

DATA STRUCTURES (BASICS)

[HASHING]

PLACEMENT PREPARATION [EXCLUSIVE NOTES]

SAVE AND SHARE

Curated By- HIMANSHU KUMAR(LINKEDIN) <https://www.linkedin.com/in/himanshukumarmahuri>

TOPICS COVERED-

- Hashing Introduction
- Hash Function, Hash Table
- Collision Handling
- Linear probing
- Quadratic probing
- Double hashing
- Some assignments for you.

INTRODUCTION TO HASHING-

Hashing is a method of storing and retrieving data from a database efficiently.

Suppose that we want to design a system for storing employee records keyed using phone numbers. And we want the following queries to be performed efficiently:

1. Insert a phone number and the corresponding information.

2. Search a phone number and fetch the information.
3. Delete a phone number and the related information.

We can think of using the following data structures to maintain information about different phone numbers.

1. An array of phone numbers and records.
2. A linked list of phone numbers and records.
3. A balanced binary search tree with phone numbers as keys.
4. A direct access table.

For **arrays and linked lists**, we need to search in a linear fashion, which can be costly in practice. If we use arrays and keep the data sorted, then a phone number can be searched in $O(\log n)$ time using Binary Search, but insert and delete operations become costly as we have to maintain sorted order.

With a **balanced binary search tree**, we get a moderate search, insert and delete time. All of these operations can be guaranteed to be in $O(\log n)$ time.

Another solution that one can think of is to use a **direct access table** where we make a big array and use phone numbers as indexes in the array. An entry in the array is NIL if the phone number is not present, else the array entry stores pointer to records corresponding to the phone number. Time complexity wise this solution is the best of all, we can do all operations in $O(1)$ time. For example, to insert a phone number, we create a record with details of the given phone number, use the phone number as an index and store the pointer to the record created in the table.

This solution has many practical limitations. The first problem with this solution is that the extra space required is huge. For example, if the phone number is of n digits, we need $O(m * 10^n)$ space for the table where m is the size of a pointer to the record. Another problem is an integer in a programming language may not store n digits.

Due to the above limitations, the Direct Access Table cannot always be used. **Hashing** is the solution that can be used in almost all such situations and performs extremely well as compared to above data structures like Array, Linked List, Balanced BST in practice. With hashing, we get $O(1)$ search time on average

(under reasonable assumptions) and $O(n)$ in the worst case.

Hashing is an improvement over Direct Access Table. The idea is to use a hash function that converts a given phone number or any other key to a smaller number and uses the small number as an index in a table called a hash table.

Hash Function:

A function that converts a given big phone number to a small practical integer value. The mapped integer value is used as an index in the hash table. In simple terms, a hash function maps a big number or string to a small integer that can be used as an index in the hash table.

A good hash function should have following properties:

1. It should be efficiently computable.
2. It should uniformly distribute the keys (Each table position be equally likely for each key).

For example, for phone numbers, a bad hash function is to take the first three digits. A better function will consider the last three digits. Please note that this may not be the best hash function. There may be better ways.

Hash Table:

An array that stores pointers to records corresponding to a given phone number. An entry in hash table is NIL if no existing phone number has hash function value equal to the index for the entry.

Collision Handling:

Since a hash function gets us a small number for a big key, there is a possibility that two keys result in the same value. The situation where a newly inserted key maps to an already occupied slot in the hash table is called collision and must be handled using some collision handling technique. Following are the ways to handle collisions:

- **Chaining:**

The idea is to make each cell of the hash table point to a linked list of records that have the same hash function value. Chaining is simple, but it requires additional memory outside the table.

- **Open Addressing:**

In open addressing, all elements are stored in the hash table itself. Each table entry contains either a record or NIL. When searching for an element, we one by one examine the table slots until the desired element is found or it is clear that the element is not present in the table.

Open Addressing-

Like separate chaining, open addressing is a method for handling collisions. In Open Addressing, all elements are stored in the hash table itself. So at any point, the size of the table must be greater than or equal to the total number of keys (Note that we can increase table size by copying old data if needed).

Important Operations:

- Insert(k): Keep probing until an empty slot is found. Once an empty slot is found, insert k.
- Search(k): Keep probing until the slot's key doesn't become equal to k or an empty slot is reached.
- Delete(k): **Delete operation is interesting.** If we simply delete a key, then the search may fail. So slots of the deleted keys are marked specially as "deleted".

Insert can insert an item in a deleted slot, but the search doesn't stop at a deleted slot.

Open Addressing is done in the following ways:

