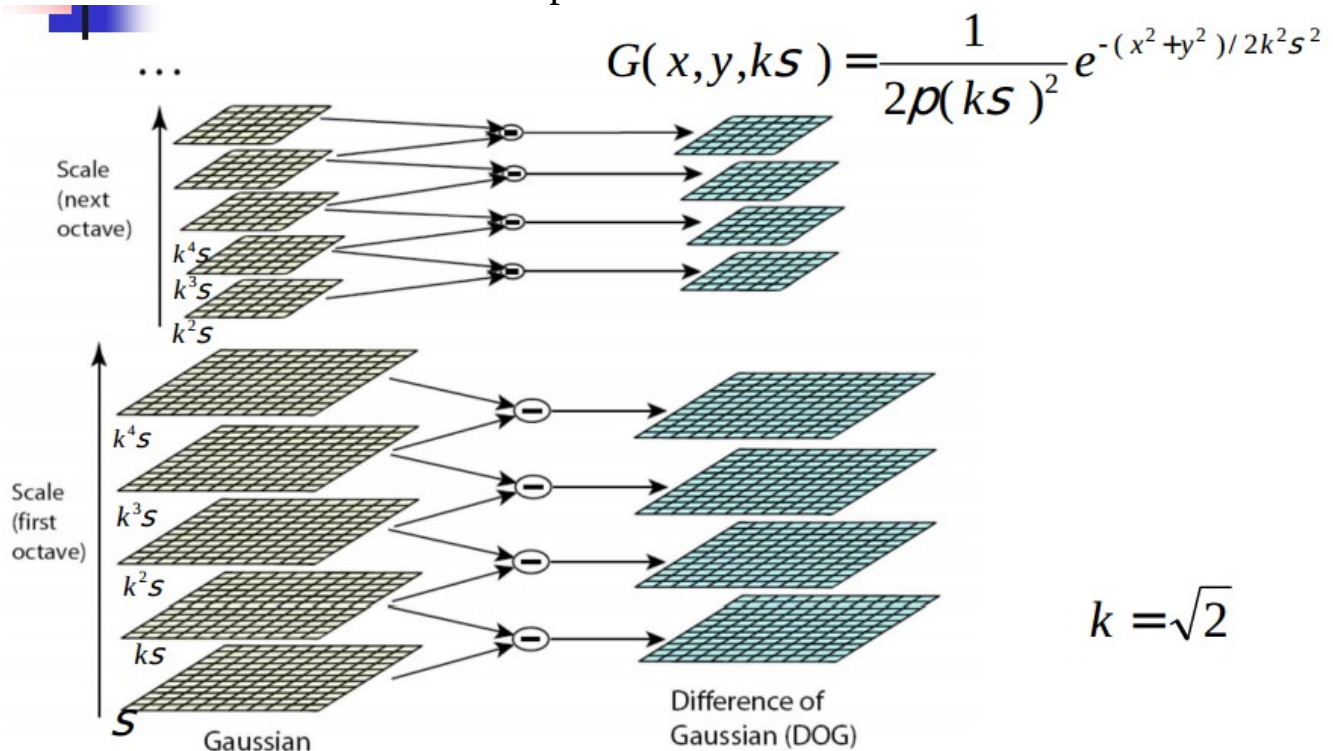


Scale Invariant Feature Transform

1. Scale space peak selection

- this step is used for finding potential location of features

- use different gaussians for capturing different scales (different scale space)
- subtract gaussians to get blob detector (laplacian of gaussian) because log can be approximated by dog and dog is computationally faster.
- find if pixel is maximum/minimum in 3x3x3 (8 pixels in same level, 9 above, 9 below) neighbourhood to select it as extrema point. (peak selection)
- use different octaves to reduce computation. For this, downscale the image, find extrema and upscale the location. But use the downscale location in the next 3/4 steps. ?



2. Key point localization

-previous step found potential points i.e. approximate location and could be features.

(a) Initial outlier rejection

- get proper location
- use Taylor series expansion to find maximum of surface
- earlier step found approximate point in the neighbourhood (because of smoothing). This step finds the exact point in that neighbourhood.

- for all the potential points $(x, y, \text{scale}, \text{octave})$, perform the following steps
- take the dog values
- calculate first derivative wrt x, y, sigma

$$X = (x, y, \sigma) \rightarrow \text{Dog}$$
$$\frac{\partial D}{\partial X} = \begin{bmatrix} \frac{\partial D}{\partial x} \\ \frac{\partial D}{\partial y} \\ \frac{\partial D}{\partial \sigma} \end{bmatrix} = \begin{bmatrix} \frac{D(x+1, y, \sigma) - D(x-1, y, \sigma)}{2} \\ \frac{D(x, y+1, \sigma) - D(x, y-1, \sigma)}{2} \\ \frac{D(x, y, \sigma+1) - D(x, y, \sigma-1)}{2} \end{bmatrix}$$

- calculate second derivative

$$H(x) = \begin{bmatrix} D_{xx} & D_{xy} & D_{x\sigma} \\ D_{yx} & D_{yy} & D_{y\sigma} \\ D_{\sigma x} & D_{\sigma y} & D_{\sigma\sigma} \end{bmatrix}$$

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$$D(\hat{x}) = D(x, y, \sigma) + \frac{1}{2} \frac{\partial D}{\partial x}^T \hat{x}$$

-this step rejects the points with less contrast.

(b) Further outlier rejection

-This step is rejecting edges.(in xy plane of dog plane – forget sigma)

-So use harris corner value.

