SOMAIYA VIDYAVIHAR UNIVERSITY

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Course: AM-I
~~Experiment~~ / assignment / ~~Tutorial No.~~ 1
Grade: Signature of the Faculty with date

① $u = e^{xyz}$ PT $\frac{d^3u}{dxdydz} = (1+3xyz+x^2y^2z^2)\,e^{xyz}$

Ans 1) $\frac{du}{dx}\left(\frac{d^2u}{dydz}\right)$

$\frac{du}{dx} = xy\,e^{xyz}$

$\frac{d^2u}{dydz} = e^{xyz}(x) + e^{xyz}(zx)(xy)$

$\frac{d^2u}{dydz} = e^{xyz}(x + x^2yz)$

$\frac{d^3u}{dxdydz} = e^{xyz}(1+2xyz) + (x+x^2yz)yz\,e^{xyz}$

$= e^{xyz}(1+3xyz+x^2y^2z^2)$

Hence proved.

② $z = x^2\tan^{-1}\left(\frac{y}{x}\right)$ PT $\frac{d^2z}{dxdy} = \frac{d^2z}{dydx} = \frac{x^2-y^2}{x^2+y^2}$

$\frac{dz}{dx} = \left[2x\tan^{-1}\left(\frac{y}{x}\right) - \frac{y}{x^2}\,x^2\left(\frac{1}{1+y^2/x^2}\right)\right] - \left[\frac{y^2}{y}\left(\frac{1}{1+x^2/y^2}\right)\right]$

$= 2x\tan^{-1}\left(\frac{y}{x}\right) - y\left(\frac{x^2}{x^2+y^2}\right) - y\left(\frac{y^2}{x^2+y^2}\right)$

$\frac{d^2z}{dydx} = \frac{d}{dy}\left(\frac{dz}{dx}\right)$

$= \frac{d}{dy}\left(2x\tan^{-1}\left(\frac{y}{x}\right)\right) - \frac{d}{dy}\left(\frac{yx^2}{x^2+y^2}\right) - \frac{d}{dy}\left(\frac{y^3}{x^2+y^2}\right)$

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$$= \frac{2x}{1+\frac{y^2}{x^2}} \times \frac{1}{x} - \left[\frac{x^2(x^2+y^2) - x^2y(2y)}{(x^2+y^2)^2}\right] - \left[\frac{3y^2(x^2+y^2) - y^3(2y)}{(x^2+y^2)^2}\right]$$

$$= \frac{2x^2}{x^2+y^2} - \left[\frac{x^4 + x^2y^2 - 2x^2y^2 + 3x^2y^2 + 3y^4 - 2y^4}{(x^2+y^2)^2}\right]$$

$$= \frac{2x^2}{x^2+y^2} - \left[\frac{x^4+y^4+2x^2y^2}{(x^2+y^2)^2}\right]$$

$$= \frac{2x^2}{x^2+y^2} - \left[\frac{(x^2+y^2)^2}{(x^2+y^2)^2}\right]$$

$$= \frac{2x^2}{x^2+y^2} - 1 = \frac{1x^2 - x^2 - y^2}{x^2+y^2} = \frac{x^2-y^2}{x^2+y^2}$$

$$\frac{d^2x}{dxdy} = \frac{d}{dx}\left(\frac{x^3}{x^2+y^2}\right) - \frac{d}{dx}\left(2y\tan^{-1}\left(\frac{x}{y}\right)\right) + \frac{d}{dx}\left(\frac{xy^2}{x^2+y^2}\right)$$

$$= \frac{3x^2(x^2+y^2) - 2x(x^3)}{(x^2+y^2)^2} - \frac{2y}{y^2+x^2} + \frac{(x^2+y^2)(y^2) - (xy)^2(2x)}{(x^2+y^2)^2}$$

$$= \frac{3x^4 + 3x^2y^2 - 2x^4 + x^2y^2 + y^4 - 2x^2y^2}{(x^2+y^2)^2} - \frac{2y^2}{x^2+y^2}$$

$$= \frac{x^4+y^4+2x^2y^2}{(x^2+y^2)^2} - \frac{2y^2}{x^2+y^2} = 1 - \frac{2y^2}{x^2+y^2}$$

$$= \frac{x^2+y^2-2y^2}{x^2+y^2} = \frac{x^2-y^2}{x^2+y^2} = \frac{d^2z}{dydz}$$

Hence proved.

