K J Somaiya College of Engineering, Vidyavihar, Mumbai

(A Constituent College of SVU)

Engineering Mechanics Notes

Module 5 – Kinetics of Particle

Module Section 5.2 – Kinetics – Work Energy Principle

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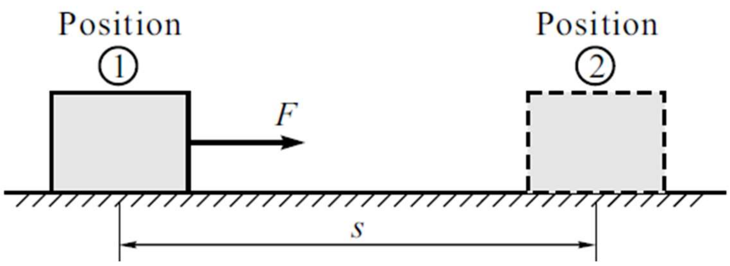
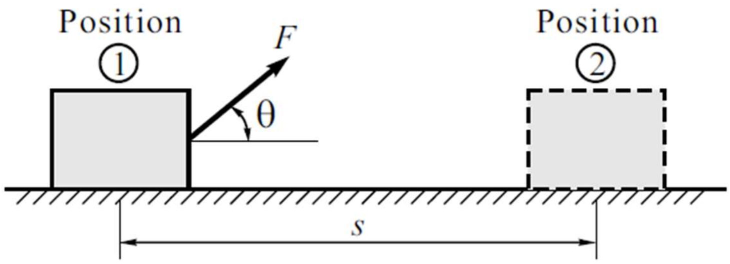
References: Engineering Mechanics, by M. D. Dayal & Engineering Mechanics – Statics and Dynamics, by N. H. Dubey.

Work: It is defined as the product of the displacement and the force in the direction of the displacement.



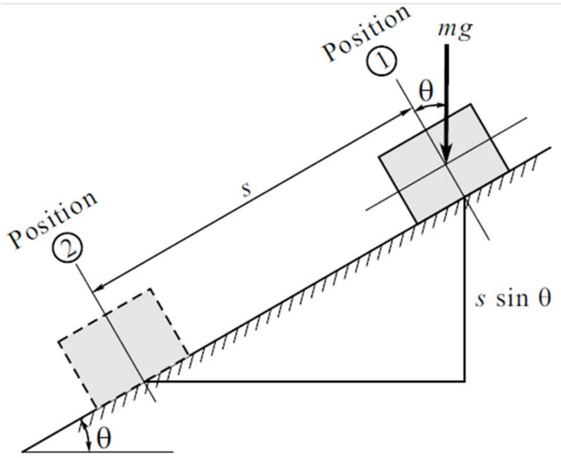
Work done = Force × Displacement $\Rightarrow U = F \times s$



If a particle is subjected to a force F at an angle θ with horizontal and the particle is displaced by s from position 1 to position 2 then work done U is the product of force component in the direction of displacement and displacement. $\Rightarrow U = F\cos\theta \times s$

1. Work done by a force is positive if the directions of force and displacement are in same direction. E.g., Work done by force of gravity is positive when a body moves from an upper position to lower position.
2. Work done by a force is negative if the directions of force displacement both are in opposite direction. E.g., Work done by force of gravity is negative when a body moves from a lower position to a higher position.
3. Work done is zero if either the displacement is zero or the force acts normal to the displacement. E.g., Gravity does not work when body moves horizontally.
4. Work is a scalar quantity.
5. Unit of work is Nm or Joule (J).

Work Done by Weight Force:



Work done = Component of weight in the direction of displacement × Displacement

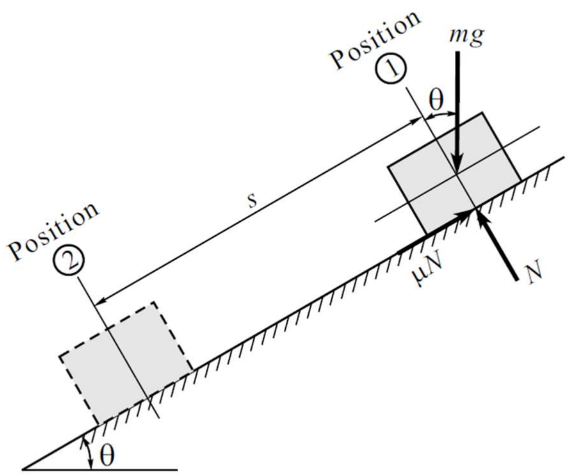
$U = mg \sin\theta \times s$

OR

Work done = Weight force × Displacement in the direction of weight force

$U = mg \times s \sin\theta$

Work Done by Frictional Force:



