

Q

Given an array & Q queries.

Amazon

In every query  $\rightarrow$  start & end indices of a Subarray.

Print the sum of the subarrays.

$A : -3, 6, 2, 4, 5, 2, 8, -9, 3, 1$

\*Q : 6

s  
e

1, 3  $\rightarrow$  12

2, 7  $\rightarrow$  12

4, 8  $\rightarrow$  9

Every Query

```

    sum = 0
    for ( i=s ; i<=e ; i++ ) {
        sum = sum + a[i],
    }
    print( sum )

```

O(N)

T.C : O(Q \* N)

Given the scores of the last 10 overs of a cricket match  
 Scores at the end of (SA vs WI)  
 WC '15

$\stackrel{41^{\text{st}}}{288}, \stackrel{42^{\text{nd}}}{312}, \stackrel{43^{\text{rd}}}{330}, \stackrel{44^{\text{th}}}{349}, \stackrel{45^{\text{th}}}{360}, \stackrel{46^{\text{th}}}{383}, \stackrel{47^{\text{th}}}{394}, \stackrel{48^{\text{th}}}{406}, \stackrel{49^{\text{th}}}{436}, \stackrel{50^{\text{th}}}{439}$

found how many runs were scored in

$$\begin{aligned} \text{In last } \overset{[46, 50]}{5 \text{ overs}} &= 439 - 360 \rightarrow O(1) \\ \text{Score in } \overset{49^{\text{th}}}{\underline{49^{\text{th}}}} \text{ over} &= 436 - 406 \rightarrow O(1) \\ \text{Score in } \overset{[42^{\text{nd}}, 45^{\text{th}}]}{\underline{42^{\text{nd}}, 45^{\text{th}}}} \text{ over} &= 360 - 288 \rightarrow O(1) \\ &\quad 45^{\text{th}} - 41^{\text{st}} \end{aligned}$$

A :  $\overset{0}{-3}, \overset{1}{6}, \overset{2}{2}, \overset{3}{4}, \overset{4}{5}, \overset{5}{2}, \overset{6}{8}, \overset{7}{-9}, \overset{8}{3}, \overset{9}{1}$

PS :  $\overset{0}{-3}, \overset{1}{3}, \overset{2}{5}, \overset{3}{9}, \overset{4}{14}, \overset{5}{16}, \overset{6}{24}, \overset{7}{15}, \overset{8}{18}, \overset{9}{19}$

### Prefix Sum

PS[i]  $\rightarrow$  Sum of all elements

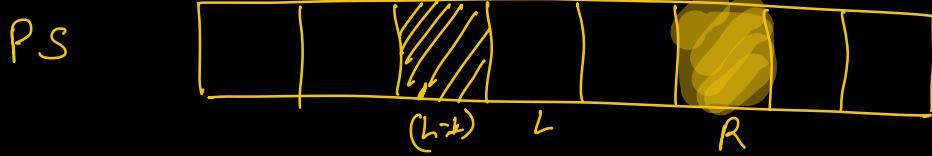
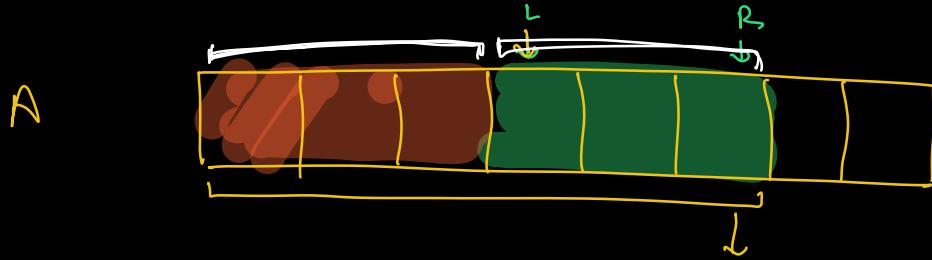
from index 0  
 to index i } inclusive

$$\frac{s}{1}, \frac{e}{3} \Rightarrow PS[3] - PS[0] = 12$$

$$\frac{2}{1}, \frac{7}{7} \Rightarrow PS[7] - PS[1] = 12$$

$$\frac{4}{1}, \frac{8}{8} \Rightarrow PS[8] - PS[3] = 9$$

$$\frac{0}{1}, \frac{5}{5} \Rightarrow PS[5] - PS[-1] \times \cancel{\text{Not required}}$$



$$\boxed{\text{Sum}(L \text{ to } R) = PS[R] - PS[L-1]} \Rightarrow O(1)$$

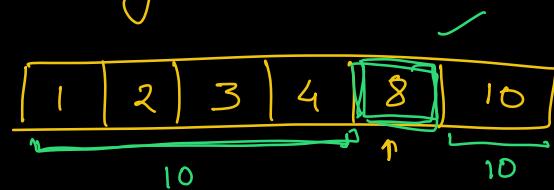
$$PS[R] = PS[L-1] + \text{Sum}(L \text{ to } R)$$

$$\Rightarrow \text{Sum}(L \text{ to } R) = PS[R] - PS[L-1]$$

$TC$	$: TC(\text{Create PS}) + TC(\text{Ans Q queries})$
	$\Downarrow$
	$O(N) + O(Q)$
$Sc$	$: O(N) \Rightarrow \text{PS Array}$

