→ Number Systems

→ Base Conversion

———— // ————

→ Intro to bit manipulation

———— // ————

$7\ 3\ 4 \Rightarrow 700 + 30 + 4$

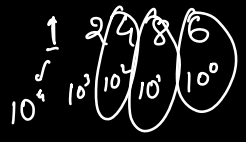
$700 + 30 + 4$

$7 \times 10^2 + 3 \times 10^1 + 4 \times 10^0$

$6\ 5\ 1\ 4 = 6 \times 10^3 + 5 \times 10^2 + 1 \times 10^1 + 4 \times 10^0$

1) Decimal No. [0 - 9]

↳ related to 'ten'



$n \longrightarrow 10^{n-1}$

2) Base 8

Octal No [0-7]

$0\ 1\ 3\ 2 \Rightarrow \underbrace{1\times8^2}_{64} + \underbrace{3\times8^1}_{24} + \underbrace{2\times8^0}_{2} = 90$

($8^3\ 8^2\ 8^1\ 8^0$)

Octal → Decimal

$(1\ 2\ 5)_8 = \underbrace{1\times8^2}_{64} + \underbrace{2\times8^1}_{16} + \underbrace{5\times8^0}_{5} = 85$

3) Base 3 [0, 1, 2] ⇒ [0, 2]

Ternary No. Syst.

$(1\ 1\ 2\ 0)_3 = \underbrace{1\times3^3}_{27} + \underbrace{1\times3^2}_{9} + \underbrace{2\times3^1}_{6} + \underbrace{0\times3^0}_{0} = 42$

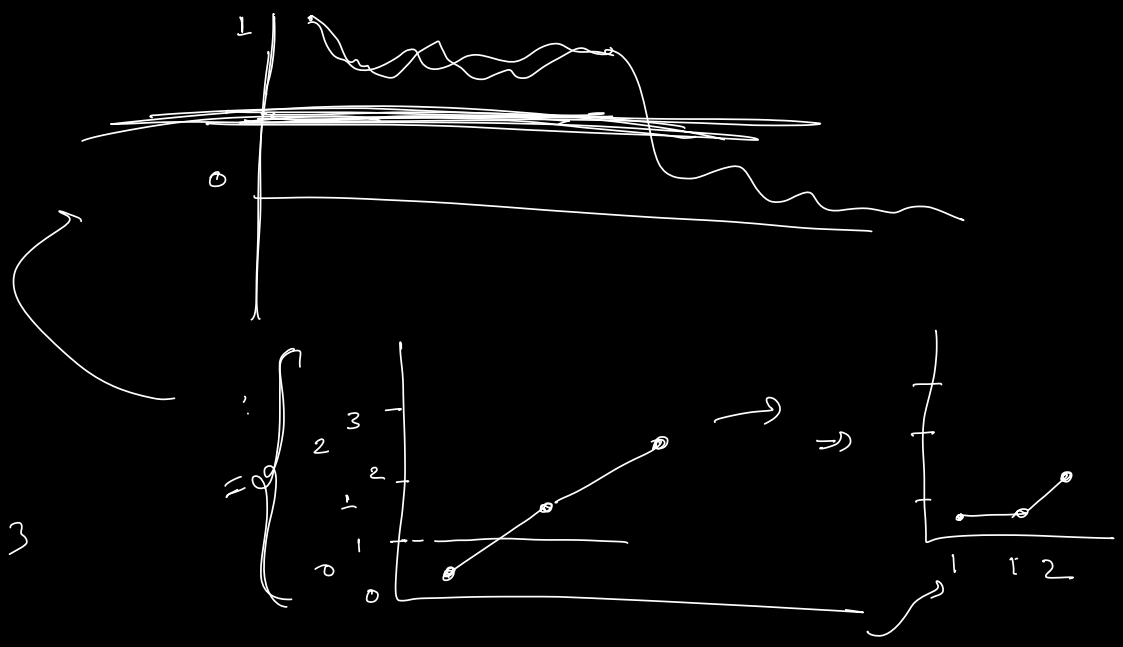
($3^3\ 3^2\ 3^1\ 3^0$)

