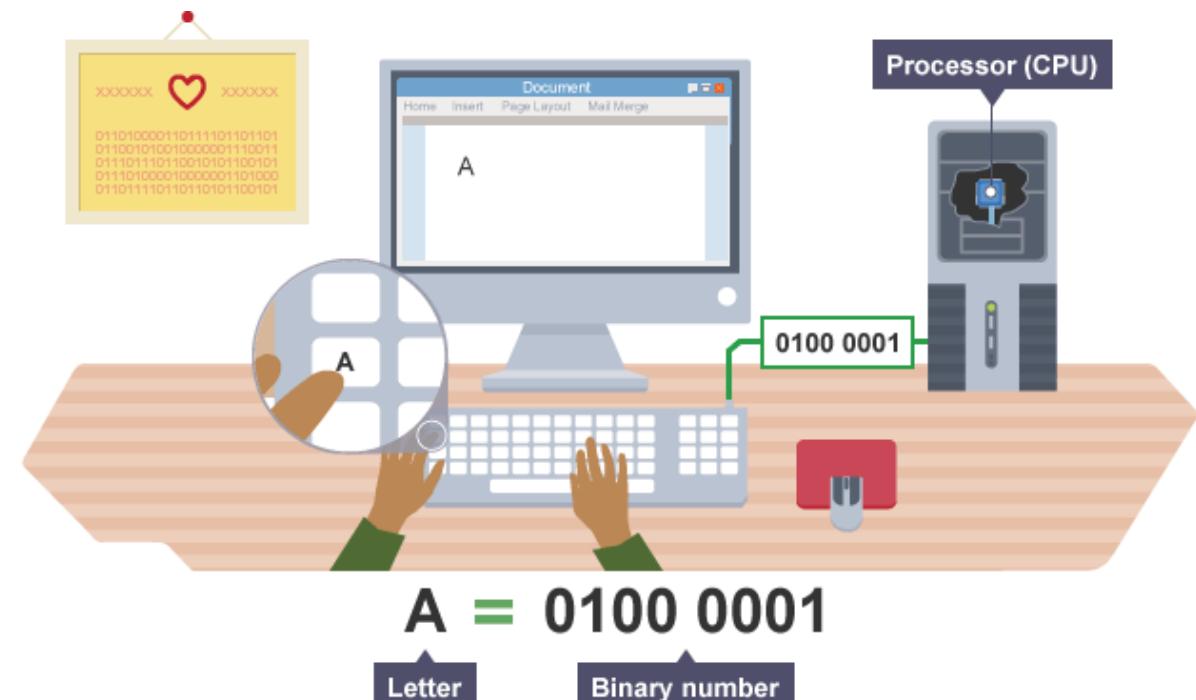


Digital Logic and Boolean Algebra

Digital devices Vs Digital Logic

- Digital devices are the devices that use digital signals
- Digital signals contains discrete levels, and it is binary
- Let's Think about computer. All the inputs given through an input device is taken as digital signal and processing of data is fully digital
- A computer system is working with binary signals or totally dealing with “1” and “0” s

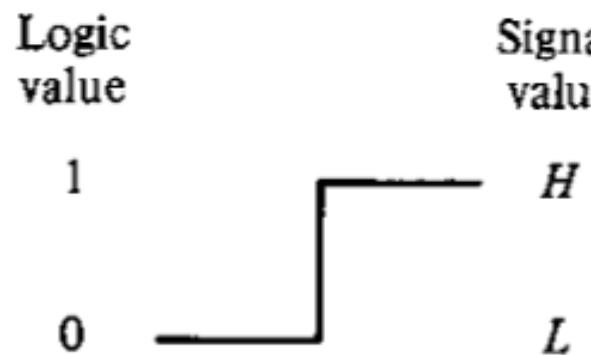


Digital Logic

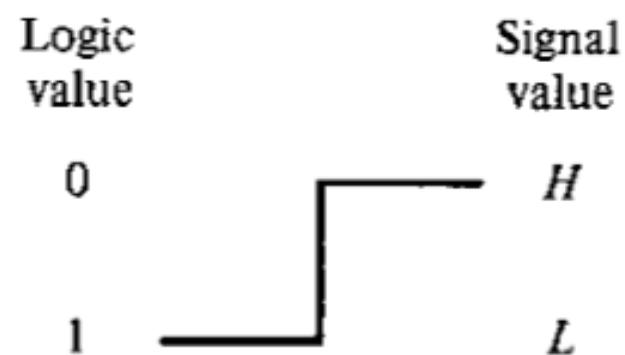
- Binary signals represent logic states.
- Digital logic, binary signals have one of two values either “1” or “0”
- So that two possible assignments of signal levels to logic values
- Because of these two states it is known as Binary
- Logic levels can be called as **LOW** level and **High** level
- These level configuration divided into two sectors according to the signal flow pattern

1. Positive Logic System: The higher signal level (H) represents logic 1, and the lower signal level (L) represents logic 0.

