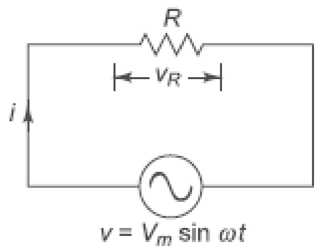
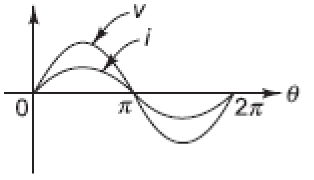
Behaviour of a pure resistor in an AC circuit



Purely resistive circuit

Waveforms The voltage and current waveforms are shown



Waveforms

Current The alternating current i is given by

$$i = \frac{v}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t \qquad \left(\because I_m = \frac{V_m}{R} \right)$$

where I_m is the maximum value of the alternating current. From the voltage and current equation, it is clear that the current is in phase with the voltage in a purely resistive circuit.

Phasor Diagram The phasor diagram is shown in Fig.

The voltage and current phasors are drawn in phase and there is no phase difference.

Impedance It is the resistance offered to the flow of current in an ac circuit.

In a purely resistive circuit,

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{V_m/R} = R$$

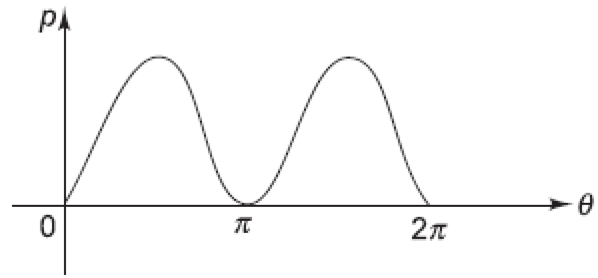
Phase Difference Since the voltage and current are in phase with each other, the phase difference is zero.

$$\phi = 0°$$

Power Factor It is defined as the cosine of the angle between the voltage and current phasors.

$$\text{Power factor} = \cos\phi = \cos(0°) = 1$$

Power



Instantaneous power p is given by

$$p = vi$$

$$= V_m \sin \omega t \, I_m \sin \omega t$$

$$= V_m I_m \sin^2 \omega t$$

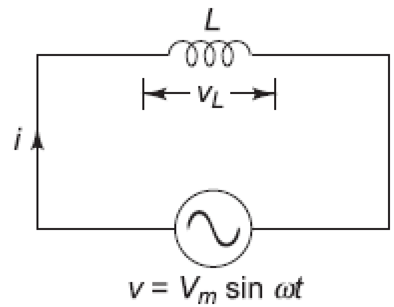
$$= \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$$

$$\text{Average power } P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = VI$$

Thus, power in a purely resistive circuit is equal to the product of rms values of voltage and current.

Behaviour of a pure inductor in an ac circuit



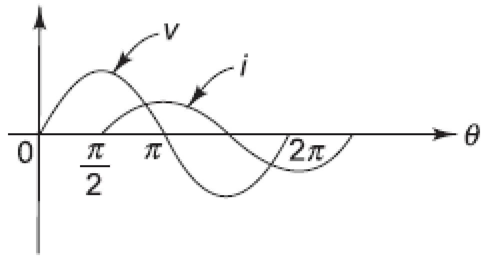
Purely inductive circuit

Current The alternating current i is given by

$$
\begin{aligned}
i &= \frac{1}{L}\int v\,dt \\
&= \frac{1}{L}\int V_m \sin \omega t\,dt \\
&= \frac{V_m}{\omega L}(-\cos \omega t) \\
&= \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) \\
&= I_m \sin\left(\omega t - \frac{\pi}{2}\right) \quad \cdots\left(I_m = \frac{V_m}{\omega L}\right)
\end{aligned}
$$

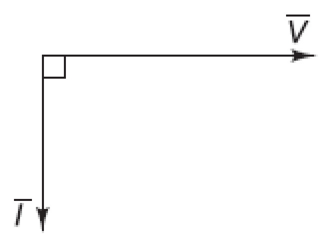
where I_m is the maximum value of the alternating current. From the voltage and current equation, it is clear that the current lags behind the voltage by 90° in a purely inductive circuit.

Waveforms



The voltage and current waveforms are shown in Fig

Phasor Diagram



The phasor diagram is shown in Fig.

Here, voltage $\overline{V}$ is chosen as reference phasor.

Current $\overline{I}$ is drawn such that it lags behind $\overline{V}$ by 90°

Impedance In a purely inductive circuit,

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{V_m/\omega L} = \omega L$$

The quantity ωL is called inductive reactance, is denoted by X_L and is measured in ohms.

For a dc supply, $f = 0 \quad \therefore \quad X_L = 0$

Thus, an inductor acts as a short circuit for a dc supply.

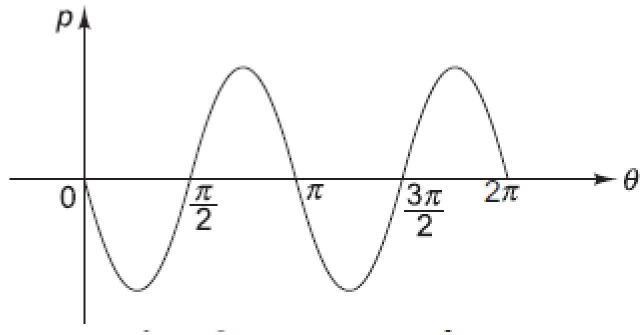
Phase Difference It is the angle between the voltage and current phasors.

$\phi = 90°$

Power Factor It is defined as the cosine of the angle between the voltage and current phasors.

$$\text{pf} = \cos\phi = \cos(90°) = 0$$

Power



Power waveform

Power Instantaneous powers p is given by

$$p = vi$$

$$= V_m \sin \omega t \, I_m \sin \left(\omega t - \frac{\pi}{2} \right)$$

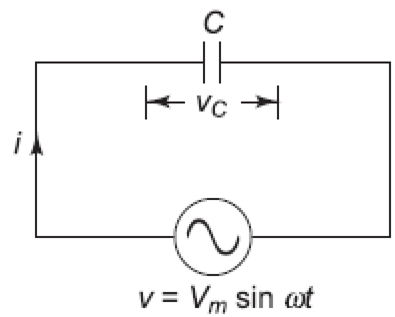
$$= -V_m I_m \sin \omega t \cos \omega t$$

$$= -\frac{V_m I_m}{2} \sin 2\omega t$$

The average power for one complete cycle, $P = 0$.