Work done = – Friction × Displacement

$U = \mu N \times s$

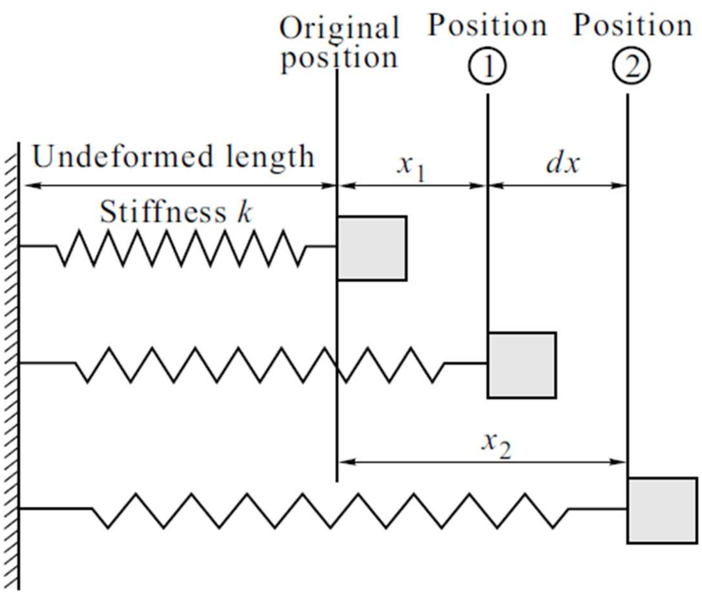
Work done by friction force is negative because direction of frictional force and displacement is opposite.

Work done by normal reaction (N) and component of weight force perpendicular to inclined plane (mg cos θ) is zero.

Work Done by Spring Force:

Consider a spring of stiffness k as shown in the figure, with some undeformed (free/original) length. Let x_1 & x_2 be deformations of spring at positions 1 & 2.

∴ Spring force $F = -k \times x$



where k is the spring stiffness (N/m)
x is the deformation of spring (m)
– ve sign indicates direction of spring force acting towards original position.

Work done = Spring force × Deformation

$$U = \int_{x_1}^{x_2} -kx\,dx$$

$$U = -\frac{1}{2}k(x_2^2 - x_1^2) \quad \text{OR} \quad U = \frac{1}{2}k(x_1^2 - x_2^2)$$

Kinetic Energy: It is the energy possessed by a particle by virtue of its motion.

$$\text{K. E.} = \frac{1}{2}\text{mv}^2$$

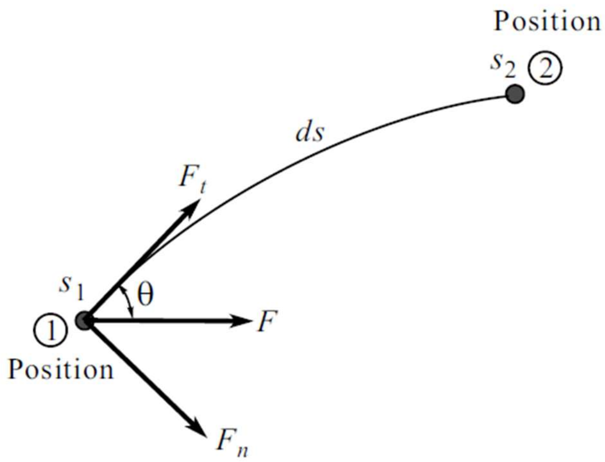
Potential Energy: It is the energy possessed by a particle by virtue of its position.

$$\text{P. E.} = +\text{mgh}, \text{if displacement is downwards}$$

$$\text{P. E.} = -\text{mgh}, \text{if displacement is upwards}$$

Work Energy Principle:

Work done by the forces acting on a particle during some displacement is equal to the change in kinetic energy during that displacement.



