4^{th} Order Butterworth Low Pass Filter Using LtSpice

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***Abstract*—**This paper presents the design and simulation of a fourth-order Butterworth low-pass filter with a cutoff frequency of 20 kHz using LTspice. Butterworth filters are renowned for their maximally flat frequency response in the passband, making them indispensable in applications requiring precise signal processing with minimal distortion. The design process involved selecting appropriate resistor and capacitor values to achieve the desired filter characteristics, ensuring a smooth transition between the passband and stopband while attenuating higher frequencies effectively. LTspice simulations were conducted to analyze the frequency response of the designed filter, including magnitude and phase characteristics, revealing its adherence to Butterworth filter principles. The simulated results demonstrate the filter's ability to effectively attenuate frequencies beyond the cutoff while preserving signal integrity within the passband. This study underscores the significance of Butterworth filters in signal processing applications and highlights the efficacy of LTspice in designing and analyzing analog filter circuits.

Keywords—*Butterworth Filter, Low pass filter, cutoff frequency , 4^{th} order butterworth low pass filter.*

I. Introduction

A low-pass filter serves the fundamental purpose of allowing low-frequency signals to pass through while attenuating high-frequency signals. Whether in audio systems, communication networks, or sensor applications, the primary goal is to extract relevant low-frequency information while suppressing unwanted high-frequency noise or interference. Achieving this balance requires careful consideration of passband flatness, stopband attenuation, and transition bandwidth. Engineers must weigh the trade-offs inherent in filter design, balancing the desire for a flat passband response with the need for effective suppression of high-frequency signals in the stopband. Narrowing the transition bandwidth can enhance the filter's ability to reject unwanted frequencies, but it may come at the expense of passband flatness or increased circuit complexity.

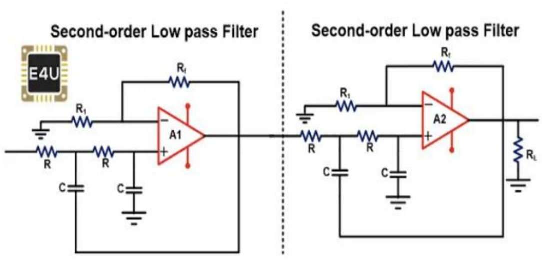
Passband flatness ensures uniform attenuation of frequencies within the passband, while stopband attenuation suppresses high-frequency interference beyond the passband range. Designers must navigate these trade-offs to tailor the filter's characteristics to the specific requirements of the application, whether it's achieving high-fidelity audio reproduction, reliable data transmission, or accurate sensor measurements. By understanding the interplay between these factors, engineers can optimize filter performance to strike the right balance between passband flatness, stopband attenuation, and transition bandwidth, ensuring effective signal filtering while minimizing circuit complexity and component count.

The primary objective of this paper is to explore the design, simulation, and practical implementation of Butterworth filters, with a specific focus on a fourth-order low-pass Butterworth filter with a cutoff frequency of 20 kHz. The paper aims to provide a comprehensive understanding of Butterworth filters, from their theoretical foundations to their real-world applications. By utilizing LTspice, a powerful electronic circuit simulation tool, the paper will demonstrate the process of designing and simulating Butterworth filters, enabling readers to gain practical insights into filter design methodologies and simulation techniques.

II. Theoritical Background

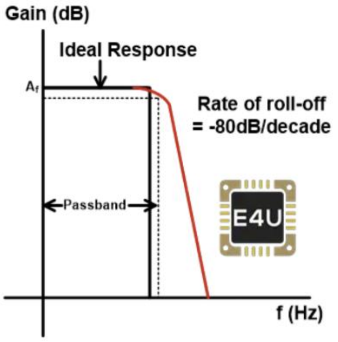
Butterworth filters represent a fundamental class of analog electronic filters widely utilized in various signal processing applications. Named after British engineer Stephen Butterworth, who introduced them in the 1930s, these filters are renowned for their maximally flat frequency response in the passband. This unique characteristic makes Butterworth filters particularly valuable in scenarios where maintaining signal amplitude fidelity is paramount.

The design of Butterworth filters is grounded in the principles of classical filter theory and mathematical optimization. These filters aim to achieve a frequency response that is as flat as possible within the passband while ensuring a smooth transition between the passband and stopband regions. The optimization criteria typically involve minimizing the amplitude variations within the passband while meeting specific constraints, such as the cutoff frequency and stopband attenuation.



