K J Somaiya College of Engineering, Vidyavihar, Mumbai

(A Constituent College of SVU)

Engineering Mechanics Notes

Module 5 – Kinetics of Particle

Module Section 5.1 – Kinetics – Newton's Second Law

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References: Engineering Mechanics, by M. D. Dayal & Engineering Mechanics – Statics and Dynamics, by N. H. Dubey.

Newton's Second Law of Motion: It states that "*the rate of change of momentum of a body is directly proportional to the resultant force and takes place in the direction of the force*".

Momentum is the quantity of motion possessed by a body, calculated by the product of its mass and velocity.

$$\frac{d}{dt}(m\bar{v}) \propto \sum\bar{F}$$

$$m\frac{d\bar{v}}{dt} = k\sum\bar{F} \text{ (for constant mass)}$$

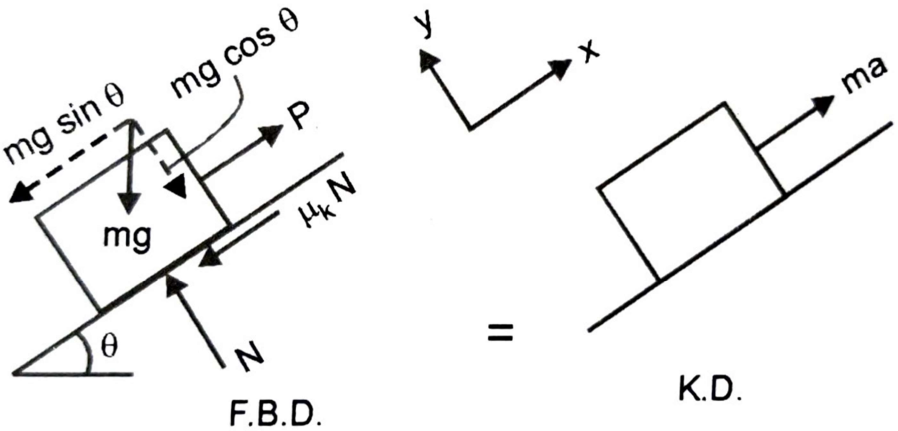
$$\Rightarrow m\bar{a} = k\sum\bar{F}$$

$$\therefore \sum\bar{F} = m\bar{a} \; (\because k = 1)$$

For rectilinear motion, or quantities in non-vector forms,

$$\sum F_x = ma_x; \sum F_y = ma_y; \sum F_z = ma_z$$

This results in another statement for the law, "*if the resultant force acting on a particle is not zero, the particle will have an acceleration proportional to the magnitude of the resultant and in the direction of the resultant force*".



For a block of mass m being pulled on an incline by a force P on a surface with kinetic coefficient of friction as μ_k, a free body diagram (FBD) is shown on the left and the corresponding kinetic diagram (KD) is shown on the right. This can be considered a visual representation of the Newton's second law.

Applying the NSL, in x and y directions as shown in the diagram, we get,

$$\sum F_x = ma_x$$

$$\Rightarrow P - mg\sin\theta - \mu_k N = ma$$

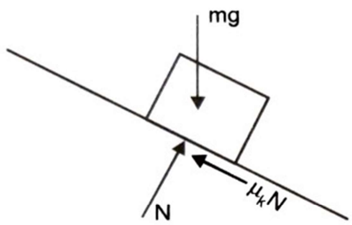
$$\sum F_y = ma_y$$

$$\Rightarrow N - mg\cos\theta = 0 \ (\because \text{no acceleration in y direction})$$

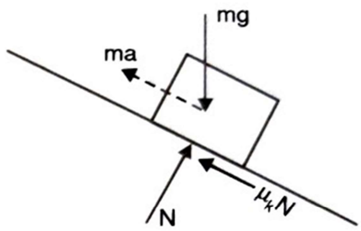
Solving the above two equations, we can find any unknowns required by the analysis, such as the acceleration.

