MIT-WPU

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MIT WORLD PEACE UNIVERSITY | PUNE
TECHNOLOGY, RESEARCH, SOCIAL INNOVATION & PARTNERSHIPS

CET2001B Advanced data Structure

S. Y. B. Tech CSE

Semester - IV

SCHOOL OF COMPUTER ENGINEERING AND TECHNOLOGY

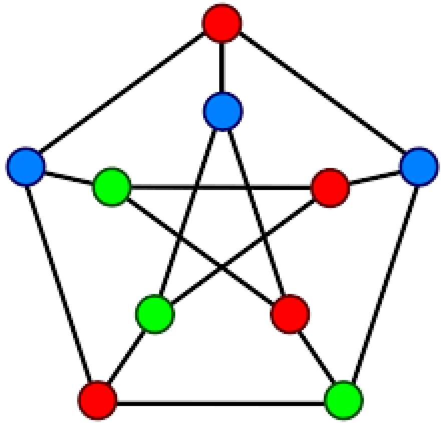
Graph

Graph- Basic Terminology, memory representation**:** Adjacency matrix, Adjacency list, Creation of Graph and Traversals,

Minimum spanning Tree- Prim’s and Kruskal’s Algorithms, Dikjtra's Single source shortest path, Topological sorting

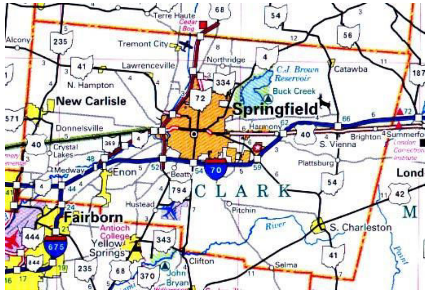
Graph

- Basic Terminology
- Memory representation
- Creation of graph and traversals
- Minimum spanning tree
- Topological sorting

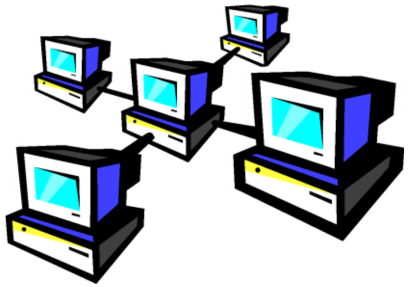


Graph Applications

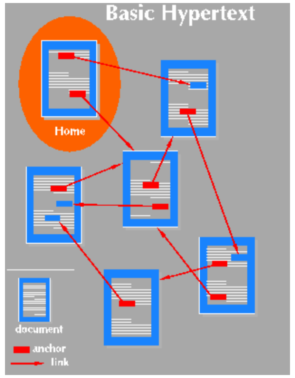
Social Network



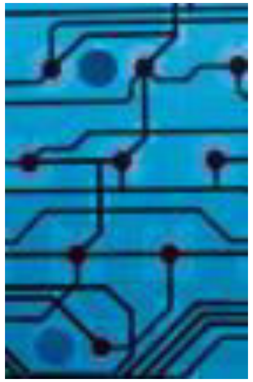
Maps



Computer Network



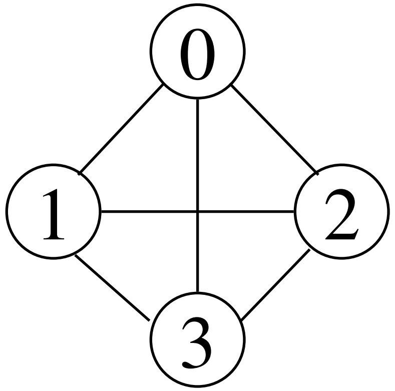
Hypertext



Circuits

Definition

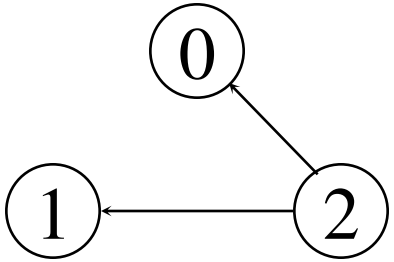
- A graph G consists of two sets
  - □a finite, nonempty set of vertices V(G)
  - □a finite, possible empty set of edges E(G)
  - □G(V,E) represents a graph



Graph G_1

Vertex Set: V(G1)={0,1,2,3}

Edge Set: E(G1)={(0,1),(0,2),(0,3),(1,2),(1,3),(2,3)}



Graph G2

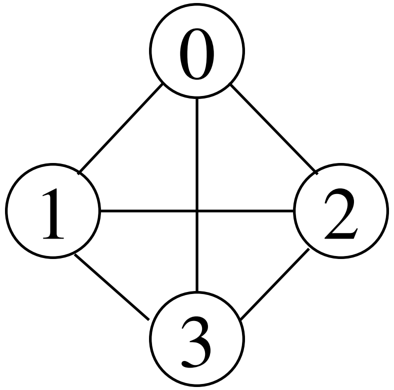
Vertex Set: V(G2)={0,1,2}

Edge Set: E(G2)={(2,0),(2,1)}

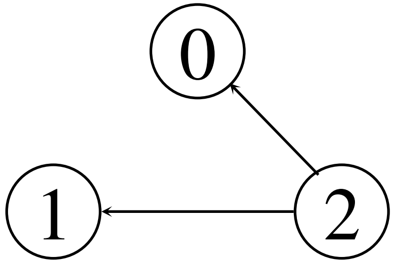
Directed and Undirected Graph

- An undirected graph is one in which the pair of vertices in an edge is unordered, (v0, v1) = (v1,v0)
- A directed graph is one in which each edge is a directed pair of vertices,

  <v0, v1> != <v1,v0>



Undirected Graph



Directed Graph

Degree

- The degree of a vertex is the number of edges incident to that vertex.
- For directed graph,

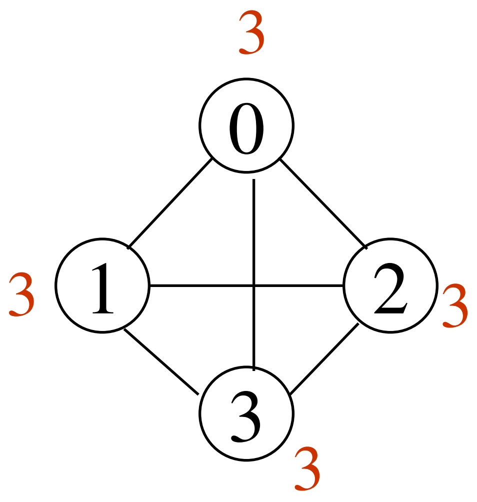
□the in-degree of a vertex *v* is the number of edges that have *v* as the head

□the out-degree of a vertex *v* is the number of edges that have *v* as the tail

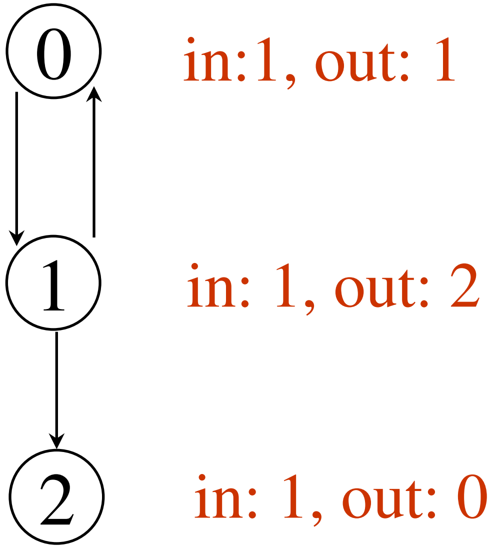
□If d_i is the degree of a vertex *i* in a graph *G* with *n* vertices and *e* edges, the number of edges (of undirected graph) are :-

$$e = \left(\sum_{i=0}^{n-1} d_i \right) / 2$$

Examples for Degree



Undirected Graph : G1



Directed Graph: G3

Adjacent and Incident

- If (v0, v1) is an edge in an undirected graph,
  - □ v0 and v1 are adjacent
  - □The edge (v0, v1) is incident on vertices v0 and v1

- If <v0, v1> is an edge in a directed graph
  - □v0 is adjacent to v1, and v1 is adjacent from v0
  - □The edge <v0, v1> is incident on v0 and v1

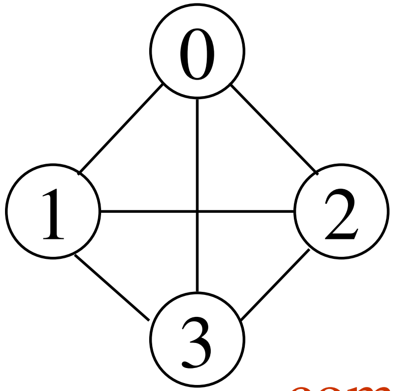
Complete graph

- A complete graph is a graph that has the maximum number of edges

□for undirected graph with n vertices, the maximum number of edges is n(n-1)/2

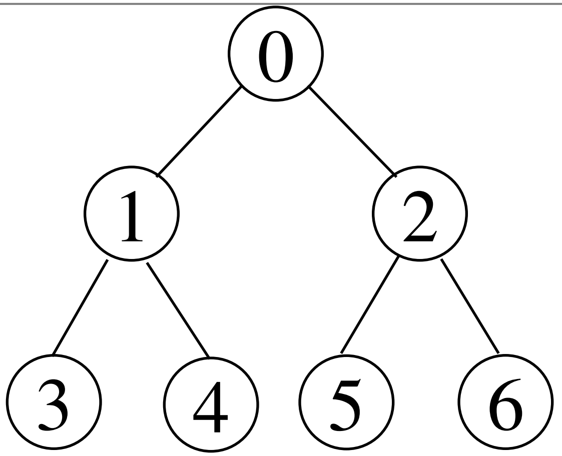
□for directed graph with n vertices, the maximum number of edges is n(n-1)

Examples for Graph

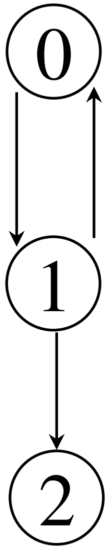


complete graph

G_1



G_2 incomplete graph



G_3

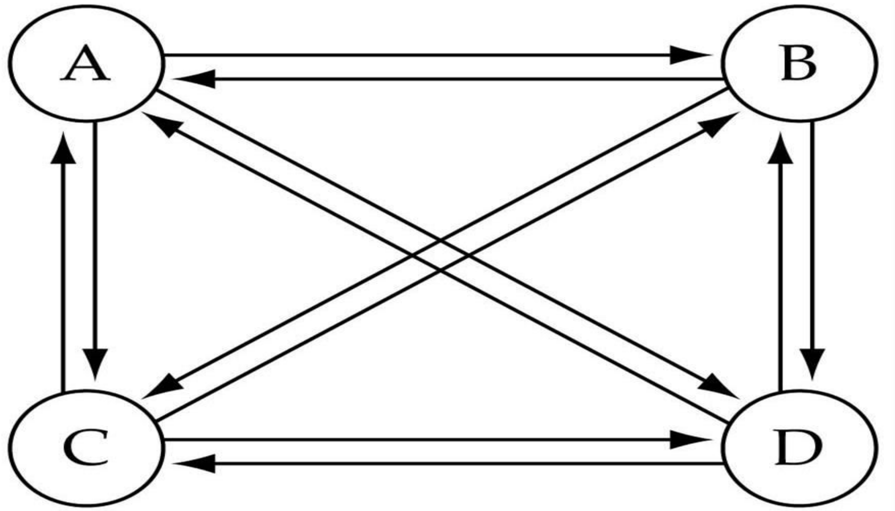
Directed graph

$V(G_1)=\{0,1,2,3\}$ $E(G_1)=\{(0,1),(0,2),(0,3),(1,2),(1,3),(2,3)\}$

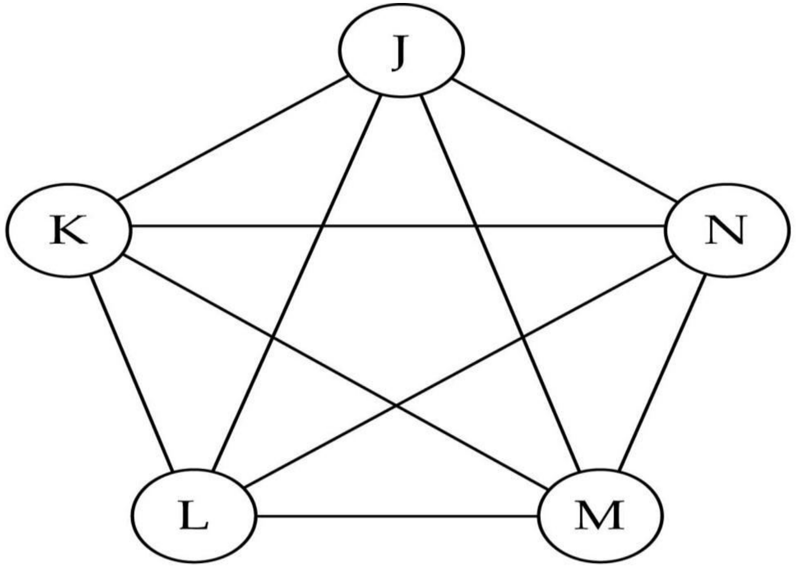
$V(G_2)=\{0,1,2,3,4,5,6\}$ $E(G_2)=\{(0,1),(0,2),(1,3),(1,4),(2,5),(2,6)\}$

$V(G_3)=\{0,1,2\}$ $E(G_3)=\{<0,1>,<1,0>,<1,2>\}$

Complete Graph



(a) Complete directed graph.



(b) Complete undirected graph.

No. of edges (complete undirected graph) : n(n-1)/2
No. of edges (complete directed graph): n(n-1)

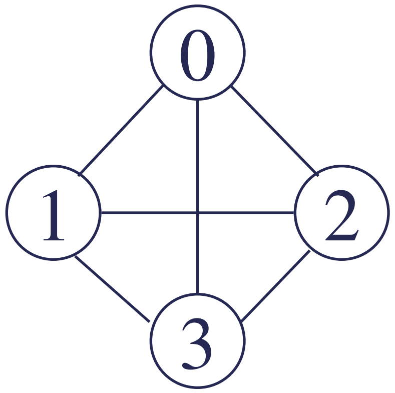
Subgraph and Path

- A subgraph of G is a graph G’ such that V(G’) is a subset of V(G) and E(G’) is a subset of E(G)

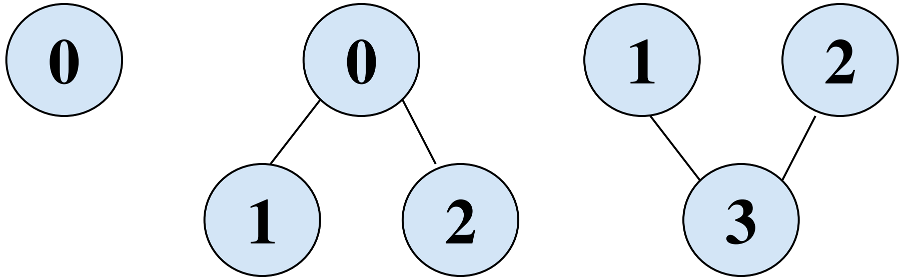
- A path from vertex v_p to vertex v_q in a graph G, is a sequence of vertices, v_p, v_{i1}, v_{i2}, ..., v_{in}, v_q, such that (v_p, v_{i1}), (v_{i1}, v_{i2}), ..., (v_{in}, v_q) are edges in an undirected graph

- The length of a path is the number of edges on it

Example for Subgraph

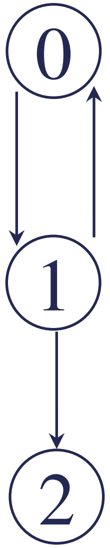


G_1

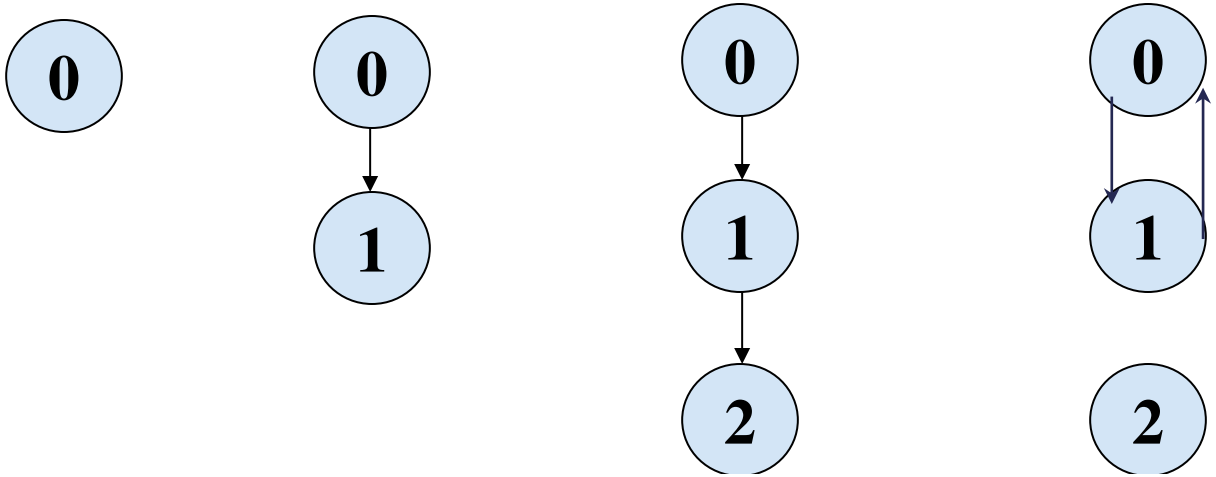


(i) (ii) (iii)

(a) Some of the subgraph of G_1



G_3



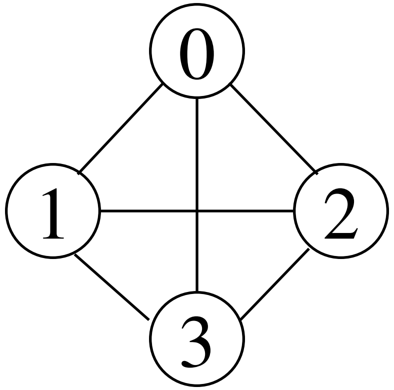
(i) (ii) (iii) (iv)

(b) Some of the subgraph of G_3

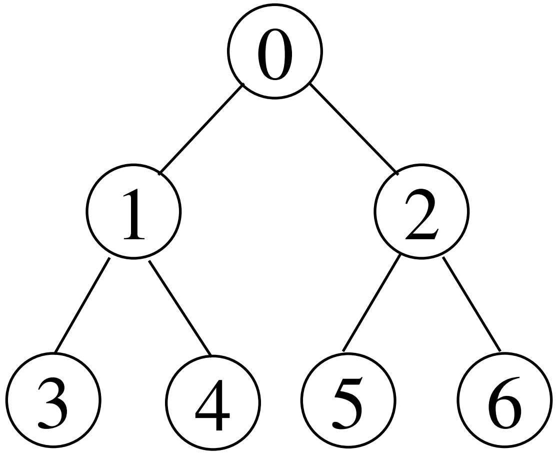
Simple path and cycle

- A simple path is a path in which all the vertices, are distinct.
- A cycle is a path, in which the first and the last vertices are same.
- In an undirected graph G, two vertices, v0 and v1, are connected if there is a path in G from v0 to v1
- An undirected graph is connected if, for every pair of distinct vertices vi, vj, there is a path from vi to vj.

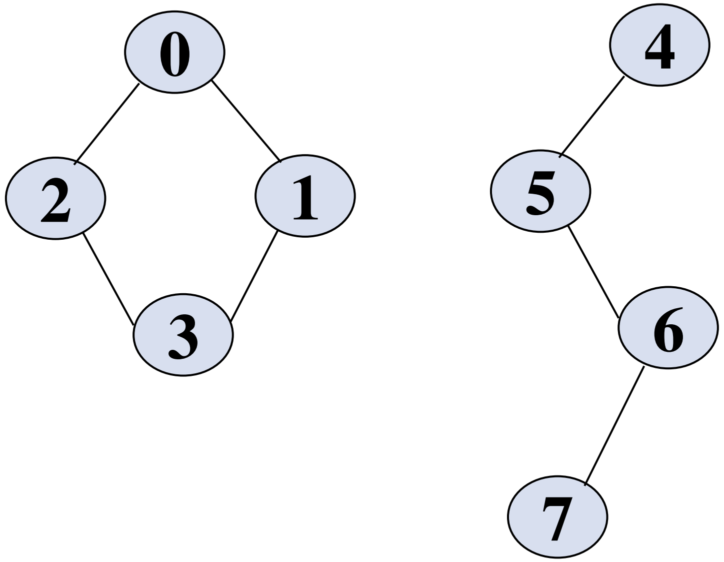
Examples for Graph



G_1



G_2



G_3

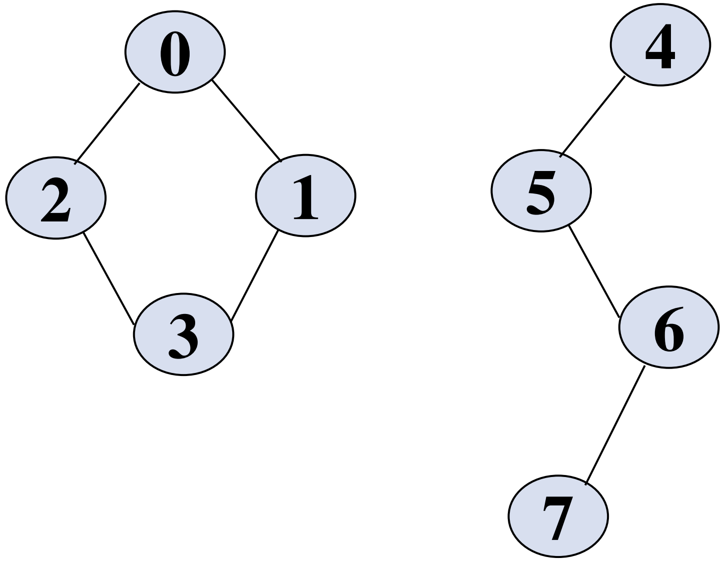
Connected Graphs: G_1 ,G_2

Graph G_3 : (not connected)

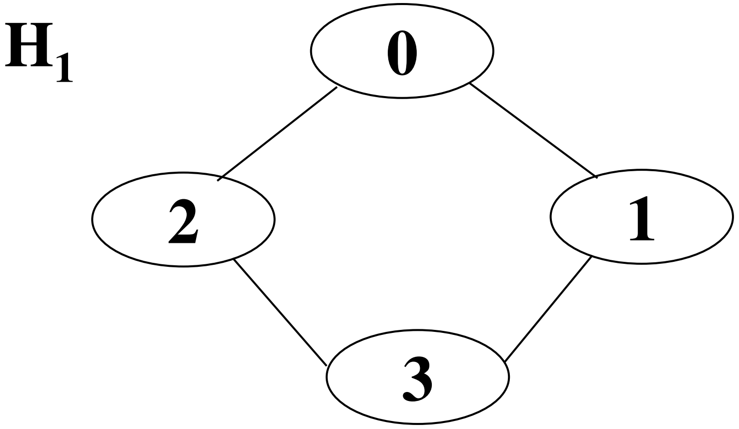
Connected Component

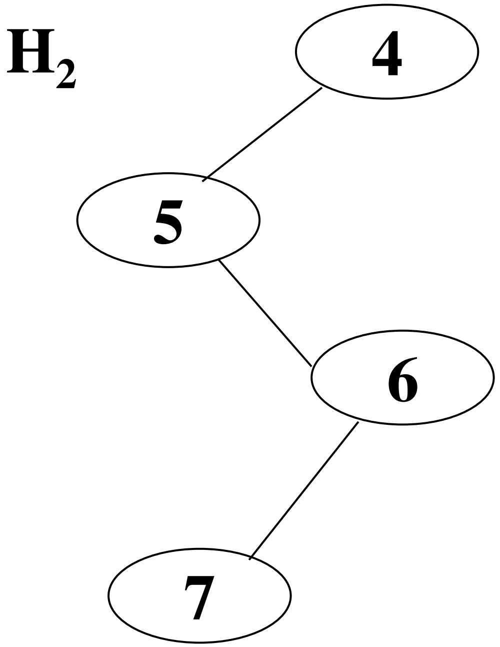
- A connected component of an undirected graph is a maximal connected subgraph.
- A tree is a graph that is connected and acyclic.
- A directed graph is strongly connected if there is a directed path from vi to vj and also from vj to vi.
- A strongly connected component is a maximal subgraph that is strongly connected.

