AM5630: Foundations of Computational Fluid Dynamics

Assignment 2 – Steady 2D Diffusion

Indian Institute of Technology Madras

A Numerical Study using Finite Volume Method (FVM) and Gauss-Seidel Iteration

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September 29, 2025

1 Introduction

The two-dimensional steady-state heat conduction equation with a source term is given as:

$$0 = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + S \tag{1}$$

where T is the temperature, k is the thermal conductivity, and S is the volumetric heat source.

We solve this equation using the Finite Volume Method (FVM), following the discretization approach from Versteeg & Malalasekara (Eq. 4.57). The resulting algebraic equations are solved iteratively using the Gauss-Seidel method.

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2 Discretization Method

2.1 Finite Volume Formulation

The computational domain is divided into finite volumes. Integrating Eq. (1) over a control volume and applying Gauss' divergence theorem gives:

$$a_P T_P = a_E T_E + a_W T_W + a_N T_N + a_S T_S + b_P \tag{2}$$

where a_P is the central coefficient, a_E, a_W, a_N, a_S are neighbor coefficients, and b_P contains the source term contribution.

The source term is linearized as:

$$S \approx S_c + S_p T_P \tag{3}$$

with $b_P = S_c \Delta V$.

2.2 Gauss-Seidel Iteration

The Gauss-Seidel update equation for each control volume is:

$$T_P^{(k+1)} = \frac{1}{a_P}\left(a_E T_E^{(k)} + a_W T_W^{(k+1)} + a_N T_N^{(k)} + a_S T_S^{(k+1)} + b_P\right) \tag{4}$$

The residual is computed as:

$$R = \sum_P |a_P T_P - (a_E T_E + a_W T_W + a_N T_N + a_S T_S + b_P)| \tag{5}$$

Convergence is achieved when $R/F < \epsilon$, where ϵ is the error tolerance and F is some flux term

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3 Case Setup (Case 3)

The computational domain and boundary conditions are provided in Figure 1 and Table 1.

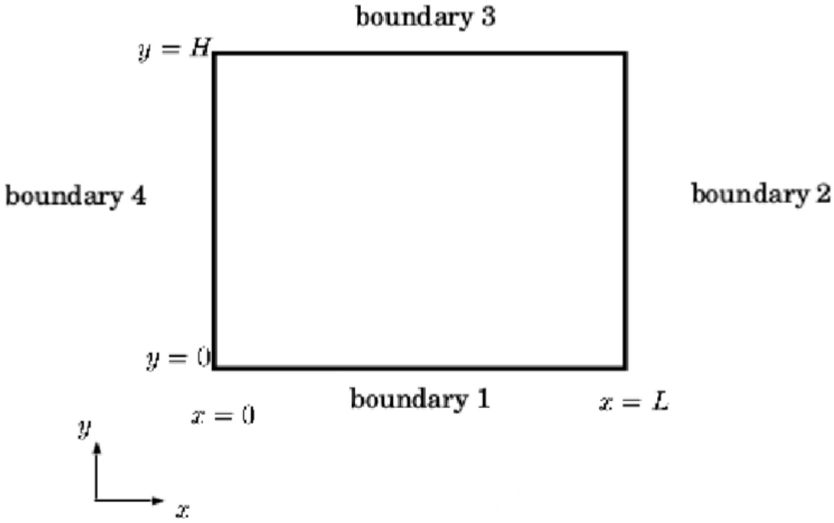

Figure 1: Computational domain for Case 3 (schematic).

Boundary	Condition
Left	$T = 15$
Top	$q = +5000$
Right	$T = 10$
Bottom	$\frac{dT}{dx} = 0$
Source	$S = -1.5$
Conductivity	$k = 16\left(\frac{y}{H} + 1\right)$

Table 1: Boundary conditions and source term for Case 3.

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4 Results

4.1 Mesh

The equation was solved on 20×20, 30×30,40×40 meshes and 60×60 meshes with some growth rate 0.95 along the x axis. This stretching was given considering gradients were higher in the right part from the results of the simulations.

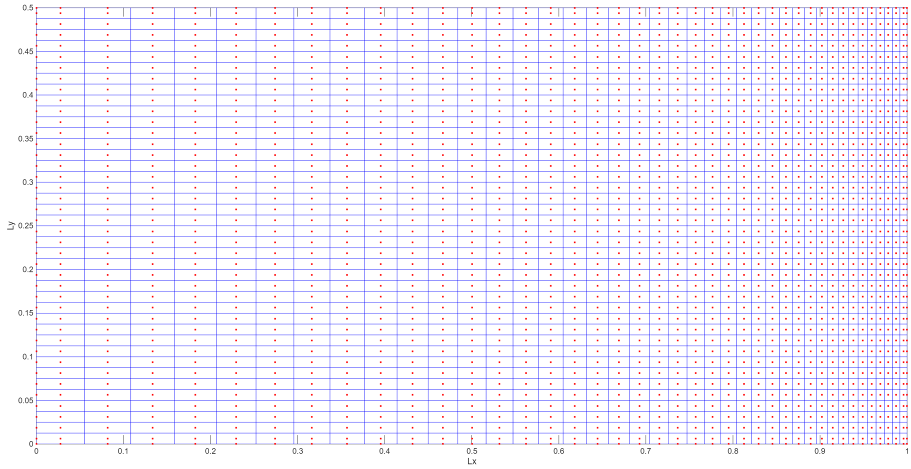


Figure 2: 40x40 mesh grid

The mesh was stretched along X axis because , from later results we find that the gradients are maximum at the right boundary.

For mesh independence study we had taken the set and plotted various quantities to illustrate that the quantities don't change much as we further increase the number of cells. Further we have plotted the number of iterations and time needed to illustrate that the computational cost goes higher with finer meshes.

