

→ Number Systems

→ Base Conversion

→ Intro to bit manipulation

$$\begin{array}{r}
 734 \\
 -\underline{-} \quad \Rightarrow \\
 + \quad \quad \quad 700 \\
 + \quad 30 \\
 \hline
 4
 \end{array}$$

$$700 + 30 + 9$$

$$7 \times \underline{10^2} + 3 \times \underline{10^1} + 4 \times \underline{10^0}$$

$$6514 = 6 \times 10^3 + 5 \times 10^2 + 1 \times 10^1 + 4 \times 10^0$$

i) Decimal No. [0 - 9] \hookrightarrow related to 'its'

$$\frac{1}{10^4} \cdot \frac{2}{10^3} \left(\frac{1}{10^4} \right) \left(\frac{1}{10^3} \right) \left(\frac{1}{10^0} \right)$$

2) Base 8

Octal No [0-7]

$$(0 \ 1 \ 3 \ 2)_8 \Rightarrow \underbrace{0 \times 8^3}_{64} + \underbrace{1 \times 8^2}_{24} + \underbrace{3 \times 8^1}_2 + \underbrace{2 \times 8^0}_1 = 90$$

Octal ↓ decimal

$$(1 \ 2 \ 5)_8 = \underbrace{1 \times 8^2}_{64} + \underbrace{2 \times 8^1}_{16} + \underbrace{5 \times 8^0}_1 = 55$$

3) Base 3

[0, 1, 2] ⇒ [0, 2]

Ternary No. Syst

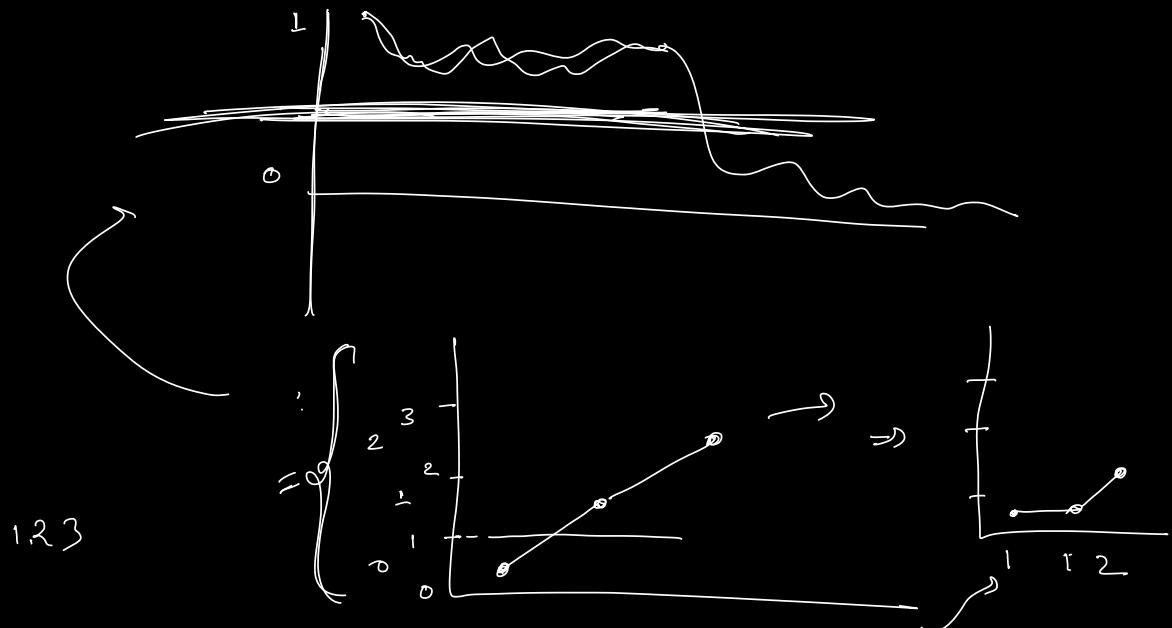
$$(1 \ 2 \ 0)_3 = \underbrace{1 \times 3^3}_{27} + \underbrace{2 \times 3^2}_9 + \underbrace{0 \times 3^1}_0 + \underbrace{0 \times 3^0}_1 = 42$$

$$2 \ 1 \ 0 \ 1 = \underbrace{2 \times 3^3}_{54} + \underbrace{1 \times 3^2}_9 + \underbrace{0 \times 3^1}_0 + \underbrace{1 \times 3^0}_1 = 61$$

