Differentiation: Level 3- Tutorial Problems

1. Let $I = [0,1]$ for $x \in \mathbb{R}$.Let $\phi(x) = dist(x, I) = \inf\{|x - y| : y \in I\}$ .Then
   (1) $\varphi(x)$ is discontinuous somewhere on $\mathbb{R}$
   (2) $\varphi(x)$ is continuous on $\mathbb{R}$ but not continuously differentiable exactly at $x = 0$
   (3) $\varphi(x)$ is continuous on $\mathbb{R}$but not continuously differentiable exactly at $x = 0, 1$
   (4) $\varphi(x)$is differentiable on $\mathbb{R}$

2. Consider the function$f(x) = |\cos x| + |\sin(2 - x)|$ .At which of the following set f is not differentiable?
   (1) $\left\{(2n+1)\frac{\pi}{2} : n \in \mathbb{Z}\right\}$
   (2) $\{n\pi \ : n \in \mathbb{Z}\}$
   (3) $\{n\pi + 2 \ : n \in \mathbb{Z}\}$
   (4) $\left\{\frac{n\pi}{2} \ : n \in \mathbb{Z}\right\}$

3. Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ 2x + x^2 & \text{if } x \geq 0 \end{cases}$. Which of the following are true?
   (1) $f"(x) = 2 \ \forall \ x \in \mathbb{R}$
   (2) $f"(0)$ does not exist.
   (3) $f"(x)$ exist for each $x \neq 0$.
   (4) $f'(0)$ does not exist.

4. Let $f(x) = \max\{\cos x, \sin x\}$. f is not differentiable only
   (1) At a single point.
   (2) At countable number of points.
   (3) Finite number of points.
   (4) At any points of R.

5. If $f(x) = x^5 - 20x^3 + 240x$, then $f(x)$is
   (1) monotonically decreasing only in$[0, \infty)$
   (2) monotonically increasing everywhere.
   (3) monotonically decreasing everywhere
   (4) monotonically increasing only in $[0, \infty)$

6. Let $f(x) = \sin x - x + \frac{x^3}{3!}$ and $g(x) = \cos x - 1 + \frac{x^2}{2!}$ $\forall x \in \ \mathbb{R}$. Which of the following is/are true?
   (1) $f(x) \geq 0 \ \forall \ x > 0$
   (2) g is an increasing function on $[0, \infty)$
   (3) g is a decreasing function on $[0, \infty)$
   (4) f is an decreasing function on $[0, \infty)$

7. Let f be a twice differentiable function on $\mathbb{R}$. Given that $f"(x) > 0 \ \forall \ x \in \mathbb{R}$. Then
   (1) $f(x) = 0$ has exactly 2 two solutions in $\mathbb{R}$
   (2) $f(x) = 0$ has a positive solution if $f(0) = 0$ and $f'(0) = 0$
   (3) $f(x) = 0$ has no positive solution if $f(0) = 0$ and $f'(0) > 0$
   (4) $f(x) = 0$ has no positive solution if $f(0) = 0$ and $f'(0) < 0$

8. The equation $11^x + 13^x + 17^x - 19^x = 0$ has
   (1) no real root
   (2) only one real root
   (3) exactly two real roots

(4) more than two real roots

9. Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a twice continuous differentiable function with $f(0) = f(1) = f'(0) = 0$. Then
   (1) f" is the zero function
   (2) $f"(0) = 0$
   (3) $f"(x) = 0$ for some$x \in (0, 1)$
   (4) $f"(x)$ never vanishes

10. Suppose P is a polynomial with real co-efficient.Which of the following is/are true?
   (1) There is no root of the derivative P' between any two roots of P.
   (2) There is exactly one root of the derivative P' between any two roots of P.
   (3) There is exactly one root of the derivative P' between any two consecutive roots of P.
   (4) There is atleast one root of p' between any two consecutive roots of P.