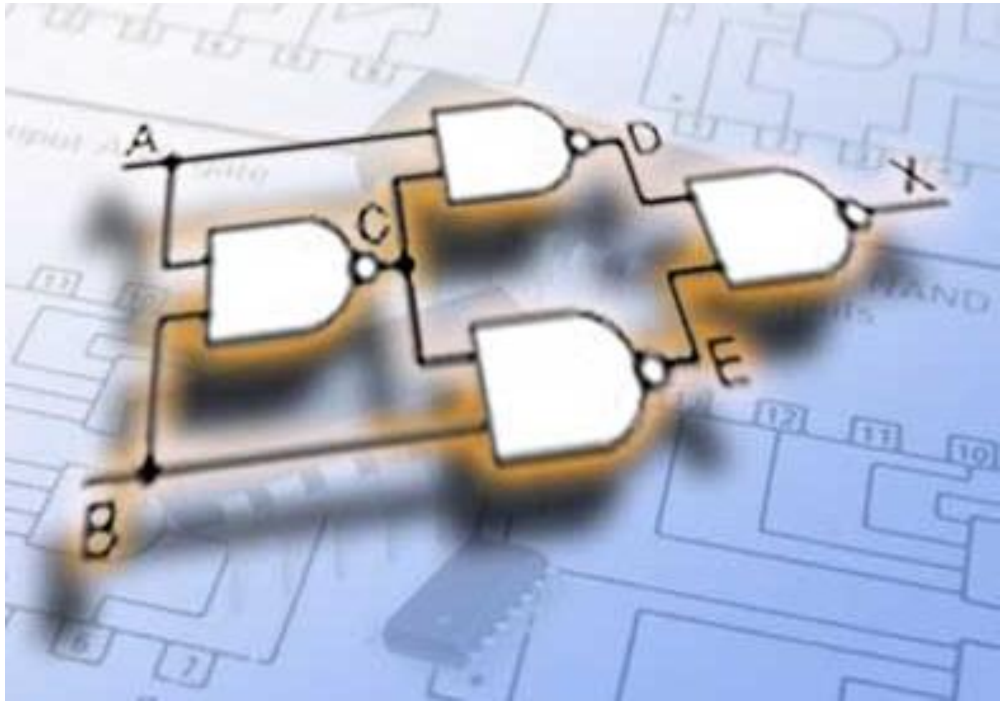
Karnaugh Map
(K-Map)



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Introduction to karnaugh Maps (K-Map)

- Karnaugh map method is a method of simplifying Boolean expressions by mapping the parameters
- K-map method is the most convenient method of simplifying Boolean expressions than using Boolean rules
- Number of cells depends with inputs and total number of combinations of given function
- SOP and POS both can be derive by using K-Map

Karnaugh Map for Two Input Variables

- Let's Consider the function

$$F(A,B) = \sum m(1,2) = A'B + AB'$$

- For two inputs there will be 4 combinations in the truth table
- Let's draw the truth table

A	B	Output	
0	0	0	A'B'
0	1	1	A'B
1	0	1	AB'
1	1	0	AB

A \ B	0	1
0	0 A'B'	1 A'B
1	1 AB'	0 AB

Karnaugh map for Three Inputs

$$F(A,B) = \sum m(2,4,6,7)$$

- For three inputs there will be 8 combinations

A	B	C	Output	
0	0	0	**0**	A'B'C'
0	0	1	**0**	A'B'C
0	1	0	**1**	A'BC'
0	1	1	**0**	A'BC
1	0	0	**1**	AB'C'
1	0	1	**0**	AB'C
1	1	0	**1**	ABC'
1	1	1	**1**	ABC

A \ BC	00	01	11	10
0	**0** A'B'C'	**0** A'B'C	**1** A'BC	**0** A'BC'
1	**1** AB'C'	**0** AB'C	**1** ABC	**1** ABC'

Karnaugh map for Four Inputs

- For four inputs there will be 16 combinations

AB \ CD	00	01	11	10
00	1 A'B'C'D'	0 A'B'C'D	0 A'B'CD	0 A'B'CD'
01	1 A'BC'D'	1 A'BC'D	0 A'BCD	0 A'BCD'
11	1 ABC'D'	0 ABC'D	1 ABCD	1 ABCD'
10	0 AB'C'D'	1 AB'C'D	0 AB'CD	0 AB'CD'

