Rank of Matrix: Level 3-Tutorial Problems

1. The rowspace of a 20×50 matrix A has dimension 13, what is the dimension of the space of solution of $Ax = 0$?

   (1) 7
   (2) 13
   (3) 33
   (4) 37

2. Let A be a 5×5 matrix and let B be obtained by changing one element of A. let r & s be the ranks of A and B respectively, which of the following statement is/are true?

   (1) $s \leq r + 1$
   (2) $r - 1 \leq s$
   (3) $s = r - 1$
   (4) $s \neq r$

3. Let A, B be $n \times n$ real matrices such that $AB = BA = 0$ and $A + B$ is invertible, which of the following statements is/are true,

   (1) Rank(A)=Rank(B)
   (2) Rank(A)+Rank(B)=n
   (3) Null(A)+Null(B)=n
   (4) (4) $A - B$ is invertible

4. Let J denote the $n \times n$ matrix with all entries 1 and let B be a $(3n) \times (3n)$ matrix given by $B = \begin{pmatrix} 0 & 0 & J \\ 0 & J & 0 \\ J & 0 & 0 \end{pmatrix}$, then the rank of B is

   (1) 2n
   (2) 3n-1
   (3) 2
   (4) 3

5. What is the rank of the following matrix?

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$$

   1. 2
   2. 3
   3. 4
   4. 5

6. If $X = [x_1, x_2, ..., x_n]^T$ is an n-tuple non-zero vector, then the n × n matrix $V = XX^T$

   (1) has rank 0
   (2) has rank 1
   (3) is orthogonal
   (4) has rank n

7. If $X = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, then the rank of $X^T X$ is:

(1) 0
(2) 1
(3) 3
(4) 4