

**Birla Institute of Technology & Science, Pilani**  
**Second Semester 2019-2020, MATH F113 (Probability & Statistics)**

**Tutorial Sheet - 2**

**Syllabus: Module 5 (Sec. 5.3-5.5: Descriptive Statistics)**

1. Students are encouraged to solve these problems by themselves before they actually attend the tutorial class.
2. During one-hour tutorial, do not expect that the tutorial instructor will provide the entire solution of a problem. Rather, you are supposed to clarify your doubts or verify your solution.

Q1. Let  $X$  denote the gasoline mileage obtained in tests on a newly designed SUV (sport utility vehicle). A sample of size 22 obtained are as

5	16	17	18	17	20	16	17	18	20	18
18	19	19	17	21	17	19	18	17	42	50

- a. Find the sample mean, sample median, sample variance and sample standard error for these data.
- b. Are there any outliers present in the data? To see the effect of outliers, drop them (if any) from data and calculate sample mean, sample median, sample variance and standard error of the sample mean.

Q2. A manufacturing company produces electric insulators. If the insulators break when in use, a short circuit is likely. To test the strength of the insulators, you carry out destructive testing to determine how much force is required to break the insulators. You measure force by observing how many pounds are applied to the insulator before it breaks. The following table lists 30 values from this experiment.

1,870	1,728	1,656	1,610	1,634	1,784	1,522	1,696	1,592	1,662
1,866	1,764	1,734	1,662	1,734	1,774	1,550	1,756	1,762	1,866
1,820	1,744	1,788	1,688	1,810	1,752	1,680	1,810	1,652	1,736

- a. Find the sample mean, sample standard deviation, and sample range for the above data.
- b. Assuming that the distribution is normal, can you provide an estimate of  $\sigma$  by using the sample range calculated in part (a)?
- c. If the distribution is not normal, can you provide an estimate of  $\sigma$  by using the sample range calculated in part (a)?

Q3. Let  $X$  be the number of packages being mailed by a randomly selected customer at a certain shipping facility. Suppose the distribution of  $X$  is as follows:

$x$	1	2	3	4
$p(x)$	0.4	0.3	0.2	0.1

- a. Consider a random sample of size  $n = 2$  (two customers), and let  $\bar{X}$  be the sample mean number of packages shipped. Obtain the probability distribution of  $\bar{X}$ .
- b. Refer to part (a) and calculate  $P(\bar{X} \leq 2.5)$ .
- c. Again consider a random sample of size  $n = 2$ , but now focus on the statistic  $R$  = the sample range (difference between the largest and smallest values in the sample). Obtain the distribution of  $R$ . [Hint: Calculate the value of  $R$  for each outcome and use the probabilities from part (a).]
- d. If a random sample of size  $n = 4$  is selected, what is  $P(\bar{X} \leq 1.5)$ ? (Hint: You should not have to list all possible outcomes, only those for which  $\bar{X} \leq 1.5$ ).

- Q4. Suppose your waiting time for a bus in the morning is uniformly distributed on  $[0,5]$ , whereas waiting time in the evening is uniformly distributed on  $[0,10]$  independent of morning waiting time.
- If you take the bus each morning and evening for a week, what is your total expected waiting time? [Hint: Define random variables  $\bar{X}_1, \bar{X}_2, \dots, \bar{X}_{10}$  and use a rule of expected value.)
  - What is the variance of your total waiting time?
  - What are the expected value and variance of the difference between morning and evening waiting times on a given day?
  - What are the expected value and variance of the difference between morning waiting time and total evening waiting time for a particular week?
- Q5. The daily amount of coffee, in litres, dispensed by a coffee vending machine located at New Delhi airport lobby is equally likely to lie between 7 to 10 litres. If daily amount of coffee from this machine is collected for a random sample of 100 days, what would be the distribution of the sample mean?
- Q6. The breaking strength of a rivet has a mean value of 10,000 psi and a standard deviation of 500 psi.
- What is the probability that the sample mean breaking strength for a random sample of 40 rivets is between 9950 and 10,250?
  - If the sample size had been 15 rather than 40, could the probability requested in part (a) be calculated from the given information?
- Q7. a. Let  $X_1$  and  $X_2$  be independent Poisson random variable with parameters  $\mu_1$  and  $\mu_2$  respectively. Let  $Y = X_1 + X_2$ . What is the distribution of  $Y$ ?
- b. Let  $X_1 \dots X_n$  be a random sample from an exponential population with parameter  $\lambda$ . Let  $Y = X_1 + \dots + X_n$ . What is the distribution of  $Y$ ?

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