Examples for Connected Component



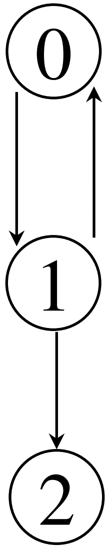
Graph G_4



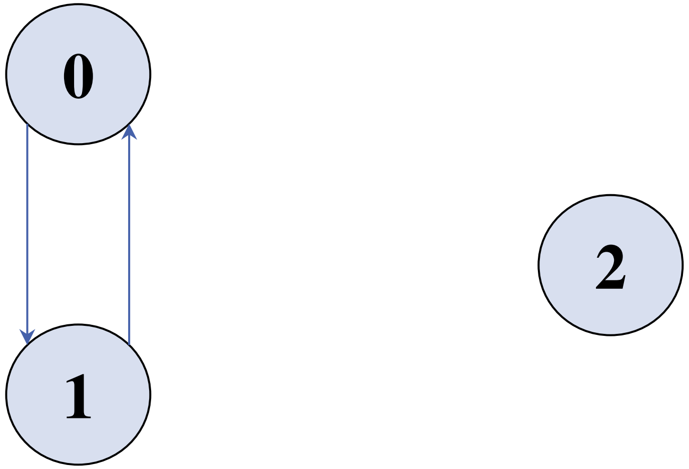


Two Connected Components for Graphs G_4: H_1 and H_2

Examples for Strongly Connected Component



G3 (Not strongly connected)



Strongly connected components of G_3

Graph Representation

- Adjacency Matrix
- Adjacency Lists

Adjacency Matrix

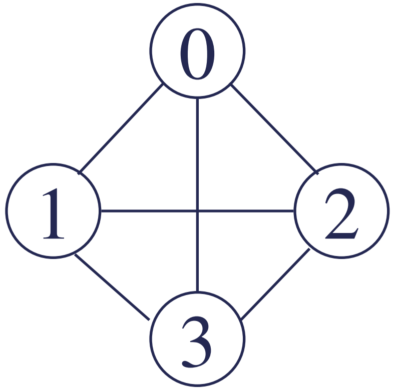
- Let G=(V,E) be a graph with n vertices.
- The adjacency matrix of G is a two-dimensional n x n array, say adj_mat

□If the edge (vi, vj) is in E(G), adj_mat[i][j]=1

□If there is no such edge in E(G), adj_mat[i][j]=0

- The adjacency matrix for an undirected graph is symmetric; the adjacency matrix for a digraph need not be symmetric

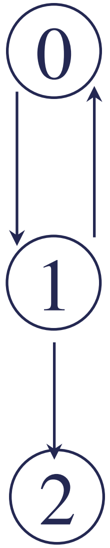
Adjacency Matrix



Graph G_1

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Adjacency Matrix for Graph G_1



Graph G_2

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Adjacency Matrix for Graph G_2

Merits: Adjacency Matrix

- From the adjacency matrix, to determine the connection of vertices is easy
- The degree of a vertex is $\sum_{j=0}^{n-1} adj\_mat[i][j]$
- For a digraph, the row sum is the out_degree, while the column sum is the in_degree

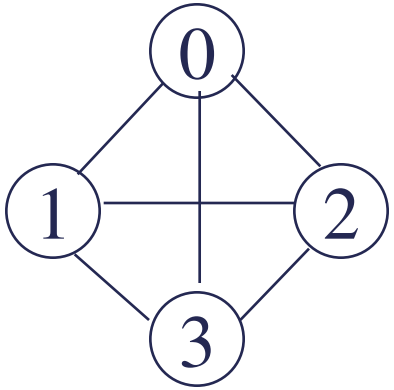
$$ind(vi) = \sum_{j=0}^{n-1} A[j,i] \qquad outd(vi) = \sum_{j=0}^{n-1} A[i,j]$$

Adjacency List: Interesting Operations

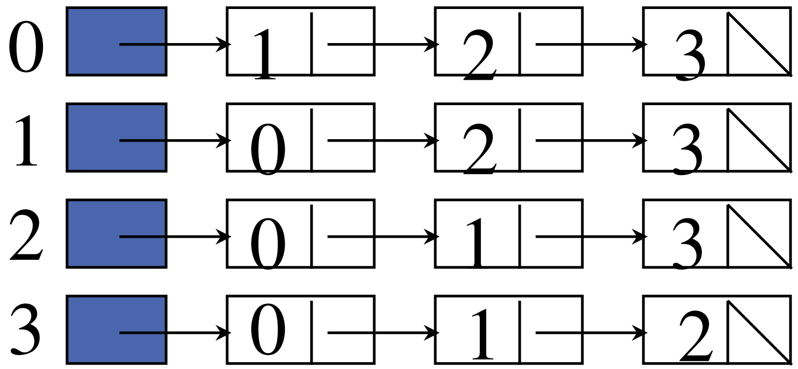
- The degree of any vertex in an undirected graph is determined by counting the no. of nodes in its adjacency list.
- No. of edges in a graph is determined in O(n+e)
- out-degree of a any vertex in a directed graph is determined by counting No. of nodes in its adjacency list.

Adjacency Lists

```
class Gnode
{   int vertex;
    node *next;
   friend class Graph;
};
class Graph
{
private:
        Gnode *Head[20];
        int n;
public:
        Graph()
        {           create head nodes for n vertices
        }
};
```



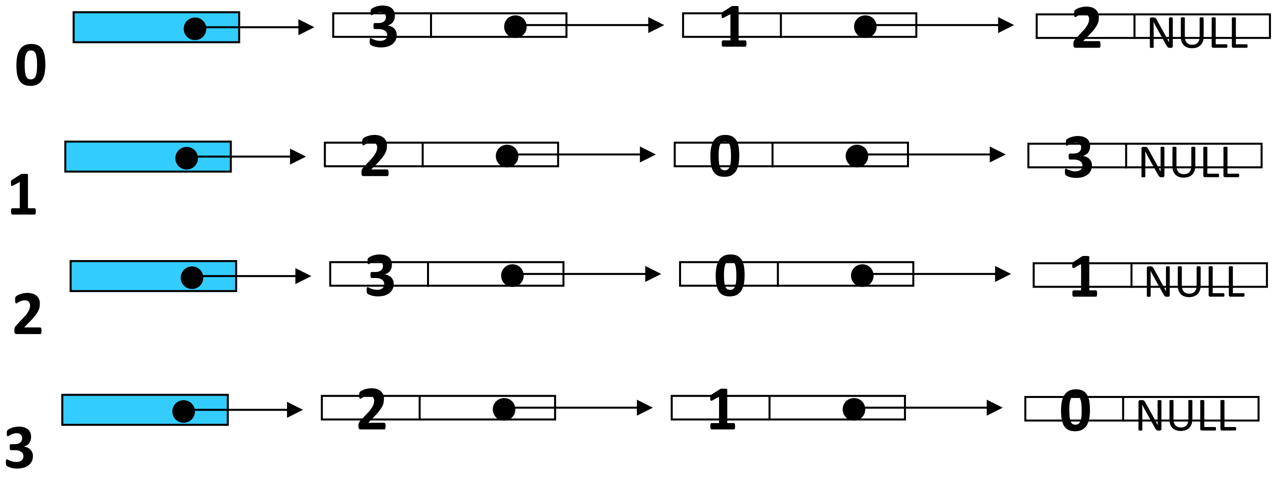
Graph G_1



Adjacency List for Graph G_1

```
graph()
{
 Accept no of vertices;
 for i=0 to n-1
    {Allocate a memory for head[i] node (array)
    head[i]->vertex=i; }
 }

create()
{
 for i=0 to n-1
 {
   temp=head[i];
   do
   {
     Accept adjacent vertex v;
    if(v==i)
         Print Self loop are not allowed;
    else
    {
      Allocate  memory for curr node;
       curr->vertex=v;
       temp->next=curr;
       temp=temp->next;
     }
     accept the choice ;
    }while(ans=='y'||ans=='Y');
  }
 }
```



Graph Traversal

- Depth First Traversal
- Breadth First Traversal

Depth First Traversal (Recursive)

Algorithm DFS()

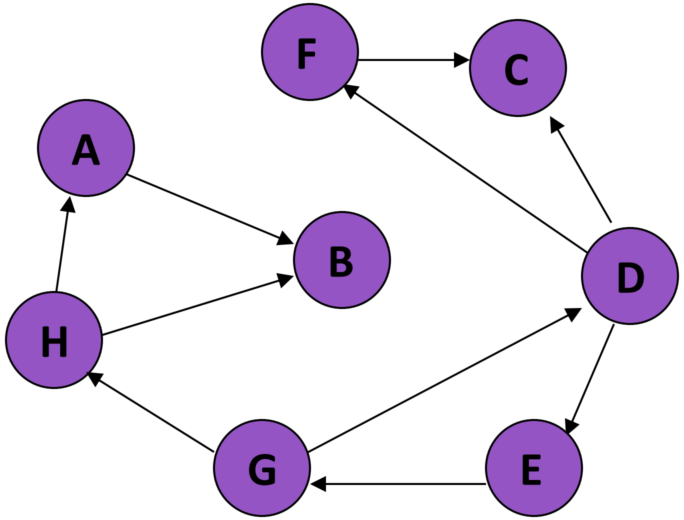
```
{
  //initially no vertex will be visited
        for( int i=0 ; i<n; i++)
                visited[i]=0;
 //start search at vertex v
     accept starting vertex v
        DFS(v);

}
```

Algorithm DFS(int v)

```
{           print v;
           visited[v]=1;
           for(each vertex w adjacent to v)
                  if(!visited[w])
                         DFS(w);
}
```

Depth First Search Traversal

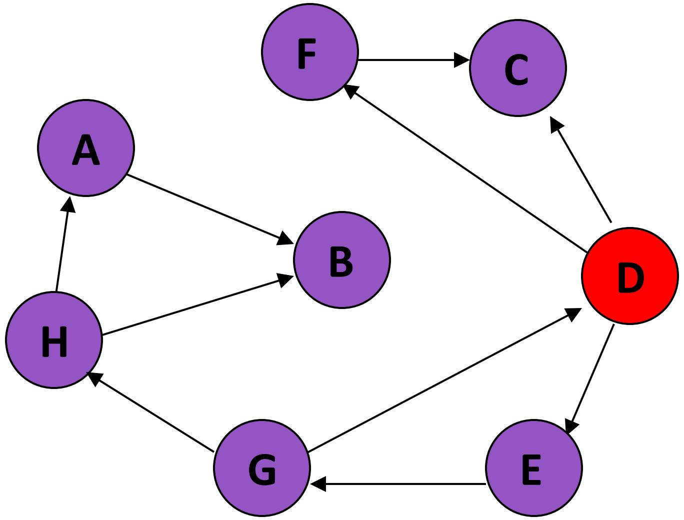


Visited Array

A	
B	
C	
D	
E	
F	
G	
H	

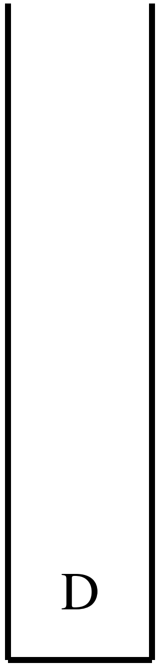
Task: Conduct a depth-first search of the graph starting with node D

Depth First SearchTraversal



Visited Array

A	
B	
C	
D	1
E	
F	
G	
H	

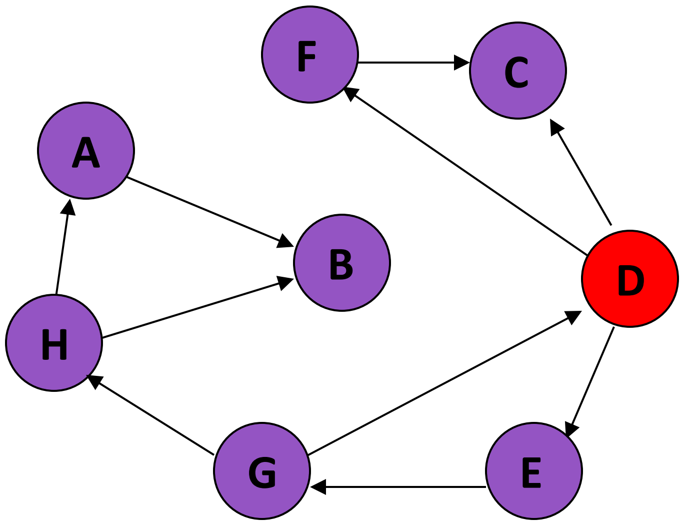


The DFT of nodes in graph :

D

Visit D

Depth First Traversal



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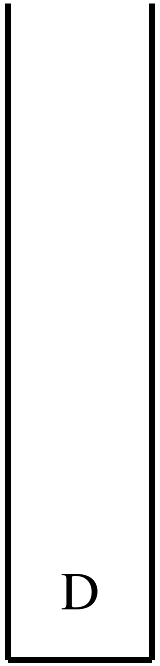
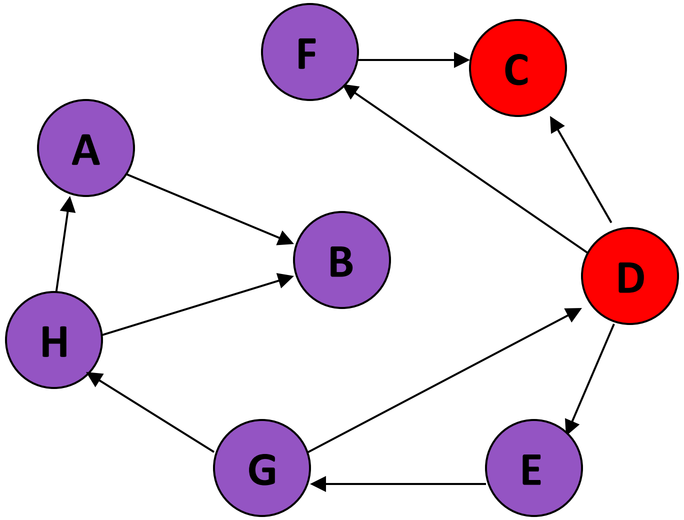


The DFT of nodes in graph :

D

Consider nodes adjacent to D, decide to visit C first (Rule: visit adjacent nodes in alphabetical order or in order of the adjacency list)

Depth First Traversal



Visited Array

A	
B	
C	1
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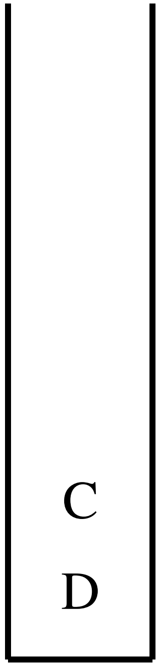
H	

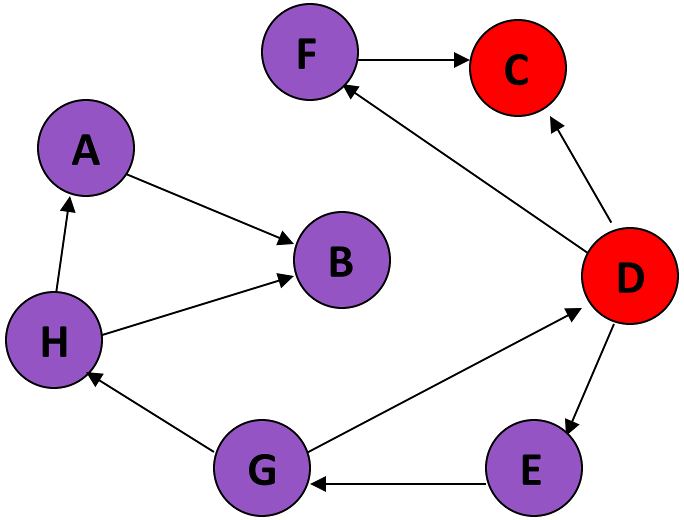


Visit C

The DFT of nodes in graph :

D, C

Depth First Traversal



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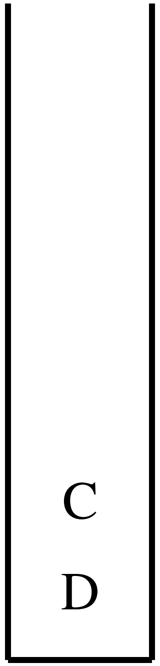
H	

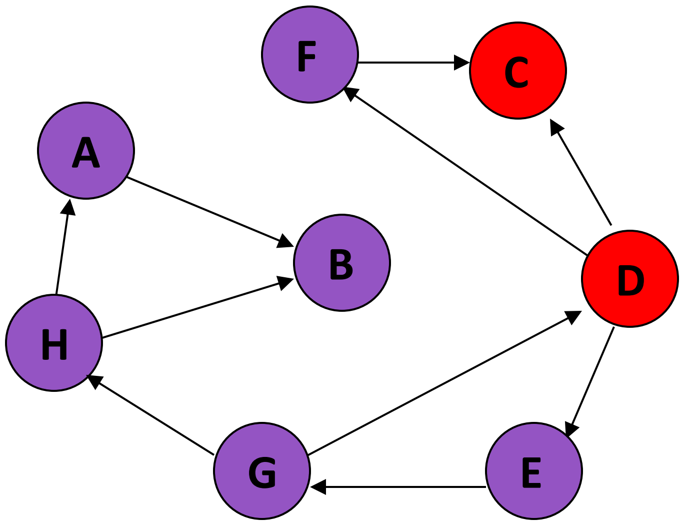


The DFT of nodes in graph :

D, C

No nodes adjacent to C; cannot continue *backtrack*, i.e., pop stack and restore previous state

Depth First Traversal



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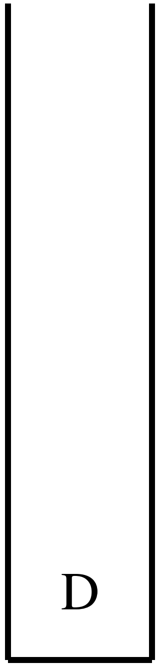
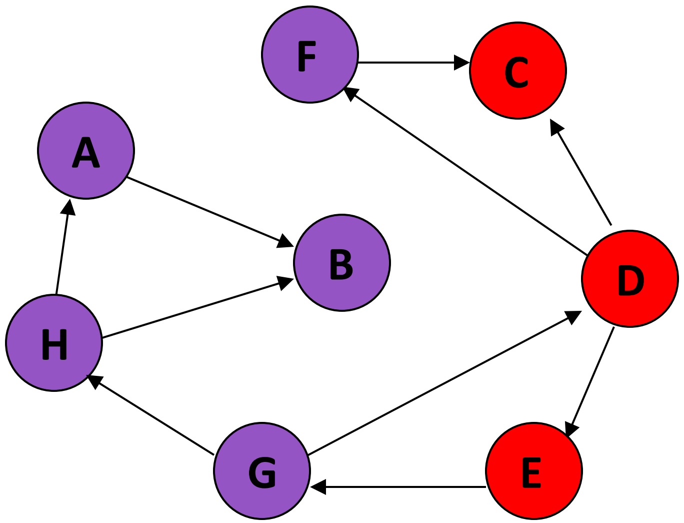


The DFT of nodes in graph :

D, C

Back to D – C has been visited, decide to visit E next

Depth First Traversal



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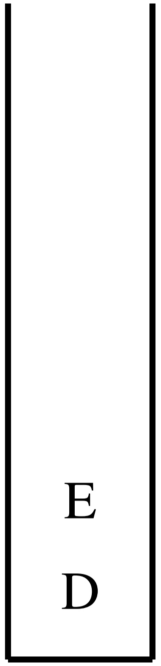
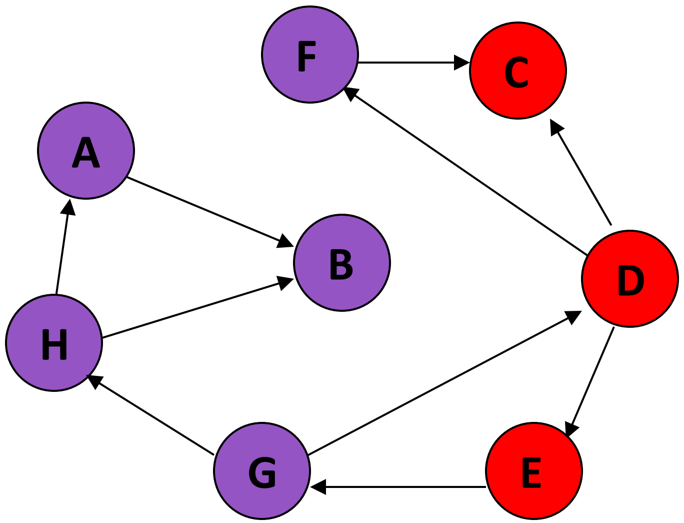


The DFT of nodes in graph :

D, C, E

Back to D – C has been visited, decide to visit E next

Depth First Traversal



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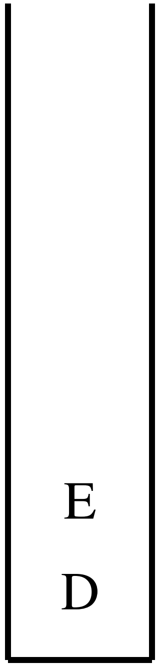
G	
H	

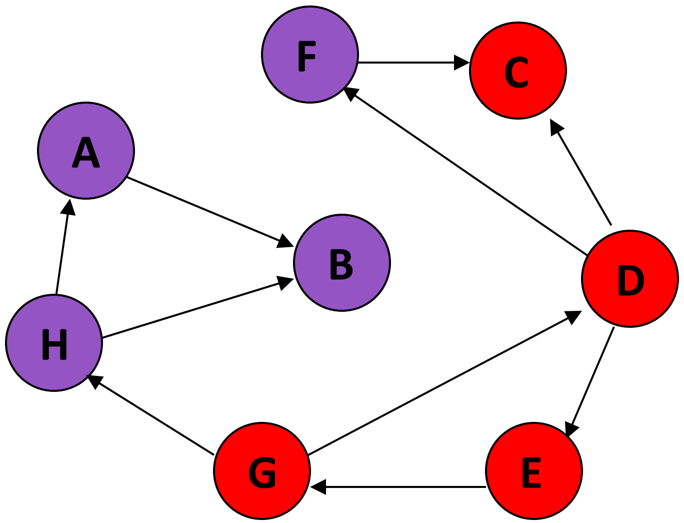


The DFT of nodes in graph :

D, C, E

Only G is adjacent to E

Depth First Traversal



Visited Array

A	
B	
C	1
D	1
E	1
F	
G	1
H	

G

E

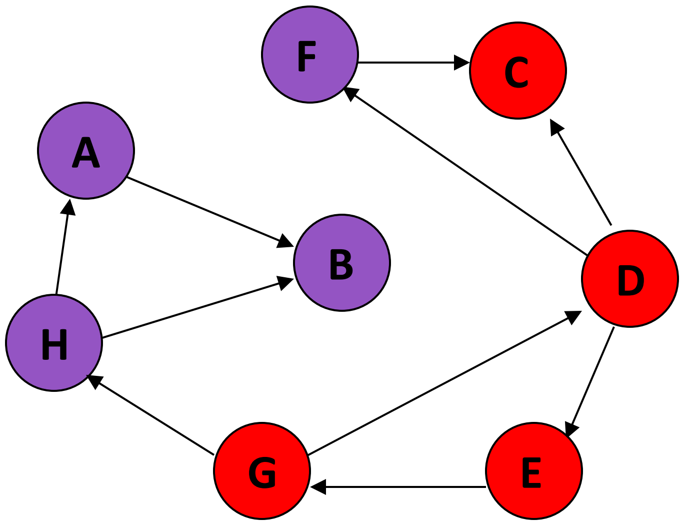
D

Visit G

The DFT of nodes in graph :

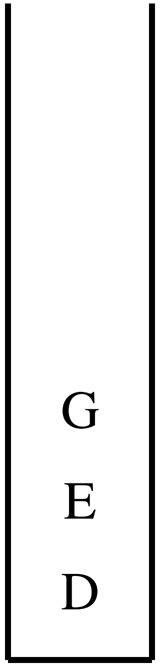
D, C, E, G

Depth First Traversal



Visited Array

A	
B	
C	**1**
D	**1**
E	**1**
F	
G	1
H	


G

E

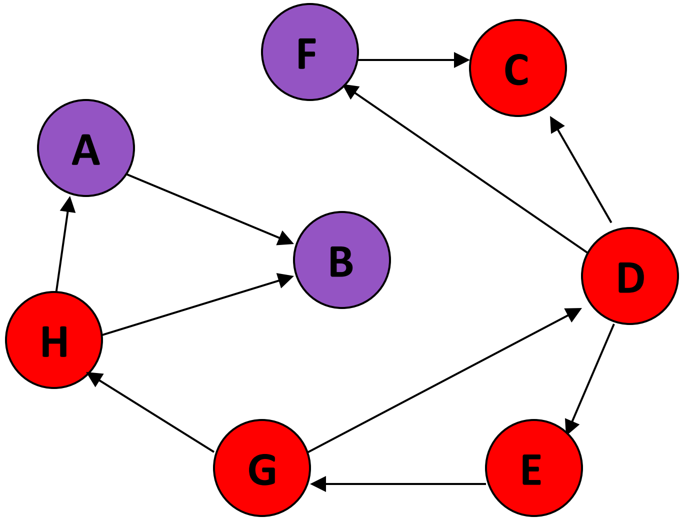
D

The DFT of nodes in graph :

D, C, E, G

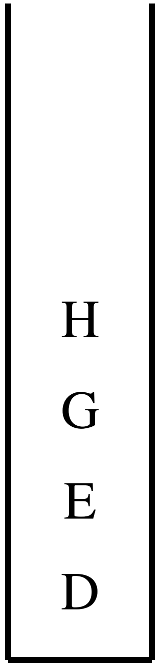
Nodes D and H are adjacent to G. D has already been visited. Decide to visit H.