A	B	C	D	Output	
0	0	0	0	1	A'B'C'D'
0	0	0	1	0	A'B'C'D
0	0	1	0	0	A'B'CD'
0	0	1	1	0	A'B'CD
0	1	0	0	1	A'BC'D'
0	1	0	1	1	A'BC'D
0	1	1	0	0	A'BCD'
0	1	1	1	0	A'BCD
1	0	0	0	0	AB'C'D'
1	0	0	1	1	AB'C'D
1	0	1	0	0	AB'CD'
1	0	1	1	0	AB'CD
1	1	0	0	1	ABC'D'
1	1	0	1	0	ABC'D
1	1	1	0	1	ABCD'
1	1	1	1	1	ABCD

Steps of drawing K-Map

1. Step 1:
   - Fill the K-Map according to the truth table or expression given
2. Step 2:
   - Encircle the octets, quads and pairs. Roll and overlap to get the largest possible group
3. Step 3: (If)
   - Eliminate the redundant group.
4. Step 4: (If)
   - Don’t cares can be taken as 1 if it leads to a larger group)
5. Step 5:
   - Write the expression looking at the variables that remains the same within the group

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- It is very important to learn about three terms for Further simplification process
- those are
    - Implicant
    - Prime Implicant
    - Essential Prime Implicant

Strategy

- Include all essential prime implicants.
- Determine the implicants that not covered by essential prime implicants
- Then add other prime implicants until all 1s are covered.

Implicant

- Any single or group of 1s that are adjacent and can be combined together is called an ***implicant***

AB \ CD	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	0	0	1	1
10	0	0	0	0

Each “1” is implicant

AB \ CD	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	0	0	1	1
10	0	0	0	0

Each group of “1” are also implicant

Prime Implicant

- prime implicant covers the large group of “1” s and combined with another implicant to eliminate a variable. Which simplifies the K-map more

AB \ CD	00	01	11	10
00	1	1	1	1
01	0	1	1	0
11	0	0	0	1
10	1	0	0	1

AB \ CD	00	01	11	10
00	1	1	1	1
01	0	1	1	0
11	0	0	0	1
10	1	0	0	1

AB \ CD	00	01	11	10
00	1	1	1	1
01	0	1	1	0
11	0	0	0	1
10	1	0	0	1

By simplifying this two prime implicant we get

A’B’C’(D’+D)

and

A’B’C(D+D’)

Where the result can be implement as A’B’(C+C’)

And it is the result of A’B’

AB \ CD	00	01	11	10
00	1	1	1	1
01	0	1	1	0
11	0	0	0	1
10	1	0	0	1

As previous by two prime implicant provides A’C’D+A’CD

Where the result provide A’D

To minimize as much as possible four adjacent implicants can be combined

AB \ CD	00	01	11	10
00	1	1	1	1
01	0	1	1	0
11	0	0	0	1
10	1	0	0	1

Implicants which are placed in edges are also adjacent and they can be rolled as one prime implicant

Note : When combining the implicants, combining three implicants is not possible. Either group or four and eight is possible

Essential Prime implicant

- A prime implicant which contains one or more 1s that are not contained in another prime implicant is called an ***essential prime implicant***.

AB \ CD	00	01	11	10
00	1	1	1	1
01	0	1	1	1
11	0	1	0	0
10	0	0	0	0

The red color "1" are not in any prime implicant
These implicants are known as essential prime implicants

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AB \ CD	00	01	11	10
00	1	1	1	1
01	0	1	1	0
11	0	0	0	1
10	1	0	0	1

Answer ?

$Z = B'D' + A'D + A'B' + ACD'$

AB \ CD	00	01	11	10
00	1	1	1	1
01	0	1	1	1
11	0	1	0	0
10	0	0	0	0

The Highlighted set of prime implicants are there in other prime implicants as well
So that it is considered as ***non essential prime implicant***

These non essential prime implicant **can be redundant**

Steps of drawing K-Map

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Example 01

- Group the prime implicants of these K- Maps get the simplified Boolean equation

