# Relational Query Languages I Relational Algebra

## What is an "Algebra"

- Mathematical system
  - Operands --- variables or values from which new values can be constructed
  - Operators --- symbols denoting procedures that construct new values from given values

## What is Relational Algebra?

- An algebra whose operands are relations or variables that represent relations
- Operators are designed to do the most common things that we need to do with relations in a database
  - The result is an algebra that can be used as a query language for relations

### Relational Query Languages (1)

- A major strength of the relational model: supports simple, powerful querying of data
- Queries can be written intuitively, and the DBMS is responsible for efficient evaluation
  - precise semantics for relational queries
  - Allows the optimizer to extensively re-order operations, and still ensure that the answer does not change.

### Relational Query Languages (2)

- Query languages: Allow manipulation and retrieval of data from a database
- Relational model supports simple, powerful QLs:
  - Strong formal foundation based on logic
  - Allows for optimization
- Query Languages != programming languages!
  - QLs not intended to be used for complex calculations
  - QLs support easy, efficient access to large data sets

### Formal Relational Query Languages

- Two mathematical Query Languages form the basis for "real" languages (e.g. SQL), and for implementation:
  - Relational Algebra: More operational, very useful for representing execution plans.
  - Relational Calculus: Lets users describe what they want, rather than how to compute it. (Non-operational, declarative.)

### The SQL Query Language (1)

- Developed by <u>IBM's System-R Project</u> in the 1970s
- Need for a standard since it is used by many vendors
- Standards:
  - SQL-86
  - SQL-89 (minor revision)
  - SQL-92 (major revision)
  - SQL-99 (major extensions, current standard)

#### The SQL Query Language (2)

- SQL has been influenced by both Relational Algebra (RA) & Relational Calculus (RC)
- The other variant of RC is Domain Relational Calculus (DRC) which has greatly influenced Query By Example (QBE)

### The SQL Query Language (3)

- SQL consists of:
  - DDL (Data Definition Language)
    - Create conceptual schema
  - DML (Data Manipulation Language)
    - Relational operators
    - Insert, Delete, Update
  - VDL (View Definition Language)
    - Specify user views & their mapping to the conceptual schema (in most DBMSs, done by DDL)
  - SDL (Storage Definition Language)
    - File organization
    - Indexes

## Operations on Relations

- Restrict/Select
- Project
- Join

**Relational Operations** 

- Divide
- Union
- Intersection
- Difference
- Product

**Set Operations** 

#### Relational Algebra Overview (Elmasri & Navathe)

#### Relational Algebra consists of several groups of operations

- Unary Relational Operations
  - SELECT (symbol: σ (sigma))
  - PROJECT (symbol: π (pi))
  - RENAME (symbol: ρ (rho))
- Relational Algebra Operations From Set Theory
  - UNION ( ∪ ), INTERSECTION ( ∩ ), DIFFERENCE (or MINUS, )
  - CARTESIAN PRODUCT (x)
- Binary Relational Operations
  - JOIN (several variations of JOIN exist)
  - DIVISION
- Additional Relational Operations
  - OUTER JOINS, OUTER UNION
  - AGGREGATE FUNCTIONS (These compute summary of information: for example, SUM, COUNT, AVG, MIN, MAX)

#### Relational Algebra

- Collection of operations on relations
- 8 operators + Rename operator
- Codd's 8 operators do not form a minimal set
- Some of them are not primitive
- Join, Intersect, & Divide can be defined in terms of other 5
- None of these 5 can be defined in terms of the remaining 4
- MINIMAL SET
- Join, Intersect, & Divide are important & should be supported directly

#### Relational Algebra

- Some common DB requests can not be performed with the RA operations
- Additional operations are developed
- Aggregate functions & additional types of joins

#### Relational Algebra

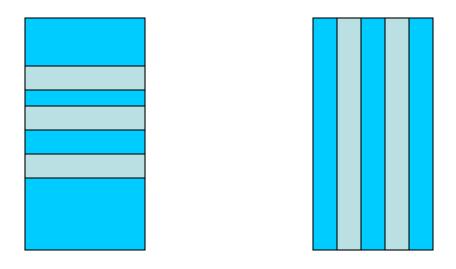
- Provides a formal foundation for relational model operations
- Plays a pivotal role in query optimization in RDBMSs
- Some of its concepts are implemented in SQL
- Expressive power of RA is used as a metric of how powerful a relational DB query language is