Consider the particle having mass m is acted upon by a force F and moving along a path as shown. Let v_1 and v_2 be the velocities of the particle at position 1 and position 2 and the corresponding displacement s_1 and s_2 respectively.

By Newton's second law in the tangential direction, we have,

$$\sum F_t = ma_t$$

$$F\cos\theta = ma_t = m\frac{dv}{dt} = m\frac{dv}{ds}\times\frac{ds}{dt} = mv\frac{dv}{ds}$$

$$F\cos\theta\, ds = mv\, dv$$

Integrating both sides, we have,

$$\int_{s_1}^{s_2} F\cos\theta\, ds = \int_{v_1}^{v_2} mv\, dv$$

$$U_{1-2} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$\therefore U = \Delta\text{K. E.}$$

Principle of Conservation of Energy:

Conservative Forces: If the work of a force is moving a particle between two positions is independent of the path followed by the particle and can be expressed as a change in its potential energy, then such forces is called as conservative forces. E.g., weight force of particle (gravity force), spring force and elastic force.

Non-Conservative Forces: The forces in which the work is dependent upon the path followed by the particles is known as non-conservative forces. E.g., frictional force

When a particle is moving from position 1 to position 2 under the action of only conservative forces (i.e., when frictional force does not exist) then by energy conservation principle we say that the total energy remains constant.

Total energy = Kinetic energy + Potential energy + Strain energy of spring

$$\text{Total energy} = \frac{1}{2}mv^2 \pm mgh + \frac{1}{2}mx^2$$

OR Total energy at position 1 = Total energy at position 2

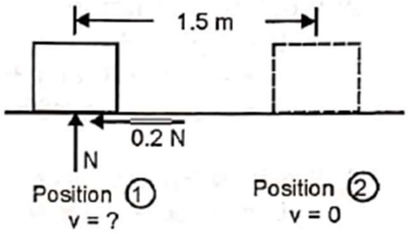
$$KE_1 + PE_1 + SE_1 = KE_2 + PE_2 + SE_2$$

Ex. 11.2 A block is pushed with an initial velocity on a horizontal surface such that it travels 1500 mm before coming to rest. If $\mu_s = 0.25$ and $\mu_k = 0.2$ find the time of travel.

Solution: Applying Work Energy Principle from position (1) to position (2)

$T_1 = ½ mv^2 = 0.5 mv^2$ J

$T_2 = 0$



U_{1-2} by frictional force

$U = -\mu_k N.s$

$= -0.2 (m \times 9.81) \times 1.5$

$= -2.943$ m J

using $T_1 + \sum U_{1-2} = T_2$

$0.5 mv^2 - 2.943 m = 0$

$\therefore$ $v = 2.426$ m/s .... Initial velocity of block

Kinematics

Block performs rectilinear motion with uniform acceleration (since forces remain constant)

$u = 2.426$ m/s , $v = 0$, $s = 1.5$ m, $a = ?$, $t = t$ sec.

using $v^2 = u^2 + 2as$

$0 = (2.426)^2 + 2 a \times 1.5$

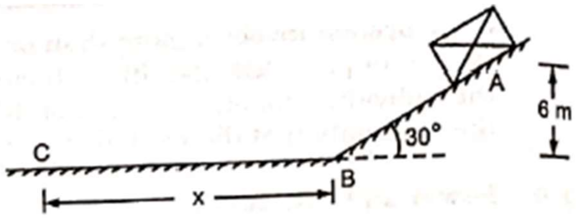
$\therefore$ $a = -1.962$ m/s^2

using $v = u + at$

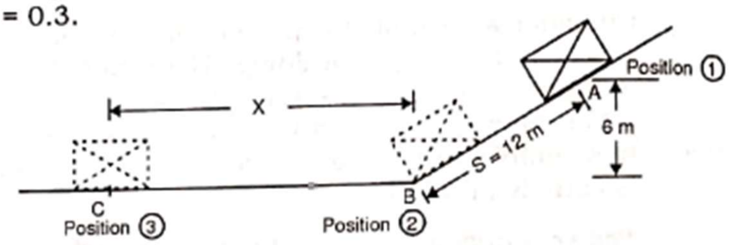
$0 = 2.426 - 1.962 t$

$\therefore$ $t = 1.236$ sec .... **Ans.**

Ex. 11.1 A 20 kg crate is released from rest on the top of incline at A. It travels on the incline and finally comes to rest on the horizontal surface at C. Find the distance x it travels on the horizontal surface and also the maximum velocity it attains during the motion. Take $\mu_k = 0.3$.



