

Prime No : Any positive no. that has exactly 2 factors.

Q Given a prime no. $p > 3$ (in a string)
Find $(p^2 - 1) \% 24$ $1 \leq p.length \leq 10^5$

$$① (13 \times 14) \% 2 = 0$$

$$② (19 \times 20) \% 2 = 0$$

$$③ (85 \times 86) \% 2 = 0$$

Product of 2 consecutive no. \rightarrow always divisible by 2

$$④ (4 \times 6) \% 8 = 0$$

$$⑤ (126 \times 128) \% 8 = 0$$

$$⑥ (10 \times 12) \% 8 = 0$$

Product of any 2 consecutive even no. \rightarrow always divisible by 8.

$$2, \overbrace{4, 6, 8, 10, 12, 14, 16, 18, 20, 22} \dots$$

$\downarrow \quad \quad \downarrow \quad \quad \downarrow \quad \quad \downarrow \quad \quad \downarrow$
 $4 \quad \quad 4 \times 2 \quad \quad 4 \times 3 \quad \quad 4 \times 4 \quad \quad 4 \times 5$

$$2i, 2i+2$$

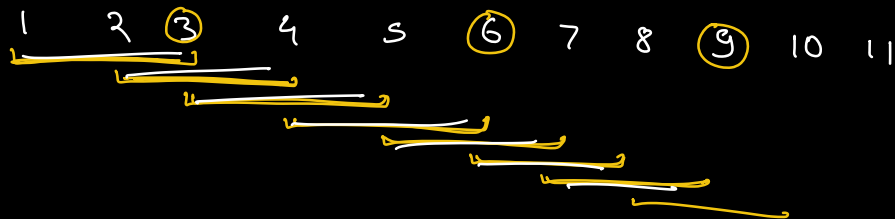
$$(2i) \times (2i+2)$$

$$2i \times 2(i+1)$$

$$4(\underline{i \times (i+1)}) \Rightarrow 4 \times (2n) \Rightarrow \underline{8n}$$

Product of any three consecutive no.

$$\boxed{(n \times (n+1) \times (n+2)) \% 3 = 0}$$



$$(p^2 - 1) \% 24 =$$

\Downarrow

$$p^2 - 1^2$$

\Downarrow

$$\boxed{(p-1) \quad (p+1)}$$

p is prime no > 3

$\Rightarrow p$ is odd

$\Rightarrow (p-1) \leftarrow (p+1)$ are even
 $\quad \quad \quad \searrow$ consecutive

$$\Rightarrow \boxed{((p-1) \times (p+1)) \% 8 = 0}$$

$$\boxed{(p-1) \quad p \quad (p+1)}$$

$\therefore p \% 3 \neq 0 \quad (p \rightarrow \text{prime no.})$

$$\therefore \boxed{((p-1) \times (p+1)) \% 3 = 0}$$

Google

Majority element

Q Given an array. ^{of size N} Return, if there exists, a no. with frequency $> N/2$

1 6, 1, 1, 2, 1 $N = 6$
ME: 1 (4)

Without using any extra space.

Brute force : • 2 loops
• count freq of all elements
• $O(N^2)$

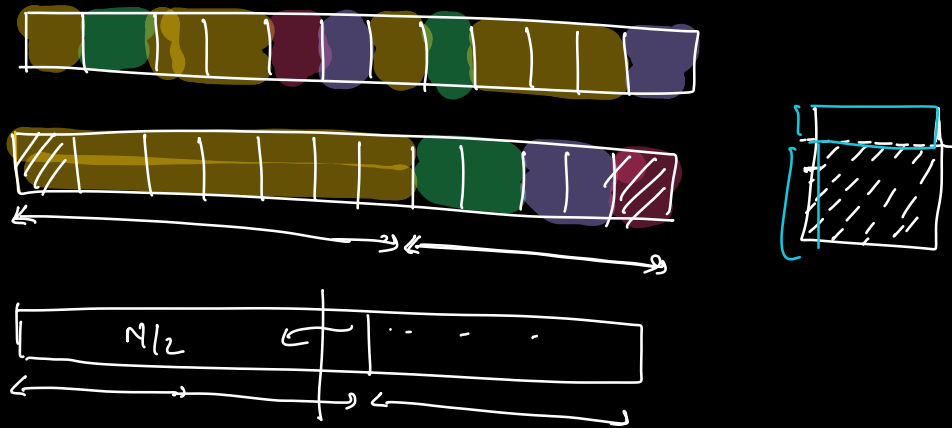
Sorting : $O(n \log n)$

(3) 4, (3) 6, 1, (3) 2, 5, (3) (3) (3)

