

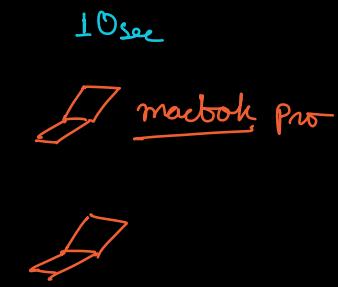
\underline{Q} for ($i=0$; $i \leq N$; $i++$) {
 | for ($j=0$; $j \leq N$; $j++$) {
 | | for ($k=0$; $k \leq j$; $k++$) {
 | | | ...
 | }
 }
 }

- How to count no. of iteration?
- How to get Big-O from no. of iteration?

- \Rightarrow Why we need TC? $\xrightarrow{\text{✓}}$
- \Rightarrow What is Big-O? \checkmark
- \Rightarrow How Big-O is related to no. of iteration? \checkmark
- \Rightarrow {What are the advantages of constraints?
What is SC?

$arr : [3, 8, 9, -1, 10, 4]$
 $\rightarrow [-1, 3, 4, 8, 9, 10]$

Ammol
"Sudha Sort"



10 sec

Python



Denyanshu
"Bestone Sort"



7 sec

C++

Language



Iterations \Rightarrow Only dependent on algo.

Aghashish

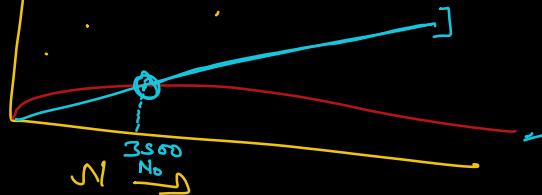
"Start Sort"

$$\frac{100 \log_{10} N}{f(N)} \in O(g(N))$$

Mukul

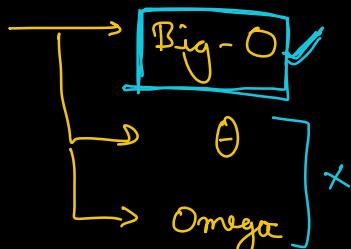
"Strong Sort"

$$\frac{N}{10} \in g(N)$$



Asymptotic Notations

$N \rightarrow \infty$



Big-O : Given a function $f(N)$, if $N \geq 0$
it is $O(g(N))$, If there exists
two constants $c & N_0$ such that

$$f(N) \leq c \cdot g(N) \quad \forall N \geq N_0$$

1st year
S2

$$f(N) = N^2 + N \quad [N >= 0] \quad \text{No of iteration}$$

$$\underline{N^2 + N} \leq \underline{N^2 + N^2} ?$$

$$\boxed{N^2 + N} \leq \boxed{2N^2} \quad N >= 0 \quad ?$$

\downarrow

$f(N)$ $g(N)$ N_0

$$\boxed{\underline{f(N)} \leq \underline{c g(N)} \quad N \geq N_0} ?$$

$\mathcal{O}(N^2)$

Q

$$f(N) = \underline{N^2} + \underline{5}$$

$$N^2 + 5 \leq \underline{N^2 + 5N^2}$$

$$\boxed{N^2 + 5} \leq \boxed{6N^2} \quad N >= \underline{1}$$

\downarrow

$f(N)$ $g(N)$ N_0

$$\boxed{\underline{f(N)} \leq \underline{c g(N)} \quad N \geq N_0}$$

$\mathcal{O}(g(N))$
 $\mathcal{O}(N^2)$

$$\underline{\mathbb{Q}} \quad f(N) = N^3 + 8\underline{N^2} + \underline{N \log N}$$

$$N^3 + 8N^2 + N \log N \leq N^3 + 8N^3 + N^3$$

$$\underbrace{N^3 + 8N^2 + N \log N}_{\begin{array}{c} f(N) \\ \downarrow \\ O(N^3) \end{array}} \leq 10N^3 \quad || \quad f(N) \leq 10N^3$$

$$f(N) \leq 10N^3$$

$$f(N) \leq 10N^{10}$$

$$\underline{\mathbb{Q}} \quad f(N) = \underline{10N^2} + 10^4 \quad N > 0$$

$$10N^2 + 10^4 \leq 10N^2 + 10^4 N^2 \quad N > 0$$

$$\begin{array}{c} \text{oval } 10N^2 + 10^4 \\ \downarrow \\ f(N) \end{array} \leq \begin{array}{c} \text{oval } 10010N^2 \\ \downarrow \\ c \end{array} \quad \begin{array}{c} \text{oval } N > 0 \\ \downarrow \\ n_0 \end{array}$$