Solution: The crate acquires maximum velocity at the lower most point B on the incline.
Applying Work Energy Principle from position (1) to (2).



$T_1 = 0$ ....since block starts from rest.
$T_2 = ½\, mv^2 = ½ \times 20 \times v^2 = 10\, v^2$ J

U_{1-2} 1) Work by weight force

$U = m\,g\,h$
$= 20 \times 9.81 \times 6 = 1177.2$ J

2) Work by frictional force

$U = -\mu_k N.s$

$\therefore \quad U = -0.3 \times 169.9 \times 12$
$= -611.64$ J

here, for the inclined surface, normal reaction,
$N = W \cos 30 = 20 \times 9.81 \cos 30 = 169.9$ N
and distance traveled by block,
$$s = \frac{6}{\sin 30} = 12 \text{ m}$$

Using $\quad T_1 + \sum U_{1-2} = T_2$

$0 + [\,1177.2 - 611.64\,] = 10\, v^2$

$\therefore \quad v = 7.52$ m/s

i.e $\quad v_{max} = 7.52$ m/s at B .... **Ans.**

To find the distance x traveled on the horizontal surface, we will apply work energy principle from position (2) to position (3)

$T_2 = ½\, mv^2 = ½ \times 20 \times (7.52)^2 = 565.56$ J
$T_3 = 0$

U_{2-3} 1) only by frictional force
$= -u_k N.s$

$\therefore \quad U_{2-3} = -0.3 \times 196.2 \times x$
$= -58.86$ x J

For the horizontal surface,
$N = W = 20 \times 9.81 = 196.2$ N
Also the distance traveled by block,
s = x meters.

Using $\quad T_2 + \Sigma U_{2-3} = T_3$

$565.56 + [-58.86\, x\,] = 0$

$\therefore \quad x = 9.608$ m .... **Ans.**

Problem 16

A 20 N block is released from rest. It slides down the inclined having $\mu = 0.2$ as shown in Fig. 14.16(a). Determine the maximum compression of the spring and the distance moved by the block when the energy is released from compressed spring. Springs constant $k = 1000$ N/m.

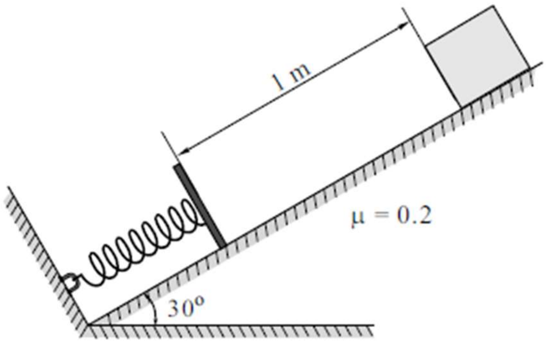


Fig. 14.16(a)

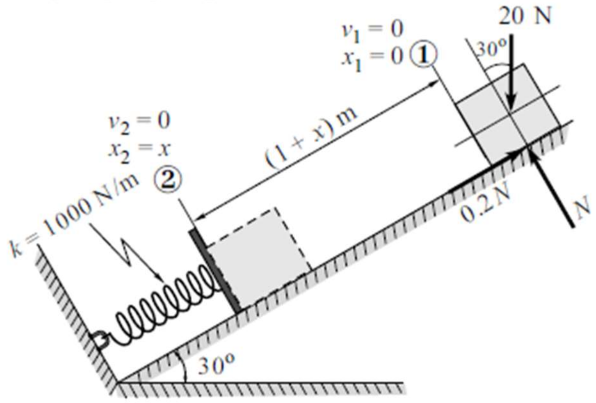


Fig. 14.16(b)

Solution

Part (i) Maximum compression of the spring

Let x be the maximum deformation of spring at position ② where the block comes to rest ($v_2 = 0$).

