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ICS 2018 Problem Sheet #3

Problem 3.1:
Distributive laws for sets (2+2 = 4 points)
Let A, B and C be sets. Proof that the following two distributive laws hold:

a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
Here
Let us consider an element x such that $x \in A \cup (B \cap C)$

If $x \in A \cup (B \cap C) \quad \rightarrow \quad x\ is\ either\ in\ A\ or\ (B \cap C)$

$x \in A\ or\ x \in (B\ and\ C)$
$x \in A\ or\ (x \in B\ and\ x \in C)$
$(x \in A)\ or\ (x \in B\ and\ x \in C)$
$(x \in A\ or\ x \in B)\ and\ (x \in A\ or\ x \in C)$
$(x \in A \cup B)\ and\ (x \in A \cup C)$
$x \in (A \cup B) \cap (A \cup C)$
$x \in (A \cup B) \cap (A \cup C)$

$So, if\ x \in A \cup (B \cap C)\ then$
$x \in (A \cup B) \cap (A \cup C)$

$\therefore A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Here
Let us consider an x element such that $x \in A \cap (B \cup C)$

If $x \in A \cap (B \cup C) \quad \rightarrow x\ is\ an\ element\ of\ A\ and\ (B \cup C)$

$(x \in A)\ and\ x \in (B \cup C)$
$(x \in A)\ and\ x \in (B \cup C)$
$(x \in A)\ and\ (x \in B\ or\ x \in C)$
$x \in (A\ and\ B)\ or\ x \in (A\ and\ C)$
$x \in (A \cap B)\ or\ x \in (A \cap C)$
$x \in (A \cap B) \cup (A \cap C)$

$So, if\ x \in A \cap (B \cup C)\ then$
$x \in (A \cap B) \cup (A \cap C)$

$\therefore A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Problem 3.2:
Reflexive, symmetric, transitive (1+1+1 = 3 points)
For each of the following relations, determine whether they are reflexive, symmetric, or transitive. Provide a reasoning.

a) $R = \{(a,b) | a, b \in \mathbb{Z} \wedge a \neq b\}$
(The numbers a and b are different.)

<u>**Reflexive Property**</u>
$\forall\, a \in \mathbb{Z}\,,\ a = a$
We have, a is equal to a. $a \neq a\ is\ False$

Thus, it is not reflexive.

<u>**Symmetric Property**</u>
$\forall (a, b) \in \mathbb{Z}\,,$
We have, $a \neq b\ and\ b \neq a$.
For Example:
$5 \neq 4\ \ and\ \ 4 \neq 5$ is True

Thus, it is symmetric.

<u>**Transitive Property**</u>
$\forall (a, b, c) \in \mathbb{Z}\,,$
We have, $a \neq b\ \wedge\ b \neq c$
In some cases, $a = c$
For Example:
$5 \neq 4\ ,4 \neq 5$
So, $5 \neq 5$ is False.

Thus, it is not transitive.

b) $R = \{(a,b) | a, b \in \mathbb{Z} \wedge |a - b| \leq 3\}$
(The absolute difference of the numbers a and b is less than or equal to 3.)

<u>**Reflexive Property**</u>
$\forall a,\ a \in \mathbb{Z}\,,$
We have, $|a - a| = 0$
$|a - a| = 0 \leq 3$

Thus, it is reflexive.

<u>**Symmetric Property**</u>
$\forall (a, b) \in \mathbb{Z}\,,$
We have, $|a - b| = |b - a|$
$|a - b| \leq 3 = |b - a| \leq 3$

Thus, it is symmetric.

<u>Transitive Property</u>
$\forall (a, b, c) \in \mathbb{Z}$,
We have,
$|a - b| \leq 3 \land |b - c| \leq 3$
$So\ |a - c| \leq 3$ May not be true
There may be conditions s.th $|a - c| > 3\ where, a = 6, b = 3\ and\ c = 1$

Thus, it is not transitive.

c) $R = \{(a, b) | a, b \in \mathbb{Z} \land (a\ mod\ 10) = (b\ mod\ 10)\}$
(The last digit of the decimal representation of the numbers a and b is the same.)

<u>Reflexive Property</u>
$\forall a,\ a \in \mathbb{Z}$,
We have,
$(a\ mod\ 10) = (a\ mod\ 10)$

Thus, it is reflexive.

<u>Symmetric Property</u>
$\forall (a, b) \in \mathbb{Z}$,
We have,
$(a\ mod\ 10) = (b\ mod\ 10)$
$\leftrightarrow (b\ mod\ 10) = (a\ mod\ 10)$
Thus, it is symmetric.

<u>Transitive Property</u>
$\forall (a, b, c) \in \mathbb{Z}$,
We have,
$(a\ mod\ 10) = (b\ mod\ 10) = (c\ mod\ 10)$
So, $(a\ mod\ 10) = (c\ mod\ 10)$

Thus, it is Transitive.