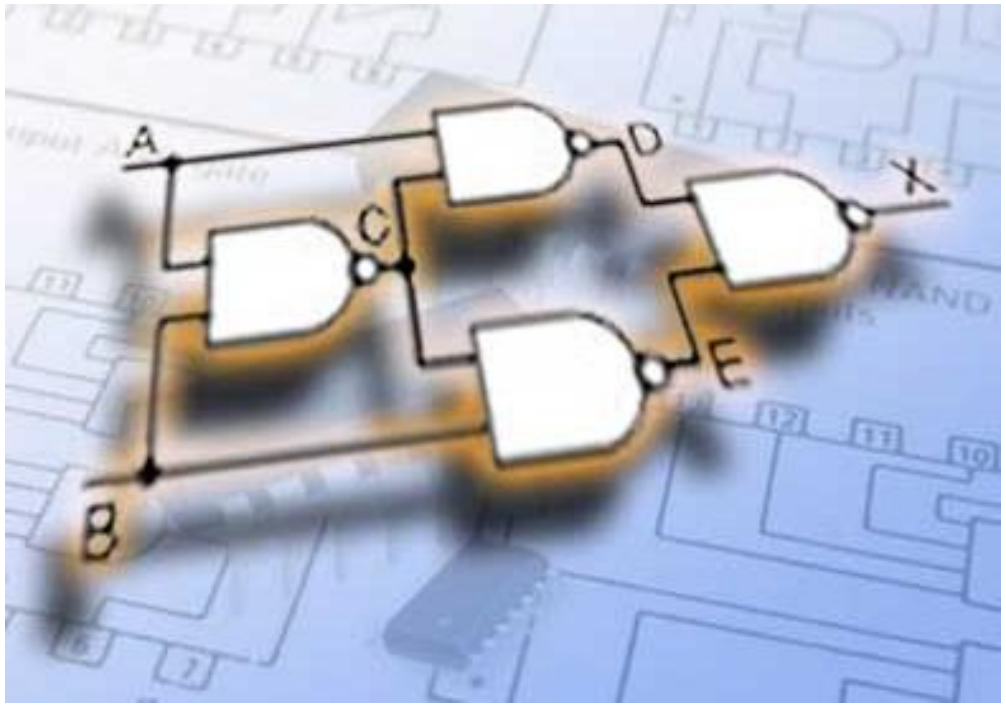
Digital Logic and Boolean Algebra

Part - 02

SOP, POS, Minterm and Maxterm



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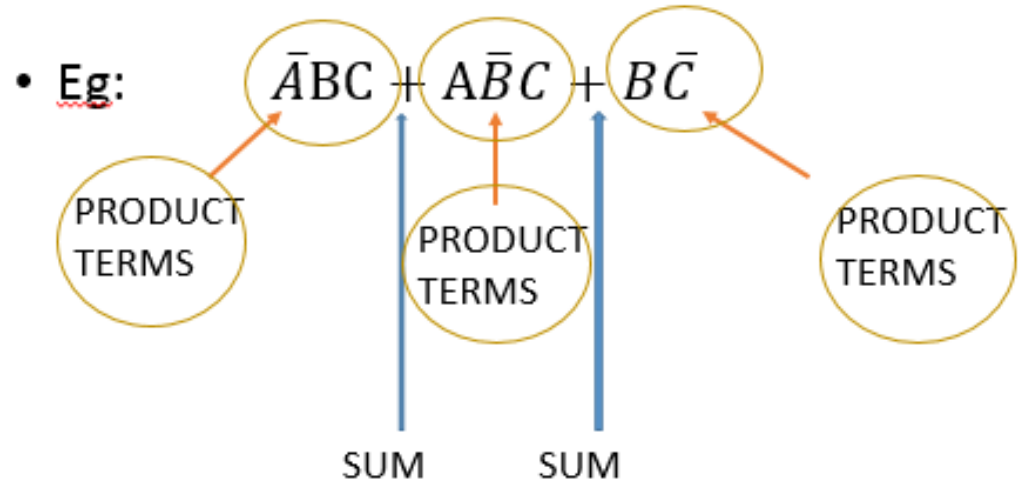
- Introduction to SOP
- Introduction to POS
- Minterm and Maxterm
- Writing SOP & POS Using Truth Table
- Canonical Representation
- Conversion between Canonical SOP to POS

