Ans 1.

(a). Let E be the Event that atmost one purchased an electric dryer. The E' is the Event that atleast two purchase electric dryer, and

$$P(E') = 1 - P(E)$$

$$= 1 - .428$$

$$= .575$$

(b) Let A = All five purchase gas

B = All five purclase electric

$\because$ Both are not related to each other, so

$$P[A \cap B] = 0$$

P[Atleast one of each type is purchased]

$$= 1 - P[A \cup B]$$

$$= 1 - [P(A) + P(B)]$$

$$= 1 - [.116 + .005]$$

$$= .879$$

Ans 2. The first letter refers to the auto deductible and the Second letter refers to the homeowner's deductible.

For example: the proportion of Individuals with both low homeowner's deductive and low auto deductible is $= 0.06$,

$$P(LL) = 0.06$$

(a). $P[\text{Medium and high auto deductible}] = 0.10$ (from table)

(b). $P[\text{low auto deductible}] = P(\{LL, LN, LM, LH\})$

$= .04 + .06 + .05 + .03$

$= 0.18$

$P[\text{low homeower's deductible}] = 0.06 + 0.10 + .03$

$= 0.19$

(c). $P(\text{Same deductible for both}) = P(\{LL, MM, HH\})$

$= .06 + .20 + .15$

$= .41$

(d). $P(\text{Deductibles are different}) = 1 - P(\{\text{Same deductible for both}\})$

$= 1 - .41$ [from (c)]

$= .59$

(e) $P(\text{Atleast one low deductible}) = P(\{LL, LN, LM, LH, ML, HL\})$

$= .04 + .06 + .05 + .03 + .10 + .03$

$= .31$

(f) $P(\text{Neither dedictible is low}) = 1 - P(\text{Atleast one low deductible})$

$= 1 - .31 = .69$

Ans 3. A_i = the Event that the system has a defect of type i.
for $i = 1,2,3$

given that

$P(A_1) = .12 \quad P(A_2) = .07 \quad P(A_3) = .05$

$P(A_1 \cup A_2) = .13 \quad P(A_1 \cup A_3) = .14$

$P(A_2 \cup A_3) = .10 \quad P(A_1 \cap A_2 \cap A_3) = .01$

(a). P(the system does not have a type 1 defect) = $P(A_1')$

$1 - P(A_1) = 1 - .12 = .88$

(b). P(the system has both type 1 and type 2 defects)

$= P(A_1 \cap A_2)$

from the addition rule

$P(A_1) + P(A_2) = P(A_1 \cup A_2) + P(A_1 \cap A_2)$

$\Rightarrow \quad P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2)$

$= 0.12 + 0.07 - 0.13$

$= 0.06$

(c). P(the system has both type 1 and type 2 defects but not a type 3 defect) = $P(\underbrace{A_1 \cap A_2}_{A} \cap \underbrace{A_3'}_{B})$

$\because \quad P(A \cap B') = P(A) - P(A \cap B)$

$= P(A_1 \cap A_2) - P(A_1 \cap A_2 \cap A_3)$

$= 0.06 - 0.01 = 0.05$

(d). P(the system has atmost two of these defects),

$\because$ The event " atmost two defects" is the Complement of " all three defects" so the $P((A_1 \cap A_2 \cap A_3)')$

$= 1 - P(A_1 \cap A_2 \cap A_3)$

$= 1 - 0.01$

$= .99$

Ans 4. With Continuation to the Que. ③.

(a). P[System has a type 2 defect given that it has a type 1 defect] = $P[A_2 / A_1]$ $\because P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$$= \frac{P[A_2 \cap A_1]}{P[A_1]}$$

from. solⁿ ③. $P(A_1 \cap A_2) = 0.06$, $P(A_1) = .12$

$$\Rightarrow \frac{0.06}{.12} = .50.$$

(b). P[system has all three type of defects given that it has a type 1 defect]

$$\Rightarrow P[A_1 \cap A_2 \cap A_3 / A_1]$$

$$= \frac{P[(A_1 \cap A_2 \cap A_3) \cap A_1]}{P[A_1]}$$ $\because P[(A \cap B) \cap A] = P[A \cap B]$

$$= \frac{P[A_1 \cap A_2 \cap A_3]}{P[A_1]}$$

$$= \frac{0.01}{.12} = 0.0833$$

(c). P[system has exactly one type of defect given that it has atleast one type of defect]