AB \ CD	00	01	11	10
00	1	1	1	0
01	1	1	1	0
11	0	0	1	1
10	0	0	1	1

AB \ CD	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	1	1	1	1
10	1	1	1	1

AB \ CD	00	01	11	10
00	1	1	1	0
01	1	1	1	0
11	0	0	1	1
10	0	0	1	1

$Z = A'C' + A'D + AC$

AB \ CD	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	1	1	1	1
10	1	1	1	1

$Z = A + CD$


Q
A

Example 02

- Simplify the K- Map s and find the SOP Expression

A \ BC	00	01	11	10
0	0	1	0	0
1	0	0	1	1

A \ BC	00	01	11	10
0	0	1	0	0
1	0	0	1	1

Out put (X) = A’B’C+AB

Example 03

A \ BC	00	01	11	10
0	1	1	1	1
1	0	0	0	1

A \ BC	00	01	11	10
0	1	1	1	1
1	0	0	0	1

Out put (X) = A'+BC'

Example 04

A \ BC	00	01	11	10
0	1	1	0	0
1	1	1	1	0

A \ BC	00	01	11	10
0	1	1	0	0
1	1	1	1	0

Out Put (X)= B'+AC

Example 05

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	1	1
11	1	0	0	1
10	1	0	0	1

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	1	1
11	1	0	0	1
10	1	0	0	1

Out Put (X) = AD’+A’BC

Simplify using K- map

1. $F(X,Y) = \sum(0,1)$
2. $F(X,Y) = \sum(0,2,3)$
3. $F(X,Y,Z) = \sum(2,3,4,5)$
4. $F(X,Y,Z) = \sum(3,4,6,7)$
5. $F(X,Y,Z) = \sum(0,2,4,5,6)$

Remaining minterms are zero

Solving K-Maps with Don’t Care Condition

- The condition don’t care indicates b symbol of “X”
- Value of this function either “0”or “1”
- Incomplete functions or can’t happen conditions are known as don’t care conditions
- For K- Map Simplification don’t care condition either include or exclude while rolling the essential prime implicant
- But getting the help from these condition may help to simplify the equation

A \ BC	00	01	11	10
0	1	1	0	X
1	0	0	1	X

A \ BC	00	01	11	10
0	1	1	0	X
1	0	0	1	X

AB \ CD	00	01	11	10
00	0	0	0	1
01	0	X	1	1
11	0	1	X	0
10	X	0	0	0

AB \ CD	00	01	11	10
00	0	0	0	1
01	0	X	1	1
11	0	1	X	0
10	X	0	0	0

Simplify using K- map

1. $F(X,Y,Z) = \sum m(0,1,4) + \sum d(2,5)$
2. $F(X,Y,Z) = \sum m(0,1,6,7) + \sum d(3,5)$
3. $F(X,Y,Z) = \sum m(1,2,5,7) + \sum d(0,4,6)$

Remaining minterms are zero

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Solving POS functions using K- map

- To get a SOP function as answer for given function, the prime implicants are grouped using “1”
- If all the groups are rounded using “0”will provide the POS function as the answer
- In this manner the output POS function is written as inverted version and answers which are getting from prime implicants is written as a normal SOP mode
- Applying Demorgan's rule and simplification will produce the POS function as answer

Example 06

- Simplify the K- Map and find the POS Expression

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	1	1	1
10	1	1	0	1

AB \ CD	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	1	1	1
10	1	1	0	1

$$\boldsymbol{\overline{X} = \overline{A} + \overline{B}CD}$$
$$\boldsymbol{\overline{\overline{X}} = \overline{\overline{A} + \overline{B}CD}}$$
$$\boldsymbol{X = \overline{\overline{(A)}}.\left(\overline{\overline{B}CD}\right)}$$
$$\boldsymbol{X = A.(B + \overline{C} + \overline{D})}$$

Example 07

- A function has output on following minterms

$$F(A,B,C,D) = \sum(0,1,2,5,8,9,10)$$

Simplify the equation and get the POS faction

AB \ CD	00	01	11	10
00	1	1	0	1
01	0	1	0	0
11	0	0	0	0
10	1	1	0	1

$$\bar{z} = B\bar{D} + CD + AB$$

$$\bar{\bar{Z}} = \overline{B\bar{D} + CD + AB}$$

$$Z = (\overline{B\bar{D}}).(\overline{CD}).(\overline{AB})$$

$$Z = (\bar{B} + D)(\bar{C} + \bar{D}).(\bar{A} + \bar{B})$$

Getting SOP and POS Expressions using K-Map

SOP

- Usually simplification of K- Map provides SOP expression
- Logic "1" are grouped to get sop

