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ICS 2018 Problem Sheet #2

Problem 2.1:

Proof by contrapositive (2 points) Let n be a natural number. If n is not divisible by 3, then n is also not divisible by 15.

Theorem: Let n be the natural number. If n is not divisible by 3 then n is also not divisible by 15.

Proof:

We prove Contrapositive.
If n is divisible by 15 then it is also divisible by 3.

Here,

If n is divisible by 15 then,
We have, $k = \frac{n}{15}$ where k is an Integer.

or, $15 \cdot k = n$ ··········· (1)

Now from (1)
We have

$$n = 15 \cdot k$$
$$n = 3 \cdot 5 \cdot k$$
$$n = 3 \cdot (5k) \qquad \cdots\cdots\cdots\cdots \quad (2)$$

Here we find 3 is a factor of number n so,
We have $\frac{n}{3} = 5k$
Therefore, it shows that n is divisible by 3 .

We proved that “If n is divisible by 15 then it is also divisible by 3” and thus its contrapositive “If n is not divisible by 3, then n is also not divisible by 15.”

Problem 2.2:
Proof by induction
Let n be a natural number with n ≥ 1. Proof that the following holds:

$$1^2 + 3^2 + 5^2 + \cdots (2n-1)^2 = \sum_{k=1}^{n} (2k-1)^2 = \frac{2n(2n-1)(2n+1)}{6}$$

Here,
Induction Basis:
Check Condition (1)

S$_1 = \frac{2n(2n-1)(2n+1)}{6} = \frac{2(2-1)(2+1)}{6} = \frac{6}{6} = 1$

Induction Step:
Assuming that Sn $= \frac{2n(2n-1)(2n+1)}{6}$ is correct for a number m.
So we have,

Sm $= \frac{2m(2m-1)(2m+1)}{6}$ .... (1)
To show that it is right for (m+1) term

Induction Hypothesis:

By construction, we know that,

$S_{(m+1)} = S_m + (m+1)^{th}$ *term*

$= S_m + (2(m+1)-1)^2$

$= \frac{2m(2m-1)(2m+1)}{6} + (2(m+1)-1)^2$

$= \frac{2m((2m)^2-1^2)}{6} + \frac{6\cdot(2m+1)^2}{6}$

$= \frac{2m(4\cdot m^2-1^2)}{6} + \frac{6\cdot(2m+1)^2}{6}$

$= \frac{2m(4\cdot m^2-1)+6\cdot(4m^2+4m+1)}{6}$

$= \frac{8m^3-2m+24m^2+24m+6}{6}$

$= \frac{8m^3+24m^2+22m+6}{6}$

$= \frac{8m^3+24m^2+22m+6}{6}$

$= \frac{2(4m^3+12m^2+11m+3)}{6}$

$= \frac{2(m+1)(4m^2+8m+3)}{6}$

$= \frac{2(m+1)(4m^2+6m+2m+3)}{6}$

$= \frac{2(m+1)(2m+1)(2m+3)}{6}$

$S_{(m+1)} = \frac{2(m+1)(2m+1)(2m+3)}{6}$ = LHS proved

Conclusion:

By mathematical induction,

When S_1 is true $\Longrightarrow$ S_2 is true

S_2 is true $\Longrightarrow$ S_3 is true

………………………………

S_n is true $\Longrightarrow$ S_{n+1} is true