**Linear Transformation : Level 3- Tutorial Questions**

1. Let $T : R^2 \to R^3$ be a linear map such that $T(-1, 2) = (1, -1, 1)$ and $T(2, -1) = (-1, 1, -1)$. Then $T(2, 1) =$
   (1) $(\frac{1}{3}, \frac{1}{3}, -\frac{1}{3})$
   (2) $(-\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$
   (3) $(\frac{1}{3}, -\frac{1}{3}, \frac{1}{3})$
   (4) $(-\frac{1}{3}, \frac{1}{3}, -\frac{1}{3})$

2. Let $T : M_2(R) \to M_2(R)$ be given by $T(A) = AB - BA$, where $B = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$. The $\dim(Ker\ T)$ is
   (1) 0
   (2) 1
   (3) 2
   (4) 3

3. If $T : R^3 \to R^3$ is a linear map given by $T(x, y, z) = (x - y, y + 3z, x + 2y)$, then $T^{-1}$ is
   (1) $\frac{1}{3}(2x + z, -x + z, \frac{x}{3} + y - \frac{z}{3})$
   (2) $\frac{1}{3}(2x + y, -x + y, \frac{x}{3} - \frac{y}{3} + z)$
   (3) $\frac{1}{3}(x + 2y, x - y, -\frac{x}{3} + \frac{y}{3} - z)$
   (4) $\frac{1}{3}(x - 2y, x + y, \frac{x}{3} - \frac{y}{3} - z)$

4. Let $T : R^2 \to R^2$ be given by $T(x, y) = (x + y, x)$. $T$ is
   (1) 1-1 but not onto
   (2) not 1-1 but onto
   (3) both 1-1 and onto
   (4) neither 1-1 nor onto

5. Consider the vector space $C$ over $R$. Let $T$ be a linear operator on $C$ defined by $T(z) = \overline{z}$. Then
   (1) $T$ is 1-1, but not onto
   (2) $T$ is onto, but not 1-1
   (3) $T$ is both 1-1 and onto
   (4) $T$ is neither 1-1 nor onto

6. Let $T : R^3 \to R^3$ be a linear maps on $R^3$ defined by $T(x, y, z) = (x, y, 0)$. Then the null space of $T$ is generated by which one of the following vector?
   (1) (0,1,0)
   (2) (0,0,1)
   (3) (1,0,0)
   (4) none of these

7. Let $T$ and $S$ be two linear maps on $R^3$ defined by $T(x, y, z) = (0, y, x)$ and $S(x, y, z) = (x, 0, 0)$. Then
   (1) $T$ is idempotent, but $S$ is not
   (2) $S$ is idempotent, but $T$ is not
   (3) both $T$ and $S$ are idempotent
   (4) neither $T$ nor $S$ is idempotent

8. Let $T : M_2(R) \to M_2(R)$ be given by $T(A) = AB - BA$, where $B = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$. The $\dim(Ker\ T)$ is
   (1) 0

(2) 1
(3) 2
(4) 3

9. Let $T : R^n \to R^n, m > n$ be a linear map. Consider the following statements about $T$
(i) $T$ can be 1-1 (ii) $T$ can be onto (iii) $\dim(T(R^n)) \geq n$

(1) only (i) is true
(2) only (ii) is false
(3) only (ii) is true
(4) only (iii) is true

10. Let $T : P_2(t) \to P_2(t)$ be given by $T(a_0 + a_1t + a_2t^2) = a_1 + 2a2t$. The matrix o f $T$ relative to the standard basis is

(1) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$
(2) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$
(3) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}$
(4) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

11. Let $T$ be the linear operator on $R^2$ given by $T(x, y) = (f_1(x, y), f_2(x, y)) = (4x - 2y, 2x + y)$, then find the matrix of $T$ in the basis $\{f_1(1, 1), f_2(1, 1)\}$

(1) $\begin{bmatrix} 3 & -2 \\ 1 & 2 \end{bmatrix}$
(2) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
(3) $\begin{bmatrix} 4 & -2 \\ 2 & 1 \end{bmatrix}$
(4) $\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$

12. If $T : R^2 \to R^3$ is a linear map such that $T(1, 0) = (2, 3, 1)$ and $T(1, 1) = (3, 0, 2)$. Then

(1) $T(x, y) = (x + y, 2x + y, 3x - 3y)$
(2) $T(x, y) = (2x + y, 3x - 3y, x + y)$
(3) $T(x, y) = (2x - y, 3x + 3y, x - y)$
(4) $T(x, y) = (x - y, 2x + y, 3x + 3y)$

13. Let $V$ be the vector space of $2 \times 2$ matrices over $R$ and let $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Let $T$ be the linear operator on $V$ defined by $T(A) = MA$, for all $A$ in $V$. Then trace of $T$ is

(1) 5
(2) 10
(3) 0
(4) none of these

14. A linear transformation $E$ on a vector space is a projection on some subspace of $V$. Then
    (1) $E^2 = 0$
    (2) $E^2 = I$
    (3) $E^2 = E$
    (4) none of these

15. Let $T$ be the linear operator on $R^3$ given by $T(x, y, z) = (x + 3y + 3z, 4y + 5z, 9z)$. Then
    (1) $T$ is diagonalizable, but $T^2$ is not
    (2) $T^2$ is diagonalizable, but $T$ is not
    (3) both $T$ and $T^2$ are diagonalizable
    (4) neither $T$ nor $T^2$ is diagonalizable

