Birla Institute of Technology & Science, Pilani

Second Semester 2019-2020, MATH F113 (Probability & Statistics)

Tutorial Sheet - 4 **Syllabus: Module 6 (7.1–7.4)**

1. Students are encouraged to solve these problems by themselves before they actually attend the tutorial class.
2. During one-hour tutorial, do not expect that the tutorial instructor will provide the entire solution of a problem. Rather, you are supposed to clarify your doubts or verify your solution.

1. Assume that the alcohol content (in percentage) in a cough syrup taken from any particular brand is normally distributed with the true standard deviation .75.
(a) Compute a 95% CI for the true average alcohol content of a certain brand if the average content for 20 specimens from the brand was 4.85.
(b) Compute a 98% CI for the true average alcohol content of a another brand based on 16 specimens with a sample average alcohol content of 4.56 .
(c) How large a sample size is necessary if the width of the 95% interval is to be .40?
(d) What sample size is necessary to estimate the true average alcohol content to within .2 with 99% confidence?

2. A random sample of $n = 15$ light bulbs of a certain type yielded the following observations on lifetime (in years):

2.0 1.3 6.0 1.9 5.1 0.4 1.0 5.3 15.7 0.7 4.8 0.9 12.2 5.3 0.6

(a) Assume that the lifetime distribution is exponential. Obtain a 95% CI for expected (true average) lifetime.
(b) What is a 95% CI for the standard deviation of the lifetime distribution?

3. A company manufactures taps. Out of 143 manufactured taps, 10 are found to be defective. Calculate a lower confidence bound at the 95% confidence level for the true proportion of such taps that are defective. Also, interpret the 95% confidence level.

4. In a survey of 2003 adults, 25% said they believed in God.
(a) Calculate and interpret a confidence interval at the 99% confidence level for the proportion of all adults who believe in God.
(b) What sample size would be required for the width of a 99% CI to be almost .05 irrespective of the value of p̂?

5. Let X_1, X_2, . . . ,X_n be a random sample from a continuous probability distribution having median $\tilde{\mu}$.
(a) Show that:

$$\mathrm{P}\left[\min X_i < \tilde{\mu} < \max X_i\right] = 1 - \frac{1}{2^{n-1}},$$

so that $(\min X_i,\ \max X_i)$ is a $100(1-\alpha)\%$ confidence interval for $\tilde{\mu}$ with $\alpha = 1/2^{n-1}$.

(c) For each of six normal male infants, the amount of the amino acid alanine (mg/100 mL) was determined while the infants were on an isoleucine-free diet, resulting in the following data:

2.84 3.54 2.80 1.44 2.94 2.70

Compute a 97% CI for the true median amount of alanine for infants on such a diet.

(b). Let $X_{(2)}$ denote the second smallest of the x_is and $X_{(n-1)}$ denote the second largest of the x_is. What is the confidence coefficient of the interval $(X_{(2)}, X_{(n-1)})$ for $\tilde{\mu}$?

6. Let $X_1, X_2, \ldots, X_n$ be a random sample from a uniform distribution on the interval $[0, \theta]$, and let $U = \frac{1}{\theta} \max X_i$.

(a) Show that the rv U has density function

$$f_U(x) = \begin{cases} nx^{n-1} & 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

(b) Verify that

$$\mathrm{P}\left[\left(\frac{\alpha}{2}\right)^{1/n} \leq U \leq \left(1 - \frac{\alpha}{2}\right)^{1/n}\right] = 1 - \alpha,$$

and

$$\mathrm{P}\left[\alpha^{1/n} \leq U \leq 1\right] = 1 - \alpha.$$

Use these identities to obtain two $100(1-\alpha)\%$ CI for θ.

(c) If the waiting time for a bus during morning hours is uniformly distributed and observed waiting times :

4.2 3.5, 1.7 1.2 2.4.

Derive a 95% CI for θ by using both intervals. Which of these two CI should be preferred?

7. Find the probability that a random sample of 25 observations, from a normal population with variance $\sigma^2 = 6$, will have a sample variance $S^2 > 9.1$.

Good luck