#### Relational Completeness

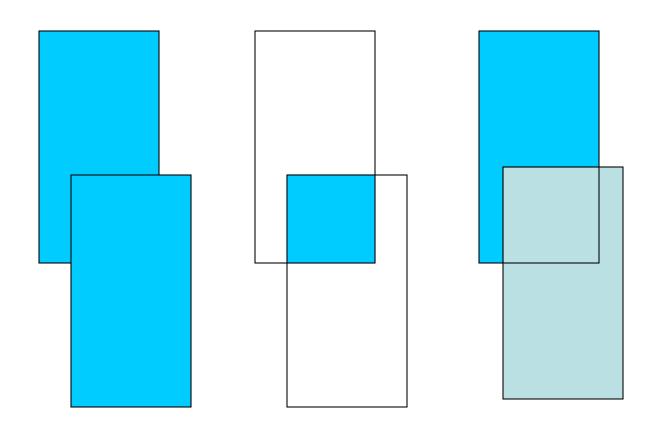
- A relational DB query language is said to be Relationally Complete, if it can express all queries that we can express in RA
- Relational DB query languages are expected to be relationally complete
- Commercial query languages like SQL, support features that allow us to express some queries that are not possible in RA

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## Restrict & Project



## Union, Intersection & Difference(1)



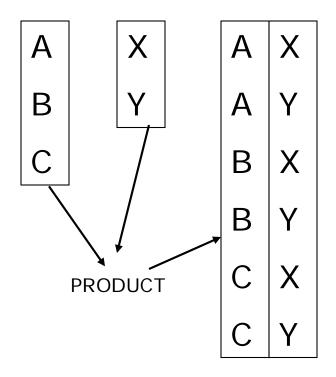
### Union, Intersection & Difference(2)

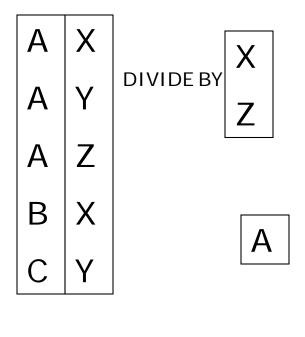
Union Compatibility: r U s is valid if:

- Relations r & s have the same arity
- Domains of the  $i^{th}$  attribute of r is the same as the domain of the  $i^{th}$  attribute of s,  $\neq i$ .

Note that r & s can be either database relations or derived relations

#### **Product & Divide**





## Divide

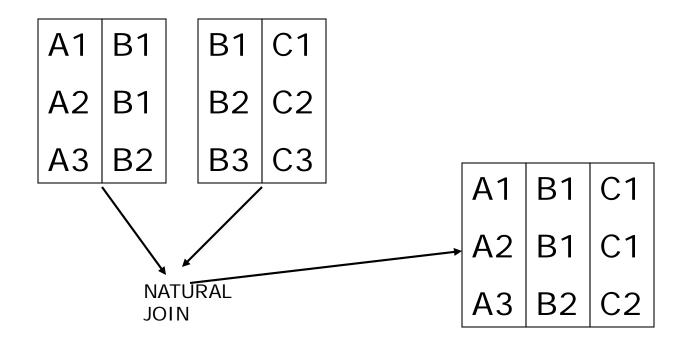
#### DIVIDE

			S1
S1	P1		S2
S1	P2	P2	
S2	P3		S3
		P2	S4
S2	P4	P4	
S2	P1	[74]	S2
S2	P2		S4
S3	P2	P1	S2
S4	P2	P2	
S4	P4	P4	

2/1/2021<sub>Shipment</sub>

Parts

#### Join



#### Closure

- A relation is closed under relational and set operators
- The result of these operators on relation(s) is another relation
- Output from one operation can become input to another
- Nested Expressions possible

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## Case Study: Sailors Database

- Sailors (<u>sid:integer</u>, sname:string,rating:integer,age:real)
- Boats (<u>bid:integer</u>,bname:string,color:string)
- Reserves (<u>sid:integer</u>, <u>bid:integer</u>, <u>day:date</u>)