Boolean Function and Representation

- According to the previous lesson Boolean functions can be implement by two ways
    - Truth table
    - Algebraic Expression
- These two implementations can be represent by two standard ways
    - SOP
    - POS

Introduction to SOP (Some of Product)

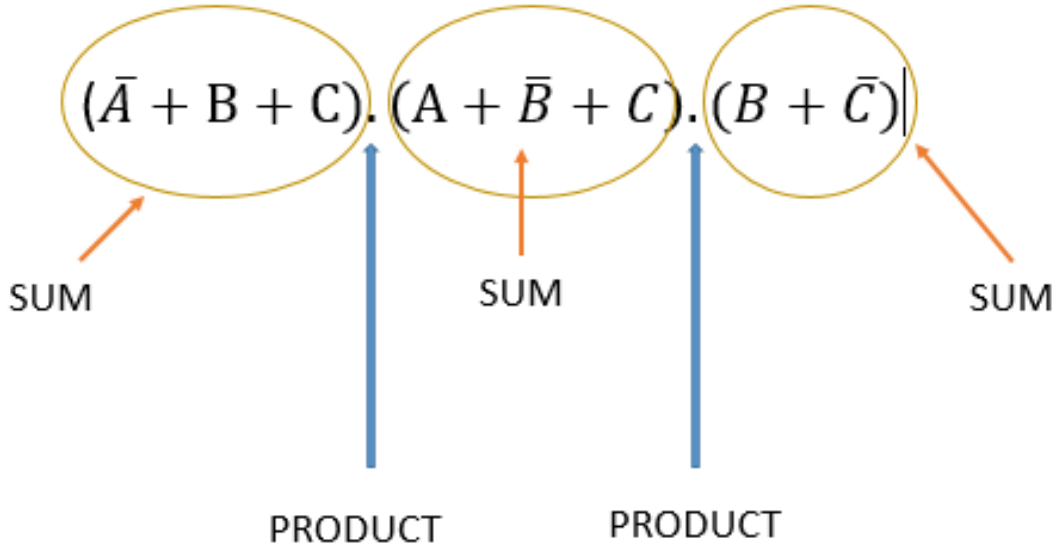
- Product of all Boolean terms are in AND notation
- Some of all products implements the total function



Introduction to POS (Product of Sum)

- Sum of all Boolean terms are in OR notation
- Sum of all Boolean terms are multiplied or product of all terms will implement the total function

- **Eg:** $(\bar{A} + B + C).(A + \bar{B} + C).(B + \bar{C})$



Minterm and Maxterm

- A Boolean function can be written in form of Boolean algebraic either truth table
- Inputs may represent “0” or “1” as logics
- The complement way describes when output is “1” where complement is for logic “0”
    - Ex: A and $\bar{A}$ B and $\bar{B}$
- The product of all inputs which represent one output, is known as minterm. Usually denoted by **“m”**
    - Ex : ABC , $\bar{B}$C , A$\bar{B}$C
- The Sum term of all inputs which represent one output, is know as Maxterm. Usually denote by **“M”**
    - Ex : (A+B+C) , ($\bar{B}$+C) , (A+$\bar{B}$+C)

Minterm and Maxterm

Minterm

- This Boolean expression has three minterms

$\bar{A}\mathrm{BC} + \mathrm{A}\bar{B}C + \bar{A}B\bar{C}$

the minterms are

$\bar{A}\mathrm{BC}$

$\mathrm{A}\bar{B}C$

$\bar{A}B\bar{C}$

Maxterm

- This Boolean expression has three maxterms

$(\bar{A} + \mathrm{B} + \mathrm{C}).(\mathrm{A} + \bar{B} + C).(\bar{A} + B + \bar{C})$

the maxterms are

$(\bar{A} + \mathrm{B} + \mathrm{C})$

$(\mathrm{A} + \bar{B} + C)$

$(\bar{A} + B + \bar{C})$

Minterm and Maxterm

- Lets consider the truth table
- There are three inputs where the truth table has 8 combinations
- So that there should be 8 minterms and maxterms
- In a Boolean function for n number of inputs there should be 2^n minterms and maxterms