$2\ 1\ 0\ 1 = \underbrace{2\times3^3}_{54} + \underbrace{1\times3^2}_{9} + \underbrace{0\times3^1}_{0} + \underbrace{1\times3^0}_{1} = 64$

4) Binary [0, 1]

$1\ 0\ 1\ 1\ 0 = 2^4 + 2^2 + 2^1 = 22$

($2^4\ 2^3\ 2^2\ 2^1\ 2^0$)



"Vivekanenda" ⟶ "Vivekanunnda"

Empathy ?

Decimal to Binary

Long division Method

2	28	0
2	14	0
2	7	1
2	3	1
2	1	1
	0	

1 1 1 0 0

2	35	1
2	17	1
2	8	0
2	4	0
2	2	0
2	1	1
	0	

1 0 0 0 1 1

2	19	1
2	9	1
2	4	0
2	2	0
2	1	1
	0	

10011

25 ⇒ ?
37 ⇒ ?
] * HW

Addition

3^rd / 4^th

i) Decimal No.

```
  1 1 1
  3 4 5 9
  2 8 4 7
-----------
0 6 3 0 6
```

```
6/10  11/10  14/10  12/10
0 6     2      3      7      8
        3      7      6      4
      ------------------------
        6      1      4      2
        ↑      ↑      ↑      ↑
     6%10   11%10  14%10  12%10
```

⇒ c = S / B

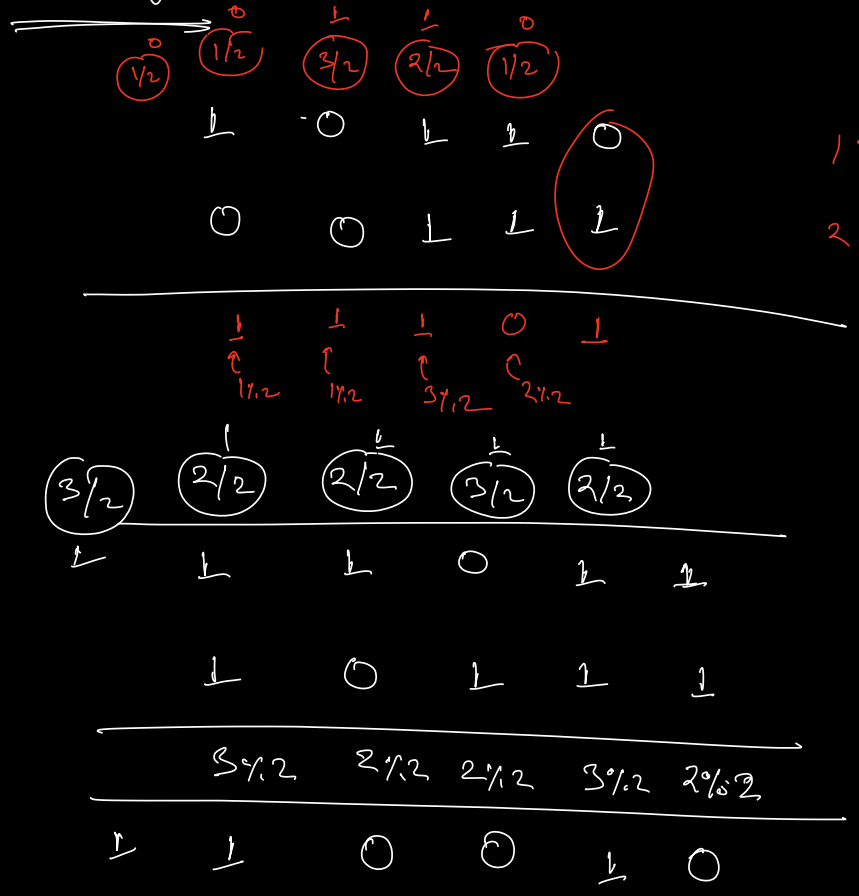
d = S % B

```
(18/10)  9
         9
        ---
        1 8
```

