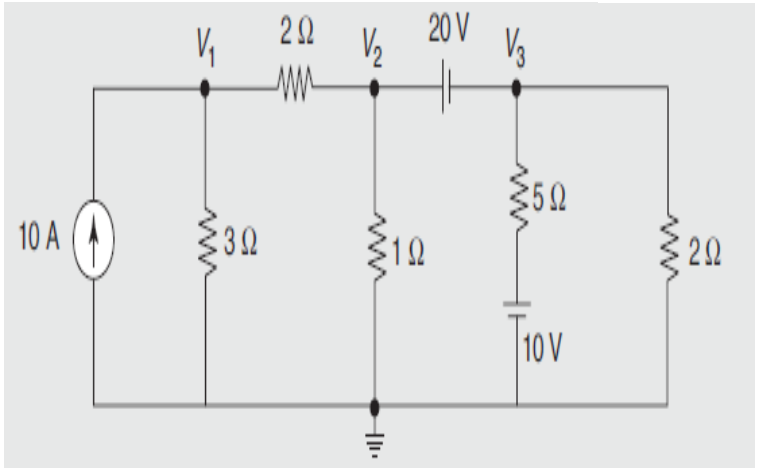
Supernode Concept

Nodes that are connected to each other by voltage sources but not to the reference node by a path of voltage sources form a super node.

Super node analysis

Ex 1:- *Determine the current in the 5 Ω resistor for the network shown in Fig*



Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node 1,

$$10 = \frac{V_1}{3} + \frac{V_1 - V_2}{2}$$

$$\left(\frac{1}{3} + \frac{1}{2}\right) V_1 - \frac{1}{2} V_2 = 10$$

$$0.83\, V_1 - 0.5\, V_2 = 10 \qquad (1)$$

Nodes 2 and 3 will form a supernode.

Writing voltage equation for the supernode,

$$V_2 - V_3 = 20 \qquad (2)$$

Applying KCL at the supernode,

$$\frac{V_2 - V_1}{2} + \frac{V_2}{1} + \frac{V_3 - 10}{5} + \frac{V_3}{2} = 0$$

$$-\frac{1}{2} V_1 + \left(\frac{1}{2} + 1\right) V_2 + \left(\frac{1}{5} + \frac{1}{2}\right) V_3 = 2$$

$$-0.5\, V_1 + 1.5\, V_2 + 0.7\, V_3 = 2 \qquad (3)$$

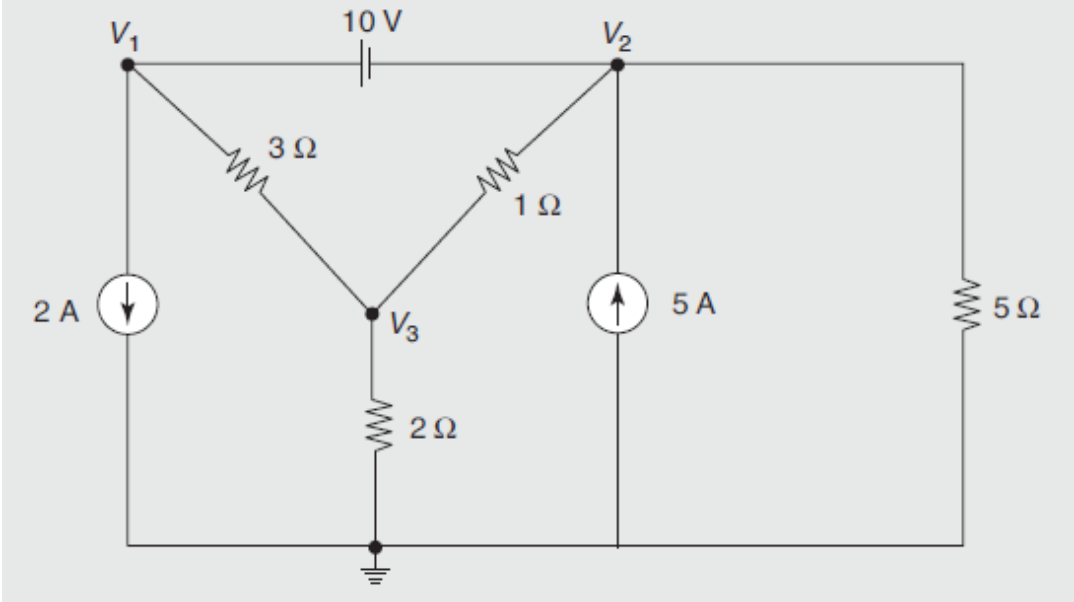
Solving Eqs (1), (2) and (3),

$$V_1 = 19.04 \text{ V}$$

$$V_2 = 11.6 \text{ V}$$

$$V_3 = -8.4 \text{ V}$$

$$I_{5\Omega} = \frac{V_3 - 10}{5} = \frac{-8.4 - 10}{5} = -3.68 \text{ A}$$


V_1
10 V
V_2
3 Ω
1 Ω
2 A
V_3
5 A
5 Ω
2 Ω

Example 2 *Find the power delivered by the 5 A current source in the network shown in Fig.*

Solution Assume that the currents are moving away from the nodes.

