BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI

Second Semester, 2020-21

MATH F113

Tutorial sheet- I

Q.1. Suppose that vehicles taking a particular freeway exit can turn right (R), turn left (L), or go straight (S). Consider observing the direction for each of three successive vehicles.

(a) List all outcomes in the event A that all three vehicles go in the same direction.
(b) List all outcomes in the event B that all three vehicles take different directions.
(c) List all outcomes in the event C that exactly two of the three vehicles turn right.
(d) List all outcomes in the event D that exactly two vehicles go in the same direction.
(e) List outcomes in D' , $C \cap D$, and $C \cup D$.

Q.2. Consider measuring the lifetime of a light bulb. Any nonnegative real number can be consider as the lifetime of the lightbulb (in hours). Find the sample space.

(a) The outcomes of the event E that the light bulb lifetime is atleast 100 hours.
(b) The outcomes of the event F that the light bulb lifetime is atmost 1000 hours.
(c) The outcomes of the event G that the light bulb lifetime is exactly 505.5 hours.

Q.3. An engineering construction firm is currently working on power plants at three different sites. Let A_i denote the event that the plant at site i is completed by the contract date. Use the operations of union, intersection, and complementation to describe each of the following events in terms of A_1, A_2, and A_3, draw a Venn diagram, and shade the region corresponding to each one.

(a) At least one plant is completed by the contract date.
(b) All plants are completed by the contract date.
(c) Only the plant at site 1 is completed by the contract date.
(d) Exactly one plant is completed by the contract date.
(e) Either the plant at site 1 or both of the other two plants are completed by the contract date.

Q.4. Computer keyboard failures can be attributed to electrical defects or mechanical defects. A repair facility currently has 25 failed keyboards, 6 of which have electrical defects and 19 of which have mechanical defects.

(a) How many ways are there to randomly select 5 of these keyboards for a thorough inspection (without regard to order)?
(b) In how many ways can a sample of 5 keyboards be selected so that exactly two have an electrical defect?

Q.5. A garage door opener has six toggle switches, each with three settings: up, center, and down.

(a) In how many ways can these switches be set?
(b) How many settings are possible in which two switches are up, two are down, and two are in the center?

Q.6. A stereo store is offering a special price on a complete set of components (receiver, compact disc player, speakers, cassette deck). A purchaser is offered a choice of manufacturer for each component:

1. **Receiver**: Kenwood, Onkyo, Pioneer, Sony, Sherwood
2. **Compact disc player**: Onkyo, Pioneer, Sony, Technics
3. **Speakers**: Boston, Infinity, Polk
4. **Cassette deck**: Onkyo, Sony, Teac, Technics

(a) In how many ways can one component of each type be selected?
(b) In how many ways can components be selected if both the receiver and the compact disc player are to be Sony?
(c) In how many ways can components be selected if none is to be Sony?
(d) In how many ways can a selection be made if at least one Sony component is to be included?

Q.7. Three molecules of type A, three of type B, three of type C, and three of type D are to be linked together to form a chain molecule. One such chain molecule is ABCDABCDABCD, and another is BCDDAAABDBCC.

(a) How many such chain molecules are there? [Hint: If the three As were distinguishable from one another A_1, A_2, A_3 and the $B's$, $C's$, and $D's$ were also, how many molecules would there be? How is this number reduced when the subscripts are removed from the $A's$?]

(b) Suppose a chain molecule of the type described is randomly selected. How many such chain are there that all three molecules of each type end up next to one another (such as in BBBAAADDDCCC)?