By work - energy principle, we have

Work done = Change in kinetic energy

$$\frac{1}{2} \times 1000(0^2 - x^2) + 20 \sin 30^\circ (1 + x) - 0.2 \times 20 \cos 30^\circ (1 + x) = 0 - 0$$

$\therefore \; x = 0.121$ m

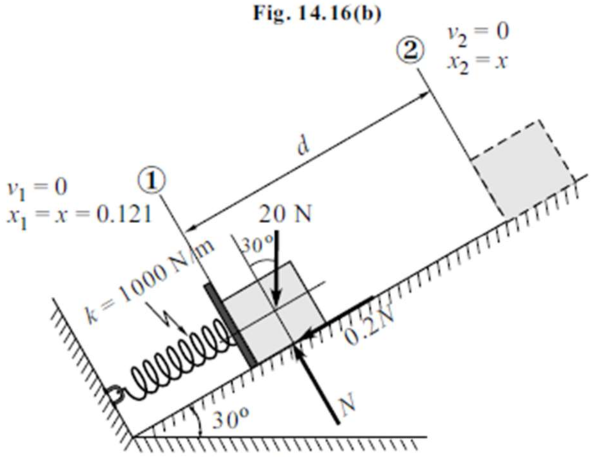


Fig. 14.16(c)

Part (ii) Distance moved by the block

By work - energy principle, we have

Work done = Change in kinetic energy

$$\frac{1}{2} \times 1000(0.121^2 - 0^2) - 20 \sin 30^\circ \times d - 0.2 \times 20 \cos 30^\circ \times d = 0 - 0$$

$\therefore \; d = 0.5437$ m

Problem 13

A 1 kg collar is attached to a spring and slides without friction along a circular rod which lies in horizontal plane as shown in Fig. 14.E13. The spring has a constant $k = 250$ N/m and is undeformed when collar is at B. Knowing that collar passes through point D with a speed of 1.8 m/s, determine the speed of the collar when it passes through point C and point B.

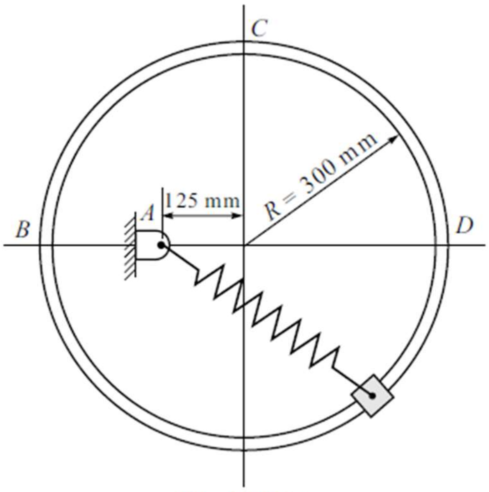


Fig. 14.13(a)

Solution

Undeformed length of spring is at B,

$$= 300 - 125 = 175 \text{ mm}$$
$$= 0.175 \text{ m}$$

Deformation of spring at position D,

$$= 125 + 300 - 175 = 250 \text{ mm}$$
$$= 0.25 \text{ m}$$

Deformation of spring at position C,

$$= \sqrt{125^2 + 300^2} - 175$$
$$= 325 - 175 = 150 \text{ mm}$$
$$= 0.15 \text{ m}$$

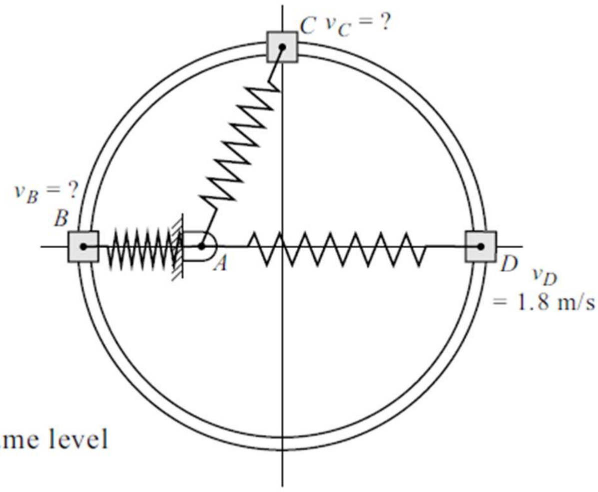


Fig. 14.13(b)

By principle of conservation of energy, we have total energy at any position remains constant.

