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Q1)

Using consequence rule, we can say that,

$$\frac{A' \Rightarrow I \qquad \{I\}\, P\, \{I \wedge i = n\} \qquad I \wedge i = n \Rightarrow r = 2^n - 1}{\underbrace{\{n \geq 0 \wedge r = 0 \wedge i = 0 \wedge p = 1\}}_{A'}\ P\ \underbrace{\{r = 2^n - 1\}}_{B'}}$$

Therefore we have,

- $A' \Rightarrow I$.
- $n \geq 0 \wedge r = 0 \wedge i = 0 \wedge p = 1 \Rightarrow p = 2^i \wedge r = 2^i - 1 \wedge i <= n$.

We also have,

$$(I \wedge i = n) \Rightarrow r = 2^n - 1.$$

$$p = 2^i \wedge r = 2^i - 1 \wedge i \leq n \wedge i = n \Rightarrow r = 2^n - 1.$$

We know that,

$$\frac{\{I \wedge b\}\ S\ \{I\}}{\{I\}\ \text{while } b \text{ do } S\ \{I \wedge \neg b\}}$$, so we can say.

$$\frac{\{I \wedge i_! = n\}\ P\ \{I\}}{A' \Rightarrow I \quad \{I\}\ P\ \{I \wedge i = n\} \quad I \wedge i = n \Rightarrow B'}$$

$$\{\underbrace{n \geq 0 \wedge r = 0 \wedge i = 0 \wedge p = 1}_{A'}\}\ P\ \{\underbrace{r = 2^n - 1}_{B'}\}.$$

Now using the rule of assignment and consequence we can write,

$$\frac{\{I[r-p/r,\ 2*p/p,\ r+p/r,\ i+1/i]\}\ P\ \{I\}}{\{I \wedge i! = n\}\ r = r - p;\ p = 2*p;\ r = r + p;\ i = i + 1\ \{I\}}$$

$$\frac{\{I \wedge i! = n\}\ P\ \{I\}}{A' \Rightarrow I \quad \{I\}\ P\ \{I \wedge i = n\}. \quad \{I \wedge i = n\} \Rightarrow B}$$

$$\{n \geq 0 \wedge r = 0 \wedge i = 0 \wedge p = 1\}\ P\ \{r = 2^n - 1\}.$$

Thus,

$$(I \wedge i != n) \Rightarrow I[r-p/r,\ 2^*p/p,\ r+p/r,\ i+1/i]$$

We can simplify the above formula:

$$(p = 2^i \wedge r = 2^i - 1 \wedge i \leq n \wedge i != n)$$

$$\Downarrow$$

$$p = 2^{i+1} \wedge r = 2^{i+1} - 1 \wedge i+1 \leq n \left[\begin{matrix} r-p/r,\ 2^*p/p \\ r+p/r \end{matrix}\right]$$

$$p = 2^{i+1} \wedge r+p = 2^{i+1} - 1 \wedge i+1 \leq n \left[r-p/r,\ 2^*p/p\right]$$

$$2^*p = 2^{i+1} \wedge r+2p = 2^{i+1} - 1 \wedge i+1 \leq n \left[r-p/r\right]$$

$$2p = 2^{i+1} \wedge r+p = 2^{i+1} - 1 \wedge i+1 \leq n$$

$$p = 2^i \wedge r = 2^i - 1 \wedge i \leq n-1$$

Let I be $p = 2^i \wedge r = 2^i - 1 \wedge i \leq n$.

Then,

1. $n \geq 0 \Rightarrow I[0/r, 0/i, 1/p]$

$n \geq 0 \Rightarrow p = 1 \wedge r = 0 \wedge n \geq 0$.

2. $(I \wedge i \mathrel{!}= n) \Rightarrow I([r+p/r, 2p/p, r-p/r, i+1/i])$

$(I \wedge i \mathrel{!}= n) \Rightarrow p = 2^i \wedge r = 2^i - 1 \wedge i+1 \leq n$.

3. $(I \wedge i = n) \Rightarrow r = 2^n - 1$.

$(p = 2^i \wedge r = 2^i - 1 \wedge i \leq n \wedge i = n) \Rightarrow r = 2^n - 1$

All constraints are valid.
Hence the program is correct.