2. Negative Logic System: The lower signal level (L) represents logic 1, and the higher signal level (H) represents logic 0.



(a) Positive logic



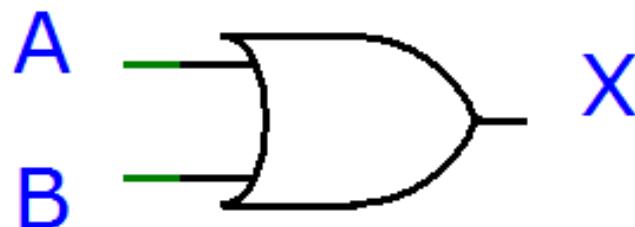
(b) Negative logic

Boolean Algebra

- Boolean Algebra is fundamental for binary systems
- A branch of algebra that deals with binary variables and logical operations.
- **Basic operations** of Boolean Algebra **is AND, OR, NOT** operation
- There are set of rules and theorems except these basic operations
- Logic level “1” and “0” is the basic **Boolean variables**

Basic Logic Operations

- OR gate
 - Represents the Logical Add operation



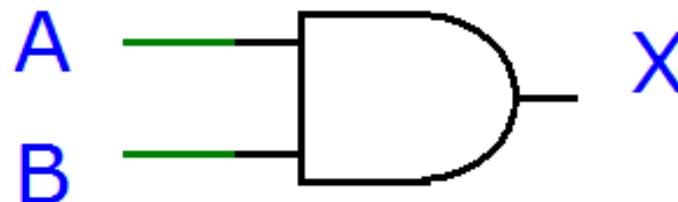
$$X = A + B$$

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

OUTPUT is Logic “1” when one or more INPUT logic are at logic “1”
OUTPUT Logic “0” when all INPUT are “0”

Basic Logic Operations

- **AND** gate
 - Represents the Logical multiplication operation



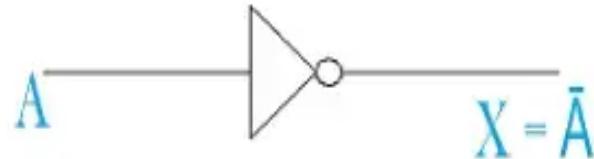
$$X = A \cdot B$$

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

OUTPUT logic is “1” when and only when and only when its INPUT logics are “1” otherwise OUTPUT is “0”

Basic Logic Operations

- NOT gate
 - Represents the Logical compliment operation

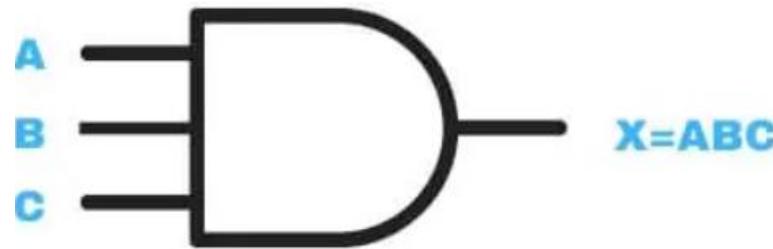


$$X = \bar{A}$$

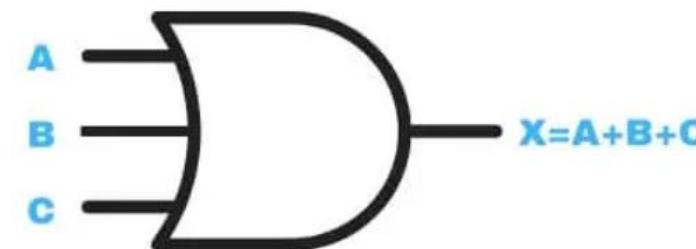
A	X
0	1
1	0

OUTPUT logic is “1” when INPUT is “0” and OUTPUT is logic “0” when INPUT is logic “1”