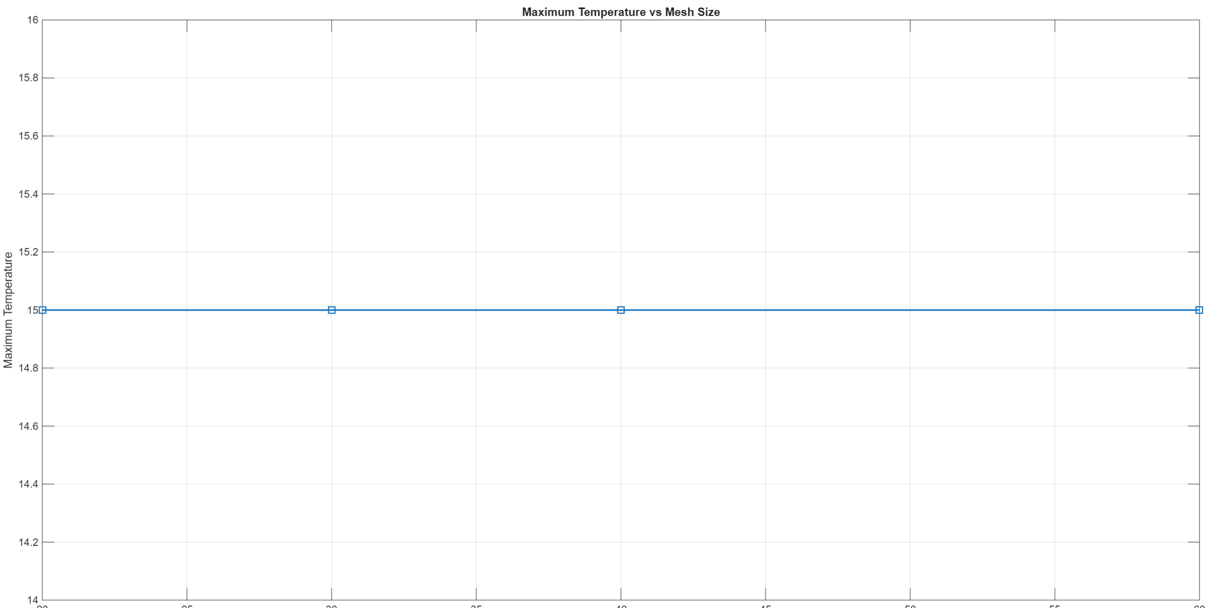


Figure 3: T max

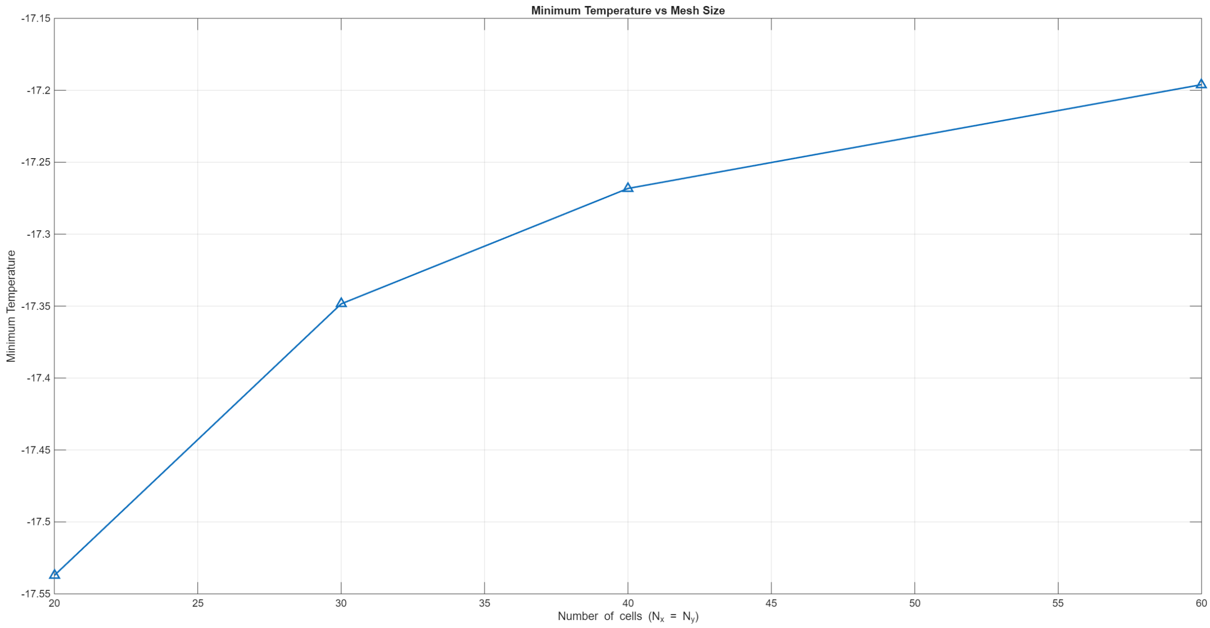


Figure 4: T min

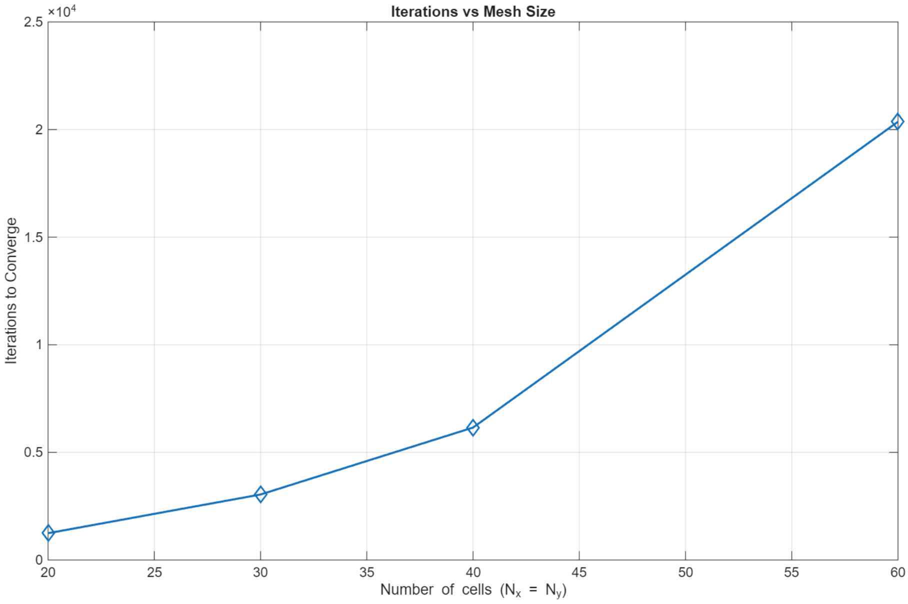


Figure 5: Iterations needed

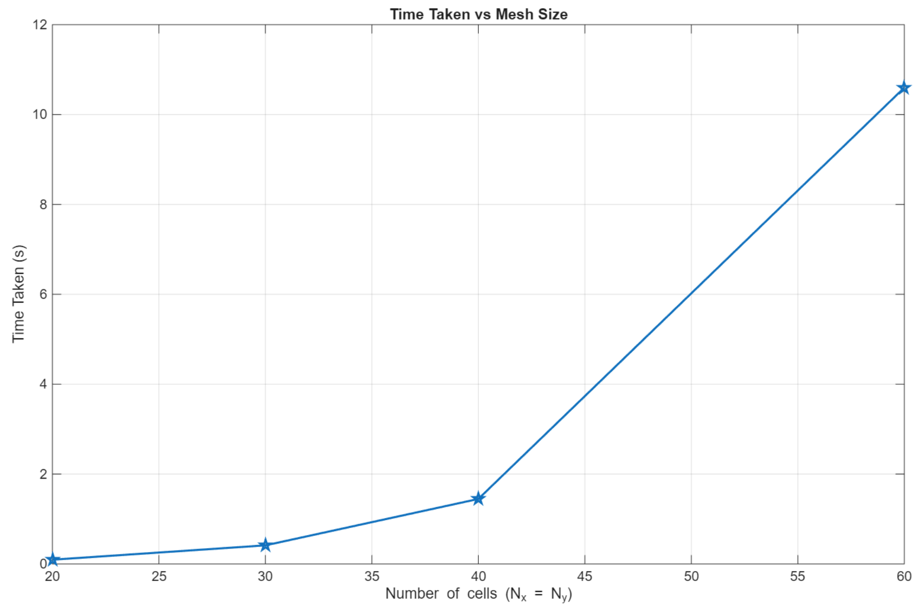


Figure 6: Time Taken increases for finer mesh

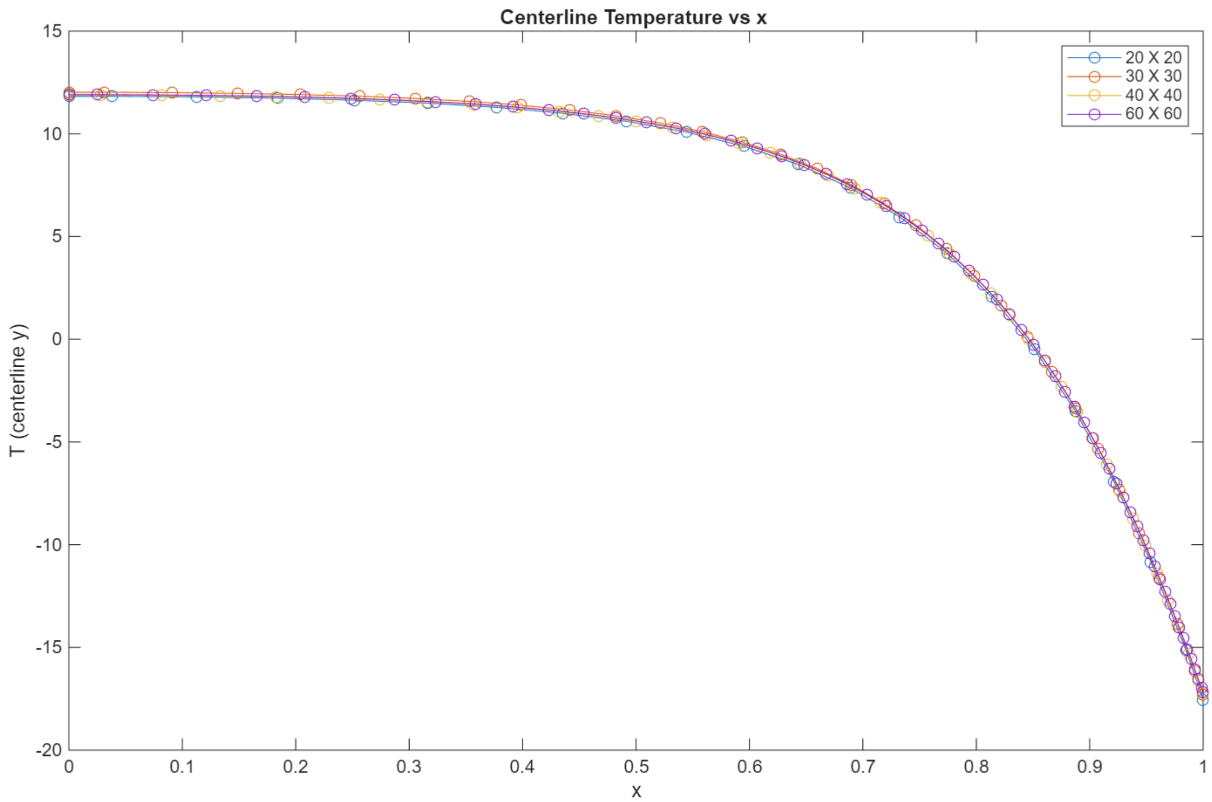


Figure 7: Centerline Temperature vs x

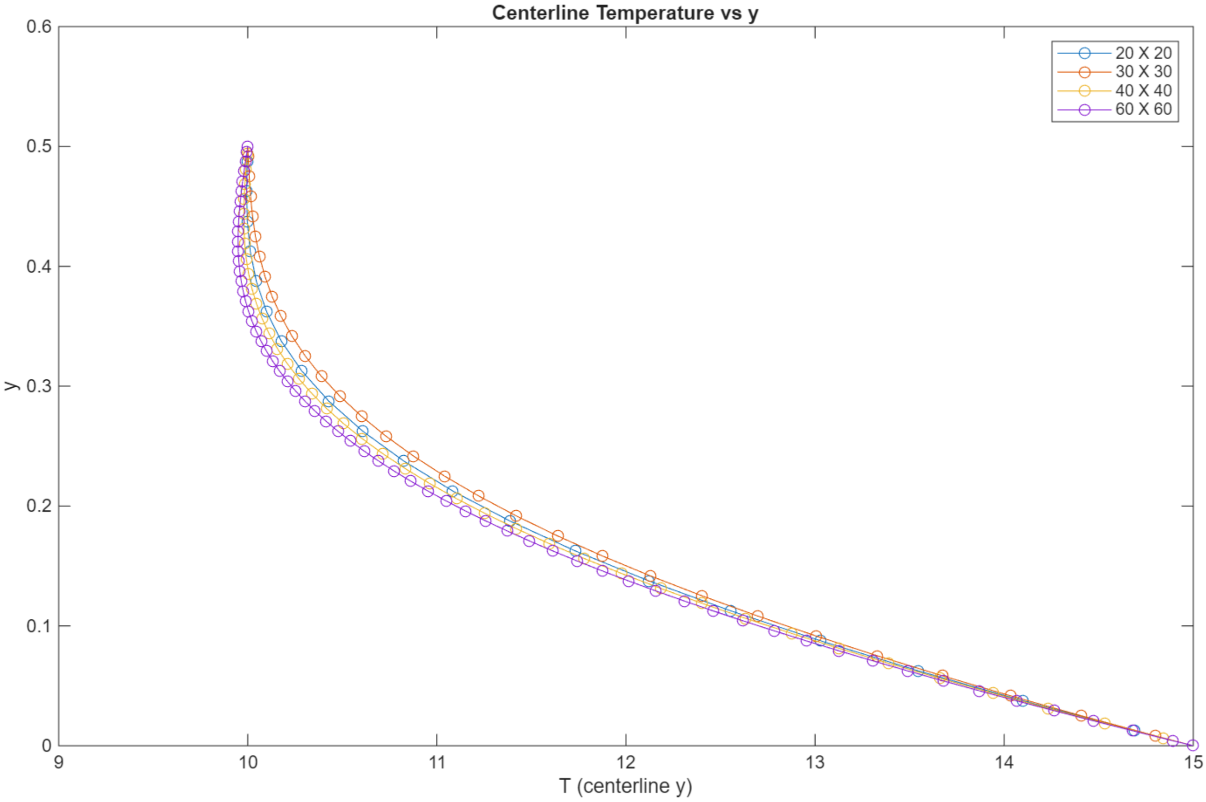


Figure 8: centerline temperaure through y

We find that T shows a convergent behavior as we tend to finer and finer meshes. The increase in computational cost outweighhs the increase in accuracy after a point. We shall stick with 40 x 40 mesh for the further analysis.

5 Residuals

As mentioned in the procedure above , we take the convergence criteria when $R/F < \epsilon$. Following is the residual plot for $\epsilon = 0.01$ and $\epsilon = 10^{-6}$

