

BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI

Second Semester, 2020-21

MATH F113

Tutorial sheet- II

Q.1. Consider the type of clothes dryer (gas or electric) purchased by each of five different customers at a certain store. Assume that one person is purchasing only one dryer.

- If the probability that at most one of these customers purchases an electric dryer is 0.428, then what is the probability that at least two customers purchase an electric dryer?
- If $P(\text{all five- purchase gas}) = 0.116$ and $P(\text{all five- purchase electric}) = 0.005$, what is the probability that at least one of each type is purchased?

Q.2. An insurance company offers four different deductible levels—none, low, medium, and high—for its homeowner's policyholders and three different levels—low, medium, and high—for its automobile policyholders. The accompanying table gives proportions for the various categories of policyholders who have both types of insurance. For example, the proportion of individuals with both low homeowner's deductible and low auto deductible is 0.06 (6% of all such individuals)

Homeowner's				
Auto	N	L	M	H
L	0.04	0.06	0.05	0.03
M	0.07	0.10	0.20	0.10
H	0.02	0.03	0.15	0.15

Suppose an individual having both types of policies is randomly selected.

- What is the probability that the individual has a medium auto deductible and a high homeowner's deductible?
- What is the probability that the individual has a low auto deductible? A low homeowner's deductible?
- What is the probability that the individual is in the same category for both auto and homeowner's deductibles?
- Based on your answer in part (c), what is the probability that the two categories are different?
- What is the probability that the individual has at least one low deductible level?
- Using the answer in part (e), what is the probability that neither deductible level is low?

Q.3. A certain system can experience three different types of defects. Let A_i ($i = 1, 2, 3$) denote the event that the system has a defect of type i . Suppose that

$$P(A_1) = 0.12, P(A_2) = 0.07, P(A_3) = 0.05$$

$$P(A_1 \cup A_2) = 0.13, P(A_1 \cup A_3) = 0.14$$

$$P(A_2 \cup A_3) = 0.10, P(A_1 \cap A_2 \cap A_3) = 0.01$$

- a. What is the probability that the system does not have a type 1 defect?
- b. What is the probability that the system has both type 1 and type 2 defects?
- c. What is the probability that the system has both type 1 and type 2 defects but not a type 3 defect?
- d. What is the probability that the system has at most two of these defects?

Q.4. Reconsider the system defect situation described in the above question.

- a. Given that the system has a type 1 defect, what is the probability that it has a type 2 defect?
- b. Given that the system has a type 1 defect, what is the probability that it has all three types of defects?
- c. Given that the system has at least one type of defect, what is the probability that it has exactly one type of defect?
- d. Given that the system has both of the first two types of defects, what is the probability that it does not have the third type of defect?

Q.5. At a certain gas station, 40% of the customers use regular gas (A_1), 35% use plus gas (A_2), and 25% use premium (A_3). Of those customers using regular gas, only 30% fill their tanks (event B). Of those customers using plus, 60% fill their tanks, whereas of those using premium, 50% fill their tanks. consider the following additional information on credit card usage:

- 70% of all regular fill-up customers use a credit card.
- 50% of all regular non-fill-up customers use a credit card.
- 60% of all plus fill-up customers use a credit card.
- 50% of all plus non-fill-up customers use a credit card.
- 50% of all premium fill-up customers use a credit card.
- 40% of all premium non-fill-up customers use a credit card.

Compute the probability of each of the following events for the next customer to arrive (Hint: tree diagram).

- a. {plus and fill-up and credit card}
- b. {premium and non-fill-up and credit card}
- c. {premium and credit card}
- d. {fill-up and credit card}
- e. {credit card}
- f. If the next customer uses a credit card, what is the probability at premium was requested?

Q.6. Suppose that five good and two defective fuses have been mixed up. To find the defective ones, we test then one by one, at random and without replacement. What is the probability that we find both of the defective fuses in exactly three tests?

- Q.7.** An urn contains 10 white and 12 red chips. Two chips are drawn at random (one by one) and without looking at their colors, are discarded.
- a.** What is the probability that a third chip drawn is red?
 - b.** If the third chips is drawn randomly and observed to be red, what is the probability that both of the discarded chips were white?
- Q.8.** Professor Stan der Deviation can take one of two routes on his way home from work. On the first route, there are four railroad crossings. The probability that he will be stopped by a train at any particular one of the crossings is 0.1, and train operate independently at the four crossings. The other route is longer but there are only two crossings, independent of one another, with the same stoppage probability for each as on the first route. On a particular day, Professor Deviation has a meeting scheduled at home for a certain time. Whichever route he takes; he calculates that he will be late if he is stopped by trains at atleast half the crossings encountered.
- a.** Which route should he take to minimize the probability of being late to the meeting?
 - b.** If he tosses a fair coin to decide on a route and he is late, what is the probability that he took the four-crossing route?