4) Binary [0, 1]

$$1 \ 0 \ 1 \ 1 \ 0 = 2^4 + 2^2 + 2^1 = 22$$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0$



"Vinekanandha"



"Vinekanandha"

Empathy ?

Decimal to Binary

Long division Method

$$\begin{array}{r}
 2 | 28 \\
 2 | 14 \\
 2 | 7 \\
 2 | 3 \\
 2 | 1
 \end{array}
 \quad
 \begin{array}{r}
 0 \\
 0 \\
 1 \\
 1 \\
 0
 \end{array}$$

$$\begin{array}{r}
 11100
 \end{array}$$

$$\begin{array}{r}
 2 | 35 \\
 2 | 17 \\
 2 | 8 \\
 2 | 4 \\
 2 | 2 \\
 2 | 1
 \end{array}
 \quad
 \begin{array}{r}
 1 \\
 1 \\
 0 \\
 0 \\
 0 \\
 1
 \end{array}$$

$$\begin{array}{r}
 10001
 \end{array}$$

$$\begin{array}{r} 19 \\ 2 \overline{)19} \\ \underline{-18} \\ 1 \end{array}$$

$$\begin{array}{r} 9 \\ 2 \overline{)9} \\ \underline{-8} \\ 1 \end{array}$$

$$\begin{array}{r} 4 \\ 2 \overline{)4} \\ \underline{-4} \\ 0 \end{array}$$

$$\begin{array}{r} 2 \\ 2 \overline{)2} \\ \underline{-2} \\ 0 \end{array}$$

$$\begin{array}{r} 1 \\ 2 \overline{)1} \\ \underline{-2} \\ 0 \end{array}$$

$$\begin{array}{r} 10011 \\ \hline \end{array}$$

$25 \rightarrow ?$
 $37 \rightarrow ?$

* HW

Addition

$$3^n / 4^n$$

i) Decimal No.

$$\begin{array}{r} . \quad \begin{array}{c} 1 \\ 3 \\ 2 \\ 8 \end{array} \quad \begin{array}{c} 1 \\ 4 \\ 9 \end{array} \quad \begin{array}{c} 1 \\ 5 \\ 9 \end{array} \quad \begin{array}{c} 1 \\ 9 \\ 7 \end{array} \\ \hline 0 \quad 6 \quad 3 \quad 0 \quad 6 \end{array}$$

$$\begin{array}{r} 6/10 \quad 11/10 \quad 14/10 \quad 12/10 \quad \Rightarrow c = 3/8 \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 2 \quad 3 \quad 7 \quad 8 \\ 3 \quad 7 \quad 6 \quad 4 \end{array}$$

$a = 3\% B$

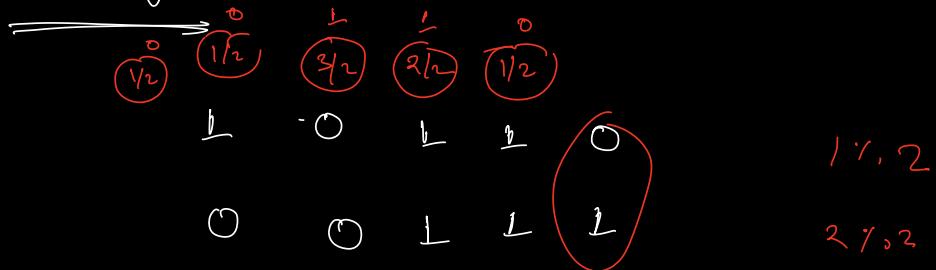
$$\begin{array}{r} 6 \quad 1 \quad 4 \quad 2 \\ \downarrow \quad \uparrow \quad \uparrow \quad \uparrow \\ 67/10 \quad 11/10 \quad 14/10 \quad 12/10 \end{array}$$