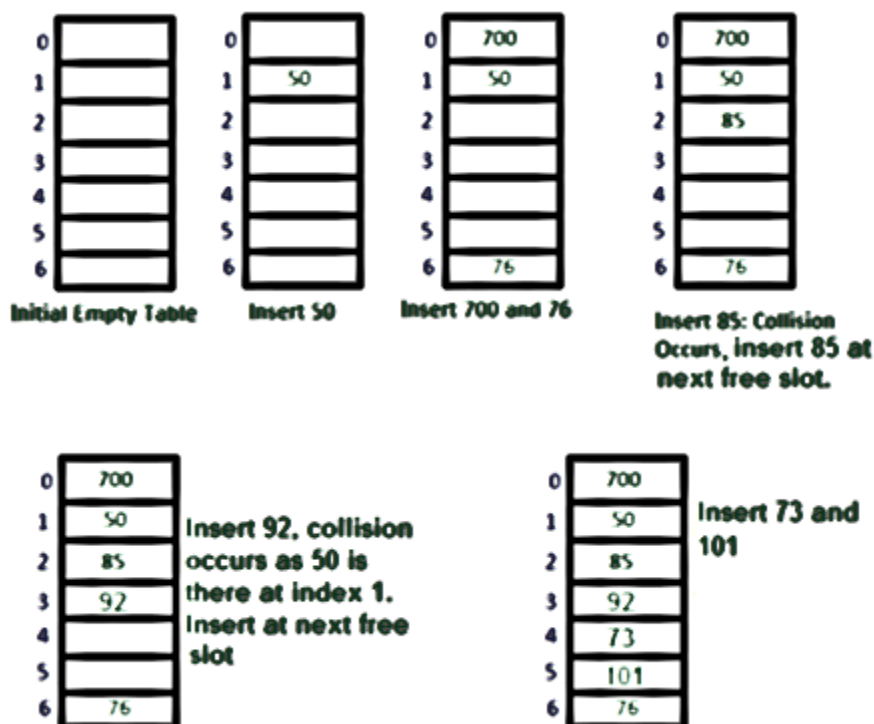
Linear Probing:

In linear probing, we linearly probe for the next slot. For example, the typical gap between the two probes is 1 as taken in the below example also.

let $\text{hash}(x)$ be the slot index computed using a hash function and S be the table size.

```
If slot  $\text{hash}(x) \% S$  is full, then we try  $(\text{hash}(x) + 1) \% S$ 
If  $(\text{hash}(x) + 1) \% S$  is also full, then we try  $(\text{hash}(x) + 2) \% S$ 
If  $(\text{hash}(x) + 2) \% S$  is also full, then we try  $(\text{hash}(x) + 3) \% S$ 
.....
.....
```

Let us consider a simple hash function as “key mod 7” and a sequence of keys as 50, 700, 76, 85, 92, 73, 101.



Clustering:

The main problem with linear probing is clustering, many consecutive elements form groups and it starts taking time to find a free slot or to search an element.

Quadratic Probing

We look for i^2 th slot in i th iteration.

let $\text{hash}(x)$ be the slot index computed using hash function.

If slot $\text{hash}(x) \% S$ is full, then we try $(\text{hash}(x) + 1*1) \% S$

If $(\text{hash}(x) + 1*1) \% S$ is also full, then we try $(\text{hash}(x) + 2*2) \% S$

If $(\text{hash}(x) + 2*2) \% S$ is also full, then we try $(\text{hash}(x) + 3*3) \% S$

Double Hashing

We use another hash function $\text{hash2}(x)$ and look for $i*\text{hash2}(x)$ slot in i th iteration.

let $\text{hash}(x)$ be the slot index computed using hash function.

If slot $\text{hash}(x) \% S$ is full, then we try $(\text{hash}(x) + 1*\text{hash2}(x)) \% S$

If $(\text{hash}(x) + 1*\text{hash2}(x)) \% S$ is also full, then we try $(\text{hash}(x) + 2*\text{hash2}(x)) \% S$

If $(\text{hash}(x) + 2*\text{hash2}(x)) \% S$ is also full, then we try $(\text{hash}(x) + 3*\text{hash2}(x)) \% S$

Comparison of above three:

- Linear probing has the best cache performance but it suffers from clustering. One more advantage of Linear probing that it is easy to compute.
- Quadratic probing lies between the two in terms of cache performance and clustering.
- Double hashing has poor cache performance but no clustering. Double hashing requires more computation time as two hash functions need to be computed.

S.No.	Seperate Chaining	Open Addressing
1.	Chaining is Simpler to implement.	Open Addressing requires more computation.
2.	In chaining, Hash table never fills up, we can always add more elements to chain.	In open addressing, table may become full.
3.	Chaining is Less sensitive to the hash function or load factors.	Open addressing requires extra care for to avoid clustering and load factor.
4.	Chaining is mostly used when it is unknown how many and how frequently keys may be inserted or deleted.	Open addressing is used when the frequency and number of keys is known.
5.	Cache performance of chaining is not good as keys are stored using linked list.	Open addressing provides better cache performance as everything is stored in the same table.
6.	Wastage of Space (Some Parts of hash table in chaining are never used).	In Open addressing, a slot can be used even if an input doesn't map to it.
7.	Chaining uses extra space for links.	No links in Open addressing

Performance of Open Addressing:

Like Chaining, the performance of hashing can be evaluated under the assumption that each key is equally likely to be hashed to any slot of the table (simple uniform hashing).

m = Number of slots in the hash table
 n = Number of keys to be inserted in the hash table

Load factor $\alpha = n/m$ (< 1)

Expected time to search/insert/delete $< 1/(1 - \alpha)$

So Search, Insert and Delete take $(1/(1 - \alpha))$ time

Assignments-

Must checkout Codes and implementation of following-

- **Direct Address Table.**
- **Implementation of Chaining.**
- **Implementation of Open Addressing.**
- **Subarray with zero sum.**
- **Longest subarray with given sum**
- **Longest Subarray with equal number of 0s and 1s**
- **Count Distinct Elements**
- **Frequencies of array elements**



HIMANSHU KUMAR(LINKEDIN)

<https://www.linkedin.com/in/himanshukumarmahuri>

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