Birla Institute of Technology & Science, Pilani

Second Semester 2019-2020, MATH F113 (Probability & Statistics)

Tutorial Sheet - 3 **Syllabus: Module 5 (Sec. 6.1, 6.2: Point Estimation)**

1. Students are encouraged to solve these problems by themselves before they actually attend the tutorial class.
2. During one-hour tutorial, do not expect that the tutorial instructor will provide the entire solution of a problem. Rather, you are supposed to clarify your doubts or verify your solution.

1. Prove or disprove: The sample standard deviation S of a random sample of size n is an unbiased estimator of the population standard deviation σ. (Hint: Any random variable has nonzero variance.)
2. A population X has $E[X] = 0, E[X^2] = 1$ and $E[X^4] = 4$. For a random sample of size 2, find the standard error of S^2.
3. The deaths of the patients with a disease occur in Poisson process with the average rate of α per day. Since the start of monitoring, the 1st death occurs after 11 hours, 2nd after 20 hours, 3rd after 26 hours and the 4th after 30 hours. Using this information, find the estimate for α using (a) method of moments, (b) method of maximum likelihood.
4. Suppose the population X is a continuous uniform distribution on the interval $[a, 2a]$ where a is the parameter of the distribution. Find the maximum likelihood estimator of a using a random sample of size n.
5. Suppose the population X is a continuous uniform distribution on the interval $[a, a+1]$ where a is the parameter of the distribution. Find the maximum likelihood estimator of a using a random sample of size n. Show that the MLE estimator of a is not unique.
6. Let $X_1, X_2, \dots, X_n$ are i.i.d random variables with density $f(x) = \frac{1}{2\sigma}\exp\left(-\frac{|x|}{\sigma}\right), \sigma > 0.$ Find estimator of σ, using (a) method of moments and (b) maximum likelihood estimation.
7. Suppose the true average growth μ of one type of plant during a 1-year period is identical to that of a second type, but the variance of growth for the first type is σ^2, whereas for the second type, the variance is $4\sigma^2$. Let $X_1, X_2, \dots, X_m$ be m independent growth observations on the first type [so $E[X_i] = \mu, V[X_i] = \sigma^2$], and let $Y_1, Y_2, \dots, Y_n$ be n independent growth observations on the second type [so $E[Y_i] = \mu, V[Y_i] = 4\sigma^2$].

   a. Show that for any δ between 0 and 1, the estimator $\hat{\mu} = \delta\bar{X}+(1-\delta)\bar{Y}$ is unbiased for μ.

   b. For fixed m and n, compute $V[\hat{\mu}]$ and then find the value of δ that minimizes $V[\hat{\mu}]$.
8. The **mean squared error** of an estimator $\hat{\theta}$ (of θ) is $MSE(\hat{\theta}) = E(\hat{\theta} - \theta)^2$. If $\hat{\theta}$ is unbiased, then $MSE(\hat{\theta}) = V(\hat{\theta})$ but in general $MSE(\hat{\theta}) = V(\hat{\theta}) + (bias)^2$. Consider the estimator $\hat{\sigma}^2 = KS^2$ where $S^2 =$ sample variance. What value of K minimizes the mean squared error of this estimator when the population distribution is normal? [*Hint:* It can be shown that $E((S^2)^2) = \frac{(n+1)\sigma^4}{(n-1)}$. In general, it is difficult to find $\hat{\theta}$ to minimize $MSE(\hat{\theta})$, which is why we look only at unbiased estimators and minimize $V(\hat{\theta})$.]

   Using the observations 0.5, 1, -1.5, 2 from the standard normal distribution, find an unbiased estimate, the maximum likelihood estimate and an estimate KS^2 for the value of K which minimizes the mean squared error.

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