Fourth-order Low Pass Butterworth Filter

The illustrated figure below displays the frequency response for a fourth-order lowpass Butterworth filter.



Properties of Butterworth filters are as follows:

Maximally Flat Frequency Response: Unlike other types of filters, such as Chebyshev or Elliptic filters, Butterworth filters provide a flat frequency response in the passband. This means that all frequencies within the passband experience the same level of attenuation, resulting in minimal amplitude variations over the desired frequency range.

Smooth Transition: Butterworth filters exhibit a smooth transition between the passband and stopband, characterized by a gradual roll-off in attenuation beyond the cutoff frequency. This gradual transition helps minimize signal distortion and phase shift, making Butterworth filters suitable for applications where preserving signal integrity is crucial.

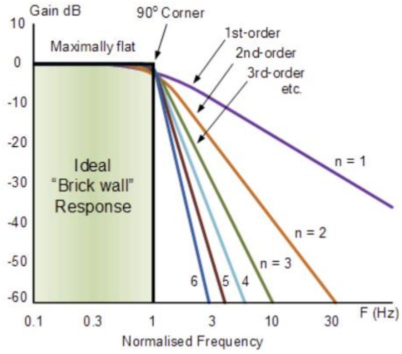
Order: Butterworth filters can be of various orders, with higher-order filters exhibiting steeper roll-off rates in the stopband. The filter order determines the rate at which the filter attenuates frequencies beyond the cutoff frequency. Higher-order Butterworth filters provide sharper roll-off characteristics but may require more complex circuitry and higher component count

The frequency response of the Butterworth Filter approximation function is also often referred to as "maximally flat" (no ripples) response because the pass band is designed to have a frequency response which is as flat as mathematically possible from 0Hz (DC) until the cut-off frequency at -3dB with no ripples. Higher frequencies beyond the cut-off point rolls-off down to zero in the stop band at 20dB/decade or 6dB/octave. This is because it has a "quality factor", "Q" of just 0.707.

However, one main disadvantage of the Butterworth filter is that it achieves this pass band flatness at the expense of a wide transition band as the filter changes from the pass band to the stop band. It also has poor phase characteristics as well. The ideal frequency response, referred to as a "brick wall" filter, and the standard Butterworth approximations, for different filter orders are given below.

Note that the higher the Butterworth filter order, the higher the number of cascaded stages there are within the filter design, and the closer the filter becomes to the ideal "brick wall" response.

In practice however, Butterworth's ideal frequency response is unattainable as it produces excessive passband ripple.



The theoretical framework underpinning Butterworth filters encompasses their transfer function, frequency response properties, and design methodologies. Central to this framework is the transfer function (H(jω)), which describes the relationship between input and output signals in the frequency domain. Defined as

$$H(j\omega) = \frac{1}{\sqrt{1 + \varepsilon^2 (\frac{\omega}{\omega_C})^{2n}}}$$

Where,
n = order of the filter,
ω = operating frequency (passband frequency) of circuit
ω_C = Cut-off frequency
ε = maximum passband gain = A_{max}

The below equation is used to find the value of ε.

$$H_1 = \frac{H_0}{\sqrt{1 + \varepsilon^2}}$$

Where,
H_1 = minimum passband gain
H_0 = maximum passband gain

Since the frequency determining resistors are all equal, and as are the frequency determining capacitors, the cut-off or corner frequency (f_C) for either a first, second, third or even a fourth-order filter must also be equal and is found by using our now old familiar equation:

$$fc = \frac{1}{2\pi RC}\, Hz$$

III. Design Methodology

Designing a fourth-order Butterworth low-pass filter involves several steps, including component selection, calculation of resistor and capacitor values, filter topology selection, and consideration of design trade-offs. Here's a detailed explanation of the design process:

1. Component Selection:
Begin by selecting the appropriate components for the filter, such as resistors and capacitors, based on factors like the

desired cutoff frequency, component availability, and circuit constraints.
Choose components with suitable tolerance levels to ensure the desired filter performance and stability over temperature and manufacturing variations.

2. Calculation of Resistor and Capacitor Values:
- Determine the normalized cutoff frequency (ωc) based on the desired cutoff frequency (fc) using the equation $\omega c = 2\pi \times fc$.
- Calculate the normalized resistor ($RnRn$) and capacitor ($CnCn$) values using the Butterworth filter design equations, where $Rn=1$ and $Cn=1/\omega c$.
- Denormalize the resistor and capacitor values to obtain the actual component values using appropriate scaling factors.