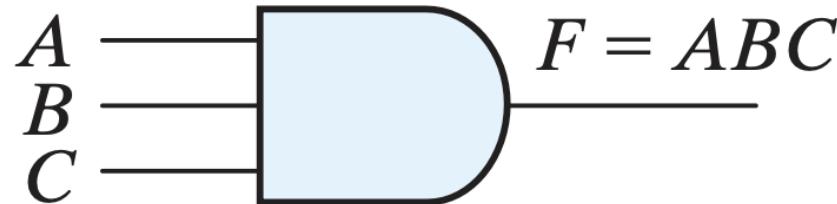
Multiple input logic gates



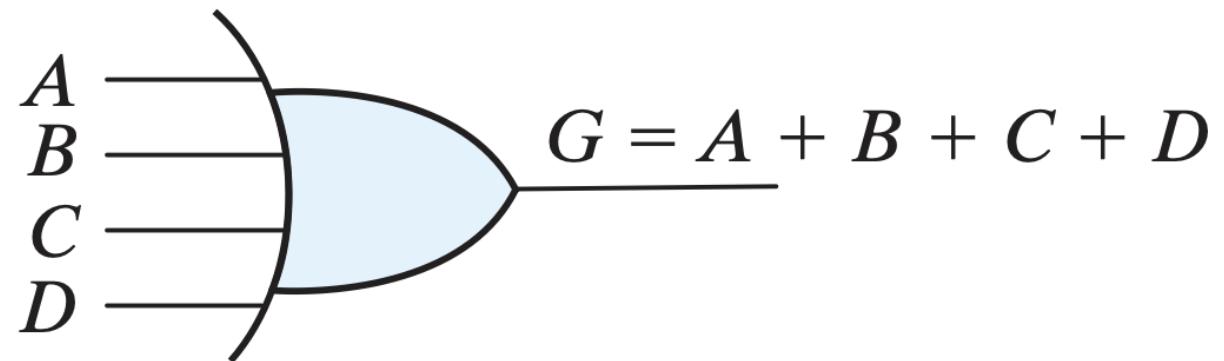
AND Gate (3 Input)



OR Gate (3 Input)



(a) Three-input AND gate



(b) Four-input OR gate

INPUT AND OUT COMBINATINS

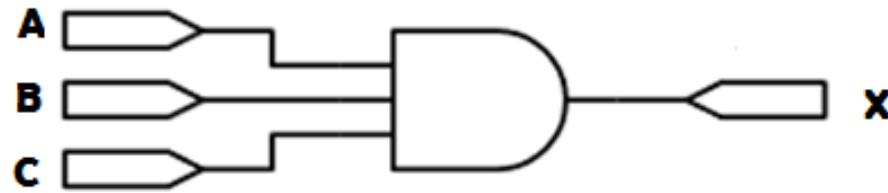
- The total number of possible combinations of binary inputs to a gate is determined by the following formula:

$$N = 2^n$$

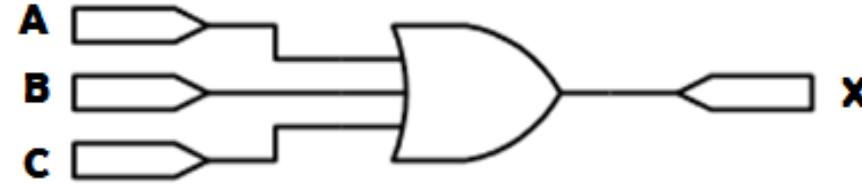
- where N is the number of possible input combinations and n is the number of input variables. To illustrate,

- For two input variables: $N = 2^2 = 4$ combinations
- For three input variables: $N = 2^3 = 8$ combinations
- For four input variables: $N = 2^4 = 16$ combinations

- Three input logic AND gate, OR gate and truth tables



$$X = A \cdot B \cdot C$$



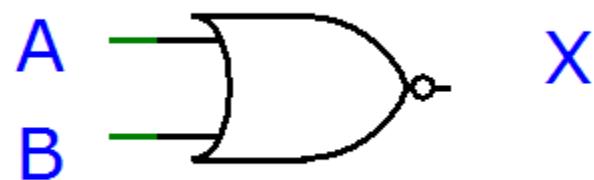
$$X = A + B + C$$

A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

A	B	C	X
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Universal and Other Logic Gates

