Knapsack 0/1

Q) Obtain the optimal solⁿ to the knapsack problem $n = 3$, $m = 20$
$(P_1, P_2, P_3) = (25, 24, 15)$ &
$(w_1, w_2, w_3) = (18, 15, 10)$.

Sol^n Given that $n=3$ & m = capacity = 20.

i	P[i]	W[i]	P[i]/W[i]
1	25	18	25/18 = 1.3
2	24	15	24/15 = 1.6
3	15	10	15/10 = 1.5

As we want $P[i]/w[i] \geq P[i+1]/w[i+1]$.
∴ Rearrange the items.

i	P[i]	W[i]	P[i]/W[i]
1	24	15	1.6
2	15	10	1.5
3	25	18	1.3

Step 1 Here we will trace knapsack backtracking algo.
Initially set final-profit = −1.
∴ BK(1, 0, 0) K=1, CP=0 CW=0
check if (CW + W[K] <= m)
i.e. if (0 + W[1] = 0+15 <= 20) → yes.
∴ set temp[1] = 1.

if (K < n) i.e. if (1 < 3) → yes

Left child ∴ BK(K+1, cp+p[K], cw + w[K]) is called

∴ Hence BK(2, 0+24, 0+15) will be called. — line 14 BK

step2 K = 2 cp = 24 cw = 15

check if(cw + w[K] <= m)
i.e. if (15 + w[2] = 15+10 <= 20) → no.

Right child ∴ Hence we will calculate ub → (upper bound).

step3. calculating upper bound —
initially set
ub = cp = 24
c = cw = 15 — line 8 & 9 — BC
for(i = K+1 to n) we will update upper bound value

consider i = k+1 = 2+1 = 3.
c = c + w[i] = 15 + w(3)
= 15 + 18
= 33

As 33 > 20 i.e. exceeding the knapack capacity, ∴ will compute ub as

ub = ub + (1 - (c-m)/w[i]) * p[i] — li 17

$\therefore ub = 24 + (1-(15-20)/w[3]) * p[3]$
$= 24 + (1+5/18) * 25$
$= \cancel{24} + \frac{\cancel{25}}{\cancel{3}} = 32.33$

$= \cancel{24+8}$

As (ub > final-profit)
i.e. $32.33 > -1$
$\therefore$ set temp[2] = 0

refer line 27 of BK, a recursive call with K=3 is made.

step 4 $K = 3$
$cp = 24 + p[2] = 24+15 = 39$
LC. $cw = 15 + w[2] = 15+10 = 25$
check if $(cw + w[K] <= m)$

i.e. if $(25 + w[3] = 25+18 <= 20) \to$ no.

Hence calculate upper bound.

step 5 Calculating UB initially set
$ub = cp = 39$
RC. $c = cw = 25$ } line 8 & 9 BC

set $i = K+1 \Rightarrow 3+1 = 4$
But $i = 4 > n$
Hence we will keep ub as ub = 39.