Depth First Traversal



Visited Array

A	
B	
C	1
D	1
E	1
F	
G	1
H	1

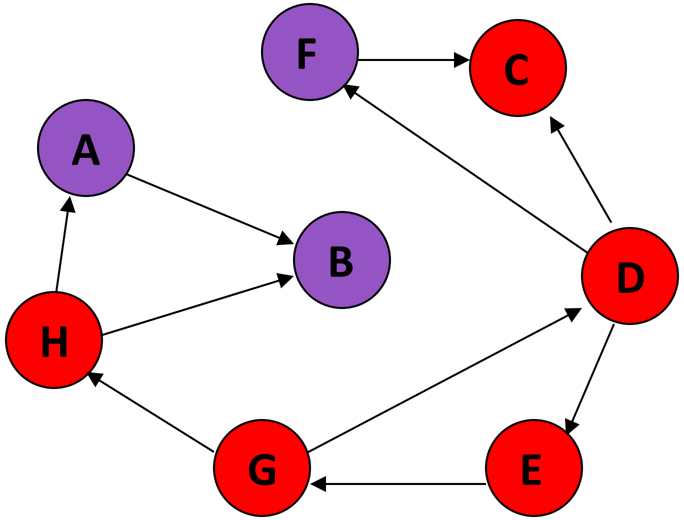


Visit H

The DFT of nodes in graph :

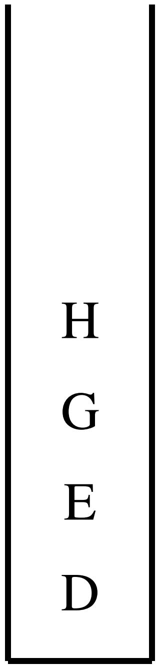
D, C, E, G, H

Depth First Traversal



Visited Array

A	
B	
C	1
D	1
E	1
F	
G	1
H	1

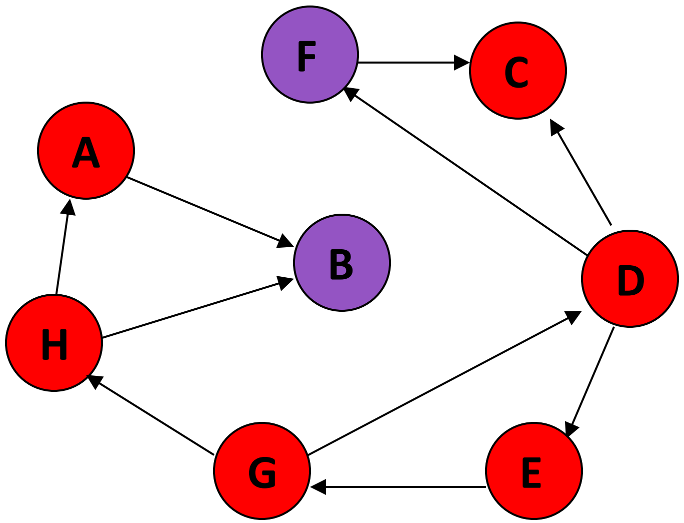


The DFT of nodes in graph :

D, C, E, G, H

Nodes A and B are adjacent to F. Decide to visit A next.

Depth First Traversal



Visited Array

A	1
B	
C	1
D	1
E	1
F	
G	1
H	1

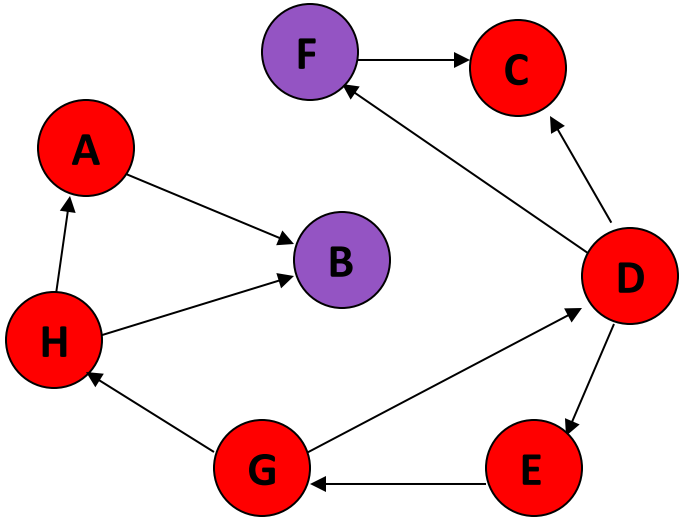
A
H
G
E
D

Visit A

The DFT of nodes in graph :

D, C, E, G, H, A

Depth First Traversal



Visited Array

A	1
B	
C	1
D	1
E	1
F	
G	1
H	1

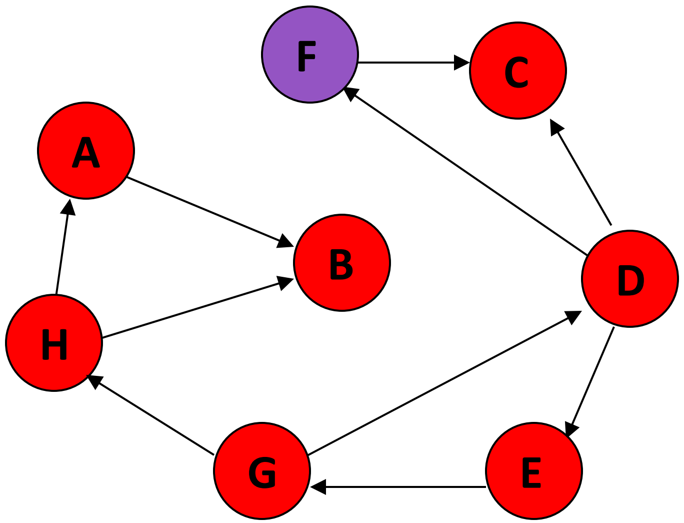
A
H
G
E
D

The DFT of nodes in graph :

D, C, E, G, H, A

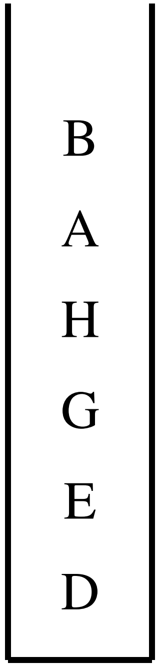
Only Node B is adjacent to A.
Decide to visit B next.

Depth First Traversal



Visited Array

A	1
B	1
C	1
D	1
E	1
F	
G	1
H	1



B

A

H

G

E

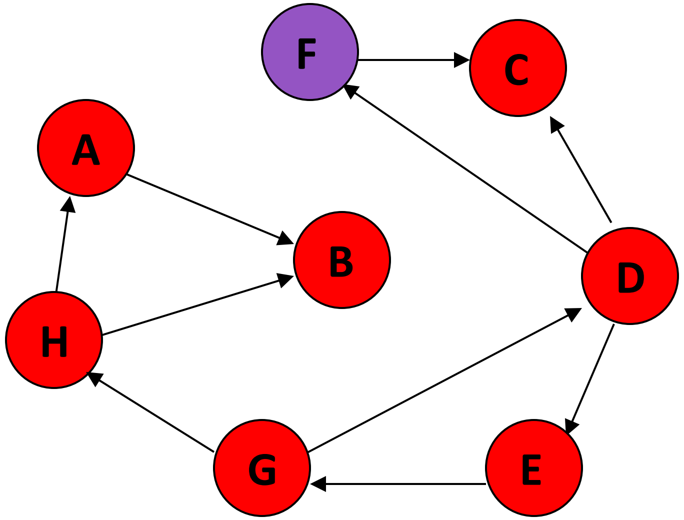
D

Visit B

The DFT of nodes in graph :

D, C, E, G, H, A, B

Depth First Traversal



Visited Array

A	1
B	1
C	1
D	1
E	1
F	
G	1
H	1

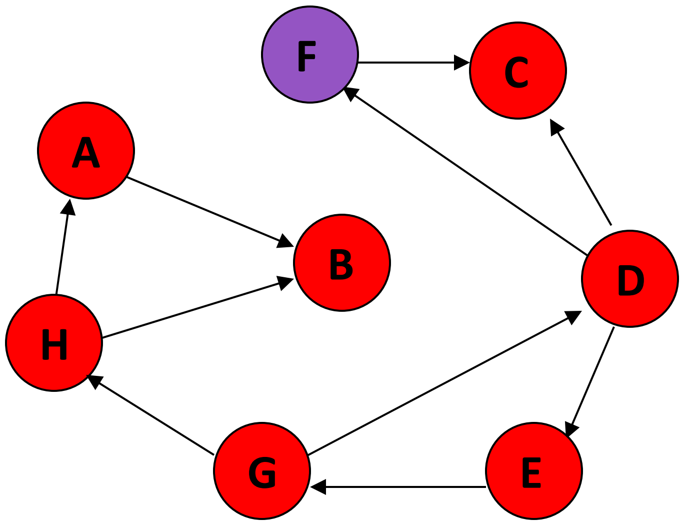
A
H
G
E
D

The DFT of nodes in graph :

D, C, E, G, H, A, B

No unvisited nodes adjacent to B. Backtrack (pop the stack).

Depth First Traversal



Visited Array

A	1
B	1
C	1
D	1
E	1
F	
G	1
H	1

H

G

E

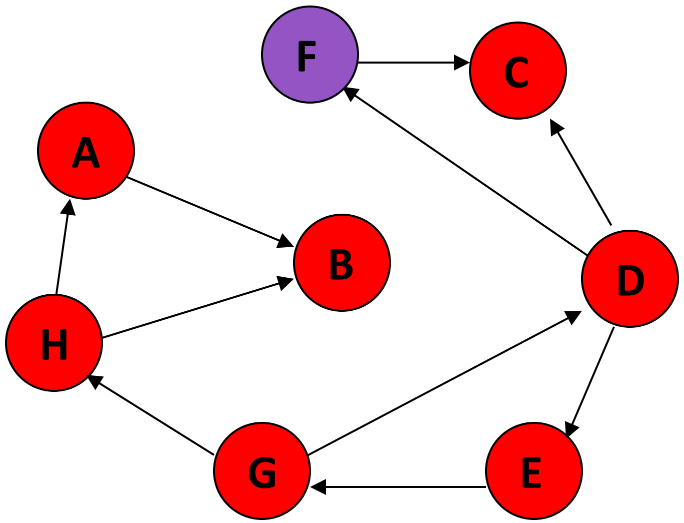
D

The DFT of nodes in graph :

D, C, E, G, H, A, B

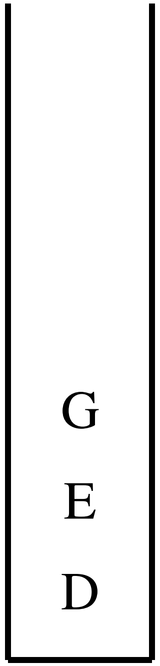
No unvisited nodes adjacent to A. Backtrack (pop the stack).

Depth First Traversal



Visited Array

A	1
B	1
C	1
D	1
E	1
F	
G	1
H	1



G

E

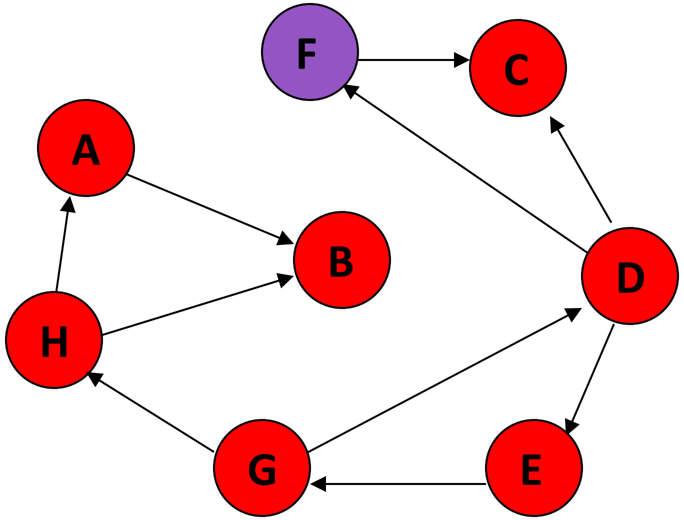
D

The DFT of nodes in graph :

D, C, E, G, H, A, B

No unvisited nodes adjacent to H. Backtrack (pop the stack).

Depth First Traversal



Visited Array

A	1
B	1
C	1
D	1
E	1
F	
G	1
H	1

E

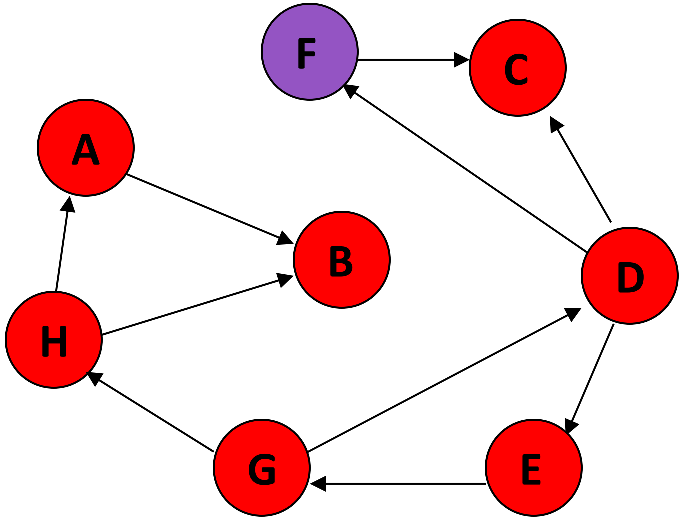
D

The DFT of nodes in graph :

D, C, E, G, H, A, B

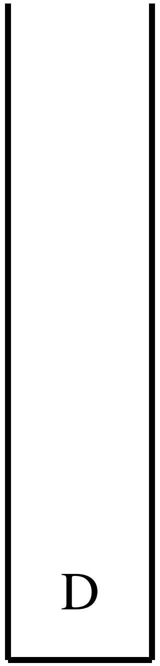
No unvisited nodes adjacent to G. Backtrack (pop the stack).

Depth First Traversal



Visited Array

A	1
B	1
C	1
D	1
E	1
F	
G	1
H	1

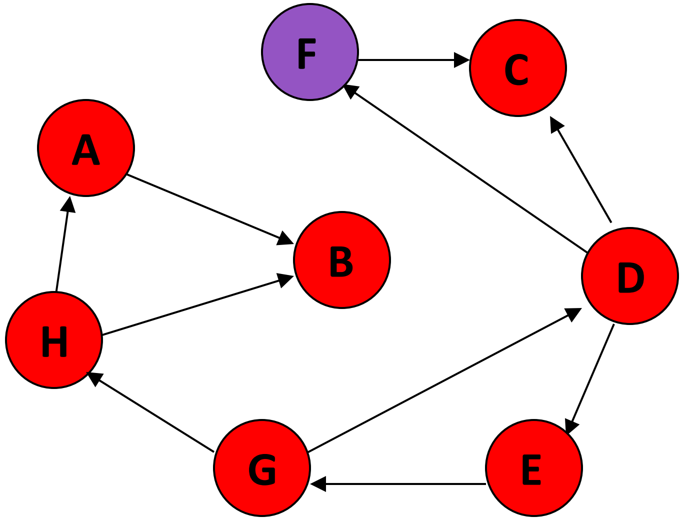


The DFT of nodes in graph :

D, C, E, G, H, A, B

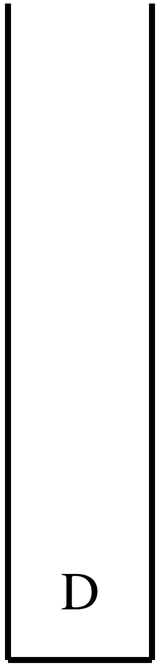
No unvisited nodes adjacent to E. Backtrack (pop the stack).

Depth First Traversal



Visited Array

A	1
B	1
C	1
D	1
E	1
F	
G	1
H	1

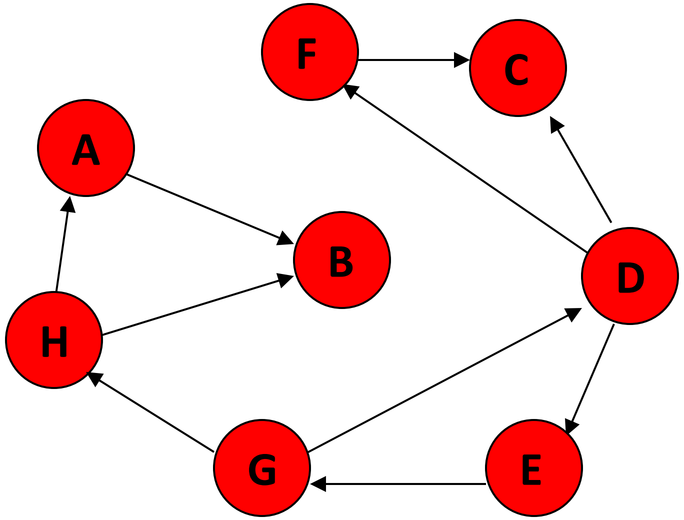


The DFT of nodes in graph :

D, C, E, G, H, A, B

F is unvisited and is adjacent to D. Decide to visit F next.

Depth First Traversal

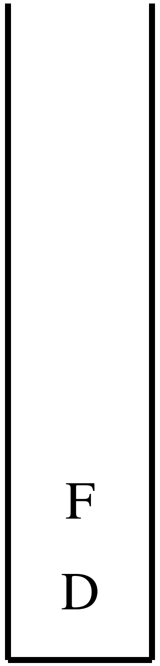


The DFT of nodes in graph :

D, C, E, G, H, A, B, F

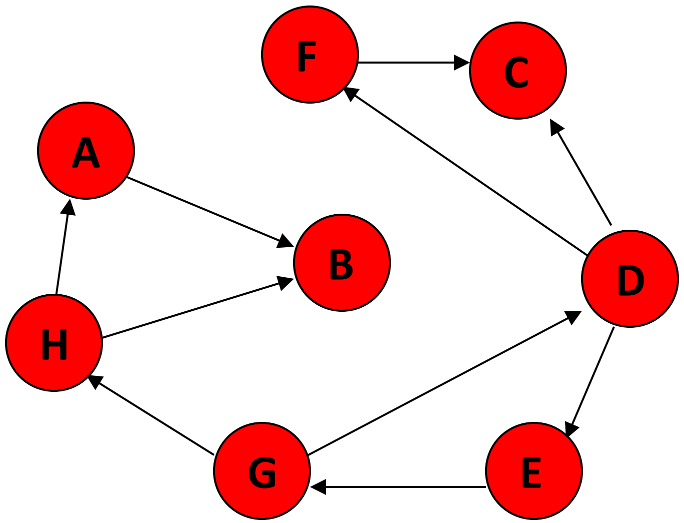
Visited Array

A	**1**
B	**1**
C	**1**
D	**1**
E	**1**
F	1
G	1
H	**1**



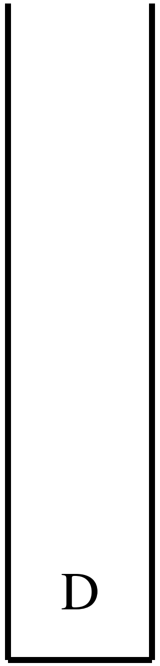
Visit F

Depth First Traversal



Visited Array

A	√
B	√
C	√
D	√
E	√
F	√
G	√
H	√

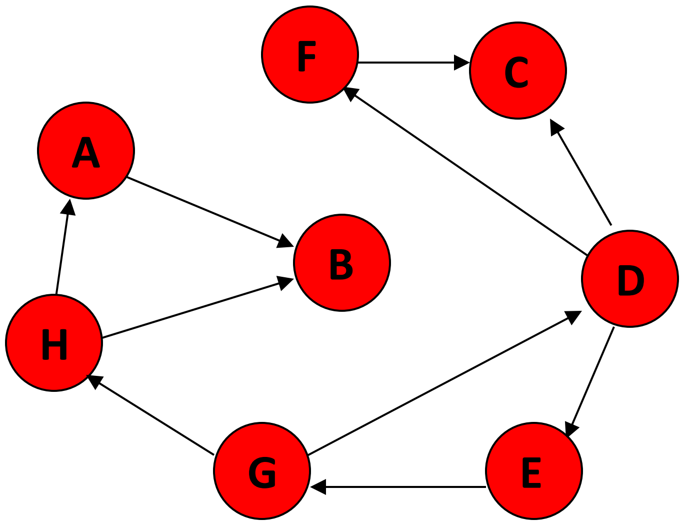


The DFT of nodes in graph :

D, C, E, G, H, A, B, F

No unvisited nodes adjacent to F. Backtrack.

Depth First Traversal



Visited Array

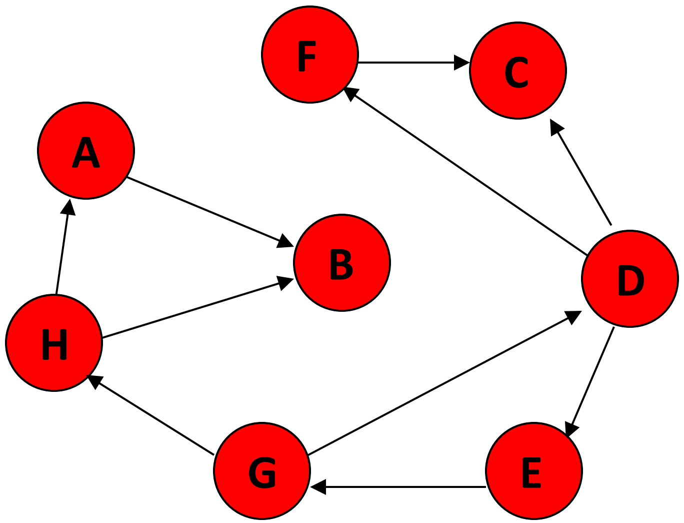
A	√
B	√
C	√
D	√
E	√
F	√
G	√
H	√

The order nodes are visited:

D, C, E, G, H, A, B, F

No unvisited nodes adjacent to D. Backtrack.

Depth First Traversal



Visited Array

A	√
B	√
C	√
D	√
E	√
F	√
G	√
H	√

The DFT of nodes in graph :

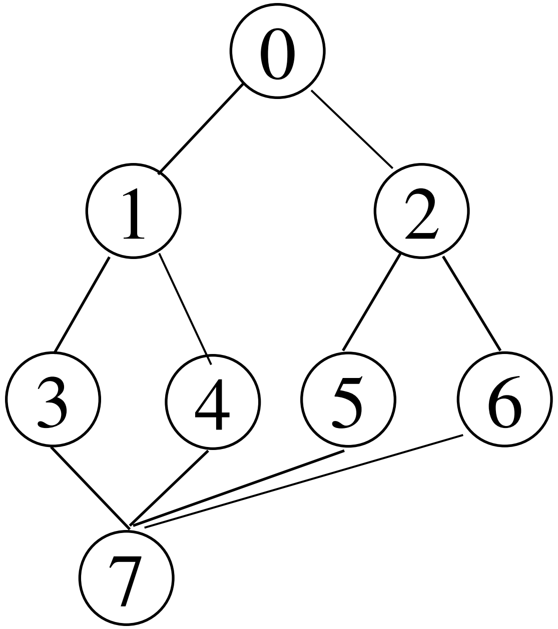
D, C, E, G, H, A, B, F

Stack is empty. Depth-first traversal is done.

