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Relations & Functions: Level 3 - Tutorial Problems

1. Consider the map $f : \mathbb{Q} \to \mathbb{R}$ defined by

   (a) $f(0) = 0$

   (b) $f(r) = \dfrac{p}{10^q}$ where $r = \dfrac{p}{q}$ with $p \in \mathbb{Z}, q \in \mathbb{N}$ and $\gcd(p, q) = 1$

   Then the map f is

   (a) one-to-one and onto

   (b) not one-to-one, but onto

   (c) onto, but not one-to-one

   (d) neither one-to-one nor onto

2. Let $\mathbb{Z}$ denote the set of integers and $\mathbb{Z}_{\geq 0}$ denote the set $\{0, 1, 2, 3, \dots\}$. Consider the map $f : \mathbb{Z}_{\geq 0} \times \mathbb{Z} \to \mathbb{Z}$ given by $f(m, n) = 2^m.(2n + 1)$. Then the map is

   (1) onto (surjective) but not one-one (injective)
   (2) one-one (injective) but not onto (surjective)
   (3) both oen-one and onto
   (4) neither one-one nor onto

3. The function $f : \mathbb{R} \longrightarrow \mathbb{R}$ given by $f(x) = (x - 1)(x - 2)(x - 3)$

   (1) one-one but not onto
   (2) onto but not one-one
   (3) one-one and onto
   (4) neither one-one nor onto

4. If $f(x) = cos([\pi^2]x) + \cos([-\pi^2]x)$. Then

   (1) $f(\frac{\pi}{2}) = -1$
   (2) $f(\pi) = 1$
   (3) $f(\frac{\pi}{2}) = 0$
   (4) $f(\frac{\pi}{4}) = 0$

5. If $f(x)$ is satisfying $f(x)f(\frac{1}{x}) = f(x) + f(\frac{1}{x})$ and if f(3)=28, then f(4) is

   (1) 63
   (2) 65
   (3) 17
   (4) None of these

6. Let $f : \mathbb{R} \to \mathbb{R}$ be a function satisfying $f(x + y) = f(x)f(y),\ \forall x, y \in \mathbb{R}$ and $\lim_{x\to 0} f(x) = 1$. Which of the following are necessarily true? [D-2017]

   (1) f is stricly increasing
   (2) f is either constant or bounded.
   (3) $f(rx) = f(x)^r$ for every rational $r \in \mathbb{Q}$
   (4) $f(x) \geq 0,\ \forall x \in \mathbb{R}$

7. The number of real roots of $x^9 + x^7 + x^5 + x^3 + x + 1$ is

   (1) 9
   (2) 5
   (3) 3
   (4) 1

8. Using the fact that $\sum\limits_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = log2$, $\sum\limits_{n=1}^{\infty} \frac{(-1)^n}{n(n+1)}$ is

   (1) $1 - 2\log 2$
   (2) $(\log 2)^3$
   (3) $1 + \log 2$
   (4) $(1 - \log 2)^2$

9. If $\sum\limits_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, then $\sum\limits_{n=1}^{\infty} \frac{1}{(2n+1)^2}$ is [J-2012]

   (1) $\frac{\pi^2}{12}$
   (2) $\frac{\pi^2}{12} - 1$
   (3) $\frac{\pi^2}{8}$
   (4) $\frac{\pi^2}{8} - 1$

10. Given that there are real constants a, b, c, d such that the identity $\lambda x^2 + 2xy + y^2 = (ax + by)^2 + (cx + dy)^2$ holds for all $x, y \in \mathbb{R}$. This implies

    (1) $\lambda = -5$
    (2) $\lambda \geq 1$
    (3) $0 < \lambda < 1$
    (4) there is no such $\lambda \in \mathbb{R}$

11. Let $f(x) = x^5 - 5x + 2$. Then [J-2018]

    (1) f has no real root
    (2) f has exactly one real root
    (3) f has exactly three real roots
    (4) all roots of f are real

12. Let $p(x)$ be a polynomial function in one variable of odd degree and g be a continuous function from $\mathbb{R}$ to $\mathbb{R}$. Then which of the following statements are true.

    A. $\exists$ a point $x_0 \in \mathbb{R}$ such that $p(x_0) = g(x_0)$
    B. If g is a polynomial function then there exists $x_0 \in \mathbb{R}$ such that $p(x_0) = g(x_0)$
    C. If g is a bounded function there exists $x_0 \in \mathbb{R}$ such that $p(x_0) = g(x_0)$
    D. There is a unique point $x_0 \in \mathbb{R}$ such that $p(x_0) = g(x_0)$