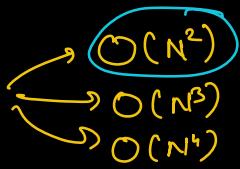
$$\begin{array}{c} \downarrow \\ g(N) \end{array}$$

$$\begin{array}{c} \text{oval } O(N^2) \\ \downarrow \end{array}$$

- ① Pick only the largest power
- ② Remove constants

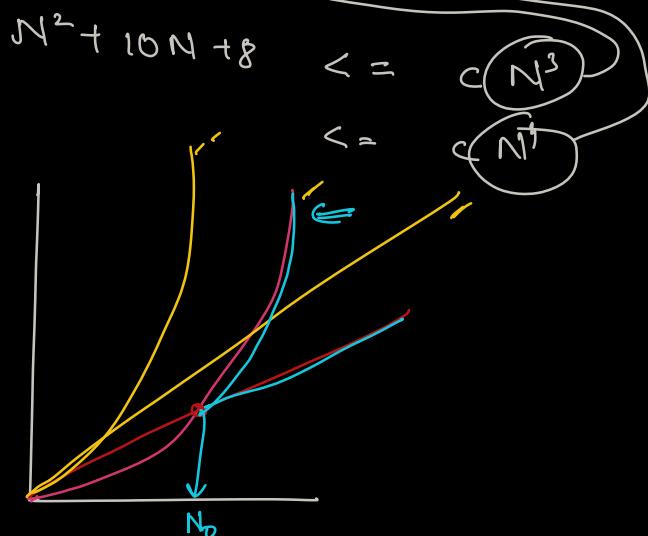
Ques

$$f(N) = N^2 + 10N + 8$$



$$f(N) \leq c \cdot g(N) \quad \forall N > N_0$$

$\hookrightarrow O(g(N))$



$$\begin{aligned} & \frac{25K}{26K} \\ & \frac{25K}{25.5K} \\ & \frac{25K}{25K} \end{aligned} \left. \begin{aligned} & \leq 50K \\ & \leq 70K \\ & \leq 100K \\ & \leq 120K \\ & \leq 180K \end{aligned} \right\} Q$$

Distances from your house

Hospital : 2 Km \Rightarrow Walk / Bycycle

Cinema : 4 Km

Airport : 25 Km \Rightarrow cab

Kashmir : 1000 Km \Rightarrow Train / flight

USA : 13000 Km \Rightarrow flight

[The Moon : $3,84,400 \text{ Km}$ \Rightarrow Rocket

Earth \rightarrow Moon : $\text{Home} \rightarrow \text{Airport} \rightarrow \text{USA Airport} \rightarrow \text{NASA} \rightarrow \text{Moon}$

$$\begin{aligned}
 & \left. \begin{aligned}
 & \text{if } N^2 \\
 & 10N^2 \quad N \rightarrow \infty \\
 & - \underline{8N^2} + \underline{6N} + 25 \\
 & \cdot 16N^2 + 49N
 \end{aligned} \right\} O(N^2) \\
 & \left. \begin{aligned}
 & N^3 + N^2 + N + 1 \\
 & N^3 \\
 & 6N^3 + 25N^2 \\
 & 25N^3 + 100
 \end{aligned} \right\} O(N^3)
 \end{aligned}$$

Break till 10:34 pm

$$\begin{aligned}
 & \text{hörd} \rightarrow \text{feind(Man)} \\
 & \boxed{\leq - | - | - | - | - | -} \Rightarrow \overbrace{O(n)}^{\uparrow} \quad \checkmark \\
 & \underbrace{\text{SDB-2}}_{\text{Sorted}}, \quad \underbrace{\text{rt A[0]}}_{\text{O}(n \log n)} \quad \leftarrow
 \end{aligned}$$