Depth First Traversal (Non-recursive)

```
Algorithm DFS(int v)
{
  for all vertices of graph
          visited[i]=0;
    push(v);
   visited[v]=1;
    do
     {
       v=pop();
      print(v);
        for(each vertex w adjacent to v)
 {
              if(!visited[w])
                  {  push(w); visited[w]=1;}
 } //end for
  } while(stack not empty)
  } //end dfs
```

Depth First Traversal



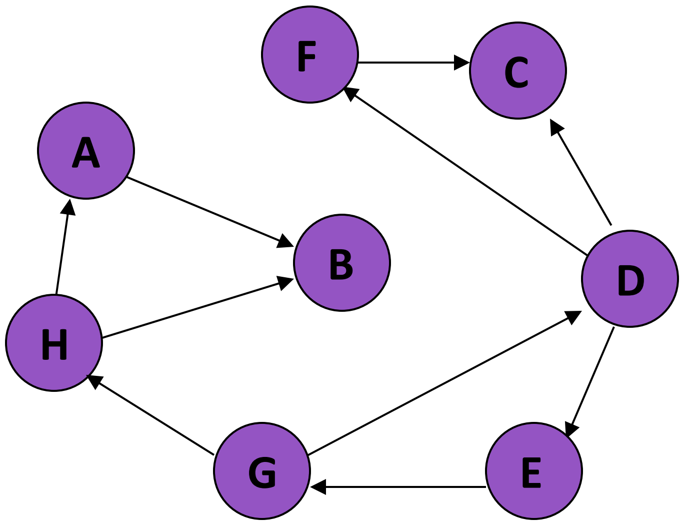
Graph G1

Find DFT for given graph G1 starting at vertex 0

Breadth First Traversal

```
Algorithm BFS(int v) {
 for(int i=0;i<n;i++)
       visited[i]=0;
         Queue  q;
         q.insert(v); visited[v]=1
         while(!q.IsEmpty())
         {
                     v=q.Delete();
                     for(all vertices w adjacent to v)
                     if(!visited[w])
                     {
                                 q.insert(w);
                                 visited[w]=1;
                     }
         }
}
```

Breadth First Traversal



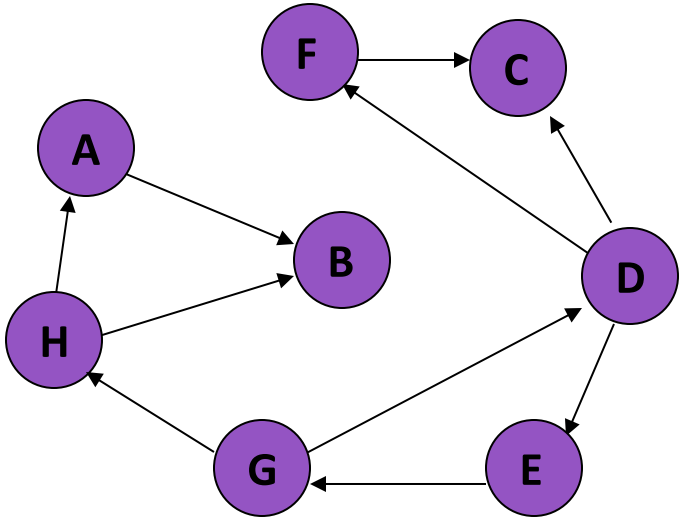
Enqueued Array

A	
B	
C	
D	
E	
F	
G	
H	

Q:

How is this accomplished? Simply replace the stack with a queue! Rules: (1) Maintain an *enqueued* array. (2) Visit node when *dequeued.*

Breadth First Traversal



Enqueued Array

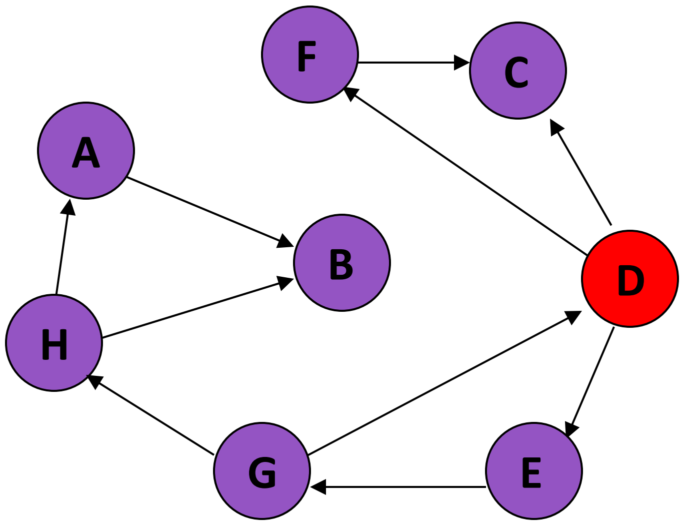
A	
B	
C	
D	√
E	
F	
G	
H	

Q :D

Nodes visited:

Enqueue D. Notice, D not yet visited.

Breadth First Traversal



Enqueued Array

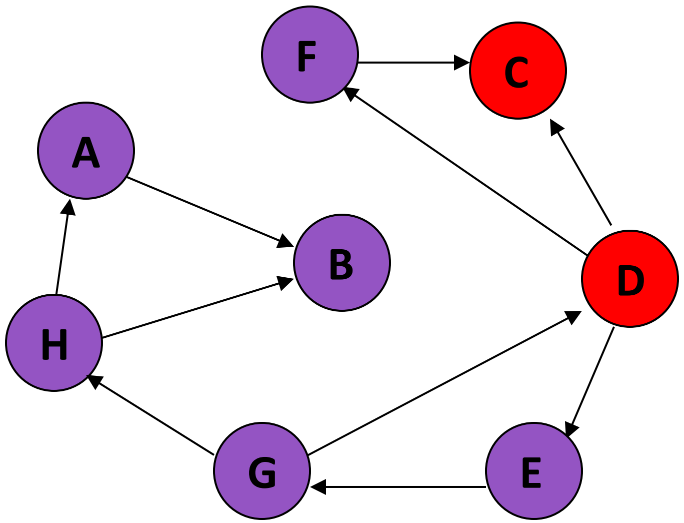
A	
B	
C	√
D	√
E	√
F	√
G	
H	

Q : C , E, F

Nodes visited: D

Dequeue D. Visit D. Enqueue unenqueued nodes adjacent to D.

Breadth First Traversal



Enqueued Array

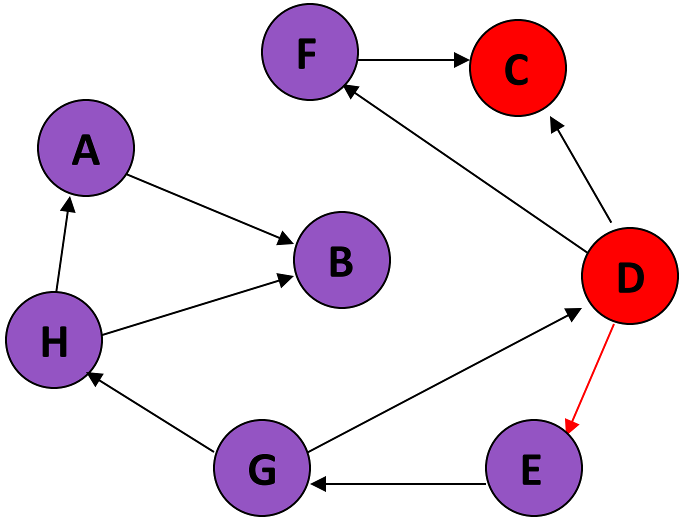
A	
B	
C	√
D	√
E	√
F	√
G	
H	

Q : E , F

Nodes visited: D, C

Dequeue C. Visit C. Enqueue unenqueued nodes adjacent to C.

Breadth First Traversal



Nodes visited: D, C, E

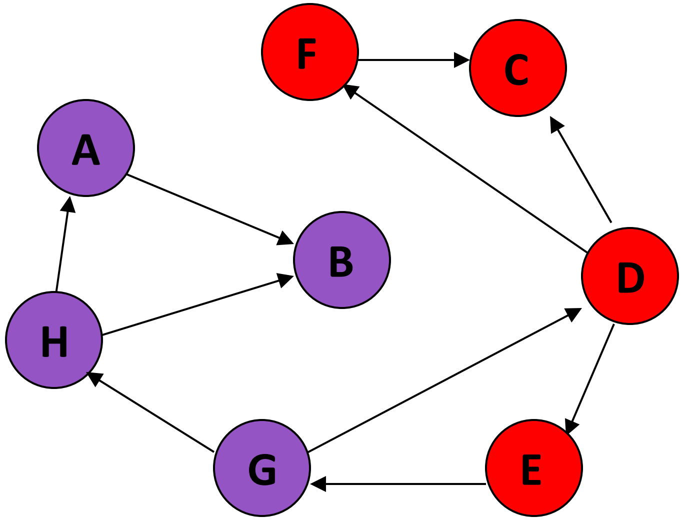
Enqueued Array

A	
B	
C	√
D	√
E	√
F	√
G	
H	

Q : F , G

Dequeue E. Visit E. Enqueue unenqueued nodes adjacent to E.

Breadth First Traversal



Enqueued Array

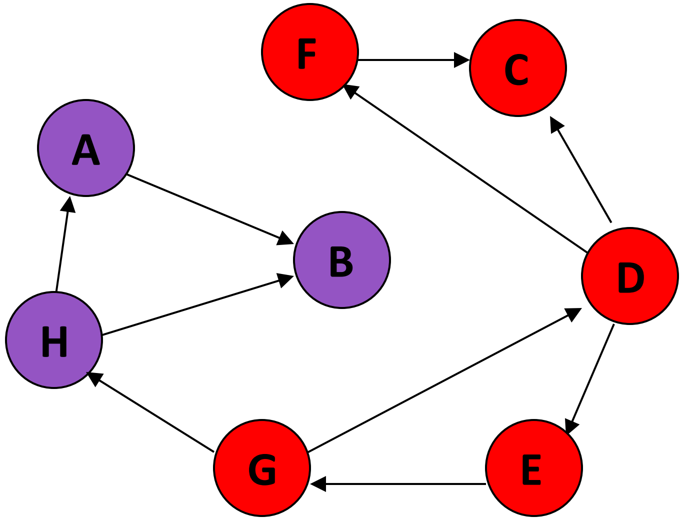
A	
B	
C	√
D	√
E	√
F	√
G	√
H	

Q : G

Nodes visited: D, C, E, F

Dequeue F. Visit F. Enqueue unenqueued nodes adjacent to F.

Breadth First Traversal



Enqueued Array

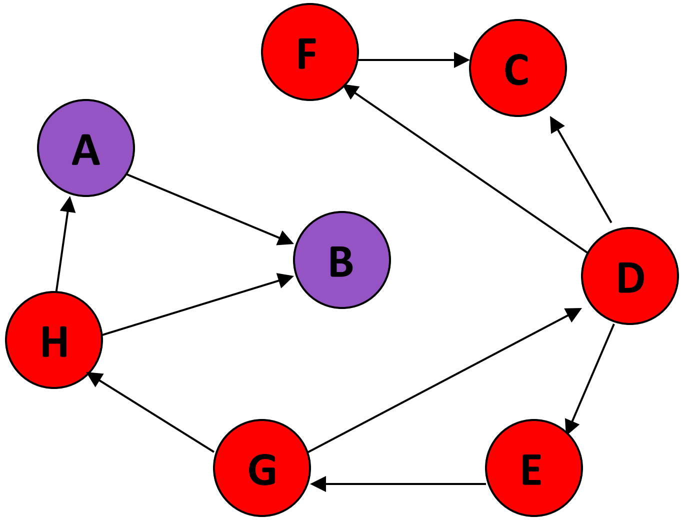
A	
B	
C	√
D	√
E	√
F	√
G	√
H	√

Q : H

Nodes visited: D, C, E, F, G

Dequeue G. Visit G. Enqueue unenqueued nodes adjacent to G.

Breadth First Traversal



Nodes visited: D, C, E, F, G, H

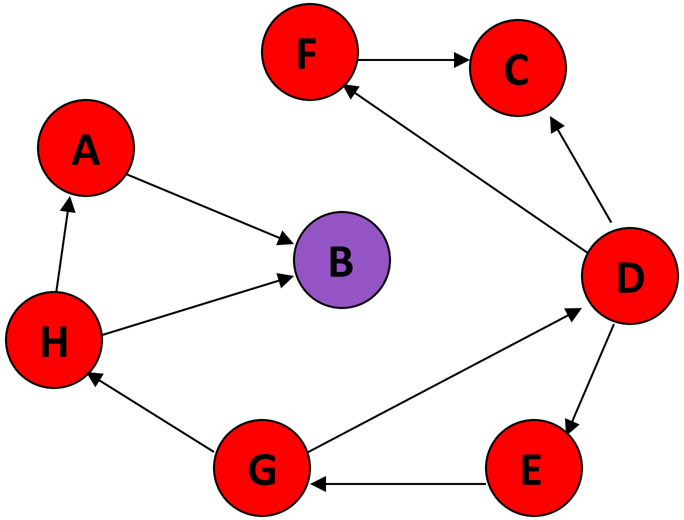
Enqueued Array

A	√
B	√
C	√
D	√
E	√
F	√
G	√
H	√

Q : A , B

Dequeue H. Visit H. Enqueue unenqueued nodes adjacent to H.

Breadth First Traversal



Enqueued Array

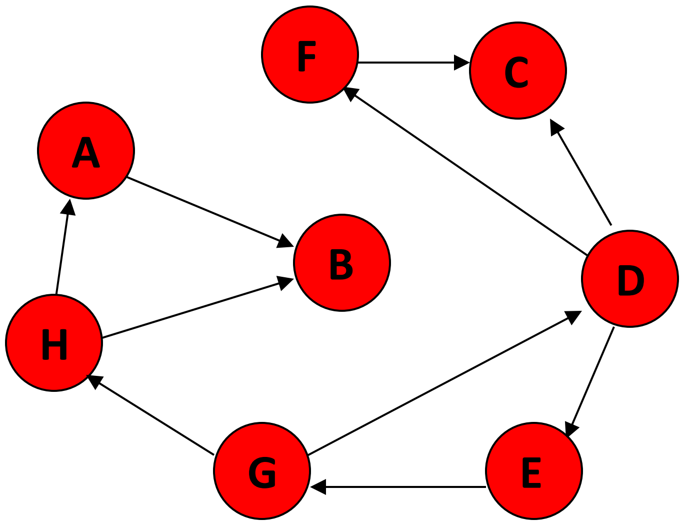
A	√
B	√
C	√
D	√
E	√
F	√
G	√
H	√

Q : B

Nodes visited: D, C, E, F, G, H, A

Dequeue A. Visit A. Enqueue unenqueued nodes adjacent to A.

Breadth First Traversal



Enqueued Array

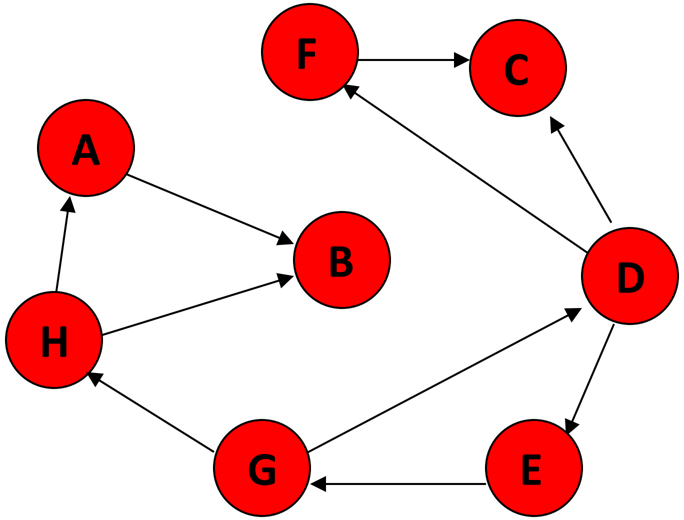
A	√
B	√
C	√
D	√
E	√
F	√
G	√
H	√

Q empty

Nodes visited: D, C, E, F, G, H, A, B

Dequeue B. Visit B. Enqueue unenqueued nodes adjacent to B.

Breadth First Traversal



Enqueued Array

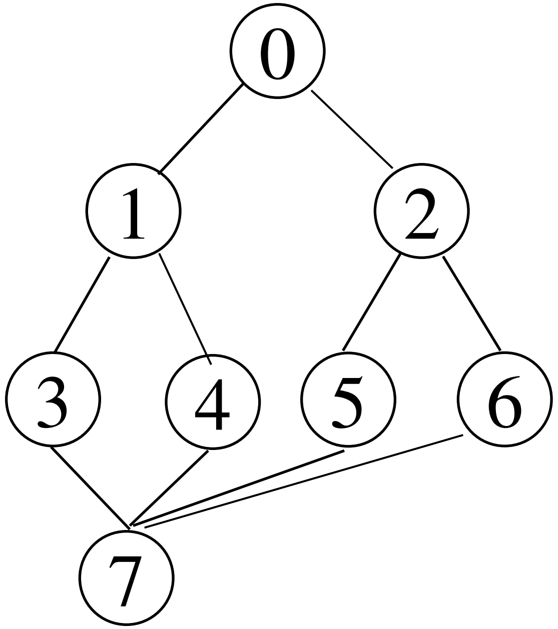
A	√
B	√
C	√
D	√
E	√
F	√
G	√
H	√

Q empty

Nodes visited: D, C, E, F, G, H, A, B

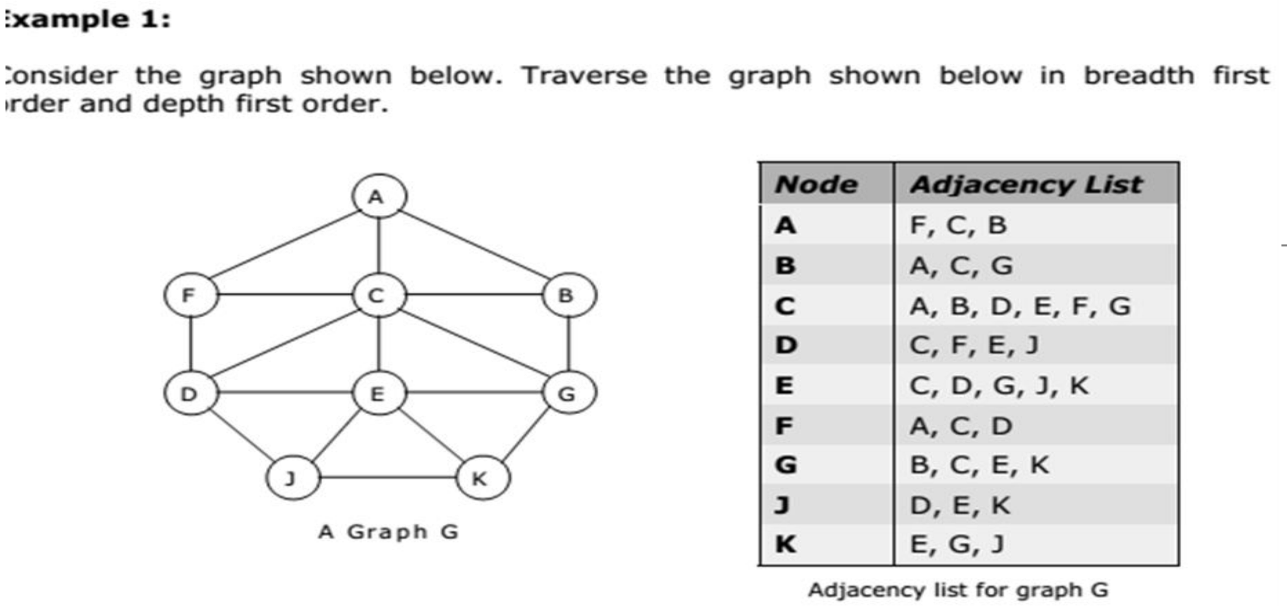
Q empty. Algorithm done.

Breadth First Traversal



Graph G1

Find BFT for given graph G1 starting at vertex 0



xample 1:

onsider the graph shown below. Traverse the graph shown below in breadth first rder and depth first order.

A Graph G

Node	Adjacency List
A	F, C, B
B	A, C, G
C	A, B, D, E, F, G
D	C, F, E, J
E	C, D, G, J, K
F	A, C, D
G	B, C, E, K
J	D, E, K
K	E, G, J

Adjacency list for graph G

BFT-A,F,C,B,D,E,G,J,K
DFT-A,F,D,J,K,G,E,C,B

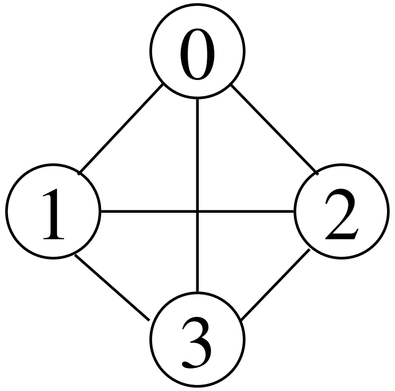
Comparison Chart

BASIS FOR COMPARISON	BFS	DFS
Basic	Vertex-based algorithm	Edge-based algorithm
Data structure used to store the nodes	Queue	Stack
Memory consumption	Inefficient	Efficient
Structure of the constructed tree	Wide and short	Narrow and long
Traversing fashion	Oldest unvisited vertices are explored at first.	Vertices along the edge are explored in the beginning.
Optimality	Optimal for finding the shortest distance,	Not optimal
Application	Examines bipartite graph, connected component and shortest path present in a graph.	Examines two-edge connected graph, strongly connected graph, acyclic graph and topological order.