D'Alembert's Principle: If the equation $\sum F = ma$ is rearranged as $\sum F - ma = 0$, treating the "– ma" as an inertia force, the system can be considered to be in dynamic equilibrium.

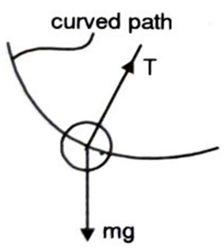
This is useful only in that COE can be used just like in static equilibrium situations, but it is not a realistic analysis.



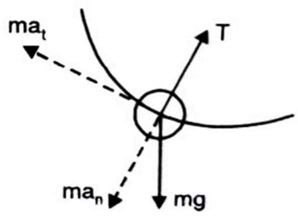
Actual forces acting on the block



Actual forces + Inertia force creates a state of dynamic equilibrium.



Actual forces acting on the pendulum



Actual forces + Inertia forces create a state of dynamic equilibrium

Applying the D'Alembert's Principle, we get a very similar equation as in NSL,

$$\sum F_x - ma_x = 0$$

$$\Rightarrow P - mg\sin\theta - \mu_k N - ma = 0$$

Ex. 10.2 A 500 N crate kept on the top of a 15° sloping surface is pushed down the plane with an initial velocity of 20 m/s. If $\mu_s = 0.5$ and $\mu_k = 0.4$, determine the distance traveled by the block and the time it will take as it comes to rest. ***(MU Dec 17)***

Solution: Applying equation of Newton's Second Law

$$\sum F_y = ma_y$$

$$N - 500\cos 15 = 0$$

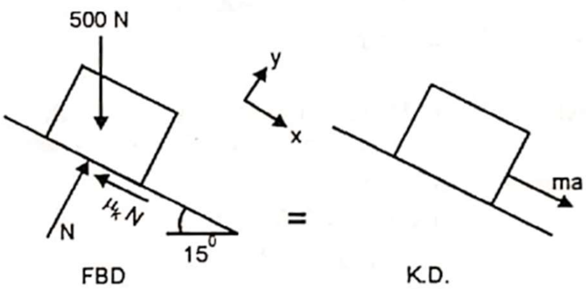
$$\therefore \quad N = 482.96 \text{ Newton}$$

$$\sum F_x = ma_x$$

$$500\sin 15 - \mu_k N = ma$$

$$500\sin 15 - 0.4\,(482.96) = \left(\frac{500}{9.81}\right)a$$

$$\therefore \quad a = -1.25 \text{ m/s}^2$$



The block travels down the slope with a – ve acceleration i.e deceleration of 1.25 m/s².

Kinematics

This is a case of Rectilinear motion – Uniform acceleration

$u = 20$ m/s, $v = 0$, $s = ?$, $a = -1.25$ m/s², $t = ?$

Using $\quad v^2 = u^2 + 2as$

$$0 = (20)^2 + 2 \times (-1.25) \times s$$

$$\therefore \quad s = 160 \text{ m} \quad \text{......... **Ans.**}$$

using $\quad v = u + at$

$$0 = 20 - 1.25 \times t$$

$$t = 16 \text{ sec} \quad \text{........ **Ans.**}$$

Ex. 10.3 An elevator with a person inside, of total mass 500 kg starts moving upwards at a constant acceleration and attains a velocity of 3 m/s after traveling a distance of 3m. i) Determine the tension in the cable ii) If after attaining the velocity of 3 m/s the elevator stops in 2.5 seconds. Find the pressure exerted by the elevator to the person weighing 600N in the elevator. ***(VJTI Nov 09)***

Solution:

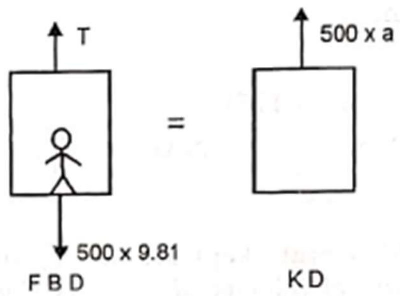
Kinematics: Motion of elevator stage (1) is rectilinear motion with uniform acceleration.