Hence, power consumed by a purely inductive circuit is zero.

Behavior of a pure capacitor in an ac circuit



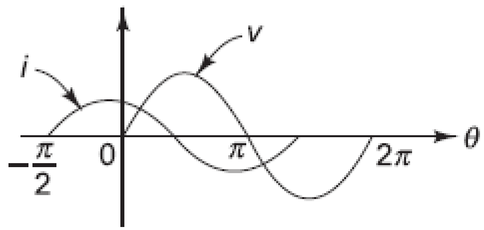
Purely capacitive circuit

Current The alternating current i is given by

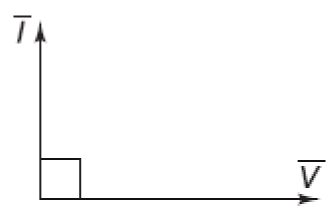
$$
\begin{aligned}
i &= C\,\frac{dv}{dt} \\
&= C\,\frac{d}{dt}(V_m \sin \omega t) \\
&= \omega C\, V_m \cos \omega t \\
&= \omega C\, V_m \sin(\omega t + 90^\circ) \\
&= I_m \sin(\omega t + 90^\circ) \quad \ldots (I_m = \omega C\, V_m)
\end{aligned}
$$

where I_m is the maximum value of the alternating current. From the voltage and current equation, it is clear that the current leads the voltage by 90° in a purely capacitive circuit.

Waveforms The voltage and current waveforms are shown in Fig.



Phasor Diagram The phasor diagram is shown in Fig.



Here, voltage $\bar{V}$ is chosen as reference phasor. Current $\bar{I}$ is drawn such that it leads $\bar{V}$ by 90°.

Impedance In a purely capacitive circuit,

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

The quantity $\frac{1}{\omega C}$ is called capacitive reactance, is denoted by X_C and is measured in ohms.

For a dc supply, $f = 0 \quad \therefore \quad X_C = \infty$

Thus, the capacitor acts as an open circuit for a dc supply.

Phase Difference

It is the angle between the voltage and current phasors.

$\phi = 90°$

Power Factor

It is defined as the cosine of the angle between the voltage and current phasors.

$$\text{pf} = \cos\phi = \cos(90°) = 0$$

Power

Instantaneous power p is given by

$$p = vi$$

$$= V_m \sin \omega t \, I_m \sin(\omega t + 90)$$

$$= V_m I_m \sin \omega t \cos \omega t$$

$$= \frac{V_m I_m}{2} \sin 2\omega t$$

The average power for one complete cycle, $P = 0$.

Hence, power consumed by a purely capacitive circuit is zero.

Example 1:

An ac circuit consists of a pure resistance of 10 ohms and is connected across an ac supply of 230 V, 50 Hz. Calculate (i) current, (ii) power consumed, (iii) power factor, and (iv) write down the equations for voltage and current.

Solution

$$R = 10\ \Omega$$
$$V = 230\ \text{V}$$
$$f = 50\ \text{Hz}$$

(i) Current

$$I = \frac{V}{R} = \frac{230}{10} = 23\ \text{A}$$

(ii) Power consumed

$$P = VI = 230 \times 23 = 5290\ \text{W}$$

(iii) Power factor

Since the voltage and current are in phase with each other, $\phi = 0°$

$$\text{pf} = \cos\phi = \cos(0°) = 1$$

(iv) Voltage and current equations

$$V_m = \sqrt{2}\,V = \sqrt{2} \times 230 = 325.27\ \text{V}$$
$$I_m = \sqrt{2}\,I = \sqrt{2} \times 23 = 32.53\ \text{A}$$
$$\omega = 2\pi f = 2\pi \times 50 = 314.16\ \text{rad/s}$$
$$v = V_m \sin \omega t = 325.27 \sin 314.16\,t$$
$$i = I_m \sin \omega t = 32.53 \sin 314.16\,t$$

Example 2

An inductive coil having negligible resistance and 0.1 henry inductance is connected across a 200 V, 50 Hz supply. Find (i) inductive reactance, (ii) rms value of current, (iii) power, (iv) power factor, and (v) equations for voltage and current.

Solution

$L = 0.1$ H
$V = 200$ V
$f = 50$ Hz

(i) Inductive reactance

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.1 = 31.42\ \Omega$$

(ii) rms value of current

$$I = \frac{V}{X_L} = \frac{200}{31.42} = 6.37\text{ A}$$

(iii) Power

Since the current lags behind the voltage by 90° in purely inductive circuit, $\phi = 90°$

$$P = VI\cos\phi = 200 \times 6.37 \times \cos(90°) = 0$$

(iv) Power factor

$$\text{pf} = \cos\phi = \cos(90°) = 0$$

(v) Equations for voltage and current

$$V_m = \sqrt{2}\,V = \sqrt{2} \times 200 = 282.84\text{ V}$$
$$I_m = \sqrt{2}\,I = \sqrt{2} \times 6.37 = 9\text{ A}$$
$$\omega = 2\pi f = 2\pi \times 50 = 314.16\text{ rad/s}$$
$$v = V_m \sin\omega t = 282.84 \sin 314.16\,t$$
$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right) = 9\sin\left(314.16\,t - \frac{\pi}{2}\right)$$

Example3:-

The voltage and current through circuit elements are

$$v = 100 \sin(314\,t + 45°) \text{ volts}$$

$$i = 10 \sin(314\,t + 315°) \text{ amperes}$$

(i) Identify the circuit elements. (ii) Find the value of the elements. (iii) Obtain an expression for power.

Solution

$$v = 100 \sin(314\,t + 45°)$$

$$i = 10 \sin(314\,t + 315°)$$

$$= 10 \sin(314\,t + 315° - 360°)$$

$$= 10 \sin(314\,t - 45°)$$

(i) Identification of elements

From voltage and current equations, it is clear that the current i lags behind the voltage by 90°. Hence, the circuit element is an inductor.