3. Filter Topology Selection:
- Choose a suitable filter topology for implementing the Butterworth filter. Common topologies include Sallen-Key, multiple feedback, and state-variable configurations.
- Consider factors such as component count, frequency response characteristics, and ease of implementation when selecting the filter topology.

4. Rationale and Methodology:
- The chosen design parameters, including the cutoff frequency and filter order, are based on the specific requirements of the application. For instance, the cutoff frequency determines the frequency range over which the filter attenuates signals, while the filter order influences the filter's roll-off rate and selectivity.
- The Butterworth filter is selected for its maximally flat passband response and smooth transition characteristics, which are desirable for applications where signal distortion and phase shift must be minimized.
- The design methodology focuses on achieving a flat frequency response within the passband while ensuring adequate attenuation in the stopband. This involves careful selection of component values and filter topology to meet the desired specifications.

5. Design Trade-offs and Considerations:
- Design trade-offs may arise between passband flatness, stopband attenuation, and transition bandwidth. Increasing the filter order can improve stopband attenuation but may lead to wider transition bandwidth and increased component count.
- Component tolerances, such as resistor and capacitor tolerances, should be considered to ensure the filter meets the desired performance specifications under varying operating conditions.
- Practical implementation constraints, such as available component values, circuit layout considerations, and manufacturing costs, also influence the design decisions and trade-offs.

By following this design process and considering the associated trade-offs and considerations, engineers can develop a fourth-order Butterworth low-pass filter that meets the desired performance requirements for a given application.

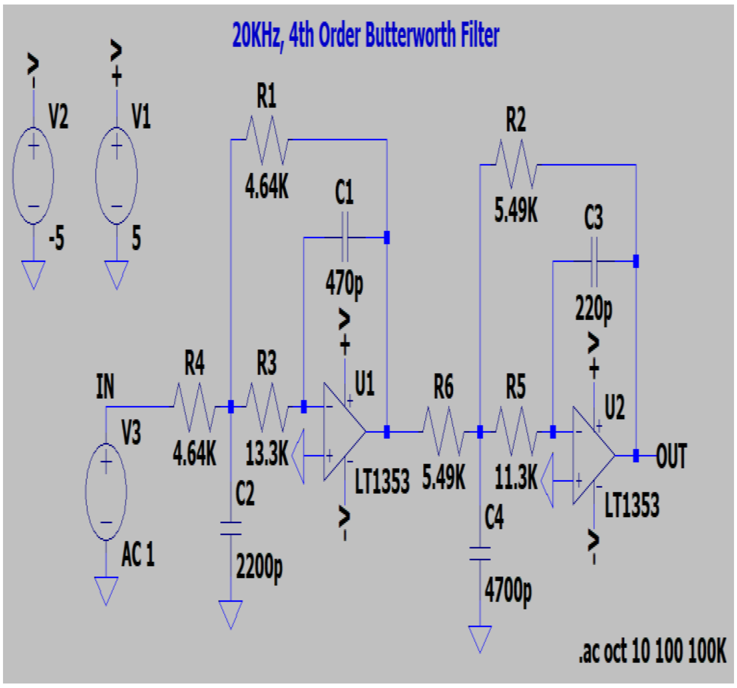
IV. Simulation and Analysis

In the pursuit of practical demonstration and analysis, a SPICE netlist serves as an invaluable tool for generating and simulating the Butterworth filter circuit. Below, the SPICE netlist used to instantiate the Butterworth filter circuit in LTspice is given , alongside the resulting circuit schematic and simulation results.