- **NOR gate**



$$X = \overline{A + B}$$

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

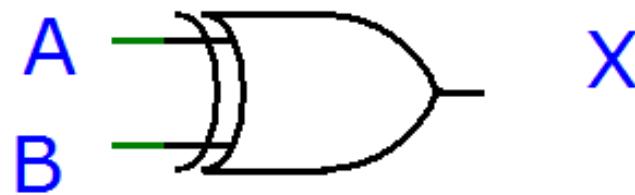
- **NAND gate**



$$X = \overline{A \cdot B}$$

A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

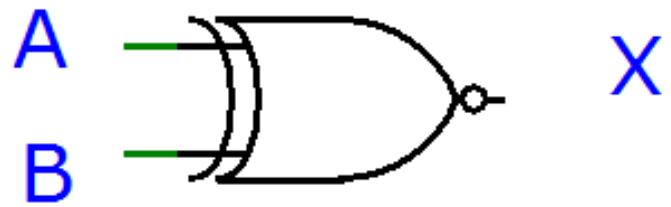
- XOR gate



$$X = A \oplus B$$
$$X = \bar{A}B + A\bar{B}$$

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

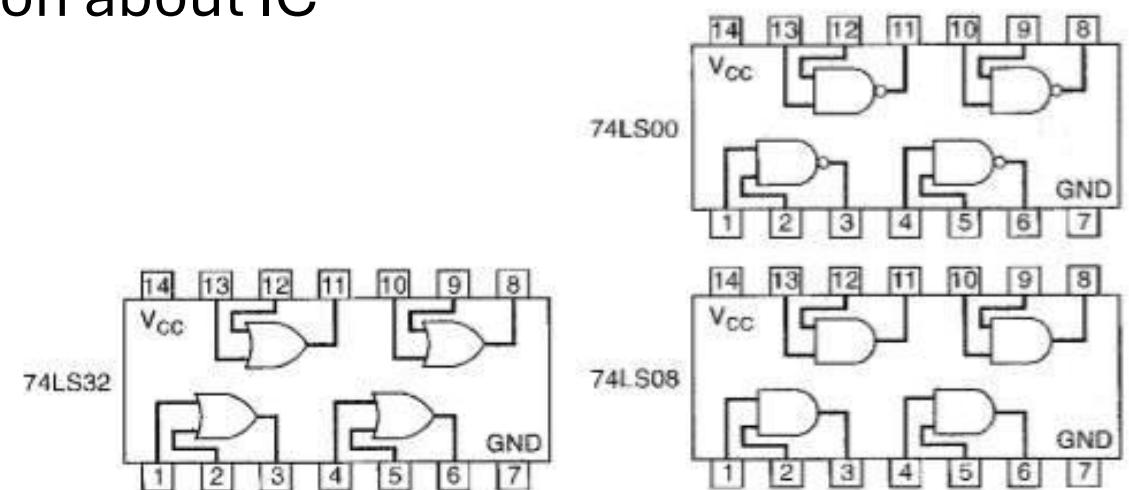
- XNOR gate



$$X = A \odot B$$
$$X = \overline{A \oplus B}$$

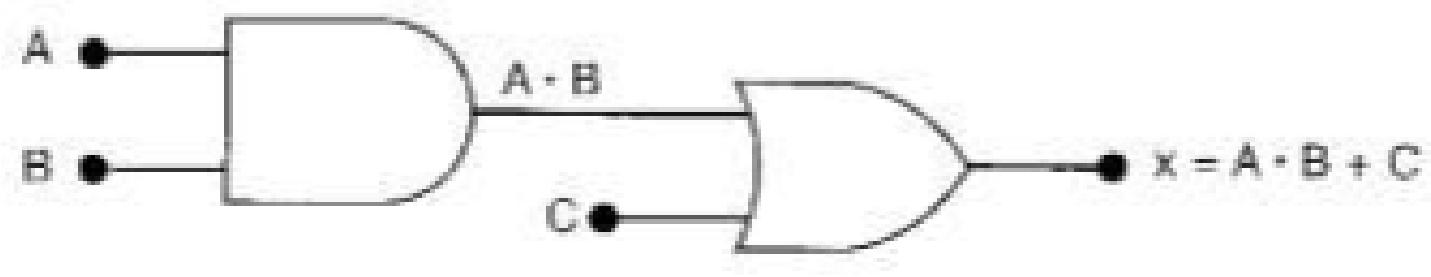
A	B	X
0	0	1
0	1	0
1	0	0
1	1	1

- ❑ IC or Integrated circuits contains logic gates physically
- ❑ The figure below shows the pin connection of three integrated circuits.
- ❑ Each IC has four identical gates in it.
- ❑ Each component tagged with a code which is known as IC's Number
- ❑ IC number helps to find the type of gate which is inbuilt with particular IC
- ❑ Each IC has its own data sheet published by the manufacturer
- ❑ The data sheet has all the features, working conditions, pin diagrams and almost all the specific information about IC