$N = 11$
 $|ME| \geq 6$ $11/2 = 5.5 \rightarrow 6$

4 6 5 3 4 5 6 4 4 4

$N = 10$
 $|ME| \geq 6$



Obs 1 Count of ME $>$ Count of all the other elements combined.

$> 50\% \Rightarrow$ Party 1 : $\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}$
Party 2-3 : $\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}$

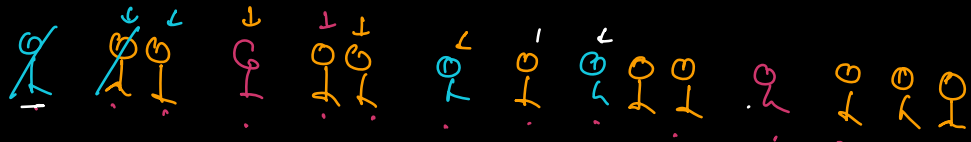
(6) (9) $\rightarrow -2$
 5 7
 (4) 5

Size	ME
<u>N</u>	<u>$\frac{N+1}{2}$</u>
N-2	$\frac{N-1}{2}$
N-4	$\frac{N-3}{2}$

$= \frac{N-3}{2} + 1$
 $= \frac{N-2+2}{2}$
 $= \frac{N}{2}$

If we remove 2 distinct elements from the array \Rightarrow ME will remain the same

Moore's Voting Algo



Winner:

Count:

3, 4, 3, 6, 1, 3, 2, 5, 3, 3, 3

ME: 3

Count:

3 4 3 6 1

ME:

Count:

Google

$$|ME| > \frac{N}{3} \quad \left. \vphantom{\frac{N}{3}} \right\}$$

$$|ME| > \frac{N}{K} \quad \left. \vphantom{\frac{N}{K}} \right\} \underline{(K-1)}$$

(2), (3), (2), (3), (2), (3), 1, (2), (3)

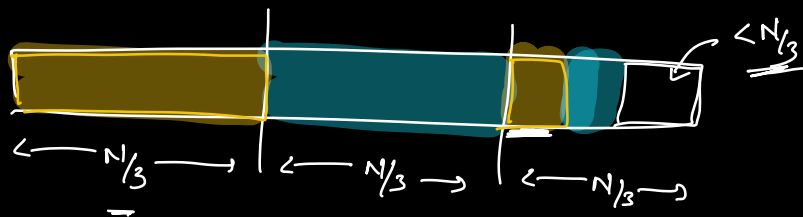
$$N = 9$$

$$|ME| \geq 4$$

$$-2 \rightarrow 4 \rightarrow 3$$

$$-3 \rightarrow 4 \rightarrow 3$$

$$1 \rightarrow 1 \rightarrow 1$$



Size $|ME|$

$$N$$

$$\frac{N}{3} + 1$$

$$N-3$$

$$\frac{N}{3}$$

$$\frac{(N-3)}{3} + 1$$

$$= \frac{N-3+3}{3}$$

$$= \frac{N}{3}$$

If we remove 3 distinct elements then the ME remains the same.