(ii) Value of elements

$$X_L = \frac{V}{I} = \frac{V_m}{I_m} = \frac{100}{10} = 10\,\Omega$$

$$X_L = \omega L$$

$$10 = 314\,L$$

$$L = 31.8 \text{ mH}$$

(iii) Expression for power

$$p = -\frac{V_m I_m}{2}\sin 2\omega t = -\frac{100 \times 10}{2}\sin(2 \times 314\,t) = -500 \sin 628\,t$$

Example 4:-

A capacitor has a capacitance of 30 microfarads which is connected across a 230 V, 50 Hz supply. Find (i) capacitive reactance, (ii) rms value of current, (iii) power, (iv) power factor, and (v) equations for voltage and current.

Solution

$C = 30\ \mu F$

$V = 230\ V$

$f = 50\ Hz$

(i) Capacitive reactance

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 30 \times 10^{-6}} = 106.1\ \Omega$$

(ii) rms value of current

$$I = \frac{V}{X_C} = \frac{230}{106.1} = 2.17\ A$$

(iii) Power

Since the current leads the voltage by 90° in purely capacitive circuit, $\phi = 90°$

$$P = VI \cos\phi = 230 \times 2.17 \times \cos(90°) = 0$$

(iv) Power factor

$$\text{pf} = \cos\phi = \cos(90°) = 0$$

(v) Equations for voltage and current

$$V_m = \sqrt{2}\, V = \sqrt{2} \times 230 = 325.27\ V$$

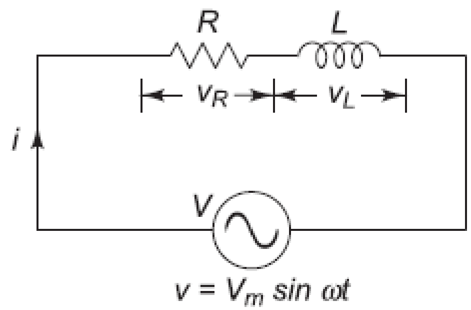
$$I_m = \sqrt{2}\, I = \sqrt{2} \times 2.17 = 3.07\ A$$

$$\omega = 2\pi f = 2\pi \times 50 = 314.16\ \text{rad/s}$$

$$v = V_m \sin\omega t = 325.27 \sin 314.16\, t$$

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right) = 3.07 \sin\left(314.16\, t + \frac{\pi}{2}\right)$$

Series RL circuit



Potential difference across the resistor = $V_R = R\,I$

Potential difference across the inductor = $V_L = X_L\,I$

The voltage $\bar{V}_R$ is in phase with the current $\bar{I}$

whereas the voltage $\bar{V}_L$ leads the current $\bar{I}$ by 90°.

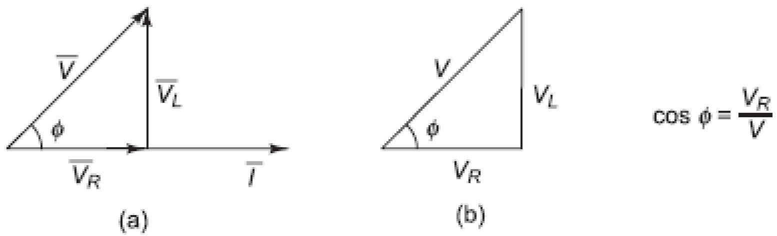
Reference: Basic electrical Engineering, By Ravish Singh, Mc Graw Hill Publication

Phasor Diagram of R-L Circuit

1. Since the same current flows through series circuit, $\overline{I}$ is taken as reference phasor.
2. Draw $\overline{V}_R$ in phase with $\overline{I}$.
3. Draw $\overline{V}_L$ such that it leads $\overline{I}$ by 90°.
4. Add $\overline{V}_R$ and $\overline{V}_L$ by triangle law of vector addition such that

$$\overline{V} = \overline{V}_R + \overline{V}_L$$

5. Mark the angle between $\overline{I}$ and $\overline{V}$ as ϕ.



(a) Phasor diagram (b) Voltage triangle

It is clear from phasor diagram that current $\overline{I}$ lags behind applied voltage $\overline{V}$ by an angle ϕ $(0° < \phi < 90°)$.

Reference: Basic electrical Engineering, By Ravish Singh, Mc Graw Hill Publication

Impedance

$$\bar{V} = \bar{V}_R + \bar{V}_L = R\bar{I} + jX_L\,\bar{I} = (R + jX_L)\,\bar{I}$$

$$\frac{\bar{V}}{\bar{I}} = R + jX_L = \bar{Z}$$

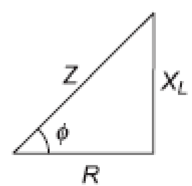
$$\bar{Z} = Z\angle\phi$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

The quantity Z is called the *complex impedance* of the *R-L* circuit.

Impedance Triangle The impedance triangle is shown in Fig.



$$\cos\phi = \frac{R}{Z}$$

Current

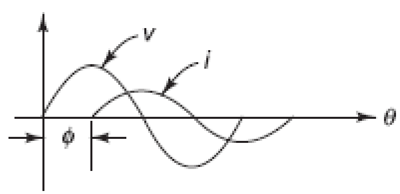
From the phasor diagram, it is clear that the current I lags behind the voltage V by an angle ϕ. If the applied voltage is given by $v = V_m \sin \omega t$ then the current equation will be

$$i = I_m \sin(\omega t - \phi)$$

where $$I_m = \frac{V_m}{Z}$$

and $$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

Waveforms The voltage and current waveforms are shown in Fig.