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3) $u = f\left(\frac{x^2}{y}\right)$ $x\frac{du}{dx} + 2y\frac{du}{dy} = 0$

$$\frac{du}{dx} = \frac{2x}{y}$$

$$\frac{du}{dy} = -\frac{x^2}{y^2}$$

Substituting the values we get

$$LHS \Rightarrow x\left(\frac{2x}{y}\right) - \left(\frac{x^2}{y^2}\right)2y$$

$$= \frac{2x^2}{y} - \frac{2x^2y}{y^2}$$

$$= \frac{2x^2y - 2x^2y}{y} = 0$$

LHS = RHS

Hence proved.

ii) PT :- $x^2\frac{d^2u}{dx^2} + 3xy\left(\frac{d^2u}{dxdy}\right) + 2y^2\left(\frac{d^2u}{dy^2}\right) = 0$

$$\frac{d}{dx}\left(\frac{du}{dx}\right) + 3xy\left(\frac{d}{dx}\left(\frac{du}{dy}\right)\right) + 2y^2\left(\frac{d}{dy}\right)\left(\frac{du}{dy}\right)$$

$$x^2 \times \frac{2x}{y} \times \frac{2x}{y} + 3x\cancel{y}\left(\frac{2x}{\cancel{y}}\right)\left(\frac{-x^2}{y^2}\right) + 2\cancel{y^2}\left(\frac{-x^2}{\cancel{y^2}}\right)\left(\frac{-2x^2}{y^2}\right)$$

$$\frac{4x^4}{y^2} - \frac{6x^4}{y^2} + \frac{2x^4}{y^2}$$

$$= \underline{\underline{0}}$$

Hence proved

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④ $u = f(e^{x-y}, e^{y-z}, e^{z-x})$

$X = e^{x-y}, \; Y = e^{y-z}, \; Z = e^{z-x}$

$$\frac{du}{dx} = \frac{du}{dX}(e^{x-y})(1) + \frac{du}{dY}(0) + \frac{du}{dZ}e^{z-x}(-1)$$

$$\frac{du}{dx} = \frac{du}{dX}e^{x-y} - \frac{du}{dZ}e^{z-x} \quad \text{— (I)}$$

$$\frac{du}{dy} = \frac{du}{dX}e^{x-y}(-1) + \frac{du}{dY}e^{y-z} + \frac{du}{dZ}(0)$$

$$= \frac{du}{dY}e^{y-z} - \frac{du}{dX}e^{x-y} \quad \text{— (II)}$$

$$\frac{du}{dz} = \frac{du}{dX}(0) + \frac{du}{dY}e^{y-z}(-1) + \frac{du}{dZ}e^{z-x}$$

$$\frac{du}{dz} = \frac{du}{dZ}e^{z-x} - \frac{du}{dY}e^{y-z} \quad \text{— (III)}$$

Adding (I), (II) and (III) we get:-

$$\frac{du}{dx} + \frac{du}{dy} + \frac{du}{dz} = 0$$

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(5) $z = f(x,y) \quad x = r\cos\theta \quad y = r\sin\theta$

TP :- $\left(\frac{dz}{dx}\right)^2 + \left(\frac{dz}{dy}\right)^2 = \left(\frac{dz}{dr}\right)^2 + \frac{1}{r^2}\left(\frac{dz}{d\theta}\right)^2$

$$\frac{dz}{dr} = \frac{dz}{dx}\cdot\frac{dx}{dr} + \frac{dz}{dy}\cdot\frac{dy}{dr}$$

$$\frac{du}{dr} = \cos\theta \quad \frac{dy}{dr} = \sin\theta$$

$$\left(\frac{dz}{dr}\right)^2 = \cos^2\theta\left(\frac{dz}{dx}\right)^2 + \sin^2\theta\left(\frac{dz}{dy}\right)^2 + 2\sin\theta\frac{dz}{dx}(\cos\theta)\frac{dz}{dy}$$

$$\frac{du}{d\theta} = -r\sin\theta \quad , \quad \frac{dy}{d\theta} = r\cos\theta$$

$$\frac{1}{r^2}\left(\frac{dz}{d\theta}\right)^2 = \frac{1}{r^2}\left(-r\sin\theta\frac{dz}{dx} + r\cos\theta\frac{dz}{dy}\right)^2$$

$$= \sin^2\theta\left(\frac{dz}{dx}\right)^2 - 2\sin\theta\frac{dz}{dx}\cos\theta\frac{dz}{dy} + \cos^2\theta\left(\frac{dz}{dy}\right)^2$$

$$= \left(\frac{dz}{dx}\right)^2(\cos^2\theta + \sin^2\theta) + \left(\frac{dz}{dy}\right)^2(\cos^2\theta + \sin^2\theta)$$

$$= \left(\frac{dz}{dx}\right)^2 + \left(\frac{dz}{dy}\right)^2 \cancel{\theta}$$

$= \underline{LHS}$

Hence proved.