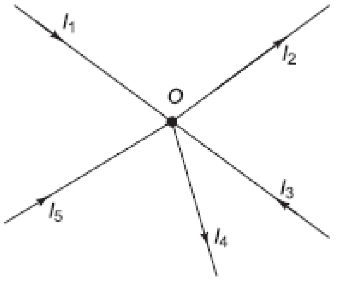
Nodal Analysis

KCL (Kirchhoff's Current Law): The algebraic sum of currents meeting at a junction or node in an electric circuit is zero.

Steps :

1. Mark all nodes.
2. Select one of the node as a reference node. Select a node as reference where a maximum elements are connected or maximum branches are meeting.
3. Assign the unknown potentials of all nodes w.r.t the reference node.
4. At each node, assume the unknown currents and mark their directions.
5. Apply the KCL at each node and write the equations in terms of node voltages. Solve the equations and find node voltages.



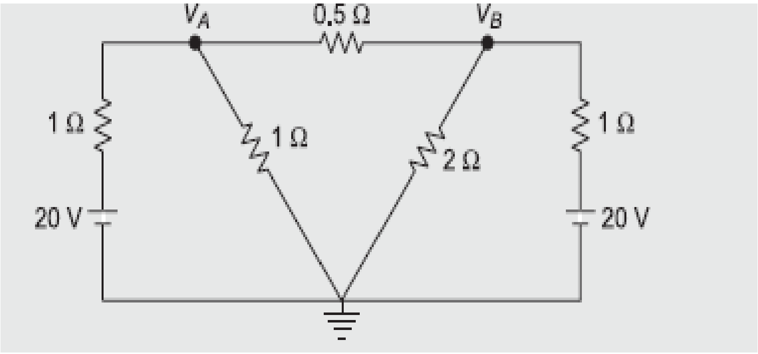
$I_1 + (-I_2) + I_3 + (-I_4) + I_5 = 0$

$I_1 - I_2 + I_3 - I_4 + I_5 = 0$

$I_1 + I_3 + I_5 = I_2 + I_4$

Ex 1: Calculate the current through 2Ω resistor for the network shown in figure

Assume the currents are moving away from the nodes.



I1 + I2 + I3 = 0

Applying KCL at node A,

$$\frac{V_A - 20}{1} + \frac{V_A}{1} + \frac{V_A - V_B}{0.5} = 0$$

$$\left(\frac{1}{1} + \frac{1}{1} + \frac{1}{0.5}\right)V_A - \frac{1}{0.5}V_B = \frac{20}{1}$$

$$4V_A - 2V_B = 20 \qquad (1)$$

I3 + I4+ I5 = 0

Applying KCL at node B,

$$\frac{V_B - V_A}{0.5} + \frac{V_B}{2} + \frac{V_B - 20}{1} = 0$$

$$-\frac{1}{0.5}V_A + \left(\frac{1}{0.5} + \frac{1}{2} + \frac{1}{1}\right)V_B = \frac{20}{1}$$

