Devang Dimri

MS QT

20246721

Solutions of Assignment 2.

Q1a. By trapezoidal method calculate $\int_0^1 \frac{4}{1+x^2}dx$. You know the answer will be π.

```
 Enter the initial and final limits a and b :
0
1
 Enter the value of n :
1000
 value of integral is :    3.1415924869231242
```

This is in approximately in correspondence with theoretical value and gets better as we increase the number of steps

b. Choose dx =0.01d0, 0.001d0, 0.0001d0, 0.00001d0 (you need to use double precision real*8). Does the error go as (1/n^2), n is the number of intervals as mentioned in lectures? Do a log-log plot of error versus n, fit a straight line in log-log plot, and confirm for yourself that the slope is 2.

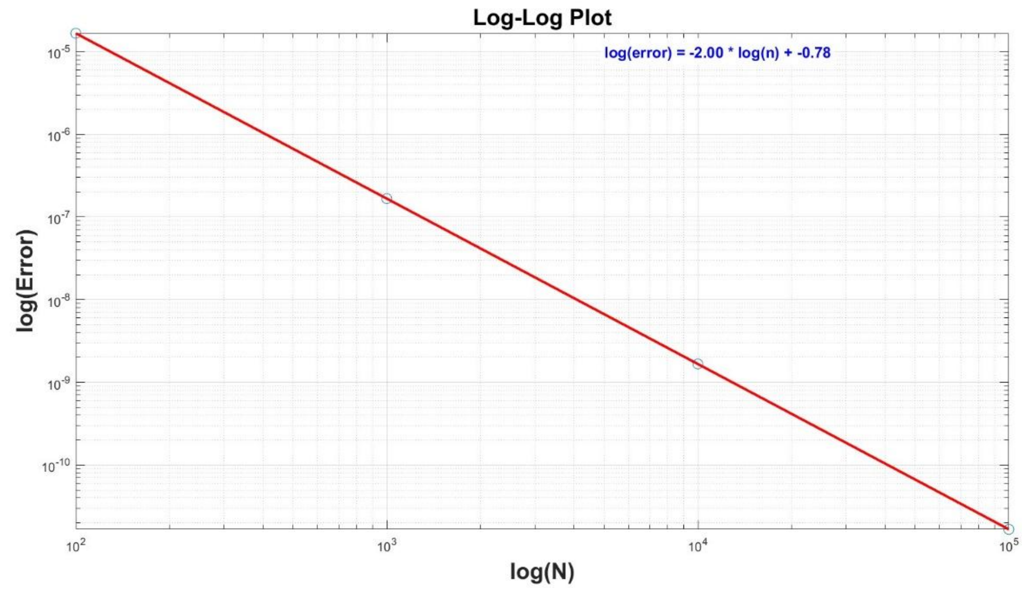
```
The value of integral for step size    1.0000000000000000E-002  is :    3.1415759869231290
error :   1.6666666664111318E-005
The value of integral for step size    1.0000000000000000E-003  is :    3.1415924869231242
error :   1.6666666891040904E-007
The value of integral for step size    1.0000000000000000E-004  is :    3.1415926519231401
error :   1.6666530378017796E-009
The value of integral for step size    1.0000000000000001E-005  is :    3.1415726533731516
error :   2.0000216641502533E-005
```

Here we have an error, for the step size = 10^{-5}, the error is not scaling as 10^{-2}

Upon manually putting the value we get the output

```
The value of integral for step size    1.0000000000000000E-002  is :    3.1415759869231290
error :   1.6666666664111318E-005
The value of integral for step size    1.0000000000000000E-003  is :    3.1415924869231242
error :   1.6666666891040904E-007
The value of integral for step size    1.0000000000000000E-004  is :    3.1415926519231401
error :   1.6666530378017796E-009
The value of integral for step size    1.0000000000000001E-005  is :    3.1415926535731526
error :   1.6640466782291696E-011
```

Loglog Plot for the same:

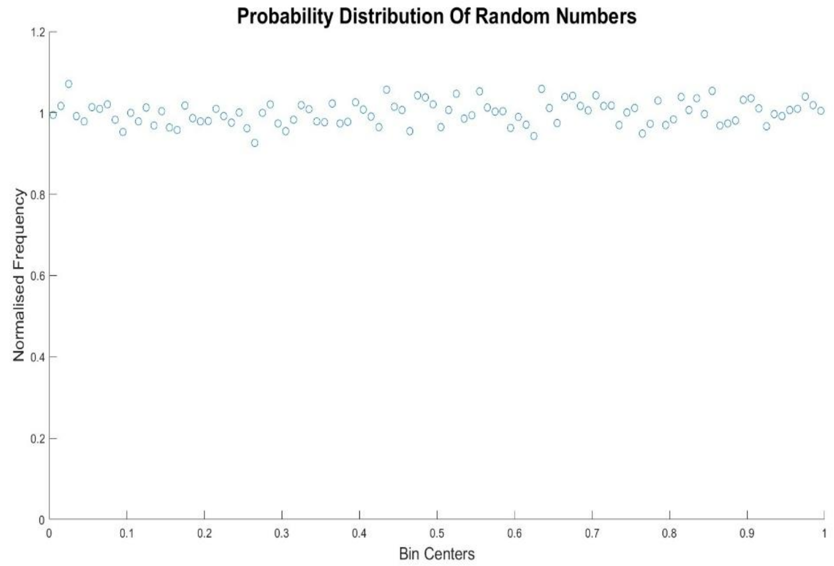


d. Now take a normalized Gaussian function $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ with standard deviation (SD=1). Integrate between -3 and +3. What value do you expect to get? Does your answer match?

```
value of integral for    600.00000000000000       bins is :    0.99729998234653661
error :    2.7000176534633935E-003
value of integral for    6000.0000000000000       bins is :    0.99730020172081357
error :    2.6997982791864272E-003
value of integral for    60000.000000000000       bins is :    0.99730020391457619
error :    2.6997960854238112E-003
value of integral for    600000.00000000000       bins is :    0.99730020393652541
error :    2.6997960634745910E-003
```

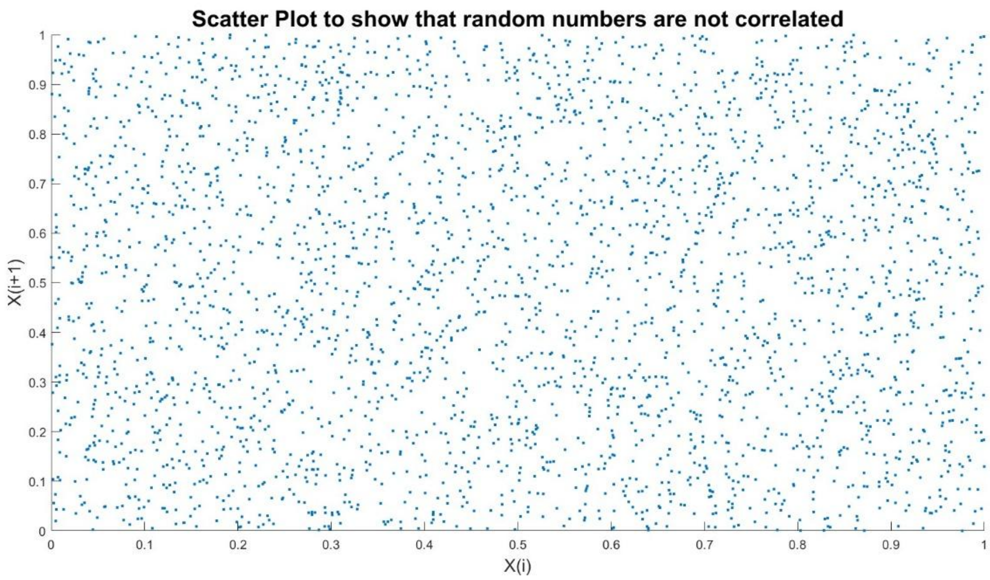
Q2. Use a random number generator which generates uniform random numbers between 0 and 1.

a. Plot probability distribution data to prove that you have random numbers with uniform deviate.