Power Instantaneous power p is given by

$$\begin{aligned} p &= v\,i \\ &= V_m \sin \omega t\, I_m \sin(\omega t - \phi) \\ &= V_m I_m \sin \omega t \sin(\omega t - \phi) \\ &= V_m I_m \left[\frac{\cos\phi - \cos(2\omega t - \phi)}{2}\right] \\ &= \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi) \end{aligned}$$

Thus, power consists of a constant part $\frac{V_m I_m}{2}$ cos ϕ and a fluctuating part $\frac{V_m I_m}{2}$ cos $(2\omega t - \phi)$. The frequency of the fluctuating part is twice the applied voltage frequency and its average value over one complete cyce is zero.

$$\text{Average power } P = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\phi = VI\cos\phi$$

Power Triangle

In terms of circuit components,

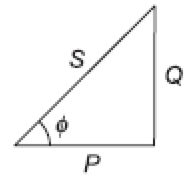
$$\cos \phi = \frac{R}{Z}$$

and $\quad V = Z\,I$

$$P = VI\cos\phi = Z\,I\,I\,\frac{R}{Z} = I^2R$$

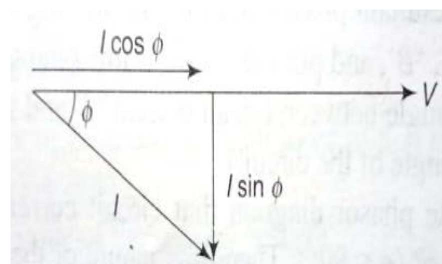
$$Q = VI\sin\phi = Z\,I\,I\,\frac{X_L}{Z} = I^2X_L$$

$$S = VI = Z\,I\,I = I^2Z$$



$$\cos\phi = \frac{P}{S}$$

Power triangle



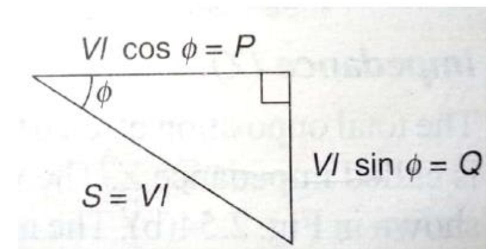


Fig.2 and 3 reference: B.R. Patil, BEEE, Oxford Publication

Power Factor It is defined as the cosine of the angle between the voltage and current phasors.

$$\text{pf} = \cos\phi$$

From voltage triangle, $\quad \text{pf} = \dfrac{V_R}{V}$

From impedance triangle, $\quad \text{pf} = \dfrac{R}{Z}$

From power triangle, $\quad \text{pf} = \dfrac{P}{S}$

In case of an *R-L* series circuit, the power factor is lagging in nature since the current lags behind the voltage by an angle ϕ.

Example1:-

When a sinusoidal voltage of 120 V (rms) is applied to a series R-L circuit, it is found that there occurs a power dissipation of 1200 W and a current flow given by $i(t) = 28.3 \sin (314t - \phi)$. Find the circuit resistance and inductance.

Solution

$$V = 120 \text{ V}$$

$$P = 1200 \text{ W}$$

$$i(t) = 28.3 \sin (314t - \phi)$$

(i) Resistance

$$I = \frac{28.3}{\sqrt{2}} = 20.01 \text{ A}$$

$$P = VI \cos \phi$$

$$1200 = 120 \times 20.01 \times \cos \phi$$

$$\cos \phi = 0.499$$

$$\phi = 60.02^\circ$$

$$Z = \frac{V}{I} = \frac{120}{20.01} = 6\ \Omega$$

$$\overline{Z} = Z \angle \phi = 6 \angle 60.02^\circ = 3 + j5.2\ \Omega$$

$$R = 3\ \Omega$$

(ii) Inductance

$$X_L = 5.2\ \Omega$$

$$X_L = \omega L$$

$$5.2 = 314 \times L$$

$$L = 0.0165 \text{ H}$$

Reference: Basic electrical Engineering, By Ravish Singh, Mc Graw Hill Publication

Example2:-

A series circuit consists of a non-inductive resistance of 6 Ω and an inductive reactance of 10 Ω. When connected to a single-phase ac supply, it draws a current i(t) = 27.89 sin (628t – 45°). Calculate (i) the voltage applied to the series circuit in the form V_m sin (ωt ± φ), (ii) inductance, and (iii) power drawn by the circuit.

Solution

$$R = 6\ \Omega$$

$$X_L = 10\ \Omega$$

$$i(t) = 27.89 \sin (628t - 45°)$$

(i) Voltage applied to the series circuit

$$\bar{Z} = R + jX_L = 6 + j10 = 11.66 \angle 59.04°\ \Omega$$

$$\bar{I} = \frac{27.89}{\sqrt{2}} \angle -45° = 19.72 \angle -45°\ \text{A}$$

$$\bar{V} = \bar{Z}\ \bar{I} = (11.66 \angle 59.04°)\ (19.72 \angle -45°) = 229.95 \angle 14.04°\ \text{V}$$

$$v = 229.95\sqrt{2} \sin (\omega t + 14.04°) = 325.2 \sin (\omega t + 14.04°)$$

(ii) Inductance

$$X_L = \omega L$$

$$10 = 628 \times L$$

$$L = 15.9\ \text{mH}$$

(iii) Power drawn by the circuit

$$P = VI \cos \phi = 229.95 \times 19.72 \times \cos (59.04°) = 2332.78\ \text{W}$$

Example3:-

A resistor of 25 Ω is connected in series with a choke coil. The series combination when connected across a 250 V, 50 Hz supply, draws a current of 4 A which lags behind the voltage by 65°. Calculate (i) resistance and inductance of the coil, (ii) total power, (iii) power consumed by resistance, and (iv) power consumed by choke coil. **[May 2014]**

