TUTORIAL QUESTIONS

Matrices, Determinants and Special Matrices

1. E, F and G are three 3×3 matrices such that $EF = G$, where $E = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

   $G = \begin{bmatrix} \cos 0 & -\sin 0 & 0 \\ \sin 0 & \cos 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then $F =$

   (1) $\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

   (2) $\begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

   (3) $\begin{bmatrix} \sin x & -\cos x & 0 \\ \cos x & \sin x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

   (4) $\begin{bmatrix} \sin x & \cos x & 0 \\ -\cos x & \sin x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2. Let A be a 3×3 and x be 3×1 real matrices. Consider the system of equations: $Ax = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, then

   (1) if the system is consistent, then it has a unique solution
   (2) if A is singular, then the system has infinitely many solutions
   (3) if the system is consistent, then $|A|$ is non-zero
   (4) if the system has a unique solution, then A is non-singular

3. Let P be an n $\times$ n idempotent matrix, that is $P^2 = P$. Which of the following is/are false?

   (1) P^t is idempotent
   (2) The possible eigen values of P can be zero
   (3) The non-diagonal entries of P can be zero
   (4) There are infinite number of n $\times$ n non- singular matrices that are idempotent

4. Let $A = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$. Then

   (1) A is idempotent
   (2) A is orthogonal
   (3) $A^{-1} = A^t$
   (4) $A^6 = I$

5. The columns of an orthogonal matrix form

   (1) an orthogonal set of vectors
   (2) an orthonormal set of vectors
   (3) a linearly independent set
   (4) all the above

6. If $A = \begin{bmatrix} -5 & -8 & 0 \\ 3 & 5 & 0 \\ 1 & 2 & -1 \end{bmatrix}$, then A^2 is:

   (1) idempotent

(2) nilpotent
(3) involutary
(4) periodic

7. Let a, b, c, d and e be real and non-zero. For the given matrix $A = \begin{bmatrix} a & 0 & b \\ 0 & c & 0 \\ d & 0 & e \end{bmatrix}$, which of the following is/are true?

(1) A is non-singular for all distinct a, b, c, d and e
(2) the minor of every element of A is equal to the cofactor of the corresponding element
(3) the number of non-zero elements of the matrix $\frac{(A-A^T)}{2}$ is 5
(4) The matrix $\frac{(A-A^T)}{2}$ is symmetric

8. If A, B and $(A+B)$ are n×n non-singular real matrices, then $[B(A+B)^{-1}A]^{-1} =$

(1) $A+B$
(2) $A^{-1}+B^{-1}$
(3) $A^{-1}+B^{-1}+I$
(4) AB

9. Let $A_{n\times n} = (a_ij),\ n \geq 3$,where $a_{ij} = (b_i^2 - b_j^2),\ i, j = 1, 2, \ldots, n$ for some distinct real numbers $b_1, b_2, \ldots, b_n$. Then $det(A)$ is: [D-2013]

(1) $\prod_{i<j}(b_i - b_j)$
(2) $\prod_{i<j}(b_i + b_j)$
(3) 0
(4) 1

10. Let $f_1(x),\ f_2(x), g_1(x),\ g_2(x)$ be differentiable functions on R. Let $F(x) = \begin{vmatrix} f_1(x) & f_2(x) \\ g_1(x) & g_2(x) \end{vmatrix}$ be the determinant if the matrix $\begin{bmatrix} f_1(x) & f_2(x) \\ g_1(x) & g_2(x) \end{bmatrix}$. Then $F'(x)$ is equal to

(1) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2'(x) & g_2(x) \end{vmatrix}$
(2) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2(x) & g_2'(x) \end{vmatrix}$
(3) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} - \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2(x) & g_2'(x) \end{vmatrix}$
(4) $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1'(x) & g_2'(x) \end{vmatrix}$