$$\begin{array}{r} 18/10 \\ \downarrow \\ 9 \\ 9 \\ \hline 1 \quad 8 \end{array}$$

$$18 \% 10$$

2) Binary No

$$B_{\text{are}} = 2$$



Handwritten diagram showing the filling of atomic orbitals for a hydrogen-like atom. The top row shows five atomic orbitals with quantum numbers $l=1$, $m_l = 1, 0, -1, 2$. The bottom row shows the corresponding electron configurations: $3/2$, $2/2$, $2/2$, $2/2$, and $2/2$. Arrows indicate the spin of each electron.

$$\begin{array}{cccccc}
 \perp & \circ & \perp & \perp & \perp \\
 \hline
 3\% & 2\% & 2\% & 3\% & 2\%
 \end{array}$$

10

Bit \Rightarrow

Bil \rightarrow $\perp \Rightarrow$ Set Bil

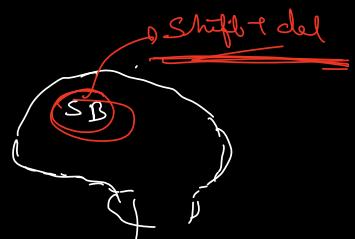
Bil → Ø ⇒ Unset Bil

1 1 0 0
1 1 0 0
1 1 1 0

$$\text{Diagram showing the binary representation of } \frac{1}{2} \text{ as } 0.\overline{1}_2 \text{ and } \frac{1}{4} \text{ as } 0.\overline{01}_2.$$

MSB \Rightarrow Most Significant Bit

$$\begin{array}{r}
 & 1 & 1 & 0 & 1 \\
 & 1 & 0 & & \\
 \hline
 & 1 & 0 & & 0 \\
 & 2 & & 2 & \\
 \hline
 & 1 & 0 & 1 & 0
 \end{array}$$



MSB

$$\begin{array}{c}
 \left(\frac{1}{-2^7} \right) \quad \left(\frac{1}{2^6} \right) \quad \left(\frac{1}{2^5} \right) \quad 0 \quad 0 \quad 0 \quad 0 \\
 \downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow \\
 1 \times (-2^7) \quad + \quad (1 \times 2^6) \quad + \quad (1 \times 2^5) \quad + \quad (1 \times 2^0)
 \end{array}$$

$$\Rightarrow -128 + 64 + 32 + 1$$

$$\Rightarrow \underline{\underline{-31}}$$

↖

$$\begin{array}{r}
 1 \\
 -2^7 \quad 0 \quad 0 \quad 0 \\
 \hline
 -2^7 \quad 2^6 \quad 2^5 \quad 2^0
 \end{array}
 \Rightarrow -8$$

$$\begin{array}{r} 0 \\ \hline -2^3 \\ \quad \quad \quad 1 \\ \quad \quad \quad \hline 2^2 \\ \quad \quad \quad 0 \\ \quad \quad \quad \hline 2^1 \\ \quad \quad \quad 1 \\ \quad \quad \quad \hline 2^0 \\ \end{array} \Rightarrow 5$$