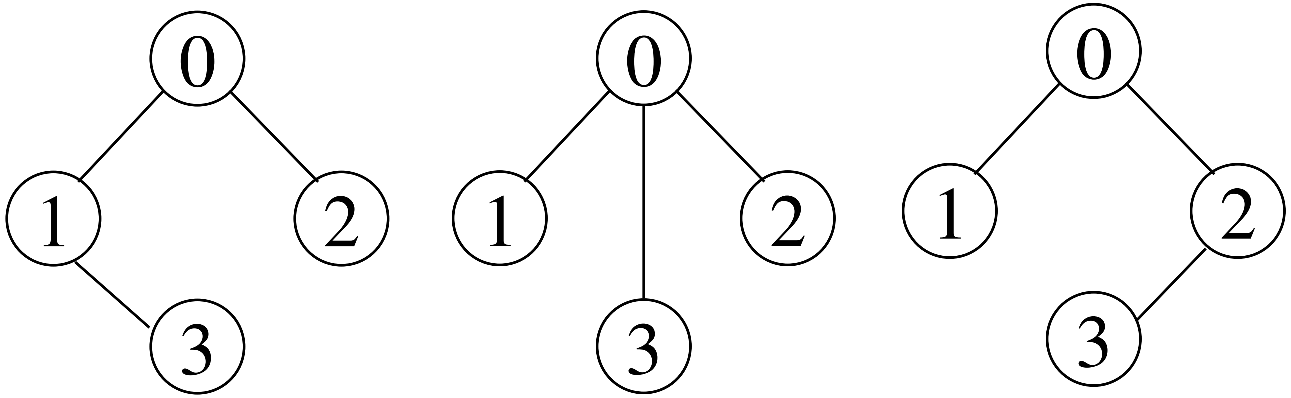
Spanning Trees

- A spanning tree is any tree that consists solely of edges in G and that includes all the vertices
- A spanning tree is a minimal subgraph, G’, of G such that V(G’)=V(G) and G’ is connected.
- Either dfs or bfs can be used to create a spanning tree
  - □When dfs is used, the resulting spanning tree is known as a depth first spanning tree
  - □When bfs is used, the resulting spanning tree is known as a breadth first spanning tree

Examples of Spanning Trees

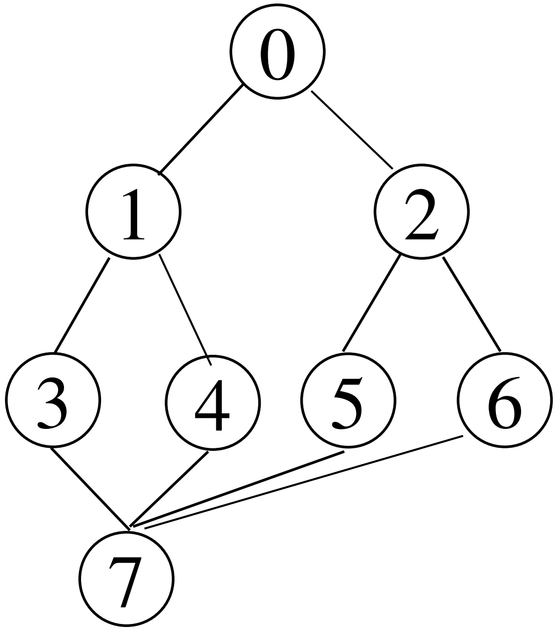


Graph G_1

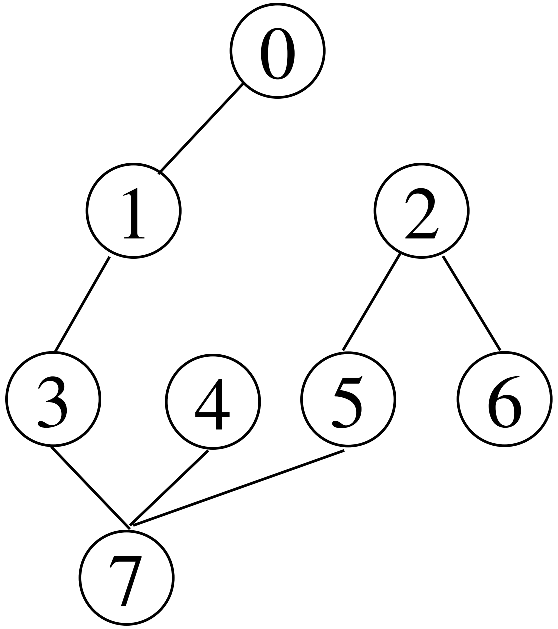


Possible spanning trees

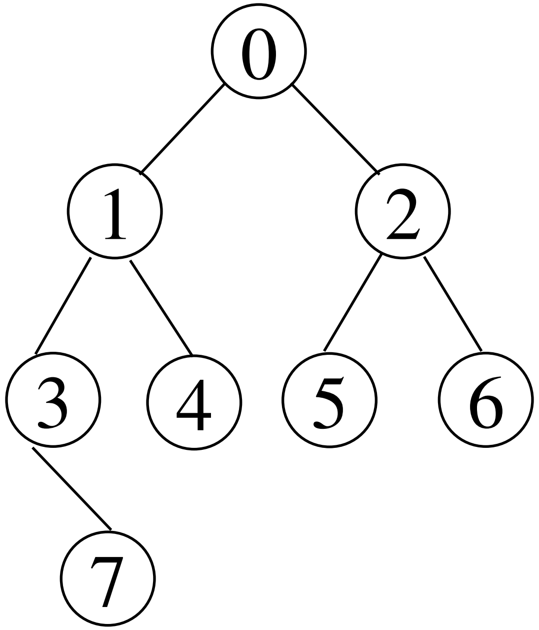
DFS VS BFS Spanning Trees



Graph



DFS Spanning Tree



BFS Spanning Tree

Minimum Spanning Tree

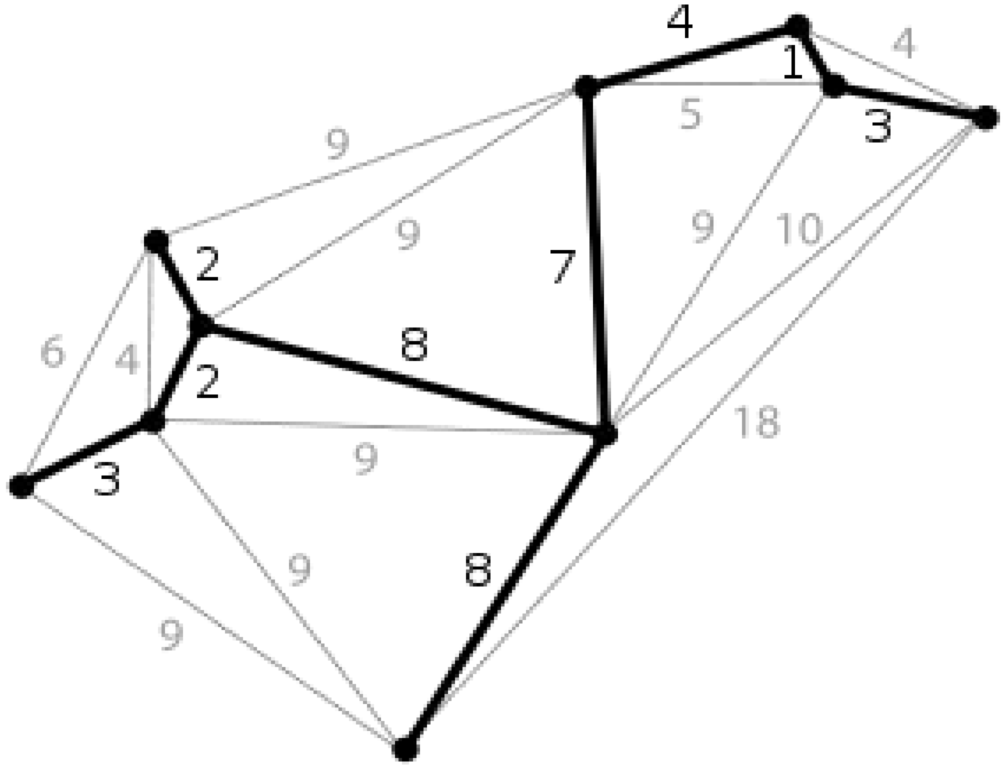
- The cost of a spanning tree of a weighted undirected graph is the sum of the costs of the edges in the spanning tree

- A minimum cost spanning tree is a spanning tree of least cost
- n-1 edges from a weighted graph of n vertices with minimum cost.

- Two different algorithms can be used
  - □Kruskal
  - □Prim

Minimum Spanning Tree

- Applications of MST in Network design
  - □Telephone
  - □Electrical
  - □TV cable
  - □Computer
  - □road



Greedy Strategy

- An optimal solution is constructed in stages
- At each stage, the best decision is made at this time
- Since this decision cannot be changed later, we make sure that the decision will result in a feasible solution
- Typically, the selection of an item at each stage is based on a least cost or a highest profit criterion

Kruskal’s Algorithm

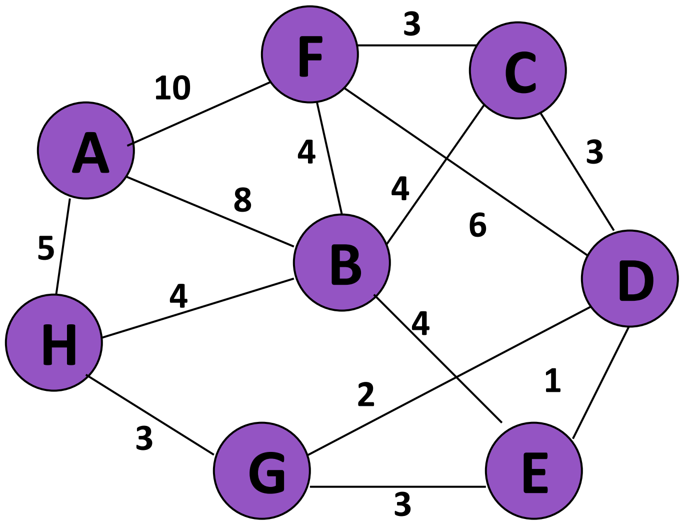
- Build a minimum cost spanning tree T by adding edges to T one at a time
- Select the edges for inclusion in T in nondecreasing order of the cost
- An edge is added to T if it does not form a cycle
- Since G is connected and has n > 0 vertices, exactly n-1 edges will be selected

Kruskal’s Algorithm

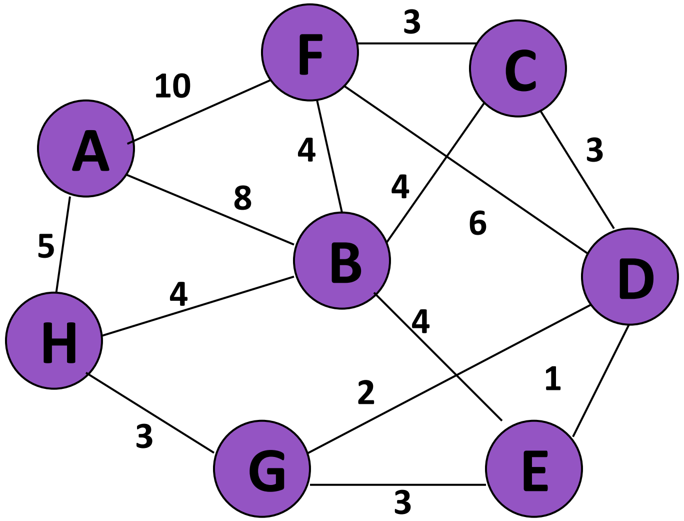
```
T= { };
while (T contains less than n-1 edges  && E is not empty)
{
     choose a least cost edge (v,w) from E;
     delete (v,w) from E;
     if ((v,w) does not create a cycle in T)
    add (v,w) to T
 else
             discard (v,w);
}
if (T contains fewer than n-1 edges)
  printf(“No spanning tree\n”);
```

Kruskal’s Algorithm

Consider an undirected, weight graph



Kruskal’s Algorithm

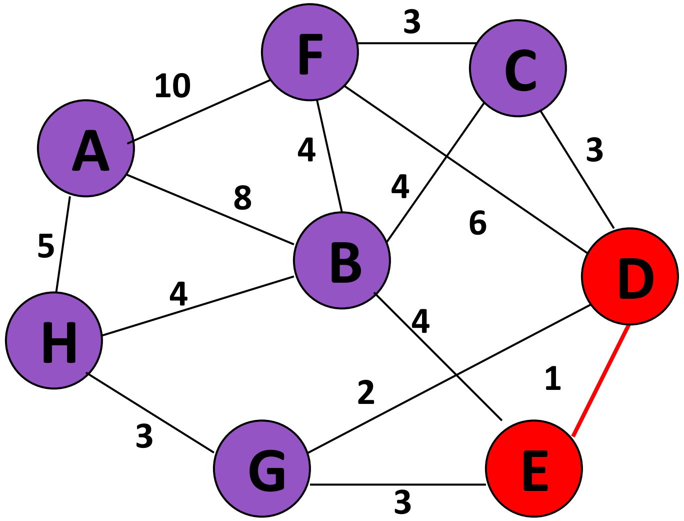


Sort the edges by increasing edge weight

edge	d_v	
(D,E)	1	
(D,G)	2	
(E,G)	3	
(C,D)	3	
(G,H)	3	
(C,F)	3	
(B,C)	4	

edge	d_v	
(B,E)	4	
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal’s Algorithm

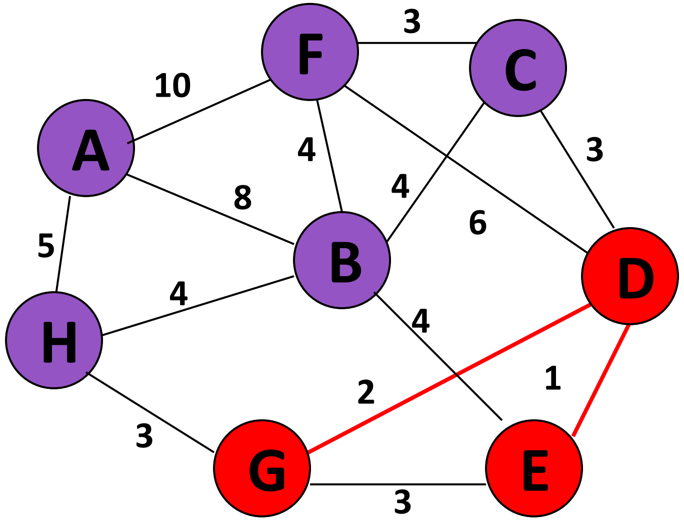


Select first |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	
(E,G)	3	
(C,D)	3	
(G,H)	3	
(C,F)	3	
(B,C)	4	

edge	d_v	
(B,E)	4	
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal’s Algorithm

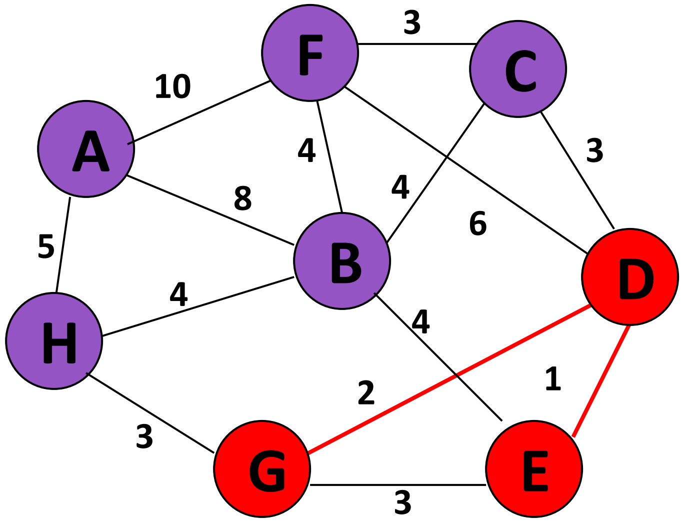


Select first |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	
(C,D)	3	
(G,H)	3	
(C,F)	3	
(B,C)	4	

edge	d_v	
(B,E)	4	
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal’s Algorithm



Select first |V|–1 edges which do not generate a cycle

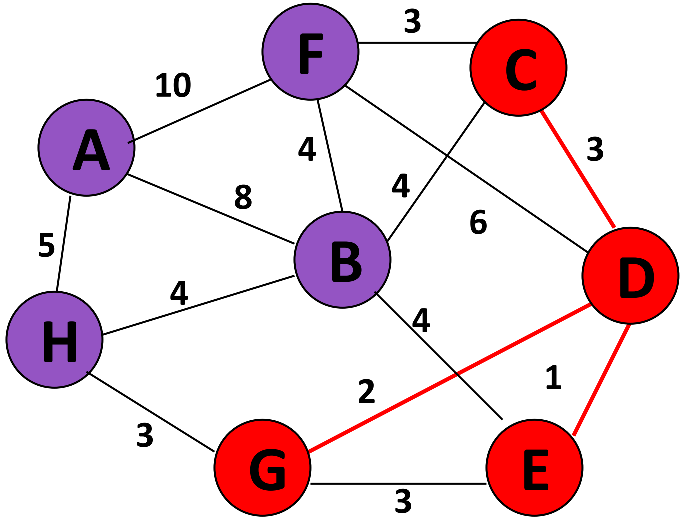
edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	
(G,H)	3	
(C,F)	3	
(B,C)	4	

edge	d_v	
(B,E)	4	
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Accepting edge (E,G) would create a cycle

Kruskal’s Algorithm

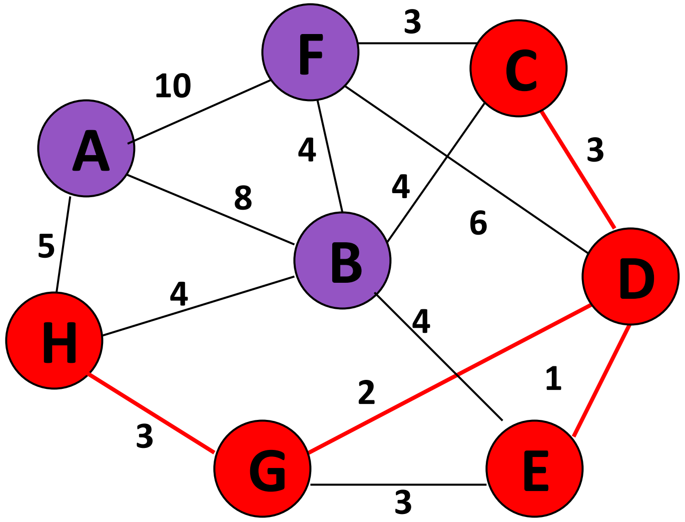
Select first |V|–1 edges which do not generate a cycle



edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	
(C,F)	3	
(B,C)	4	

edge	d_v	
(B,E)	4	
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal’s Algorithm

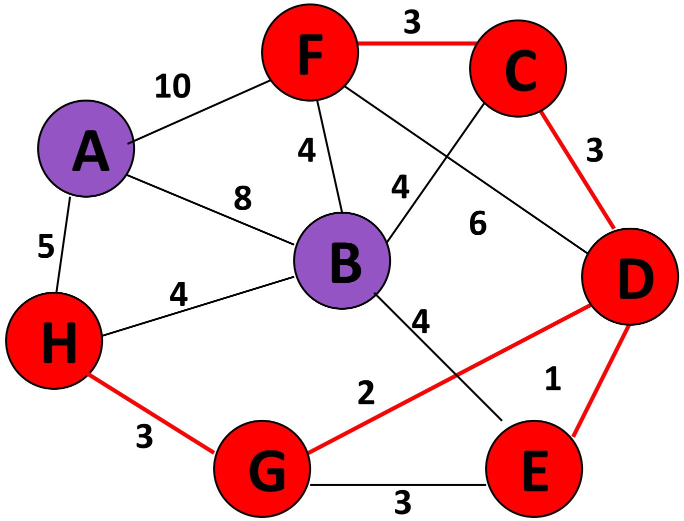


Select first |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	√
(C,F)	3	
(B,C)	4	

edge	d_v	
(B,E)	4	
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal’s Algorithm

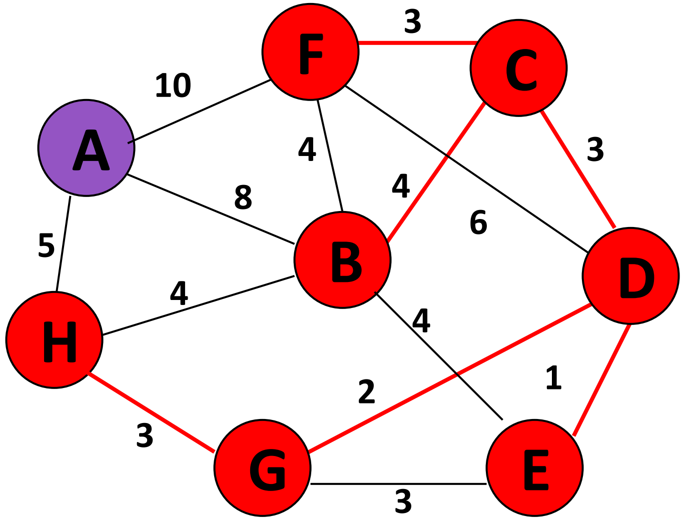


Select first |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	√
(C,F)	3	√
(B,C)	4	

edge	d_v	
(B,E)	4	
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal’s Algorithm

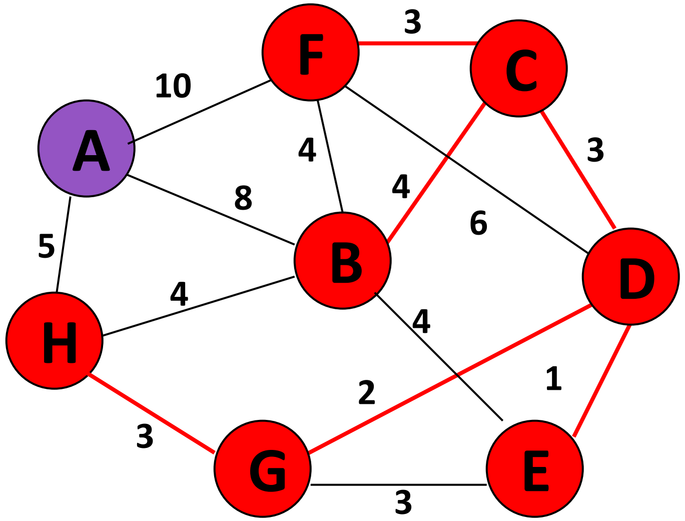


Select first |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	√
(C,F)	3	√
(B,C)	4	√

edge	d_v	
(B,E)	4	
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal’s Algorithm

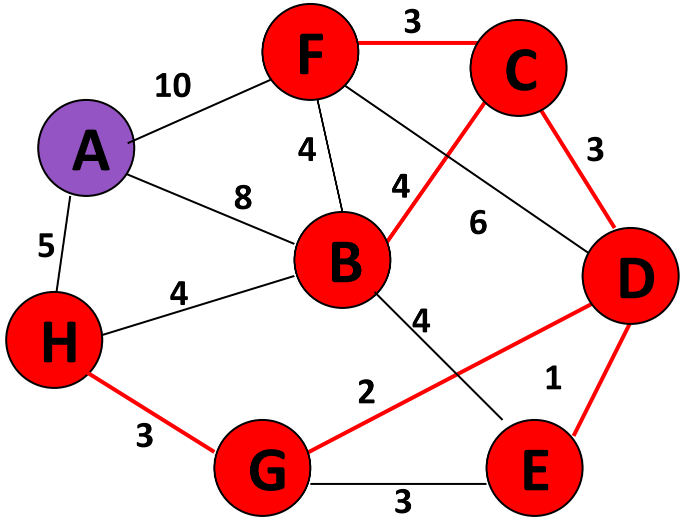


Select first |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	√
(C,F)	3	√
(B,C)	4	√

edge	d_v	
(B,E)	4	χ
(B,F)	4	
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal’s Algorithm

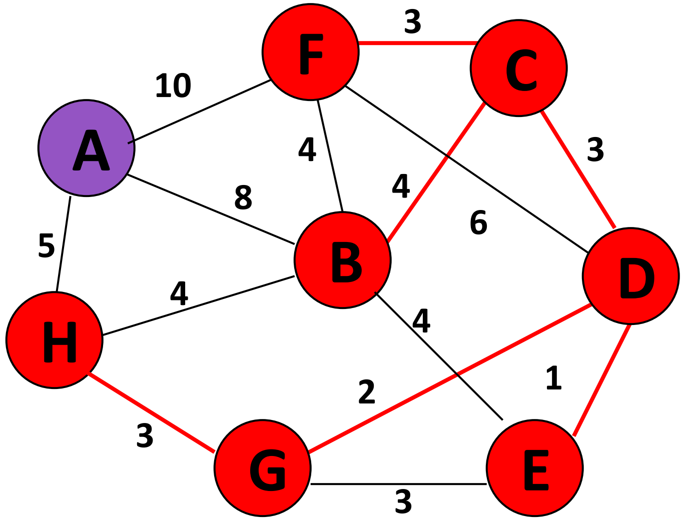


Select first |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	√
(C,F)	3	√
(B,C)	4	√

edge	d_v	
(B,E)	4	χ
(B,F)	4	χ
(B,H)	4	
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal’s Algorithm

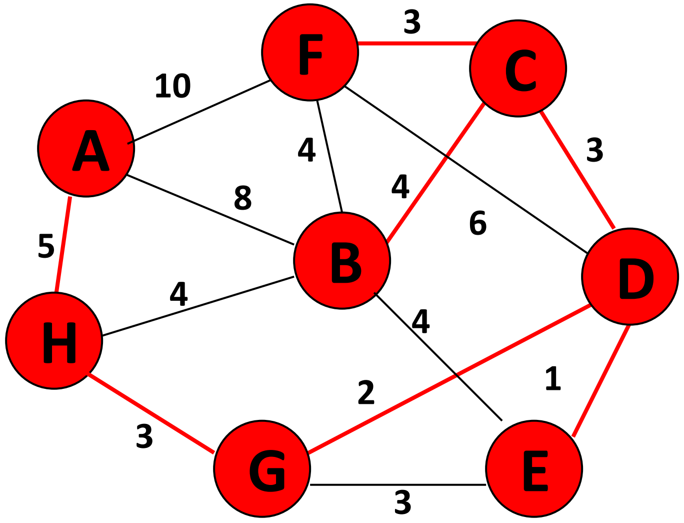


Select first |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	√
(C,F)	3	√
(B,C)	4	√

edge	d_v	
(B,E)	4	χ
(B,F)	4	χ
(B,H)	4	χ
(A,H)	5	
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal’s Algorithm