Let E = " exactly one defect "

from venn diagram.

$$P(E) = .04 + .00 + .01 = .05$$

P(atleast one defect) = $P(A_1 \cup A_2 \cup A_3)$

$$\because \quad P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C) \quad -(1)$$

By using eqn (1)

$$P(A_1 \cup A_2 \cup A_3) = .14$$

So,

$$P[E / A_1 \cup A_2 \cup A_3] = \frac{P[(E \cap [A_1 \cup A_2 \cup A_3])]}{P[A_1 \cup A_2 \cup A_3]}$$

$$= \frac{0.05}{.14}$$

$$= .3571$$

(d). P[system has no third type defect given that it has both of first two type of defects] = $P[A_3' \cap A_1 \cap A_2]$

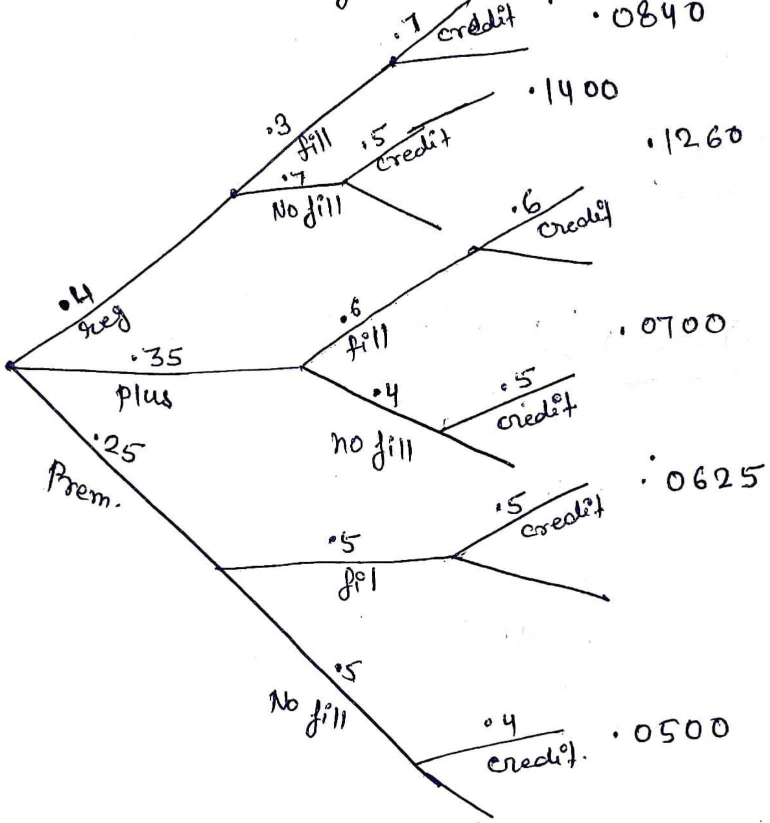
$$= \frac{P[A_3' \cap [A_1 \cap A_2]]}{P[A_1 \cap A_2]}$$

from ~~eq~~ soln (3).

$P[A_3' \cap (A_1 \cap A_2)] = .05$

$$\Rightarrow \frac{.05}{.06} = .8333$$

Ans 5. With the help of tree diagram.



(a) P{ plus and fill-up and credit card)

$= P(\text{plus} \cap \text{fill} \cap \text{credit})$

$= (.35)(.6)(.6) = .1260.$

(b). $P(\text{Premium} \cap \text{no fill} \cap \text{credit}) = (.25)(.5)(.4) = .05$

(c). $P(\text{premium and credit}) = .0625 + .0500 = .1125$ (using diagram)

(d). $P(\text{fill} \cap \text{credit}) = .0840 + .1260 + .0625 = .2725$

(e). $P(\text{credit}) = .0840 + .1400 + .1260 + .0700 + .0625 + .0500$

$= .5325$

(f) $P[\text{Premium} \mid \text{credit}] = \dfrac{P[\text{premium} \cap \text{credit}]}{P[\text{credit}]}$

$= \dfrac{.1125}{.5325}$

$= .2113$

Ans 6. Let

D_1 = first fuses tested is defective

D_2 = Second " " "

D_3 = Third " " "

G_1 = First fuses tested is good

G_2 = Second " " "

G_3 = Third " " "

