BITS Pilani
Second Semester 2020-21
Math F113
Tutsheet-3

1. Consider X = the number of toys chosen from a shop, with $F(0) = 0.32,\ F(1) = 0.44,\ F(2) = 0.58,\ F(3) = 0.76,$ and$F(4) = 0.82$. Compute the following:

a. $P(1 \leq X \leq 4)$.

b. $P(X = 2)$.

2. Consider X is a random variable and the cdf of X is,

$$F_X(x) = \begin{cases} 0 & \text{for } x < 0 \\ \frac{1}{4} & \text{for } 0 \leq x < 1 \\ \frac{3}{4} & \text{for } 1 \leq x < 2 \\ 1 & \text{for } x \geq 2 \end{cases}$$

   Find its pmf?

3. Let X be a random variable that takes values from 0 to 9 with equal probability $\frac{1}{10}$. Find the pmf of the random variable $Y = X$ mod (3).

4. Suppose 3 people all toss a hat into a box and then proceed to randomly pick out a hat without replacement. What is the expected number of people to get their own hat back.

5. Consider an observation on the gender of newborn child at a hospital until a boy is born. Let $p = P(B)$ being the probability that a boy is born, and the births are independent scenario. If we define the random variable X by $x =$ number of births observed. Find the pmf for the above.

6. The pmf of the number X of persons praying at church was given as $p(1) = 0.30,\ p(2) = 0.25,\ p(3) = 0.15,\ p(4) = 0.05,\ p(5) = 0.10,$ and $p(6) = 0.15$. Find the variance.

7. Using MGFs prove that if $X \sim Binomial(m, p)$ and $Y \sim Binomial(n, p)$ are independent, then $(X + Y) \sim Binomial(m + n, p)$.

8. If X has the discrete Uniform distribution $f(x) = \frac{1}{\mathrm{K}}$ for $x = 1, 2, \cdots, K$. Find the corresponding mgf & $E(X^2)$ using mgf.