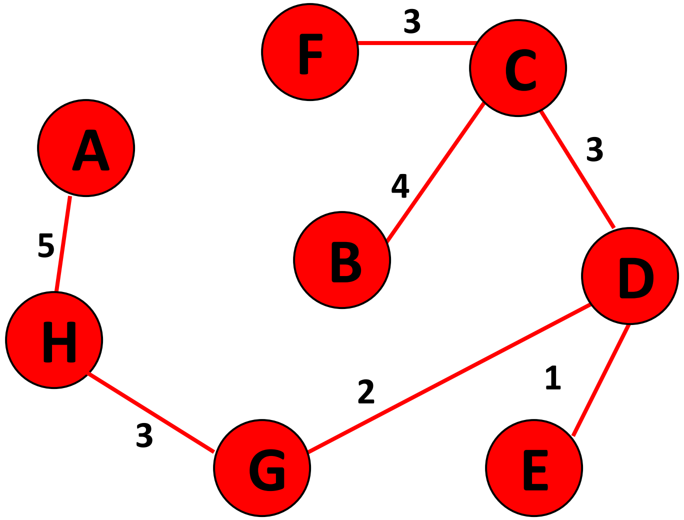


Select first |V|–1 edges which do not generate a cycle

edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	√
(C,F)	3	√
(B,C)	4	√
(B,E)	4	χ
(B,F)	4	χ
(B,H)	4	χ
(A,H)	5	√
(D,F)	6	
(A,B)	8	
(A,F)	10	

Kruskal’s Algorithm

Select first |V|–1 edges which do not generate a cycle



edge	d_v	
(D,E)	1	√
(D,G)	2	√
(E,G)	3	χ
(C,D)	3	√
(G,H)	3	√
(C,F)	3	√
(B,C)	4	√

edge	d_v	
(B,E)	4	χ
(B,F)	4	χ
(B,H)	4	χ
(A,H)	5	√
(D,F)	6	
(A,B)	8	
(A,F)	10	

} not considered

Done

Total Cost = Σ ***d_v* = 21**

∉

Prim’s Algorithm

```
//Assume G has at least one vertex
TV={0}; //start with vertex 0 and no edges

for (T=$\emptyset$; T contains less than n-1 edges; add(u,v) to T)
{
  let (u,v) be a least cost edge such that $u \in TV$ and $v \notin TV$;
  if (there is no such edge ) break;
  add v to TV;
}

if (T contains fewer than n-1 edges)
cout<<“No spanning tree\n”;
```

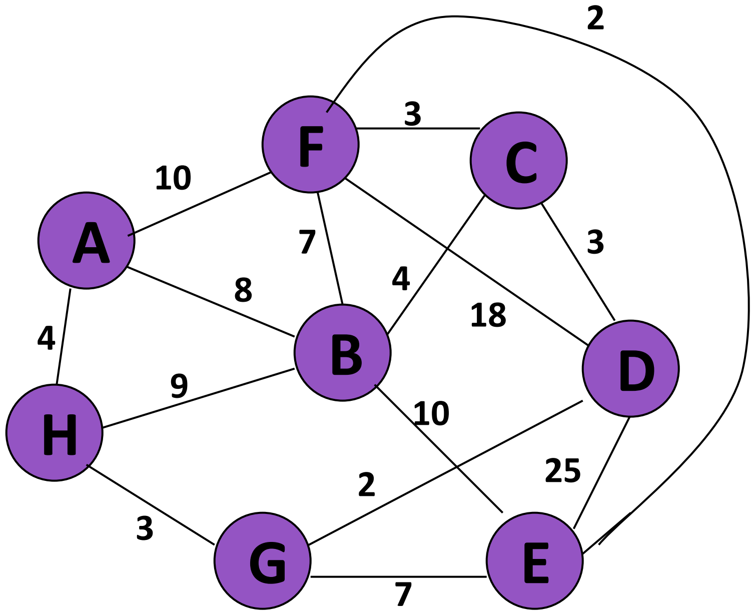
Prim's Algorithm

```
Algorithm prims(start_v){
 //cost[i][j] is either +ve or infinity.
//A MST is  computed & stored as a set of edges in the
//array  t[n][1].  t[i][0], t[i][1]) is an edge in the MST
//where 0<i<n.
// start_v be the starting vertex
{
//Initialize nearest
 nearest [start_v] =-1;
 for i=0 to n-1 do
 {          if(i!=start_v)
                   nearest[i]= start_v;
 }
 r=0;

for  i=1 to n-1 do
{  //find n-1 additional edges for t
    min= ∞
   for k=0 to n-1
   {  // find  j :  vertex such that;
             if (nearest[k]!= -1 and cost[k, nearest[k]] <min)
             {  j=k ; min= cost[k, nearest[k]];}
    }
//update tree and total cost
   t[ r][0]=j , t[r][1]=nearest[j];  r=r+1;
   mincost = mincost +cost[j, nearest[j]);
   nearest[j]=-1;

//update nearest for remaining vertices
   for k=0 to n-1
  {            if(nearest[k]!= -1 and (cost[k, nearest[k])> cost[k, j]
             nearest[k]=j;
  }
    return mincost;
   }  //end for i=1 to n-1
}
```