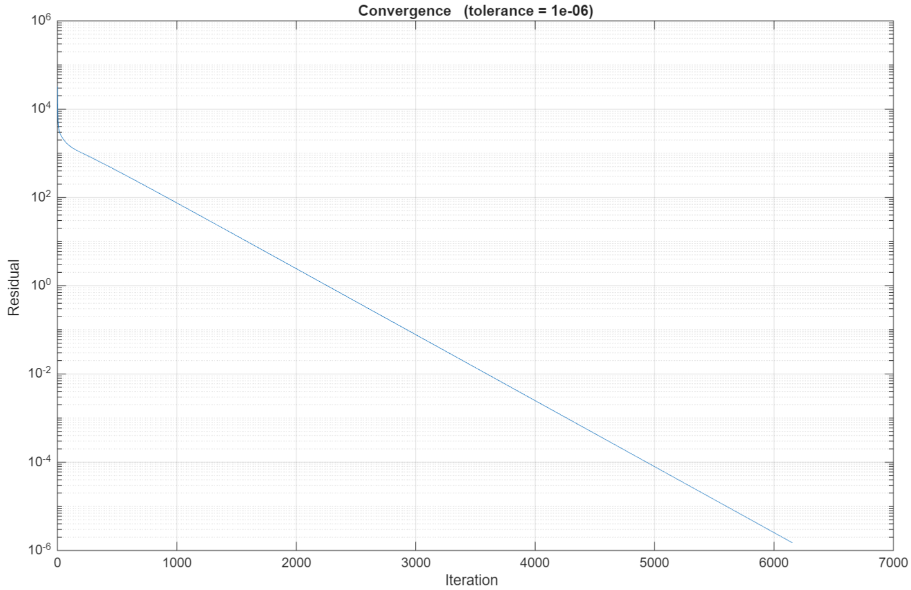


Figure 10: $\epsilon = 10^{-6}$

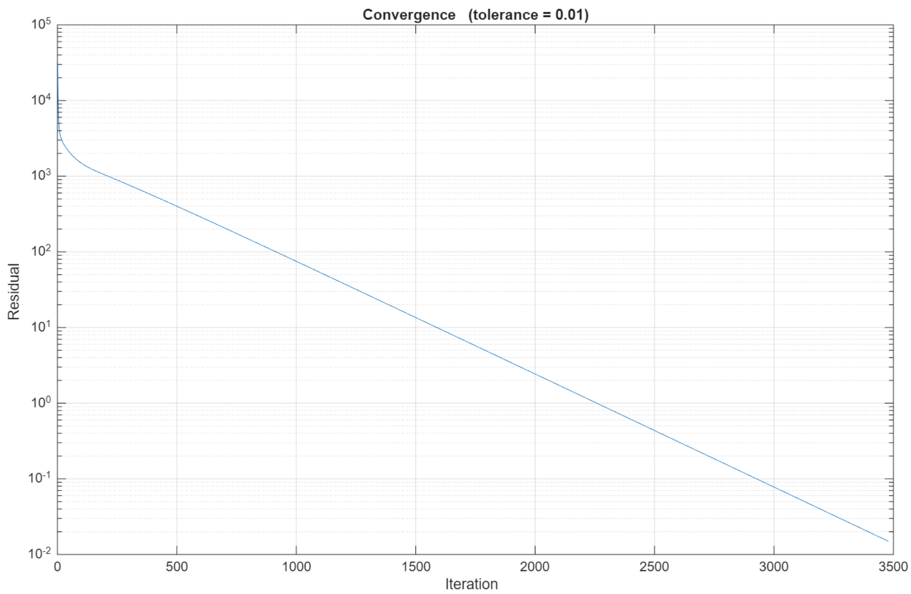


Figure 9: $\epsilon = 0.01$

- Residuals decrease monotonically with iteration.
- Smaller tolerance ($\epsilon = 10^{-6}$) requires more iterations but gives accurate fluxes.
- Relaxed tolerance ($\epsilon = 10^{-3}$) converges faster but yields less accurate results.

6 RESULTS

For the given boundary conditions the contour of temperature is plotted. Heat flux is also calculated as plotted as a vector plot on top of the temperature contour.