18 % 10

2) Binary No

Base = 2



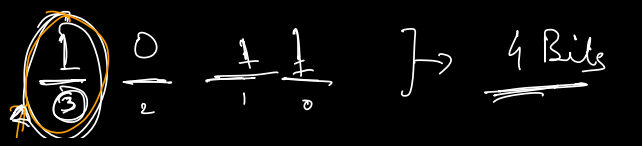
± / 0

Bit ⟹ ± / 0

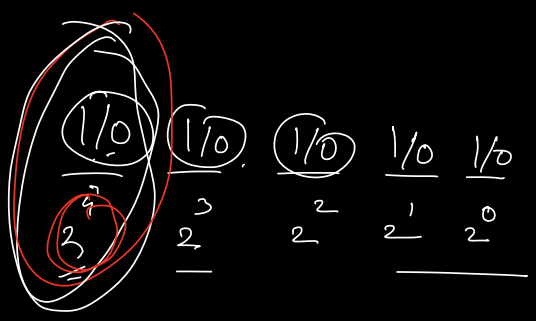
Bit → ± ⇒ Set Bit

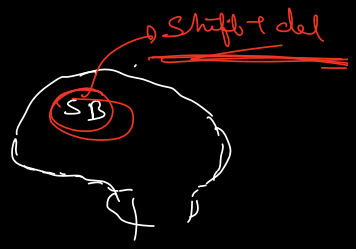
Bit → 0 ⇒ Unset Bit

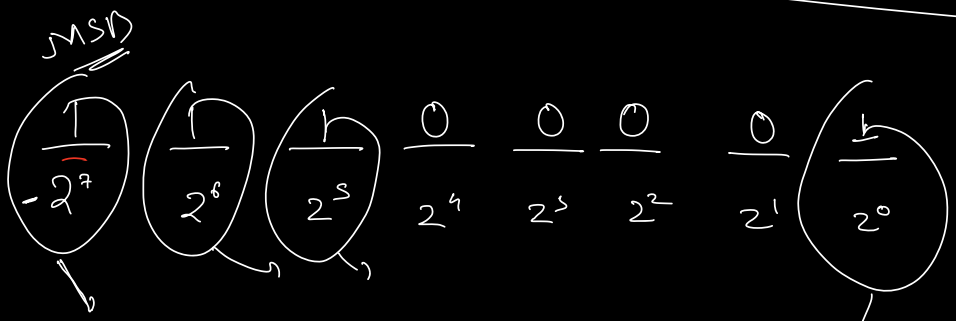
1101 → Unset → 1100 → Set → 1110



MSB => Most Significant Bit



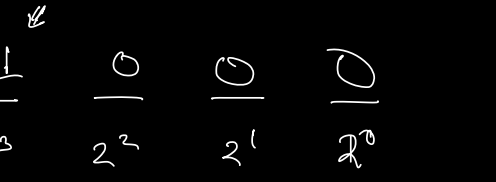




$1 \times (-2^7) + (1 \times 2^6) + (1 \times 2^5) + (1 \times 2^0)$

$\Rightarrow -128 + 64 + 32 + 1$

$\Rightarrow \underline{-31}$



$\Rightarrow -8$

$\underset{-2^3}{0}\quad \underset{2^2}{1}\quad \underset{2^1}{0}\quad \underset{2^0}{1} \Rightarrow 5$

$\underset{2^3}{1}\quad \underset{2^2}{1'}\quad \underset{2^1}{1'}\quad \underset{2^0}{1'} \Rightarrow -8+4+2+1$

