Exercise Chapter 3

Ex1 (3.1.9) The sample space of a random experiment is {a, b, c, d, e, f}, and each outcome is equally likely. A random variable is defined as follows:

outcome	a	b	c	d	e	f
x	0	0	1.5	1.5	2	3

Determine the probability mass function of X. Use the probability mass function to determine the following probabilities:

a. $P(X = 1.5)$
b. $P(0.5 < X < 2.7)$
c. $P(X > 3)$
d. $P(0 \le X < 2)$
e. $P(X = 0 \text{ or } X = 2)$

Ex2 (3.1.13) The following function is probability mass function, and determine the requested probabilities:

x	1.25	1.5	1.75	2	2.25
$f(x)$	0.2	0.4	0.1	0.2	0.1

a. $P(X \ge 2)$
b. $P(X < 1.65)$
c. $P(X = 1.5)$
d. $P(X < 1.3 \text{ or } X > 2.1)$

Ex3 (3.1.15) In a semiconductor manufacturing process, three wafers from a lot are tested. Each wafer is classified as pass or fail. Assume that the probability that a wafer passes the test is 0.8 and that wafers are independent. Determine the probability mass function of the number of wafers from a lot that pass the test.

Ex4 (3.1.17) An assembly consists of three mechanical components. Suppose that the probabilities that the first, second, and third components meet specifications are 0.95, 0.98, and 0.99, respectively. Assume that the components are independent. Determine the probability mass function of the number of components in the assembly that meet specifications.

Ex5 (3.1.18) The distribution of the time until a Web site changes is important to Web crawlers that search engines use to maintain current information about Web sites. The distribution of the time until change (in days) of a Web site is approximated in the following table.

Days until Changes	Probability
1.5	0.05
3.0	0.25
4.5	0.35
5.0	0.20
7.0	0.15

Calculate the probability mass function of the days until change.

Ex6 Determine the cumulative distribution function for the random variable in **Ex3**.

Ex7 (3.2.7) The following function is cumulative distribution function, and determine the probability mass function and the requested probabilities.

$$F(x) = \begin{cases} 0 & x < 1 \\ 0.5 & 1 \le x < 3 \\ 1 & 3 \le x \end{cases}$$

a. $P(X \le 3)$
b. $P(X \le 2)$
c. $P(1 \le X \le 2)$
d. $P(X > 2)$

Ex8 (3.2.8) The following function is cumulative distribution function, and determine the probability mass function and the requested probabilities.

$$F(x) = \begin{cases} 0 & x < -10 \\ 0.25 & -10 \le x < 30 \\ 0.75 & 30 \le x < 50 \\ 1 & 50 \le x \end{cases}$$

a. $P(X \le 50)$
b. $P(X \le 40)$
c. $P(40 \le X \le 60)$
d. $P(X < 0)$
e. $P(0 \le X < 10)$
f. $P(-10 < X < 10)$

Ex9 (3.3.1) If the range of X is the set {0, 1, 2, 3, 4} and P(X = x) = 0.2, determine the mean and variance of the random variable.

Ex10 (3.3.6) In a NiCd battery, a fully charged cell is composed of nickelic hydroxide. Nickel is an element that has multiple oxidation states. Assume the following proportions of the states:

Nickel Charge	Proportions Found
0	0.17
+2	0.35
+3	0.33
+4	0.15

a. Determine the cumulative distribution function of the nickel charge.
b. Determine the mean and variance of the nickel charge.

Ex11 (3.3.8) Trees are subjected to different levels of carbon dioxide atmosphere with 6% of them in a minimal growth condition at 350 parts per million (ppm), 10% at 450 ppm (slow growth), 47% at 550 ppm (moderate growth), and 37% at 650 ppm (rapid growth). What are the mean and standard deviation of the carbon dioxide atmosphere (in ppm) for these trees in ppm?

Ex12 (3.4.1) Assume that the wavelengths of photosynthetically active radiations (PAR) are uniformly distributed at integer nanometers in the red spectrum from 675 to 700 nm.

a. What are the mean and variance of the wavelength distribution for this radiation?
b. If the wavelengths are uniformly distributed at integer nanometers from 75 to 100 nanometers, how do the mean and variance of the wavelength distribution compare to the previous part? Explain.

Ex13 (3.4.6) Each multiple-choice question on an exam has four choices. Suppose that there are 10 questions and the choice is selected randomly and independently for each question. Let X denote the number of questions answered correctly. Does X have a discrete uniform distribution? Why or why not?