u = 0, v = 3 m/s, s = 3 m, a = ?, t = -

Using $v^2 = u^2 + 2as$

$3^2 = 0 + 2 \times a \times s$

$\therefore \quad a = 1.5\ m/s^2$



Kinetics of elevator

Let T be the tension entire cable

Applying NSL

$\Sigma F_y = m a_y \quad \uparrow +ve$

$T - 500 \times 9.81 = 500 \times a$

$T - 4905 = 500 \times 1.5$

or $\quad T = 5655\,N$ .... **Ans.**

Kinematics: Motion of elevator stage (2)

u = 3 m/s, v = 0, s = –, a = ?, t = 2.5 sec.

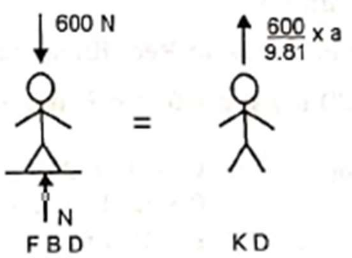
Using $v = u + at$

$0 = 3 + a \times 2.5$

or $\quad a = -1.2\ m/s^2$ .... **Ans.**

Kinetics of person inside the elevator

Let us isolate the person. Let N be the normal reaction the person receives from the floor of the elevator.



Applying N S L

$\Sigma F_y = m a_y \quad \uparrow +ve$

$N - 600 = \frac{600}{9.81} \times a$

$N - 600 = 61.62 \times (-1.2)$

or $\quad N = 526.6\ N$ .... **Ans.**

Problem 4

Two blocks A (10 kg mass), B (28 kg mass) are separated by 12 m as shown in Fig. 13.4(a). If the blocks start moving, find the time 't' when the blocks collide. Assume $\mu = 0.25$ for block A and plane and $\mu = 0.10$ for block B and plane.

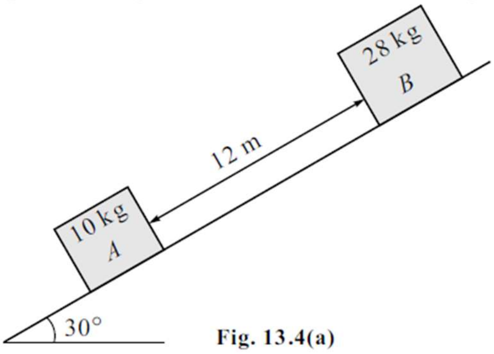


Fig. 13.4(a)

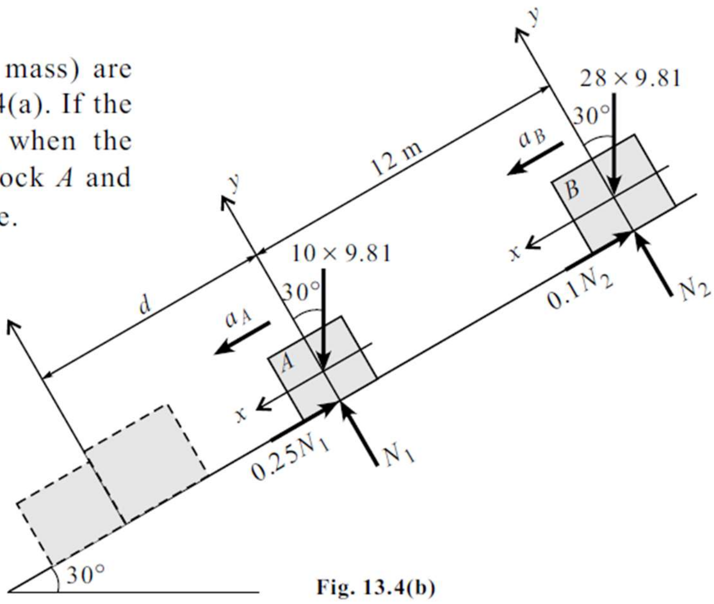


Fig. 13.4(b)

Solution

Refer to Fig. 13.4(b).