- Form the hessian matrix for the dog surface at that point

$$H = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix}$$

- find eigenvalues λ_1, λ_2
- calculate corner value as
$$r = \lambda_1 / \lambda_2$$
- reject the points if $r > 10$

3. Orientation assignment

-find dominant orientations of the keypoint. This is for converting the feature into rotation invariant.

- find the magnitude and orientation for every point in 4x4 neighbourhood around that point for that scale.

L = smoothed image at scale = gaussian (not dog)

$$dx = L(x+1, y) - L(x-1, y) \quad \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

$$dy = L(x, y+1) - L(x, y-1) \quad \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

$$\text{magnitude}(x, y) = \sqrt{dx^2 + dy^2}$$

$$\text{orientation}(x, y) = \text{atan}(dy/dx)$$


- create a weighted direction histogram.

36 bins (meaning each bin covers 10 degrees i.e. 0-10, 10-20, 20-30, etc.)

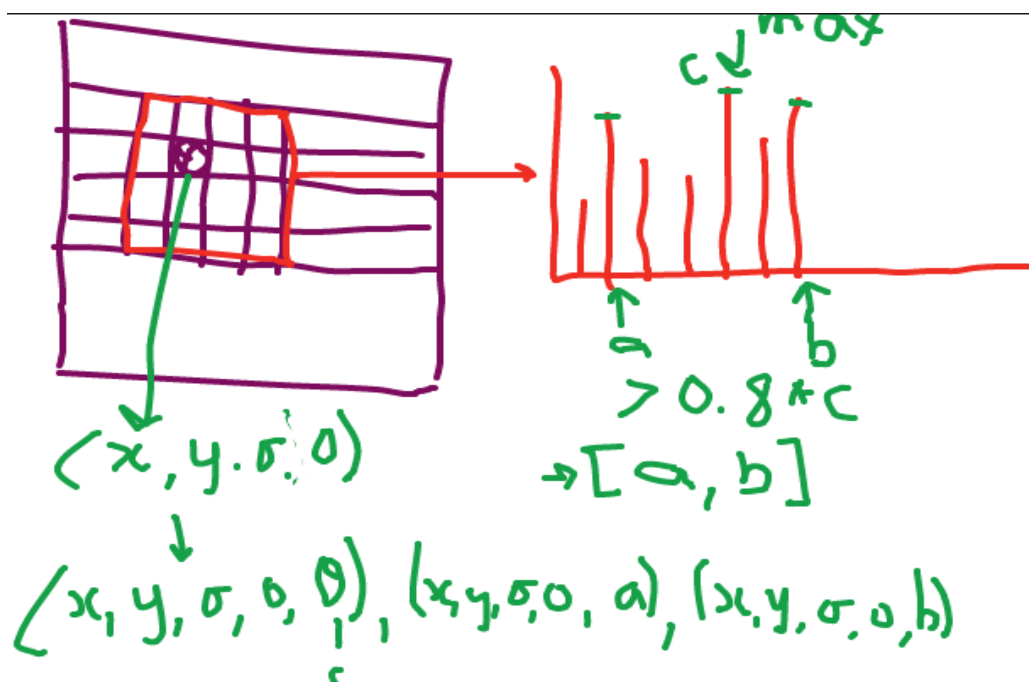
'orientation' = orientation/10 -> assign to the bin

weight = magnitude and spatial gaussian filter with $\sigma = 1.5 * \text{scale of point}$

$m=5, P=(2, 3, 3)$
 $\theta=72^\circ$
 $\text{gauss1D}(\frac{nt}{\sigma} * 1.5) = [-, -, -, -]$
 $xy = [a, b, c, d]$
 $72^\circ \rightarrow 7 \text{ bin}$
 $H[7] + m$
 $H[5] + ma, H[6] + mb, H[7] + mc, H[8] + md$



- find the dominant directions in the histogram. Maximum = dominant direction. Any direction having $> 0.8 * \text{dominant direction}$, consider them as potential points also.



4. Keypoint descriptor

-describing the key point as a high dimensional vector

- take 16x16 pixel window around the point
- compute magnitude and relative orientation (=orientation -dominant direction) for every pixel
- 16x16 divide into 4x4 regions. So, every block contains 4x4 pixels

4x4 pixels	4x4	4x4	4x4

- for every block, construct weighted histogram (8 bins, so 0-45,45-90, etc.,weight=magnitude and spatial gaussian)
- concatenate all the histograms. 8Bins x 16 blocks=> 128 vector

postprocessing

- normalize 128D vector to unit vector

[20,40,30,10] -> [20,40,30,10]/ 100

-> [0.2,0.4,0.3,0.1]

- for non-linear intensity transform, remove any value greater than 0.2 in the normalized vector and again renormalize ?

[0.2,0,0,0.1] -> [0.66,0,0,0.33]

5. keypoints matching

-match the keypoints against database or another image

feature1 -> 128D vector in image 1

feature2 -> 128D vector in image2

euclidean distance = $\sqrt{(f1[1]-f2[1])^2 + (f1[2] -f2[2])^2 + \dots)}$

image1 – features_{i1}, features_{i2}, features_{i3},...

image2 – features_{j1}, features_{j2}, features_{j3},...

- take features_{i1}
- compute euclidean distance wrt features_{j1}, features_{j2}, features_{j3},...
- find feature with minimum euclidean distance --> matching feature1
- ratio = distance best match / distance 2nd best match
- consider the second best match (2nd minimum distance) if ratio ≤ 0.8
- threshold distance, so both have to be less than threshold distance