SOMAIYA
VIDYAVIHAR UNIVERSITY

Batch: P-3-2 Roll No.: 28
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Course: AM - I
Experiment / assignment / tutorial No. 9
Grade: Signature of the Faculty with date

Q1) Given equation ; $x^2 - 2x + 2 = 0$

$$x = 1+i,\ 1-i$$

let $\alpha = 1+i$, $\beta = 1-i$

α and β can also be written as

$$\alpha = \sqrt{2}\cos\frac{\pi}{4} + i\sqrt{2}\sin\frac{\pi}{4}$$

$$\beta = \sqrt{2}\cos\frac{\pi}{4} - i\sqrt{2}\sin\frac{\pi}{4}$$

for $\alpha^n + \beta^n = \left[\sqrt{2}\cos\frac{\pi}{4} + i\sqrt{2}\sin\frac{\pi}{4}\right]^n + \left[\sqrt{2}\cos\frac{\pi}{4} - i\sqrt{2}\sin\frac{\pi}{4}\right]^n$

By DMVT

$$= \left[\sqrt{2}^n\cos\frac{\pi}{4}n + i\sqrt{2}^n\sin n\frac{\pi}{4}\right] + \left[\sqrt{2}^n\cos n\frac{\pi}{4} - i\sqrt{2}^n\sin\frac{n\pi}{4}\right]$$

$$= 2\left[\sqrt{2}^n\cos\frac{n\pi}{4}\right]$$

$$\alpha^n + \beta^n = 2\cdot 2^{n/2}\cdot\cos\frac{n\pi}{4}$$

for $\alpha^8 + \beta^8 = 2\cdot 2^{8/2}\cos 2\pi$

$$\alpha^8 + \beta^8 = 32 \qquad [\cos 2\pi = 1]$$

② $[1+i]^{3/4} = \left(\sqrt{2}\cos\frac{\pi}{4} + i\sqrt{2}\sin\frac{\pi}{4}\right)^{3/4}$

$= (\sqrt{2})^{3/4}\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]^{3/4}$

$= (\sqrt{2})^{3/4}\left[\cos\left(2k\pi+\frac{\pi}{4}\right) + i\sin\left(2k\pi+\frac{\pi}{4}\right)\right]^{3/4}$

$= (\sqrt{2})^{3/4}\left[\cos\frac{8k\pi+\pi}{4}\right) + i\sin\frac{8k\pi+\pi}{4}\right]$

$= (\sqrt{2})^{3/4}\left[\cos\frac{24k\pi+3\pi \times 3}{16} + i\sin\frac{24k\pi+3\pi}{16}\right]$

for values of $K = 0,1,2,3$

$x_0 = (\sqrt{2})^{3/4}\left[\cos\frac{3\pi}{16} + i\sin\frac{3\pi}{16}\right]$

$x_1 = (\sqrt{2})^{3/4}\left[\cos\frac{27\pi}{16} + i\sin\frac{27\pi}{16}\right]$

$x_2 = (\sqrt{2})^{3/4}\left[\cos\frac{51\pi}{16} + i\sin\frac{51\pi}{16}\right]$

$x_3 = (\sqrt{2})^{3/4}\left[\cos\frac{75\pi}{16} + i\sin\frac{75\pi}{16}\right]$

$\therefore x_0x_1x_2x_3 = (\sqrt{2})^3 \cdot \left[\cos\left(\frac{3\pi+27\pi+51\pi+75\pi}{16}\right)\right]$

$+ i\sin\left(\frac{3\pi+27\pi+51\pi+75\pi}{16}\right)$

$= 2\sqrt{2}\left[\cos\left(\frac{156\pi}{16}\right) + i\sin\left(\frac{156\pi}{16}\right)\right]$

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③ $u = \log \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$

(i) $u = \log \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$

$e^u = \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$ [antilog]

$e^u = \frac{1 + \tan \pi/2}{1 - \tan \pi/2}$ — ①

$e^{-u} = \frac{1 - \tan \theta/2}{1 + \tan \theta/2}$ — ②

Let, $\cos u = \frac{e^u + e^{-u}}{2}$

$= \left[\frac{\frac{1+\tan\pi/2}{1-\tan\pi/2} + \frac{1-\tan\theta/2}{1+\tan\theta/2}}{2}\right]$

$= \left[\frac{(1+\tan\theta/2)^2 + (1-\tan\theta/2)^2}{2 - 2\left(\tan^2\frac{\theta}{2}\right)}\right]$