Solution

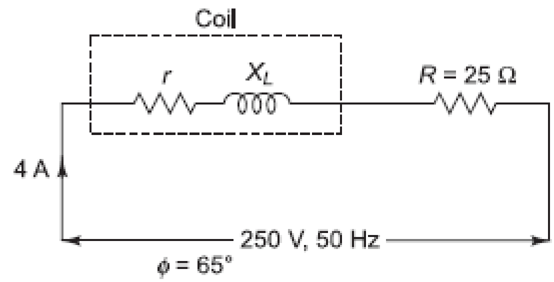
$R = 25\ \Omega$

$V = 250\text{ V}$

$f = 50\text{ Hz}$

$I = 4\text{ A}$

$\phi = 65°$



(i) Resistance and inductance of the coil

$$Z = \frac{V}{I} = \frac{250}{4} = 62.5\ \Omega$$

$$\bar{Z} = Z\angle\phi = 62.5\angle 65° = 26.41 + j56.64\ \Omega$$

But $$\bar{Z} = (R + r) + jX_L$$

$$X_L = 56.64\ \Omega$$

$$R + r = 26.41$$

$$r = 26.41 - 25 = 1.41\ \Omega$$

$$X_L = 2\pi f L$$

$$56.64 = 2\pi \times 50 \times L$$

$$L = 0.18\text{ H}$$

(ii) Total power

$$P = I^2(R + r) = (4)^2 \times 26.41 = 422.56\text{ W}$$

(iii) Power consumed by resistance

$$P_R = I^2 R = (4)^2 \times 25 = 400\text{ W}$$

(iv) Power consumed by choke coil

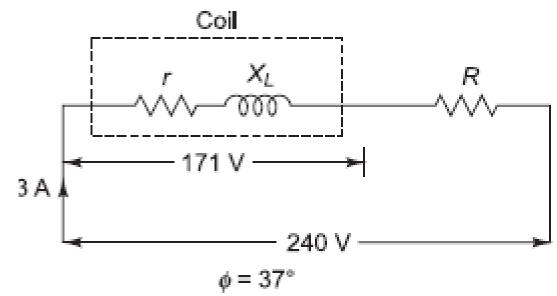
$$P_{coil} = I^2 r = (4)^2 \times 1.41 = 22.56\text{ W}$$

Example4:-

When a resistor and a coil in series are connected to a 240 V supply, a current of 3 A flows, lagging 37° behind the supply voltage. The voltage across the coil is 171 volts. Find the resistance and reactance of the coil, and the resistance of the resistor. **[May 2014, Dec 2015]**

Solution

$$V = 240\text{ V}$$
$$I = 3\text{ A}$$
$$\phi = 37°$$
$$V_{\text{coil}} = 171\text{ V}$$



(i) Resistance and reactance of the coil

$$Z_{\text{coil}} = \frac{V_{\text{coil}}}{I} = \frac{171}{3} = 57\ \Omega$$

$$\sqrt{r^2 + X_L^2} = 57$$

$$r^2 + X_L^2 = 3249$$

$$Z = \frac{V}{I} = \frac{240}{3} = 80\ \Omega$$

$$\bar{Z} = Z\angle\phi = 80\ \angle 37° = 63.89 + j48.15\ \Omega$$

But $$\bar{Z} = (R + r) + jX_L$$

$$r^2 + X_L^2 = 3249$$
$$r^2 + (48.15)^2 = 3249$$
$$r^2 = 931.04$$
$$r = 30.51\ \Omega$$

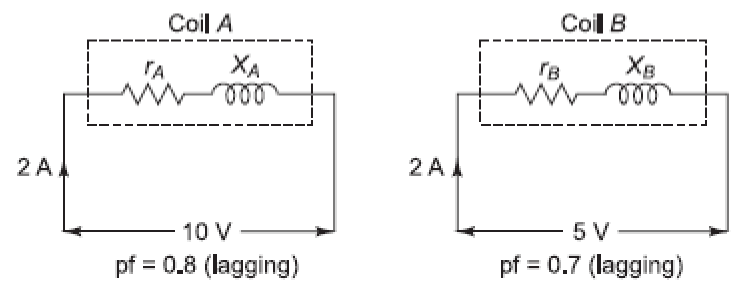
(ii) Resistance of the resistor

$$R + r = 63.89$$
$$R + 30.51 = 63.89$$
$$R = 33.38\ \Omega$$

Example5:- *A coil A takes 2 A at a power factor of 0.8 lagging with an applied p.d. of 10 V. A second coil B takes 2 A with a power factor of 0.7 lagging with an applied voltage of 5 V. What voltage will be required to produce a total current of 2 A with coils A and B in series? Find the power factor in this case.*

Solution

$I = 2\text{ A}$



Coil A: $I_A = 2\text{ A}$, $\text{pf}_A = 0.8$ (lagging), $V_A = 10\text{ V}$

Coil B: $I_B = 2\text{ A}$, $\text{pf}_B = 0.7$ (lagging), $V_B = 5\text{ V}$

For Coil A,

$$\phi_A = \cos^{-1}(0.8) = 36.87°$$

$$Z_A = \frac{V_A}{I_A} = \frac{10}{2} = 5\ \Omega$$

$$\bar{Z}_A = Z_A \angle\phi_A = 5\ \angle 36.87° = 4 + j3\ \Omega$$

$$r_A = 4\ \Omega$$

$$X_A = 3\ \Omega$$

For Coil B,

$$\phi_B = \cos^{-1}(0.7) = 45.57°$$

$$Z_B = \frac{V_B}{I_B} = \frac{5}{2} = 2.5\ \Omega$$

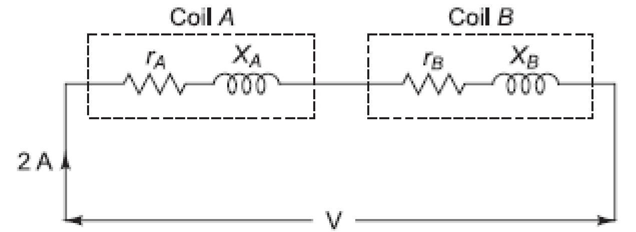
$$\bar{Z}_B = Z_B \angle\phi_B = 2.5\ \angle 45.57° = 1.75 + j1.78\ \Omega$$

$$r_B = 1.75\ \Omega$$

$$X_B = 1.78\ \Omega$$

Continue………………

When coils A and B are connected in series,



$$\bar{Z} = r_A + jX_A + r_B + jX_B = 4 + j3 + 1.75 + j1.78$$

$$= 5.75 + j4.78 = 7.48 \angle 39.74° \ \Omega$$

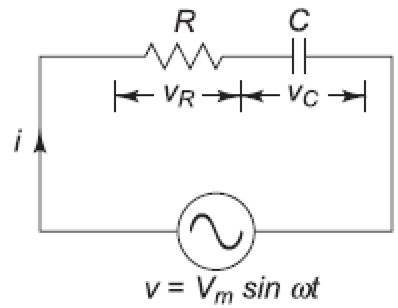
$$Z = 7.48 \ \Omega$$

$$\phi = 39.74°$$

$$V = ZI = 7.48 \times 2 = 14.96 \text{ V}$$

$$\text{pf} = \cos\phi = \cos(39.74°) = 0.77 \text{ (lagging)}$$

Series R-C circuit



Let V and I be the rms values of applied voltage and current.