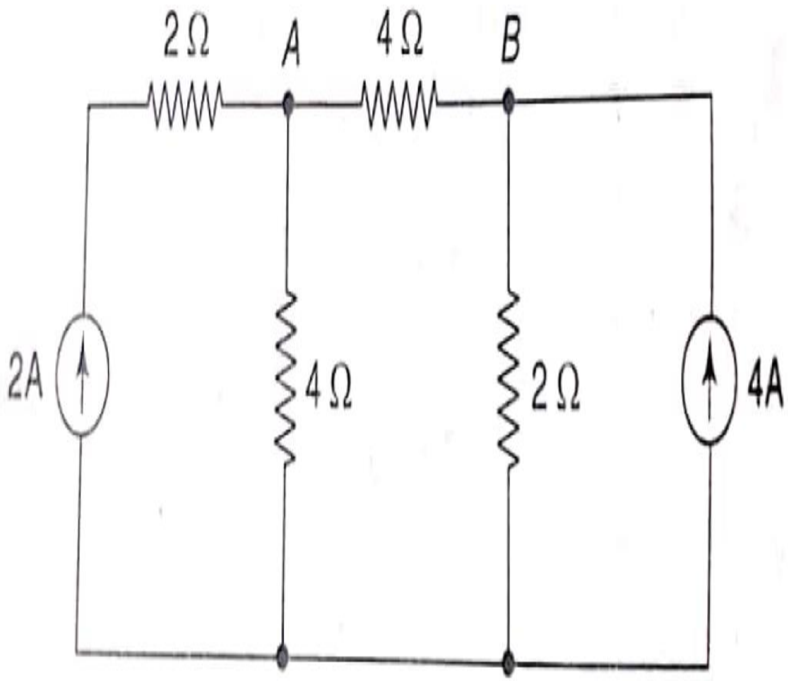
$$-2V_A + 3.5V_B = 20 \qquad (2)$$

Solving Eqs (1) and (2),

$$V_A = 11 \text{ V}$$
$$V_B = 12 \text{ V}$$

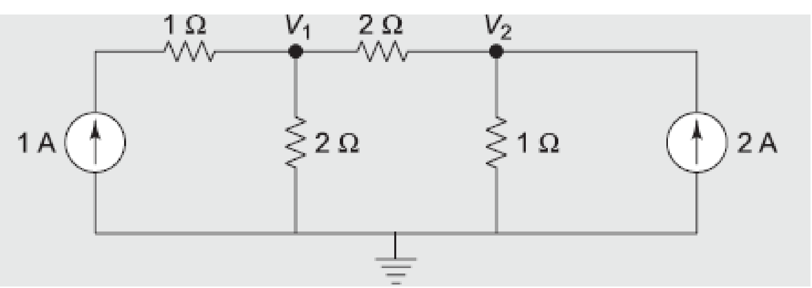
$$\text{Current through 2 Ω resistor} = \frac{V_B}{2} = \frac{12}{2} = 6 \text{ A}$$

Ex. Find node voltages in the given network.



Example 2: *Find the voltage at nodes 1 and 2.*

Solution Assume that the currents are moving away from the nodes.



Applying KCL at Node 1,

$$1 = \frac{V_1}{2} + \frac{V_1 - V_2}{2}$$

$$\left(\frac{1}{2} + \frac{1}{2}\right)V_1 - \frac{1}{2}V_2 = 1$$

$$V_1 - 0.5\ V_2 = 1 \qquad (1)$$

Applying KCL at Node 2,

$$2 = \frac{V_2}{1} + \frac{V_2 - V_1}{2}$$

$$-\frac{1}{2}V_1 + \left(\frac{1}{1} + \frac{1}{2}\right)V_2 = 2$$

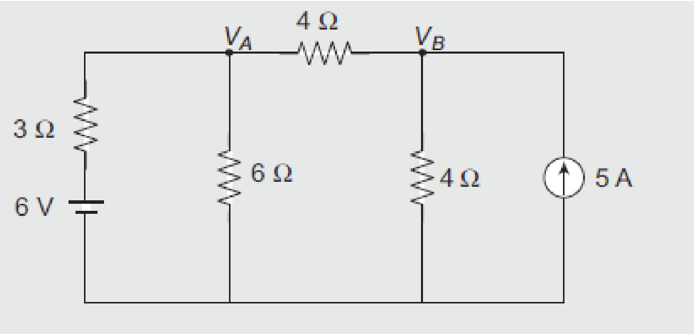
$$-0.5\ V_1 + 1.5\ V_2 = 4 \qquad (2)$$

Solving Eqs (1) and (2),

$$V_1 = 2\ \text{V}$$
$$V_2 = 2\ \text{V}$$

Example 3:- *Find the value of current flowing through 4 Ω resister.*

Solution Assume that the currents are moving away from the nodes.



Applying KCL at Node *A*,

$$\frac{V_A - 6}{3} + \frac{V_A}{6} + \frac{V_A - V_B}{4} = 0$$

$$\left(\frac{1}{3} + \frac{1}{6} + \frac{1}{4}\right)V_A - \frac{1}{4}V_B = 2$$

$$0.75\ V_A - 0.25\ V_B = 2 \qquad (1)$$

Applying KCL at Node *B*,

$$\frac{V_B - V_A}{4} + \frac{V_B}{2} = 5$$

$$-\frac{1}{4}V_A + \left(\frac{1}{4} + \frac{1}{2}\right)V_B = 5$$

$$-0.25\ V_A + 0.75\ V_B = 5 \qquad (2)$$

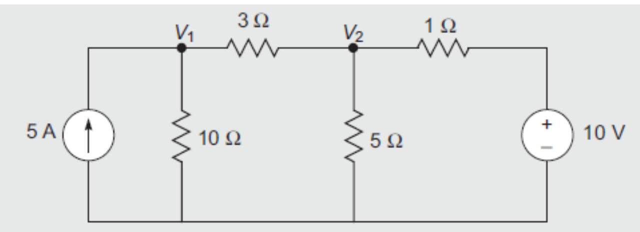
Solving Eqs (1) and (2),

$$V_A = 5.5\text{ V}$$

$$V_B = 8.5\text{ V}$$

$$I_{4\Omega} = \frac{V_B - V_A}{4} = \frac{8.5 - 5.5}{4} = 0.75\text{ A}$$

Example 4:- *For the network given below, find value of current flowing through the 3 Ω resistor.*



Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node 1,

$$5 = \frac{V_1}{10} + \frac{V_1 - V_2}{3}$$

$$\left(\frac{1}{10} + \frac{1}{3}\right) V_1 - \frac{1}{3} V_2 = 5$$

$$0.433V_1 - 0.333V_2 = 5 \qquad (1)$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{3} + \frac{V_2}{5} + \frac{V_2 - 10}{1} = 0$$

$$-\frac{1}{3} V_1 + \left(\frac{1}{3} + \frac{1}{5} + 1\right) V_2 = 10$$

$$-0.333V_1 + 1.533V_2 = 10 \qquad (2)$$

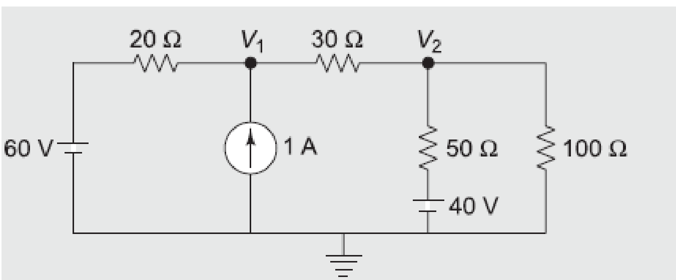
Solving Eqs (1) and (2),

$$V_1 = 19.89 \text{ V}$$

$$V_2 = 10.84 \text{ V}$$

$$I_{3\,\Omega} = \frac{V_1 - V_2}{3} = \frac{10.89 - 10.84}{3} = 3.02 \text{ A}$$

Example 5:- *Find the value of current flowing in the 100 Ω resistor.*



Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node 1,

$$\frac{V_1 - 60}{20} + \frac{V_1 - V_2}{30} = 1$$

$$\left(\frac{1}{20} + \frac{1}{30}\right)V_1 - \frac{1}{30}V_2 = \frac{60}{20} + 1$$

$$0.083\, V_1 - 0.033\, V_2 = 4 \qquad (1)$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{30} + \frac{V_2 - 40}{50} + \frac{V_2}{100} = 0$$

$$-\frac{1}{30}V_1 + \left(\frac{1}{30} + \frac{1}{50} + \frac{1}{100}\right)V_2 = \frac{40}{50}$$

$$-0.033\, V_1 + 0.063\, V_2 = 0.8 \qquad (2)$$

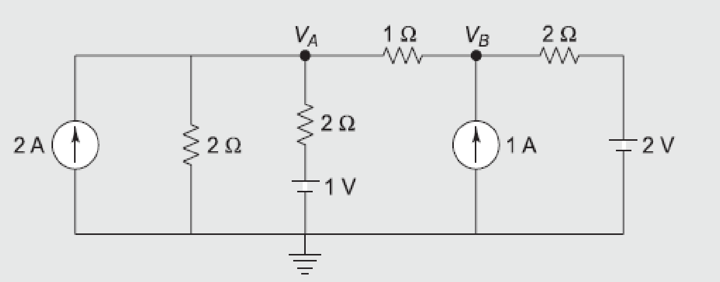
Solving Eqs (1) and (2),

$$V_1 = 67.25 \text{ V}$$

$$V_2 = 48 \text{ V}$$

$$\text{Current through the 100 } \Omega \text{ resistor} = \frac{V_2}{100} = \frac{48}{100} = 0.48 \text{ A}$$

