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Panel - C

DMGT Problem set -2

1) $A = \{\phi, \{\phi\}, \{\phi\{\phi\}\}\}$
   i) $\phi \in A \rightarrow$ True $\Rightarrow \phi$ is a element in A
   ii) $\{\phi\} \subseteq A \rightarrow$ True $\rightarrow \{\phi\}$ is a ~~subject~~ subset of A
   iii) $\{\phi\} \in A \rightarrow$ True $\rightarrow \{\phi\}$ a element of A
   iv) $\{\phi, \{\phi\}\} \subseteq A \rightarrow$ True $\rightarrow \{\phi, \{\phi\}\}$ is a subset of A
   v) $\{\{\phi\}\} \in A \rightarrow$ False $\rightarrow \{\{\phi\}\}$ is not a element in A

2) $U = \{1,2,3,4,5,6,7,8,9\}$
   $A = \{1,2,4,6,8\}$
   $B = \{2,4,5,9\}$
   $C = \{1,2,3,4\}$
   $D = \{7,8\}$

   i) $A \oplus B = (A \cup B) - (A \cap B)$
   $= \{1,2,4,5,6,8,9\} - \{2,4\}$
   $= \{1,5,6,8,9\}$

   $C \oplus B = (C \cup B) - (C \cap B)$
   $= \{1,2,3,4,5,9\} - \{2,4\}$
   $= \{1,3,5,9\}$

   ii) ① $A - B = \{1,2,4,6,8\} - \{2,4,5,9\}$
   $= \{1,6,8\}$

   ② $B - A = \{2,4,5,9\} - \{1,2,4,6,8\}$
   $= \{5,9\}$

③ $C - D = \{1,2,3,4\} - \{7,8\}$
$= \{1,2,3,4\}$

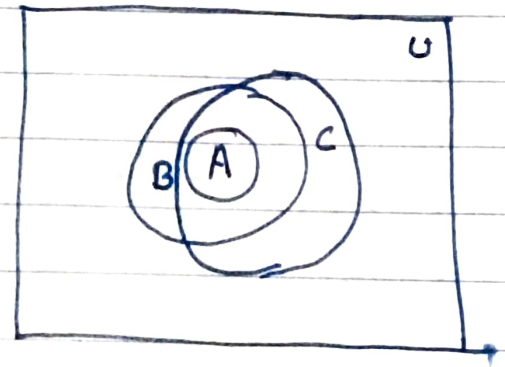
iii) $\overline{(A \cup B)} \rightarrow (A \cup B) = \{1,2,4,5,6,8,9\}$
$\therefore (A \cup B)' = U - (A \cup B)$
$= \{1,2,3,4,5,6,7,8,9\} - \{1,2,4,5,6,8,9\}$
$= \{3,7\}$

$\overline{(A \cap B)} = A \cap B = \{2,4\}$
$(A \cap B)' = U - (A \cap B)$
$= \{1,2,3,4,5,6,7,8,9\} - \{2,4\}$
$= \{1,3,5,6,7,8,9\}$

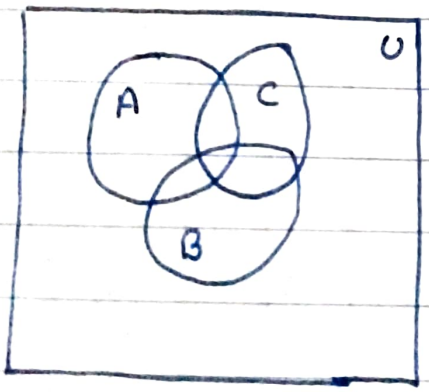
iv) $A \cap (\bar{C} \cup D) \rightarrow \bar{C} \cup D = \{5,6,7,8\}$
$A \cap (\bar{C} \cup D) = \{6,8\}$

5) Venn diagrams:

i)



ii)



7) i) $(\overline{\bar{A} \cup \bar{B}}) \cup (\bar{A} \cap B)$

$= [(\overline{\bar{A} \cup B}) = \bar{A} \cap \bar{B}]$

$(\bar{A} \cap \bar{B}) \cup (\bar{A} \cap \bar{B}) = (\bar{A} \cap (\bar{B} \cup B))$