Potential difference across the resistor = $V_R = R\,I$

Potential difference across the capacitor = $V_C = X_C\,I$

The voltage $\overline{V}_R$ is in phase with the current $\overline{I}$ whereas voltage $\overline{V}_C$ lags behind the current $\overline{I}$ by 90°.

$$\overline{V} = \overline{V}_R + \overline{V}_C$$

Reference: Basic electrical Engineering, By Ravish Singh, Mc Graw Hill Publication

Phasor Diagram for R-C circuit

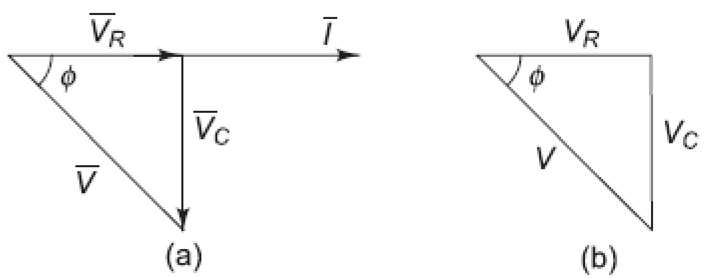
Steps for drawing phasor diagram

1. Since the same current flows through series circuit, $\bar{I}$ is taken as reference phasor.
2. Draw $\bar{V}_R$ in phase with $\bar{I}$.
3. Draw $\bar{V}_C$ such that it lags behind $\bar{I}$ by 90°.
4. Add $\bar{V}_R$ and $\bar{V}_C$ by triangle law of addition such that

$$\bar{V} = \bar{V}_R + \bar{V}_C$$

5. Mark the angle $\bar{I}$ and $\bar{V}$ as ϕ.

It is clear from phasor diagram that current $\bar{I}$ leads applied voltage $\bar{V}$ by an angle ϕ ($0° < \phi < 90°$).



$$\cos\phi = \frac{V_R}{V}$$

(a) Phasor diagram (b) Voltage triangle

Impedance

$$\bar{V} = \bar{V}_R + \bar{V}_C$$

$$= R\bar{I} - jX_C\bar{I}$$

$$= (R - jX_C)\bar{I}$$

$$\frac{\bar{V}}{\bar{I}} = R - jX_C = \bar{Z}$$

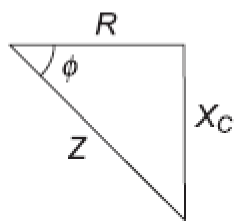
$$\bar{Z} = Z\angle -\phi$$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

The quantity $\bar{Z}$ is called the *complex impedance* of the *R-C* circuit.

Impedance Triangle The impedance triangle is shown in Fig.



$$\cos \phi = \frac{R}{Z}$$

Impedance triangle

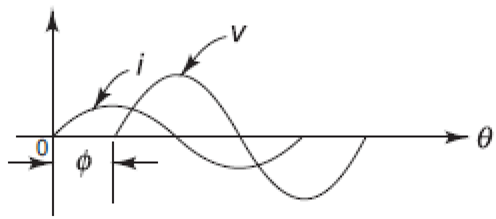
Current From the phasor diagram, it is clear that the current I leads the voltage V by an angle ϕ. If the applied voltage is given by $v = V_m \sin \omega t$ then the current equation will be

$$i = I_m \sin(\omega t + \phi)$$

where
$$I_m = \frac{V_m}{Z}$$

and
$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

Waveforms The voltage and current waveforms are shown in Fig.



Power

Active power $P = VI\cos\phi = I^2R$

Reactive power $Q = VI\sin\phi = I^2X_C$

Apparent power $S = VI = I^2Z$

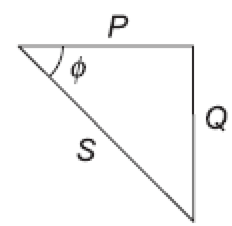
Power Factor It is defined as the cosine of the angle between voltage and current phasors.

From voltage triangle, $\text{pf} = \dfrac{V_R}{V}$

From impedance triangle $\text{pf} = \dfrac{R}{Z}$

From power triangle, $\text{pf} = \dfrac{P}{S}$

Power Triangle The power triangle is shown in Fig.



$\cos\phi = \dfrac{P}{S}$

In case of an *R-C* series circuit, the power factor is leading in nature since the current leads the voltage by an angle ϕ.

Example1:-

The voltage applied to a circuit is $e = 100 \sin(\omega t + 30°)$ and the current flowing in the circuit is $i = 15 \sin(\omega t + 60°)$. Determine impedance, resistance, reactance, power factor and power.

Solution

$$e = 100 \sin(\omega t + 30°)$$
$$i = 15 \sin(\omega t + 60°)$$

(i) Impedance

$$\bar{E} = \frac{100}{\sqrt{2}} \angle 30° \text{ V}$$

$$\bar{I} = \frac{15}{\sqrt{2}} \angle 60° \text{ A}$$

$$\bar{Z} = \frac{\bar{E}}{\bar{I}} = \frac{\frac{100}{\sqrt{2}} \angle 30°}{\frac{15}{\sqrt{2}} \angle 60°} = 6.67 \angle -30° = 5.77 - j3.33 = R - j X_C$$

$$Z = 6.67 \ \Omega$$

(ii) Resistance

$$R = 5.77 \ \Omega$$

(iii) Reactance

$$X_C = 3.33 \ \Omega$$

(iv) Power factor

$$\text{pf} = \cos\phi = \cos(30°) = 0.866 \text{ (leading)}$$

(v) Power

$$P = EI \cos\phi = \frac{100}{\sqrt{2}} \times \frac{15}{\sqrt{2}} \times 0.866 = 649.5 \text{ W}$$

Example2:-

A voltage of 125 V at 50 Hz is applied across a non-inductive resistor connected in series with a capacitor. The current is 2.2 A. The power loss in the resistor is 96.8 W. Calculate the resistance and capacitance.