Q

for ($i = 1$; $i \leq N$; $i + 1$) {

for ($j = 1$; $j \leq N$; $j++$) { $\Rightarrow O(N^3)$

$\left\{ f_k \mid k = 1, 2, \dots, N \right\}$

- 1 -

1

3

1

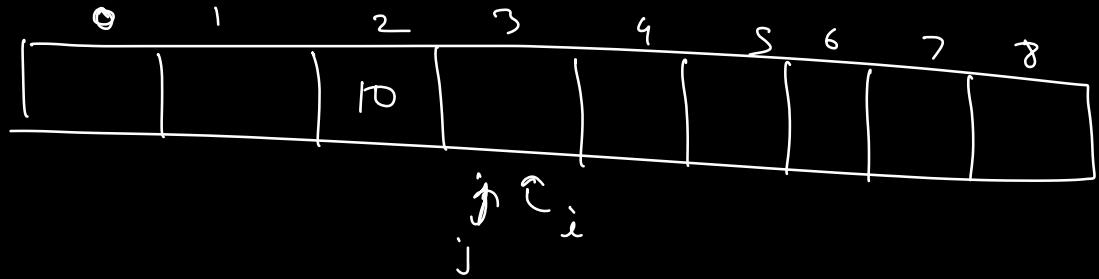
```
void fn( arr[], N) {  
    j = 0;
```

```

N
    {
        for ( i = 0; i < N; i++ ) {
            i = j
            while ( j < N && an[i] < an[j] ) {
                an[i] = an[j]
                j++
            }
        }
    }

```

a_0	a_1	\dots	a_{n-2}	a_{n-1}	i	j	No of iterations
0	1	\vdots	j	i	0	0	$0 \rightarrow 1$
(a_0)	(a_0)	\vdots	j	i	1	1	\vdots



$$i = 0$$

$$\rightarrow 0$$

$$a(i) = 10$$

$$i = 1$$

$$\rightarrow 1$$

$$a(j) = 10$$

$$i = 2$$

$$\rightarrow 1$$

$$a(i) < a(j)$$

$$i = 3$$

$$\rightarrow 1$$

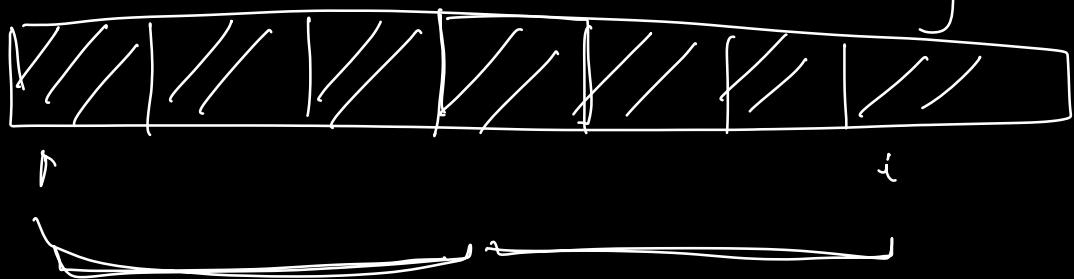
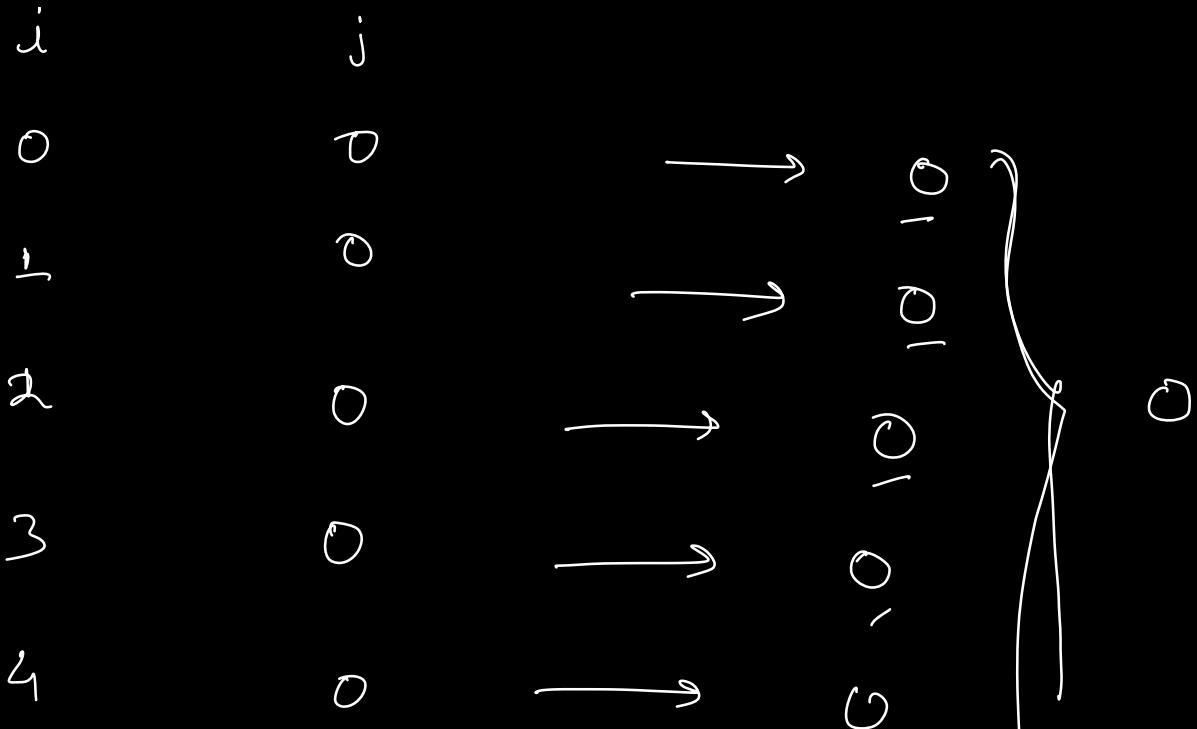
$$i = 4$$

