* Assignment - 1 *

1.1 (a) Sample space $(\Omega) = \{HH, HT, TH, TT\}$

(b) Event space $= \{\phi, \{HH\}, \{HT\}, \{TH\}, \{TT\},$
$\{HH, HT\}, \{HH, TH\}, \{HH, TT\}, \{HT, TH\}, \{HT, TT\},$
$\{TH, TT\}, \{HH, HT, TH\}, \{HT, TH, TT\}, \{HH, HT, TT\}$
$\{HH, TH, TT\}, \{HH, TH, HT, TT\}\}.$

(c) (i) $P(\{HH\}) = P(\{HT\}) = P(\{TH\}) = P(\{TT\})$
$= 1/4$

(ii) $P(\text{at least 1 Head}) = P(\{HH, TH, HT\})$
$= 3/4$

(iii) $P(\text{exactly 1 head}) = P(\{HT, TH\})$
$= 2/4$

2.1 given $f(K, n, p) = {}^{n}C_{K}\, p^{K} (1-p)^{n-K}$

According to question $p = 0.9$ and $n = 50$ words to Recognize words correctly $(K) = 45$

$\Rightarrow$

$$P(K = 45) = f(45, 50, 0.9)$$
$$= {}^{50}C_{45} (0.9)^{45} (0.1)^{5}$$
$$= 0.1849$$

2.2 given probability of K accidents is given by PMF $f(K, \lambda) = \frac{\lambda^{K} e^{-\lambda}}{K!}$ where $\lambda = 10$