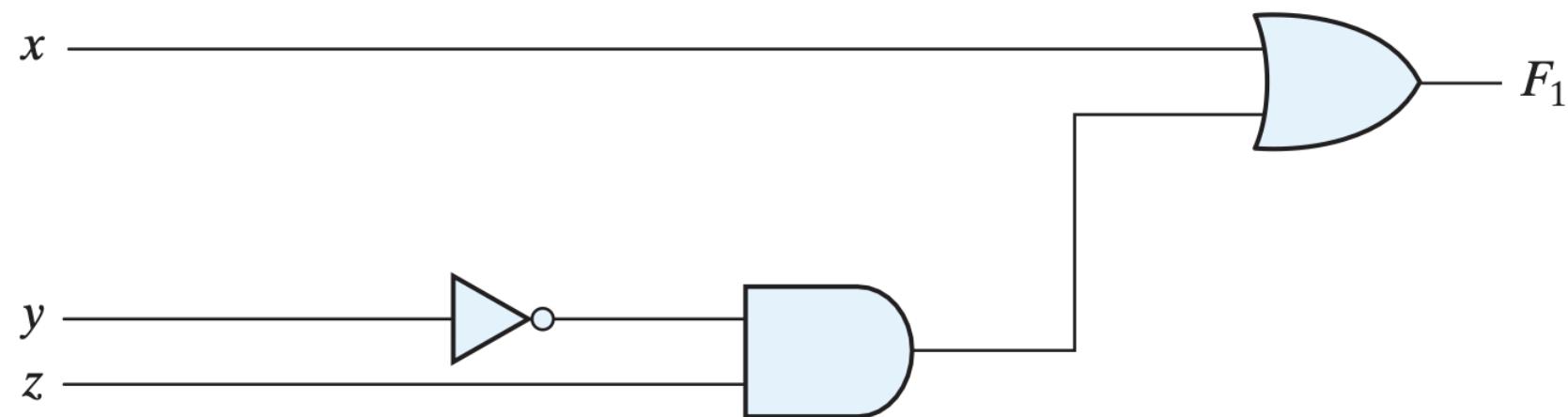


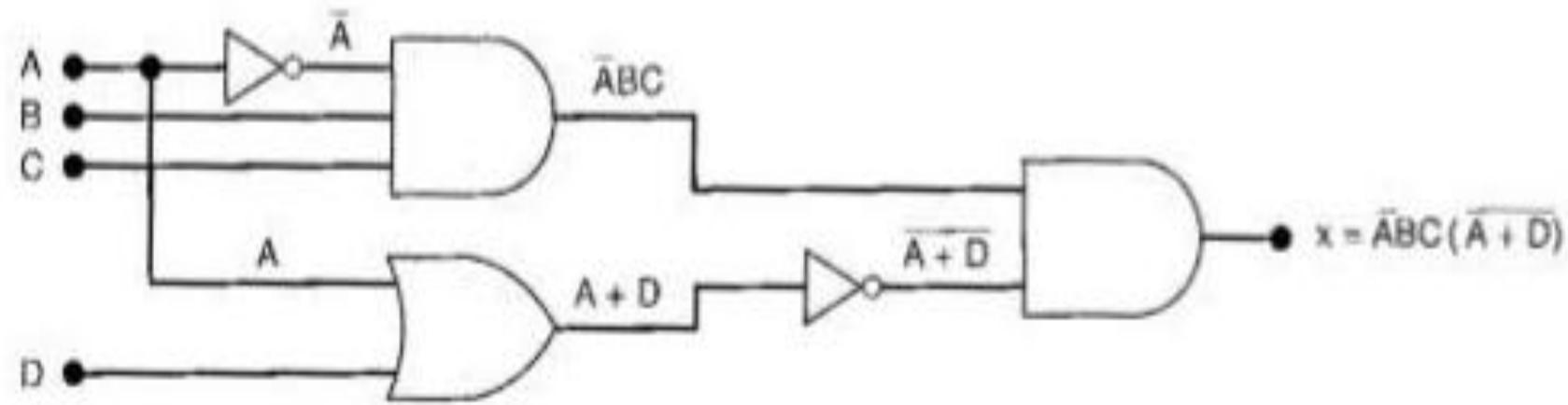
CONSTRUCTING BOOLEAN EXPRESSION

- Initially identify the inputs and outputs
- write outputs for each gate
- Write all the out puts in an order with logic expressions
- End of the main output you will get a expression for the whole circuit



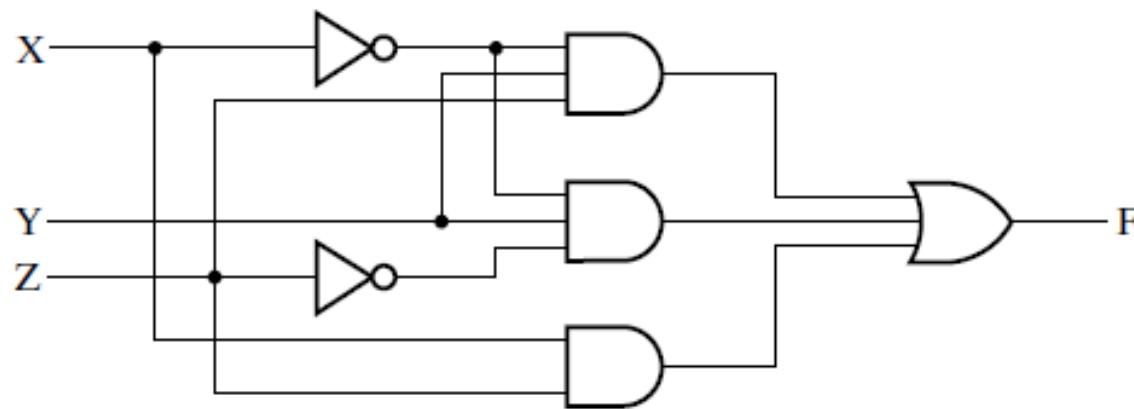
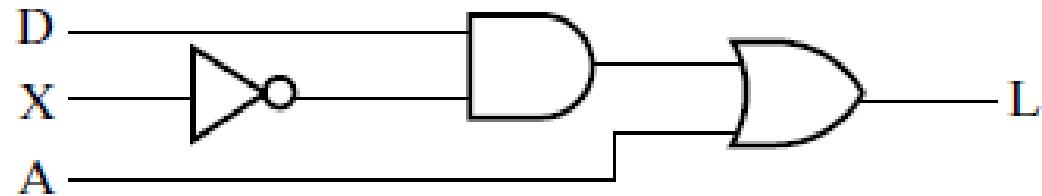
$$F_1 = x + y'z$$