As (39 > final profit)
i.e. 39 > −1

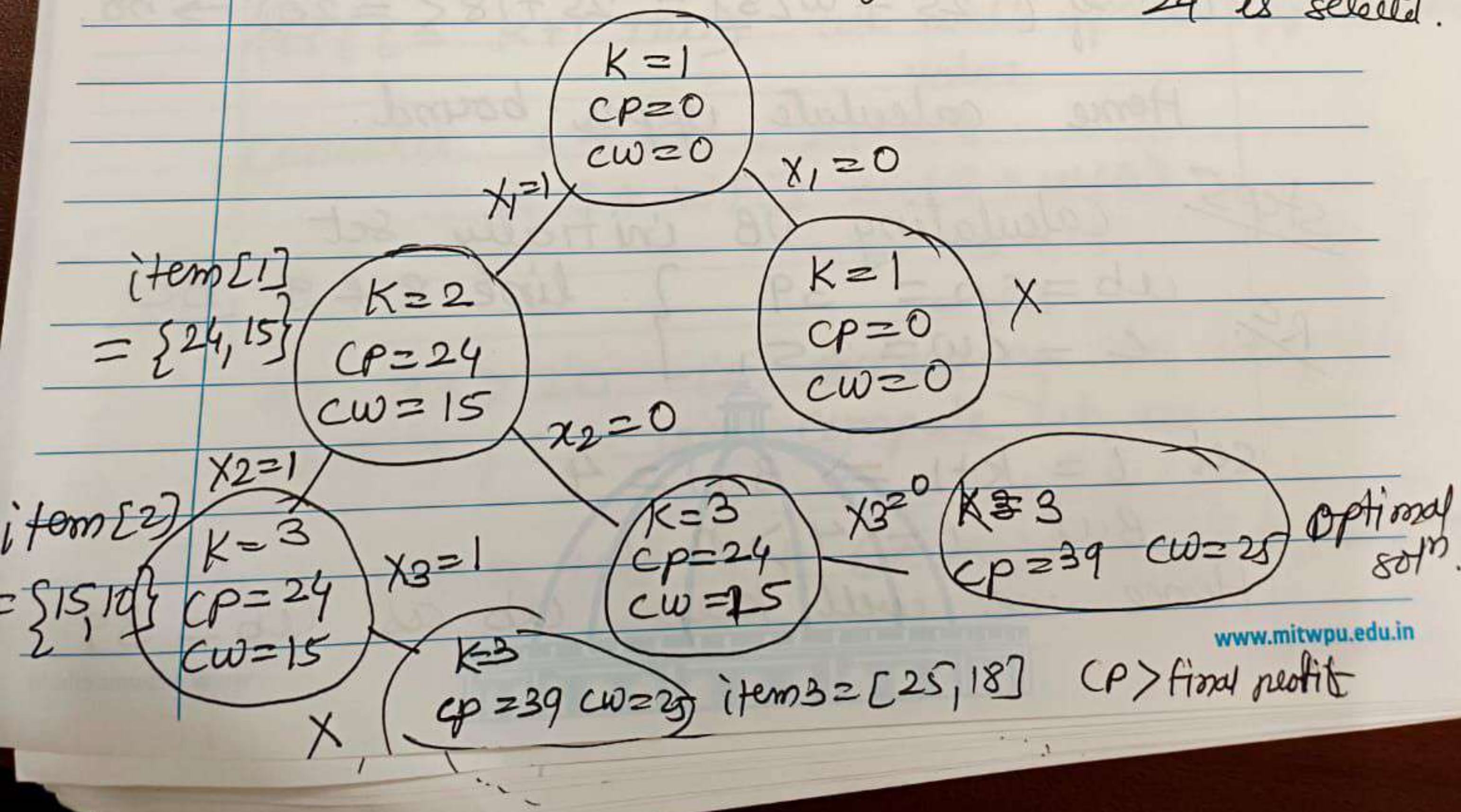
∴ set temp[k] = temp[3] = 0.

Now k = n = 3 & cp = 39 > final_profit
> −1.

∴ final profit = cp = 39 } line 30 & 31
final wt = cw = 25 } BK

copy all contents of temp to an array
X[] ⟶ line 32, 33 of BK.

∴ X = {1, 0, 0}
i.e.
item 1 with weight = 15 & profit = 24 is selected.



Here the left branch indicates inclusion of item & right branch indicates exclusion of items.

Hence $x_1 = 1$ means 1^{st} item selected

$\left.\begin{matrix} x_2 = 0 \\ x_3 = 0 \end{matrix}\right\}$ items not selected.

The state space tree can be created in DFS manner.

Ex 2. Consider the following instance for knapsack problem using backtracking –

$n = 8$

$P = \{11, 21, 31, 33, 43, 53, 55, 65\}$

$w = \{1, 11, 21, 23, 33, 43, 45, 55\}$

$m = 110.$

Sol^n. we will arrange all the items $\frac{P_1}{w_1} > \frac{P_{i+1}}{W_{i+1}} > \frac{P_{i+2}}{W_{i+2}} > \dots$

items	Pi	wi	Pi/wi
1	11	1	11
2	21	11	1.9
3	31	21	1.47
4	33	23	1.43
5	43	33	1.30
6	53	43	1.23
7	55	45	1.22
8	65	55	1.18

let $m = 110$

initially total-profit $= -1$

$cp = 0$, $cw = 0$, let $k = 0$ $\quad P[1] = 11 \quad w[1] = 1$

$cp = cp + 11 = 0 + 11 = 11$

$cw = cw + 1 = 0 + 1 = 1 < m$ ∴ select item 1

$K = 1 \quad P[2] = 21 \quad w[2] = 11$

$cp = 11 + 21 = 32$

$cw = 1 + 11 = 12 < 110$ ∴ select item 2

$K = 2 \quad P[3] = 31 \quad w[3] = 21$

$cp = 32 + 31 = 63$

$cw = 12 + 21 = 33 < 110$ select item 3

$K = 3 \quad P[4] = 33 \quad w[4] = 23$

∴ $cp = 63 + 33 = 96$

$cw = 33 + 23 = 56 < 110$ select item $= 4$

$K = 4 \quad P[5] = 43 \quad w[5] = 33$

$cp = 96 + 43 = 139$

$cw = 56 + 33 = 89 < 110$ ∴ select item $= 5$

$K = 5 \quad P[6] = 53 \quad w[6] = 43$

∴ $cp = 139 + 53 = 192$

$cw = 89 + 43 = 132 > 110$ ∴ not to select item 6.