$$\Rightarrow \quad P(X = K) = \frac{10^{K} e^{-10}}{K!}$$

(a) For $X = 0$

$$\Rightarrow P(X=0) = e^{-10}$$

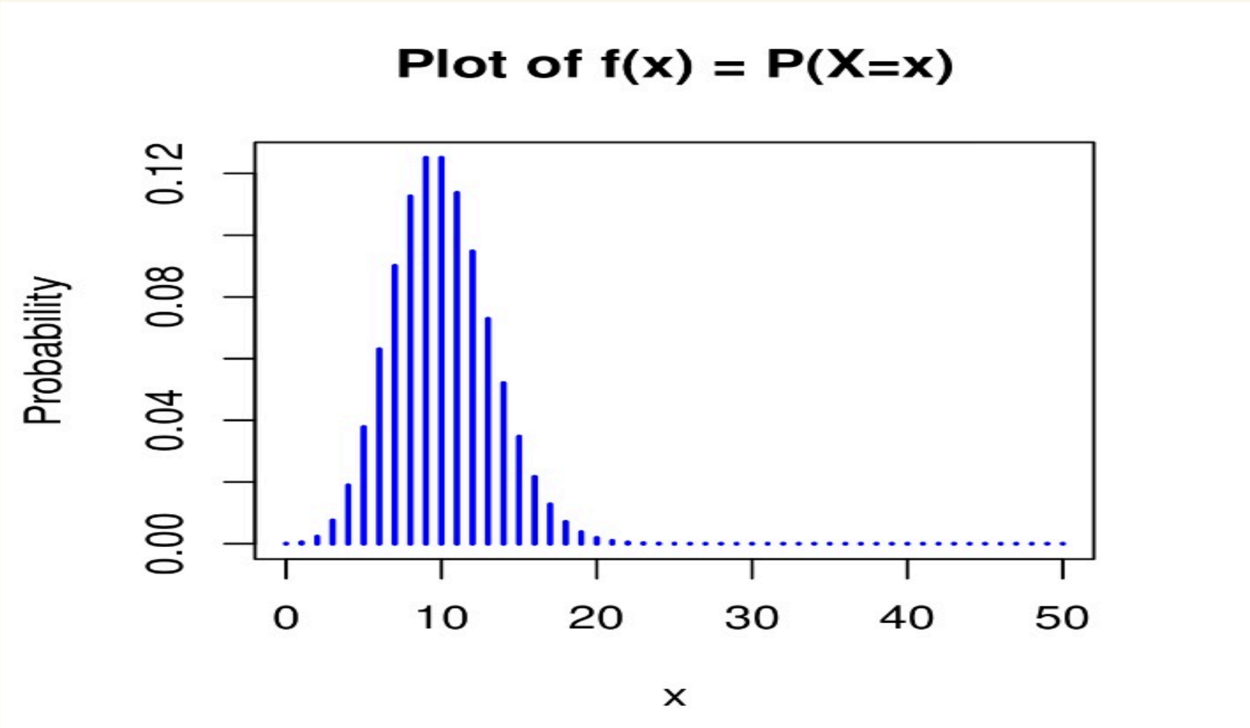
(b) $7 < X < 10$

$$\Rightarrow P(7<X<10) = P(X=8) + P(X=9)$$

$$= e^{-10}\left(\frac{10^8}{8!} + \frac{10^9}{9!}\right) = \frac{10^8 e^{-10}}{8!}\frac{19}{9}$$

$$= 0.2377$$

(C) $$f(x) = P(X=x) = \frac{10^x e^{-10}}{x!}$$



3.1. given $$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

(a) given $\mu = 1, \sigma = 1$

$$f(0) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}}$$

(b) given $\mu = 0, \sigma = 1$

$$f(1) = \frac{1}{\sqrt{2\pi}} e^{-1/2}$$

(C) PDF = $f(x)$

Cumulative Distribution function

$$F_X(x) = \int_{-\infty}^{x} f(x) = P(X \le x)$$

given that

$$P(x_1 \le X \le x_2) = 0.3$$

$$\int_{-\infty}^{x_2} f(x)\,dx - \int_{-\infty}^{x_1} f(x)\,dx = 0.3$$

$$\Rightarrow \int_{x_1}^{x_2} f(x)\,dx = 0.3 \quad \text{—} \; (1)$$

&

$$P(x_1 \le X \le x_3) = 0.45$$

$$\Rightarrow \int_{-\infty}^{x_3} f(x)\,dx - \int_{-\infty}^{x_1} f(x)\,dx = 0.45$$

$$\Rightarrow \int_{x_1}^{x_3} f(x)\,dx = 0.45 \quad \text{—} \; (2)$$

$$\Rightarrow P(x_2 \le X \le x_3) = \int_{x_2}^{x_3} f(x)\,dx$$

$$= \int_{x_1}^{x_3} f(x)\,dx - \int_{x_1}^{x_2} f(x)\,dx$$

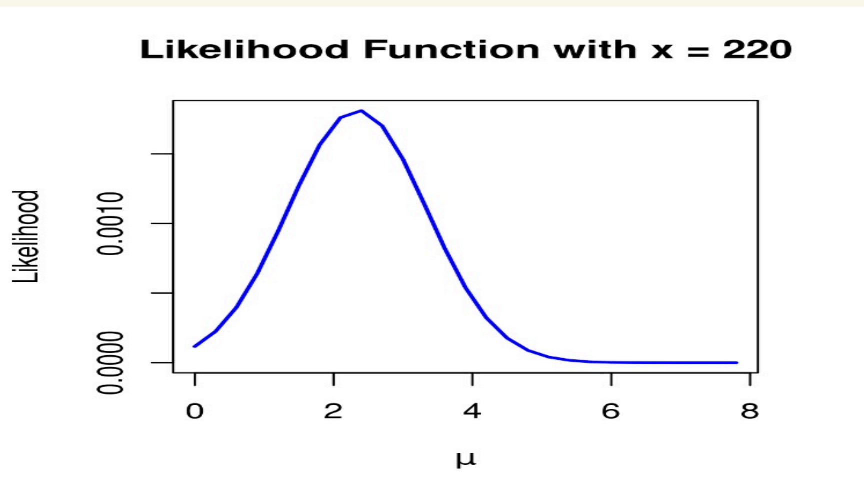
$$= 0.45 - 0.3$$

$$= \underline{0.15}$$ Ans

4.1 $X \sim f(x, \mu)$

$$f(x,\mu) = \frac{1}{x\sqrt{2\pi}} e^{-\frac{(\log_{10} x - \mu)^2}{2}}$$

(a)