Solution

$$V = 125 \text{ V}$$
$$f = 50 \text{ Hz}$$
$$I = 2.2 \text{ A}$$
$$P = 96.8 \text{ W}$$

(i) Resistance

$$Z = \frac{V}{I} = \frac{125}{2.2} = 56.82 \text{ A}$$

$$P = I^2 R$$

$$96.8 = (2.2)^2 \times R$$

$$R = 20\ \Omega$$

(ii) Capacitance

$$X_C = \sqrt{Z^2 - R^2} = \sqrt{(56.82)^2 - (20)^2} = 53.18\ \Omega$$

$$X_C = \frac{1}{2\pi f C}$$

$$53.18 = \frac{1}{2\pi \times 50 \times C}$$

$$C = 59.85\ \mu\text{F}$$

Example3:-

A resistor and a capacitor are connected across a 250 V supply. When the supply frequency is 50 Hz, the current drawn is 5 A. When the frequency is increased to 60 Hz, it draws 5.8 A. Find the values of R and C and power drawn in the second case.

Solution

$$V = 250 \text{ V}$$
$$f_1 = 50 \text{ Hz}$$
$$I_1 = 5 \text{ A}$$
$$f_2 = 60 \text{ Hz}$$
$$I_2 = 5.8 \text{ A}$$

(i) Values of R and C

For $f_1 = 50$ Hz,

$$Z_1 = \frac{V}{I_1} = \frac{250}{5} = 50\ \Omega$$

$$Z_1 = \sqrt{R^2 + \left(\frac{1}{2\pi f_1 C}\right)^2} = \sqrt{R^2 + \left(\frac{1}{100\pi C}\right)^2}$$

$$R^2 + \left(\frac{1}{100\pi C}\right)^2 = 2500$$

For $f_2 = 60$ Hz,

$$Z_2 = \frac{V}{I_2} = \frac{250}{5.8} = 43.1\ \Omega$$

$$Z_2 = \sqrt{R^2 + \left(\frac{1}{2\pi f_2 C}\right)^2} = \sqrt{R^2 + \left(\frac{1}{120\pi C}\right)^2}$$

$$R^2 + \left(\frac{1}{120\pi C}\right)^2 = 1857.9\ \Omega$$

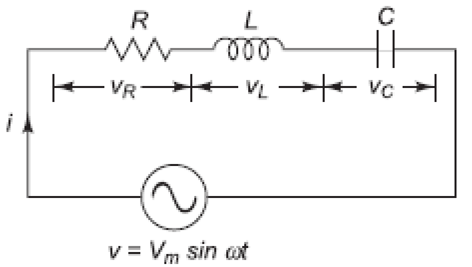
Solving Eqs (1) and (2),

$$R = 19.96\ \Omega$$
$$C = 69.4\ \mu\text{F}$$

(ii) Power drawn in the second case

$$P_2 = I_2^2 R = (5.8)^2 \times 19.96 = 671.45 \text{ W}$$

Series R-L-C Circuit



Let V and I be the rms values of the applied voltage and current.

Potential difference across the resistor = $V_R = R\ I$

Potential difference across the inductor = $V_L = X_L\ I$

The voltage $\overline{V}_R$ is in phase with the current $\overline{I}$, the voltage $\overline{V}_L$ leads the current $\overline{I}$ by 90° and the voltage $\overline{V}_C$ lags behind the current $\overline{I}$ by 90°.

$$\overline{V} = \overline{V}_R + \overline{V}_L + \overline{V}_C$$

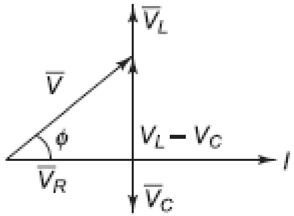
Reference: Basic electrical Engineering, By Ravish Singh, Mc Graw Hill Publication

Phasor diagram of R-L-C Circuit

Phasor Diagram Since the same current flows through R, L and C, the current I is taken as a reference phasor.

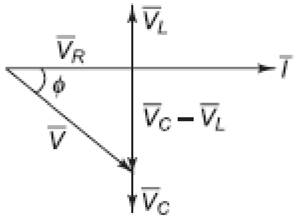
Case (i) $X_L > X_C$

The reactance X will be inductive in nature and the circuit will behave like an R-L circuit.



Case (ii) $X_C > X_L$

The reactance X will be capacitive in nature and the circuit will behave like an R-C circuit.



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Impedance

$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C = R\bar{I} + jX_L\bar{I} - jX_C\bar{I} = [R + j(X_L - X_C)]\,\bar{I}$$

$$\frac{\bar{V}}{\bar{I}} = R + j(X_L - X_C) = \bar{Z}$$

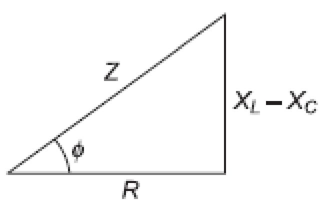
$$\bar{Z} = Z\angle\phi$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

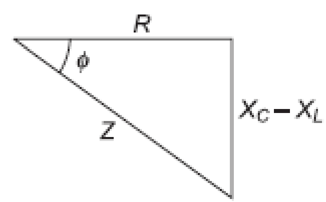
$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Impedance Triangles Impedance triangles are shown in Fig. 4.42.

Case (i) $X_L > X_C$



Case (ii) $X_C > X_L$



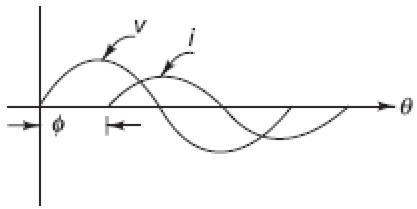
Current Equation If the applied voltage is given by $v = V_m \sin \omega t$ then current equation will be

$$i = I_m \sin(\omega t \pm \phi)$$

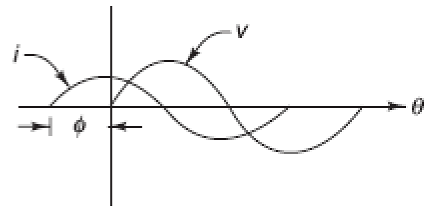
'–' sign is used when $X_L > X_C$.
'+' sign is used when $X_C > X_L$.