A	B	C	MINTERM		MAXTERMS	
			output	Terms	output	Terms
0	0	0	$\bar{A}.\bar{B}.\bar{C}$	m_0	A+B+C	M_0
0	0	1	$\bar{A}.\bar{B}.C$	m_1	A+B+ $\bar{C}$	M_1
0	1	0	$\bar{A}$.B.$\bar{C}$	m_2	A+$\bar{B}+C$	M_2
0	1	1	$\bar{A}$.B.C	m_3	A+ $\bar{B}+\bar{C}$	M_3
1	0	0	A.$\bar{B}.\bar{C}$	m_4	$\bar{A}$ +B+C	M_4
1	0	1	A.$\bar{B}.C$	m_5	$\bar{A}$ +B+ $\bar{C}$	M_5
1	1	0	A.B.$\bar{C}$	m_6	$\bar{A}+\bar{B}+C$	M_6
1	1	1	A.B.C.	m_7	$\bar{A}+\bar{B}+\bar{C}$	M_7

Writing SOP & POS Using Truth Table

A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

- Consider the truth table and write SOP for when output is logic “1”

SOP = $m_0+m_4+m_7$

$= \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + ABC$

X(A,B,C,)= $\sum$ (0,4,7)

Abbreviated form of writing SOP & POS

- Write POS when output is logic ”0”

POS = $M_1M_2M_3M_5M_6$

= (A+B+C’)(A+B’+C)(A+B’+C’)(A’+B+C’)(A’+B’+C)

X(A,B,C,)= $\sum$ (1,2,3,5,6)

A	B	C	X	X’
0	0	0	0	1
0	0	1	0	1
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	1	0
1	1	1	1	0

SOP for X’

So that $\bar{X} = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\mathrm{C} + \bar{A}BC + A\bar{B}C$

Let consider the compliment of this

$\bar{\bar{X}} = \overline{\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\mathrm{C} + \bar{A}BC + A\bar{B}C}$

According Demorgan’s rule compliment of x’ can be written as

$X = (\overline{\bar{A}\bar{B}\bar{C}}).(\overline{\bar{A}\bar{B}\mathrm{C}}).(\overline{\bar{A}BC}).(\overline{A\bar{B}C})$

$X = (\bar{\bar{A}} + \bar{\bar{B}} + \bar{\bar{C}}).(\bar{\bar{A}} + \bar{\bar{B}} + \bar{C}).(\bar{\bar{A}} + \bar{B} + \bar{C}).(\bar{A} + \bar{\bar{B}} + \bar{C})$

$X = (A+B+C).(A+B+\bar{C}).(A+\bar{B} + \bar{C}).(\bar{A} + B + \bar{C})$

$X = M_7 . M_6 . M_5 . M_3$

Now SOP of x’ ; m_2, m_4, m_6, m_7 is converted POS of X

As a conclusion all the **“1”** of out puts are **SOP** of truth table where all the**”0”** can be written as **POS**


