Exercises Chapter 2

Ex1. The sample space of a random experiment is {a, b, c, d, e} with probabilities 0.1, 0.1, 0.2, 0.4, and 0.2, respectively. Let A denote the event {a, b, c}, and let B denote the event {c, d, e}. Determine the following:

a. $P(A \cup B)$ b. $P(A)$ c. $P(B)$ d. $P(A \cap B)$

Ex2. A part selected for testing is equally likely to have been produced on any one of six cutting tools.

a. What is the sample space?
b. What is the probability that the part is from tool 1?
c. What is the probability that the part is from tool 3 or tool 5?
d. What is the probability that the part is not from tool 4?

Ex3. In a NiCd battery, a fully charged cell is composed of nickelic hydroxide. Nickel is an element that has multiple oxidation states and that is usually found in the following states:

Nickel Charge	Proportions Found
0	0.17
+2	0.35
+3	0.33
+4	0.15

a. What is the probability that a cell has at least one of the positive nickel-charged options?
b. What is the probability that a cell is not composed of a positive nickel charge greater than +3?

Ex4. (2.3.8) Disks of polycarbonate plastic from a supplier are analyzed for scratch and shock resistance. The results from 100 disks are summarized as follows:

		Shock Resistance	
		High	Low
Scratch	High	70	9
Resistance	Low	16	5

Let A denote the event that a disk has high shock resistance, and let B denote the event that a disk has high scratch resistance. If a disk is selected at random, determine the following probabilities:

a. P(A)
b. P(B)
c. $P(A')$
d. $P(A \cap B)$
e. $P(A \cup B)$
f. $P(A' \cup B)$

Ex5. (2.3.17) A computer system uses passwords that contain exactly eight characters, and each character is one of 26 lowercase letters (a–z) or 26 uppercase letters (A–Z) or 10 integers (0–9). Let Ω denote the set of all possible passwords, and let A and B denote the events that consist of passwords with only letters or only integers, respectively. Suppose that all passwords in Ω are equally likely. Determine the probability of each of the following:

a. A
b. B
c. A password contains at least 1 integer.
d. A password contains exactly 2 integers.

Ex6. (2.4.1) If A, B, and C are mutually exclusive events with P(A) = 0.2, P(B) = 0.3, and P(C) = 0.4, determine the following probabilities:

a. $P(A \cup B \cup C)$
b. $P(A \cap B \cap C)$
c. $P(A \cap B)$
d. $P[(A \cup B) \cap C]$
e. $P(A' \cap B' \cap C')$

Ex7. (2.4.2) If P(A) = 0.3, P(B) = 0.2, and $P(A \cap B) = 0.1$, determine the following probabilities:

a. $P(A')$
b. $P(A \cup B)$
c. $P(A' \cap B)$
d. $P(A \cap B')$
e. $P[(A \cup B)']$
f. $P(A' \cup B)$

Ex8. (2.4.3) A manufacturer of front lights for automobiles tests lamps under a high-humidity, high-temperature environment using intensity and useful life as the responses of interest. The following table shows the performance of 130 lamps:

		Useful life	
		Satisfactory	Unsatisfactory
Intensity	Satisfactory	117	3
	Unsatisfactory	8	2

a. Find the probability that a randomly selected lamp will yield unsatisfactory results under any criteria.
b. The customers for these lamps demand 95% satisfactory results. Can the lamp manufacturer meet this demand?

Ex9. (2.5.1) The analysis of results from a leaf transmutation experiment (turning a leaf into a petal) is summarized by type of transformation completed:

		Total Textural Transformation	
		Yes	No
Total color transformation	Yes	243	26
	No	13	18

a. If a leaf completes the color transformation, what is the probability that it will complete the textural transformation?
b. If a leaf does not complete the textural transformation, what is the probability it will complete the color transformation?

Ex10. (2.5.2) Samples of skin experiencing desquamation are analyzed for both moisture and melanin content. The results from 100 skin samples are as follows:

		Melanin Content	
		High	Low
Moisture content	High	13	7
	Low	48	32

Let A denote the event that a sample has low melanin content, and let B denote the event that a sample has high moisture content. Determine the following probabilities:

P(A) P(B) P(A | B) P(B | A)

Ex11. (2.5.3) The following table summarizes the number of deceased beetles under autolysis (the destruction of a cell after its death by the action of its own enzymes) and putrefaction (decomposition of organic matter, especially protein, by microorganisms, resulting in production of foul-smelling matter):

		Autolysis	
		High	Low
Putrefaction	High	14	59
	Low	18	9

a. If the autolysis of a sample is high, what is the probability that the putrefaction is low?
b. If the putrefaction of a sample is high, what is the probability that the autolysis is high?
c. If the putrefaction of a sample is low, what is the probability that the autolysis is low?

