Problem 7.1:
A Boolean function ϕ is defined using the following sum of minterms:

$$\phi(A, B, C, D, E) = m_0 + m_2 + m_4 + m_6 + m_9 + m_{10} + m_{13} + m_{14} + m_{15} + m_{16} + m_{17} + m_{21} + m_{26} + m_{28} + m_{30} + m_{31}$$

a) Calculate the prime implicants of φ.

Minterm	Pattern	Used	Minterm	Pattern	Used	Minterm	Pattern	Used
m_0	00000	✓	$m_{0,2}$	000-0	✓	$m_{0,2,4,6}$	00--0	
			$m_{0,4}$	00-00	✓			
			$m_{0,16}$	-0000				
m_2	00010	✓	$m_{2,6}$	00-10	✓	$m_{2,6,10,14}$	0--10	
m_4	00100	✓	$m_{2,10}$	0-010	✓			
m_{16}	10000	✓	$m_{4,6}$	001-0	✓			
			$m_{16,17}$	1000-				
m_6	00110	✓	$m_{6,14}$	0-110	✓			
m_9	01001	✓	$m_{9,13}$	01-01				
m_{10}	01010	✓	$m_{10,26}$	-1010	✓	$m_{10,26,14,30}$	-1-10	
m_{17}	10001	✓	$m_{10,14}$	01-10	✓			
			$m_{17,21}$	10-01				
m_{13}	01101	✓	$m_{13,15}$	011-1				
m_{14}	01110	✓	$m_{14,15}$	0111-	✓	$m_{14,15,30,31}$	-111-	
m_{21}	10101	✓	$m_{14,30}$	-1110	✓			
m_{26}	11010	✓	$m_{26,30}$	11-10	✓			
m_{28}	11100	✓	$m_{28,30}$	111-0				
m_{15}	01111	✓	$m_{15,31}$	-1111	✓			
m_{30}	11110	✓	$m_{30,31}$	1111-	✓			
m_{31}	11111	✓						

This gives us the prime implicants :

$m_{0,2,4,6}$ $= (\neg A \wedge \neg B \wedge \neg E)$
$m_{0,16}$ $= (\neg B \wedge \neg C \wedge \neg D \wedge \neg E)$
$m_{2,6,10,14}$ $= (\neg A \wedge D \wedge \neg E)$
$m_{16,17}$ $= (A \wedge \neg B \wedge \neg C \wedge \neg D)$
$m_{9,13}$ $= (\neg A \wedge B \wedge \neg D \wedge E)$
$m_{10,26,14,30}$ $= (B \wedge D \wedge \neg E)$
$m_{17,21}$ $= (A \wedge \neg B \wedge \neg D \wedge E)$
$m_{13,15}$ $= (\neg A \wedge B \wedge C \wedge E)$
$m_{14,15,30,31}$ $= (B \wedge C \wedge D)$
$m_{28,30}$ $= (A \wedge B \wedge C \wedge \neg E)$

b) Construct the prime implicant chart and identify the essential prime implicants.

	m_0	m_2	m_4	m_6	m_9	m_{10}	m_{13}	m_{14}	m_{15}	m_{16}	m_{17}	m_{21}	m_{26}	m_{28}	m_{30}	m_{31}
$m_{0,2,4,6}$	✓	✓	✓	✓												
$m_{0,16}$	✓									✓						
$m_{2,6,10,14}$		✓		✓		✓		✓								
$m_{16,17}$										✓	✓					
$m_{9,13}$					✓		✓									
$m_{10,26,14,30}$						✓		✓					✓		✓	
$m_{17,21}$											✓	✓				
$m_{13,15}$							✓		✓							
$m_{14,15,30,31}$								✓	✓						✓	✓
$m_{28,30}$														✓	✓	

Essential prime implicants:

$m_{0,2,4,6} = (\neg A \wedge \neg B \wedge \neg E)$
$m_{16,17} = (A \wedge \neg B \wedge \neg C \wedge \neg D)$
$m_{9,13} = (\neg A \wedge B \wedge \neg D \wedge E)$
$m_{10,26,14,30} = (B \wedge D \wedge \neg E)$
$m_{17,21} = (A \wedge \neg B \wedge \neg D \wedge E)$
$m_{14,15,30,31} = (B \wedge C \wedge D)$
$m_{28,30} = (A \wedge B \wedge C \wedge \neg E)$

c) Write out the minimal boolean expressions defining ф.

$\phi(A, B, C, D, E) = (\neg A \wedge \neg B \wedge \neg E) \vee (A \wedge \neg B \wedge \neg C \wedge \neg D) \vee (\neg A \wedge B \wedge \neg D \wedge E) \vee (B \wedge D \wedge \neg E) \vee$
$(A \wedge \neg B \wedge \neg D \wedge E) \vee (B \wedge C \wedge D) \vee (A \wedge B \wedge C \wedge \neg E)$