Q Given an array (N-1) containing all numbers from 1 to N except one number.

Find the missing no.

1, 3, 4, 5 ⇒ 2

4, 5, 1, 3, 6 N ⇒ 6

⇒ 8, 9, 2, 4, 5, 6, 1, 7 N = 9

✓ 1, 2, 3, 4, 5, 6 ⇒ N = 7 [1, 7]

3, 6, 1, 7, 4, 9, 8 N = 8 [1, 8]

2, 5

Sort : 1, 2, 4, 5, 6, 7, 8, 9 ⇒ $O(n \log n)$

Counting sort $O(N)$

If no number was missing

$$Sum_N = \frac{N(N+1)}{2} \Rightarrow O(1)$$

$Sum_N - Sum_{Array}$ ⇒ Missing No.

$\boxed{TC : O(N)}$

$$\begin{array}{r} \cancel{1} + \cancel{2} + \cancel{3} + 4 + \cancel{5} \\ - \quad \cancel{3} + \cancel{5} + \cancel{2} + \cancel{1} \\ \hline \end{array}$$

$N \approx 10^6$

$$Sum_N = \frac{10^6(10^6+1)}{2} \approx \frac{10^{12}}{2} \Leftarrow \text{integer?}$$

↑ Overflow

Approach 2

[3 ^ 4 ^ 1 ^ 5 ^ 6 ^ 1 ^ 2 ^ 3 ^ 4 ^ 5 ^ 6]

$a \wedge a = 0$

$\cancel{a} \wedge b \wedge \cancel{c} \wedge \cancel{d} \wedge \cancel{e} \wedge \cancel{c} \wedge \cancel{d}$

$\boxed{TC: O(N)}$

$\boxed{\% \Rightarrow \text{Modulus (Remainder)}}$

$a \% b$ => Remainder when a/b

$19 \div 4 \Rightarrow 4 \mid 3$

$19 = 4 \times 4 + 3$

$\boxed{\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}}$

→ Max multiple of divisor ≤ dividend

// $\boxed{\text{Remainder} = \text{Dividend} - (\text{Max Multiple of divisor}) \le \text{dividend}}$ //

Quizes.

$150 \% 11 = 150 - (11 \times 13)$

$= 7$

$100 \% 7 = 100 - (7 \times 14)$

$= 2$

$-40 \% 7 = -40 - \underbrace{(7 \times -6)}$

↳ Max multiple of 7 less than -40

-42

$-40 - (-42)$

$= 2$

C / C++ / Java / JS / C#

Python ✓

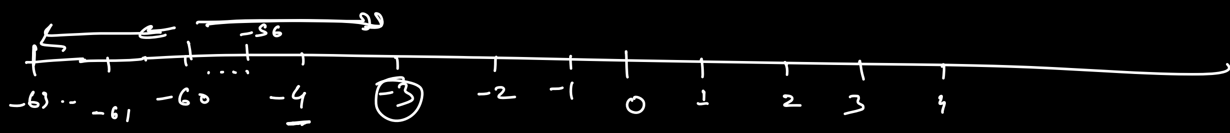
[- x - = +]

$-60 \% 9 = -60 - (-63)$

$= -60 + 63$

$= 3$

$\boxed{\begin{array}{l} a < 0 \\ a \% M = a \% M + M \end{array}}$



$-4 < -3$

$-56 > -60$

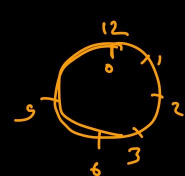
$a \% M \Rightarrow [0, M-1]$

$4268012399 \% 10$ $\quad [0, 9]$

$\_\_\_\_ \% 12 \Rightarrow [0 - 11]$

$$
\begin{array}{cccccccccc}
0 & 1 & 2 & \cdots & 12 & 13 & \cdots & 24 & \cdots 27 \cdots & 108890 \\
\downarrow & \downarrow & \downarrow & & \downarrow & \downarrow & & \downarrow & & \\
0 & 1 & 2 & & 0 & 1 & & 0 & &
\end{array}
$$