Ex 12. (2.5.6) A batch of 500 containers for frozen orange juice contains 5 that are defective. Three are selected, at random, without replacement from the batch.

a. What is the probability that the second one selected is defective given that the first one was defective?
b. What is the probability that the first two selected are defective?
c. What is the probability that the first two selected are both acceptable?
d. What is the probability that the third one selected is defective given that the first and second ones selected were defective?
e. What is the probability that the third one selected is defective given that the first one selected was defective and the second one selected was okay?
f. What is the probability that all three selected ones are defective?

Ex 13. (2.6.1) Suppose that $P(A \mid B) = 0.4$ and $P(B) = 0.5$. Determine the following:

a. $P(A \cap B)$
b. $P(A' \cap B)$

Ex 14. (2.6.2) Suppose that $P(A \mid B) = 0.2$, $P(A \mid B') = 0.3$, and $P(B) = 0.8$. What is $P(A)$?

Ex 15. (2.6.3) The probability is 1% that an electrical connector that is kept dry fails during the warranty period. If the connector is ever wet, the probability of a failure during the warranty period is 5%. If 90% of the connectors are kept dry and 10% are wet, what proportion of connectors fail during the warranty period?

Ex 16. (2.6.4) Heart failures are due to either natural occurrences (87%) or outside factors (13%). Outside factors are related to induced substances (73%) or foreign objects (27%). Natural occurrences are caused by arterial blockage (56%), disease (27%), and infection (e.g., staph infection) (17%).

a. Determine the probability that a failure is due to an induced substance.
b. Determine the probability that a failure is due to disease or infection.

Ex 17. (2.6.5) The edge roughness of slit paper products increases as knife blades wear. Only 1% of products slit with new blades have rough edges, 3% of products slit with blades of average sharpness exhibit roughness, and 5% of products slit with worn blades exhibit roughness. If 25% of the blades in manufacturing are new, 60% are of average sharpness, and 15% are worn, what is the proportion of products that exhibit edge roughness?

Ex 18. (2.6.7) Computer keyboard failures are due to faulty electrical connects (12%) or mechanical defects (88%). Mechanical defects are related to loose keys (27%) or improper assembly (73%). Electrical connect defects are caused by defective wires (35%), improper connections (13%), or poorly welded wires (52%).

a. Find the probability that a failure is due to loose keys.
b. Find the probability that a failure is due to improperly connected or poorly welded wires.

Ex 19. (2.7.1) If $P(A \mid B) = 0.3$, $P(B) = 0.8$, and $P(A) = 0.3$, are the events B and the complement of A independent?

Ex 20. (2.7.2) If $P(A) = 0.2$, $P(B) = 0.2$, and A and B are mutually exclusive, are they independent?

Ex 21. (2.8.1) Customers are used to evaluate preliminary product designs. In the past, 95% of highly successful products received good reviews, 60% of moderately successful products received good reviews, and 10% of poor products received good reviews. In addition, 40% of products have been highly successful, 35% have been moderately successful, and 25% have been poor products.

a. What is the probability that a product attains a good review?
b. If a new design attains a good review, what is the probability that it will be a highly successful product?

c. If a product does not attain a good review, what is the probability that it will be a highly successful product?

Ex 22. (2.8.2) Suppose that P(A | B) = 0.7, P(A) = 0.5, and P(B) = 0.2. Determine P(B | A).

Ex 23. (2.8.7) Two Web colors are used for a site advertisement. If a site visitor arrives from an affiliate, the probabilities of the blue or green colors being used in the advertisement are 0.8 and 0.2, respectively. If the site visitor arrives from a search site, the probabilities of blue and green colors in the advertisement are 0.4 and 0.6, respectively. The proportions of visitors from affiliates and search sites are 0.3 and 0.7, respectively. What is the probability that a visitor is from a search site given that the blue ad was viewed?

Ex 24. (2.8.9) An e-mail filter is planned to separate valid e-mails from spam. The word free occurs in 60% of the spam messages and only 4% of the valid messages. Also, 20% of the messages are spam. Determine the following probabilities:

a. The message contains free.
b. The message is spam given that it contains free.
c. The message is valid given that it does not contain free