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## Example Schema

R1

sid	bid	day
22	101	10/10/96
58	103	11/12/96

**S**1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

**S**2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

## **Projection**

- Deletes attributes that are not in projection list.
- Schema of result contains exactly the fields in the projection list, with the same names that they had in the (only) input relation.
- Projection operator has to eliminate duplicates! (Why??)
  - Note: real systems typically don't do duplicate elimination unless the user explicitly asks for it.

sname	rating
yuppy	9
lubber	8
guppy	5
rusty	10

 $\pi_{sname,rating}^{(S2)}$ 

age 35.0 55.5

$$\pi_{age}(S2)$$

#### Selection

- Selects rows that satisfy selection condition.
- No duplicates in result!
- Schema of result identical to schema of (only) input relation.
- Result relation can be the input for another relational algebra operation! (Operator composition.)

sid	sname	rating	age
28	yuppy	9	35.0
58	rusty	10	35.0

$$\sigma_{rating>8}(S2)$$

sname	rating
yuppy	9
rusty	10

$$\pi_{sname,rating}(\sigma_{rating} > 8^{(S2)})$$

### Union, Intersection, Set-Difference

- All of these operations take two input relations, which must be <u>union-compatible</u>:
  - Same number of fields
  - Corresponding' fields have the same domain/data type

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0
44	guppy	5	35.0
28	yuppy	9	35.0

 $S1 \cup S2$ 

sid	sname	rating	age
22	dustin	7	45.0

$$S1-S2$$

sid	sname	rating	age
31	lubber	8	55.5
58	rusty	10	35.0

$$S1 \cap S2$$

#### Union, Intersection & Difference

Union Compatibility: r U s is valid if:

- Relations r & s have the same arity
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Note that r & s can be either database relations or derived relations

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#### **Cross-Product**

- Each row of S1 is paired with each row of R1.
- Result schema has one field per field of S1 and R1, with field names 'inherited' if possible.
  - Conflict: Both S1 and R1 have a field called sid.

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	22	101	10/10/96
22	dustin	7	45.0	58	103	11/12/96
31	lubber	8	55.5	22	101	10/10/96
31	lubber	8	55.5	58	103	11/12/96
58	rusty	10	35.0	22	101	10/10/96
58	rusty	10	35.0	58	103	11/12/96

Renaming operator:

$$\rho(C(1 \rightarrow sid1,5 \rightarrow sid2),S1 \times R1)$$

#### Joins

• Condition Join:  $R \bowtie_{c} S = \sigma_{c}(R \times S)$ 

(sid)	sname	rating	age	(sid)	bid	day
22	dustin	7	45.0	58	103	11/12/96
31	lubber	8	55.5	58	103	11/12/96

$$S1 \bowtie_{S1.sid < R1.sid} R1$$

- Result schema same as that of cross-product.
- Fewer tuples than cross-product, might be able to compute more efficiently
- Sometimes called a theta-join.

#### **Joins**

<u>Equi-Join</u>: A special case of condition join where the condition c contains only <u>equalities</u>

sid	sname	rating	age	bid	day
22	dustin	7	45.0	101	10/10/96
58	rusty	10	35.0	103	11/12/96

$$S1 \bowtie_{sid} R1$$

- Result schema similar to cross-product, but only one copy of fields for which equality is specified.
- Natural Join: Equijoin on all common fields.

## Examples of Division A/B

	sno	pno	pno	pno	pno
	s1	p1	p2	p2	p1
	s1		B1	p4	p2
	s1	p2 p3 p4 p1 p2 p2 p2 p4	D1	<i>B</i> 2	p4
	s1	p4		DZ	В3
	s2	p1	sno		DS
	s2	p2	s1		
	s3	p2	s2	sno	
	<ul><li>s2</li><li>s2</li><li>s3</li><li>s4</li></ul>	p2	s3	s1	sno
	s4	p4	s4	s4	s1
		4	A/B1	A/B2	A/B3
4	0004				00

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### Extended Operators of RA

- Duplicate-Elimination Operator δ
- Aggregation Operators
  - Sum
  - Count
  - Average
  - Max
  - Min
- Grouping Operator
- Sorting Operator τ
- Extended Projection
- Outerjoin Operator

### Aggregate Functions & Operations

 Aggregation function takes a collection of values and returns a single value as a result.

avg: average value min: minimum value max: maximum value sum: sum of values

count: number of values

Aggregate operation in relational algebra: General Form

$$g_{G_1,G_2,\ldots,G_n} g_{F_1(A_1),F_2(A_2,\ldots,F_n(A_n))}(E)$$

E is any relation

- $-G_1, G_2 \dots, G_n$  is a list of attributes on which to group (optional)
- Each  $F_i$  is an aggregate function
- Each A<sub>i</sub> is an attribute name

## Aggregate Operation – Example

Relation /.