$\Rightarrow -1$

↳ MSB

$\underset{-2^3}{1}\quad \underset{2^2}{0}\quad \underset{2^1}{1}\quad \underset{2^0}{0} \qquad -2^3+2^1$

$\Rightarrow -6$

↳ 10 X

↳ −10 X

Limit on (Max) & (Min)

# bits		Min	Max
2	$\underset{-2^1}{1}\ \underset{2^0}{0}$	−2	1
	0 1		

		Min	Max
3	$\underline{0}\ \ \underline{1}\ \ \underline{1}$ $-(2^2)\ \ 2^1\ \ 2^0$	-4	3
4	$\underline{0}\ \ \underline{1}\ \ \underline{1}\ \ \underline{1}$ $3\ \ 2\ \ 1\ \ 0$	-8 ,	7
n	$\underline{1}\ \ \underline{\quad}\ \ \underline{\quad}\ \ \underline{\quad}$ $(n-1)\ \ \ldots\ \ 2\ \ 1\ \ 0$	Min $-2^{(n-1)}$	Max $(2^{n-1}-1)$

$$\underbrace{\underline{0}\ \ \underline{1}\ \ \underline{1}\ \ \ldots\ \ \underline{1}\ \ \underline{1}}_{2^{(n-2)}+2^{(n-3)}+\ldots 2^0}$$
$(n-1)\quad (n-2)\qquad\qquad 0$

$0 \rightarrow (n-2)$

$0, 1, 2, 3, 4, 5$

$0, 1$

$1, 2, 4, 8.$

$\frac{2}{1} = \frac{4}{2} = \frac{8}{4} = 2$

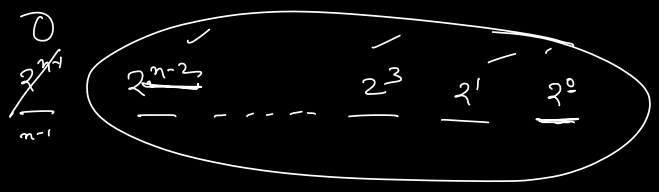
$2^0 + 2^1 + 2^3 + \ldots 2^{(n-3)} + 2^{(n-2)}$

GP

GP

$a = 1, \quad r = 2, \quad t = (n-1)$

$$\text{Sum} = \frac{a \times (r^t - 1)}{r-1} = \frac{1 \times 2^{n-1} - 1}{2-1} = (2^{n-1} - 1)$$



$0 \longrightarrow n-2$

$n = 4$ $\quad 0, 1, 2 \quad \Rightarrow (n-1) = 3$

$n = 5$ $\quad 0, 1, 2, 3 \quad \Rightarrow 4 \rightarrow (5-1)$

---

1 Byte = 8 Bit $\quad [-2^{(8-1)}, 2^{(8-1)}-1]$

$[-128, 127]$

2 Byte = 16 Bit = $[-2^{15}, 2^{15}-1]$

$-32768 \quad 32767$

4 Byte = 32 Bit = $[-2^{31}, 2^{31}-1]$

int, long, unsigned int

$-2,147,483,648$ — Integer - Min

$2,147,483,647$ — Integer - Max

$\Rightarrow 1\ 0\ 0\ 1\ 0\ 0\ 1\ 0$

$7\ 6\ 5\ 4\ 3\ 2\ 1\ 0$

$= -(2^7) + 2^4 + 2^1$

$-128 + 16 + 2$

Bit Manipulation

Bitwise operations

&	|	~	^
and	or	not	xor

[<< >>]

a	b	a & b	a | b	a ^ b	~a
0	0	0	0	0	1
0	1	0	1	1	1
1	0	0	1	1	0
1	1	1	1	0	0

4 & 3 = 0

100
011
000 => 0

4 | 3 = 7

100
011
111 => 7

4 ^ 3 = 7

100
011
111 => 7

Decimal to Binary for -ve no. ?