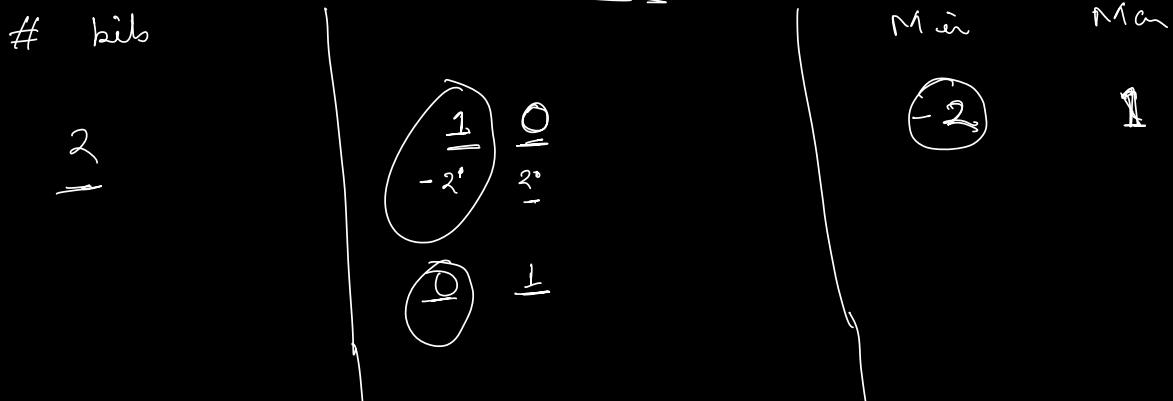
$$\begin{array}{r} 1 \\ \hline -2^3 \\ \quad \quad \quad 1 \\ \quad \quad \quad \hline 2^2 \\ \quad \quad \quad 1 \\ \quad \quad \quad \hline 2^1 \\ \quad \quad \quad 1 \\ \quad \quad \quad \hline 2^0 \\ \end{array} \Rightarrow -8 + 4 + 2 + 1 \Rightarrow -1$$

JMSD

$$\begin{array}{r} 1 \\ \hline -2^3 \\ \quad \quad \quad 0 \\ \quad \quad \quad \hline 2^2 \\ \quad \quad \quad 1 \\ \quad \quad \quad \hline 2^1 \\ \quad \quad \quad 0 \\ \quad \quad \quad \hline 2^0 \\ \end{array} \begin{array}{l} -2^3 + 2^1 \\ \Rightarrow -6 \end{array}$$

↔ 10 X
↔ -10 X

Limited on Min & Max



3

$$\begin{array}{c} 0 \\ \underline{-2^2)} \\ - \end{array} \quad \begin{array}{c} 1 \\ 2^1 \\ - \end{array} \quad \begin{array}{c} 1 \\ 2^0 \\ - \end{array}$$

Min

Max

-4

3

4

$$\begin{array}{c} 0 \\ \underline{-3} \\ - \end{array} \quad \begin{array}{c} 1 \\ 2 \\ - \end{array} \quad \begin{array}{c} 1 \\ 1 \\ - \end{array} \quad \begin{array}{c} 1 \\ 0 \\ - \end{array}$$

-8, 7

n

$$\begin{array}{c} 1 \\ \underline{(n-1)} \\ - \end{array} \quad \begin{array}{c} - \\ 2 \\ - \end{array} \quad \begin{array}{c} - \\ 2 \\ - \end{array} \quad \begin{array}{c} - \\ 1 \\ - \end{array} \quad \begin{array}{c} - \\ 0 \\ - \end{array}$$

Min

Max

$$\begin{array}{c} -2^{(n-1)} \\ \underline{-} \\ \underline{(2^{n-1}-1)} \end{array}$$

$$\underbrace{1, 2, 4, 8}_{\text{Sum}} = \frac{4}{2} = \frac{8}{4} = \underline{\underline{2}}$$

$$\begin{array}{c} 0 \\ \underline{(n-2)} \\ - \end{array} \quad \begin{array}{c} 1 \\ (n-2) \\ - \end{array} \quad \begin{array}{c} 1 \\ - \end{array} \quad \dots \quad \begin{array}{c} 1 \\ 0 \\ - \end{array}$$

$$\underbrace{2^{(n-2)} + 2^{(n-3)} + \dots + 2^0}_{=}$$

$$\begin{array}{c} 0 \xrightarrow{(n-2)} \\ \underline{\underline{1, 2, 3}} \\ 4, 5 \end{array}$$

