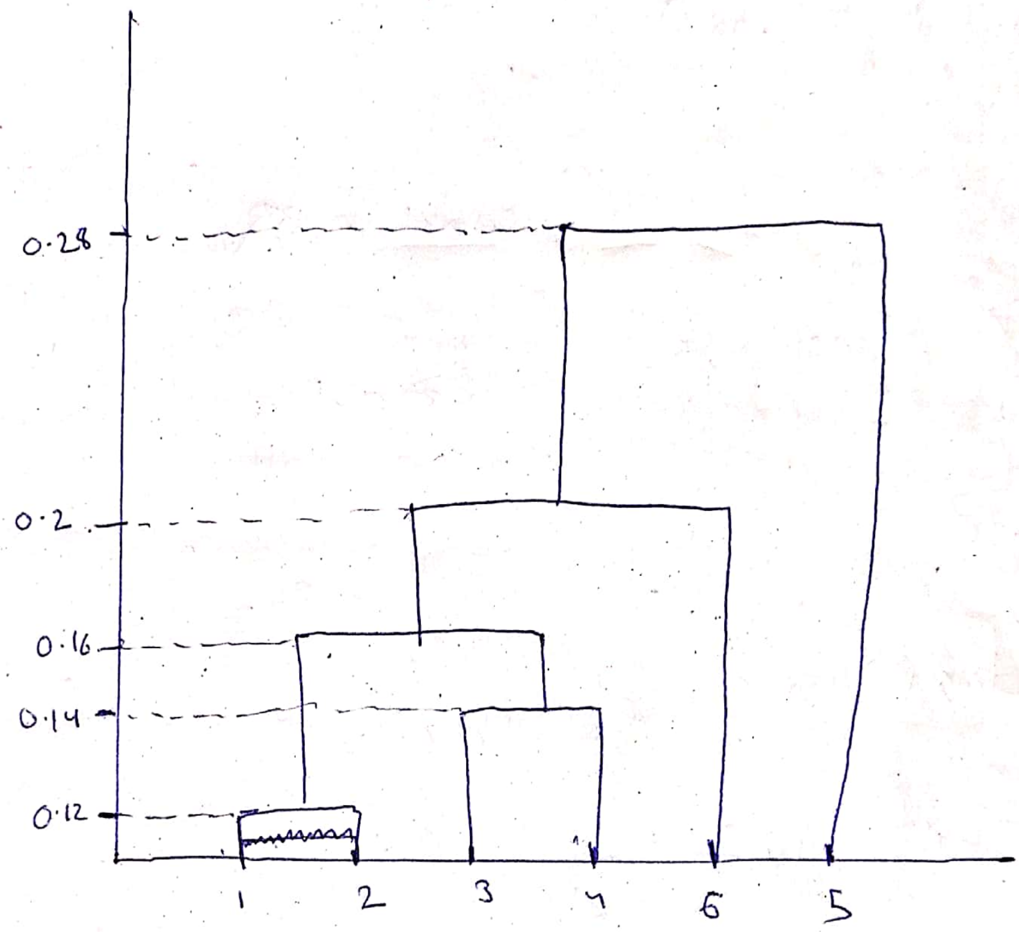
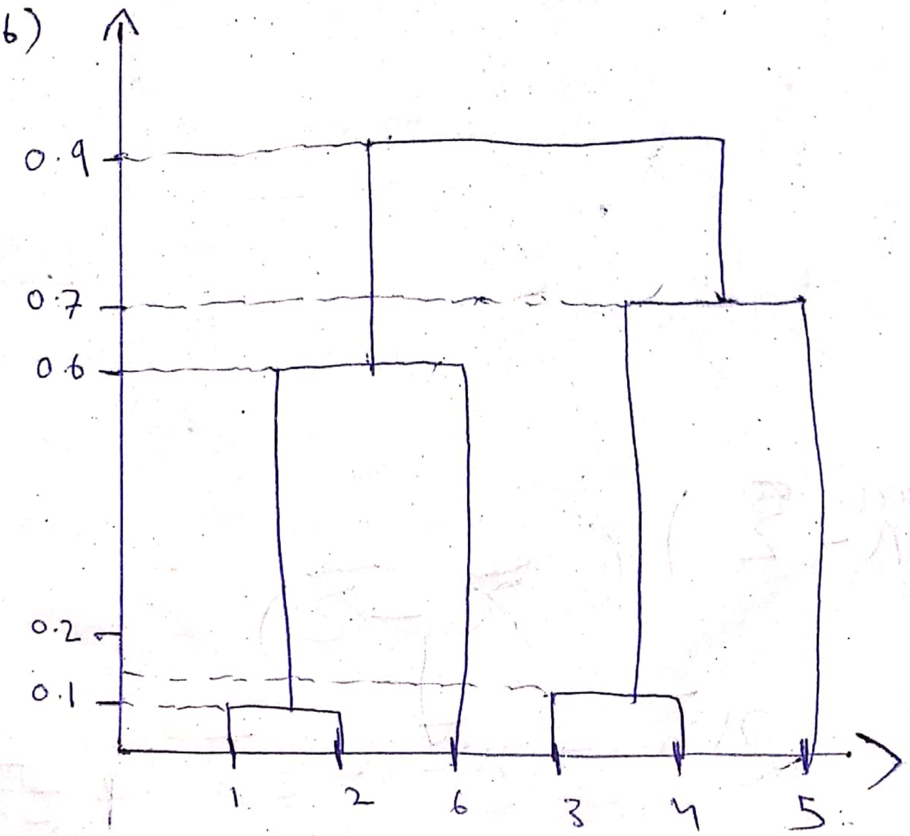
A1(a)