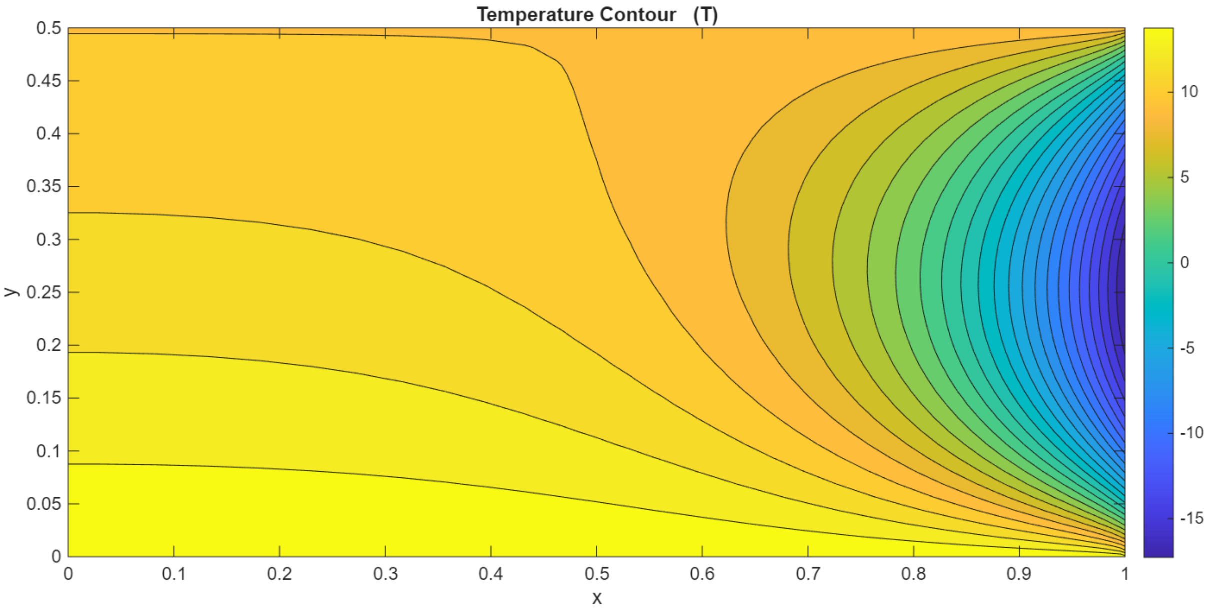


Figure 11: Contour of Temperature

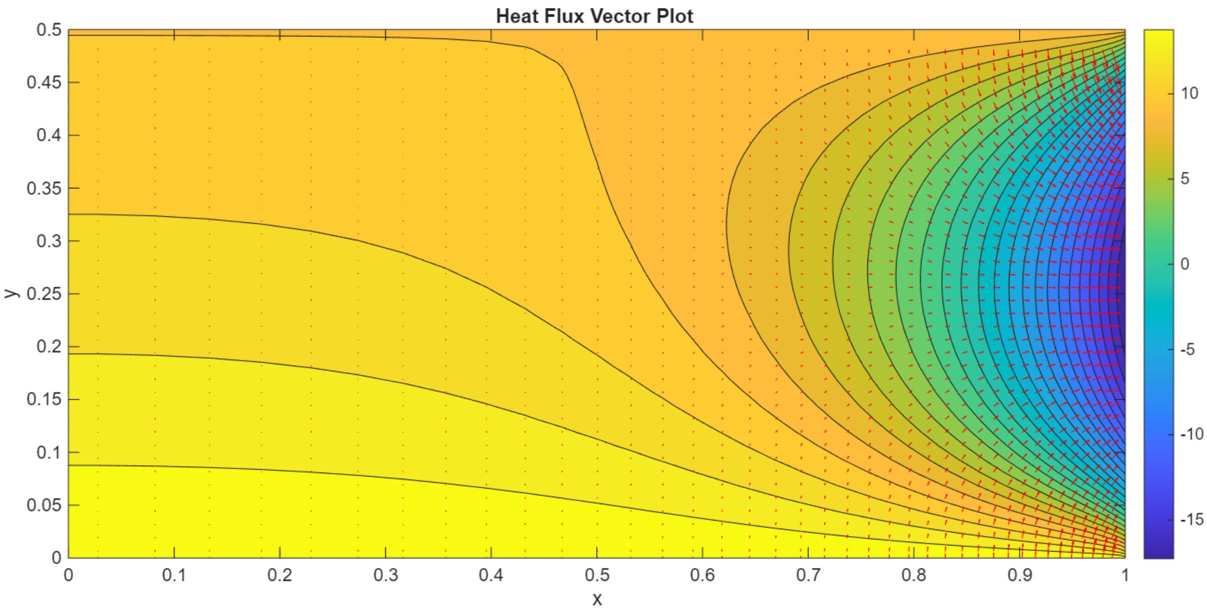


Figure 12: Heat Flux Vector Plot

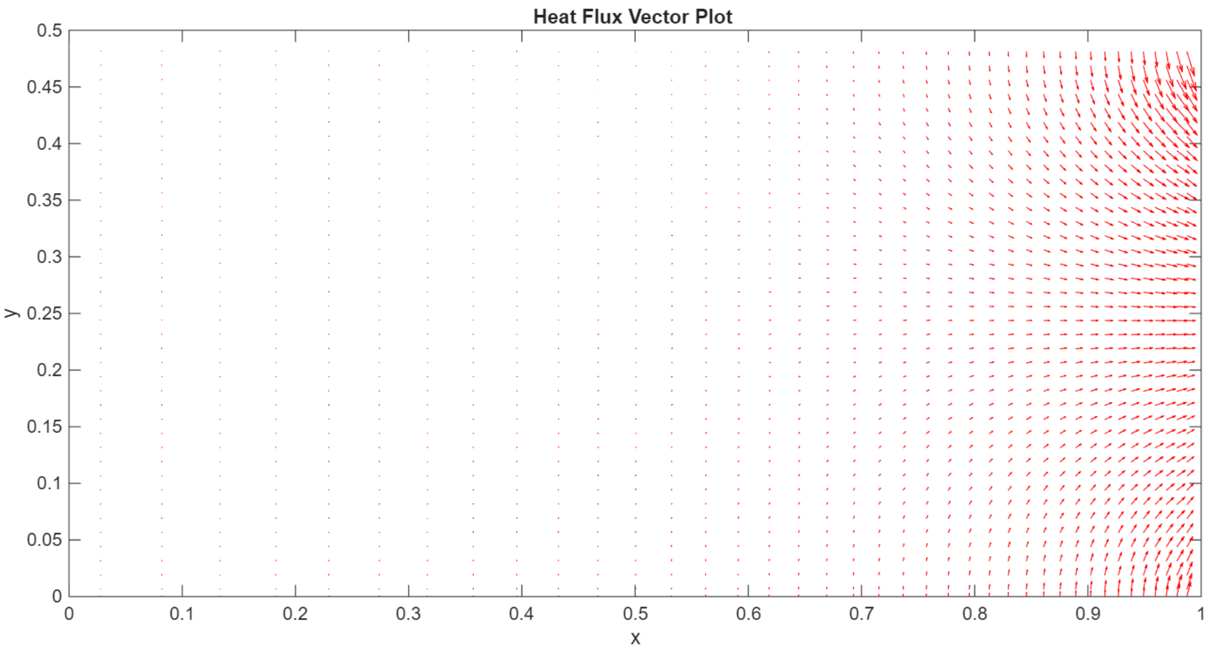
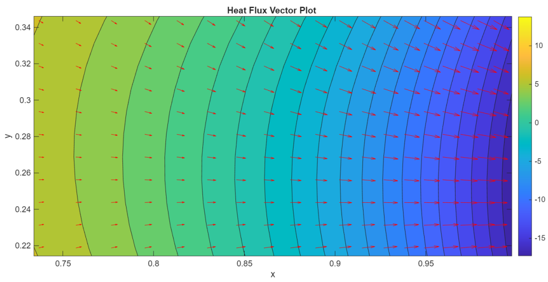
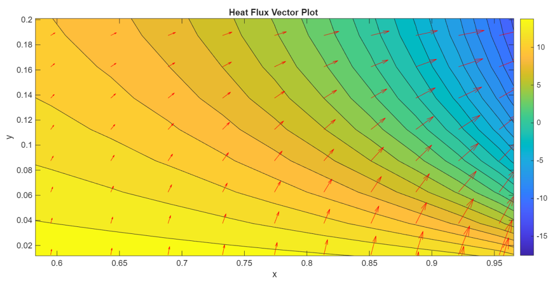


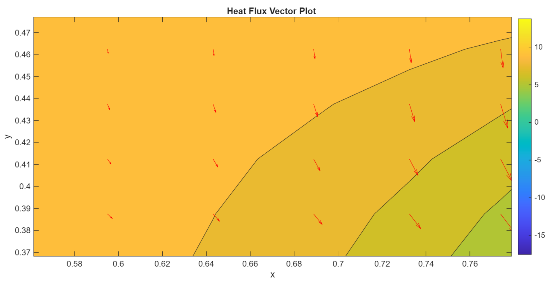
Figure 13: Vector Plot of Heat Flux



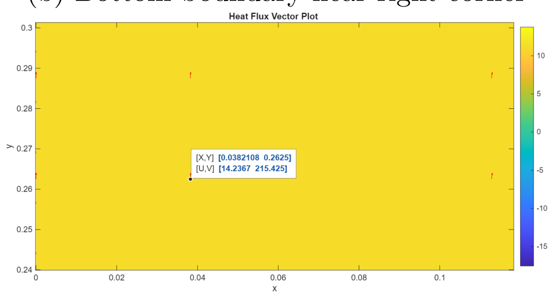
(a) Near the middle of right boundary



(b) Bottom boundary near right corner



(c) Top boundary near right corner



(d) Left boundary near centre

Figure 14: A zoomed in view of the different boundaries.