Q
A

Q & A

A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

- Considering the truth table's out put write all the SOP and POSs

SOP are **X(A,B,C)= $\sum$ (0,2,4,6,)**

where ; SOP = $m_0+m_2+m_4+m_6$

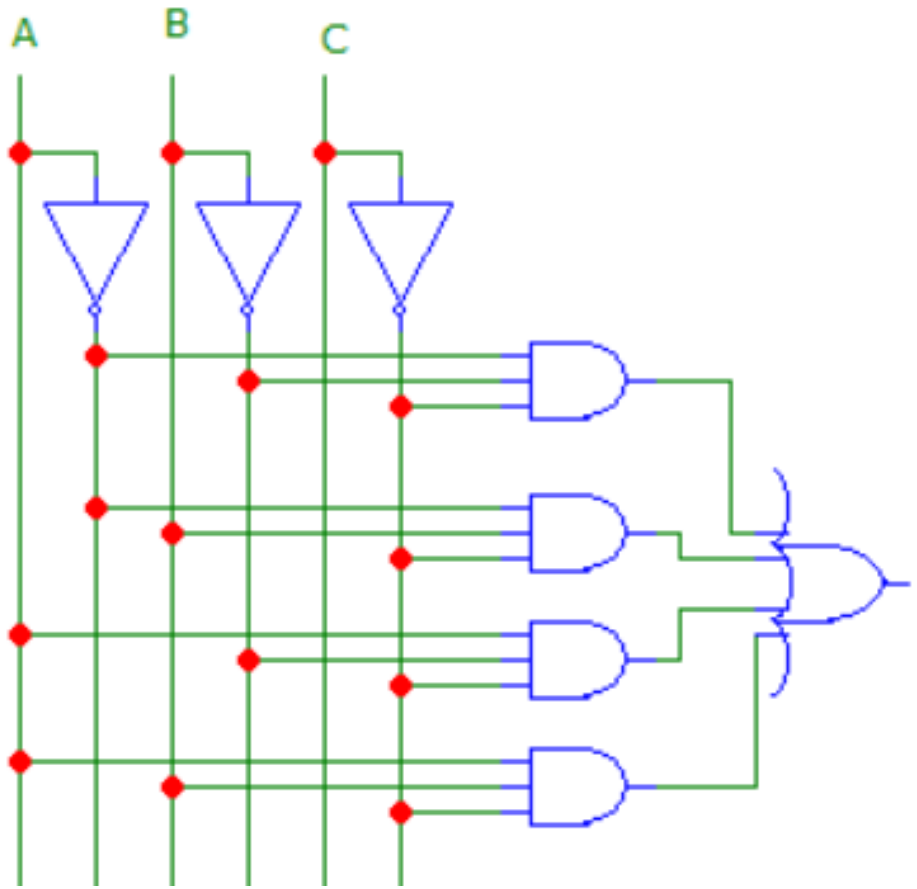
$= \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$

