**System of Linear Equations: Level 3- Tutorial Problems**

1. Let $A$ be a $5 \times 4$ matrix with real entries such that $AX = 0 \Leftrightarrow X = 0$, where $X$ is a $4 \times 1$ vector, $'0'$ is the null vector, then $Rank(A)$ is

   (1) 4
   (2) 5
   (3) 2
   (4) 1

2. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 2 & 3 & \alpha \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 3 \\ \beta \end{bmatrix}$. Then the system $AX = b$ over the real numbers has

   (1) no solution whenever $\beta \neq 7$
   (2) an infinite number of solutions whenever $\alpha \neq 2$
   (3) an infinite number of solutions if $\alpha = 2$
   (4) a unique solution if $\alpha \neq 2$

3. The system of equations $x + y + z = 1,\ 2x + 3y - z = 5,\ x + 2y - kz = 4$, where $k \in \mathbb{R}$ has an infinite number of solutions for

   (1) k=0
   (2) k=1
   (3) k=2
   (4) k=3

4. The system of equations:
   1. $x + 2.x^2 + 3.xy + 0.y = 6$
   2. $x + 1.x^2 + 3.xy + 1.y = 5$
   1. $x - 1.x^2 + 0.xy + 1.y = 7$

   (1) has solutions in rational numbers
   (2) has solutions in real numbers
   (3) has solutions in complex numbers
   (4) has no solution

5. Let $M = \begin{bmatrix} 2 & 0 & 3 & 2 & 0 & -2 \\ 0 & 1 & 0 & -1 & 3 & 4 \\ 0 & 0 & 1 & 0 & 4 & 4 \\ 1 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$, $b_1 = \begin{bmatrix} 5 \\ 1 \\ 1 \\ 4 \end{bmatrix}$ and $b_2 = \begin{bmatrix} 5 \\ 1 \\ 3 \\ 3 \end{bmatrix}$. Then which of the following are true?

   1. both systems $MX = b_1$ and $MX = b_2$ are inconsistent
   2. both systems $MX = b_1$ and $MX = b_2$ are consistent
   3. the system $MX = b_1 - b_2$ is consistent
   4. the system $MX = b_1 - b_2$ is inconsistent

6. Let $A = \begin{pmatrix} 2 & 0 & 5 \\ 1 & 2 & 3 \\ -1 & 5 & 1 \end{pmatrix}$. The system of linear equations $AX = Y$ has a solution

(a) only for $Y = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}, x \in \mathbb{R}$

(b) only for $Y = \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix}, y \in \mathbb{R}$

(c) only for $Y = \begin{pmatrix} 0 \\ y \\ z \end{pmatrix}, y, z \in \mathbb{R}$

(d) for all $Y \in \mathbb{R}^3$