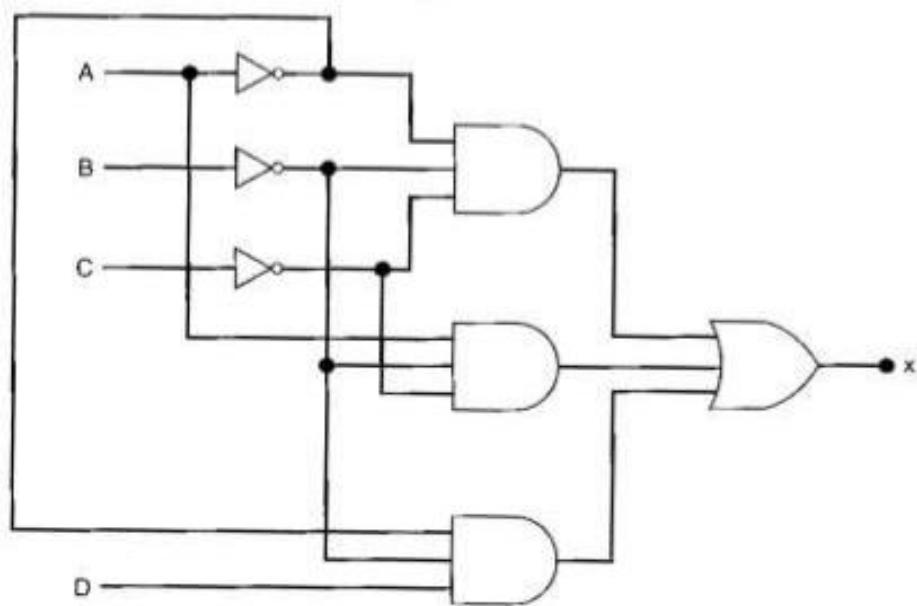
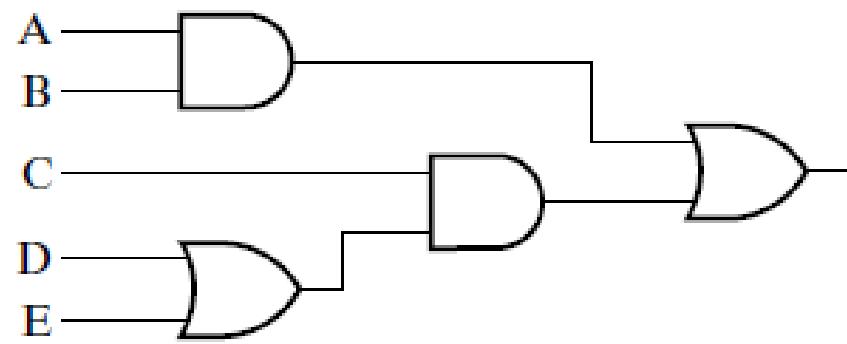






Q1. Write the Boolean expression for below circuits





Question

- Implement the circuit for the following Boolean expression using basic logic gates