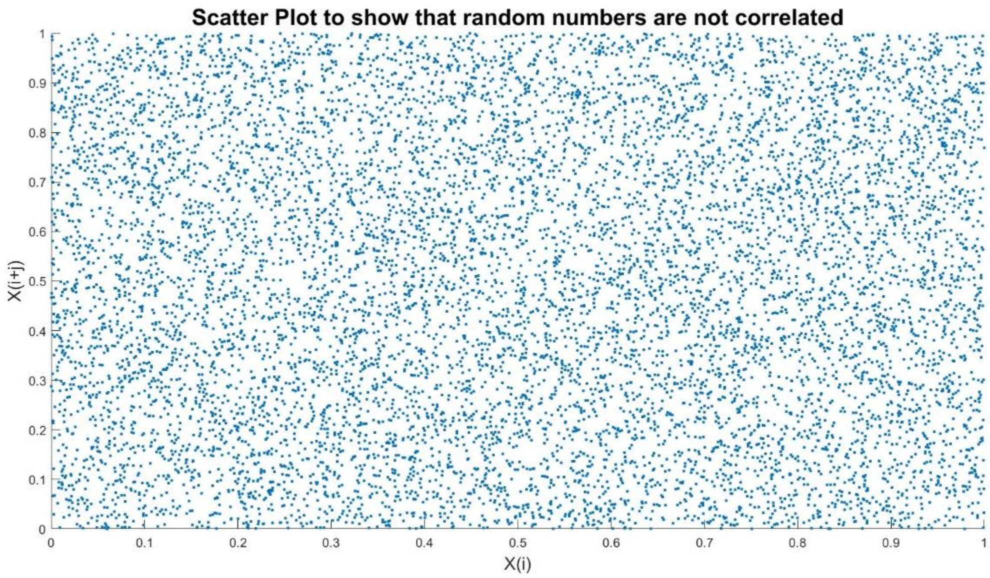


b. Do a scatter plot to show that the random numbers are uncorrelated.

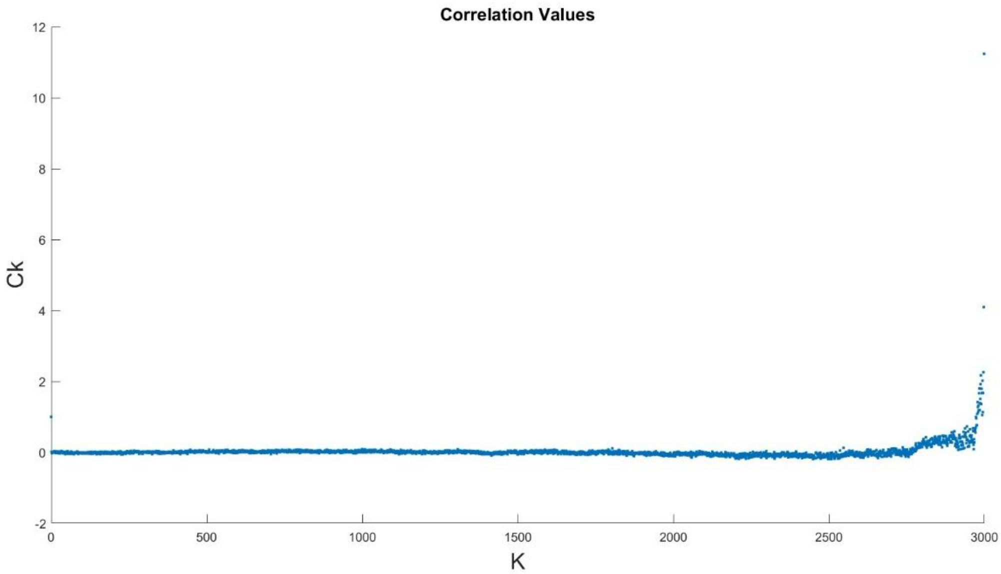
For 3000 Random Numbers



For 10000 Random Numbers



c. Calculate the correlation function to convince yourself that random numbers have no correlation.