(i) Consider the F.B.D. of block *A*

By Newton's second law, we have

$$\sum F_x = ma_x$$

$$10 \times 9.81 \sin 30^\circ - 0.25 \times 10 \times 9.81 \cos 30^\circ = 10a_A$$

$$a_A = 2.781 \text{ m/s}^2 \ (30^\circ \swarrow)$$

(ii) Consider the F.B.D. of block *B*

By Newton's second law, we have

$$\sum F_x = ma_x$$

$$29 \times 9.81 \sin 30^\circ - 0.1 \times 28 \times 9.81 \cos 30^\circ = 28a_B$$

$$a_B = 4.055 \text{ m/s}^2 \ (30^\circ \swarrow)$$

(iii) Motion of block *A*

$$d = 0 + \frac{1}{2}a_A t^2 \qquad \text{... (I)}$$

(iv) Motion of block *B*

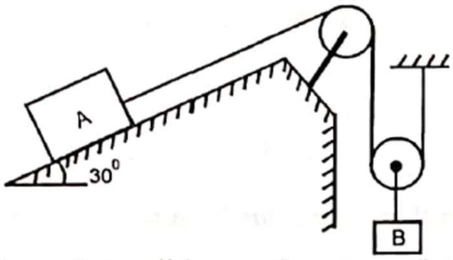
$$d + 12 = 0 + \frac{1}{2}a_B t^2 \qquad \text{... (II)}$$

(v) From Eqs. (I) and (II), we get

$$\frac{1}{2} \times 2.781 \times t^2 + 12 = \frac{1}{2} \times 4.055 \times t^2$$

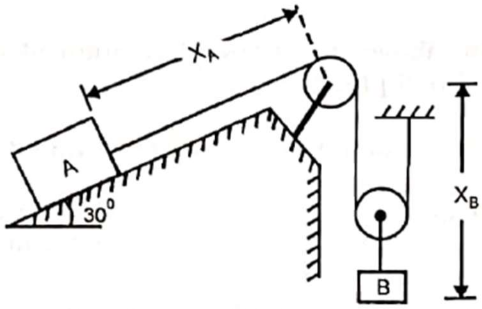
$\therefore \ t = 4.34$ s (Time when the blocks collide)

Ex. 10.5 A package A of mass 25 kg is being pulled up the incline by a load B of mass 60 kg connected to it by an inextensible rope passing over frictionless pulleys. Determine the accelerations of the two blocks and the tension in the connecting rope. Take $\mu_s = 0.4$ and $\mu_k = 0.3$ between the incline and A.



Solution: Downward movement of load B causes package A to slide up the plane. Let us develop the relation between the accelerations of A and B using Constant String Length Method (CSLM).

Let variables x_A and x_B define the positions of A and B. As x_B increases, x_A would decrease. If L is the length of string, then the length L is the sum of string portions in terms of x_A and x_B and plus/minus constants (constants are the string portions which don't change during motion like the length of cord wrapped over the pulleys).



$\therefore$ $L = (2\,x_B) + (-\,x_A) \pm$ constants (x_A is – ve because it reduces with increase in x_B)

Differentiating w.r.t time

$$0 = 2\,v_B - v_A$$

Differentiating again w.r.t time

$$0 = 2\,a_B - a_A \quad \text{or} \quad a_A = 2\,a_B \quad \text{.......... (1)}$$