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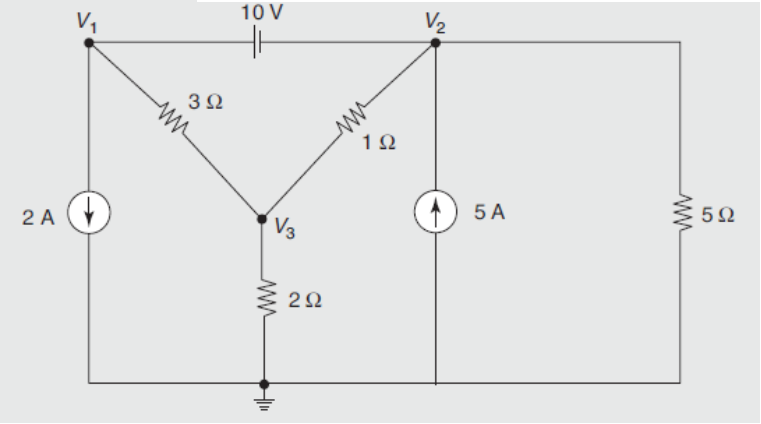


Nodes 1 and 2 will form a supernode.

Writing voltage equation for the supernode,

$$V_1 - V_2 = 10 \qquad (1)$$

Applying KCL at the supernode,

$$2 + \frac{V_1 - V_3}{3} + \frac{V_2}{5} + \frac{V_2 - V_3}{1} = 5$$

$$\frac{1}{3}V_1 + \left(\frac{1}{5} + 1\right)V_2 - \left(\frac{1}{3} + 1\right)V_3 = 3$$

$$0.33V_1 + 1.2V_2 - 1.33V_3 = 3 \qquad (2)$$

Applying KCL at Node 3,

$$\frac{V_3 - V_1}{3} + \frac{V_3 - V_2}{1} + \frac{V_3}{2} = 0$$

$$-\frac{1}{3}V_1 - V_2 + \left(\frac{1}{3} + 1 + \frac{1}{2}\right)V_3 = 0$$

$$-0.33V_1 - V_2 + 1.83V_3 = 0 \qquad (3)$$

Solving Eqs (1), (2) and (3),

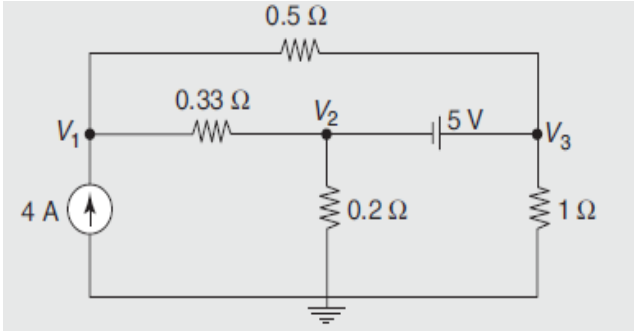
$$V_1 = 13.72 \text{ V}$$

$$V_2 = 3.72 \text{ V}$$

$$V_3 = 4.51 \text{ V}$$

Power delivered by the 5 A source $= 5\ V_2 = 5 \times 3.72 = 18.6$ W

Example 3:- *find the node voltages V_1, V_2 and V_3.*



Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node 1,

$$4 = \frac{V_1 - V_2}{0.33} + \frac{V_1 - V_3}{0.5}$$

$$\left(\frac{1}{0.33} + \frac{1}{0.5}\right)V_1 - \frac{1}{0.33}V_2 - \frac{1}{0.5}V_3 = 4$$

$$5.03\,V_1 - 3.03\,V_2 - 2\,V_3 = 4 \qquad (1)$$

Nodes 2 and 3 will form a supernode.

Writing voltage equation for the supernode,

$$V_3 - V_2 = 5 \qquad (2)$$

Applying KCL at the supernode,

$$\frac{V_2 - V_1}{0.33} + \frac{V_2}{0.2} + \frac{V_3}{1} + \frac{V_3 - V_1}{0.5} = 0$$

$$\left(-\frac{1}{0.33} - \frac{1}{0.5}\right)V_1 + \left(\frac{1}{0.33} + \frac{1}{0.2}\right)V_2 + \left(1 + \frac{1}{0.5}\right)V_3 = 0$$

$$-5.03\,V_1 + 8.03\,V_2 + 3\,V_3 = 0 \qquad (3)$$

Solving Eqs (1), (2) and (3),

$$V_1 = 2.62\ \text{V}$$

$$V_2 = -0.17\ \text{V}$$

$$V_3 = 4.83\ \text{V}$$

Homework

Find the value of current flowing through the branch *ab*.