$$1. A = X + Y\bar{Z}$$

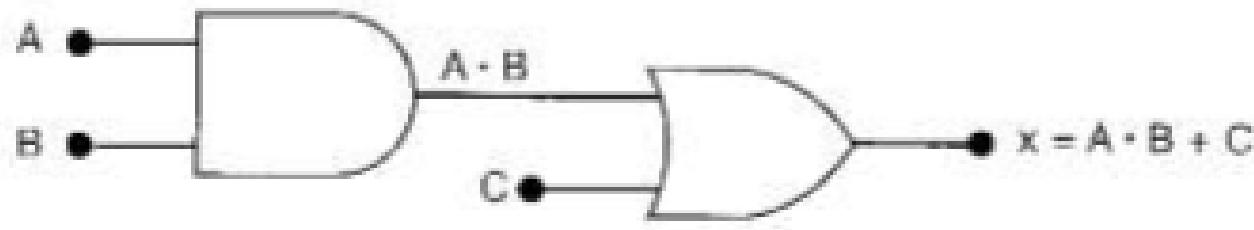
$$2. A = (\bar{X} + Y) \cdot \bar{Z}$$

$$3. Z = A(B + C) + \bar{B}D$$

$$4. Z = [(\overline{A + B}) \cdot C] + [(\overline{AB}) + C]$$

$$5. Y = (\bar{A}\bar{B} + A\bar{B}) + (C \cdot D)$$

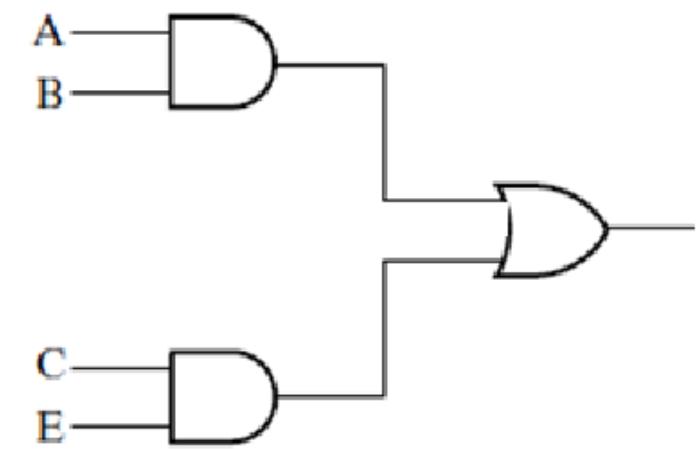
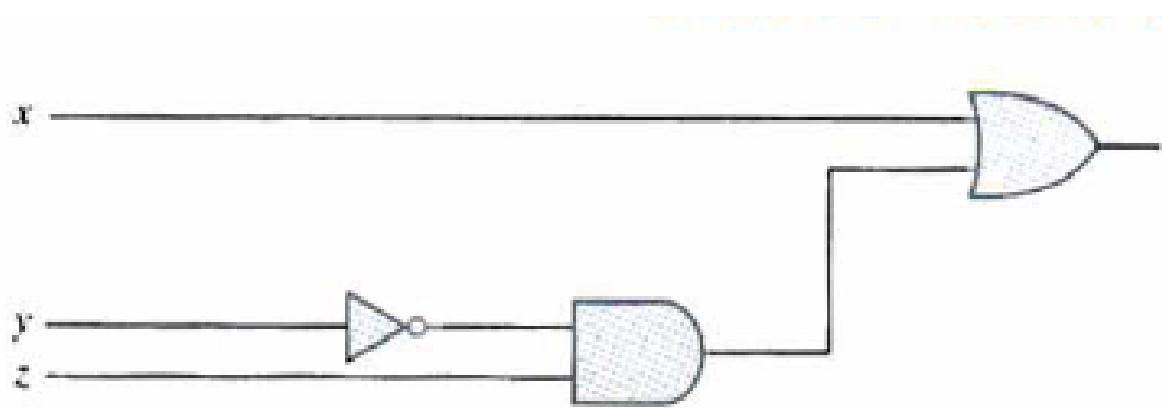
ANALYSING COMBINATIONAL LOGIC CIRCUIT USING TRUTH TABLE



- The Combination logic circuit can be analyzed by the help of truth table
- Remember when building the truth table, lists all possible inputs and therefore fully describes the operation of the logic

A	B	C	A.B	x
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	1

Q2. Draw the truth table for following circuits



BOOLIAN RULES

1.	Law of Identity	$A = A$ $\overline{A} = \overline{A}$
2.	Commutative Law	$A \cdot B = B \cdot A$ $A + B = B + A$
3.	Associative Law	$A \cdot (B \cdot C) = A \cdot B \cdot C$ $A + (B + C) = A + B + C$
4.	Idempotent Law	$A \cdot A = A$ $A + A = A$
5.	Double Negative Law	$\overline{\overline{A}} = A$
6.	Complementary Law	$A \cdot \overline{A} = 0$ $A + \overline{A} = 1$
7.	Law of Intersection	$A \cdot 1 = A$ $A \cdot 0 = 0$
8.	Law of Union	$A + 1 = 1$ $A + 0 = A$

9.	DeMorgan's Theorem	$\begin{aligned}\overline{AB} &= \overline{A} + \overline{B} \\ \overline{A+B} &= \overline{\overline{A}} \overline{\overline{B}}\end{aligned}$
10.	Distributive Law	$\begin{aligned}A \cdot (B + C) &= (A \cdot B) + (A \cdot C) \\ A + (BC) &= (A + B) \cdot (A + C)\end{aligned}$
11.	Law of Absorption	$\begin{aligned}A \cdot (A + B) &= A \\ A + (AB) &= A\end{aligned}$
12.	Law of Common Identities	$\begin{aligned}A \cdot (\overline{A} + B) &= AB \\ A + (\overline{AB}) &= A + B\end{aligned}$

- Prove the following rules using truth table
 - Cumulative law
 - Associative law

Commutative Law

$$\begin{aligned} A \cdot B &= B \cdot A \\ A + B &= B + A \end{aligned}$$

$$A \cdot B = B \cdot A$$

A	B	A.B	B.A
0	0	0	0
0	1	0	0
1	0	0	0
1	1	1	1

$$A + B = B + A$$

A	B	A+B	B+A
0	0	0	0
0	1	1	1
1	0	1	1
1	1	1	1

SO THAT CUMULATIVE LAW IS TRUTH

Associative Law

$$A \cdot (B \cdot C) = A \cdot B \cdot C$$

$$A + (B + C) = A + B + C$$

A	B	C	B.C	A.(B.C)	A.B.C
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	1	0	0
1	0	0	0	0	0
1	0	1	0	0	0
1	1	0	0	0	0
1	1	1	1	1	1