$[A \cap (B \cup C) = (A \cap B) \cup (A \cap C)]$

$\therefore (\bar{A}) \cap (U) = \bar{A}$

ii) $[(A \cap B) \cup (A \cap \bar{B}) \cup (\bar{A} \cap B)] \cap B$

$=$

Using $(A \cap B) \cup (A \cap C) = A \cap (B \cup C)$

$[(A \cap (\bar{B} \cup B) \cup (\bar{A} \cap B)] \cap B$

$[A \cup (\bar{A} \cap B)] \cap B$

$[A \cup (B - A)] \cap B$

$[(B \cup A) - (A - A)] \cap B$

$= (B \cup A) \cap B$

$= B$

iii) $[(A \cup B) \cap \bar{A}) \cup (\overline{B \cap \bar{A}})$

$(B - A) \cup (\overline{B \cap \bar{A}})$

$(B - A) \cup (\bar{B} \cup \bar{A})$

$U - (A \cap B)$

$(\overline{\bar{A} \cap \bar{B}}) - \bar{A} \cap \bar{B}$

iv) $[\overline{(A \cap B) \cup C}] \cap \bar{B}$

$[\overline{(A \cap B) \cup C}] \cup B$

$(A \cap B) \cup B \cup C = \overline{B \cup C} = \bar{B} \cap \bar{C}$

8]

→ i) $A - B = B$?

→ Not possible, as $A - B$ means the element of A which are not present in B. so, how can $A - B = B$ when it should be the subset of A.

ii) $A - B = B - A$?

→ only possible if set A = set B

iii) $A \oplus B = A$?

→ $(A \cup B) - (A \cap B) = A$

only possible if $B = \{\}$

$A - \phi = A$.

Example 8)

→ $n(U) = 80$

Cobol, $n(cb) = 50$

C, $n(c) = 55$

Pascal, $n(P) = 46$

$n(c \cap cb) = 37$, $n(c \cap P) = 28$, $n(cb \cap P) = 2$

$n(c \cup cb \cup P)' = 7$

$n(c \cup cb \cup P) = 80 - 7 = 73$

i) Find $n(c \cap cb \cap P)$

$|B \cap C \cap P| = |B \cup C \cup P| - (|B| + |C| + |P| - |B \cap C| - |B \cap P| - |C \cap P|)$

$= 73 - 50 - 55 - 46 + 37 + 28 + 25$

$= 12$

ii) Exactly 2;

$|B \cap C \cap \bar{P}| + |B \cap \bar{C} \cap P| + |\bar{B} \cap C \cap P|$

$= (37 - 12) + (25 - 12) + (28 - 12)$

$= 25 + 13 + 16$

$= 54$

iii) Exactly 1,

Exactly cobol + Exactly Pascal + exactly C

$(50 - 37 - 25 + 12) + (55 - 37 - 28 + 12) + (46 - 28 - 25 + 12)$

$= 0 + 2 + 5 = 7$

11]

→ $n(U) = 250$

div. by 3, $n(A)$

div. by 5, $n(B)$

div. by 7, $n(C)$

i) 3 or 5, or 7 $= n(A) + n(B) + n(C) -$ commo

$$n(A) = \left[\frac{250}{3}\right] = 83$$

$$n(B) = \left[\frac{250}{5}\right] = 50$$

$$n(C) = \left[\frac{250}{7}\right] = 35$$

$$n(A \cap B) = \left[\frac{250}{15}\right] = 16$$

$$n(B \cap C) = \left[\frac{250}{35}\right] = 7$$

$$n(A \cap C) = \left[\frac{250}{21}\right] = 11$$

$$n(A \cap B \cap C) = \left[\frac{250}{105}\right] = 2$$

$\therefore n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

$= 83 + 50 + 35 - 16 - 7 - 11 + 2$

$= 13$

ii) 3 or 7 but not 5.

