0_, xn_}, {y_, y0_}, steps_] :=
 Block[{xold = x0, yold = y0, sollist = {{x0, y0}}, x, y, h}, h = N[(xn - x0) / steps];
  Do[xnew = xold + h;
   ynew = yold + h * (f /. {x → xold, y → yold});
   sollist = Append[sollist, {xnew, ynew}];
   xold = xnew;
   yold = ynew, {steps}];
  Return[sollist]]
```

Ques 1: Approximate the solution of $\frac{dy}{dx} = x + 2y$, with $0 \le x \le 1$, y(0) =0 using four steps.

In[∘]:=
```
eular[x + 2 y, {x, 0, 1}, {y, 0}, 4]
```
Out[∘]=
```
{{0, 0}, {0.25, 0.}, {0.5, 0.0625}, {0.75, 0.21875}, {1., 0.515625}}
```

Ques 2: Approximate the solution of $\frac{dy}{dx} = 1 + \frac{y}{x}$, with $1 \le x \le 6$, y(1) =1 using 10 steps.

In[∘]:=
```
eular[1 + y/x, {x, 1, 6}, {y, 1}, 10]
```
Out[∘]=
```
{{1, 1}, {1.5, 2.}, {2., 3.16667}, {2.5, 4.45833}, {3., 5.85}, {3.5, 7.325},
 {4., 8.87143}, {4.5, 10.4804}, {5., 12.1448}, {5.5, 13.8593}, {6., 15.6193}}
```

Ques 3: Approximate the solution of $\frac{dy}{dx} = \frac{y}{x}$, with $1 \le x \le 5$, y(1) =1 using 10 steps.

In[∘]:=
```
eular[y/x, {x, 1, 5}, {y, 1}, 10]
```
Out[∘]=
```
{{1, 1}, {1.4, 1.4}, {1.8, 1.8}, {2.2, 2.2}, {2.6, 2.6},
 {3., 3.}, {3.4, 3.4}, {3.8, 3.8}, {4.2, 4.2}, {4.6, 4.6}, {5., 5.}}
```

Ques 4: Approximate the solution of $\frac{dy}{dx} = xy^3 - y$, with $0 \le x \le 1$, y(0) =1 using 4 steps.

In[∘]:=
```
eular[x y^3 - y, {x, 0, 1}, {y, 1}, 4]
```
Out[∘]=
```
{{0, 1}, {0.25, 0.75}, {0.5, 0.588867}, {0.75, 0.467175}, {1., 0.369499}}
```

Practical 12 - Runge Kutta Method

RK4 BLOCK

In[∘]:=
```
RK4[f_, {x0_, xn_}, y0_, steps_] :=
 Block[{xold = x0, yold = y0, sollist = {{x0, y0}}, x, y, n,
   h, k1, k2, k3, k4, fn, ynew, xnew}, h = N[(xn - x0) / steps];
  Do[xnew = xold + h;
   k1 = h * f[xold, yold];
   k2 = h * f[xold + h/2, yold + k1/2];
   k3 = h * f[xold + h/2, yold + k2/2];
   k4 = h * f[xold + h, yold + k3];
   ynew = yold + 1/6 * (k1 + 2 k2 + 2 k3 + k4);
   sollist = Append[sollist, {xnew, ynew}];
   xold = xnew;
   yold = ynew, {steps}];
  Return[sollist]]
```

Ques. 1:- Approximate the solution $\frac{dy}{dx} = x + y$, with 1≤x≤2, y(1)=1, using 10 steps.

In[ ]:=
```
f[x_, y_] := x + y;
RK4[f, {1, 2}, 1, 10]
```

Out[ ]=
```
{{1, 1}, {1.1, 1.21551}, {1.2, 1.46421}, {1.3, 1.74958}, {1.4, 2.07547}, {1.5, 2.44616},
 {1.6, 2.86635}, {1.7, 3.34125}, {1.8, 3.87662}, {1.9, 4.4788}, {2., 5.15484}}
```

Ques. 2:- Approximate the solution $\frac{dy}{dx} = 1 + \frac{y}{x}$, with 1≤x≤6, y(1)=1, using 5 steps.

In[ ]:=
```
f[x_, y_] := 1 + y/x;
RK4[f, {1, 6}, 1, 5]
```

Out[ ]=
```
{{1, 1}, {2., 3.37963}, {3., 6.285}, {4., 9.53051}, {5., 13.0288}, {6., 16.7284}}
```

Ques. 3:- Approximate the solution $\frac{dy}{dx} = \frac{x}{y}$, with 0≤x≤5, y(0)=1, when h=0.5

In[ ]:=
```
f[x_, y_] := x/y;
RK4[f, {0, 5}, 1, 10]
```

Out[ ]=
```
{{0, 1}, {0.5, 1.11816}, {1., 1.4144}, {1.5, 1.80294}, {2., 2.2362}, {2.5, 2.69269},
 {3., 3.16237}, {3.5, 3.64014}, {4., 4.12318}, {4.5, 4.60984}, {5., 5.09908}}
```