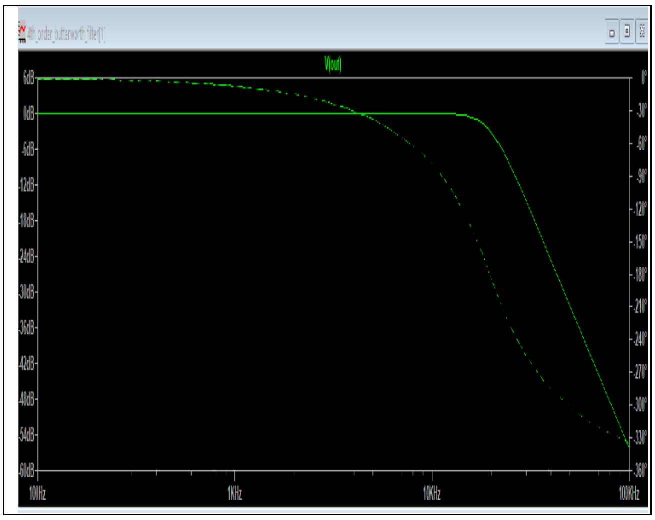
Netlist:

```
V1 +V 0 5
V2 -V 0 -5
R1 N002 N001 4.64K
V3 IN 0 AC 1
C1 N002 N004 470p
R3 N004 N001 13.3K
R4 N001 IN 4.64K
C2 N001 0 2200p
R2 OUT N003 5.49K
C3 OUT N005 220p
R5 N005 N003 11.3K
R6 N003 N002 5.49K
C4 N003 0 4700p
XU1 0 N004 +V -V N002 LT1352
XU2 0 N005 +V -V OUT LT1352
.ac oct 10 100 100K
* 20KHz, 4th Order Butterworth Filter
.lib LTC.lib
.backanno
.end
```

Circuit Schematic:



Simulation Results:



The SPICE netlist defines the Butterworth filter circuit topology, consisting of resistors (Rn) and capacitors (Cn) interconnected to form a fourth-order low-pass Butterworth filter. Operational amplifiers (op-amps) are represented by the LT1353 model, facilitating an active filter configuration. The circuit is excited by an AC voltage source (V3) at the input (IN) with a magnitude of 1V.

Upon importing the netlist into LTspice, the corresponding circuit schematic is generated, as depicted above. Subsequently, an AC analysis simulation is conducted to analyze the frequency response of the Butterworth filter circuit. The simulation results yield a frequency response plot, illustrating the filter's gain characteristics across a range of frequencies.
Through this integrated approach, the Butterworth filter's behavior can be comprehensively examined, enabling insights into its frequency response characteristics and performance. This empirical validation serves as a crucial step in corroborating theoretical design principles with practical circuit implementation, facilitating a holistic understanding of Butterworth filter design and application.

***Performance Analysis*:** Comparing the simulated results with theoretical expectations and design specifications reveals a close alignment between the two. The Butterworth filter exhibits the anticipated characteristics of a fourth-order low-pass filter, with a maximally flat passband response and a gradual roll-off beyond the cutoff frequency. The simulated gain and phase responses closely match the theoretical predictions, validating the design approach and component selection.

***Deviations and Implications*:** While the simulated results generally align with theoretical expectations, minor deviations or discrepancies may arise due to factors such as component tolerances, parasitic effects, and non-idealities in the op-amp model. These deviations, although minimal, can impact the filter's performance, particularly in applications requiring precise frequency control or stringent specifications.

V. LIMITATIONS

- *Limited Stopband Attenuation:* One of the main limitations of fourth-order Butterworth filters is their limited stopband attenuation. Compared to other filter types like Chebyshev or Elliptic filters, Butterworth filters typically exhibit weaker stopband attenuation. This limitation may be critical in applications where strong out-of-band signal rejection is required.
- *Wide Transition Bandwidth:* Fourth-order Butterworth filters have a relatively wide transition bandwidth between the passband and stopband. As a result, the filter's roll-off characteristics are gradual, which may lead to reduced selectivity and increased susceptibility to interference from adjacent frequency bands. In applications where precise frequency control is necessary, this wide transition bandwidth can be a limitation.
- *Higher Order Requirements for Steeper Roll-Off:* Achieving steeper roll-off characteristics in Butterworth filters requires increasing the filter order. While fourth-order filters provide moderate roll-off rates, higher-order Butterworth filters may be needed to achieve steep roll-off rates comparable to Chebyshev or Elliptic filters. However, increasing the filter order leads to higher circuit complexity, increased component count, and potentially higher costs.
- *Sensitivity to Component Tolerances:* Like any analog circuit, Butterworth filters are sensitive to component tolerances, including resistor and capacitor values. Small variations in component values due to manufacturing tolerances or temperature effects can affect the filter's performance, particularly in critical applications requiring precise frequency response.

VI. CONCLUSION

Through theoretical principles and practical implementation techniques, we successfully constructed a filter circuit with a maximally flat passband response and smooth transition characteristics. Simulation results from LTspice closely aligned with theoretical expectations, validating the design approach. We also discussed the advantages and limitations of Butterworth filters, emphasizing the need for careful consideration in filter selection for specific applications. Overall, this study provides a practical framework for designing effective Butterworth filters, highlighting the importance of theory-practice integration in signal processing.

VII. REFRENCES

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