- Prove the rules of
 - Law of absorption (redundancy)
 - Law of common identities

Using Boolean rules

Law of Redundancy

$$\begin{aligned}
 A(A + B) &= A \\
 \text{L.H.S=} &= A \cdot A + AB \\
 &= A + AB \\
 &= A(1+B) \\
 &= A
 \end{aligned}$$

Law of common identities

$$\begin{aligned}
 A + (\bar{A}B) &= A + B \\
 \text{L.H.S=} &= A(1+B) + (\bar{A}B) \\
 &= A + AB + \bar{A}B \\
 &= A + B(A + \bar{A}) \\
 &= A + B
 \end{aligned}$$

CIRCUIT SIMPLIFICATION USING BOOLEAN RULES

$$\begin{aligned} X &= A \cdot (B + A)' \cdot B' + A \\ &= A \cdot (B' \cdot A') \cdot B' + A \\ &= A \cdot B' \cdot A' \cdot B' + A \\ &= A \cdot A' \cdot B' \cdot B' + A \\ &= 0 \cdot B' \cdot B' + A \\ &= 0 + A \\ &= A \end{aligned}$$

$$\begin{aligned} X &= (A+B) \cdot (A+C) \\ &= AA + AC + AB + BC \\ &= A + AC + AB + BC \\ &= A(1 + C) + AB + BC \\ &= A + AB + BC \\ &= A(1 + B) + BC \\ &= A + BC \end{aligned}$$

Application of De Morgan's Rule

$$\overline{(A \cdot B)} = \overline{A} + \overline{B}$$

$$\overline{(A+B)} = \overline{A} \cdot \overline{B}$$

$$\overline{(A \cdot B \cdot C)} = \overline{A} + \overline{B} + \overline{C}$$

$$\overline{(A+B+C)} = \overline{A} \cdot \overline{B} \cdot \overline{C}$$

EX:

$$\begin{aligned}\bullet \overline{A + \bar{B}C} &= \overline{A} \cdot \overline{\bar{B}C} \\ &= \overline{A} [\bar{\bar{B}} + \bar{C}] \\ &= \overline{A} (B + \bar{C})\end{aligned}$$

$$\overline{A + \overline{BC}}$$



$$\overline{A + (\overline{B} + \overline{\overline{C}})}$$



$$\overline{A + \overline{B} + \overline{\overline{C}}}$$



$$\overline{A} \overline{\overline{B}} \overline{\overline{C}}$$

$$\downarrow$$

$$\overline{ABC}$$

Breaking shortest bar
(multiplication changes to addition)

Applying associative property
to remove parentheses

Breaking long bar in two places,
between 1st and 2nd terms;
between 2nd and 3rd terms

Applying identity $\overline{\overline{A}} = A$
to $\overline{\overline{B}}$ and $\overline{\overline{C}}$

Let's consider the Boolean function of

$$X = A \cdot B \cdot C + A \bar{B} (\bar{\bar{A}} \cdot \bar{C})$$

Simplify using Boolean rules

$$X = A \cdot B \cdot C + A \bar{B} (\bar{\bar{A}} \cdot \bar{\bar{C}})$$

$$X = A \cdot B \cdot C + A \bar{B} (\bar{\bar{A}} + \bar{\bar{C}})$$

$$X = A \cdot B \cdot C + A \bar{B} (A + C)$$

$$X = A \cdot B \cdot C + A \bar{B} A + A \bar{B} C$$

$$X = A \cdot B \cdot C + A \bar{B} (1 + C) \quad \leftarrow$$

$$X = A \cdot B \cdot C + A \bar{B}$$

$$X = A(B \cdot C + \bar{B})$$

$$X = A(BC + \bar{B})$$

According to common identities rule
 $AB' + AB'C = AB'(1+C) = AB'$

Use DeMorgan's Theorem, as well as any other applicable rules of Boolean algebra, to simplify the following expression so there are no more complementation bars extending over multiple variable:

1. $\overline{A + \bar{B}C}$
2. $\overline{\overline{AB} + \overline{AC}}$
3. $\overline{(A + B) . (\bar{C} + D)}$
4. $\overline{\bar{X}}. \overline{Y} + \overline{XY}$
5. $\overline{(A + BC). (D + EF)}$
6. $\overline{X\bar{Y}} \overline{(\bar{W}+\bar{Y})}$
7. $\overline{A.\overline{AB}} + \overline{B.\overline{AB}}$



Simplify the Boolean functions using Boolean rules

(a) $\overline{A} \overline{C} + \overline{A}BC + \overline{B}C$

(b) $\overline{(A + B + C)} \cdot \overline{ABC}$

(c) $AB\overline{C} + AC$