$$\rightarrow 1$$

$$\vdots \quad \vdots \quad \vdots$$

$O(N)$

$\frac{4}{0}, \frac{20}{1}, \frac{21}{2}, \frac{6}{3}, \frac{10}{4}$ $a(i) < a(j)$
 $i < j$



$O(N)$

Space Complexity

1 int \Rightarrow 4 Byte
1 long \Rightarrow 8 Bytes

void fn (int N) {

[int $a = N \times N;$ // 4 Bytes]
[long $b = N \times N \times N;$ // 8 Bytes] 12 Bytes.

[int $c[N] = \text{new int}[N];$ // $4N$ Bytes] $O(1)$

[int $m[N][N],$

} \rightarrow
 $\text{Space} = 12B + 4NB$ $O(N)$ =

$$\text{Space} = 12 + 4N + N^2 \Rightarrow \underline{\underline{O(N^2)}}$$

fn (int N, M) }

int $c[N][M]$

}

$O(NM)$

~~CP~~
 Spoj
 Codechef
 Codeforces
 etc.

~~1s / 2s~~ ?
 Online Judges

~~TLE~~ \Rightarrow Time limit exceeded

1 Giga \longrightarrow 10^9 machine level instruction

1 iteration \longrightarrow 10 MLI
 (appr.) \rightarrow 100MLI
 (appr.)

1 GHz Processor can perform $\Rightarrow (10^7 - 10^8)$ iterations

~~for (i=0; i<N; i++) {~~
 } {
 ↓

~~in 1 sec~~

in $\boxed{1 \text{ sec}}$ \rightarrow $10^7 - 10^8$

$O(N)$ \Rightarrow

$$N \Rightarrow \underline{10^2}$$

Solve a Problem

- \rightarrow Read the problem statement
- \Rightarrow Analyse Constraints
- \rightarrow Logic
- \rightarrow Write Code
- \rightarrow Calculate TC