Discussion

- The solution had converged giving a feel of a real result. There is no high temperature shoot up anywhere in the domain
- The most influential boundary is the right boundary because it has a high flux of q = 5000 , which takes away the heat from the system. A Neumann boundary condition was employed here.

- The temperature near the boundary is low around -15° and it increases on all sides, rising to 10 in the top and 15 in the bottom boundary , as expected.
- The temperature along the bottom and top boundaries are a constatn 10 and 15 , as we have enforced them
- But the temperature in the left boundary is not constant. It has a gradient and increases from 10° to 15° from top to bottom
- The **heat flux** shows the show of heat to illustrate the direction of heat flow. We observe high fluxes near left boundary as expected, due to the high flux enforced in that boundary conditon.
- The heat flows from higher temperature to lower temperature throughout the domain and the magnitude is higher where there is a more faster change in temperature. This is implemented numerically by $\dot{q}_x = -k\frac{\partial T}{\partial x}, \qquad \dot{q}_y = -k\frac{\partial T}{\partial y}$
- We observe that the **heat flux along x is nearly zero** in the right boundary as expected physically because there is no temperature gradient there and hence no heat flow. We had employed this numerically with the Neumann boundary condition $T_W = T_P$ at the west boundary.
- The **heat flux is non zero** in the top and bottom boundaries even though the temperature is a constant . This is because there is still a temperature gradient , especially arouond the right side . We had employed a Dirchelet bouondary condition in these places .
- The heat flux is very high near the corners, given the large temperature gradient as observed in the contours. Numerically this is because the two boundary conditions need to be satisfied simultaneously.

7 Effect of Change in Boundary Conditions

7.1 Changing the left boundary to a Dirchelet

Changing the Neumann boundary condition in right boundary with a constant temperature dirchelet boundary condition of $T = -15^\circ$. 15° was set for example because that is the temperature near the centre of that boundary in the previous B.C implementation .

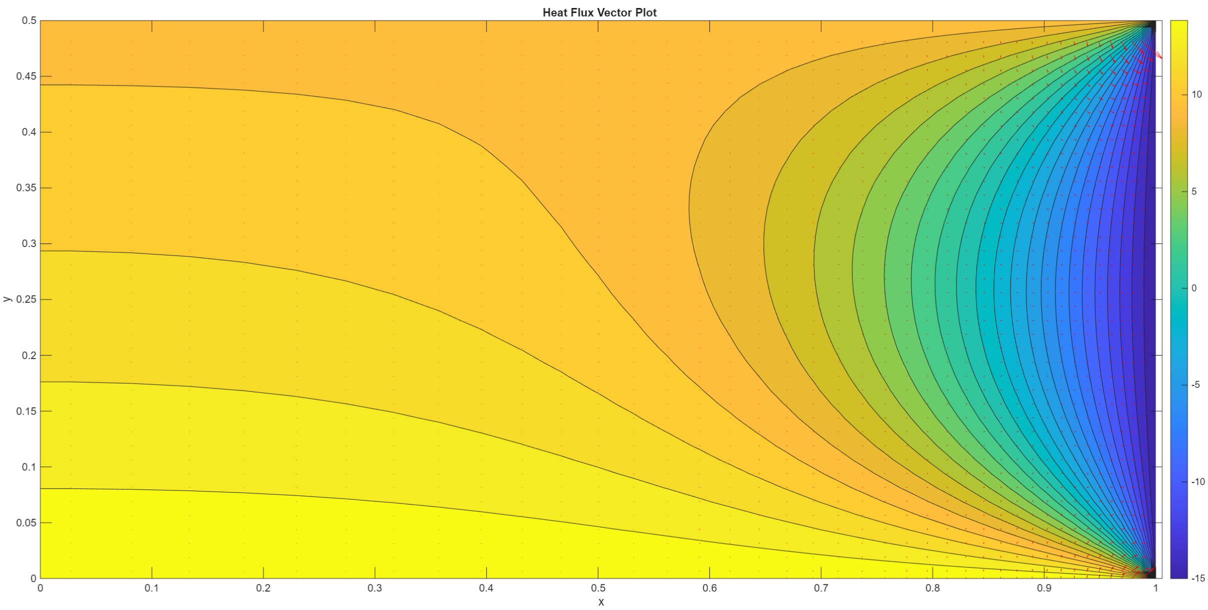


Figure 15: Temperature contour with constant Temperature in right wall

At first glace the contour plot seems similar to the previous implementation, but when we observe the heat fluxes and the contours near the corners we see a large difference from ouur previous implementation.

