**SUBJECT: Linear Algebra - TOPIC: Solution of System of linear equations**

**Solution of system of linear equations:**
Let $A$ be an $m \times n$ matrix, $X$ be an $n \times 1$ matrix and $B$ be an $m \times 1$ matrix, then the matrix equation $AX = B$ represents a system of $m$ linear equations in $n$ unknowns, where $A$ is called the coefficient matrix, $X$ is the variable or unknown matrix and $B$ is the constant matrix.

**Case(i)** $AX = B$, $B \neq 0$ $- - - - - - - - - - (1)$

Such a system is called non homogenous system.

1. The system (1) has a unique solution iff $rank(A) = rank(A : B) = n$, where $n$ is the number of unknowns or number of columns.
2. The system (1) has infinite number of solutions iff $rank(A) = rank(A : B) = r < n$. In this case we have to take $(n - r)$ variables as free, out of the $n$ variables.
3. The system (1) has no solution iff $rank(A) \neq rank(A : B)$

Eg: 1) $x + 2y - z = 3$, $3x - y + 2z = 1$, $2x - 2y + 3z = 2$, $x - y + z = -1$

Eg: 2) $5x + 3y + 7z = 4$, $3x + 26y + 2z = 9$, $7x + 2y + 10z = 5$

Eg: 3) $x + y + z = 1$, $2x + 2y + 2z = 3$

**Case(ii)** $AX = B$, $B = 0$ $- - - - - - - - - - (2)$

Such a system is called homogenous system.

1. In this case $rank(A) = rank(A : B)$ always, then the system is consistent and has atleast one solution.
2. $X = 0$ is always a solution of (2), this solution is called trivial solution.
3. System (2) has a unique solution ($X = 0$) iff $rank(A) = n$, number of columns or number of unknowns.
4. System (2) has infinite number of solutions iff $rank(A) = r < n$, in this case there will be $(n - r)$ free variables and the solution of (2) is called the null space of matrix $A$ given by $N(A) = \{x : Ax = 0, x \in \mathbb{R}^n\}$ and $dim(N(A)) = n - r$.
5. If $m < n$ then system (2) has infinitely many solutions.