Do NOT Type WHILE I (Instruction) IS SPEAKING

Q Given an array. Find an equilibrium index in this array.



$$\begin{array}{cccccc} -7, & 1, & 5, & 2, & -4, & 3, & 0 \\ -7, & -6, & -1, & \frac{1}{2}, & -3, & 0, & 0 \end{array}$$

$PS[0] = A[0],$

$\left. \begin{array}{l} \text{for } (i=1; i < N; i++) \{ \\ \quad PS[i] = PS[i-1] + A[i], \\ \} \\ // \text{find equilibrium index.} \end{array} \right\} \Rightarrow O(N)$

$i < N-1$

$i <= N-2$

$\left. \begin{array}{l} \text{for } (i=1; i < (N-1); i++) \{ \\ \quad \text{sumLeft} = PS[i-1], \\ \quad \text{sumRight} = PS[N-1] - PS[i], \\ \quad \text{if } (\text{sumLeft} == \text{sumRight}) \{ \\ \quad \quad \text{return } i; \\ \} \\ \} \\ \text{return } -1; \end{array} \right\} O(N)$

$TC : O(N) \quad | \quad SC : O(N)$

HW Try to implement without using any extra space.

30 min

(at least)

25 mins  $\rightarrow$  Hint / clues

& without modifying the given array.

↳ Stack

Q

Given an array of size N with all elements equal to zero.

Given Q queries having an index & a value.

Add the given val to all the elements from i to  $N-1$ .

A : 0 0 0 0 0 0 0

Q 0 3 3 3 3 3 3 =

i val 0 3 3 3 5 5 5

1 3 0 3 4 4 6 6 6

4 2

2 1

Return the final array.

for every query {

for ( $i = \underline{\text{enden}}$ ;  $i < N$ ;  $i \rightarrow i$ ) {  
 $A[i] = A[i] + \text{val},$   
}

$\Rightarrow O(N)$

)

TC :  $O(q^*N)$   
SC :  $\underline{\underline{O(1)}}$   
(<sub>Extra</sub>  
<sub>Space</sub>)

0	1	2	3	4	5	6	7
0	3	0	7	0	1	0	0

$i$       val       $P_3$        $P_2$        $P_1$

3      2       $P_5$

5      1

1      3       $\Rightarrow$

3      5

0	3	3	10	10	11	11	11
---	---	---	----	----	----	----	----

OBS. Prefix sum propagates value at index  $i$  to all the indices  $> i$ ,

$O(Q)$  [for every query  $\in$

$$O(1) \Rightarrow A[i] = A[i] + \text{val}_i,$$
 ]

$O(N)$  // Take PS  $\Rightarrow$  ans.

TC:  $O(Q+N) \Rightarrow O(\max(Q, N))$

SC:  $O(1)$

i, v	0	0	2	3	4
2, 5	0	0	5	0	0
1, 6	0	6	5	0	0
2, 1	0	6	6	0	0
	0	6	12	12	12

Break till 10:54 pm

Q Given an array of size  $N$  with all elements equal to zero.

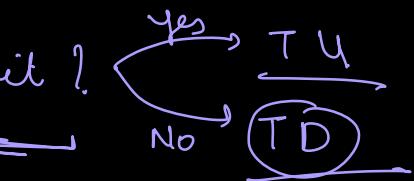
Q queries

In every query  $s, e & \text{val}$ .

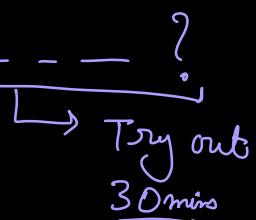
Add the val to the subarray from  $s$  to  $e$ .

$s$	$e$	$\text{val}$	0	0	2	3	4	5
2	4	2	0	0	2	2	2	0
1	3	1	0	1	3	3	2	0
0	2	3	3	4	6	3	2	0

<u>s</u>	<u>e</u>	<u>val</u>	0	0	2	0	0	0	0	0								
2	4	2	0	0	2	0	0	-2										
1	3	1	0	1	2	0	-1	-2										
0	2	3	3	1	2	-3	-1	-2										
3	5	4	3	1	2	1	-1	-2										
<u>T<sub>(n-1)</sub></u>																		
<hr/>																		
3      4      6      7      6      4																		

Did you get it? 