P.E. throughout the ring is zero because it is at same level (horizontal).

Total energy at position D = Total energy at position B

(K.E. + P.E. + S.E.) at D = (K.E. + P.E. + Spring energy) at B

$$\frac{1}{2} \times 1 \times 1.8^2 + 0 + \frac{1}{2} \times 250 \times 0.25^2 = \frac{1}{2} \times 1 \times v_B^2 + 0 + \frac{1}{2} \times 250 \times 0^2$$

$$9.43 = 0.5 v_B^2$$

$$v_B = 4.343 \text{ m/s}$$

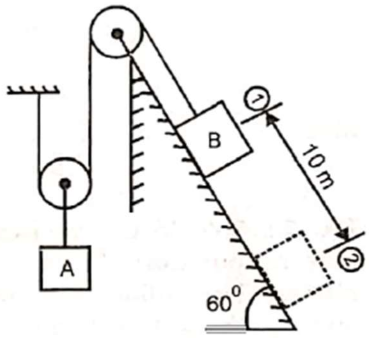
Total energy at position D = Total energy at position C

$$9.43 = \frac{1}{2} \times 1 \times v_C^2 + 0 + \frac{1}{2} \times 250 \times 0.15^2$$

$$9.43 = 0.5 v_C^2 + 2.8125$$

$$\therefore \; v_C = 3.638 \text{ m/s}$$

Ex. 11.5 Block B of weight 3000 N having a speed of 2 m/s in position (1) travels 10 m along and down the slope. Block A of weight 1000 N is connected to it by an inextensible string. Find the velocities of the blocks in the new position. Take $\mu_s = 0.35$ and $\mu_k = 0.3$ at the inclined surface.



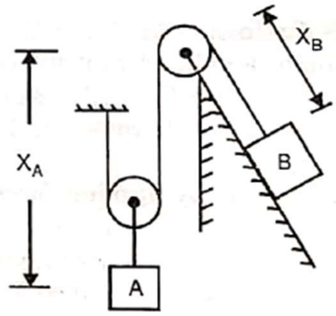
Solution: We shall first find the relation between the velocities and the distance traveled by the two connected blocks.

Using constant string length method (CSLM)

If x_A and x_B are the variable positions of A and B measured from a fixed reference point, we have the length L of string in terms of x_A and x_B as

$L = (-2\,x_A) + x_B \pm$ constants

[x_A is – ve since with increase in x_B, x_A would decrease]



Differentiating w. r. to time

$$0 = -2\,v_A + v_B$$

or $v_B = 2\,v_A$ ...relation between the velocities

since the time interval is the same, we have

$x_B = 2\,x_A$ ....relation between distance traveled

Applying Work Energy Principle to the system of A and B from position (1) to (2).

$$T_1 = \tfrac{1}{2} m_A v_A^2 + \tfrac{1}{2} m_B v_B^2$$
$$= \tfrac{1}{2}(101.94) \times (0.5\,v_B)^2 + \tfrac{1}{2}(305.81)\,v_B^2$$
$$= 165.65\,v_B^2$$
$$= 165.65\,(2)^2 = 662.6 \text{ J}$$

$$T_2 = \tfrac{1}{2} m_A v_A^2 + \tfrac{1}{2} m_B v_B^2$$
$$= 165.65\,v_B^2 \text{ J}$$

U_{1-2}

1) by weight of block B

$U = +\,m\,g\,h = 3000 \times 8.66$
$= 25981$ J
$= 25981$ J

(+ ve since displacement is downwards) Block B moves vertically down by $h = 10 \sin 60 = 8.66$ m

