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Panel - C

Subject - DMGT

### DMGT Problem Set - 2

1)

$$\rightarrow A = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$$

- i)  $\emptyset \in A \rightarrow$  True  $\Rightarrow \emptyset$  is an element in A.
- ii)  $\{\emptyset\} \subseteq A \rightarrow$  True  $\Rightarrow \{\emptyset\}$  is a subset of A.
- iii)  $\{\emptyset\} \in A \rightarrow$  True  $\Rightarrow \{\emptyset\}$  is an element of A.
- iv)  $\{\emptyset, \{\emptyset\}\} \subseteq A \rightarrow$  True  $\Rightarrow \{\emptyset, \{\emptyset\}\}$  is a subset of A.
- v)  $\{\{\emptyset\}\} \in A \rightarrow$  False  $\Rightarrow \{\{\emptyset\}\}$  is not an element in A.

2)

$$\rightarrow U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\},$$

$$A = \{1, 2, 4, 6, 8\}$$

$$B = \{2, 4, 5, 9\}$$

$$C = \{1, 2, 3, 4\}$$

$$D = \{7, 8\}$$

$$\begin{aligned} \text{i) } A \oplus B &= (A \cup B) - (A \cap B) \\ &= \{1, 2, 4, 5, 6, 8, 9\} - \{2, 4\} \\ &= \{1, 5, 6, 8, 9\} \end{aligned}$$

$$\begin{aligned} C \oplus D &= (C \cup D) - (C \cap D) \\ &= \{1, 2, 3, 4, 7, 8\} - \emptyset \\ &= \{1, 2, 3, 4, 7, 8\} \end{aligned}$$

$$\begin{aligned} \text{ii) } A - B &= \{1, 2, 4, 6, 8\} - \{2, 4, 5, 9\} \\ &= \{1, 6, 8\} \end{aligned}$$

$$\begin{aligned} B - A &= \{2, 4, 5, 9\} - \{1, 2, 4, 6, 8\} \\ &= \{5, 9\} \end{aligned}$$

$$\begin{aligned} C - D &= \{1, 2, 3, 4\} - \{7, 8\} \\ &= \{1, 2, 3, 4\} \end{aligned}$$

$$\begin{aligned} \text{iii) } (\overline{A \cup B}) &\Rightarrow (A \cup B) = \{1, 2, 4, 5, 6, 8, 9\} \\ \therefore (A \cup B)' &= U - (A \cup B) \\ &= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \\ &\quad \{1, 2, 4, 5, 6, 8, 9\} \\ &= \{3, 7\} \end{aligned}$$

$$\begin{aligned} \overline{A \cap B} &\Rightarrow A \cap B = \{2, 4\} \\ \therefore (A \cap B)' &= U - (A \cap B) \\ &= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \\ &\quad \{2, 4\} \end{aligned}$$

$$= \{1, 3, 5, 6, 7, 8, 9\}$$

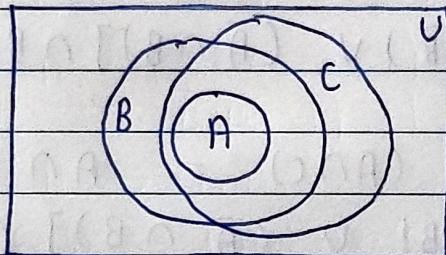
$$\text{iv) } A \cap (\bar{C} \cup D) \Rightarrow \bar{C} \cup D = \{5, 6, 7, 8\}$$

$$\therefore A \cap (\bar{C} \cup D) = \{6, 8\}$$

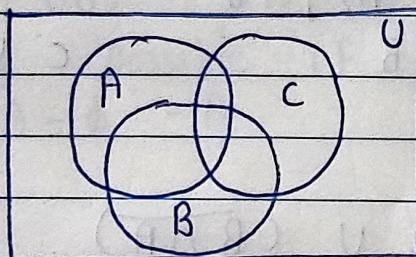
5) Venn diagrams :-



- i) A, B, C are sets such that  $A \subseteq B$ ,  $A \subseteq C$ ,  $(B \cap C) \subseteq A$  and  $A \subseteq (B \cap C)$ .