2's Complement

$$-a = \sim a + 1$$

$\sim a$ → 1's complement

Ex 8 bit representation of -10

$a = 10$: 0 0 0 0 1 0 1 0

$\sim a$ → : 1 1 1 1 0 1 0 1

$+1$: 0 0 0 0 0 0 0 1

(-10) : 1 1 1 1 0 1 1 0

(1 → bit 7; bits 6 5 4 3 2 1 0)

$$-(2^7) + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1$$

$$-128 + 64 + 32 + 16 + 4 + 2$$

$$-10$$

$$-(2^{n-1})$$

a = 13 : 1 1 0 1

b = 10 : 1 0 1 0

a & b 1 0 0 0 => 8

a | b 1 1 1 1 => 15

a ^ b 0 1 1 1 => 7

---

a | 1

Even → a = 10 → (a+1)

Odd → a = 11 → a

10 => 10

11 => 11

11 => 11

---

a ^ 1

Even → (a+1)

Odd → (a-1)

$a \& b = b \& a$

$a \mid b = b \mid a$

$a \wedge b = b \wedge a$

} Commutative

$a - b \neq b - a$

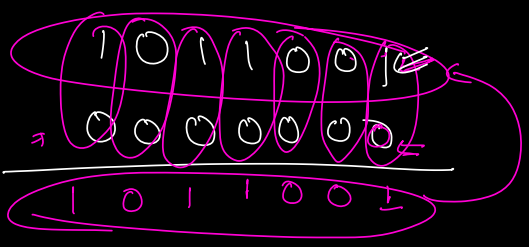
$a + b = b + a$

$(a \wedge b \wedge c) = a \wedge (b \wedge c)$

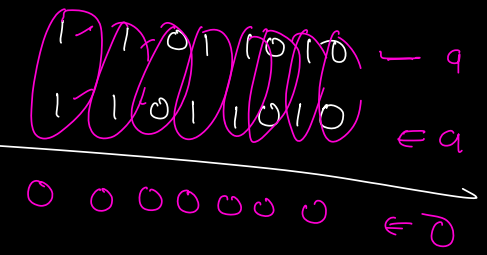
$= (a \wedge b) \wedge c$

$= (a \wedge c) \wedge b$

$\boxed{a \wedge 0 = a}$



$\boxed{a \wedge a = 0}$



$a \wedge b \wedge a = (a \wedge a) \wedge b$

$= 0 \wedge b$

$= b$

$a \wedge c \wedge a \wedge b \wedge c = b$

Amazon MS Adobe

Q Single No

Given N elements. All elements are appear twice except one element.

Find the single no.

$10^9, 2, 3, 1, 5, 2, 1, 3, 6, 6, 10^9$

$\Rightarrow 5$

Java: HashMap — Python: Dict — C++: unordered_map — Map — Extra Space

Key: a(i) — val: freq

Sorting

$1, 1, 2, 3, 3, 5, 6, 6, 10^9, 10^9$

$(i, (i+1))$

Library Sorting fn $\Rightarrow$ Extra time

$O(n \log n)$

$10^9 \wedge 2 \wedge 3 \wedge 1 \wedge 5 \wedge 2 \wedge 1 \wedge 3 \wedge 6 \wedge 6 \wedge 10^9$

```
int ans = 0;
for (i=0; i<n; i++){
    ans = a[i] ^ ans;
}
```

Std: 6

4 | 3

30 min → find ans

Octal (8) (10)

3
7

10 % 8 (12)

