A Clan Seeking the Unknown Horizons of Mathematics

Infinite Series: Level 3 - Tutorial Questions

1. The sum of the series $\sum_{n=1}^{\infty}\left[(n+1)^{\frac{1}{5}}-n^{\frac{1}{5}}\right]$ is
   (a) less than -1
   (b) equal to -1
   (c) greater than -1 but less than 2
   (d) none of the above

2. $\lim_{n\to\infty}\sum_{k=0}^{n}\frac{2k}{k^2+n^2}=$
   (a) 0
   (b) $\log 2$
   (c) 2
   (d) ∞

3. Which of the following series are convergent?
   (a) $\sum_{n=0}^{\infty}\frac{\log n}{n^{\frac{3}{2}}}$
   (b) $\sum_{n=0}^{\infty}\frac{n^2}{n^{n!}}$
   (c) $\sum_{n=0}^{\infty}\frac{1}{n\log n}$
   (d) $\sum_{n=0}^{\infty}\frac{e^n}{n^{100}}$

4. Let $x_n \in \mathbb{R}$ such that $\sum_{n=1}^{\infty}x_n=-5$. Then
   (a) $\lim_{n\to\infty}x_n=0$
   (b) there exists an $m\in\mathbb{N}$ such that $x_n\leq 0$ for all $n>m$
   (c) $\sum_{n=1}^{\infty}|x_n|=5$
   (d) $|x_n|\leq 5$ for all $n\in\mathbb{N}$

5. Which of the following series are convergent?
   (a) $\sum_{n=1}^{\infty}\frac{(-1)^n+\frac{1}{2}}{n}$
   (b) $\sum_{n=1}^{\infty}e^{-n}n^2$

(c) $\sum_{n=1}^{\infty} \frac{1+2+\cdots+n}{1^2+2^2+\cdots+n^2}$

(d) $\sum_{n=1}^{\infty} \frac{1.2.3}{4.5.6} + \frac{7.8.9}{10.11.12} + \ldots$

6. The largest interval I such that the series $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$ converges whenever $x \in I$ is equal to

   (a) $[-1,1]$
   (b) $[-1.1)$
   (c) $(-1,1]$
   (d) $(-1,1)$

7. Let $\sum a_n$ be a convergent series. Let $b_n = a_{n+1} - a_n$ for all $n \in \mathbb{N}$. Then

   (a) $\sum b_n$ should also be convergent and $(b_n) \to 0$ as $n \to \infty$
   (b) $\sum b_n$ need not be convergent and $(b_n) \to 0$ as $n \to \infty$
   (c) $\sum b_n$ is convergent but (b_n) need not tend to zero as $n \to \infty$
   (d) none of the above statement is true

8. Consider the real sequences (a_n) and (b_n) such that $\sum a_n b_n$ converges. Which of the following statements is true?

   (a) If $\sum a_n$ converges, then (b_n) is bounded
   (b) If $\sum b_n$ converges, then (a_n) is bounded
   (c) If (a_n) is bounded, then (b_n) converges
   (d) If (a_n) is unbounded, then (b_n) bounded

9. Which of the following series converge?

   (a) $\sum_{n=1}^{\infty} \left(\frac{\log n}{n^{1+2\epsilon}}\right)$

   (b) $\sum_{n=1}^{\infty} \left(\frac{(\log n)^2}{n^{1+2\epsilon}}\right)$

   (c) $\sum_{n=1}^{\infty} \left(\frac{n^2+1}{n^3+n}\right)$

   (d) $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n$

10. The set of all values of a for which the series $\sum_{n=1}^{\infty} \frac{a^n}{n!}$ converges is

   (a) $(0,\infty)$
   (b) $(-\infty,0]$
   (c) $(-\infty,\infty)$
   (d) $(-1,1)$

11. Consider the following statements.

   S_1: $\sum_{n=3}^{\infty} \frac{1}{(\log\log n)^{\log n}}$ is a convergent series

   S_2: $\sum_{n=3}^{\infty} \frac{1}{n^{\log\log\log n}}$ is a convergent series.

   Which of the following statements are true?

   (a) S_1 and S_2 are true

(b) S_1 is true but not S_2
(c) S_2 is true but not S_1
(d) Neither S_1 nor S_2 is true

12. Let $\sum_{n=0}^{\infty} a_n$ be a divergent series of positive terms. Then it follows that

(a) $\sum_{n=0}^{\infty} a_n^2$ is also divergent
(b) the sequence (a_n) does not converge to 0
(c) the sequence (a_n) is not bounded
(d) $\sum_{n=0}^{\infty} \sqrt{a_n}$ is also divergent

13. The series $\sum_{n=1}^{\infty} \frac{n+1}{n^p}$ is convergent for

(a) $0 < p < 1$
(b) $1 < p < 2$
(c) $p = 2$
(d) $p > 2$

14. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ is

(a) convergent
(b) divergent
(c) conditionally convergent
(d) absolutely convergent

15. The series $\frac{2}{1^2} + \frac{3}{2^2} + \frac{4}{3^2} + \frac{5}{4^2} + \frac{6}{5^2} + \dots$ is

(a) conditionally convergent
(b) absolutely convergent
(c) absolutely divergent
(d) none of these

16. Consider the statements
(i) The series $\sum \sin \frac{1}{n}$ is convergent
(ii) The series $\frac{1.2}{3^2.4^2} + \frac{3.4}{5^6.6^2} + \frac{5.6}{7^2.8^2} + \dots$ is convergent
Then

(a) both the statements (i) and (ii) are true
(b) (i) is true and (b) is false
(c) (i) is false and (b) is true
(d) neither (i) nor (ii) is true

17. If $u_n = \sqrt{n+1} - \sqrt{n}$ and $v_n = \sqrt{n^4+1} - n^2$, then

(a) $\sum_{n=1}^{\infty} u_n$ converges but $\sum_{n=1}^{\infty} v_n$ diverges
(b) $\sum_{n=1}^{\infty} u_n$ diverges but $\sum_{n=1}^{\infty} v_n$ converges
(c) $\sum_{n=1}^{\infty} u_n$ and $\sum_{n=1}^{\infty} v_n$ both converges

(d) $\sum_{n=1}^{\infty} u_n$ and $\sum_{n=1}^{\infty} v_n$ both diverges

18. For the value of x the infinite series $\sum \frac{\left(1+\frac{1}{n}\right)^{n^2}}{x^n}$ converges?

   (a) $x < e$
   (b) $x > e$
   (c) $x = e$
   (d) $x = 1$

19. Let $a_n = \sin \frac{1}{n^2}$, $n = 1, 2, \ldots$, then

   (a) $\lim_{n\to\infty} a_n = 1$
   (b) $\sum_{n=1}^{\infty} a_n$ converges
   (c) $\lim_{n\to\infty} \sup a_n \neq \lim_{n\to\infty} \inf a_n$
   (d) $\sum_{n=1}^{\infty} a_n$ diverges

20. Which of the following is a convergent series?

   (a) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}-\sqrt{n}}$
   (b) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$
   (c) $\sum_{n=1}^{\infty} (-1)^n \log n$
   (d) $\sum_{n=1}^{\infty} \frac{\log n}{n}$