Let us isolate A and B and perform kinetic analysis of each of them

Kinetics of package A

Applying equations of Newton's second law to A

$$\sum F_y = ma_y$$

$$N - 245.25\cos 30 = 0$$

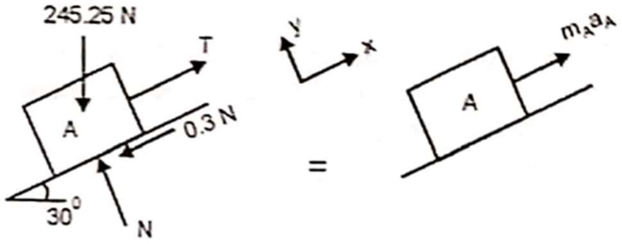
$$N = 212.39 \text{ Newton}$$

$$\sum F_x = ma_x$$

$$T - 245.25\sin 30 - 0.3\,N = m_A a_A$$

$$T - 245.25\sin 30 - 0.3(212.39) = 25\,a_A$$

$$T - 186.34 = 25\,a_A \quad \text{.......(2)}$$



F.B.D. K.D.

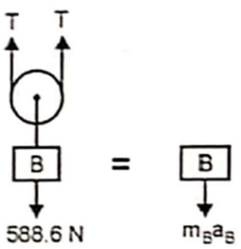
Kinetics of load B

Applying equations of Newton's Second Law to B

$$\sum F_y = ma_y \uparrow \quad +\text{ve}$$

$$2T - 588.6 = -\,m_B a_B$$

$$2T - 588.6 = -\,60\,a_B \quad \text{.........(3)}$$



FBD KD

Solving equations (1), (2) and (3), we get

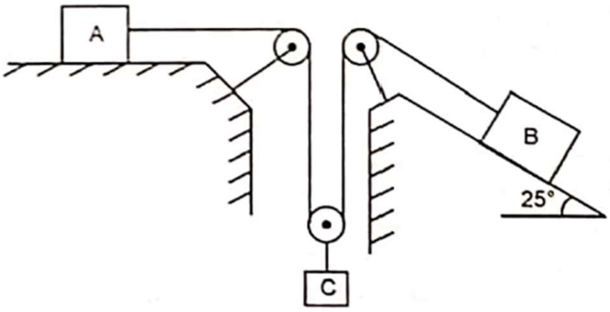
$a_A = 2.7\ m/s^2$ .... **Ans.**

$a_B = 1.35\ m/s^2$ .... **Ans.**

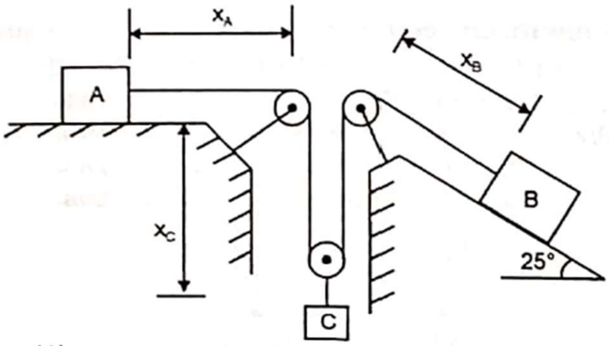
$T = 253.84$ N .... **Ans.**

Ex. 10.6 Find acceleration of block A, B and C shown in figure when the system is released from rest. Mass of blocks A, B and C is 5 kg, 10 kg and 50 kg respectively. Coefficient of friction for blocks A and B is 0.3. Neglect weight of pulley and rope friction.

(MU Dec 07)



Solution: Blocks A, B and C are connected to each other by a string and perform dependent motion. We need to first find the relation between the acceleration of the three blocks using CSLM.