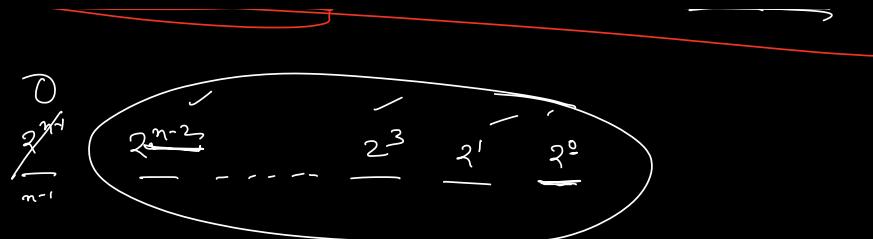
$$\underbrace{2^0 + 2^1 + 2^2 + \dots + 2^{(n-2)} + 2^{(n-1)}}_{=}$$

GP

GP

$a = 1$, $r = 2$, $t = (n-1)$

Sum = $\frac{a \times (r^t - 1)}{r - 1}$ = $\frac{1 \times 2^{n-1} - 1}{2 - 1}$ = $(2^{n-1} - 1)$



$$0 \rightarrow n-2$$

$$n = 4$$

$$\underbrace{0, 1, 2}_{\rightarrow (n-1) = 3}$$

$$n = 5$$

$$0, 1, 2, 3 \Rightarrow 4 \rightarrow (5-1) = 4$$

$$\underline{1 \text{ Byte}} = \underline{8 \text{ Bit}} = \left[\frac{-2^{(8-1)}}{1}, 2^{(8-1)} - 1 \right] = [-128, 127]$$

$$\underline{2 \text{ Byte}} = \underline{16 \text{ Bits}} = \left[\frac{-2^{15}}{1}, 2^{15} - 1 \right] = [-32768, 32767]$$

4 Byte = 32 Bits = $\left[\frac{-2^{31}}{1}, 2^{31} - 1 \right]$

int \downarrow log.

unsigned int

$\rightarrow \begin{matrix} & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ & 7 & 6 & 5 & ; & 3 & 2 & 1 & 0 \end{matrix}$

$- (2^7) + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0$

$- 128 + 64 + 32 + 16 + 8 + 4 + 2 + 1$

Integ. - Min $\rightarrow -2, 147, 483, 648$

Integ. - Max $\rightarrow 2, 147, 483, 647$

Bit Manipulation

Bitwise operators

$\{ \begin{array}{l} \& | \sim \wedge \\ \text{and} & \text{or} & \text{not} & \text{xor} \end{array} \}$

$(\ll \gg) \leftarrow$

a	b	$a \& b$	$a b$	$a \wedge b$	$\sim a$
0	0	0	0	0	1
0	1	0	1	0	0
1	0	0	1	1	0
1	1	1	1	1	0

$$4 \& 3 = 0 \ 1 - \underline{\quad} \ 1 \ 0 \ 0 \ 0]$$

$$\underline{\quad} \ 0 \ 0 \ 1 \ 1]$$

$$\underline{\quad} \ 0 \ 0 \ 0$$

$$4 | 3 = 7 \quad \underline{\quad} \ 0 \ 0 \ 0$$

$$\underline{\quad} \ 0 \ 1 \ 1]$$

$$\underline{\quad} \ 1 \ 1 \ 1 \Rightarrow 7$$

$$4 \wedge 3 = 7 \quad \underline{\quad} \ 0 \ 0 \ 0$$

$$\underline{\quad} \ 0 \ 1 \ 1]$$

$$\underline{\quad} \ 1 \ 1 \ 1 \Rightarrow 7$$

Decimal to Binary for -ve no. ?