$\% 12$



$$
\left.\begin{array}{c} -\infty \\ \vdots \\ +\infty \end{array}\right\} \% n \left\{\begin{array}{c} 0 \\ \vdots \\ n-1 \end{array}\right.
$$

Modular Arithmetic

$$\boxed{(a+b)\,\%\,M = (a\,\%\,M + b\,\%\,M)\,\%\,M}$$

$a = 7$
$b = 5$
$M = 10$

$((7\,\%\,10) + (5\,\%\,10))$

$7 + 5$

$= 12$ ×

$$\boxed{(a \times b)\,\%\,M = (a\,\%\,M \times b\,\%\,M)\,\%\,M}$$

HW

$(a-b)\,\%\,M$

$(a/b)\,\%\,M$

Advance classes

Q Given three nos, a, n & p.

find $\boxed{a^n\,\%\,p}$

```
int PowerMod(a, n, p){
    int ans = 1;
    for(i=1; i<=N; i++){
        ans = (ans * a) % P;
    }
    ret ans % P;
}
```

O(N)

$a = a \times a$

i	a_i	a_f
1	a	a^2
2	a^2	a^4
3	a^4	a^8
4	a^8	a^{16}

$$ans = (ans \times a) \% p$$

$(\;)\ \% p \Rightarrow [0, p-1]$

$$\Big(\underbrace{(ans \% p)}_{(p-1)} \times \underbrace{(a \% p)}_{(p-1)}\Big) \% p$$

$\approx p^2 \% p$ ← int

10^9+7

Break till 10:46 p

Divisibility Rules

Rule for 3

a No N is divisible by 3
if sum of digits of N is divisible by 3

$(35632) \% 3$

$= (3 \times 10^4 + 5 \times 10^3 + 6 \times 10^2 + 3 \times 10 + 2) \% 3$

$= ((3 \times 10^4) \% 3 + (5 \times 10^3) \% 3 + (6 \times 10^2) \% 3 + (3 \times 10) \% 3 + 2 \% 3) \% 3$

$$((3\%3) \times (10^4\%3))\%3 \Rightarrow (3\%3)\%3$$

$$+ ((5\%3) \times (10^3\%3))\%3 \Rightarrow (5\%3)\%3$$

$$+ ((6\%3) \times (10^2\%3))\%3 \Rightarrow (6\%3)\%3$$

$$+ ((3\%3) \times (10\%3))\%3 \Rightarrow (3\%3)\%3$$

$$+ (2\%3) \Rightarrow (2\%3)$$

$$\%3$$

$$\left.\begin{array}{l} 10\%3 \\ 10^2\%3 \\ 10^3\%3 \\ 10^4\%3 \\ \vdots \\ 10^x\%3 \end{array}\right\} = 1$$

$$(\underbrace{a\%M}_{[0, M-1)})\%M = a\%M$$

$$((3\%3) + (5\%3) + (6\%3) + (3\%3) + (2\%3))\%3$$

$$\Downarrow \quad (a\%M + b\%M)\%M = (a+b)\%M$$

$$\underbrace{(3+5+6+3+2)}_{\text{Sum of digits}}\%3$$

Hence Proved

Rule for 4

if the no. formed by the last two digits is divisible by 4 then the complete no. will also be divisible.

$$(4577)\,\%\,4$$

$$\Rightarrow (4\times 10^3 + 5\times 10^2 + 7\times 10 + 7)\,\%\,4$$

$(a+b)\,\%\,M \Rightarrow$

$$= \big((4\times 10^3)\,\%\,4 + (5\times 10^2)\,\%\,4 + (7\times 10)\,\%\,4 + 7\,\%\,4\big)\,\%\,4$$

