Sheet 8

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Problem 8

A full adder digital circuit was introduced in class.It is defined by the following two Boolean functions:

$$S = A \oplus B \oplus C_{in}$$
$$C_{out} = (A \land B) \lor (C_{in} \land (A \oplus B))$$

Making the truth table :

A	B	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

a) Write both functions as a (short) disjunction of product terms.

Here,

We know that,

$$\begin{aligned} A \oplus B &= \neg(A \land B) \land (A \lor B) \\ &= (\neg A \lor \neg B) \land (A \lor B) \\ &= (A \land \neg B) \lor (\neg A \land B) \end{aligned}$$

$S = A \oplus B \oplus C_{in}$

We can also write it as :

$$\begin{aligned} S &= A \oplus B \oplus C_{in} \\ &= ((A \land \neg B) \lor (\neg A \land B)) \oplus C_{in} \\ &= (\neg((A \land \neg B) \lor (\neg A \land B)) \land C_{in}) \lor (((A \land \neg B) \lor (\neg A \land B)) \land \neg C_{in}) \\ &= ((\neg(A \land \neg B) \land \neg(\neg A \land B)) \land C_{in}) \lor (((A \land \neg B) \lor (\neg A \land B)) \land \neg C_{in}) \\ &= (((\neg A \lor B) \land (A \lor \neg B)) \land C_{in}) \lor (((A \land \neg B) \lor (\neg A \land B)) \land \neg C_{in}) \\ &= (((\neg A \land (A \lor \neg B)) \lor (B \land (A \lor \neg B))) \land C_{in}) \lor (((A \land \neg B) \lor (\neg A \land B)) \land \neg C_{in}) \\ &= (((\neg A \land \neg B) \lor (B \land A)) \land C_{in}) \lor (((A \land \neg B) \lor (\neg A \land B)) \land \neg C_{in}) \\ &= (\neg A \land \neg B \land C_{in}) \lor (A \land B \land C_{in}) \lor (A \land \neg B \land \neg C_{in}) \lor (\neg A \land B \land \neg C_{in}) \end{aligned}$$

For the Second term :

$$
\begin{aligned}
C_{out} &= (A \wedge B) \vee (C_{in} \wedge (A \oplus B)) \\
&= (A \wedge B) \vee (C_{in} \wedge ((A \wedge \neg B) \vee (\neg A \wedge B))) \\
&= (A \wedge B) \vee ((C_{in} \wedge (A \wedge \neg B)) \vee (C_{in} \wedge (\neg A \wedge B))) \\
&= (A \wedge B) \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B)) \\
&= (A \wedge B) \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B)) \\
&= (A \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B))) \wedge (B \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B))) \\
&= (A \vee (C_{in} \wedge A \wedge \neg B) \vee A \vee (C_{in} \wedge \neg A \wedge B)) \wedge (B \vee (C_{in} \wedge A \wedge \neg B) \vee B \vee (C_{in} \wedge \neg A \wedge B)) \\
&= (A \vee A \vee (C_{in} \wedge \neg A \wedge B)) \wedge (B \vee B \vee (C_{in} \wedge A \wedge \neg B)) \\
&= (A \vee (C_{in} \wedge \neg A \wedge B)) \wedge (B \vee (C_{in} \wedge A \wedge \neg B)) \\
&= ((A \vee C_{in}) \wedge (A \vee \neg A) \wedge (A \vee B)) \wedge ((B \vee C_{in}) \wedge (B \vee A) \wedge (B \vee \neg B)) \\
&= (A \vee C_{in}) \wedge (A \vee B) \wedge (B \vee C_{in}) \wedge (B \vee A) \\
&= (A \vee C_{in}) \wedge (A \vee B) \wedge (B \vee C_{in}) \\
&= (A \wedge B) \vee (C_{in} \wedge A) \vee (C_{in} \wedge B)
\end{aligned}
$$

b) Write both functions as a (short) conjunction of sum terms

Now writing the Boolean functions in CNF Form :

We can reverse the DNF boolean opearators ($\wedge$ *to* $\vee$) and ($\vee$ *to* $\wedge$) to get CNF :

Or we can construct CNF looking at the table. Either way we get the following result.

$$
\begin{aligned}
S &= A \oplus B \oplus C_{in} \\
&= (\neg A \vee \neg B \vee C_{in}) \wedge (A \vee B \vee C_{in}) \wedge (A \vee \neg B \vee \neg C_{in}) \wedge (\neg A \vee B \vee \neg C_{in})
\end{aligned}
$$

$$
\begin{aligned}
C_{out} &= (A \wedge B) \vee (C_{in} \wedge (A \oplus B)) \\
&= (A \wedge B) \vee (C_{in} \wedge ((A \wedge \neg B) \vee (\neg A \wedge B))) \\
&= (A \wedge B) \vee ((C_{in} \wedge (A \wedge \neg B)) \vee (C_{in} \wedge (\neg A \wedge B))) \\
&= (A \wedge B) \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B)) \\
&= (A \wedge B) \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B)) \\
&= (A \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B))) \wedge (B \vee ((C_{in} \wedge A \wedge \neg B) \vee (C_{in} \wedge \neg A \wedge B))) \\
&= (A \vee (C_{in} \wedge A \wedge \neg B) \vee A \vee (C_{in} \wedge \neg A \wedge B)) \wedge (B \vee (C_{in} \wedge A \wedge \neg B) \vee B \vee (C_{in} \wedge \neg A \wedge B)) \\
&= (A \vee A \vee (C_{in} \wedge \neg A \wedge B)) \wedge (B \vee B \vee (C_{in} \wedge A \wedge \neg B)) \\
&= (A \vee (C_{in} \wedge \neg A \wedge B)) \wedge (B \vee (C_{in} \wedge A \wedge \neg B)) \\
&= ((A \vee C_{in}) \wedge (A \vee \neg A) \wedge (A \vee B)) \wedge ((B \vee C_{in}) \wedge (B \vee A) \wedge (B \vee \neg B)) \\
&= (A \vee C_{in}) \wedge (A \vee B) \wedge (B \vee C_{in}) \wedge (B \vee A) \\
&= (A \vee C_{in}) \wedge (A \vee B) \wedge (B \vee C_{in})
\end{aligned}
$$

c) Write both functions using only not ($\neg$) and not-and ($\uparrow$) operations.