2) by weight of block A

$U = -\,m\,g\,h = -\,1000 \times 5$
$= -\,5000$ J

(– ve since displacement is upwards) since $x_A = 0.5\,x_B$, displacement of block A $= 0.5 \times 10 = 5$ m

3) by frictional force at the inclined surface

$U = -\,\mu k N \times s$
$U = -\,0.3 \times 1500 \times 10$
$= -\,4500$ J

block B travels a distance $s = 10$ m Normal reaction on the inclined surface $N = W \cos\theta$ $= 3000 \cos 60$ $= 1500$ Newton

Using $T_1 + \sum U_{1-2} = T_2$

$$662.6 + [25981 - 5000 - 4500] = 165.65\,v_B^2$$

$\therefore$ $v_B = 10.17$ m/s .... **Ans.**

also $v_A = 0.5\,v_B$

$\therefore$ $v_A = 0.5 \times 10.17 = 5.086$ m/s .... **Ans.**

Problem 22

Two springs each having stiffness of 0.5 N/cm are connected to ball B having a mass of 5 kg in a horizontal position producing initial tension of 1.5 m in each spring as shown in Fig. 14.E22. If the ball is allowed to fall from rest what will be its velocity after it has fallen through a height of 15 cm.

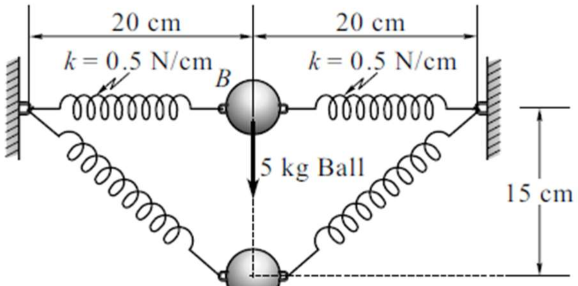


Fig. 14.22(a)

Solution

Method I

Initial position tension = 1.5 N

$T = kx$

$1.5 = (0.5)(x)$

$x = 3$ cm (Deformation in initial position)

$\therefore$ Free length of spring $= 20 - 3 = 17$ cm

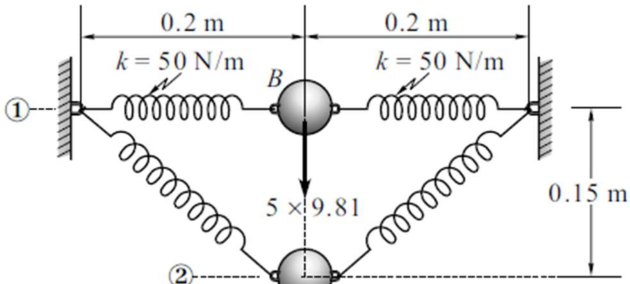


Fig. 14.22(b)

At position ①

$v_1 = 0$

$x_1 = 3$ cm

$\therefore\ x_1 = 0.03$ m

Displacement $h = 15$ cm

$\therefore\ h = 0.15$ m

At position ②

$v_2 = ?$

$x_2 = (25 - 17) = 8$ cm

$x_2 = 0.08$ m

Spring constant $k = 0.5$ N/cm

$\therefore\ k = 50$ N/m

By principle of work - energy, we have

Work done = Change in kinetic energy

$$5 \times 9.81 \times 0.15 + \left[\frac{1}{2} \times 50(0.03^2 - 0.08^2)\right] \times 2 = \frac{1}{2} \times 5 \times v_2^2 - 0$$

$v_2 = 1.68$ m/s

Method II

By principle of conservation of energy

Total energy = K.E. + P.E. + S.E.

Total energy remains constant at any position.

Total energy at position ① = Total energy at position ②

(K.E. + P.E. + S.E.) at position ① = (K.E. + P.E. + S.E.) at position ②

$$\frac{1}{2} \times 5 \times 0^2 + 5 \times 9.81 \times 0 + \frac{1}{2} \times 50 \times 0.03^2 = \frac{1}{2} \times 5 \times v_2^2 - 5 \times 9.81 \times 0.15 + \frac{1}{2} \times 50 \times 0.08^2$$

$$0.0225 = 2.5v_2^2 - 7.3575 + 0.16$$

$v_2 = 1.69$ m/s