A \ BC	00	01	11	10
0	0	0	1	1
1	0	1	1	0

Output(X)= A'B+AC

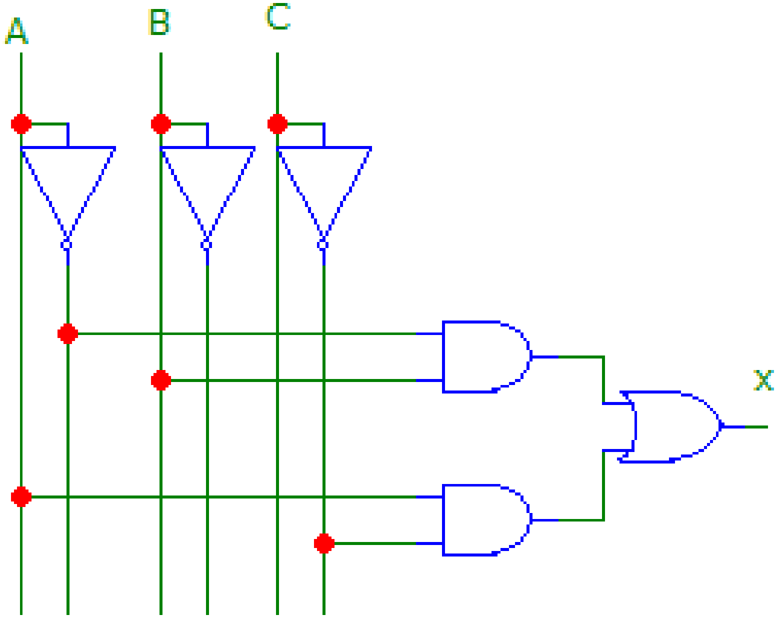
POS

- To get POS cover all the "0" logic(X') of K- Map
- Get X' in SOP form
- Compliment the both side

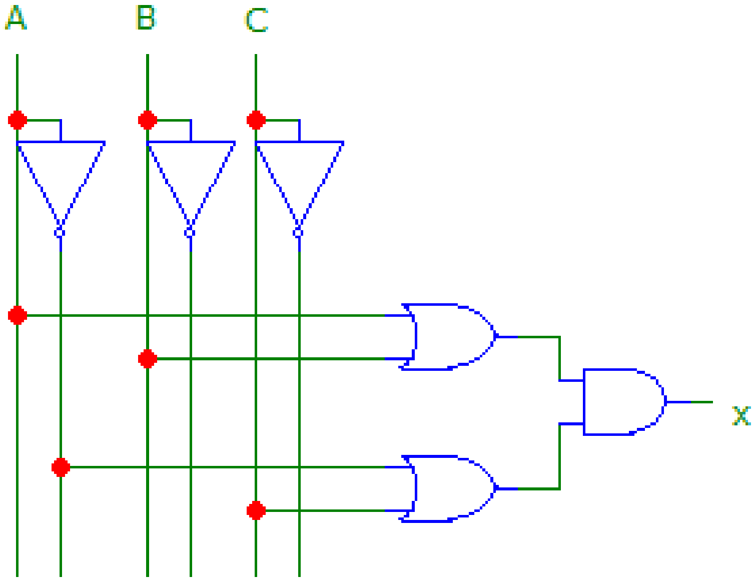
A \ BC	00	01	11	10
0	0	0	1	1
1	0	1	1	0

$$\bar{X} = \bar{A}\bar{B} + A\bar{C}$$
$$\bar{\bar{X}} = \overline{\bar{A}\bar{B} + A\bar{C}}$$
$$X = (\overline{\bar{A}\bar{B}}).(\overline{A\bar{C}})$$
$$X = (A + B).(\bar{A} + C)$$

Coresponding Digital System for SOP & POS



SOP FUNCTION= A'B+AC



POS FUNCTION $X = (A + B).(\overline{A} + C)$

1. $F(A,B,C,D) = \sum m(0,1,2,5,7,8,9,10,13,15)$
2. $F(W,X,Y,Z) = \sum m(0,1,3,5,7,8,9,11,13,15)$
3. $F(P,Q,R,S) = \sum m(3,4,5,7,9,13,14,15)$
4. $F(P,Q,R,S) = \sum m(1,3,4,6,9,11,12,14)$
5. $F(A,B,C,D) = \sum m(1,3,4,6,8,9,11,13,15) + \sum d(0,2,14)$
6. $F(A,B,C,D) = \sum m(0,2,8,10,14) + \sum d(5,15)$

Remaining minterms are zero