$n(A) + n(C) - n(B \cap C) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$

$= 83 + 35 - 7 - 16 - 11 + 2$

$= 86$

12)

→ $n(U) = 2000$

2, $n(A) = 1000$ [2000/2]

3, $n(B) = 666$ [2000/3]

5, $n(C) = 400$ [2000/5]

7, $n(D) = 285$ [2000/7]

$n(A \cap B) = 333$ [2006/6]

$n(A \cap C) = 200$ [2000/10]

$n(A \cap D) = 142$ [2000/14]

$n(B \cap C) = 133$ [2000/15]

$n(B \cap D) = 95$ [2000/21]

$n(C \cap D) = 57$ [2000/35]

$n(B \cap C \cap D) = 19$ [2000/105]

$n(A \cap B \cap C) = 66$ [2000/30]

$n(A \cap C \cap D) = 28$ [2000/70]

$n(A \cap B \cap C \cap D) = 9$ [2000/10]

$n(A \cap B \cap D) = 47$ [2000/42]

$$n(A \cup B \cup (C \cup D)) = n(A) + n(B) + n(C) + n(D)$$
$$- n(A \cap B) - n(A \cap C) - n(A \cap D)$$
$$- n(B \cap C) - n(B \cap D) - n(C \cap D)$$
$$+ n(A \cap B \cap C) + n(A \cap B \cap D)$$
$$+ n(A \cap C \cap D) + n(B \cap C \cap D)$$
$$- n(A \cap B \cap C \cap D)$$
$$= 1000 + 666 + 400 + 285 - 333 - 200 - 14$$
$$- 133 - 95 - 57 + 66 + 47 + 28 + 1$$
$$- 9$$
$$= 1551 - 9$$
$$= 1542$$

13]

$\longrightarrow$ $n(U) = 119$

D. S., $n(A) = 96$

Fou. $n(B) = 53$

A.L., $n(C) = 39$

$n(A \cap B) = 38$, $n(A \cap B \cap C) = 22$

$n(B \cap C) = 31$

$n(A \cap C) = 32$,

$$|A \cup B \cup C| = 96 + 53 + 39 - 31 - 32 - 38 + 22$$
$$= 109 < 119$$

$\therefore$ Info. not correct.

14]

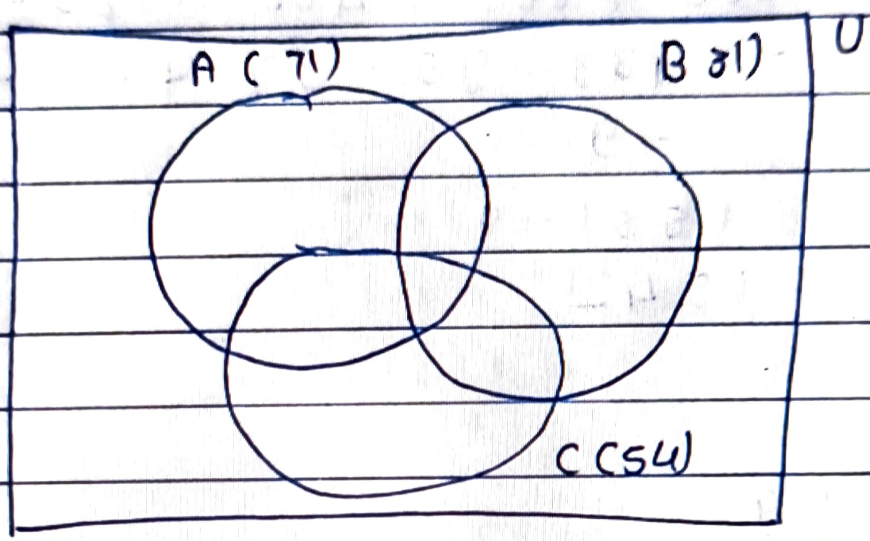
→ $n(U) = 100$

India today, $n(A) = 70$

Fortune, $n(B) = 31$

Business India, $n(C) = 54$

$n(B \cap C) = 0$



15]

→ $|A| = 40$, $|B| = 32$

Available mem. $= 64 - 16 = 48$

$|A \cup B| = |A| + |B| - |A \cap B|$

$|A \cup B| \leq 48$

$|A| + |B| - |A \cap B| \leq 48$

$|A \cap B| \geq |A| + |B| - 48$

$= 40 + 32 - 48 = 24$

∴ Minimum 24 K. required.