2, 5, 2, 3, 2, 5, 2, 1, 3, 6

ME1: 2

Cont 1: 1 → 2 → 1
 ↓
 2
 ↓
 3 → 2 → 1

ME2: 3 New

Cont 2: 10

2

3

6

Code ⇒ Tricky

-2, 4, 3, 6, 3

ME1: 10

Cont 1: 1

4, 7, 3, 10, 3

ME2: 3

Cont 2: 1

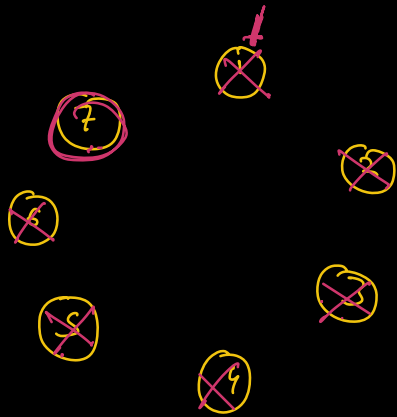
-2, 4, 3
 4, 3, 7
 3, 4, 3

Break till LL.Oop

Q
DeShaw

Josephus

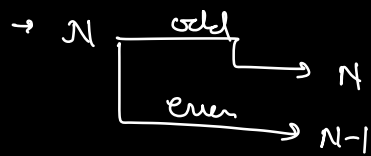
$N = 7$



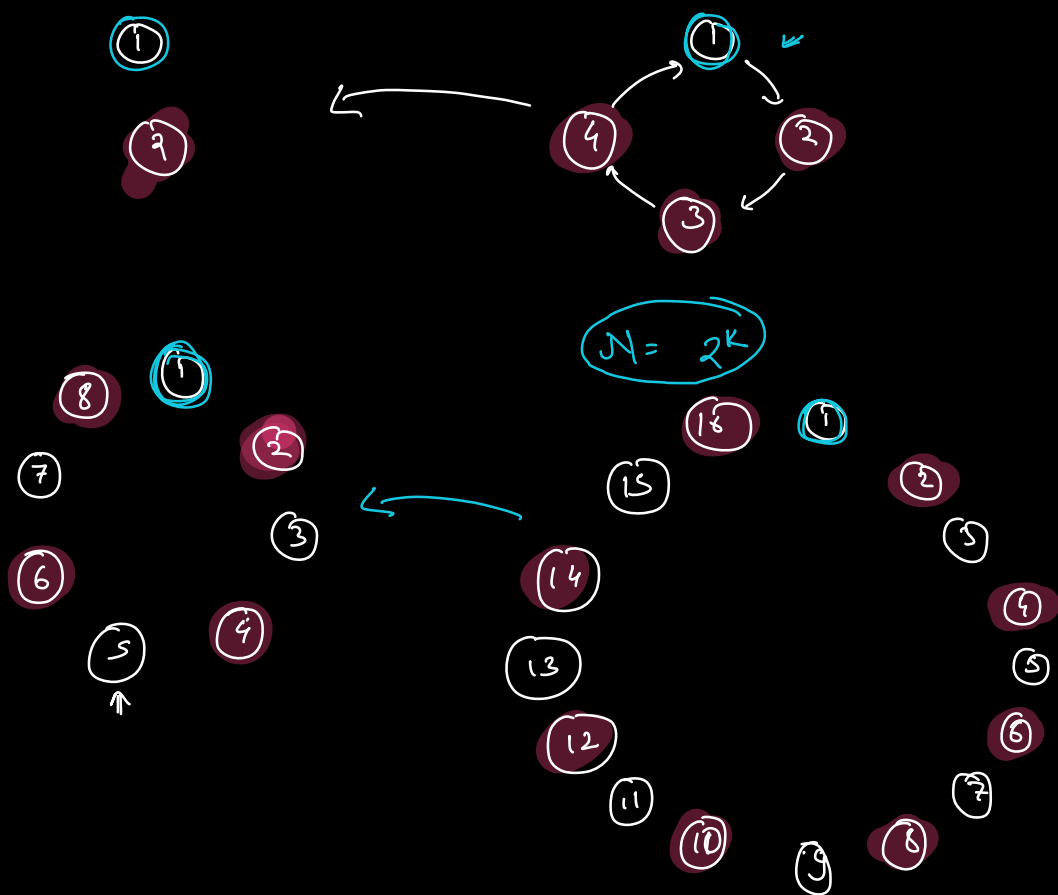
$N = 6$



→ Last odd no.