We take the DNF of ther functions :

So, now

For term $S = A \oplus B \oplus C_{in}$

$$\begin{aligned} S &= A \oplus B \oplus C_{in} \\ &= (\neg A \wedge \neg B \wedge C_{in}) \vee (A \wedge B \wedge C_{in}) \vee (A \wedge \neg B \wedge \neg C_{in}) \vee (\neg A \wedge B \wedge \neg C_{in}) \\ &= \neg\neg((\neg A \wedge \neg B \wedge C_{in}) \vee (A \wedge B \wedge C_{in}) \vee (A \wedge \neg B \wedge \neg C_{in}) \vee (\neg A \wedge B \wedge \neg C_{in})) \\ &= \neg(\neg(\neg A \wedge \neg B \wedge C_{in}) \wedge \neg(A \wedge B \wedge C_{in}) \wedge \neg(A \wedge \neg B \wedge \neg C_{in}) \wedge \neg(\neg A \wedge B \wedge \neg C_{in})) \\ &= (\neg(\neg A \wedge \neg B \wedge C_{in}) \uparrow \neg(A \wedge B \wedge C_{in}) \uparrow \neg(A \wedge \neg B \wedge \neg C_{in}) \uparrow \neg(\neg A \wedge B \wedge \neg C_{in})) \\ &= (\neg A \uparrow \neg B \uparrow C_{in}) \uparrow (A \uparrow B \uparrow C_{in}) \uparrow (A \uparrow \neg B \uparrow \neg C_{in}) \uparrow (\neg A \uparrow B \uparrow \neg C_{in}) \end{aligned}$$

For the Second term:

$$\begin{aligned} C_{out} &= (A \wedge B) \vee (C_{in} \wedge (A \oplus B)) \\ &= (A \wedge B) \vee (C_{in} \wedge A) \vee (C_{in} \wedge B) \\ &= \neg\neg((A \wedge B) \vee (C_{in} \wedge A) \vee (C_{in} \wedge B)) \\ &= \neg(\neg(A \wedge B) \wedge \neg(C_{in} \wedge A) \wedge \neg(C_{in} \wedge B)) \\ &= \neg((A \uparrow B) \wedge (C_{in} \uparrow A) \wedge (C_{in} \uparrow B)) \\ &= ((A \uparrow B) \uparrow (C_{in} \uparrow A) \uparrow (C_{in} \uparrow B)) \\ &= (A \uparrow B) \uparrow (C_{in} \uparrow A) \uparrow (C_{in} \uparrow B) \end{aligned}$$

d) In a digital circuit, we can easily reuse common terms. Draw a small digital circuit implementing S and C_{out} using NAND gates only.

- S=A$\oplus B \oplus C_{in}$

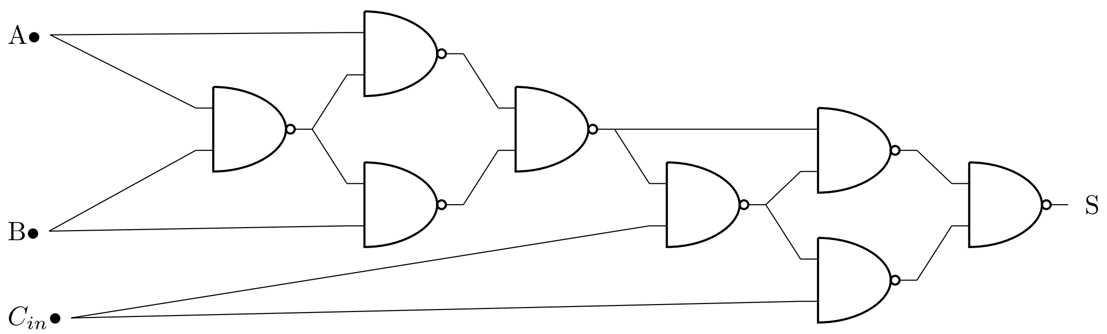

Fig 1

- $C_{out} = (A \wedge B) \vee (C_{in} \wedge (A \oplus B))$

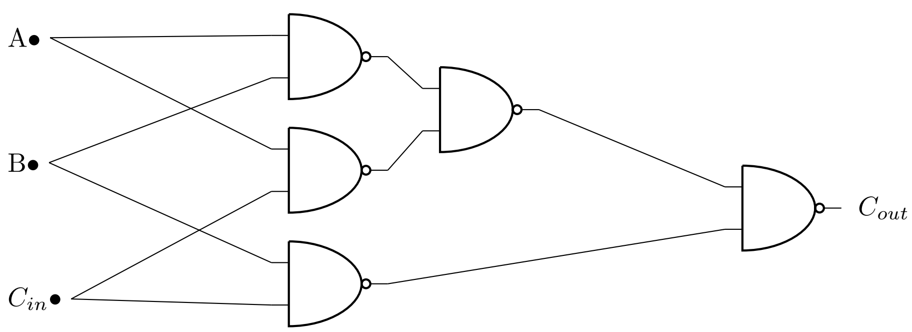

Fig 2

Note : Here in the Fig 2 any input originating from A B C do not intersect with the inputs originating from any other different inputs.(ie:there is no intersection of two different inputs).