Ex14 (3.5.2) Let X be a binomial random variable with p = 0.1 and n = 10. Calculate the following probabilities.

a. $P(X \le 2)$
b. $P(X > 8)$
c. $P(X = 4)$
d. $P(5 \le X \le 7)$

Ex15 (3.5.4) The random variable X has a binomial distribution with n = 10 and p = 0.5. Sketch the probability mass function of X.

a. What value of X is most likely?
b. What value(s) of X is(are) least likely?

Ex16 (3.5.7) The phone lines to an airline reservation system are occupied 40% of the time. Assume that the events that the lines are occupied on successive calls are independent. Assume that 10 calls are placed to the airline.

a. What is the probability that for exactly three calls, the lines are occupied?
b. What is the probability that for at least one call, the lines are not occupied?
c. What is the expected number of calls in which the lines are all occupied?

Ex17 (3.5.8) A multiple-choice test contains 25 questions, each with four answers. Assume that a student just guesses on each question.

a. What is the probability that the student answers more than 20 questions correctly?
b. What is the probability that the student answers fewer than 5 questions correctly?

Ex18 (3.5.9) Samples of rejuvenated mitochondria are mutated (defective) in 1% of cases. Suppose that 15 samples are studied and can be considered to be independent for mutation. Determine the following probabilities.

a. No samples are mutated.
b. At most one sample is mutated.
c. More than half the samples are mutated.

Ex19 (3.5.13) Because all airline passengers do not show up for their reserved seat, an airline sells 125 tickets for a flight that holds only 120 passengers. The probability that a passenger does not show up is 0.10, and the passengers behave independently.

a. What is the probability that every passenger who shows up can take the flight?
b. What is the probability that the flight departs with empty seats?

Ex20 (3.6.2) Suppose that X is a negative binomial random variable with p = 0.2 and r = 4. Determine the following:

a. $E(X)$
b. $P(X = 20)$
c. $P(X = 19)$
d. $P(X = 21)$

Ex21 (3.6.3) Consider a sequence of independent Bernoulli trials with p = 0.2.

a. What is the expected number of trials to obtain the first success?
b. After the eighth success occurs, what is the expected number of trials to obtain the ninth success?

Ex22 (3.6.5) Assume that each of your calls to a popular radio station has a probability of 0.02 of connecting, that is, of not obtaining a busy signal. Assume that your calls are independent.

a. What is the probability that your first call that connects is your 10th call?
b. What is the probability that it requires more than five calls for you to connect?
c. What is the mean number of calls needed to connect?

Ex23 (3.6.9) A trading company uses eight computers to trade on the New York Stock Exchange (NYSE). The probability of a computer failing in a day is 0.005, and the computers fail independently. Computers are repaired in the evening, and each day is an independent trial.

a. What is the probability that all eight computers fail in a day?
b. What is the mean number of days until a specific computer fails?
c. What is the mean number of days until all eight computers fail on the same day?

Ex24 (3.7.2) Suppose that X has a hypergeometric distribution with N = 10, n = 3, and K = 4. Sketch the probability mass function of X. Determine the cumulative distribution function for X.

Ex25 (3.8.3) Suppose that the number of customers who enter a store in an hour is a Poisson random variable, and suppose that P(X = 0) = 0.05. Determine the mean and variance of X.

Ex26 (3.8.8) The number of views of a page on a Web site follows a Poisson distribution with a mean of 1.5 per minute.

a. What is the probability of no views in a minute?
b. What is the probability of two or fewer views in 10 minutes?

Ex27 (3.s29) Determine the probability mass function for the random variable with the following cumulative distribution function:

$$F(x) = \begin{cases} 0 & x < 2 \\ 0.2 & 2 \leq x < 5.7 \\ 0.5 & 5.7 \leq x < 6.5 \\ 0.8 & 6.5 \leq x < 8.5 \\ 1 & 8.5 \leq x \end{cases}$$

Ex28 (3.s35) Saguaro cacti are large cacti indigenous to the southwestern United States and Mexico. Assume that the number of saguaro cacti in a region follows a Poisson distribution with a mean of 280 per square kilometer. Determine the following:

a. Mean number of cacti per 10,000 square meters.
b. Probability of no cacti in 10,000 square meters.

c. Area of a region such that the probability of at least two cacti in the region is 0.9.

Ex29 (3. S30) The random variable X has the following probability distribution:

x	2	3	5	8
Probability	0.2	0.4	0.3	0.1

Determine the following:

a. $P(X \leq 3)$

b. $P(X > 2.5)$

c. $P(2.7 < X < 5.1)$

d. $E(X)$

e. $V(X)$