Let x_A, x_B and x_C be the variable positions of blocks A, B and C.
Applying CSLM
Total length of string

$L = x_A + x_B + 2x_C \quad \pm$ constant .... (1)

If C moves down then A moves to the left and B moves up the slope, causing variable x_C to increase with time and variables x_A and x_B to decrease with time.
Therefore correcting the above equation (1) we get

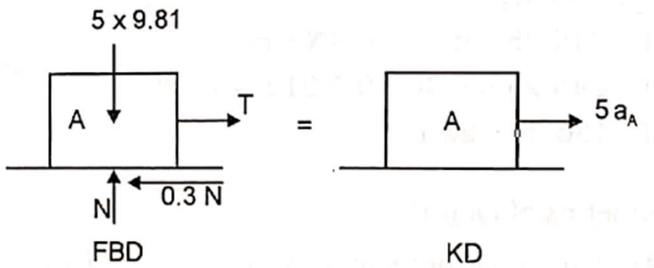
$L = -x_A - x_B + 2x_C \quad \pm$ constants .... (2)

Differentiating equation (2) twice w.r.t. time, we get the acceleration relation as

$0 = -a_A - a_B + 2a_C$ .... (3)

Let us now isolate the blocks A, B and C and perform kinetic analysis using NSL to each of them separately. Let T be the tension in the string.

<u>Kinetics of block A</u>



Applying NSL

$\Sigma F_y = m.a_y \uparrow +$ ve

$N - 5 \times 9.81 = 0$

$\therefore \quad N = 49.05$ N

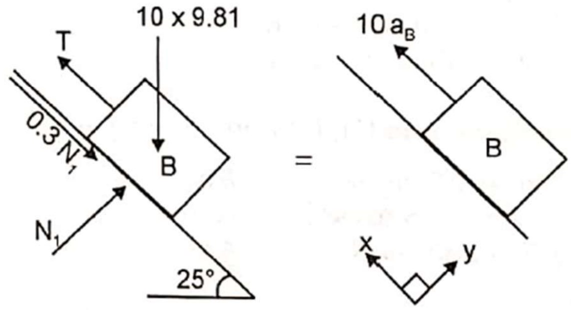
$\Sigma F_x = m.a_x \rightarrow +$ve

$T - 0.3\,N = 5\,a_A$

$T - 0.3\,(49.05) = 5\,a_A$

$\therefore \quad a_A = 0.2\,T - 2.943$ .... (4)

<u>Kinetics of block B</u>



Applying NSL

$\Sigma F_y = m.a_y$

$N_1 - 10 \times 9.81 \cos 25 = 0$

$\therefore \; N_1 = 88.91$ N

$\Sigma F_x = m.a_x$

$T - 0.3\,N_1 - 10 \times 9.81 \sin 25 = 10\,a_B$

$T - 0.3 \times 88.91 - 41.459 = 10\,a_B$

$\therefore \; a_B = 0.1\,T - 6.813$ .... (5)

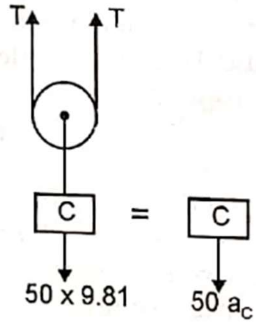
<u>Kinetics of block C</u>

Applying NSL

$\Sigma F_y = m.a_y \uparrow +ve$

$T + T - 50 \times 9.81 = -50\ a_C$

$\therefore \quad a_C = -0.04\ T + 9.81$ .... (6)



Substituting equations (4), (5) and (6) in equation (3)

$0 = -(0.2\ T - 2.943) - (0.1\ T - 6.813) + 2(-0.04\ T + 9.81)$

$\therefore \quad T = 77.3$ N .... **Ans.**

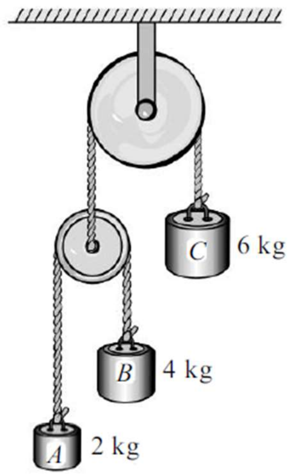
also $\quad a_A = 12.517\ m/s^2$ .... **Ans.**

$a_B = 0.917\ m/s^2$ .... **Ans.**

$a_C = 6.718\ m/s^2$ .... **Ans.**

Problem 12

In the system of pulleys, the pulleys are massless and the strings are inextensible. Mass of A = 2 kg, mass of B = 4 kg and mass = 6 kg as shown in Fig. 13.12(a). If the system is released from rest, find **(i)** tension in each of the three string, and **(ii)** acceleration of each of the three masses.



