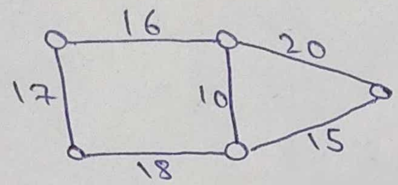
Tutorial-6

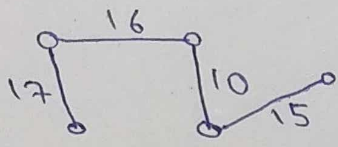
① **Minimum Spanning Tree :**

A spanning Tree of an undirected graph is a subgraph that is a tree Joined by all vertices. One of those tree which has minimum total cost would be its minimum spanning Tree.

eg :-



Minimum cost spanning Tree



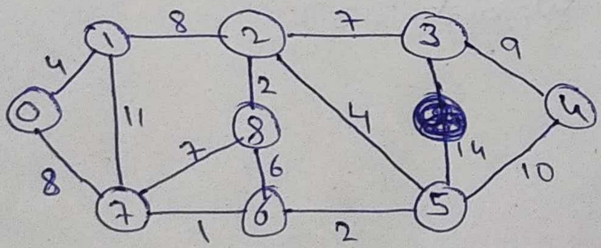
Application of MST –

It has direct applications in the design of networks including computer networks, telecommunication networks, transportation networks etc.

②

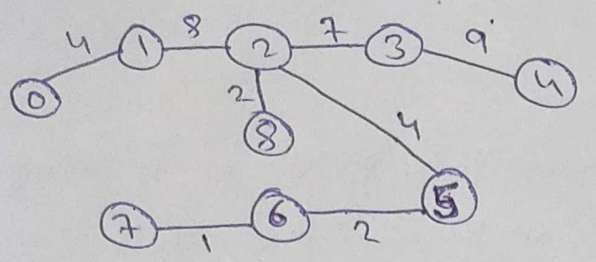
	Prim's Algorithm	Krushkal Algo	Dikstra's Algo	Bellman Ford's Algo
TC	$O(V^2)$	$O(E \log V)$	$O(V + E\log V)$	$O(VE)$
SC	$O(V+E)$	$O(\lvert E\rvert + \lvert V\rvert)$	$O(V^2)$	$O(V2)$

③



Prim's Algorithm

0	1	2	3	4	5	6	7	8
$\boxed{0}$	~~∞~~	∞	∞	∞	∞	∞	∞	∞
	$\boxed{4}$	$\boxed{8}$	$\boxed{7}$		$\boxed{4}$	~~6~~	~~7~~	$\boxed{2}$
						$\boxed{2}$	$\boxed{1}$	
				~~10~~ $\boxed{9}$				



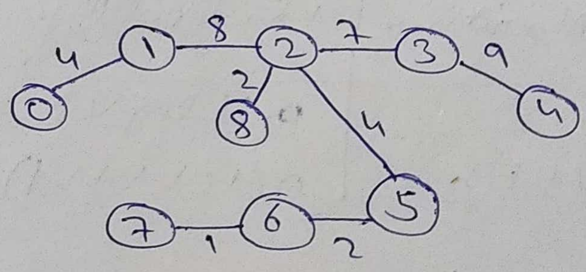
Prim Weight = 37

Parent :	0	1	2	3	4	5	6	7	8
	-1	~~-1~~	~~-1~~	~~-1~~	-1	~~-1~~	-1	~~-1~~	~~-1~~
		0	1	2		2		~~0~~	2
							~~8~~	~~8~~	
					~~5~~ 3		5	6	

Parent : -1 0 1 2 3 2 5 6 2

Kruskal's Algorithm :

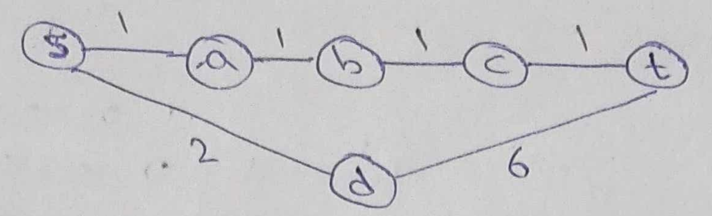
u	v	w	
7	6	1	✓
6	5	2	✓
2	8	2	✓
2	5	4	✓
0	1	4	✓
8	6	6	×
7	8	7	×
2	3	7	✓
1	2	8	✓
0	7	8	×
3	4	9	✓
5	4	10	×
1	7	11	×
3	5	14	×



Weight = 37

(4) i) If 10 units is added to each edge, the overall weight of the path may change.