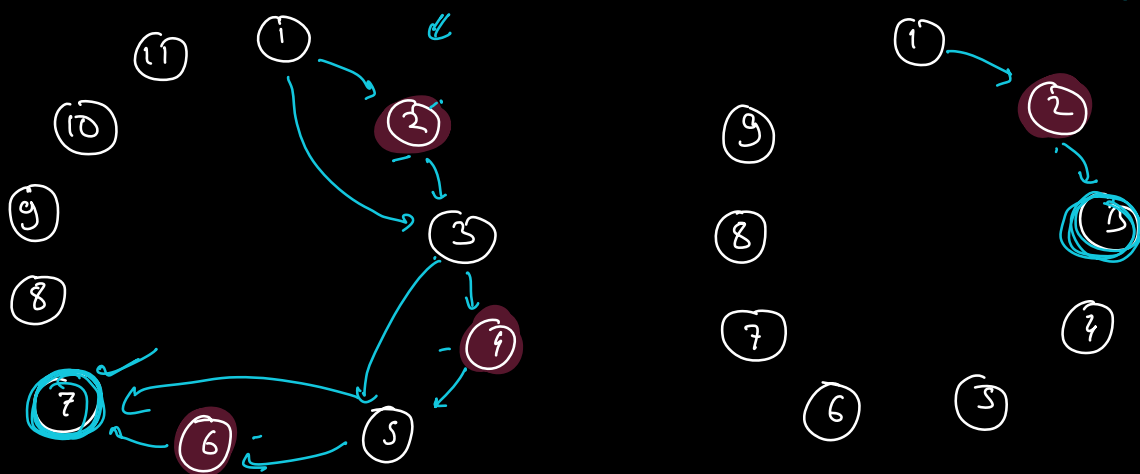


→ Last prime no.



If $N = 2^k$

\Rightarrow Person starting the killings remains alive.



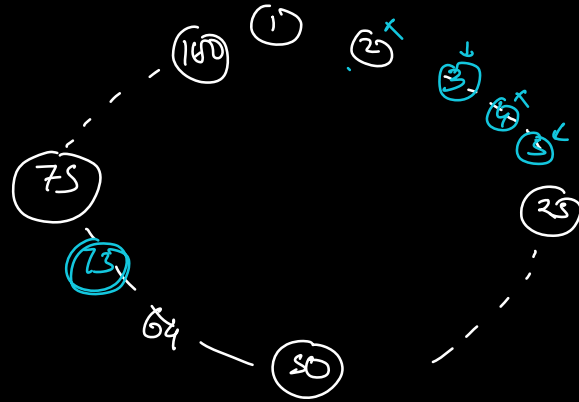
$$N = 100$$

$$\text{Closest Power of } 2 = \underline{\underline{64}}$$

$$100 - 64 \Rightarrow \underline{\underline{36}}$$

$$2 \times 36 + 1$$

$$= \underline{\underline{73}}$$



$$N = 1000;$$

$$1000 = 2^k$$

$$\log_2 1000 = k$$

$$N = 10$$

$$\log_2 2 = 3.14 = 2^3 \Rightarrow \underline{\underline{8}}$$

$$\log_2 100 = 6.6 = 2^6 \Rightarrow \underline{\underline{64}}$$

$$\log_2 50 = 5.46 = 2^5 = \underline{\underline{32}}$$

$$N = 100$$

$$2 \rightarrow 4 \rightarrow 8 \rightarrow 16 \rightarrow 32 \rightarrow \underline{\underline{64}} \rightarrow 128$$

$$O(\log N)$$

$$N = 50$$

$$2 \rightarrow 4 \rightarrow 8 \rightarrow 16 \rightarrow \underline{\underline{32}} \rightarrow 64$$

$$O(\log N)$$

$$2 \times (N - 2^k) + 1$$

Doubts

$$S_{m,N} - S_{m,n} = m$$

$$\begin{array}{r} 1 + 2 + 3 + \dots + m + \cancel{n} \dots \cancel{n} \\ - \quad \cancel{1} + \cancel{2} \dots \cancel{n} + \cancel{n} \dots \cancel{n} \end{array}$$

$$\boxed{(m-n) = S_N - S_{m,n}} \quad (i)$$

$$\begin{array}{r} 1^2 + 2^2 + 3^2 + \dots + m^2 + \cancel{n^2} \dots \cancel{n^2} \\ - \quad \cancel{1^2} + \cancel{2^2} + \dots \cancel{n^2} + \cancel{n^2} \dots \cancel{n^2} \end{array}$$

$$m^2 - n^2 = S_{2N} - S_{2m,n}$$

$$\underline{(m-n)(m+n)} = S_{2N} - S_{2m,n}$$

$$\boxed{(m+n) = \frac{S_{2N} - S_{2m,n}}{S_N - S_{m,n}}} \quad (ii)$$

All problems → Bookmark