16. Let $T : R^4 \to R^4$ be given by $T(x, y, z, t) = (y + 2z, x + z, 2x + y + 2t, 2z)$. Then
    (1) $T$ has no real eigen values
    (2) all real eigen values of $T$ are positive
    (3) all real eigen values of $T$ are negative
    (4) $T$ has both positive and negative real eigen values

17. Let $V$ be a four dimensional vector space with basis$\{v_1, v_2, v_3, v_4\}$.Let $T : V \to V$be a linear map defined by $T(v_1) = v_2, T(v_2) = v_3, T(v_3) = v_4$ and $T(v_4) = 5v_1 - 2v_2 - \frac{1}{2}v_3 - v_4$.The minimal polynomial is
    (1) $x^3 - \frac{x^2}{2} + 2x - 5$
    (2) $\frac{x^2}{2} + 2x - 5$
    (3) $x^4 + x^3 + \frac{x^2}{2} + 2x - 5$
    (4) $2x - 5$

18. Let $T$ be a linear operator on $R^3$ defined by $T(x, y, z) = (x, y, 0)$. Then
    (1) $(0, 0, 4)$ is in the range space
    (2) $(2, \frac{1}{\sqrt{2}}, 0)$ is in the zero space
    (3) $(1, 1, 1)$ is in the range space
    (4) $(0, 0, 1)$ is in the zero space

19. Let $T : R^4 \to R^3$ be the linear transformation given by $T(x, y, z, w) = (x - y + z + w, x + 2z - w, x + y + 3z - 3w)$. Then the dimension of the range space of $T$ is:
    (1) 0
    (2) 1
    (3) 2
    (4) 3

20. Let $M = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ and $T : V \to V$ be the linear map defined by $T(A) = AM$, where $V$ be the vector space of all $2\times2$ real matrices. Then rank and nullity of $T$, respectively are:
    (1) 1,3
    (2) 3,1
    (3) 2,2
    (4) 4,0

21. Let $T_1$ and $T_2$ be linear operators on $R^2$ defined as $T_1(x, y) = (y, x)$ and $T_2(x, y) = (x, 0)$. Then
    (1) $T_1T_2 \neq T_2T_1$
    (2) $T_1T_2 = T_2T_1$
    (3) $T_1T_2(x, y) = (0, x)$

(4) $T_2T_1(x, y) = (y, 0)$

22. Given $T : V \to W$ is a linear map
    (1) $T$ is 1-1 $\Rightarrow$ nullity $(T) = 0$
    (2) $T$ is 1-1 $\Rightarrow$ nullity $(T) \neq 0$
    (3) nullity $(T) = 0 \Rightarrow T$ is 1-1
    (4) nullity $(T) = 0 \Leftrightarrow T$ is 1-1

23. Let $T$ be a linear operator on a finite dimensional vector space $V$. If $m(t) = t^r + a_{r-1}t^{r-1} + a_{r-2}t^{r-2} + .... + a_1t + a_0, a_0 \neq 0$ is the minimal polynomial of $T$, then
    (1) $T$ is invertible
    (2) $T$ is not singular
    (3) 0 is not a root of $m(t)$
    (4) 0 is not an eigen value of $T$

24. Let $T$ be a linear operator on $R^2$ satisfying $T^2 - T + 1 = 0$. Then
    (1) $T$ is 1-1
    (2) $T$ is not 1-1
    (3) Trace $T$ is in $R$
    (4) In some cases Trace $(T)$ in $R$ and in some cases Trace $(T)$ not in $R$

25. Le t$T$ be a map on $F[X]$. Then which of the following is/are linear transformation(s?)
    (1) $T(f(x)) = -f(x)$
    (2) $T(f(x)) = f(-x)$
    (3) $T(f(x)) = f(0)$
    (4) $T(f(x)) = f(x) + f(-x)$

26. Let $T$ and $S$ be two invertible linear operators on a vector space $V$ over a field $F$. Then
    (1) $(TS)^{-1} = T^{-1}S^{-1}$
    (2) $(ST)^{-1} = S^{-1}T^{-1}$
    (3) $ST = TS$
    (4) None of these

27. Let $T$ be a linear operator on an $n$ dimensional vector space $V$ over a field $F$ and let $T$ has $n$ distinct eigen values. Then
    (1) $T$ is diagonalizable
    (2) $T$ is invertible
    (3) $T$ is invertible as well as diagonalizable
    (4) $T$ is not invertible

28. Let $P$ be the vector space of all polynomials $f(x)$ with real coefficents defines on $[0, 1]$. Let $D$ and $T$ be two linear operators on $P$ defined by $Df(x) = \frac{d}{dx}(f(x))$ and $T(f(x)) = x.f(x)$. Then
    (1) $DT = TD$
    (2) $DT \neq TD$
    (3) $DT(f(x) = f(x) + x.\frac{d}{dx}(f(x))$
    (4) $TD(f(x)) = x.\frac{d}{dx}(f(x))$

29. Let $T$ be a linear operator on a 3 dimensional vector space $V$ satisfying $T^3 - T^2 - T + I = 0$. Then
    (1) $T$ is 1-1
    (2) $T$ is onto
    (3) $T$ is invertible

(4) $T^{-1} = I + T - T^2$

30. Let V be the vector space of polynomials over R of degree at most n. Define $T : V \to V$ by $T(p(x)) = x.\frac{d}{dx}(p(x))$. Then

(1) T is 1-1
(2) T is onto
(3) T is not invertible
(4) Rank of T is n

31. The geometrical effect of the linear transformation associated with the matrix $\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$ is:

(1) a rotation by an angle 90°
(2) a stretching along $X$ axis
(3) a reflexion w.r.t X axis
(4) stretching along Y axis and a reflection w.r.t Y axis