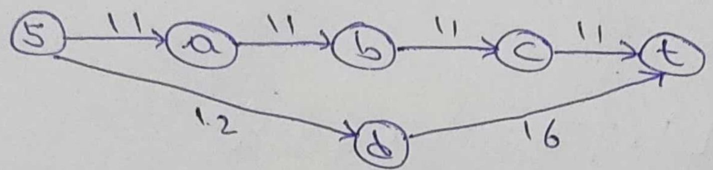
eg:-



Shortest path is $s \to a \to b \to c \to t$

Weight $= 1 + 1 + 1 + 1 = 4$

Now if 10 units weight is added to each edge.

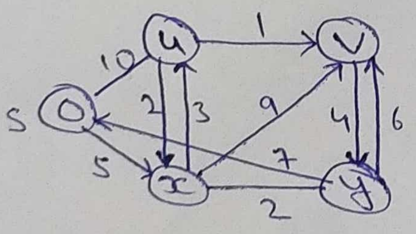


Shortest path changed to $S \to d \to t$

Weight $= 28$

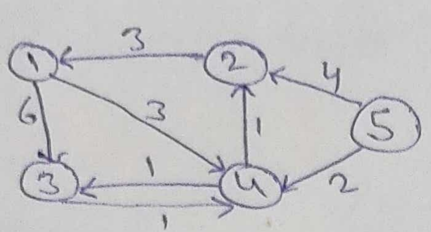
ii) Multiplying the weight of each by 10 will have no impact on the shortest path.

(5)



s	u	v	x	y
0	∞	∞	∞	∞
0	10	∞	5	∞
0	10	11	5	∞
0	10	11	5	7

⑥ All pair shortest path Algorithm - Floyd Warshall



$$A^0 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \infty & 6 & 3 & \infty \\ 2 & 3 & 0 & \infty & \infty & \infty \\ 3 & \infty & \infty & 0 & 2 & \infty \\ 4 & \infty & 1 & 1 & 0 & \infty \\ 5 & \infty & 4 & \infty & 2 & 0 \end{array}$$

$$A^1 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \infty & 6 & 3 & \infty \\ 2 & 3 & 0 & 9 & 6 & \infty \\ 3 & \infty & \infty & 0 & 2 & \infty \\ 4 & \infty & 1 & 1 & 0 & \infty \\ 5 & \infty & 4 & \infty & 2 & 0 \end{array}$$

$A^0[2,3] = \infty$

$A^0[2,1] + A^0[1,3] = 3 + 6 = 9$

$9 < \infty$

Similarly,

$A^0[2,4] = \infty$

$A^0[2,1] + A^0[1,4] = 3 + 3 = 6 < \infty$

$A^0[2,5] = \infty$

$A^0[2,1] + A^0[1,5] = 3 + \infty$

$$A^2 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \infty & 6 & 3 & \infty \\ 2 & 3 & 0 & 9 & 6 & \infty \\ 3 & \infty & \infty & 0 & 2 & \infty \\ 4 & \infty & 1 & 1 & 0 & \infty \\ 5 & 7 & 4 & 13 & 2 & 0 \end{array}$$

$A^1[1,3] = 6$

$A^1[1,2] + A^1[2+3] = \infty + 9$

$6 < \infty + 9$

~~A° [2,5]~~

$$A^3 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \infty & 6 & 3 & \infty \\ 2 & 3 & 0 & 9 & 6 & \infty \\ 3 & \infty & \infty & 0 & 2 & \infty \\ 4 & \infty & 1 & 1 & 0 & \infty \\ 5 & 7 & 4 & 13 & 2 & 0 \end{array}$$

$$A^{9} = \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ \infty & 3 & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 3 & 3 & 2 & 0 \end{bmatrix} \end{array}$$

$$A_{5} = \begin{array}{c|c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 4 & 4 & 3 & \infty \\ 3 & 0 & 7 & 6 & \infty \\ \infty & 3 & 0 & 2 & \infty \\ \infty & 1 & 1 & 0 & \infty \\ 7 & 3 & 3 & 2 & 0 \end{bmatrix} \end{array}$$