POS are **X(A,B,C)= $\sum$ (1,3,5,7,)**

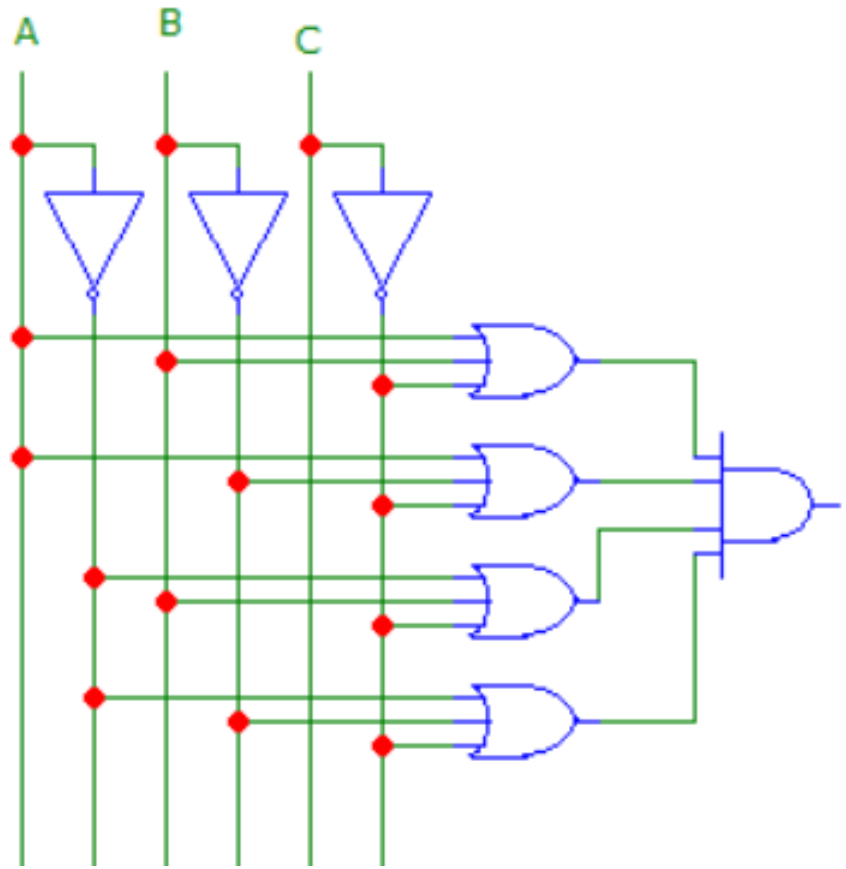
where ; POS = $M_1 . M_3 . M_5 . M_7$

$= (A + B + C').(A + B' + C')(A' + B + C')(A' + B' + C')$

Draw the logic gate diagram for both SOP and POS functions



SOP function

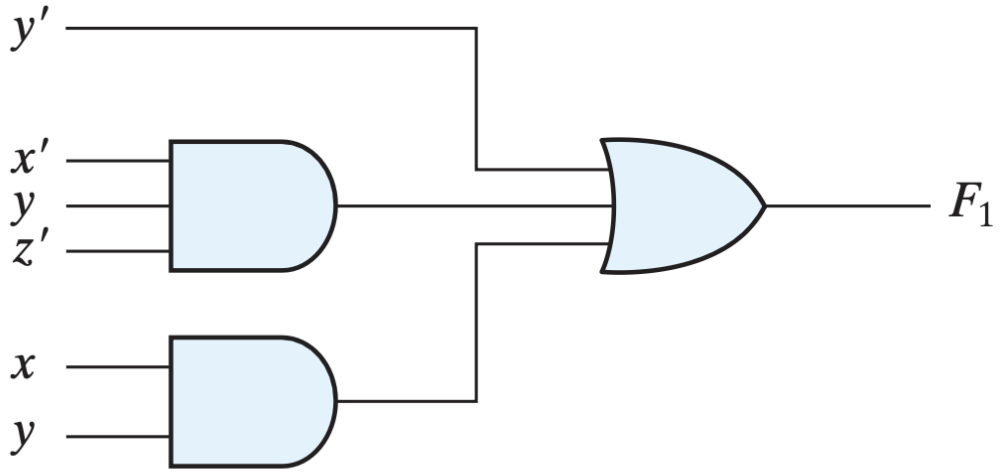


POS function

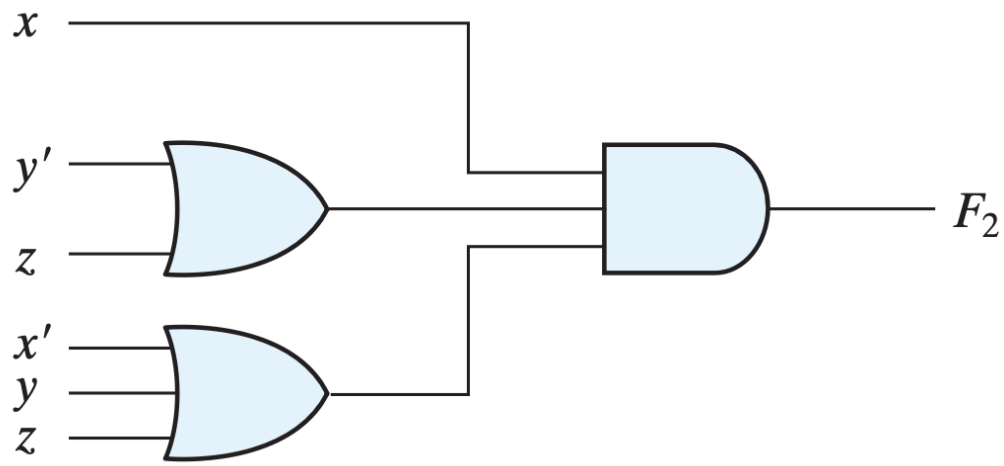
I. Identify the function F1 and F2 whether a SOP or POS fuction

II. Draw the Logic gate diagram for POS and SOP functions

$$F_1 = y' + xy + x'yz' \quad \textbf{and} \quad F_2 = x(y' + z)(x' + y + z')$$



(a) Sum of Products



(b) Product of Sums


Q
A

Q1

- A system has three inputs and provides the output f(x) when

  When f(x)= $\sum(0,1,2,3,7)$

i. Draw the truth table
ii. Write SOP form in canonical representation
iii. simplify the SOP term and draw equivalent circuit
iv. Write the POS term

Q2

- Express the **compliment** of following function in sum of minterms form
  - F(A,B,C,D) = $\sum(3,5,9,11,15)$
  - F(A,B,C,D) = $\sum(2,4,5,7)$