Homework - Introduction to computer Science.
Digdarshan Kunwar
Sheet - 4

Assignment - 4
4.1
Given
p is a prefix of $w \in \Sigma^*$ if there is a word $q \in \Sigma^*$ s.th $w = pq$.
p is a proper prefix of w if $p \neq w$.

a) $\preceq \subseteq \Sigma^* \times \Sigma^*$
$p \preceq w$ for $p, w \in \Sigma^*$ and p is a prefix of w.

we have to show $\preceq$ is a partial order:

Answer:

Reflexivity
The relation is reflexive a $\forall p \in \Sigma^*. \ (p, p) \in \preceq$

Symmetric property / Antisymmetric Property.
for every $p, w \in \Sigma^*$
$(p, w) \in \preceq$ and $(w, p) \in \preceq \Rightarrow p = w$
So, as p is not a proper prefix of w so p can be equal to w. Therefore it is anti-symmetric.

Transitive properperty.
Here, If
$\Sigma^*. (p, q) \in \preceq \ \wedge \ \Sigma^*. (q, w) \in \preceq$
Then,
$\Sigma^*. (p, w) \in \preceq$.
Therefore the relation is transitive.

Therefore,
we find that the relation is reflexive, anti-symmetric and transitive, So we can say that relation $\preceq$ is a partial order.

b.) Here, $\prec \subset \Sigma^* \times \Sigma^*$
where, $p \prec w$ for $p, w \in \Sigma^*$ and p is a proper prefix of w.
we have to show that $\prec$ is a strictly partial order.
Answer:

Reflexive property.
Here,
for $\forall a \notin \prec$
Therefore it is not reflexive; ie It is irreflexive.

Symmetric property.
We have,
for $\forall p, w \in \Sigma^*$. $(p, w) \in \prec$
But here, $p \neq w$ since $p \neq w$, we can say.
~~So,~~ $(p, w) \neq (w, p)$.
So, $(x, y) \notin \prec$
Therefore it is asymmetric..

Transitive property.
As $(p, q) \in \Sigma^* \in \prec$ and $(q, w) \in \Sigma^* \in \prec$
we can claim that
$(p, w) \in \Sigma^* \in \prec$

So, the relation is transitive.

As they have irreflexivity, asymmetry and transitive relation, we can claim that $\prec$ is a strict partial order.

c. So,

from 4.1 a

$\preceq$ is total as it is a partial order relation.

$(p, w) \in \preceq$ and $(w, p) \in \preceq$

from 4.1 b

$\prec$ is not a total as it is strict partial order relation

$(p, w) \in \prec$ and $(p, w) \notin \prec$

Also, it is not reflexive

.

.

Problem 4.2

Here,

$f: A \mapsto B \wedge g: B \mapsto C$

a) To prove: Given that $g \circ f$ is bijective, the f is injective and g is surjective.

So, we have each element of set A mapped to exactly one element of set C

So, let $a \in A$, $b \in B$ and $c \in C$.

Then, there exists $a \in A$ and with $g \circ f(a) = c$

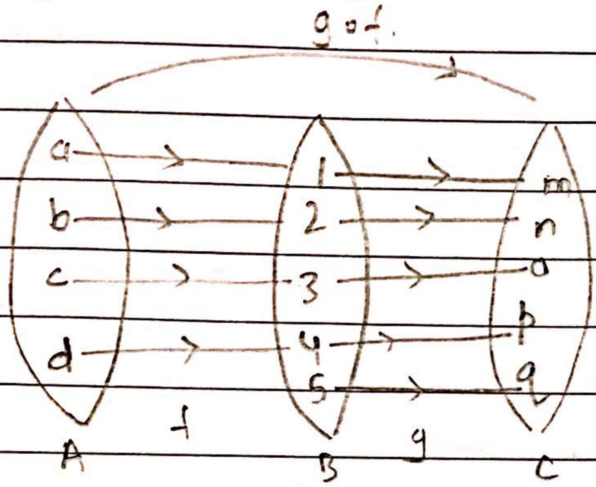
and $g(f(a)) = c$

Then if we take $b \in f(a) \in B$, $g(b) = c$
As we have $c \in C$ is an image of g. c must be mapped by atleast one of the element of B. So, this implies that g must be surgjective
$\therefore$ g is surjective.

Now for function f.
To have an injective property the function f should ~~have~~ map atmost one element of B.
Condition considiration.
Given the image of $g \circ f(x) = g \circ f(y)$ and $x \neq y$ then $g \circ f$ is not bijective. Thus every image of f should have a unique image, leading to unique image in $g \circ f$.
Thus the function f is injective.

b) Here,
If.
$f: A\{a, b, c, d\} \longmapsto B\{1, 2, 3, 4, 5\}$.
f is injective
$g: B\{1, 2, 3, 4, 5\} \longmapsto C\{m, n, o, p, q\}$.
g is surjective.
So,
Now,
$g \circ f: A\{a, b, c, d\} \longmapsto C\{m, n, o, p, q\}$
Here,
$g \circ f$ is not bijective because not every element of C gets mapped to. Also, $g \circ f$ is not surjective. Since to be bijective a function needs

to be injective and surjective. gof, here is not surjective ∴ gof is not bijective.



c) Here,

f: A{a, b, c} $\longmapsto$ B{4,1, 2, 3}

Function f is not ~~injective~~ surjective.

g: B{4,1, 2, 3} $\longmapsto$ C{m, n, o}.

Function g is not injective

So, Now,

gof : A{a, b, c} $\longmapsto$ C{m, n, o}.

Here,

gof is bijective because every element of C gets mapped to. Also, for every gof the injectivity and surjective property holds true.

Thus gof is bijective.