A	В	С
α	α	7
α	β	7
β	β	3
β	β	10

 $g_{\text{sum(c)}}(\mathbf{r})$ 

sum(C)

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## Aggregate Operation – Example

#### Relation *account* grouped by *branch-name*:

branch_name	account_number	balance
Perryridge	A-102	400
Perryridge	A-201	900
Brighton	A-217	750
Brighton	A-215	750
Redwood	A-222	700

## branch\_name $g_{\text{sum(balance)}}$ (account)

branch_name	sum(balance)
Perryridge	1300
Brighton	1500
Redwood	700

### Aggregate Functions (Cont.)

- Result of aggregation does not have a name
  - Can use rename operation to give it a name
  - For convenience, we permit renaming as part of aggregate operation

branch\_name 9 sum(balance) as sum\_balance (account)

### Generalized Projection

 Extends the projection operation by allowing arithmetic functions to be used in the projection list.

$$\prod_{F_1, F_2}, ..., F_n(E)$$

- E is any relational-algebra expression
- Each of  $F_1$ ,  $F_2$ , ...,  $F_n$  are are arithmetic expressions involving constants and attributes in the schema of E.
- Given relation credit\_info(customer\_name, limit, credit\_balance), find how much more each person can spend:

 $\Pi_{customer\_name, \ limit - credit\_balance}$  (credit\_info)

### **Generalized Projection**

Cust_name	limit	Credit_bal
Curry	2000	1750
Hayes	1500	1500
Jones	6000	700
Smith	2000	400

Given relation *credit\_info(customer\_name, limit, credit\_balance)*, find how much more each person can spend:

 $\Pi_{customer\_name, (limit-credit\_balance)}$  as credit\_available (credit\_info)

Cust_name	Credit_available
Curry	250
Hayes	0
Jones	5300
Smith	1600

#### **Outer Join**

- An extension of the join operation that avoids loss of information.
- Computes the join and then adds tuples form one relation that does not match tuples in the other relation to the result of the join.
- Uses null values:
  - null signifies that the value is unknown or does not exist

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## Outer Join – Example

#### Relation loan

loan_number	branch_name	amount
L-170	Downtown	3000
L-230	Redwood	4000
L-260	Perryridge	1700

#### Relation borrower

customer_name	loan_number
Jones	L-170
Smith	L-230
Hayes	L-155

## Outer Join – Example

Inner Join

*loan* ⋈ *Borrower* 

loan_number	branch_name	amount	customer_name
L-170	Downtown	3000	Jones
L-230	Redwood	4000	Smith

Left Outer Join

*loan* → Borrower

loan_number	branch_name	amount	customer_name
L-170	Downtown	3000	Jones
L-230	Redwood	4000	Smith
L-260	Perryridge	1700	null

## Outer Join – Example

#### Right Outer Join

*loan* ⋈ *borrower* 

loan_number	branch_name	amount	customer_name
L-170 L-230	Downtown Redwood	3000 4000	Jones Smith
L-250 L-155	null	null	Hayes

#### Full Outer Join

*loan*⊐⊠ *borrower* 

loan_number	branch_name	amount	customer_name
L-170	Downtown	3000	Jones
L-230	Redwood	4000	Smith
L-260	Perryridge	1700	null
L-155	null	null	Hayes

#### **Null Values**

- It is possible for tuples to have a null value, denoted by null, for some of their attributes
- null signifies an unknown value or that a value does not exist or something that is NA
- Aggregate functions simply ignore null values (as in SQL)
- For duplicate elimination and grouping, null is treated like any other value, and two nulls are assumed to be the same (as in SQL)