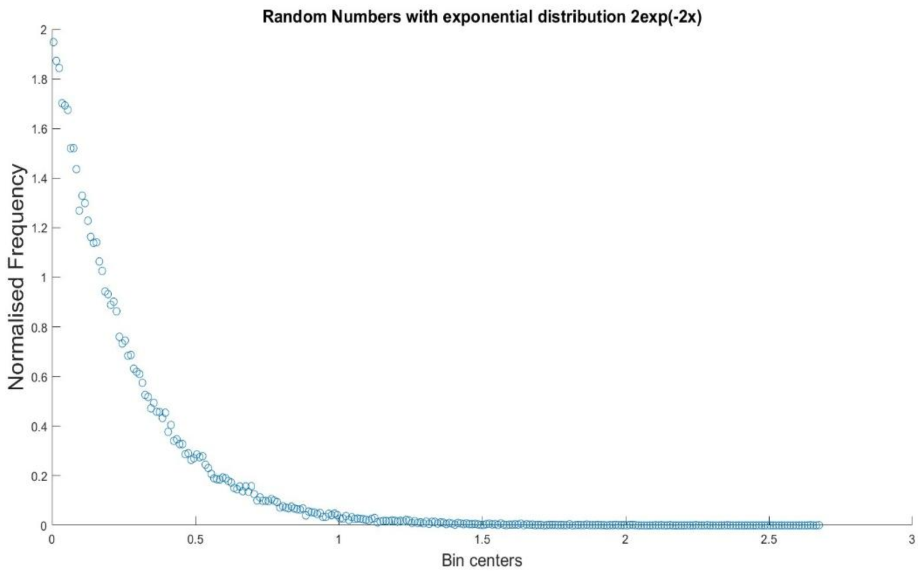


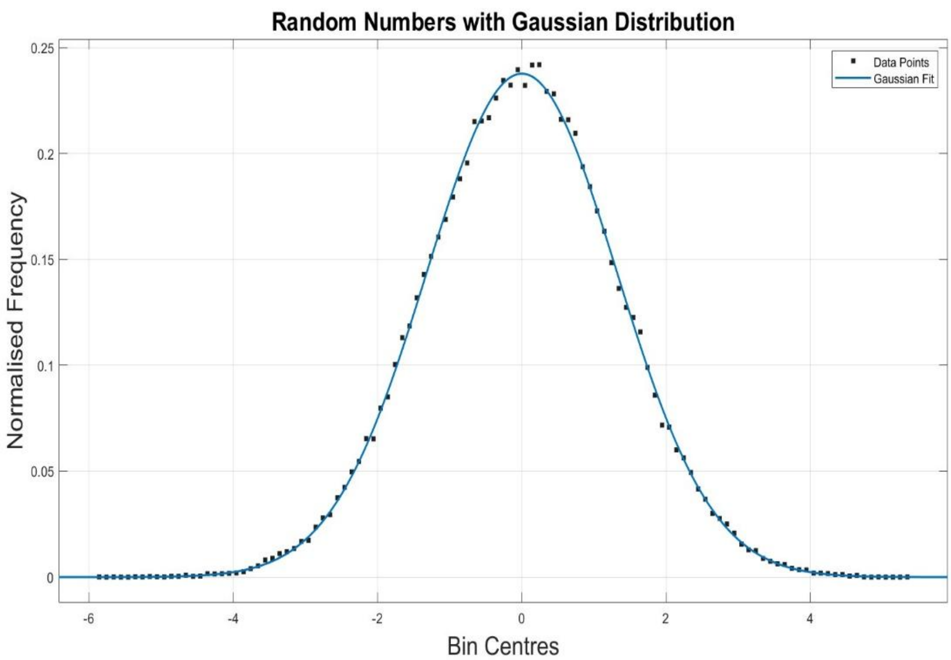
d. Calculate the standard deviation (SD) of the random numbers about the mean.

```
 How many random numbers are needed?
3000
 Value of mean   0.50158754588312371
 Value of Standard Deviation about mean is    0.28992974445865000
```

```
 How many random numbers are needed?
10000
 Value of mean   0.50301976897202116
 Value of Standard Deviation about mean is    0.28894278482732355
```

Q4. Generate random numbers with (a) exponential (e^{-2x}) and (b) Gaussian (with SD=2) distributions.





Q5. Write a program that computes the following multi-dimensional integral using the (a) brute force and (b) importance sampling Monte Carlo integration methods:

$$I = \int_{-\infty}^{\infty} d^3\vec{x}\, d^3\vec{y}\, g(\vec{x},\vec{y})$$

where $g(\vec{x},\vec{y}) = \exp(-\vec{x}^2 - \vec{y}^2 - \frac{(\vec{x}-\vec{y})^2}{2})$

Suppose you do not know the exact value of the integral. How do you compute the error? Compare the efficiency of the two methods.

Using Brute Force Method

Value of N	Value of Integral	Standard Deviation
10	1.8578151277768273E-019	1.1774897837460271E-019
100	5.6654962913766433E-005	4.7751790622709463E-005
1000	18.438920932205505	15.155126458701508
10000	8.9386639204157472	5.5180742407212922
100000	7.4493785341981686	2.9305630885588818
1000000	11.941676165891147	1.4126515734867313
10000000	11.055497390629194	0.36021550374703309
100000000	10.826071005236610	0.11398406850420724

Using Importance Sampling

Value of N	Value of Integral	Standard Deviation
10	12.230445708199349	2.8322296426049718
100	11.614296298906252	0.84303741231598217
1000	10.934067231392655	0.25688975489851240
10000	10.981624976000667	8.0724337975353708E-002
100000	10.977911107983942	2.5535766038086421E-002
1000000	10.976090385814022	8.0563348021678912E-003
10000000	10.957425967474840	2.5458782852731845E-003
100000000	10.960770688080057	8.0522851616618169E-004