Thus $\text{upperbound} = 11 + 21 + 31 + 33 + 43 + \left(\frac{110 - 89}{43}\right) * 53$

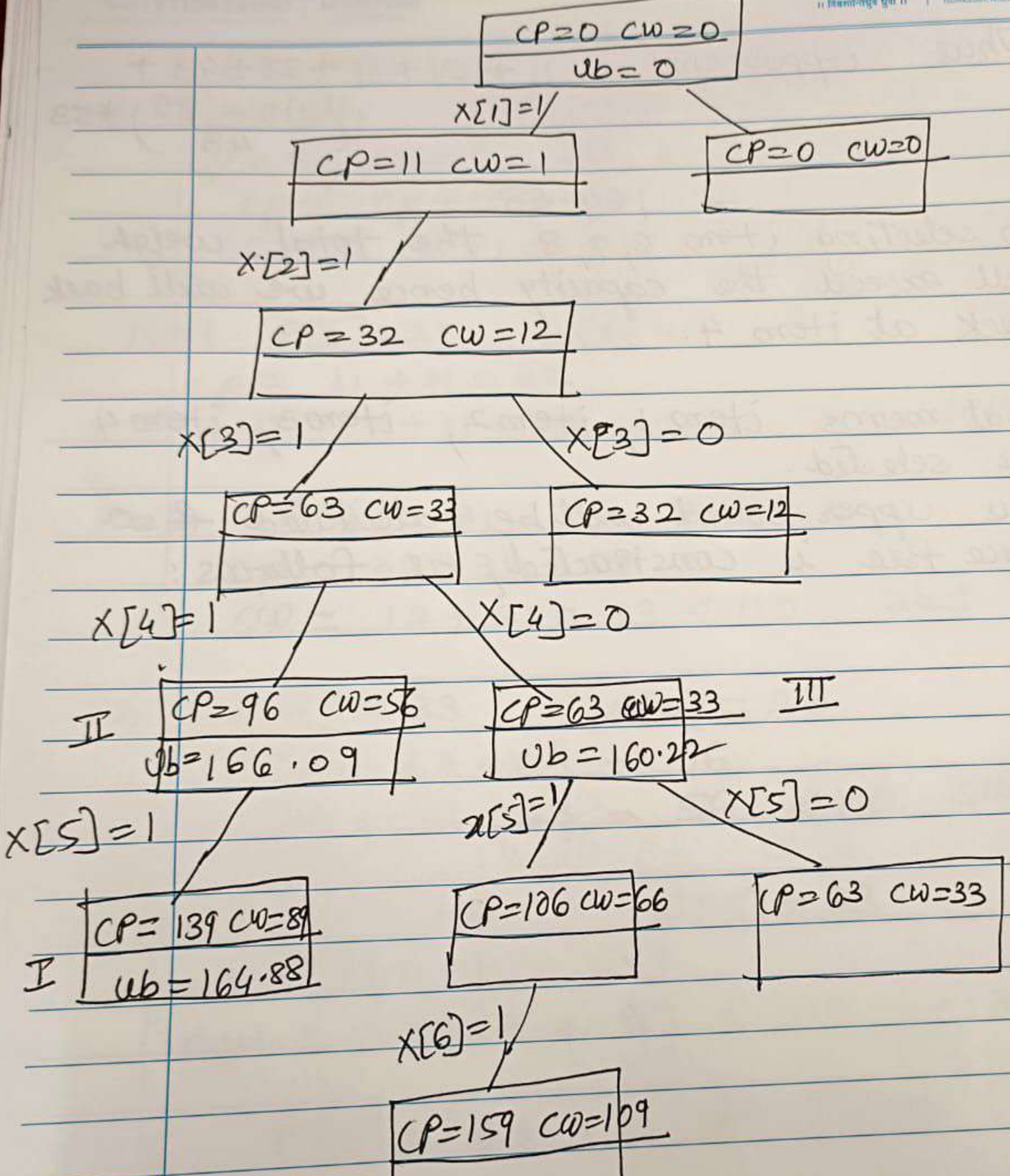
$= 164.88$

on selecting item 6, 7, 8 the total weigh will exceed the capacity hence we will back track at item 4.

That means item 1, item 2, item 3, item 4 are selected.

Thus upper bound will be calculated & a space tree is constructed, as follows:

PTO



sol^n

∴ sol^n give max. profit by selection of items = 1, 2, 3, 5, 6.

Computation at

node I $\Rightarrow$ $ub = cp + \left(\frac{m-cw}{w_{i+1}}\right) * p_{i+1}$

$= 11 + 21 + 31 + 33 + 43 + \left(\frac{110-89}{43}\right) * 55$

$= 164.88$

node II $ub = 11 + 21 + 31 + 33 + \left(\frac{110-56}{33}\right) * 43$

$= 166.09$

node III

$ub = 11 + 21 + 31 + 43 + 53 + \left(\frac{110-109}{45}\right) * 55$

$= 160.22$