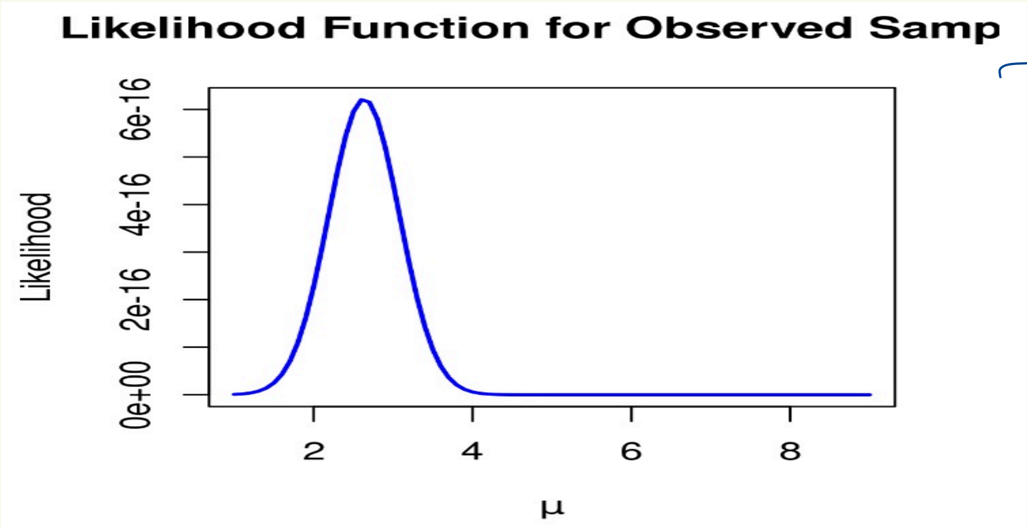


← Plot likelihood vs μ

Assuming log base 10

(b) for observed sample $x = [\underset{x_1}{303.25}, \underset{x_2}{443}, \underset{x_3}{220}, \underset{x_4}{560}, \underset{x_5}{880}]$

$$f(x_{1:5}, \mu) = \frac{1}{\left(\prod_{i=1}^{5} x_i\right)\left(\sqrt{2\pi}\right)^5} e^{-\frac{\sum_{i=1}^{5}(\log_{10}(x_i)-\mu)^2}{2}}$$



→ likelihood plot for observed sample

(c) to find max^m of $f(x_{1:5}, \mu)$ for observed sample with the help of plot it is around $\mu = 2.5$ or 2.6

or by solving

$$\text{Max}^m f(x_{1:5}, \mu) = \text{Min}^m \left(\sum_{i=1}^{5} (\log(x_i) - \mu)^2 \right)$$

(decreasing monotonic function e^{-x})

$$\Rightarrow \text{Min}^m \text{ of } \sum_{i=1}^{5} (\log(x_i) - \mu)^2$$

$$-2\sum_{i=1}^{5} (\log x_i - \mu) = 0$$

$$\Rightarrow \quad \mu = \frac{\log_{10}(x_1 x_2 x_3 x_4 x_5)}{5}$$

$$= \underline{\underline{2.63}} \quad \text{Ans}$$

Codes Used

2.2
(c)

```
lambda <- 10
x_values <- 0:50
poisson_probabilities <- dpois(x_values, lambda)
plot(x_values, poisson_probabilities, type="h", lwd=2, col="blue",
     xlab="x", ylab="Probability",
     main="Plot of f(x) = P(X=x)")
```

4.1
(a)

```
x <- 220
mu_values <- seq(0, 8, by=0.3)
likelihood_function <- function(x, mu) {
  return ((1 / (x * sqrt(2 * pi))) * exp(-(( ( log(x)/log(10) ) - mu)^2) / 2))
}
likelihood_values <- sapply(mu_values, function(mu) likelihood_function(x, mu))
plot(mu_values, likelihood_values, type="l", lwd=2, col="blue",
     xlab=expression(mu), ylab="Likelihood",
     main="Likelihood Function with x = 220")
```

4.1
(b)

```
x_values <- c(303.25, 443, 220, 560, 880)
mu_values <- seq(1, 9, by=0.1)
likelihood_function <- function(x_values, mu) {
  n <- length(x_values)
  product_x <- prod(x_values)
  sum_log_diff <- sum((log10(x_values) - mu)^2)
  likelihood <- (1 / (product_x * (sqrt(2 * pi)^n))) * exp(-0.5 * sum_log_diff)
  return(likelihood)
}

likelihood_values <- sapply(mu_values, function(mu) likelihood_function(x_values, mu))
plot(mu_values, likelihood_values, type="l", lwd=2, col="blue",
     xlab=expression(mu), ylab="Likelihood",
     main="Likelihood Function for Observed Sample")
```