A1 b)



A1(c) 1st step for which complete link clustering differs from single link is where AB & F are grouped together by dist (AB, F) = dist (B, F) = 0.61

We want dist (AB, CD) = dist (A, D) to be smaller than this value, such as 0.53. And we also want dist (ABCD, F) = dis(C, F) = 0.93 to be the smallest so that ABCD & F are grouped together. These changes make both of them equal.

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A2) $X = a\,\delta_k = (0, 0 \ldots 0, a, 0, 0 \ldots 0)^T$

K is uniformly distributed

a is arbitrarly distribuited

$C_{ij} = E[\underbrace{[E[X_i] - X_i][E[X_j] - X_j]}]$

$E[X_i] \cdot E[X_j] - X_i E[X_j]$
$- X_j E[X_j] + X_i X_j$

$\Rightarrow E[X_i] = \frac{M}{n}$; M is the mean of distribution a.

n is the dimension of X

$\Rightarrow E[E[X_i] E[X_j]] = \frac{M^2}{n^2}$

$E[E[X_j] X_i] = \frac{M^2}{n^2} = E[E[X_i] X_j]$

$E[X_i X_j] =$ If $i = j$ $E[X_i^2] = \sigma^2 + \frac{M^2}{n^2}$

else 0

$$C = \begin{bmatrix} \sigma^2 & -\frac{\mu^2}{n^2} & \cdots & \cdots \\ -\frac{\mu^2}{n} & \sigma^2 & & \\ -\frac{\mu^2}{n^2} & & \ddots & \\ \vdots & & & \\ -\frac{\mu^2}{n^2} & & & \sigma^2 \end{bmatrix}$$

i.e. $C_{ij} = \sigma^2$ If $i = j$

$C_{ij} = -\frac{\mu^2}{n^2}$ If $i \,!= j$

A2(b) Consider $C \times U$ where

U is unit vector = $\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$

$$C \times U = \begin{bmatrix} (n-1) \times -\frac{\mu^2}{\sigma^2} + \sigma^2 \\ (n-1) \times \frac{\mu^2}{\sigma^2} + \sigma^2 \\ \vdots \\ (n-1) \times \frac{\mu^2}{\sigma^2} + \sigma^2 \end{bmatrix}$$

$$C \times V = \left[(n-1)\frac{-\mu^2}{\sigma^2} + \sigma^2\right] V$$

$$\lambda = (n-1)\frac{-\mu^2}{n\sigma^2} + \sigma^2$$, Unit vecto V is eigen vector

Let $a = \sigma^2$, $b = \frac{-\mu^2}{n^2}$

$$\Rightarrow C = \begin{bmatrix} a & b & b & \cdots & b & b \\ b & a & & & & \vdots \\ \vdots & & \ddots & & & b \\ b & \cdots & & & b & a \end{bmatrix}$$

$$\det(C - \lambda I) = \begin{vmatrix} a-\lambda & b & \cdots & b \\ b & a-\lambda & & \vdots \\ \vdots & & \ddots & b \\ b & \cdots & b & a-\lambda \end{vmatrix}$$

$$= \begin{vmatrix} a-\lambda-b & 0 & 0 & \cdots & b-a+\lambda \\ 0 & a-\lambda-b & & & \\ \vdots & & & & \\ b & b & b & \cdots & a-\lambda \end{vmatrix}$$

$$= \cancel{(a-\lambda-b)}\,(a-\lambda-b)^{n-1} \begin{vmatrix} 1 & 0 & 0 & \cdots & -1 \\ 0 & 1 & 0 & \cdots & -1 \\ \vdots & & & & \vdots \\ b & b & \cdots & & \alpha-1 \end{vmatrix}$$

$\lambda = a + b$, repeats $n-1$ times

$= \left(\sigma^2 - \frac{\mu^2}{\sigma^2} \right)$; $n-1$ eigen values are same.

& n^{th} root is $(n-1) - \frac{\mu^2}{\sigma^2} + \sigma^2$

A 2(c) PCA is not a good way ~~to select~~ for this dataset as all the $(n-1)$ eigenvalues are same but the eigen vectors may be different in different directions. Hence choosing a few of them may cause ~~at~~ a lot of loss in information.

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