Birla Institute of Technology & Science, Pilani

Second Semester 2019-2020, MATH F113 (Probability & Statistics)

Tutorial Sheet - 1 **Syllabus: Module 4 (Joint Distributions)**

1. Students are encouraged to solve these problems by themselves before they actually attend the tutorial class.
2. During one-hour tutorial, do not expect that the tutorial instructor will provide the entire solution of a problem. Rather, you are supposed to clarify your doubts or verify your solution.

1. Let X denote the number of times a photocopy machine will malfunction: 0, 1, 2 or 3 times, on any given month. Let Y denote the number of times (0, 1 or 2) a technician is called on an emergency service. The joint pmf is given as: $p(0,0) = 0.15$, $p(0,1) = 0.05$, $p(0,2) = 0$, $p(1,0) = 0.30$, $p(1,1) = 0.15$, $p(1,2) = 0.05$, $p(2,0) = 0.05$, $p(2,1) = 0.05$, $p(2,2) = 0.10$, $p(3,0) = 0$, $p(3,1) = 0.05$, and $p(3,2) = 0.05$. Find **(i)** $P(X < Y)$, **(ii)** the marginal pdfs of X and Y, and **(iii)** $\text{Cov}(X, Y)$.

2. Consider two continuous random variables X and Y with pdf

$$f(x,y) = \begin{cases} \frac{2}{81}x^2y; & 0 < x < k, 0 < y < k \\ 0; & \text{otherwise} \end{cases}$$

Find **(i)** k, **(ii)** $P(X < Y)$, **(iii)** $P(X + Y > 3)$, and **(iv)** the marginal pdfs of X and Y. Are X and Y independent?

3. Consider two continuous random variables X and Y with pdf

$$f(x,y) = \begin{cases} k(x+y); & x > 0, y > 0, 3x + y < 3 \\ 0; & \text{otherwise} \end{cases}$$

(a) Find **(i)** k, **(ii)** $P(X > 3Y)$, **(iii)** the marginal pdfs of X and Y, and **(iv)** the conditional pdfs of X and Y. Are X and Y independent?
(b) Compute **(i)** $\text{Cov}(X+2, Y-3)$, **(ii)** $\text{Corr}(-2X+3, 2Y+7)$, and **(iii)**$\text{Cov}(-2X+3Y-4, 4X+7Y+5)$.

4. Consider two continuous random variables X and Y with pdf

$$f(x,y) = \begin{cases} ke^{-y}; & 0 < y < \infty, -y < x < y \\ 0; & \text{otherwise} \end{cases}$$

(a) Find **(i)** k, **(ii)** the marginal pdfs of X and Y, and **(iii)** the conditional pdfs of X and Y.
(b) Compute $\text{Cov}(X, Y)$ and hence comment whether X and Y are independent or not.

5. Dick and Jane have agreed to meet for lunch between noon (0:00 pm) to 1:00 pm. Denote Jane's arrival time by X, Dick's by Y, and suppose X and Y are independent with density functions

$$f_X(x) = \begin{cases} 3x^2; & 0 < x < 1 \\ 0; & \text{otherwise} \end{cases}$$

$$f_Y(y) = \begin{cases} 2y; & 0 < y < 1 \\ 0; & \text{otherwise} \end{cases}$$

(a) Find the probability that Jane arrives before Dick, and hence compute the expected amount of time Jane would have to wait for Dick to arrive.
(b) If they have pre-decided on a condition that whoever comes first will only wait for 15 minutes for the other, what is the probability that they will meet for lunch?

6. Let T_1, T_2, ..., T_k be independent exponential random variables with mean values $1/\lambda_1$, $1/\lambda_2$, ..., $1/\lambda_k$, respectively. Denote $T_{min} = \min(T_1, T_2, ..., T_k)$. Show that T_{min} has an exponential distribution. What is the mean of T_{min}?

Stay Safe!