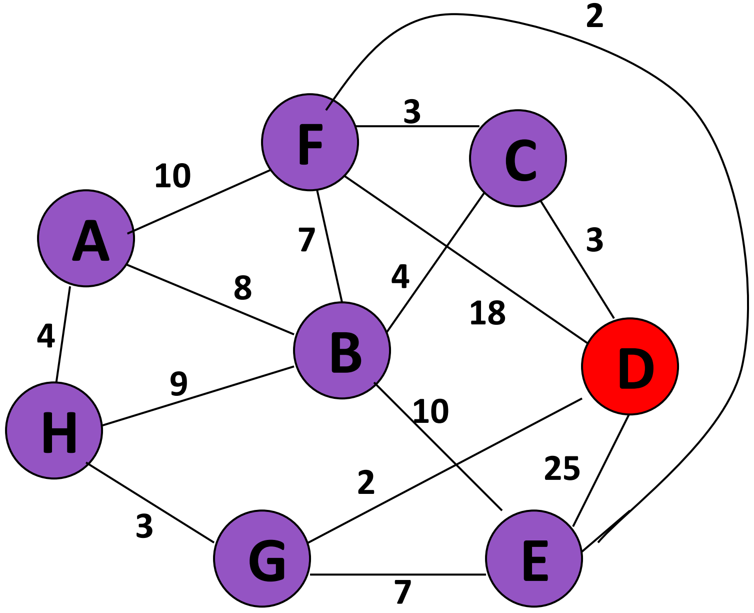
Prim's Algorithm



Initialize array

	K	d_v	p_v
A	F	∞	–
B	F	∞	–
C	F	∞	–
D	F	∞	–
E	F	∞	–
F	F	∞	–
G	F	∞	–
H	F	∞	–

Prim’s Algorithm

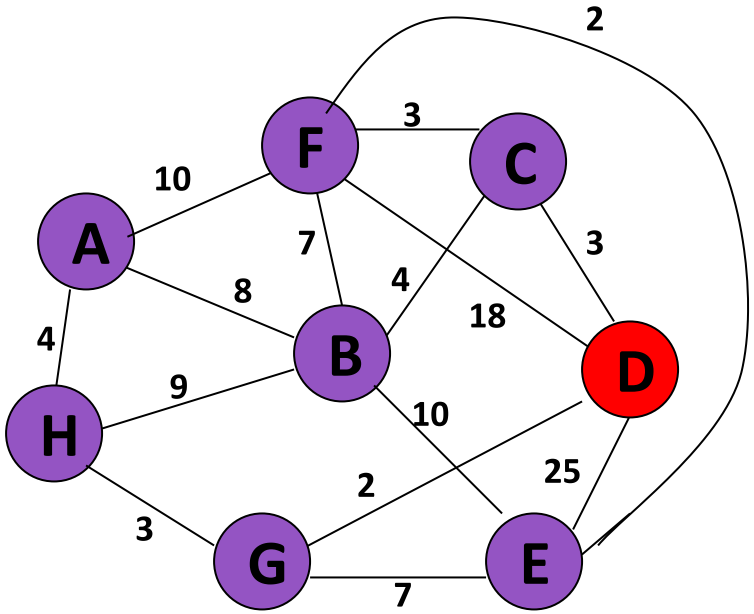


Start with any node, say D

	K	d_v	p_v
A			
B			
C			
D	T	0	–
E			
F			
G			
H			

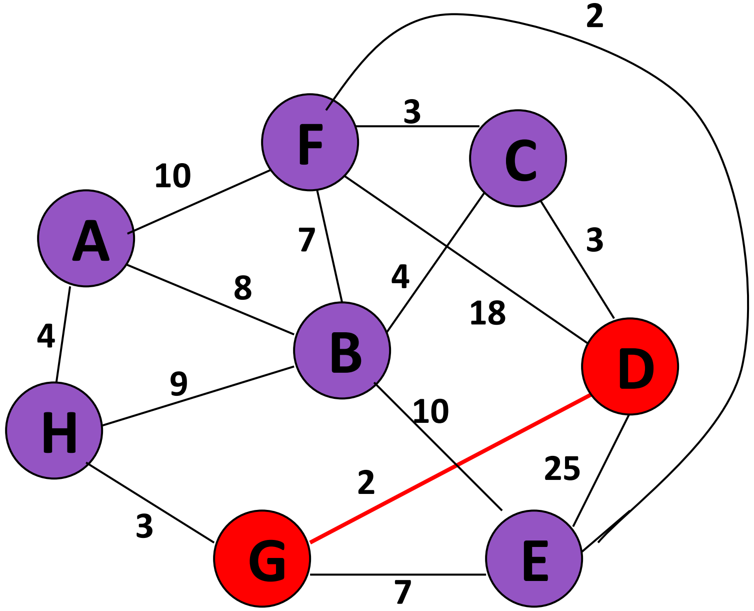
Prim’s Algorithm

Update distances of adjacent, unselected nodes



	K	d_v	p_v
A			
B			
C		3	D
D	T	0	–
E		25	D
F		18	D
G		2	D
H			

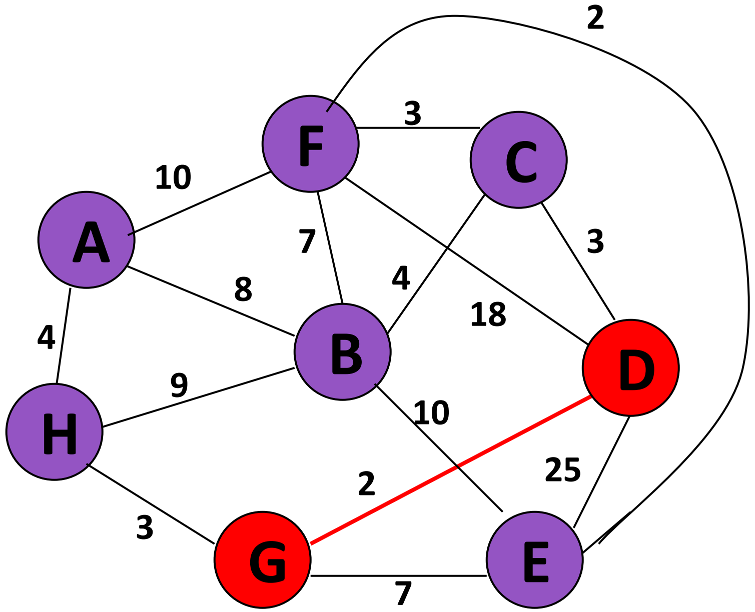
Prim’s Algorithm



Select node with minimum distance

	K	d_v	p_v
A			
B			
C		3	D
D	T	0	–
E		25	D
F		18	D
G	T	2	D
H			

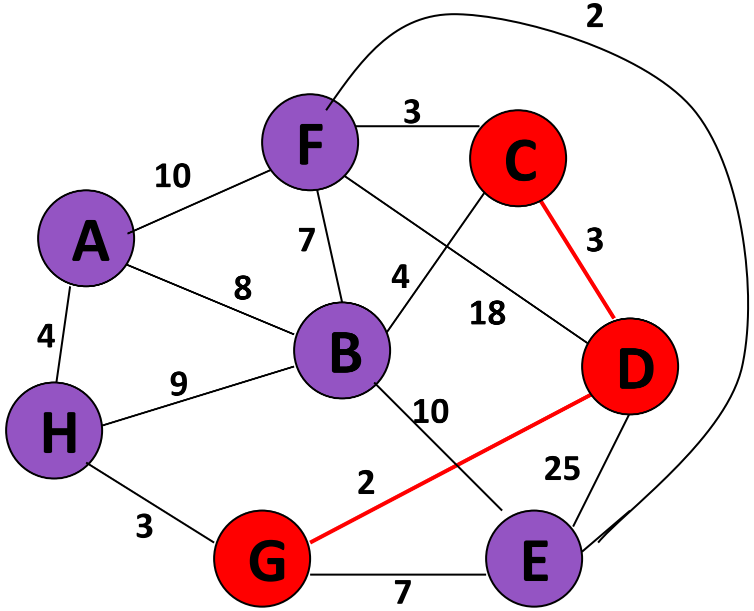
Prim’s Algorithm



Update distances of adjacent, unselected nodes

	K	d_v	p_v
A			
B			
C		3	D
D	T	0	–
E		7	G
F		18	D
G	T	2	D
H		3	G

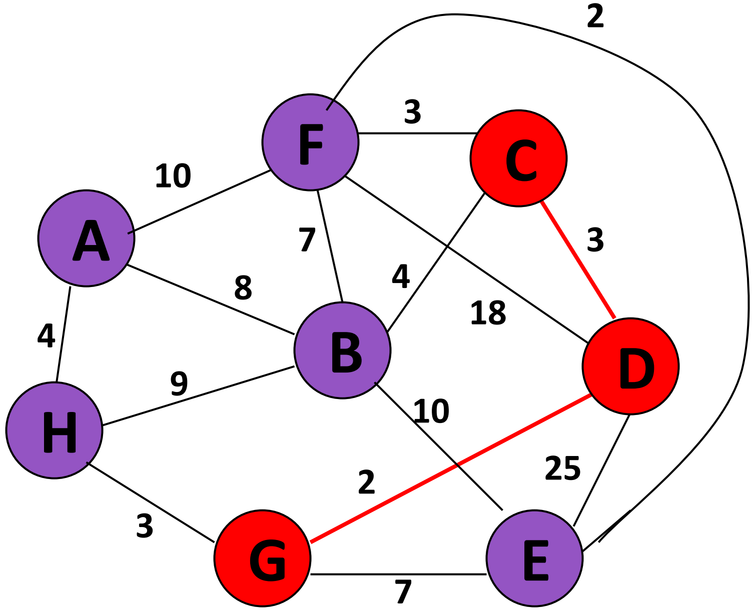
Prim's Algorithm



Select node with minimum distance

	K	d_v	p_v
A			
B			
C	T	3	D
D	T	0	–
E		7	G
F		18	D
G	T	2	D
H		3	G

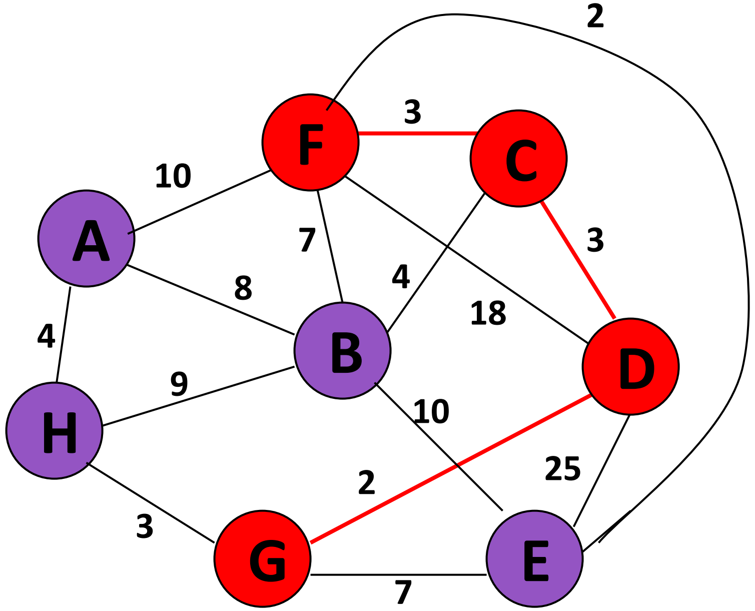
Prim’s Algorithm



Update distances of adjacent, unselected nodes

	K	d_v	p_v
A			
B		4	C
C	T	3	D
D	T	0	–
E		7	G
F		3	C
G	T	2	D
H		3	G

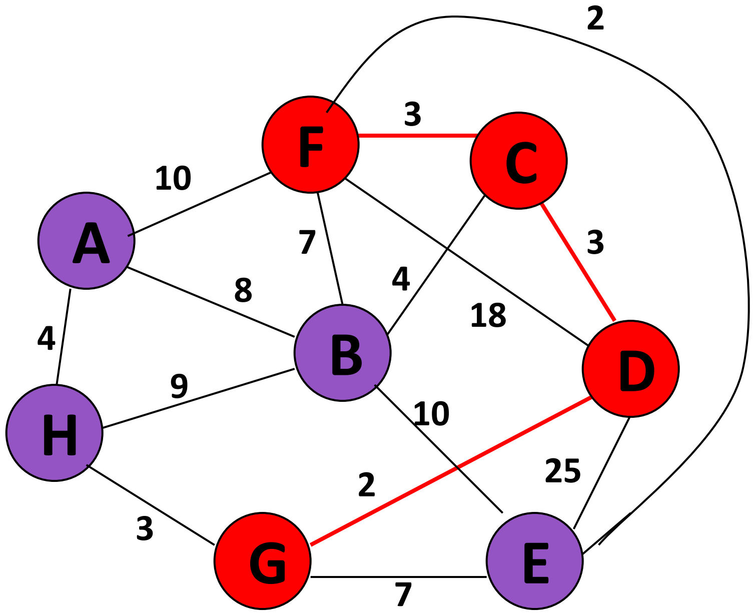
Prim’s Algorithm



Select node with minimum distance

	K	d_v	p_v
A			
B		4	C
C	T	3	D
D	T	0	–
E		7	G
F	T	3	C
G	T	2	D
H		3	G

Prim’s Algorithm

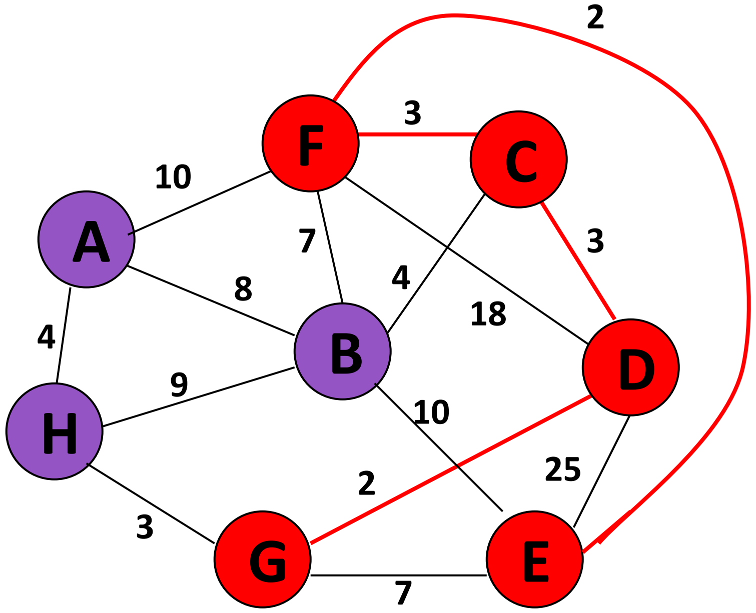


Update distances of adjacent, unselected nodes

	K	d_v	p_v
A		10	F
B		4	C
C	T	3	D
D	T	0	–
E		2	F
F	T	3	C
G	T	2	D
H		3	G

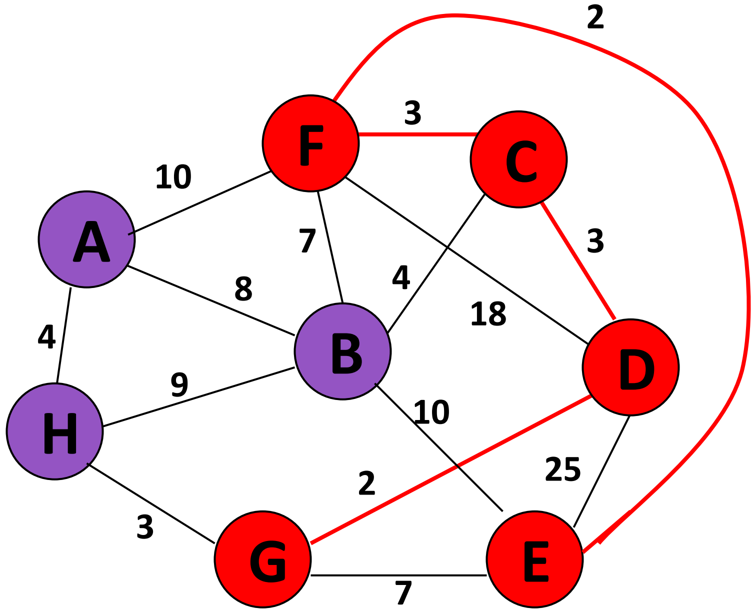
Prim’s Algorithm

Select node with minimum distance



	K	d_v	p_v
A		10	F
B		4	C
C	T	3	D
D	T	0	–
E	T	2	F
F	T	3	C
G	T	2	D
H		3	G

Prim's Algorithm

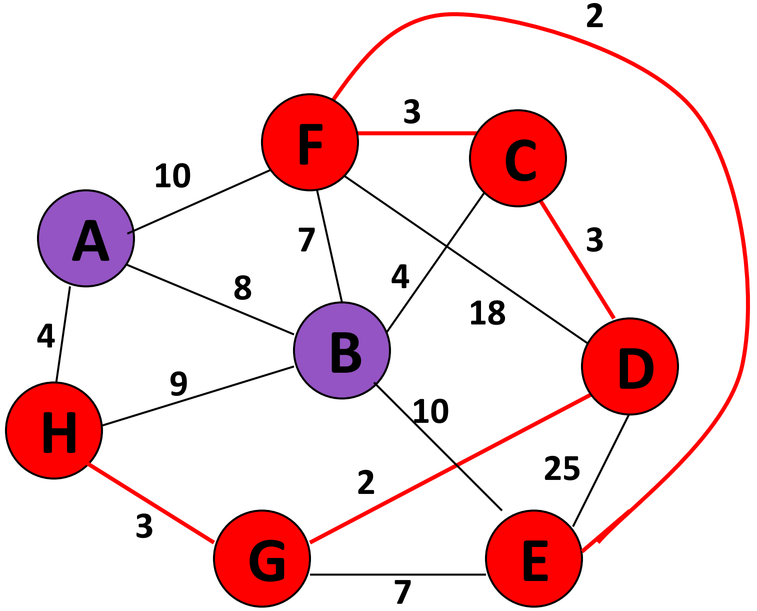


Update distances of adjacent, unselected nodes

	K	d_v	p_v
A		10	F
B		4	C
C	T	3	D
D	T	0	–
E	T	2	F
F	T	3	C
G	T	2	D
H		3	G

Table entries unchanged

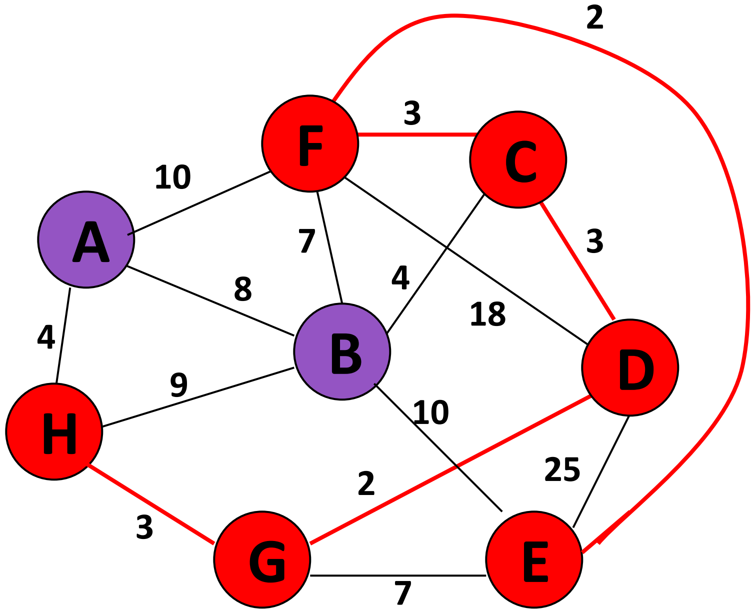
Prim’s Algorithm



Select node with minimum distance

	K	d_v	p_v
A		10	F
B		4	C
C	T	3	D
D	T	0	–
E	T	2	F
F	T	3	C
G	T	2	D
H	T	3	G

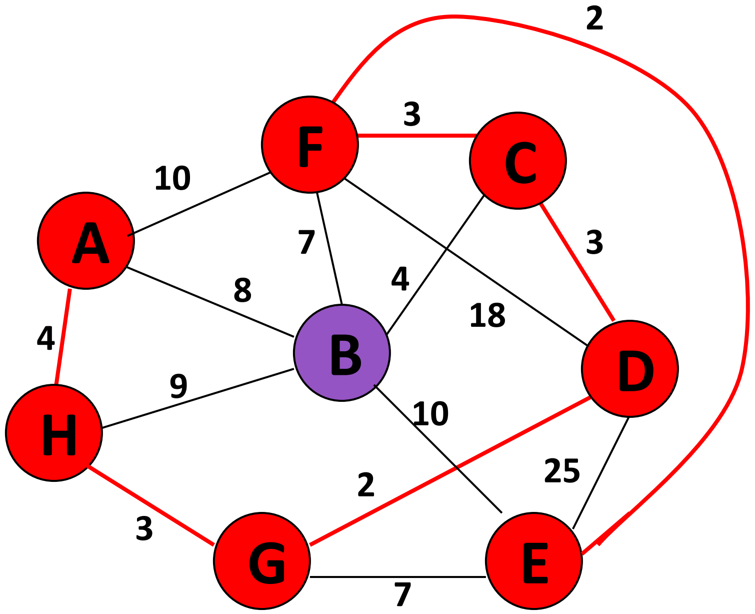
Prim's Algorithm



Update distances of adjacent, unselected nodes

	K	d_v	p_v
A		4	H
B		4	C
C	T	3	D
D	T	0	–
E	T	2	F
F	T	3	C
G	T	2	D
H	T	3	G

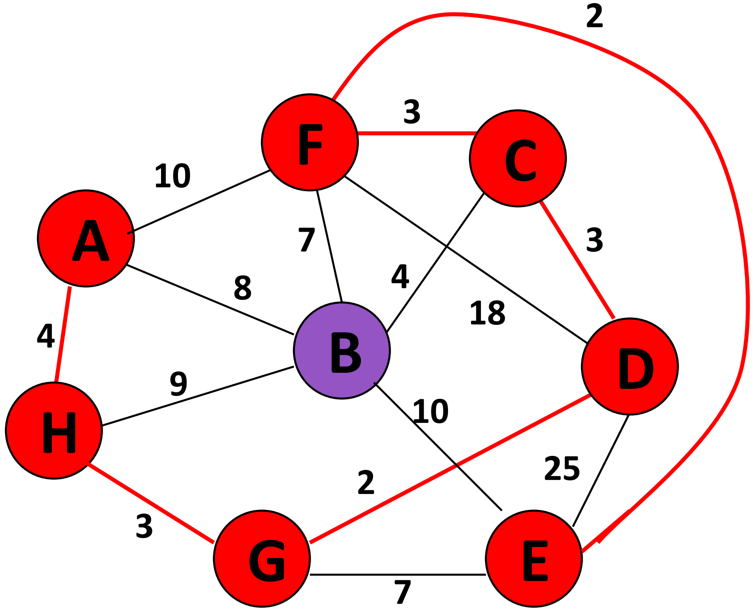
Prim's Algorithm



Select node with minimum distance

	K	d_v	p_v
A	T	4	H
B		4	C
C	T	3	D
D	T	0	–
E	T	2	F
F	T	3	C
G	T	2	D
H	T	3	G

Prim's Algorithm

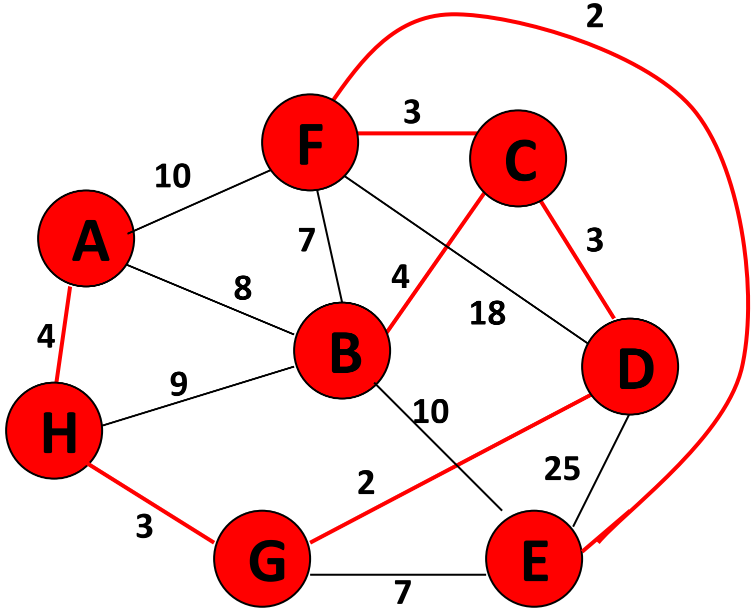


Update distances of adjacent, unselected nodes

	K	d_v	p_v
A	T	4	H
B		4	C
C	T	3	D
D	T	0	–
E	T	2	F
F	T	3	C
G	T	2	D
H	T	3	G

Table entries unchanged

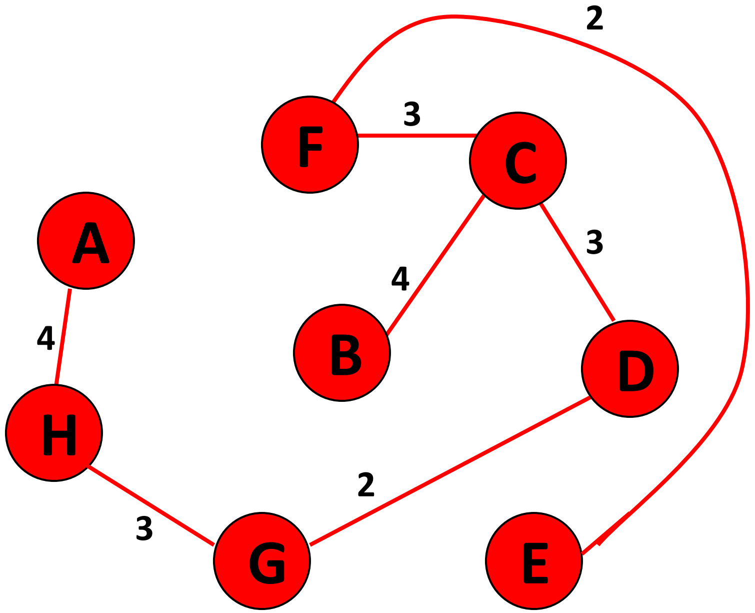
Prim’s Algorithm



Select node with minimum distance

	K	d_v	p_v
A	T	4	H
B	T	4	C
C	T	3	D
D	T	0	–
E	T	2	F
F	T	3	C
G	T	2	D
H	T	3	G

Prim’s Algorithm



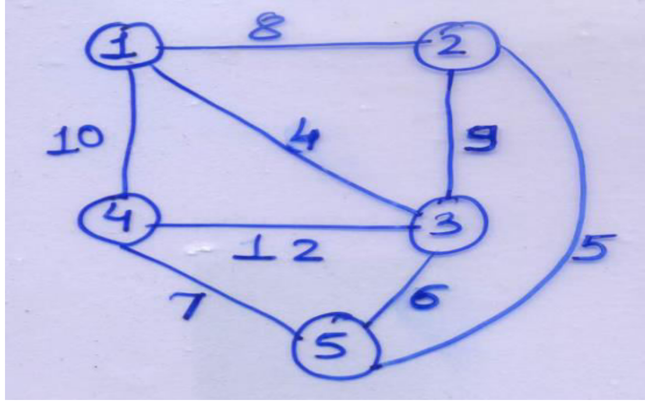
Cost of Minimum Spanning Tree = Σ d_v = 21

	K	d_v	p_v
A	T	4	H
B	T	4	C
C	T	3	D
D	T	0	–
E	T	2	F
F	T	3	C
G	T	2	D
H	T	3	G

Done

Algorithm prims(E,cost,n,t)

```
{   // Stv be the starting vertex
      nearest [Stv] =-1;
     for i=1 to n do     //Initialize nearest
     {     if(i!=Stv)
               nearest[i]=Stv;
     }  r=1;
    for  i=1 to n-1 do
    {  //find n-1 additional edges for t
          min= ∞
        for k=1cto n //find minimum
        {  if (nearest[k]!= -1 and cost[k, nearest[k]] <min)
                    j=k ; min= cost[k, nearest[k]];
        } //end of k loop
#       find  j : the index(or vertex)such that;
          t[r][1]=j , t[r][2]=nearest[j];  r=r+1;
        mincost = mincost +cost[j, nearest[j]);
              nearest[j]=-1;
        for k=1 to n     //update nearest
        {  if(nearest[k]!= -1 and (cost[k, nearest[k])>
                                   cost[k, j]  then
                    nearest[k]=j;
          } //end of k loop
      } //end of i loop
        return mincost;
}//end of algorithm
```



Cost Adjacency Matrix

	1	2	3	4	5
1	∞	8	4	10	∞
2	8	∞	9	∞	5
3	4	9	∞	12	6
4	10	∞	12	∞	7
5	∞	5	6	7	∞

Initial values of Near

Stv=1	
Near[1]	= -1
Near[2]	= 1
Near[3]	= 1
Near[4]	= 1
Near[5]	= 1

For i=1 find minimum edge connecting to v1

Stv=1	
cost[2,near[2]]	=8
cost[3,near[3]]	=4
cost[4,near[4]]	=10
Cost[5,near[5]]	= ∞

j=3

Put near[1]= -1 as it has been included in ST

	1	2	3	4	5
1	∞	8	4	10	∞
2	8	∞	9	∞	5
3	4	9	∞	12	6
4	10	∞	12	∞	7
5	∞	5	6	7	∞

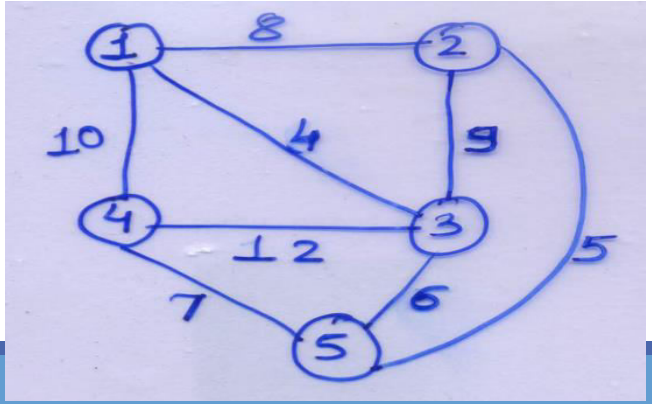
Update Near matrix i.e. (cost [i][j] < cost[j][near[j]])

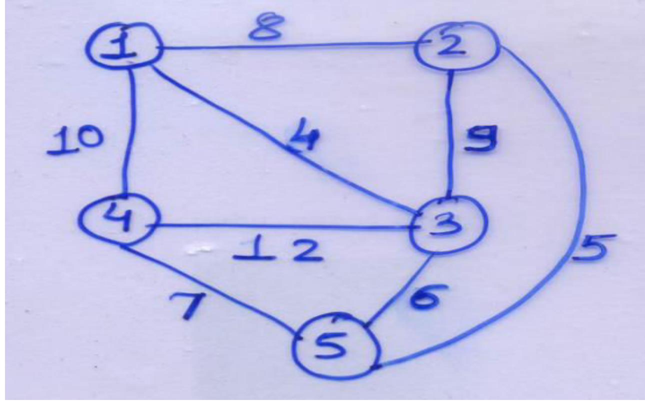
J=3	
cost[2,3] < cost[2,near[2]]	9 < 8
Cost[4,3] < cost[4,near[4]]	12< 10
Cost[5,3] < Cost[5,near[5]]	6 < ∞

j=3	
Near[1]	= -1
Near[2]	= 1
Near[3]	= -1
Near[4]	= 1
Near[5]	= 3

T	1 (v1)	2 (v2)	3 (cost)
1	1	3	4
2			
3			
4			
5			
6			

Put near[3]= -1 as it has been included in ST





--

For i=2 find minimum edge connecting to v1 / v3

Find next j

Cost Adjacancy Matrix

	1	2	3	4	5
1	∞	8	4	10	∞
2	8	∞	9	∞	5
3	4	9	∞	12	6
4	10	∞	12	∞	7
5	∞	5	6	7	∞

j=3	
Near[1]	= -1
Near[2]	= 1
Near[3]	= -1
Near[4]	= 1
Near[5]	= 3

j=3	
cost[2,near[2]]	=8
cost[4,near[4]]	=10
Cost[5,near[5]]	= 6

j=5

Cost Adjacency Matrix

	1	2	3	4	5
1	∞	8	4	10	∞
2	8	∞	9	∞	5
3	4	9	∞	12	6
4	10	∞	12	∞	7
5	∞	5	6	7	∞

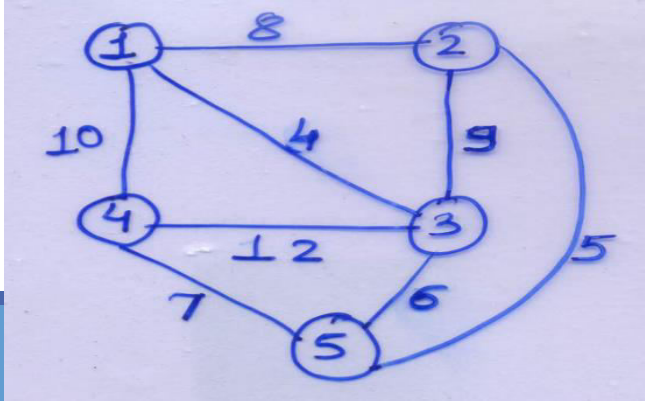
Update Near matrix i.e. (cost [i][j] < cost[j][near[j]])

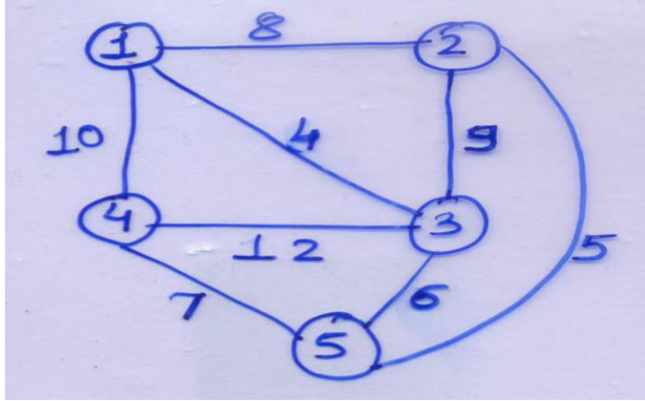
J=5	
cost[2,5] < cost[2,near[2]]	5 < 8
Cost[4,5] < cost[4,near[4]]	7< 10

j=5	
Near[1]	= -1
Near[2]	= 5
Near[3]	= -1
Near[4]	= 5
Near[5]	= -1

T	1 (v1)	2 (v2)	3 (cost)
1	1	3	4
2	3	5	6
3			
4			
5			
6			

Put near[5]= -1 as it has been included in ST





For i=3 find minimum edge connecting to v1 / v3/v5

Find next j

Cost Adjacency Matrix

	1	2	3	4	5
1	∞	8	4	10	∞
2	8	∞	9	∞	5
3	4	9	∞	12	6
4	10	∞	12	∞	7
5	∞	5	6	7	∞

j=5	
Near[1]	= -1
Near[2]	= 5
Near[3]	= -1
Near[4]	= 5
Near[5]	= -1

j=5	
cost[2,near[2]]	=5
cost[4,near[4]]	=7

j=2

Put near[2]= -1 as it has been included in ST

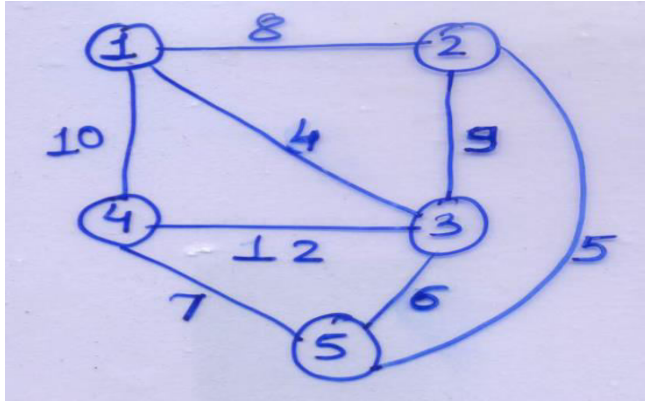
	1	2	3	4	5
1	∞	8	4	10	∞
2	8	∞	9	∞	5
3	4	9	∞	12	6
4	10	∞	12	∞	7
5	∞	5	6	7	∞

J=2	
Cost[4,2] < cost[4,near[4]]	∞ < 7

j=2	
Near[1]	= -1
Near[2]	= -1
Near[3]	= -1
Near[4]	= 5
Near[5]	= -1

T	1 (v1)	2 (v2)	3 (cost)
1	1	3	4
2	3	5	6
3	2	5	5
4	4	5	7
5			
6			

Put near[4]= -1 as it has been included in ST



	1	2	3	4	5
1	∞	8	4	10	∞
2	8	∞	9	∞	5
3	4	9	∞	12	6
4	10	∞	12	∞	7
5	∞	5	6	7	∞

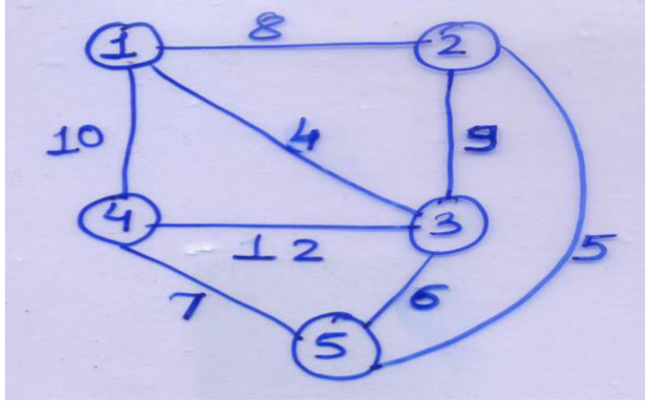
For i=4 find minimum edge connecting to v1 / v3/v5/v2

J=2	
Cost[4,2] < cost[4,near[4]]	∞ < 7

j=2	
Near[1]	= -1
Near[2]	= -1
Near[3]	= -1
Near[4]	= 5 j=4
Near[5]	= -1

T	1 (v1)	2 (v2)	3 (cost)
1	1	3	4
2	3	5	6
3	2	5	5
4	4	5	7
5			
6			

Put near[4]= -1 as it has been included in ST



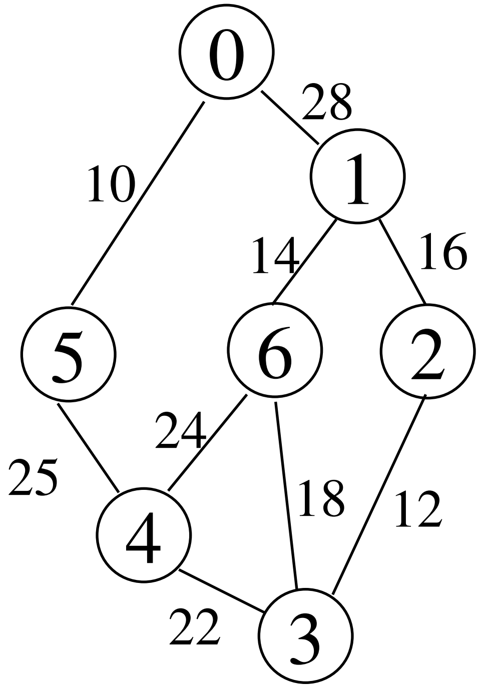
Analysis of Prim’s Algorithm

- Run time will be O(n^2) .

- Unlike Kruskal’s, it doesn’t need to see all of the graph at once. It can deal with it one piece at a time. It also doesn’t need to worry if adding an edge will create a cycle since this algorithm deals primarily with the nodes, and not the edges.

Home Assignment

Find MST for given Graph G1 using Prim’s and Kruskal Algorithm



Comparison Prim’s and Kruskal’s Algorithm

Prim’s Algorithm	Kruskal’s Algorithm
Starts to build the Minimum Spanning Tree from any vertex in the graph.	Starts to build the Minimum Spanning Tree from the vertex carrying minimum weight in the graph.
Time complexity of O(V^2)	Time complexity is O(E log V)
Gives connected component as well as it works only on connected graph.	Generate forest(disconnected components) at any instant as well as it can work on disconnected components
Runs faster in dense graphs.	Runs faster in sparse graphs.
Prefer list data structures.	Prefer heap data structures.
Applications -Travelling Salesman Problem, Network for roads and Rail tracks connecting all the cities etc.	Applications -e LAN connection, TV Network etc.

Shortest Path Problems

- Directed weighted graph.
- Path length is sum of weights of edges on path.
- The vertex at which the path begins is the source vertex.
- The vertex at which the path ends is the destination vertex.

Shortest Path Problems

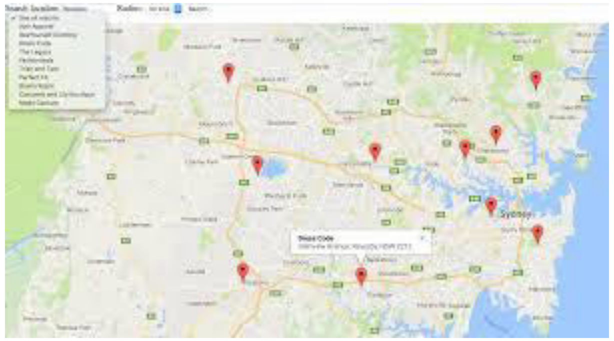
- Single source single destination.
- Single source all destinations.
- All pairs (every vertex is a source and destination).

Dijkstra Algorithm

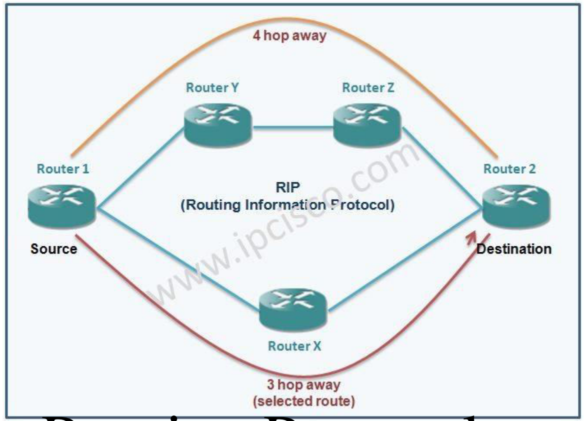
- Finds Single source all destination shortest paths
- Uses Greedy Method
- No negative weights are allowed
- Application

□ Routing protocols in computer networks

□ Google Maps and many more..