Waveforms The voltage and current waveforms are shown in Fig.

Case (i) $X_L > X_C$



Case (ii) $X_C > X_L$



Power

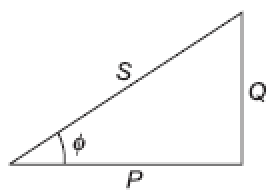
Average power $P = VI \cos \phi = I^2R$

Reactive power $Q = VI \sin \phi = I^2X$

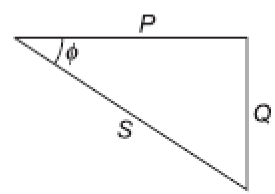
Apparent power $S = VI = I^2Z$

Power Triangles Power triangles are shown in Fig.

Case (i) $X_L > X_C$



Case (ii) $X_C > X_L$



Power Factor It is defined as the cosine of the angle between voltage and current phasors.

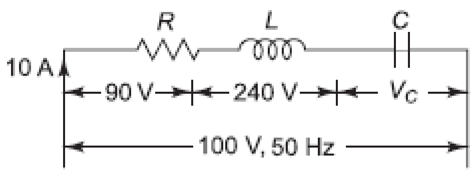
$$\text{pf} = \cos \phi$$

$$\text{pf} = \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$$

Example 1:-

A circuit consists of a pure inductor, a pure resistor and a capacitor connected in series. When the circuit is supplied with 100 V, 50 Hz supply, the voltages across inductor and resistor are 240 V and 90 V respectively. If the circuit takes a 10 A leading current, calculate (i) value of inductance, resistance and capacitance, (ii) power factor of the circuit, and (iii) voltage across the capacitor.

Solution



$$V = 100 \text{ V}$$
$$f = 50 \text{ Hz}$$
$$V_L = 240 \text{ V}$$
$$V_R = 90 \text{ V}$$
$$I = 10 \text{ A}$$

(i) Value of inductance, resistance and capacitance

$$R = \frac{V_R}{I} = \frac{90}{10} = 9\ \Omega$$

$$X_L = \frac{V_L}{I} = \frac{240}{10} = 24\ \Omega$$

$$Z = \frac{V}{I} = \frac{100}{10} = 10\ \Omega$$

Continue…..

$$Z = \frac{V}{I} = \frac{100}{10} = 10\,\Omega$$

$$\bar{Z} = R + j\,X_L - j\,X_C = R - j(X_C - X_L)$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$10 = \sqrt{(9)^2 + (X_C - 24)^2}$$

$$X_C = 28.36\,\Omega$$

$$X_L = 2\pi f L$$

$$24 = 2\pi \times 50 \times L$$

$$L = 0.076\ \text{H}$$

$$X_C = \frac{1}{2\pi f C}$$

$$28.36 = \frac{1}{2\pi \times 50 \times C}$$

$$\text{C} = 112.24\ \mu\text{F}$$

(ii) Power factor of the circuit

$$\text{pf} = \frac{R}{Z} = \frac{9}{10} = 0.9\ \text{(leading)}$$

(iii) Voltage across the capacitor

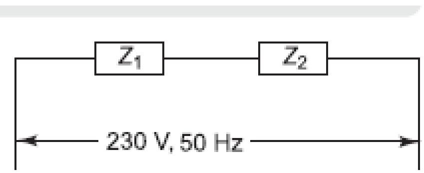
$$V_C = X_C I = 28.36 \times 10 = 283.6\ \text{V}$$

Example2:-

Two impedances $Z_1 = 40\angle 30°\ \Omega$ and $Z_2 = 30\ \angle 60°\ \Omega$ are connected in series across a single-phase 230 V, 50 Hz supply. Calculate the (i) current drawn, (ii) pf, and (iii) power consumed by the circuit.

Solution

$$\bar{Z}_1 = 40\ \angle 30°\ \Omega$$
$$\bar{Z}_2 = 30\ \angle 60°\ \Omega$$
$$V = 230\ \text{V}$$



(i) Current drawn

$$\bar{Z} = \bar{Z}_1 + \bar{Z}_2 = 40\ \angle 30° + 30\ \angle 60° = 67.66\ \angle 42.81°\ \Omega$$
$$I = \frac{V}{Z} = \frac{230}{67.66} = 3.4\ \text{A}$$

(ii) Power factor

$$\text{pf} = \cos\phi = \cos(42.81°) = 0.734\ \text{(lagging)}$$

(iii) Power consumed

$$P = VI\cos\phi = 230 \times 3.4 \times 0.734 = 573.99\ \text{W}$$

Example3:-

A coil of 3 Ω resistance and an inductance of 0.22 H is connected in series with an imperfect capacitor. When such a series circuit is connected across a 200 V, 50 Hz supply, it has been observed that their combined impedance is (3.8 + j6.4) Ω. Calculate the resistance and capacitance of the imperfect capacitor.

Solution

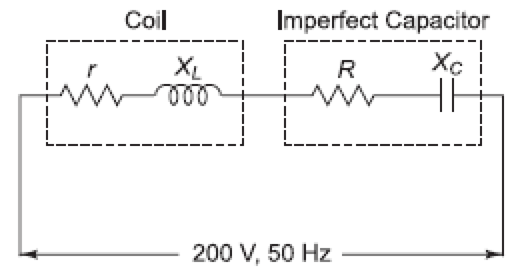
$r = 3\ \Omega$

$L = 0.22\ \text{H}$

$V = 200\ \text{V}$

$f = 50\ \text{Hz}$

$Z = 3.8 + j\,6.4\ \Omega$



(i) Resistance of the imperfect capacitor

$$\bar{Z} = 3.8 + j6.4\ \Omega$$

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.22 = 69.12\ \Omega$$

$$3 + R = 3.8$$

$$R = 0.8\ \Omega$$

(ii) Capacitance of the imperfect capacitor

$$69.12 - X_C = 6.4$$

$$X_C = 62.72\ \Omega$$

$$X_C = \frac{1}{2\pi f C}$$

$$C = 50.75\ \mu\text{F}$$