$(a\times b)\,\%\,M$

$$= \Big(\underbrace{\big((4\,\%\,4)\times(\underset{0}{10^3\,\%\,4})\big)\,\%\,4}_{0} + \underbrace{\big((5\,\%\,4)\times(\underset{0}{10^2\,\%\,4})\big)\,\%\,4}_{0} + \underbrace{(7\,\%\,4\times 10\,\%\,4)\,\%\,4}_{} + 7\,\%\,4\Big)\,\%\,4$$

$$+ \quad + \quad + (7\times 10)\,\%\,4 + 7\,\%\,4)\,\%\,4$$

$100\,\%\,4 = 0$

$(10\times 100)\,\%\,4 = 0$

$10^4\,\%\,4 = 0$

$(x\,100)\,\%\,4 = 0$

$10^x\,\%\,4 = 0$

$x \geq 2$

$$\boxed{HW \Rightarrow 5,\ 8,\ 9}$$

$$(0 + 0 + (7\times 10)\,\%\,4 + 7\,\%\,4)\,\%\,4$$

$$= \big((7\times 10)\,\%\,4 + 7\,\%\,4\big)\,\%\,4$$

$$(70\,\%\,4 + 7\,\%\,4)\,\%\,4$$

$$(a\,\%\,M + b\,\%\,M)\,\%\,M$$

$$\Rightarrow (a+b)\,\%\,M$$

$$(70+7)\,\%\,4$$

$$\Rightarrow 77\,\%\,4$$

Google* Given a very large no S in the form of a string.
& an integer p [length of S : $[1, 10^5]$]
find $S \% p$

S ; "101"
P ; 10 $\Rightarrow$ 1

(String to int) % P ✕

INT Max $\approx 10^9$ ↳ 10 digits

1000000000

" 1 0 2 1 1 0 4 3 7 8 9 - - - - - - "
10^5

$\Rightarrow$ $S = S_{N-1} \ldots S_3 \; S_2 \; S_1 \; S_0$

$$S \% P = (S_{N-1} \times 10^{N-1} + S_{N-2} \times 10^{N-2} + \ldots + S_1 \times 10 + S_0) \% p$$

$$(a+b) \% M = (a \% M + b \% M) \% M$$

$$= ((S_{N-1} \times 10^{N-1}) \% P + (S_{N-2} \times 10^{N-2}) \% P + \ldots + (S_1 \times 10) \% P + S_0 \% P) \% P$$

p^{10^5}

$O(N)$

$$((S_{N-1} \% P) \times (10^{N-1} \% P)) \% P$$

$O(1)$ $O(N)$

$\boxed{S_i : [0, 9]}$

$\boxed{TC : O(N^2)}$

$(10^{N-1} \% P)$ $\quad - - - \quad 10^{N-2} \% P \quad 10^{N-3} \% P \quad - - - - \quad 1 \% P$

$\Downarrow$

$O(N) \times N$

$\Rightarrow O(N^2)$

$$\left[\left(10^{100000} \right) \% P \right]$$

$\Downarrow$

Integer $\quad O(N)$

1 2 3 4 4 5 6 2 3

$$(1 \times 10^8 + 2 \times 10^7 + 3 \times 10^6 \; --- \; + 3 \times 10^0) \% P$$

$(1 \% P \times 10^8 \% P) \% P \quad 10^7 \% P \quad 10^6 \% 7$

$O(1) \quad O(N) \quad O(N \quad O(N)$

Carry forward

$10^{9-1} \% P$

$10^9 \% P = (10 \times 10^{9-1}) \% P$

$= \left(\underbrace{10 \% P}_{O(1)} \times \underbrace{10^{9-1} \% P}_{O(1)} \right) \% P$

$10^4 \% P$

$10^5 \% P = (10 \% P \times 10^4 \% P) \% P$

TC : $O(N)$

$O(N^2)$

```
for ( i = N-1; i >= 0; i--) {
    ans = ans + (S[i] % P * PowerMod(10, i, P))
    ans = ans % P;
}
```

$0 \longrightarrow N$

$1 \% P$

$100 = (10 \times 10) \% P$

$1000 = (100 \times 10) \% P$

$10^3 \% P = x$

$1000 \% P$

$(10 \times 100) \% P$

$(10 \% P \times (100) \% P) \% P$

$\times \quad 10 \% P \Rightarrow x$

$(10^{n}) \% P = x$

Interview Bit (2017)

→ 2 ACM ICPC WF

Anshu

Scaler Alumni

- Sataxai ; Amazon April 2019
- Naman Bhalla ; Google April 2019
- Mohit Sharma ;

Money + Learning + Ownership + People + environment

+ Aligned with mission