

$$1. \rightarrow T(n) = 3T(n/2) + n^2$$

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

$$a = 3, \quad b = 2$$

$$c = \log_2 3 = 1.58$$

$$n^c = n^{1.58}$$

$$f(n) = n^2$$

By case 3: $f(n) > n^c$

$$T(n) = \Theta(f(n)) = \Theta(n^2)$$

$$2. \rightarrow T(n) = 4T(n/2) + n^2$$

$$T(n) = aT(n/b) + f(n)$$

$$a = 4, \quad b = 2$$

$$c = \log_2 4 = 2$$

$$n^c = n^2$$

$$f(n) = n^2$$

By case 2: $f(n) = n^c$

$$T(n) = \Theta(n^c \log n) = \Theta(n^2 \log n)$$

$$3. \rightarrow T(n) = T(n/2) + 2^n$$

$$T(n) = aT(n/b) + f(n)$$

$$a = 1, \quad b = 2$$

$$c = \log_2 1 = 0$$

$$n^c = n^0 = 1, \quad f(n) = 2^n$$

By case 1: $f(n) > n^c$

$$T(n) = \Theta(f(n)) = \Theta(2^n)$$

$$4. \rightarrow T(n) = 2^n T(n/2) + n^n$$

$$T(n) = aT(n/b) + f(n)$$

$$a = 2^n, b = 2$$

$$c = \log_2 2^n = n$$

$$n^c = n^n, f(n) = n^n$$

$$f(n) = n^c$$

$$T(n) = \Theta(n^c \log n)$$

$$T(n) = \Theta(n^n \log n)$$

$$5. \rightarrow T(n) = 16T(n/4) + n$$

$$T(n) = aT(n/b) + f(n)$$

$$a = 16, b = 4$$

$$c = \log_4 16 = 2$$

$$n^c = n^2, f(n) = n$$

$$f(n) < n^c$$

$$T(n) = \Theta(n^c)$$

$$T(n) = \Theta(n^2)$$

$$6. \rightarrow T(n) = 2T(n/2) + n \log n$$

$$T(n) = aT(n/b) + f(n)$$

$$a = 2, b = 2$$

$$c = \log_2 2 = 1$$

$$n^c = n, f(n) = n \log n$$

$$f(n) > n^c$$

$$T(n) = \Theta(f(n))$$

$$T(n) = \Theta(n \log n)$$

$$7.) T(n) = 2T(n/2) + n/\log n$$

$$a=2, b=2$$

$$c = \log_2 2 = 1$$

$$n^c = n^1, f(n) = n/\log n$$

$$n^c > f(n)$$

$$T(n) = \Theta(n^c)$$

$$T(n) = \Theta(n)$$

$$8.) T(n) = 2T(n/4) + n^{0.51}$$

$$T(n) = aT(n/b) + f(n)$$

$$a=2, b=4$$

$$c = \log_4 2 = 0.5$$

$$n^c = n^{0.5}, f(n) = n^{0.51}$$

$$f(n) > n^c$$

$$T(n) = \Theta(f(n))$$

$$T(n) = \Theta(n^{0.51})$$

$$9.) T(n) = 0.5T(n/2) + 1/n$$

$$T(n) = aT(n/b) + f(n)$$

$$a=0.5, b=2$$

$$c = \log_2 0.5 = -1$$

$$n^c = n^{-1} = 1/n, f(n) = 1/n$$

$$f(n) = n^c$$

$$T(n) = \Theta(n^c \log n)$$

$$= \Theta(\log n/n)$$

$$10.) T(n) = 16T(n/4) + n!$$

$$T(n) = aT(n/b) + f(n)$$

$$a=16, b=4$$

$$c = \log_4 16 = 2$$

$$n^c = n^2, f(n) = n!$$

$$f(n) > n^c$$

$$T(n) = \Theta(f(n))$$

$$T(n) = \Theta(n!)$$

$$11.) T(n) = 4T(n/2) + \log n$$

$$a = 4, b = 2$$

$$c = \log_2 4 = 2$$