<u>s.</u>	<u>e</u> ,	<u>val</u>	0	4	3	2	0	1	-4	-2	0									
2	4	2																		
1	3	4																		
2	6	1																		
<u>T<sub>(n-1)</sub></u>			0, 4, 7, 7, 3, 1, 0																	
<hr/>																				
↓ PS																				

What if ... - - - ? 

Q

Given an array of size N.

{Return two arrays leftMan[] & rightMan  
where

Every index  $i$  {  
 $\underline{\text{leftMan}[i]} \rightarrow \text{Man of all elements from } 0 \text{ to } i$   
 $\underline{\text{rightMan}[i]} \rightarrow \text{Man of all the elements from } i \text{ to } (N-1)$

LeftMan :      -3, 6, 2, 4, 5, 2, 8, -9, 3, 1  
                   -3, 6, 6, 6, 6, 6, 8, 8, 8, 8

RightMan :      8, 8, 8, 8, 8, 8, 8, 3, 3, 1

$\underline{\text{LM}[0]} \rightarrow \text{Man}(0 \rightarrow 0) \Rightarrow A[0]$

$\underline{\text{LM}[1]} \rightarrow \text{Man}(0 \rightarrow 1) \Rightarrow \text{Man}(\underline{\text{LM}[0]}, A[1])$

$\underline{\text{LM}[2]} \rightarrow \text{Man}(0 \rightarrow 2) \Rightarrow \text{Man}(\underline{\text{LM}[1]}, A[2])$

$\underline{\text{LM}[3]} \rightarrow \text{Man}(0 \rightarrow 3) \Rightarrow \text{Man}(\underline{\text{LM}[2]}, A[3])$

⋮

$\boxed{\underline{\text{LM}[i]} \rightarrow \text{Man}(\underline{\text{LM}[i-1]}, A[i])}$

{  
 for ( $i=1$ ;  $i < N$ ;  $i++$ ) {  
 $\underline{\text{LM}[i]} = \text{Man}(\underline{\text{LM}[i-1]}, A[i]);$   
 }

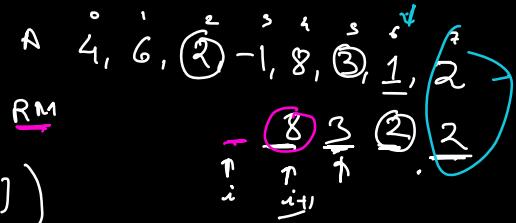
Man till now

$$RM[N-1] = A[N-1]$$

$$RM[N-2] = \text{Max}(RM[N-1], A[N-2])$$

⋮

$$RM[i] = \text{Max}(RM[i+1], A[i])$$



```

for (i=N-2; i>=0; i--) {
    RM[i] = Max(RM[i+1], A[i]),
}

```

$TC : O(N)$
$SC : O(N)$

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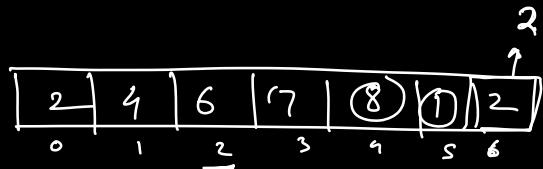
### Take Aways

① What is prefix sum & how it could be useful to get sum of any subarray in  $O(1)$  TC.

② How to use carry-forward tech. to construct LeftMax & RightMax arrays. (Will be used in future classes).

Doubts

RM



$$\begin{aligned} \underline{\text{RM}}[i] &\Rightarrow \underline{\text{Max}}(\underline{i} \rightarrow \underline{N-1}) \\ &= \text{Max}(N-1 \rightarrow i) \end{aligned}$$

$$PS[i_0] = A[i_0]$$

$$\overbrace{i=1 \longrightarrow N}$$

Coding Test

V<sub>s</sub>

Online Judge

↳ Constraints  
 ↳ O(N)  
 ↳ O(log N)  
 ↳ O(n<sup>r</sup>)

Interviews

↳ BF  
 ↳ Try to optimize

- Understand [Try out ex]
- BF => (2, 3, 4, 5, 6 loops)

$$\begin{array}{ccccccccc}
 -3, & 6, & 2, & 4, & 5, & 2, & 8, & -9, & 3, & 1 \\
 \text{RM} & & & & 8 & 8 & 8 & 3 & 3 & 1 \\
 & & & & \swarrow & \searrow & & & & \swarrow \searrow
 \end{array}$$

$\underline{\text{RM}} \quad \underline{\text{A(N-1)}}$

$\boxed{Q} \Rightarrow \textcircled{1s} \rightarrow \boxed{\square} \text{ Scals. Senn}$