2' Complement

Ex

$$\boxed{1 \overline{\overline{1}} -a = \sim a + 1} \quad \text{≡}$$

↓'s complement

8 bit representation of -10

$$a = 10 : \underline{0} \underline{0} \underline{0} \underline{0} \underline{1} \underline{0} \underline{1} \underline{0}$$

$$\begin{array}{r} \sim a \\ \rightarrow \\ : \quad \underline{1} \underline{1} \underline{1} \underline{1} \underline{0} \underline{1} \underline{0} \underline{1} \\ + 1 \\ : \quad \underline{0} \underline{0} \underline{0} \underline{0} \underline{0} \underline{0} \underline{0} \underline{1} \end{array}$$

$$(-10) : \quad \begin{array}{cccccccc} & & & & & & & \\ & \text{7} & \text{6} & \text{5} & \text{4} & \text{3} & \text{2} & \text{1} \\ \text{7} & \text{1} & \text{1} & \text{1} & \text{0} & \text{1} & \text{1} & \text{0} \end{array} \quad \equiv$$

$$\rightarrow (2^7) + 2^6 + 2^5 + 2^4 + 2^2 + 2^1$$

$$-128 + 64 + 32 + 16 + 4 + 2$$

$$\underline{\underline{\rightarrow 10}}$$

$$\begin{array}{c} \text{7} \\ \text{6} \\ \text{5} \\ \text{4} \\ \text{3} \\ \text{2} \\ \text{1} \\ \text{n-1} \end{array} \quad \underline{\underline{\quad \quad \quad}}$$

$$- (2^{n-1})$$

$$a = 13 \quad ; \quad \overbrace{\begin{array}{cccc} 1 & 0 & 1 & 0 \end{array}}^{(a+1)} \\ b = 10 \quad ; \quad \begin{array}{cccc} 1 & 0 & 0 & 0 \end{array} \Rightarrow 8$$

$$a \mid b \quad \begin{array}{cccc} 1 & 1 & 1 & 1 \end{array} \Rightarrow 15$$

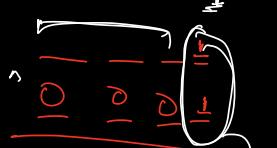
~~$$a \wedge b \quad \begin{array}{cccc} 0 & 1 & 1 & 1 \end{array} \Rightarrow 7$$~~

$$a \mid 1$$

Even $\rightarrow a = 10$ $(a+1)$
 Odd $\rightarrow a = 1$, a

$\begin{array}{c} 10 \\ 00 \\ \hline 10 \end{array} \Rightarrow 10$
 $\begin{array}{c} 10 \\ 00 \\ \hline 10 \end{array} \Rightarrow 11$
 $\begin{array}{c} 10 \\ 00 \\ \hline 10 \end{array} \Rightarrow 11$

$$a \wedge i$$

Even \rightarrow 
 Odd \rightarrow 

$(a+1)$ $(a-1)$

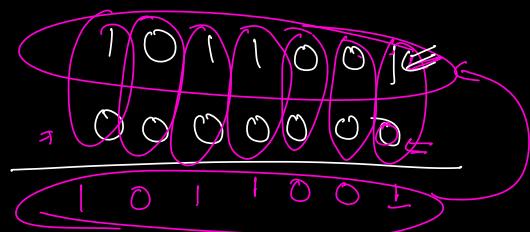
$$\begin{array}{lcl}
 a \& b &= b \& a \\
 \overline{a \mid b} &=& b \mid a \\
 a \wedge b &=& b \wedge a
 \end{array}
 \quad \left. \begin{array}{c} \\ \\ \end{array} \right\} \text{Commutation} \quad a - b \neq b - a$$

$$(a \wedge b \wedge c) = a \wedge (b \wedge c)$$

$$\begin{aligned}
 &= (a \wedge b) \wedge c \\
 &= (a \wedge c) \wedge b
 \end{aligned}$$

$$\overbrace{a+b} = \overbrace{b+a}$$

$$a \wedge 0 = a$$



$$a \wedge a = 0$$

$$\begin{array}{r}
 1 0 1 1 0 0 0 1 \leftarrow a \\
 1 1 0 1 1 0 1 0 \leftarrow a \\
 \hline
 0 0 0 0 0 0 0 \leftarrow 0
 \end{array}$$