$$n^2 = n^2, f(n) = \log n$$

$$f(n) < n^c$$

$$T(n) = \Theta(n^c)$$

$$T(n) = \Theta(n^2)$$

$$12.) T(n) = \text{Sqrt}(n)T(n/2) + \log n$$

$$T(n) = aT(n/b) + f(n)$$

$$a = n^{1/2}, b = 2, f(n) = \log n$$

$$c = \log_2 n^{1/2} = \frac{1}{2} \log n$$

$$n^c = n^{1/2} \log n$$

$$f(n) \leq n^c$$

$$\therefore T(n) = \Theta(n^c)$$

$$13.) T(n) = 3T(n/2) + n$$

$$a = 3, b = 2$$

$$c = \log_2 3 = 1.58$$

$$n^c = n^{1.58}, f(n) = n$$

$$f(n) < n^c$$

$$T(n) = \Theta(n^c)$$

$$T(n) = \Theta(n^{1.58})$$

$$14.) T(n) = 3T(n/3) + \text{sqxt}(n)$$

$$a = 3, b = 3$$

$$c = \log_3 3 = 1$$

$$n^c = n^1, f(n) = n^{1/2}$$

$$f(n) < n^c$$

$$T(n) = \Theta(n^c)$$

$$T(n) = \Theta(n)$$

$$15.) T(n) = 4T(n/2) + cn$$

$$a = 4, b = 2$$

$$c = \log_2 4 = 2$$

$$n^c = n^2, f(n) = cn = n$$

$$f(n) < n^c$$

$$T(n) = \Theta(n^c)$$

$$T(n) = \Theta(n^2)$$

$$16.) T(n) = 3T(n/4) + n \log n$$

$$a = 3, b = 4$$

$$c = \log_4 3 = 0.79$$

$$n^c = n^{0.79}, f(n) = n \log n$$

$$f(n) > n^c$$

$$T(n) = \Theta(f(n))$$

$$T(n) = \Theta(n \log n)$$

$$17.) T(n) = 3T(n/3) + n/2$$

$$a = 3, b = 3$$

$$c = \log_3 3 = 1$$

$$n^c = n^1, f(n) = n/2$$

$$f(n) = \Theta(n^c)$$

$$T(n) = \Theta(n^c)$$

$$T(n) = \Theta(n)$$

$$18. \rightarrow T(n) = 6T(n/3) + n^2 \log n$$

$$a=6, b=3$$

$$c = \log_3 6 = 1.63$$

$$n^c = n^{1.63}, f(n) = n^2 \log n$$

$$f(n) > n^c$$

$$T(n) = \Theta(f(n))$$

$$T(n) = \Theta(n^2 \log n)$$

$$19. \rightarrow T(n) = 4T(n/2) + n/\log n$$

$$a=4, b=2, c = \log_2 4 = 2$$

$$n^c = n^2, f(n) = n/\log n$$

$$f(n) < n^c$$

$$T(n) = \Theta(n^c) = \Theta(n^2)$$

$$20. \rightarrow T(n) = 64T(n/8) - n^2 \log n$$

$$a=64, b=8, c = \log_8 64 = 2$$

$$n^c = n^2, f(n) = -n^2 \log n = n^2 \log n - 1 = n^2 \log 1/n$$

$$f(n) > n^c$$

$$T(n) = \Theta(f(n)) = \Theta(n^2 \log 1/n)$$

$$21. \rightarrow T(n) = 7T(n/3) + n^2$$

$$a=7, b=3, c = \log_3 7 = 1.77$$

$$n^c = n^{1.77}, f(n) = n^2$$

$$f(n) > n^c$$

$$T(n) = \Theta(f(n)) = \Theta(n^2)$$

$$22. \rightarrow T(n) = T(n/2) + n(2 - \cos n)$$

$$a=1, b=2, c = \log 1 = 0$$

$$n^c = n^0 = 1, f(n) = n(2 - \cos n)$$

$$f(n) > n^c$$

$$T(n) = \Theta(f(n))$$

$$T(n) = \Theta(n(2 - \cos n))$$