$O(N^3)$

Importance of

Constraints

$\mathcal{O}(N^3)$ algo

$$1 \leq N \leq \underline{10^9}$$

$$(10^4)^3 = \frac{10^{12}}{\cancel{10^8}}$$

$$1 \leq N \leq \underline{100}$$

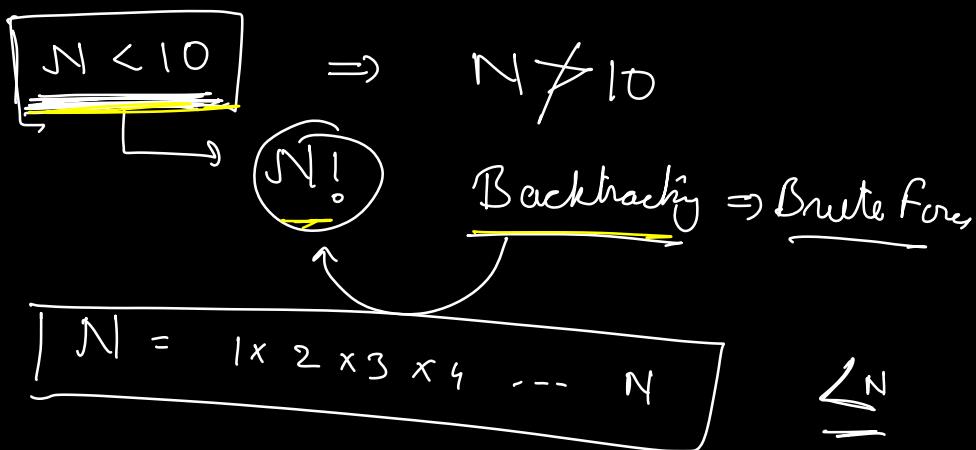
$$(100)^3 = \underline{10^6}$$

T_C	$\underline{\log N}$	\sqrt{N}	$\underline{N \log N}$	N^2	$\frac{2^N}{\cancel{N=20}}$	$\underline{N!}$
$N = \underline{10^6}$	$\log 10^6$ \downarrow 20	$\sqrt{10^6}$ 10 ³	$10^6 \log 10^6$ 2×10^7 2×10^2	10^{12} \times	2^{20} 10^6	$\boxed{N=10}$ 3×10^7

$$2^{10} = 1024 \approx 1000$$

$$2^{30} \approx 10^9$$

$$\underline{2^{20}} \approx 10^6$$



Q Given an array.

Find sum of all the elements?

```

int fn(an[], n) {
    int s = 0;
    for (i=0; i<n; i++) {
        s = s + an[i];
    }
    return s;
}
    
```

$$\begin{cases} 1 \leq N \leq 10^5 \\ 1 \leq an[i] \leq 10^6 \end{cases}$$

$$[10^6] [10^6] \cdots [10^6]$$

$$S = 10^6 \times 10^5$$

$$S = 10^{11} \Leftarrow \text{int?}$$

Overflow

$$TC: O(N) \Rightarrow 10^5$$

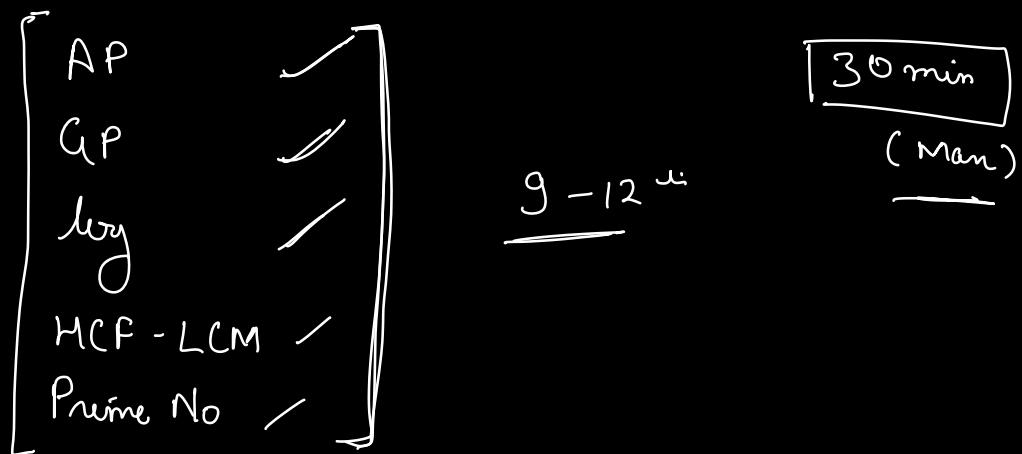
$$SC: O(N)$$

Extra/Space: $O(1)$ \rightarrow

$$\log_b N = \Theta \log_d N$$

$$\boxed{\log_2 N \approx \log_3 N \approx \log_{10} N}$$

$$+ \quad \quad \quad + + + \quad \quad \quad O \log_2 N \quad \quad \quad O \log_3 N$$



$$N \leq \underline{10^6}$$

$$TC : \underline{\underline{O(N!)}} \quad \quad \quad \underline{(10^6!) \rightarrow \text{iterati}}$$

$$N = 100 \times \xrightarrow{\quad \text{Optimize} \quad}$$

$$N \leq 10 \swarrow$$

CP \Rightarrow Maths \Rightarrow { Permutation
Probability
Geometry
Concen. Hull } X