2 ⇒ 10

1 + 1 = 2 % 2 → 10

7 % 10

10 % 10

10 | 11

- 4 20
- 2 11
- 3 34
- 3 42

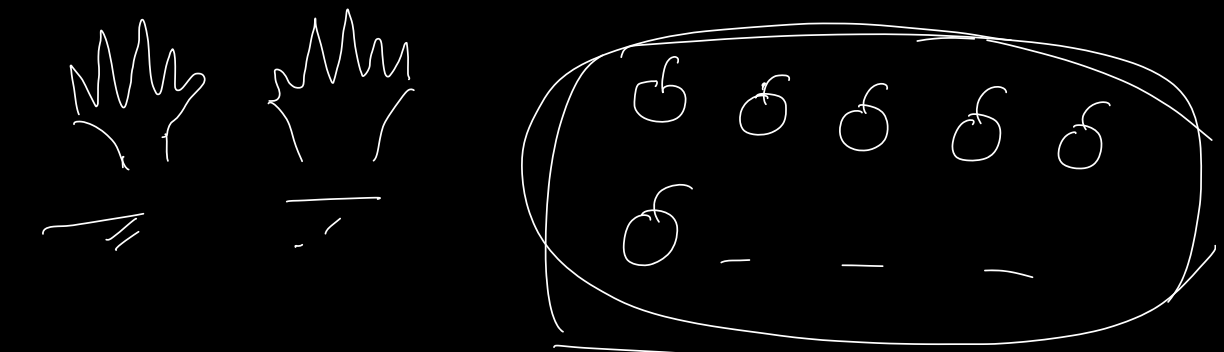
07

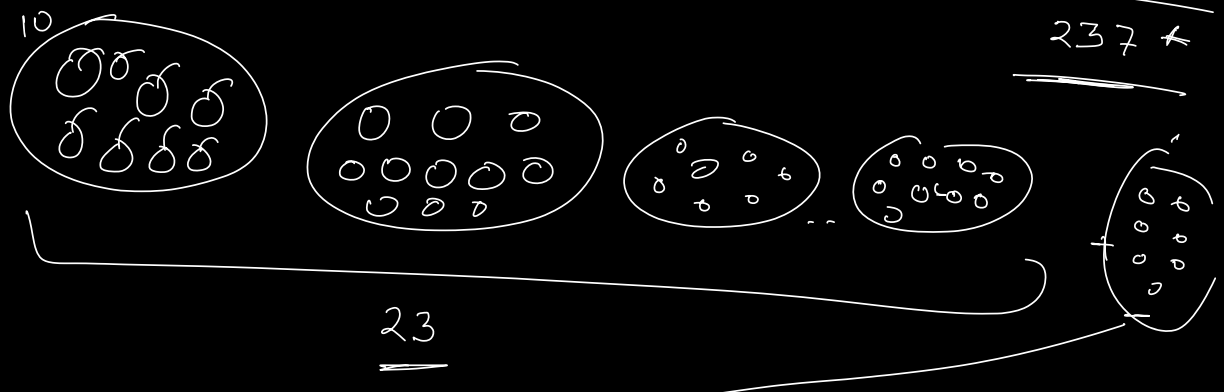
Hexadecimal (16)

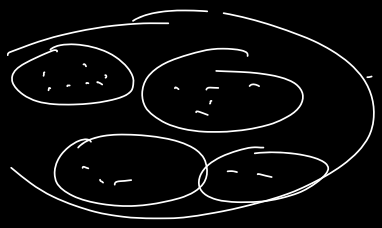
[0-9] [A-F]

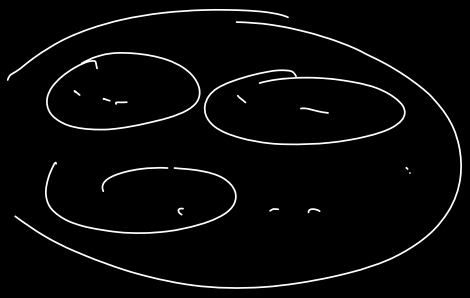
int a = 1 0 1 1 0 0 0

1 0 1 0 => 10









237 = $2 \times 10^2 + 3 \times 10^1 + 7 \times 10^0$

0 → ϕ
1 → ε
2 → ||
3 → |||
⋮
9

0 1 2 3 4 5 6 7 8 9

1/0

% ⇒ mod