If fuses are tested one by one, at random and without replacement then

P[Both of the defective fuces in exactly three test]

$= P[G_1 D_2 D_3 \cup D_1 G_2 D_3]$ (selection step)

$= P[G_1 D_2 D_3] + P[D_1 G_2 D_3)$

$= P(G_1) P(D_2|G_1) P(D_3|G_1 D_2) + P(D_1) P(G_2|D_1) P(D_3/D_1 G_1)$

Using probability rules

$$= \frac{5}{7} \times \frac{2}{6} \times \frac{1}{5} + \frac{2}{7} \times \frac{5}{6} \times \frac{1}{5}$$

$$= 0.095$$

Ans 7. (a) # of white chips = 10

# of red chips = 12

Let $i \geq 1$, R_i = the Event the i^{th} chip drawn is red

W_i = the Event the i^{th} " " is white.

If two chips are drawn at random and discared. the

$P(R_1) = 12/22.$

$P(W_1) = \frac{10}{22}$

Then the Sample space of drawn two chips is

$\{R_2W_1, W_2R_1, R_2R_1, W_2W_1)$

$P(R_3)$ = P[third chip drawn is red

$= P[R_3/R_2W_1] \cdot P[R_2W_1] + P[R_3|W_2R_1]P(W_2R_1]$

$+ P(R_3|R_2R_1)P[R_2R_1] + P[R_3|W_2W_1)P[W_2W_1]$

$\because P[R_2W_1] = P[R_2|W_1] \cdot P(W_1) = \frac{12}{21} \times \frac{10}{22} = \frac{20}{77}$

$P[W_2R_1] = \frac{10}{21} \times \frac{12}{22} = \frac{20}{77}$

$P[R_2R_1] = \frac{11}{21} \times \frac{12}{22} = \frac{22}{77}$

$P[W_2W_1] = \frac{9}{21} \times \frac{10}{22} = \frac{15}{77}$

$\Rightarrow P[R_3] = \frac{11}{20} \times \frac{20}{77} + \frac{11}{20} \times \frac{20}{77} + \frac{10}{20} \times \frac{22}{77} + \frac{12}{20} \times \frac{15}{77}$

$= \frac{12}{22}$

(b) If the third chips is drawn randomly and observed R then

P[Both of the discarded chips are white given that third chips is red]

$$= P[W_1 W_2 / R_3] = \frac{P[R_3/W_1W_2] \cdot P[W_1 W_2]}{P[R_3]}$$

$$\because P[R_3/W_1W_2] = \frac{12}{20}$$

$$P[W_1 W_2] = \frac{15}{77}$$

$$P[R_3] = \frac{12}{20} \times \frac{15}{77} + \frac{11}{20} \times \frac{20}{77} + \frac{11}{20} \times \frac{20}{77} + \frac{10}{20} \times \frac{22}{77}$$

$$\Rightarrow P[W_1W_2/R_3] = \frac{\frac{12}{20} \times \frac{15}{77}}{\frac{12}{22}}$$

$$= \frac{3}{14}$$

given that, P(train stopped) = .01

Ans 8. For route # 1.

P[late] = P[stopped at 2 or 3 or 4 crossings]

= 1 − P[stopped at 0 time or 1 time]

= 1 − P [Not stopped] + P[stopping at once]

∵ P[Not stopped] = (.9) (.9) (.9)(.9)

∵ P[stopping at once (at crossing #1, #2, #3), or #4]

$= 4.(.9)^3 (0.1)$

P[late] at route # 1 = $1 - (.9)^4 - 4(.9)^3(.1)$

= .0523

for route #2

P(late) = P(stopped at 1 or 2 crossings)

= 1 − P(stopped at none)

$= 1 - (.9)^4 = 1 - .81 = .19$

Thus: P(late) at route #1 < P(late) at route #2

so, route #1 should be taken

(b) $P(\text{4- crossing route} / \text{late}) = \dfrac{P[\text{4-C route} \cap \text{late}]}{P[\text{late}]}$

from Bayes' theorem

$$\frac{P[\text{4-c route}) \, P[\text{late} / \text{4-c route}]}{P[\text{4-c route}] \, P[\text{late} / \text{4-c route}] + P[\text{2-c route}) \, P(\text{late} / \text{2-c route}]}$$

$$= \frac{(.5)(.0523)}{(.5)(.0523) + (.5)(.19)}$$

$$= .216$$

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