$= \frac{2 + 2\tan^2\theta/2}{2 - 2\tan^2\theta/2}$

$\cosh u = \frac{1}{\cos\theta}$

Hence, $\cosh u = \sec\theta$ — ①

We know that,

$$\cancel{\sin}\ \sinh u = \sqrt{\cosh^2 u - 1}$$

$$\sinh u = \sqrt{\sec^2\theta - 1} \qquad \text{from (1)}$$

$$\sinh u = \sqrt{\tan^2\theta}$$

$$\sinh u = \tan\theta \quad \text{--- (2)}$$

$$\tanh u = \frac{\sinh u}{\cosh u}$$

$$= \frac{\tan\theta}{\sec\theta} \qquad \text{from (1) \& (2)}$$

$$= \frac{\sin\theta}{\cos\theta} \cdot \cos\theta$$

$$\tanh u = \sin\theta \quad \text{--- (3)}$$

Consider,

$$\tanh\frac{u}{2} = \frac{\sinh u/2}{\cosh u/2}$$

Multiply by $2\cosh u/2$

$$= \frac{2\sinh u}{2\cosh^2 u/2} = \frac{\sinh u}{1+\cosh u}$$

$$= \frac{\tan\theta}{1+\sec\theta} \quad \text{--- from (1) \& (2)}$$

$$= \frac{\sin\theta / \cos\theta}{(\cos\theta+1)/\cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta+1}$$

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$$= \frac{2\sin\theta/2\ \cos\theta/2}{2\cos^2\theta/2}$$

$$= \frac{\sin\theta/2}{\cos\theta/2}$$

Hence Proved,

$$\tan\frac{u}{2} = \tan\frac{\theta}{2} \quad \text{—④}$$

Q4] $\cos(x+iy) = \cos x\cos iy - \sin x\sin iy$

$\cos\alpha + i\sin\alpha = \cos x\cosh y - i\sin x\sinh y$

$\therefore$ Separating, $\cos\alpha = \cos x\cosh y$

$\sin\alpha = -\sin x\ \sinh y$

Squaring & adding

$\cos^2\alpha + \sin^2\alpha = \cos^2 x\cosh^2 y + \sin^2 x\sinh^2 y$

$1 = \cos^2 x\cosh^2 y + \sin^2 x\sinh^2 y$

$1 = (1+\sinh^2 y)(1-\sin^2 x) + \sin^2 x\cdot\sinh^2 y$

$1 = 1-\sin^2 x + \sinh^2 y - \sinh^2 y.\sin^2 y + \sin^2 x\sinh^2 y$

$0 = -\sinh^2 y - \sin^2 x$

$\pm\sinh y = \pm\sin^2 x \quad$ —①

② $\cos 2x + \cosh 2y = 2$ (prove)

$\cos 2x + \cosh 2y = (1-\sin^2.2) + (1+2\sinh^2 y)$

$= 1-2\sin^2 x + 1 + 2\sinh^2 y$

but $\sin^2 x = \sinh^2 y$ from ①.

$\therefore \cos 2x + \cosh^2 y = 2$

Hence proved

5Q] $\cosh^{-1}(\sqrt{1+x^2}) = \tanh^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$

LHS: $= \cosh^{-1}(\sqrt{1+x^2}) = a$

$$\sqrt{1+x^2} = \cosh a$$

$$1+x^2 = \cosh^2 a$$

But $1+\sinh^2 a = \cosh^2 a$

$$\therefore x = \sinh a \quad —(1)$$

Consider:

$$\frac{x}{\sqrt{1+x^2}} = \frac{\sinh a}{\cosh a}$$

$$\therefore \frac{\sinh a}{\cosh a} = \tanh a$$

$$\therefore \tanh^{-1}\left(\frac{\sinh a}{\cosh a}\right) = a$$

$$\therefore \tan^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) = a$$

$= $ RHS

Hence proved

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6Q) Find $\log[\sin(x+iy)]$

We know that

$$\sin(x+iy) = \sin x \cos iy + \cos x \sin iy$$
$$= \sin x \cosh y + i \cos x \sinh y$$
$$\therefore \log[\sin x \cosh y + i\cos x \sinh y]$$

We know that,

$$\log[x+iy] = \frac{1}{2}\log(x^2+y^2) + i\tan^{-1}\left(\frac{y}{x}\right)$$

$$\therefore \log(\sin x \cosh y + i\cos x \sinh y)$$
$$= \frac{1}{2}\log[\cosh^2 y \sin^2 x + \cos^2 x \sinh^2 y] + i\tan^{-1}\left(\frac{\sin x \cosh y}{\cos x \sinh y}\right)$$
$$= \frac{1}{2}\log[\cosh^2 y(1-\cos^2 x) + \cos^2 x(\cosh^2 y - 1)] + i\tan^{-1}(\cot x \tanh y)$$
$$= \frac{1}{2}\log[\cosh^2 y - \cos^2 x \cosh^2 y + \cos^2 x \cosh^2 y - \cos^2 x]$$
$$+ i\tan^{-1}(\cot x \tanh y)$$
$$= \frac{1}{2}\log[\cosh^2 y - \cos^2 x] + i\tan^{-1}(\cot x \cdot \tanh y)$$
$$= \frac{1}{2}\log\left[\left(\frac{1+\cosh 2y}{2}\right) - \left(\frac{1+\cos 2x}{2}\right)\right] + i\tan^{-1}(\cot x \cdot \tanh y)$$
$$= \frac{1}{2}\log\left[\frac{1}{2}(\cosh 2y - \cos 2x)\right] + i\tan^{-1}(\cot x \cdot \tanh y)$$

Hence proved.