$$a \wedge b \wedge a = (a \wedge a) \wedge b$$

$$= 0 \wedge b$$

$$= 0$$

$$a \wedge c \wedge a \wedge b \wedge c = b$$

Amager
MS
Adobe

Q

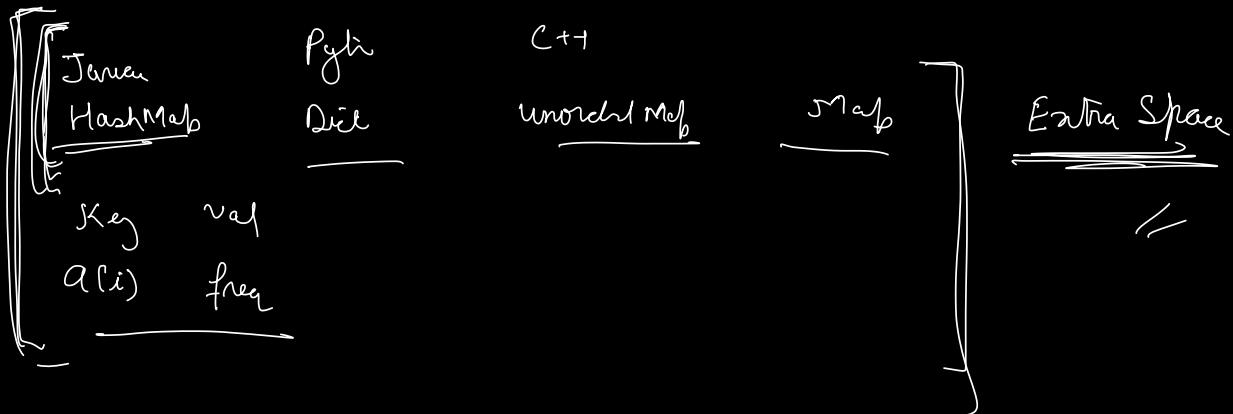
Single No

Given n elements. All elements are appear twice except one element.

Find the single no.

10⁹, 2, 3, 1, 5, 2, 1, 3, 6, 6, 10⁹

$\Rightarrow 5$



Sorting
 $\frac{1}{(i, (i+1))}, \frac{1}{2}, \frac{2}{3}, \frac{3}{3}, \frac{5}{6}, \frac{6}{6}, \frac{10^9}{10^9}$

Library Sort f_n \Rightarrow Extra Time
 $O(n \log n)$

10⁹ 2 3 1 5 2 1 3 6 6 10⁹

```

int ans = 0;
for (i=0; i<n; i++) {
    ans = arr[i] ^ ans;
}

```

4 1 3 4

$$2 \quad , \quad N \quad (2)_{\text{ch}}$$

A hand-drawn diagram consisting of a large oval. Inside the oval, there are two vertical lines, each with a horizontal line through it, both labeled with the number '1'. There are also two short horizontal lines extending from the right side of the oval.

Oct 18 (10)

37

10%, 12

$$2 \begin{array}{c} \swarrow \\[-1ex] \text{L} \\[-1ex] \searrow \end{array} - \textcircled{6} \leftarrow$$

