**Vector space, Linear independence, Basis,Dimension: Level 3- Tutorial Questions**

1. Which of the following is/are subspaces of the vector space of all real functions,
   (1) set of all one-one functions
   (2) set of all bijective functions
   (3) set of all onto functions
   (4) none of the above

2. Let $M$ be the vector space of all $3 \times 3$ real matrices and let $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then which of the following is/are subspaces of $M$, [J-2011]
   (1) $W_1 = \{X \in M \mid XA = AX\}$
   (2) $W_2 = \{X \in M \mid X + A = A + X\}$
   (3) $W_3 = \{X \in M \mid Trace(AX) = 0\}$
   (4) $W_4 = \{X \in M \mid \det(AX) = 0\}$

3. Let $n$ be a +ve integer and $n \geq 3$ and let $\{u_1, u_2, ..., u_n\}$ be $n$ linearly independent elements in a vector space over $\mathbb{R}$. set $u_0 = 0$ and $u_{n+1} = u_1$, define $v_i = u_i + u_{i+1}$ and $w_i = u_{i-1} + u_i,\ i = 1, 2, ..., n$, then
   (1) $v_1, v_2, ..., v_n$ are linearly independent if $n = 2010$.
   (2) $v_1, v_2, ..., v_n$ are linearly independent if $n = 2011$.
   (3) $w_1, w_2, ..., w_n$ are linearly independent if $n = 2010$.
   (4) $w_1, w_2, ..., w_n$ are linearly independent if $n = 2011$.

4. Let $x = (x_1, x_2, x_3), y = (y_1, y_2, y_3) \in \mathbb{R}^3$ be linearly independent. let $\delta_1 = x_2y_3 - y_2x_3,\quad \delta_2 = x_1y_3 - y_1x_3,\ \delta_3 = x_1y_2 - y_1x_2$, if $V$ is the span of $x, y$, then
   (1) $V = \{(u, v, w) : \delta_1 u - \delta_2 v + \delta_3 w = 0\}$
   (2) $V = \{(u, v, w) : -\delta_1 u + \delta_2 v + \delta_3 w = 0\}$
   (3) $V = \{(u, v, w) : \delta_1 u + \delta_2 v - \delta_3 w = 0\}$
   (4) $V = \{(u, v, w) : \delta_1 u + \delta_2 v + \delta_3 w = 0\}$.

5. The dimension of the vector space formed by the solutions of the system of equations:$x + y + z = 0, x + 2y = 0, y - z = 0$ is
   (1) 1
   (2) 2
   (3) 3
   (4) 0

6. Let $U = \{(0, b, c, d, e) \in R^5\}$ and $W = \{(a, 0, c, d, e) \in R^5\}$ be subspaces of $R^5$. Then $\dim(U \cap W)$ is
   (1) 5
   (2) 4
   (3) 3
   (4) 2

7. If $S = \{(1, 1, 0), (2, 1, 3)\}$, then which of the following vectors of $R^3$ in not in Span $(S)$?
   (1) $(0, 0, 0)$
   (2) $(3, 2, 3)$
   (3) $(1, 2, 3)$
   (4) $(\frac{4}{3}, 1, 1)$

8. Let $v$ be the vector 5 space of ordered pairs of complex numbers over the field $R$. then dimension of $v$ is

(1) 1
(2) 2
(3) 3
(4) 4

9. Let $V$ be the vector space of polynomials of degree $\leq 3$; $U_1 < V$ be the subspace of polynomials of degree $\leq 2$; $U_2 < V$ be the subspace of polynomials vanishing at 0. Then which of the following is not true?

(1) span of $U_1$ and $U_2$ is $V$.
(2) $U_1$ is isomorphic to $U_2$
(3) $\dim(U_1 \cap U_2) = 2$
(4) $\dim U_1 < \dim U_2$

10. Consider the set $A = \{(1, 0, -i), (1 + i, 1 - i, 1), (i, i, 1)\}$ of $C^3$. Which of the following is/are true?

(1) $L(A)$ has dimension 1
(2) $L(A)$ has dimension 2
(3) $A$ is a basis for $C^3$
(4) Each element of $C^3$ can be generated as a linear combination of elements of $A$

11. Let $U$ and $W$ be subspaces of a vector space $V$ over a field $F$ and let $\dim V = 12, \dim U = 6$, and $\dim W = 8$. Then

(1) $\dim(U \cap W) \leq 6$ and $\dim(U + W) \geq 8$
(2) $\dim(U \cap W) \geq 6$ and $\dim(U + W) \geq 8$
(3) $\dim(U \cap W) \geq 6$ and $\dim(U + W) \leq 8$
(4) $\dim(U \cap W) \leq 6$ and $\dim(U + W) \geq 12$

12. Let $V$ be a vector space of dimension 3 and let $A$ and $B$ be its two disjoint subspaces having dimensions 2 and 1 respectively. Then

(1) $V = A \cap B$
(2) $V = A \cup B$
(3) $V = A + B$
(4) Nothing can be said

13. If a finite dimensional vector space $V$ over a field $F$ is the direct sum of its two subspaces $U$ and $W$, then

(1) $\dim(U + W) = \dim U + \dim W$
(2) $U \cap W = \{0\}$
(3) $\dim V = \dim U + \dim W$
(4) $\dim(U \cap W) = 0$

14. Let $n$ be a positive integer and let $H_n$ be the space of all $n \times n$ matrices $A = (a_{ij})$ with entries in $\mathbb{R}$ satisfying $a_{ij} = a_{rs}$ whenever $i + j = r + s$ for $i, j, r, s = 1, , 2, ..., n$, then the dimension of $H_n$ as a vector space over $\mathbb{R}$ is, [D-2011]

(1) $n^2$
(2) $n^2 - n + 1$
(3) $2n + 1$
(4) $2n - 1$