- ii)  $(A \cap B \cap C) = \emptyset$ ,  $A \cap B \neq \emptyset$ ,  $B \cap C \neq \emptyset$ ,  $A \cap C \neq \emptyset$



7)

$$\rightarrow \text{i) } (\overline{A \cup B}) \cup (\overline{A} \cap B)$$

=

$$[(\overline{A \cup B})] = (\overline{A} \cap \overline{B})$$

$$(\overline{A} \cap \overline{B}) \cup (\overline{A} \cap B) = (\overline{A} \cap (\overline{B} \cup B))$$

$$[(A \cap (\overline{B} \cup C)) = (A \cap B) \cup (A \cap C)]$$

$$\therefore (\overline{A}) \cap (U) = \overline{\overline{A}}$$

$$\text{ii) } [(A \cap B) \cup (A \cap \overline{B}) \cup (\overline{A} \cap B)] \cap B$$

=

$$\text{Using } (A \cap B) \cup (A \cap C) = A \cap (B \cup C)$$

$$[(A \cap (\overline{B} \cup B)) \cup (\overline{A} \cap B)] \cap B$$

$$[A \cap U (\overline{A} \cap B)] \cap B$$

$$[A \cap U (B - A)] \cap B = [(\overline{A} \cap B) = B - A]$$

$$[(B \cup A) - (A - A)] \cap B$$

$$= (B \cup A) \cap B$$

$$= B$$

$$C \cup (B - A) = (B \cup C) - (A - C)$$

$$\text{iii) } [(A \cup B) \cap \overline{A}] \cup (\overline{B} \cap \overline{A})$$

$$\rightarrow (B - A) \cup (\overline{B} \cap \overline{A})$$

$$(B - A) \cup (\overline{B} \cup \overline{A})$$

$$\cup - (A \cap B)$$

$$(\overline{A \cap B}) = \overline{A} \cup \overline{B}$$

iv)  $\overline{[(A \cap B) \cup C]} \cap \overline{B}$

$$\overline{[(A \cap B) \cup C] \cup B} = \overline{\overline{B} \cup C} = \overline{B} \cap \overline{C}$$

8]

→ i)  $A - B = B$  ?

→ Not possible, as  $A - B$  means the element of A which are not present in B. so, how can  $A - B = B$  when it should be the subset of A.

ii)  $A - B = B - A$  ?

→ only possible if set A = set B

iii)  $A \oplus B = A$  ?

→  $(A \cup B) - (A \cap B) = A$

only possible if  $B = \{\}$

$A - \emptyset = A$ .

Example 8)

→  $n(U) = 80$

coboL,  $n(Cb) = 50$

C,  $n(C) = 55$

Pascal,  $n(P) = 46$

$n(C \cap Cb) = 37$ ,  $n(C \cap P) = 28$ ,  $n(Cb \cap P) = 25$

$n(C \cup Cb \cup P)' = 7$

$$n(C \cup \bar{C} \cap B \cup P) = 80 - 7 = 73$$

i) Find  $n(C \cap C \cap B \cap P)$

$$\begin{aligned} |B \cap C \cap P| &= |B \cup C \cup P| - (|B| + |C| + |P|) \\ &\quad - |B \cap C| - |B \cap P| - |C \cap P| \\ &= 73 - 50 - 55 - 46 + 37 + 28 + 25 \\ &= 12 \end{aligned}$$

ii) Exactly 2,

$$\begin{aligned} |B \cap C \cap \bar{P}| + |B \cap \bar{C} \cap P| + |\bar{B} \cap C \cap P| \\ = (37 - 12) + (25 - 12) + (28 - 12) \\ = 25 + 13 + 16 \end{aligned}$$

iii) Exactly 1,

$$\begin{aligned} \text{Exactly cobol} + \text{Exactly Pascal} + \text{Exactly C} \\ (50 - 37 - 25 + 12) + (55 - 37 - 28 + 12) \\ + (46 - 28 - 25 + 12) \\ = 0 + 2 + 5 = 7 \end{aligned}$$

11]

$$\rightarrow n(U) = 250$$

div. by 3,  $n(A)$

div. by 5,  $n(B)$

div. by 7,  $n(C)$

$$i) 3 \text{ or } 5, \text{ or } 7 = n(A) + n(B) + n(C) - \text{common}$$

$$n(A) = \left[ \frac{250}{3} \right] = 83$$

$$n(B) = \left[ \frac{250}{5} \right] = 50$$

$$n(C) = \left[ \frac{250}{7} \right] = 35$$

$$n(A \cap B) = \left[ \frac{250}{15} \right] = 16$$

$$n(B \cap C) = \left[ \frac{250}{35} \right] = 7$$

$$n(A \cap C) = \left[ \frac{250}{21} \right] = 11$$

$$n(A \cap B \cap C) = \left[ \frac{250}{105} \right] = 2$$

$$\therefore n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) \\ + n(A \cap B \cap C)$$

$$= 83 + 50 + 35 - 16 - 7 - 11 + 2$$

$$= 13$$

ii) 3 or 7 but not 5.

