Digdarshan Kunwar
Computer Science

Sheet 5

Problem 5.1

Here,

$\alpha^2 = \alpha$

$\alpha + \alpha = \beta$

$\gamma^2 = \gamma$

$\gamma + \gamma = \gamma$

$\delta^2 = \beta\beta$

$\delta + \delta = \alpha\alpha$

Now, from the equations abhove we can devise that.

α is a product unit

so, $\alpha = 1$

γ is a sum unit

so, $\gamma = 0$

Now, from the abhove, we can say that.

$\beta = \alpha + \alpha = 2$

Now,

$\delta^2 = \beta\beta = 22$ — (1)

$\delta + \delta = \alpha\alpha = 11$ — (2)

Now converting ~~(22)~~ $(22)_b$ to decimal and supposing it as x_{10}

so,

$(x)_{10} = 2 \times b^1 + 2 \times b^0$

So, we have,

$$(x)_{10} = 2b+2 \quad \text{——} \quad (3)$$

Now, converting $(11)_b$ to decimal and supposing it as y

So,

$$(y)_{10} = 1b^1 + 1 \cdot b^0$$

&

$$(y)_{10} = 1b+1 = 2\delta$$

So,

$$\delta_b = \left(\frac{1b+1}{2}\right)_{10} \quad \text{——} \quad (4)$$

Now, equating (1) ~~and~~, (3) and (4)

$$\left(\frac{1b+1}{2}\right)^2 = 2b+2$$

$$(b+1)^2 = 4(2b+2)$$

or, $(b+1)^2 = 8b+8$

or, $b^2+2b+1-8b-8=0$

or, $b^2-6b-7=0$

or, $b^2-7b+b-7=0$

or $b(b-7)+1(b-7)=0$

or, $(b+1)(b-7)=0$

$\therefore$ $b=-1$ or $b=7$

Since there are no -ve bases so,

The base for the utopian people is 7.

Therefore decimal number 99 would be

$$\begin{array}{r|l}
7 & 99 \rightarrow 1 \\
7 & 14 \rightarrow 0 \\
7 & 2 \rightarrow 2 \\
 & 0
\end{array}$$

So, $(99)_{10}$ is $(201)_7$.

which is $(\beta \gamma \alpha)$ for the utopians.

Problem 5.2

a) $(-1)_{10} = (?)_5$

firstly representing $(1)_{10}$ in base 5
0001

The b-1 complement of $(1)_5$
4443

The b's complement of $(1)_5$

$$\begin{array}{r}
4443 \\
+ \quad 1 \\
\hline
4444
\end{array}$$

So,
$(-1)_{10} = (4444)_5$

$(-8)_{10} = (?)_5$

firstly representing $(8)_{10}$ in base 5

$$\begin{array}{r|l}
5 & 8 \rightarrow 3 \\
5 & 1 \rightarrow 1 \\
 & 0
\end{array}$$

$(8)_{10} = (0013)_5$

Now $(b-1)$ complement of $(0013)_5$
$(4431)_2$

b's complement of $(0013)_5$

$$\begin{array}{r} 4431 \\ +\quad 1 \\ \hline (4432)_5 \end{array}$$

Therefore $(-8)_{10} = (4432)_5$

(b) $(-8)_{10} + (-1)_{10}$

So,
$(4432)_5 + (4444)_5$

$$\begin{array}{r} 4432 \\ 4444 \\ \hline 4431 \end{array}$$

So,
$(-8)_{10} + (-1)_{10} = (4431)_5$

Converting $(4431)_5$ to decimal :

$(b-1)$ complemet is 0013
b's complement is 0014

Converting $(0014)_5$ to base 10 we have
$(0014)_5 = (1\times5+4\times5^0)_{10}$
$= (9)_{10}$

Here, as we are finding the complement of $(4431)_5$ and converting it to decimal we know the sign is -ve.

Therefore $(4431)_5 = (-9)_{10}$

the converted decimal is -9.

Problem 5.3

Here, we have. -273.15_{10}

Since the number is negative the sign bit at the first is 1.

So, dealing the calculations with 273.15_{10} for the rest of the part we have,

$(273.15)_{10} = (?)_2$

Considering the integer part of the decimal

$2 \,|\, 273 \rightarrow 1$
$2 \,|\, 136 \rightarrow 0$
$2 \,|\, 68 \rightarrow 0$
$2 \,|\, 34 \rightarrow 0$
$2 \,|\, 17 \rightarrow 1$
$2 \,|\, 8 \rightarrow 0$
$2 \,|\, 4 \rightarrow 0$
$2 \,|\, 2 \rightarrow 0$
$2 \,|\, 1 \rightarrow 1$
$\;\;\; 0$

So,

$(273)_{10} = (100010001)_2$

Now considering the part after the decimal

$0.15 \times 2 = 0.30$ 0

$0.30 \times 2 = 0.60$ 0

$0.60 \times 2 = 1.20$ 1

$0.20 \times 2 = 0.40$ 0

$0.40 \times 2 = 0.80$ 0

$0.80 \times 2 = 1.60$ 1

$0.60 \times 2 = 1.20$ 1

$0.20 \times 2 = 0.40$ 0

So,

$$0.15_{10} = (00\overline{1001})_2$$

So,

$$273.15_{10} = (100010001.00\overline{1001})_2$$

Now,

In exponent form $(1.00010001001\overline{1001}) * 2^{1000}$

$$(1.00010001001\overline{1001}) * 2^{1000}$$

That is

$$1.00010001001\overline{1001} \; E \; 8$$

The exponent is represented by $(8 + 127) = 135$ in a single precision floating system.

Converting

$2 \,|\, 135 \rightarrow 1$

$2 \,|\, 67 \rightarrow 1$

$2 \,|\, 33 \rightarrow 1$

$2 \,|\, 16 \rightarrow 0$

$2 \,|\, 8 \rightarrow 0$

$2 \,|\, 4 \rightarrow 0$

$2 \,|\, 2 \rightarrow 0$

$2 \,|\, 1 \rightarrow 1$

0

So, $(135)_{10} = (10000111)_2$

Now,

Sign bit is 1

Exponent is 10000111

Mantissa is 00010001001001100110011

So, Floating point representation.

let S = Sign

E = Exponent

M = Mantissa

S	E	M
1	10000111	00010001001001100110011

(b). Now, the decimal fraction from the answer.

from

$(100010001.00\overline{1001})_2$

$(100010001.00100110011001)_2$ is 23 bit.

So, converting the decimal part.

.001001100110011

$$= \frac{1}{2^{15}} + 1\times\frac{1}{2^3} + 1\times\frac{1}{2^6} + \frac{1}{2^7}\times 1 + 1\times\frac{1}{2^{10}} + 1\times\frac{1}{2^{11}} + 1\times\frac{1}{2^{14}}$$

$$= \frac{2457}{16384} + \frac{1}{2^{15}} = \frac{4915}{32768}$$

$= \cancel{0.1499633789}$ 0.14999389648

So, the number actually stored is $-\cancel{273.1499633789}$
-273.1499389648