Example 6:- *Find V_A and V_B.*



Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node A,

$$2 = \frac{V_A}{2} + \frac{V_A - 1}{2} + \frac{V_A - V_B}{1}$$

$$\left(\frac{1}{2} + \frac{1}{2} + \frac{1}{1}\right)V_A - \left(\frac{1}{1}\right)V_B = 2 + \frac{1}{2}$$

$$2V_A - V_B = 2.5 \qquad (1)$$

Applying KCL at Node B,

$$\frac{V_B - V_A}{1} + \frac{V_B - 2}{2} = 1$$

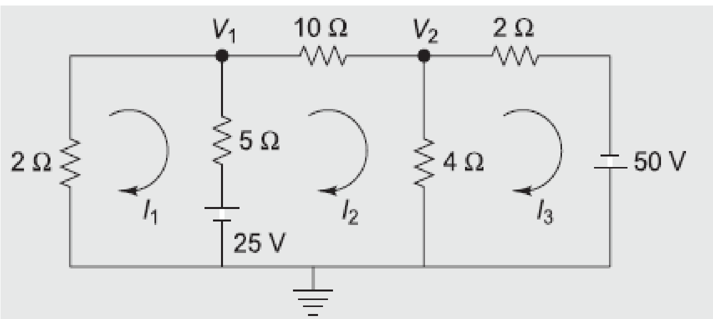
$$-\left(\frac{1}{1}\right)V_A + \left(\frac{1}{1} + \frac{1}{2}\right)V_B = 1 + \frac{2}{2}$$

$$-V_A + 1.5\,V_B = 2 \qquad (2)$$

Solving Eqs (1) and (2),

$$V_A = 2.875 \text{ V}$$
$$V_B = 3.25 \text{ V}$$

Example 7:- *Find the values of currents I_1, I_2 and I_3.*



Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node 1,

$$\frac{V_1}{2} + \frac{V_1 - 25}{5} + \frac{V_1 - V_2}{10} = 0$$

$$\left(\frac{1}{2} + \frac{1}{5} + \frac{1}{10}\right)V_1 - \frac{1}{10}V_2 = \frac{25}{5}$$

$$0.8\ V_1 - 0.1\ V_2 = 5 \qquad (1)$$

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{10} + \frac{V_2}{4} + \frac{V_2 - (-50)}{2} = 0$$

$$-\frac{1}{10}V_1 + \left(\frac{1}{10} + \frac{1}{4} + \frac{1}{2}\right)V_2 = -\frac{50}{2}$$

$$-0.1\ V_1 + 0.85\ V_2 = -25 \qquad (2)$$

Solving Eqs (1) and (2),

$$V_1 = 2.61\text{ V}$$

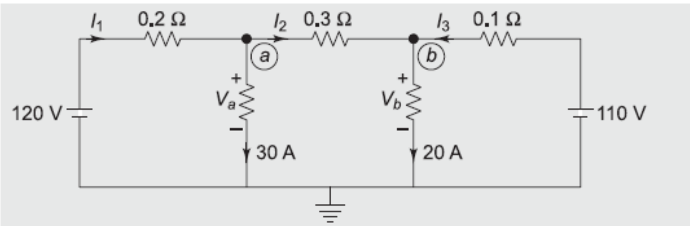
$$V_2 = -29.1\text{ V}$$

$$I_1 = -\frac{V_1}{2} = \frac{-2.61}{2} = -1.31\text{ A}$$

$$I_2 = \frac{V_1 - V_2}{10} = \frac{2.61 - (-29.1)}{10} = 3.17\text{ A}$$

$$I_3 = \frac{V_2 + 50}{2} = \frac{-29.1 + 50}{2} = 10.45\text{ A}$$

Example 8:- *Find the values of currents I_1, I_2 and I_3 and voltages V_a and V_b.*



Solution Applying KCL at Node a,

$$I_1 = 30 + I_2$$

$$\frac{120 - V_a}{0.2} = 30 + \frac{V_a - V_b}{0.3}$$

$$36 - 0.3V_a = 1.8 + 0.2V_a - 0.2V_b$$

$$0.5V_a - 0.2V_b = 34.2 \tag{1}$$

Applying KCL at Node b,

$$I_2 + I_3 = 20$$

$$\frac{V_a - V_b}{0.3} + \frac{110 - V_b}{0.1} = 20$$

$$\frac{0.1V_a - 0.1V_b + 33 - 0.3V_b}{0.03} = 20$$

$$0.1V_a - 0.4V_b = -32.4 \tag{2}$$

Solving Eqs (1) and (2),

$$V_a = 112 \text{ V}$$

$$V_b = 109 \text{ V}$$

$$I_1 = \frac{120 - V_a}{0.2} = \frac{120 - 112}{0.2} = 40 \text{ A}$$

$$I_2 = \frac{V_a - V_b}{0.3} = \frac{112 - 109}{0.3} = 10 \text{ A}$$

$$I_3 = \frac{110 - V_b}{0.1} = \frac{110 - 109}{0.1} = 10 \text{ A}$$