$$\begin{aligned}
 n(A) + n(C) - n(B \cap C) - n(A \cap B) - n(A \cap C) \\
 + n(A \cap B \cap C) \\
 = 83 + 35 - 7 - 16 - 11 + 2 \\
 = 86
 \end{aligned}$$

(2)

$$\rightarrow n(U) = 2000$$

$$2, n(A) = 1000$$

[ 2000 / 2 ]

$$3, n(B) = 666$$

[ 2000 / 3 ]

$$53, n(C) = 400$$

[ 2000 / 5 ]

$$7, n(D) = 285$$

[ 2000 / 7 ]

$$n(A \cap B) = 333$$

[ 2000 / 6 ]

$$n(A \cap C) = 200$$

[ 2000 / 10 ]

$$n(A \cap D) = 142$$

[ 2000 / 14 ]

$$n(B \cap C) = 133$$

[ 2000 / 15 ]

$$n(B \cap D) = 95$$

[ 2000 / 21 ]

$$n(C \cap D) = 57$$

[ 2000 / 35 ]

$$n(B \cap C \cap D) = 19$$

[ 2000 / 105 ]

$$n(A \cap B \cap C) = 66$$

[ 2000 / 80 ]

$$n(A \cap C \cap D) = 28$$

[ 2000 / 47 ] 70 ]

$$n(A \cap B \cap C \cap D) = 9$$

[ 2000 / 10 ]

$$n(A \cap B \cap D) = 47$$

[ 2000 / 42 ]

$$\begin{aligned}
n(A \cup B \cup C \cup D) &= n(A) + n(B) + n(C) + n(D) \\
&\quad - n(A \cap B) - n(A \cap C) - n(A \cap D) \\
&\quad - n(B \cap C) - n(B \cap D) - n(C \cap D) \\
&\quad + n(A \cap B \cap C) + n(A \cap B \cap D) \\
&\quad + n(A \cap C \cap D) + n(B \cap C \cap D) \\
&\quad - n(A \cap B \cap C \cap D) \\
&= 1000 + 666 + 400 + 285 - 333 - 200 - 142 \\
&\quad - 133 - 95 - 57 + 66 + 47 + 28 + 19 \\
&\quad - 9 \\
&= 1551 - 9 \\
&= 1542
\end{aligned}$$

13]

$$\rightarrow n(V) = 119$$

$$\text{D.S.}, \quad n(A) = 96$$

$$\text{Fou.} \quad n(B) = 53$$

$$\text{A.L.}, \quad n(C) = 39$$

$$n(A \cap B) = 38, \quad n(A \cap B \cap C) = 22$$

$$n(B \cap C) = 31$$

$$n(A \cap C) = 32,$$

$$\begin{aligned}
|A \cup B \cup C| &= 96 + 53 + 39 - 31 - 32 - 38 + 22 \\
&= 109 < 119
\end{aligned}$$

$\therefore$  Info. not correct.

14]

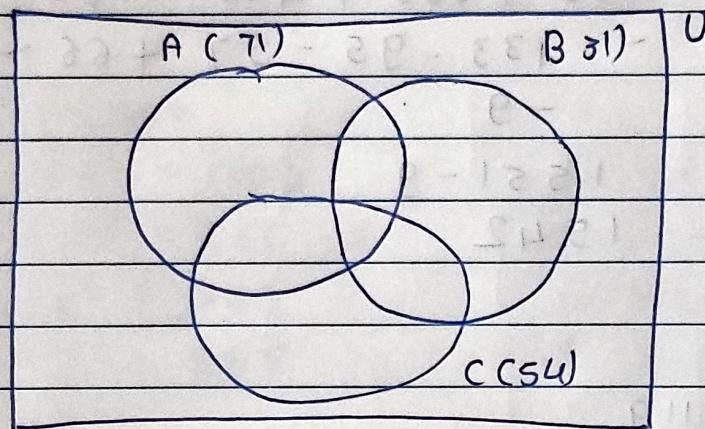
$$\rightarrow n(W) = 100$$

India today,  $n(A) = 70$

Fortune,  $n(B) = 31$

Business India,  $n(C) = 54$

$$n(B \cap C) = 0$$



15]

$$\rightarrow |A| = 40, |B| = 32$$

$$\text{Available mem.} = 64 - 16 = 48$$

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$|A \cup B| \leq 48$$

$$|A| + |B| - |A \cap B| \leq 48$$

$$|A \cap B| \geq |A| + |B| - 48$$

$$= 40 + 32 - 48 = 24$$

$\therefore$  Minimum 24 k. required.