Solution

Assume the direction of motion of all block as above.

(i) Kinematic relation [Fig. 13.12(b)]

$$Tx_A + Tx_B + 2Tx_C = 0$$

$$x_A + x_B + 2x_C = 0$$

Differentiating w.r.t. t

$$v_A + v_B + 2v_C = 0$$

Differentiating w.r.t. t again,

$$a_A + a_B + 2a_C = 0 \quad ..... \text{(I)}$$

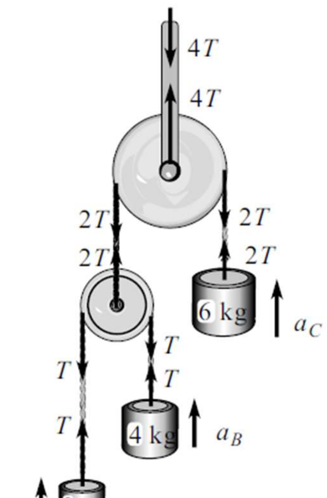


Fig. 13.12(b)

(ii) Consider the F.B.D. of block A [Fig. 13.12(c)]

$$\Sigma F_y = ma_y$$

$$T = 2 \times 9.81 = 2a_A$$

$$a_A = 0.5T - 9.81 \quad ..... \text{(II)}$$

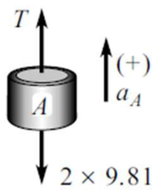


Fig. 13.12(c) : F.B.D. of Block A

(iii) Consider the F.B.D. of block B [Fig. 13.12(d)]

$$\Sigma F_y = ma_y$$

$$T - 4 \times 9.81 = 4a_B$$

$$a_B = 0.25T - 9.81 \quad ..... \text{(III)}$$

(iv) Consider the F.B.D. of block C [Fig. 13.12(e)]

$$\sum F_y = ma_y$$

$$2T - 6 \times 9.81 = 6a_C$$

$$a_C = 0.33T - 9.81 \quad \text{..... (IV)}$$

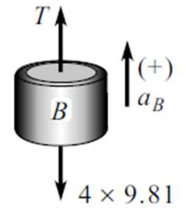


Fig. 13.12(d) : F.B.D. of Block *B*

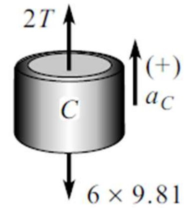


Fig. 13.12(e) : F.B.D. of Block *C*

(v) Putting Eqs. (II), (III) and (IV) in Eq. (I),

$$a_A + a_B + a_C = 0$$

$$(0.5T - 9.81) + (0.25T - 9.81) + (0.33T - 9.81) = 0$$

$$0.5T + 0.25T + 0.33T - 9.81 - 9.81 - 9.81 = 0$$

$$1.08T - 29.43 = 0$$

$$T = 27.25 \text{ N}$$

(vi) From Eq. (I),

$$a_A = 0.5 \times 27.25 - 9.81$$

$$a_A = 3.82 \text{ m/s}^2 \ (\uparrow)$$

(vii) From Eq. (II),

$$a_B = 0.25 \times 27.25 - 9.81$$

$$a_B = -3 \text{ m/s}^2 \text{ (Wrong assumed direction)}$$

$$a_B = 3 \text{ m/s}^2 \ (\downarrow)$$

(viii) From Eq. (III),

$$a_C = 0.33 \times 27.25 - 9.81$$

$$a_C = -0.82 \text{ m/s}^2 \text{ (Wrong assumed direction)}$$

$$a_C = 0.82 \text{ m/s}^2 \ (\downarrow)$$