

Limit: Level 3- Tutorial Problems

1. $\lim_{n \rightarrow \infty} (1 - \frac{1}{n^2})^n =$

- (1) 1
- (2) $e^{-\frac{1}{2}}$
- (3) e^{-2}
- (4) e^{-1}

2. Which of the following are true?

- (1) $(1 + \frac{1}{n})^{n+1} \rightarrow e$ as $n \rightarrow \infty$
- (2) $(1 + \frac{1}{n+1})^n \rightarrow e$ as $n \rightarrow \infty$
- (3) $(1 + \frac{1}{n})^{n^2} \rightarrow e$ as $n \rightarrow \infty$
- (4) $(1 + \frac{1}{n^2})^n \rightarrow e$ as $n \rightarrow \infty$

3. Which of the following are true?

- (1) $\lim_{x \rightarrow \infty} \frac{\log x}{x^{\frac{1}{2}}} = 0$ and $\lim_{x \rightarrow \infty} \frac{\log x}{x} = \infty$
- (2) $\lim_{x \rightarrow \infty} \frac{\log x}{x^{\frac{1}{2}}} = \infty$ and $\lim_{x \rightarrow \infty} \frac{\log x}{x} = 0$
- (3) $\lim_{x \rightarrow \infty} \frac{\log x}{x^{\frac{1}{2}}} = 0$ and $\lim_{x \rightarrow \infty} \frac{\log x}{x} = 0$
- (4) $\lim_{x \rightarrow \infty} \frac{\log x}{x^{\frac{1}{2}}} = 0$ and $\lim_{x \rightarrow \infty} \frac{\log x}{x}$ does not exist.

4. $\lim_{x \rightarrow 0} \frac{1}{x} \int_x^{2x} e^{-t^2} dt$

- (1) does not exist
- (2) is infinite
- (3) exist and equal to 1
- (4) exist and equal to 0

5. Let $f(x) = \begin{cases} x^2, & \text{if } x \in [0, \frac{1}{2}] \\ x, & \text{if } x \in (\frac{1}{2}, 1] \end{cases}$, then $\lim_{x \rightarrow \frac{1}{2}} f(x) =$

- (1) $\frac{1}{2}$
- (2) 2
- (3) 1
- (4) does not exist

6. Suppose $\{x\} = x - [x]$ where $[x]$ denotes the greatest integer less than or equal to x , then $\lim_{n \rightarrow 0} \frac{(x) + (2x) + \dots + (nx)}{n^2}$ is

- A) x B) $x/2$ C) $x/3$ D) $x/4$

7. $\lim_{x \rightarrow \infty} x \left(\log(1 + \frac{x}{2}) - \log \frac{x}{2} \right)$ is:

- (1) 0
- (2) 1
- (3) 2
- (4) None of these