$$2 \Rightarrow \underline{\underline{10}}$$

$$1+1=2$$

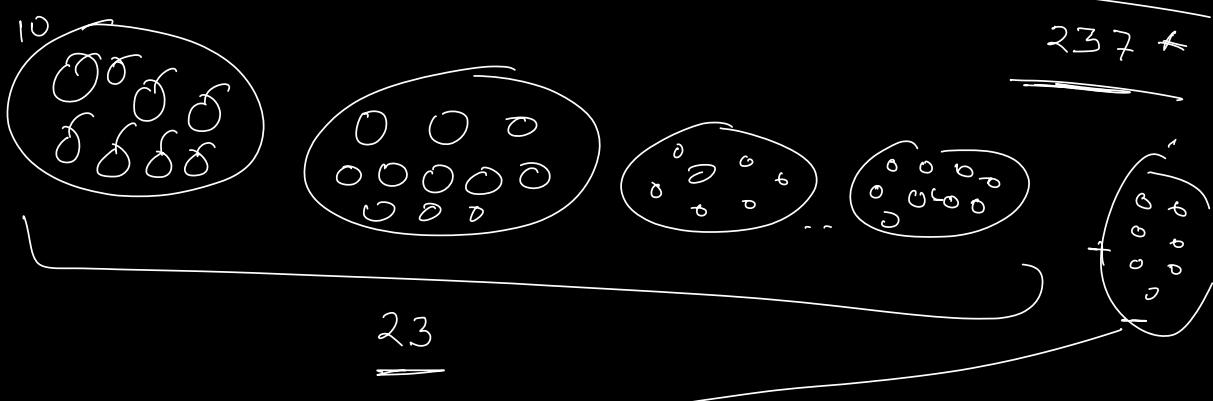
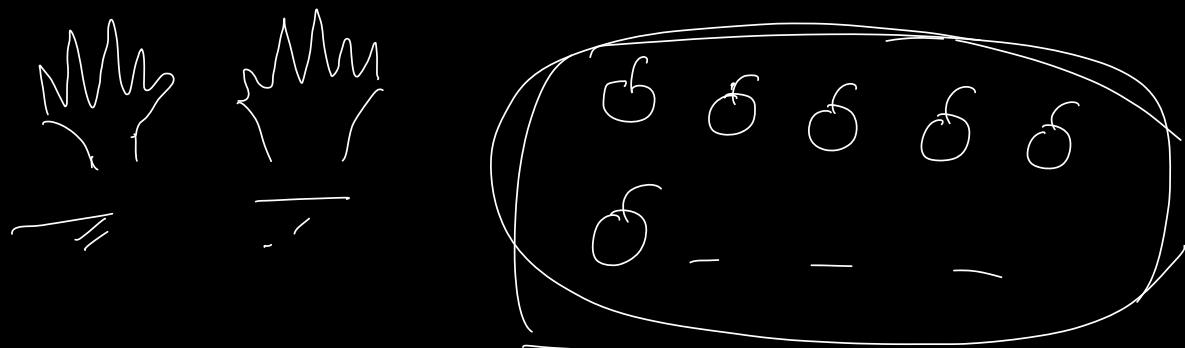
Hexadecimal

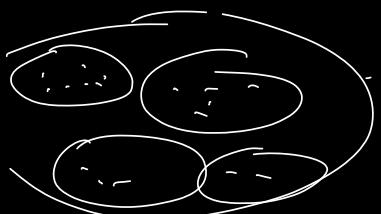
16

[0 - 9] [A - F]

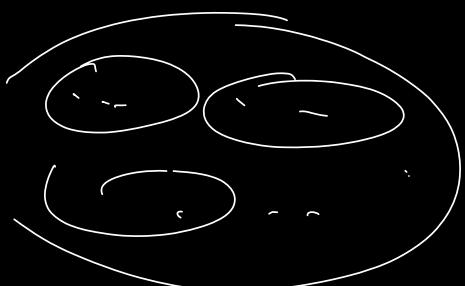
int a = 10110000

1010 \Rightarrow 10





$$\boxed{237} = \underline{\underline{2 \times 10^2}} + \underline{\underline{3 \times 10^1}} + \underline{\underline{7 \times 10^0}}$$



$$\left(\begin{array}{c|c} \begin{matrix} 0 \rightarrow \phi \\ 1 \rightarrow e \\ 2 \rightarrow \\ 3 \rightarrow \\ \vdots \\ g \end{matrix} & \begin{matrix} 0 \\ 9 \\ 2 \\ 3 \\ 2 \\ 5 \\ 5 \\ 2 \\ 3 \\ 2 \\ 1 \\ 6 \\ 5 \\ 3 \\ 2 \\ 1 \\ 0 \\ 9 \\ 0 \end{matrix} \\ \hline \end{array} \right)$$

$$\underline{\underline{1/0}}$$

$$\underbrace{\%}_{\text{---}} \rightarrow \underbrace{mole}$$