Eigen Values and Eigen Vectors: Level 3- Tutorial Problems

1. For the matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, A^{-1} is

   (1) $A^2 - 2A$
   (2) $A^2 + 2A + 3I$
   (3) $A^2 - 2A - I$
   (4) $A - 3I$

2. Consider the system of equations $Ax = cx$, where A is an n $\times$ n real matrix, x is an n $\times$ 1 real matrix, and c is a real scalar. Let (c_i, x_i) be an eign pair of an eign value and its corresponding eigen vector of A . Which of the following statements is not correct?

   (1) for homogenous n$\times$n system of linear equations, $(A - cI)x = 0$ having a non-trivial solution, the rank of $(A - cI) < n$
   (2) for the matrix A^m, m beign a positive integer, (c_i^m, x_i^m) will be the eigen pair for all i
   (3) If $A^T = A^{-1}$, then $|c_i| = 1$, for all i
   (4) If $A^T = A$, then c_i is real for all i

3. The minimal polynomial of the matrix $A = \begin{bmatrix} 0 & 0 & 0 & -4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ is

   (1) $(x^2 - 1)(x^2 - 4)$
   (2) $(x^2 + 1)(x^2 - 4)$
   (3) $(x^2 - 1)(x^2 + 4)$
   (4) $(x^2 + 1)(x^2 + 4)$

4. Let P, M and N be $n \times n$ matrices such that M and N are non-singular. If x is an eigen vector of P corresponding to the eigen value λ, then an eigen vector of $N^{-1}MPM^{-1}N$ corresponding to λ is

   (1) $MN^{-1}x$
   (2) $M^{-1}Nx$
   (3) $NM^{-1}x$
   (4) $N^{-1}Mx$

5. If $A = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$ where a, b, c are non-zero real numbers, then A has

   (1) three non-zero real eigen values
   (2) purely imaginary eigen values
   (3) two non-zero real eigen values
   (4) only one non-zero real eigen value

6. If $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$, $0 < x < \pi$, then A

   (1) is skew symmetric
   (2) is orthogonal
   (3) is symmetric
   (4) has no real eigen value

7. Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ and let α_n and β_n denote the two eigenvalues of A^n such that $|\alpha_n| \geq |\beta_n|$. Then

   (1) $\alpha_n \to \infty$ as $n \to \infty$
   (2) $\beta_n \to 0$ as $n \to \infty$
   (3) β_n is positive if n is even
   (4) β_n is negative if n is odd.

8. Let n be an odd number ≥ 7. Let $A = [a_{ij}]$ be an $n \times n$ matrix with $a_{i,i+1} = 1$ for all $i = 1, 2, ..., n-1$ and $a_{n,1} = 1$. Let $a_{ij} = 0$ for all the other pairs (i, j). Then we can conclude that

   (1) A has 1 as an eigenvalue.
   (2) A has -1 as an eigenvalue.
   (3) A has atleast one eigenvalue with multiplicity ≥ 2
   (4) A has no real eigenvalues.

9. Let A be an $n \times n$ matrix with $n \geq 2$ with chara-polynomial $x^{n-2}(x^2 - 1)$, then

   (1) $A^n = A^{n-2}$
   (2) $Rank(A) \geq 2$
   (3) $Rank(A) = 2$
   (4) there exists non-zero vectors x & y such that $A(x + y) = x - y$

10. Let A be a real symmetric matrix and $B = I + iA$, where $i^2 = -1$. Then

    (1) B is invertible if and only if A is invertible.
    (2) All eigen values of B are necessarily real.
    (3) $B - I$ is necessarily invertible
    (4) B is necessarily invertible

11. Consider the matrix
    $A(x) = \begin{pmatrix} 1 + x^2 & 7 & 11 \\ 3x & 2x & 4 \\ 8x & 17 & 13 \end{pmatrix}$; $x \in \mathbb{R}$. Then

    (1) $A(x)$ has eigenvalue 0 for some $x \in \mathbb{R}$
    (2) 0 is not an eigenvalue of $A(x)$ for any $x \in \mathbb{R}$
    (3) $A(x)$ has eigenvalue 0 for all $x \in \mathbb{R}$
    (4) $A(x)$ is invertible for every $x \in \mathbb{R}$

12. Let $P_A(x)$ denote the characteristic polynomial of a matrix A. Then for which of the following matrices, $P_A(x) - P_{A^{-1}}(x)$ is a constant?

    1. $\begin{pmatrix} 3 & 3 \\ 2 & 4 \end{pmatrix}$
    2. $\begin{pmatrix} 4 & 3 \\ 2 & 3 \end{pmatrix}$
    3. $\begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix}$
    4. $\begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}$