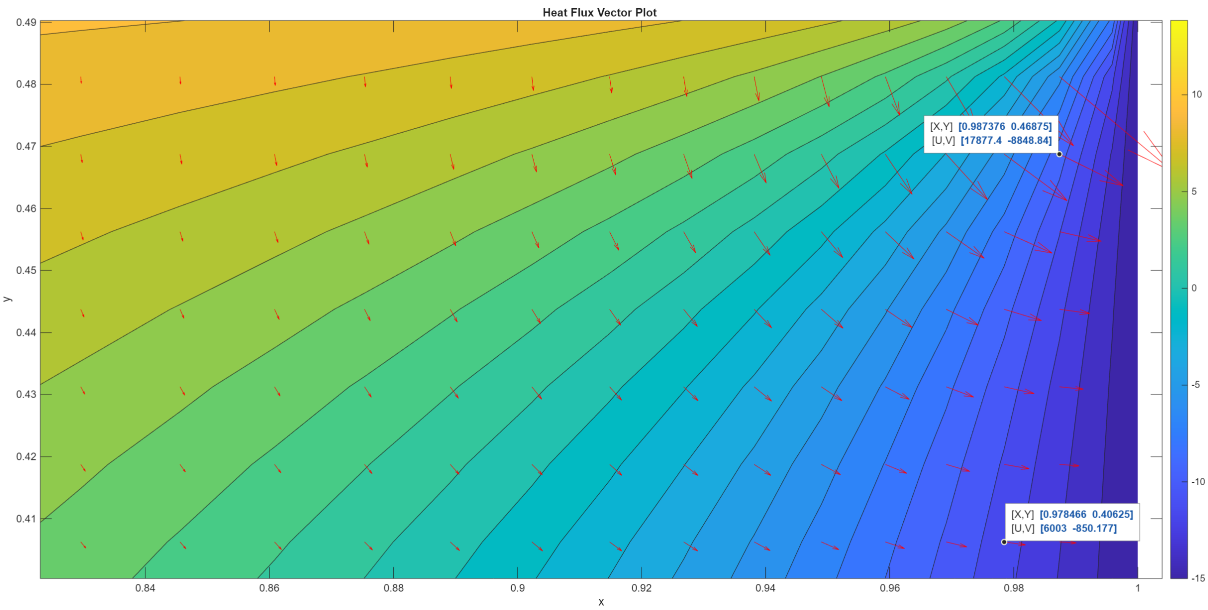


Figure 16: Large heat fluxes near the corners

The corners have unrealistically high fluxes in order to satisfy both the boundaries in the corners. This leads to an **unphysical** result. This concludes that **boundary conditions can effect the solution significantly** and careful implementation needs to be done . It also illustrates that CFD results , no matter how low the tolerances are **does not guarantee a real solution**. Physically possible boundary conditions and fine meshing should be done in problematic areas are essential for a true solution.

7.2 Changing the top boundary to a Neumann

Changing the boundary condition in the top to $q = 5000$ gives the following solution

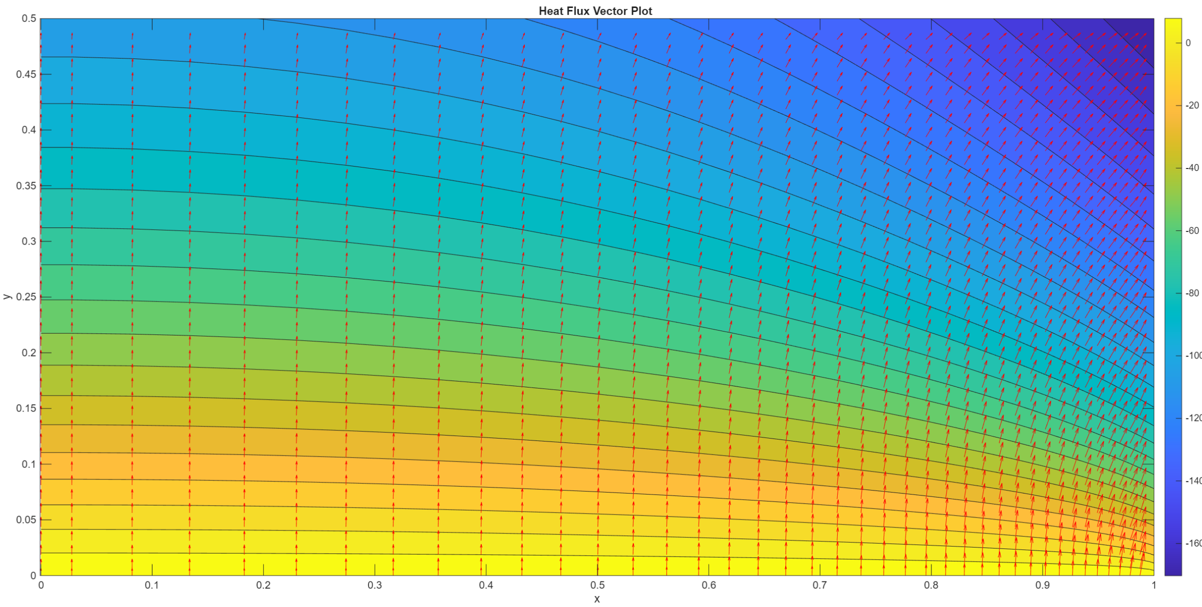


Figure 17: Top boundary changed to a Neumann condition

We observe that the temperature field was completely changeed, where the top section also takes heat away from the system . The minimum temmperature is around $-170°$. But the other boundary conditions remain like earlier. This example illustrates how the change in boundary condition can significantly affect the solution.

8 Conclusions

- Mesh independence study illustrated that the mesh resolution of 40x40 was good enough for accurate results.
- As we go more fine with our mesh resolution , the computational cost increases , leading to higher computation times.
- Stretching along regions with higher gradients give a more accurate result.
- Gauss-Seidel iteration is simple and robust and convergence is achieved for the type of conditions we employ in these examplse.
- Changing boundary conditions significantly alters the temperature field and energy balance.
- A converged solution does not imply a real world result. We need to employ physically real boundary conditions and finer meshing in places with big changes in order to accurately capture the field.