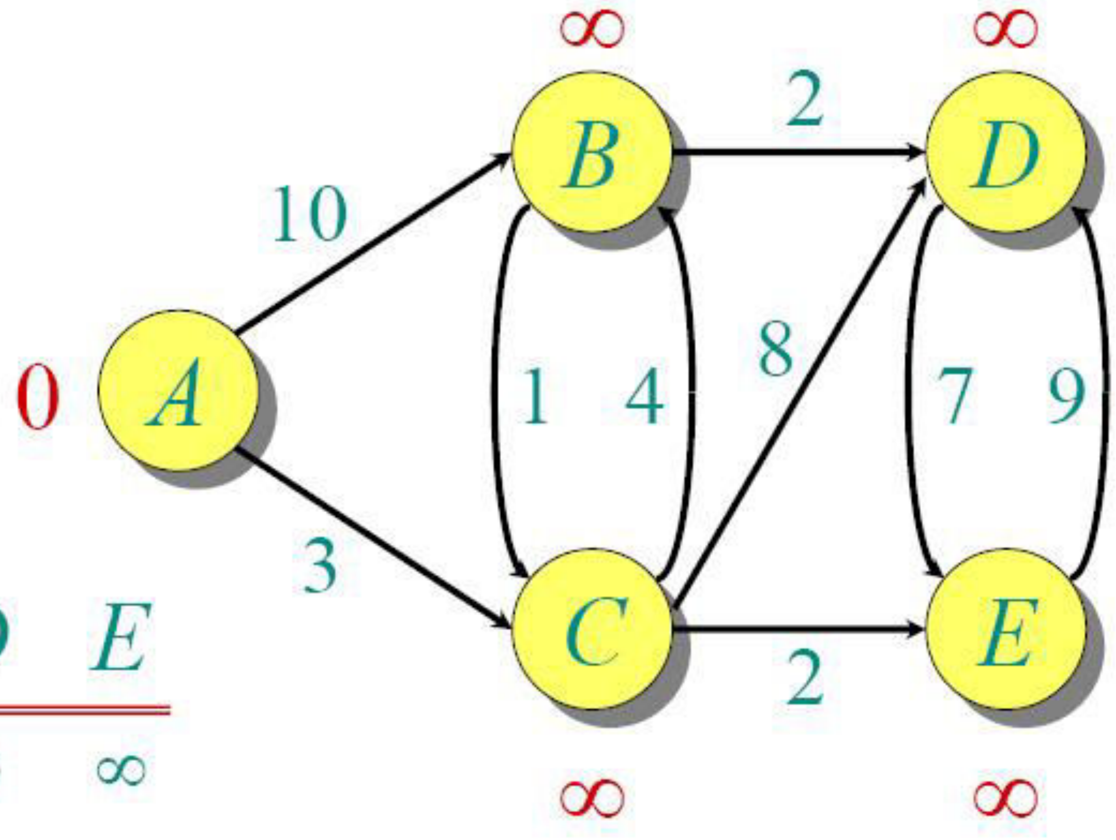
Google Maps



Routing Protocols

Dijkstra Example

Initialize:

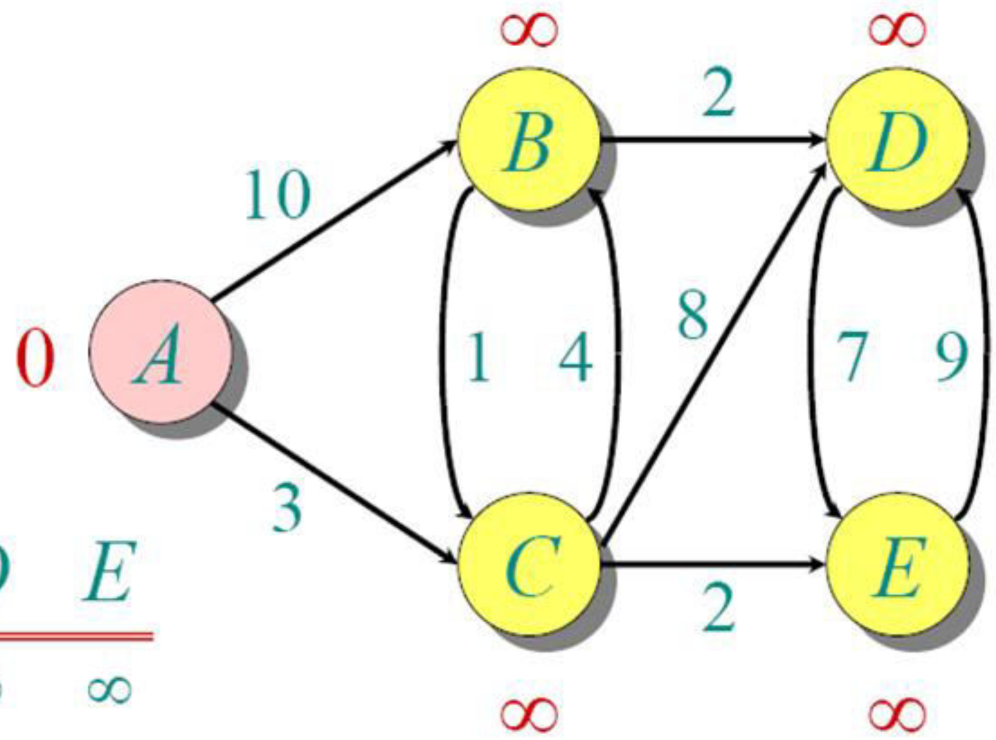


Q:	A	B	C	D	E
	0	∞	∞	∞	∞

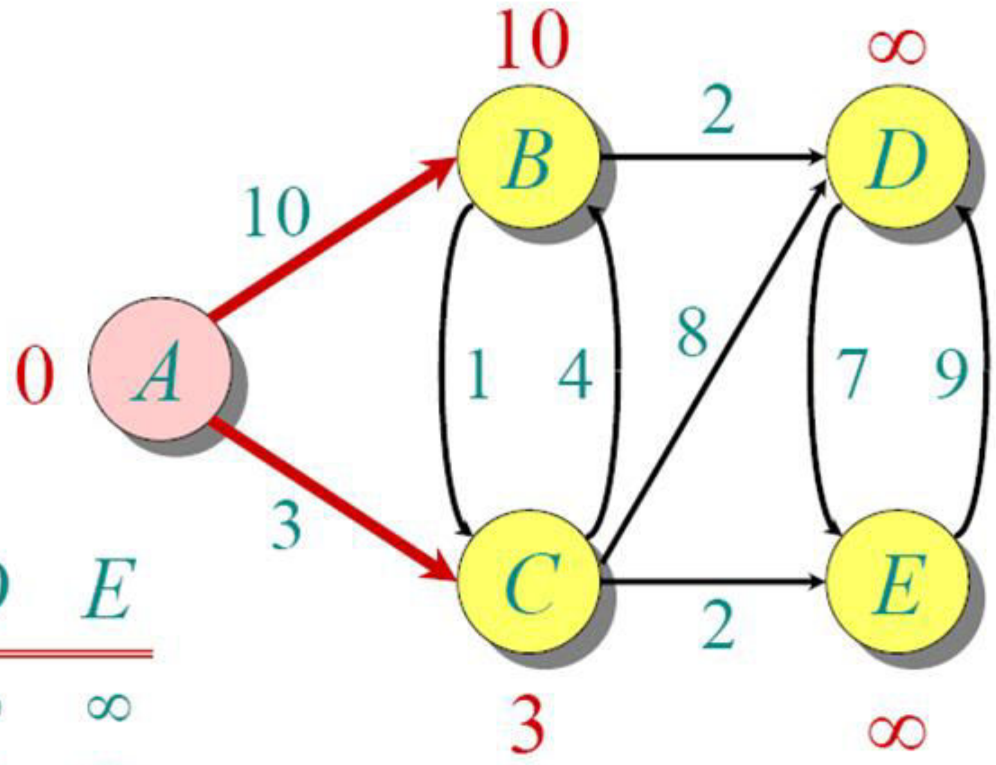
S: {}

Dijkstra Example

Q:	A	B	C	D	E
	0	∞	∞	∞	∞



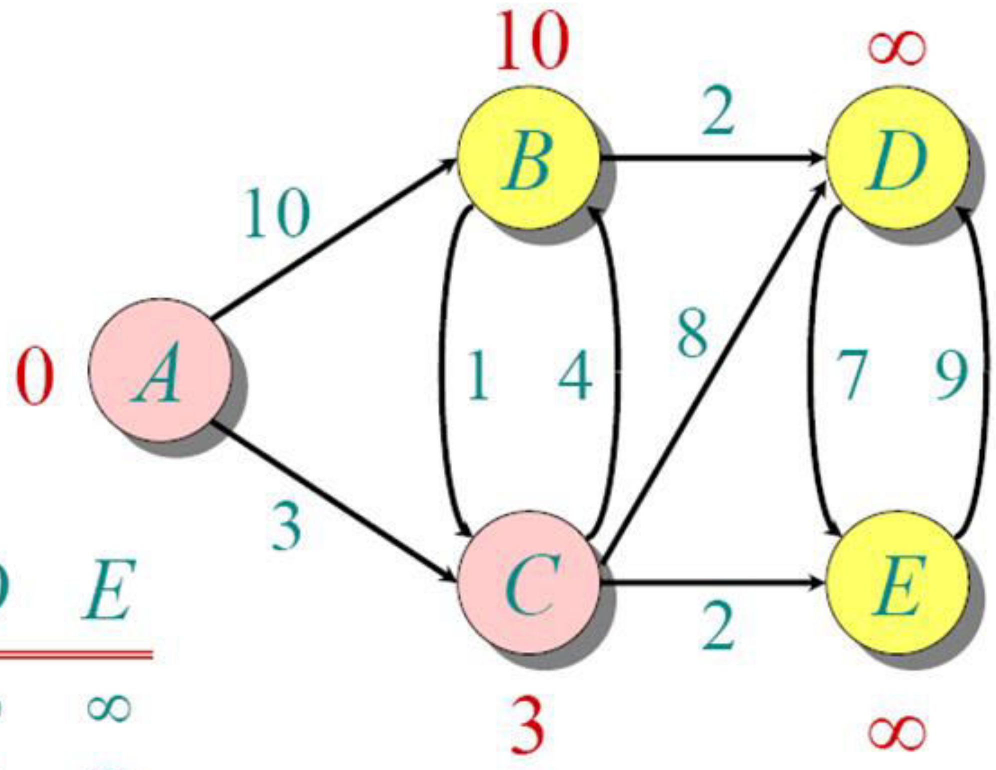
Dijkstra Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞

S: { A }

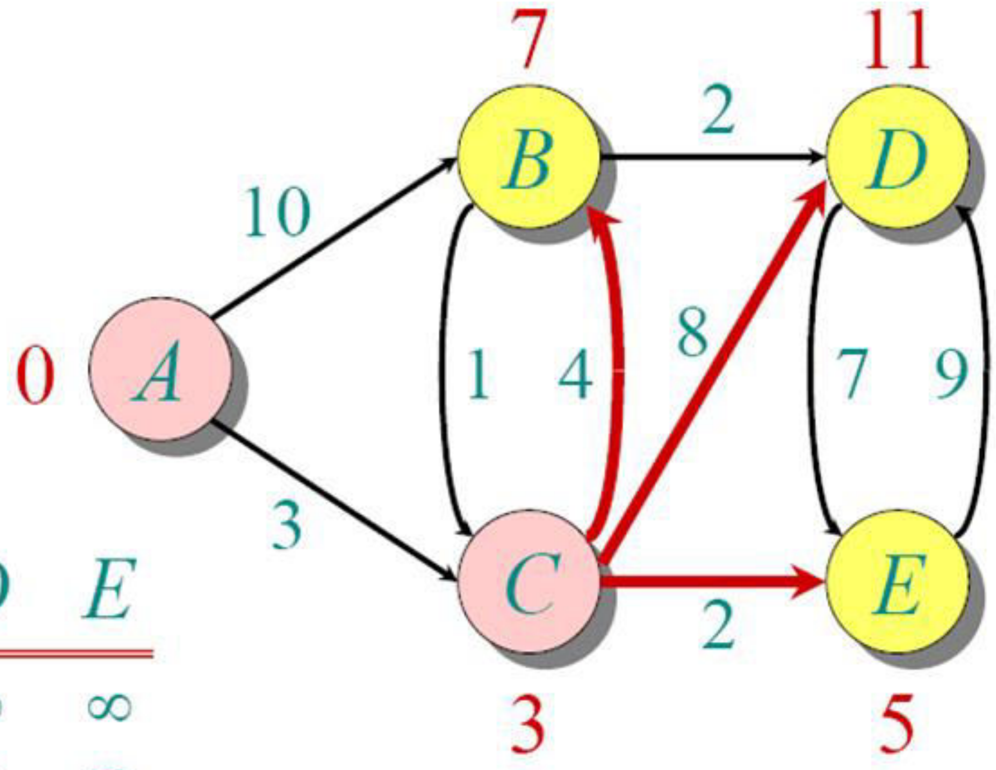
Dijkstra Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞

$S: \{ A, C \}$

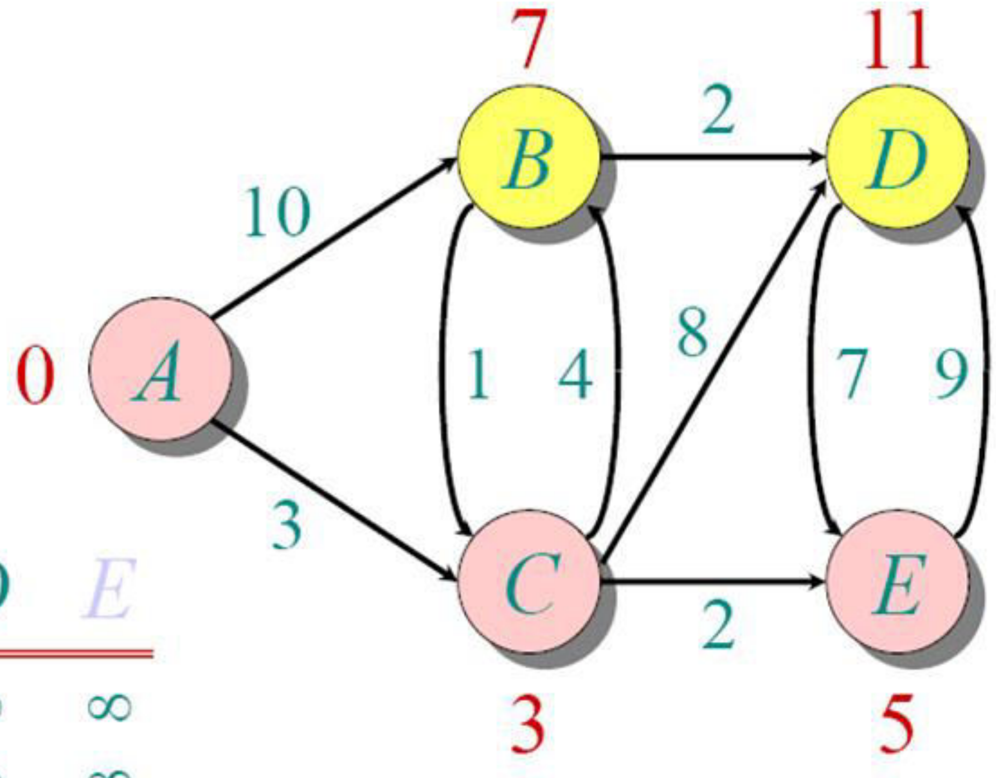
Dijkstra Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5

S: { A, C }

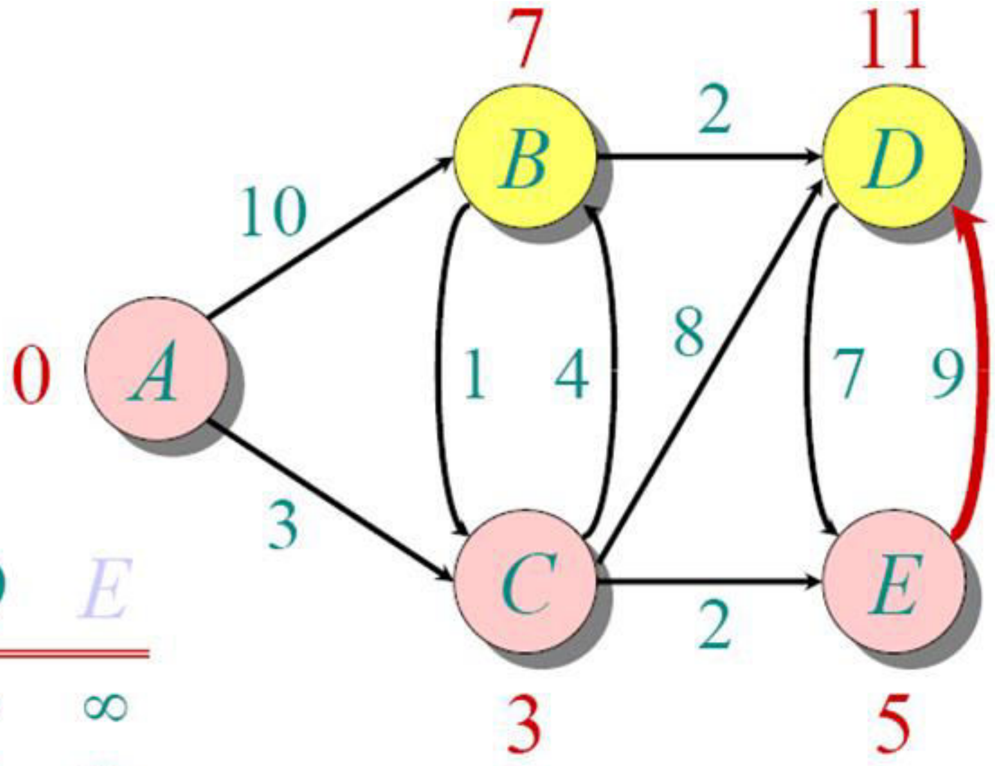
Dijkstra Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5

S: { A, C, E }

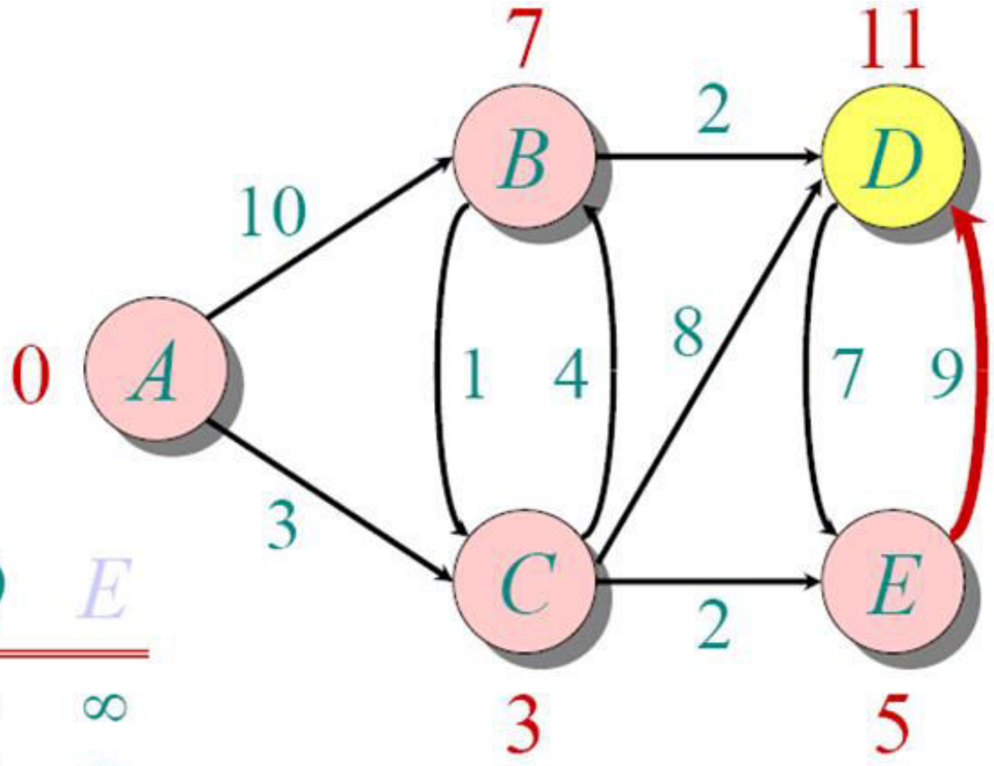
Dijkstra Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	

S: { A, C, E }

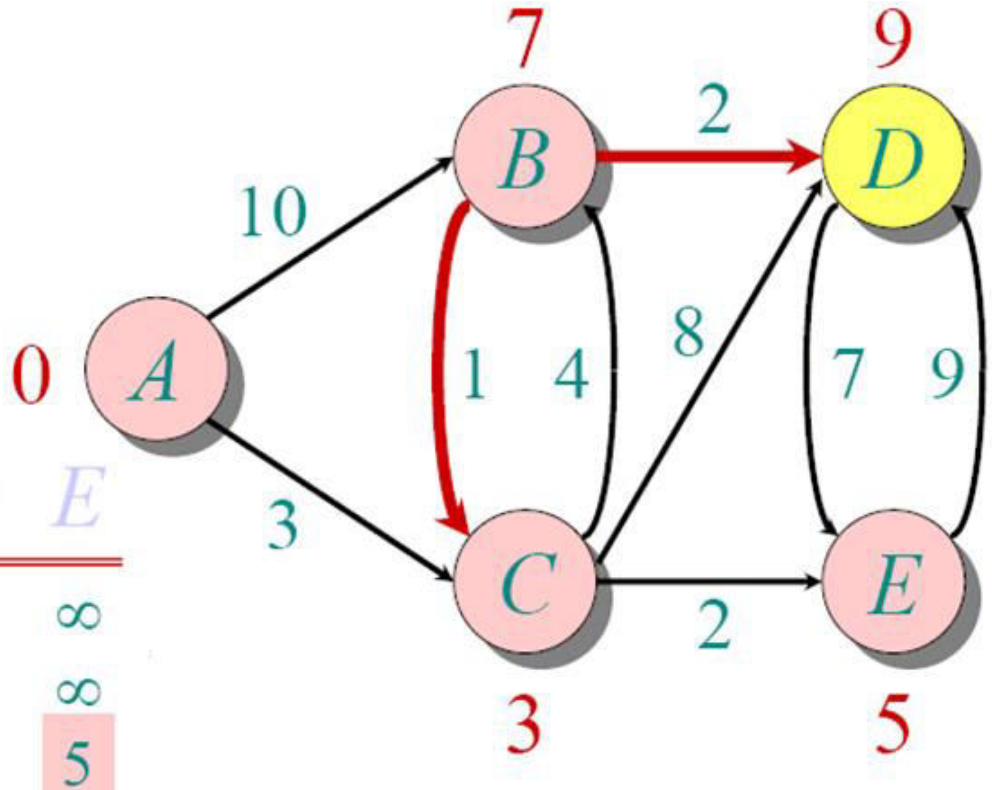
Dijkstra Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	

S: { A, C, E, B }

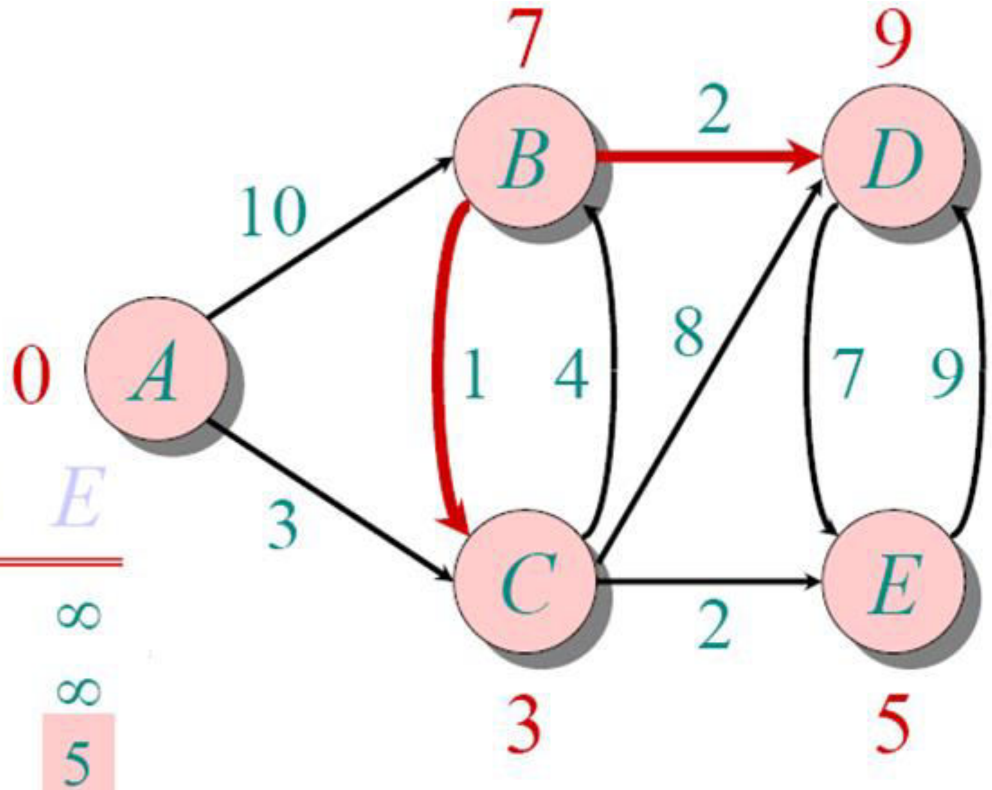
Dijkstra Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	
				9	

S: { A, C, E, B }

Dijkstra Example



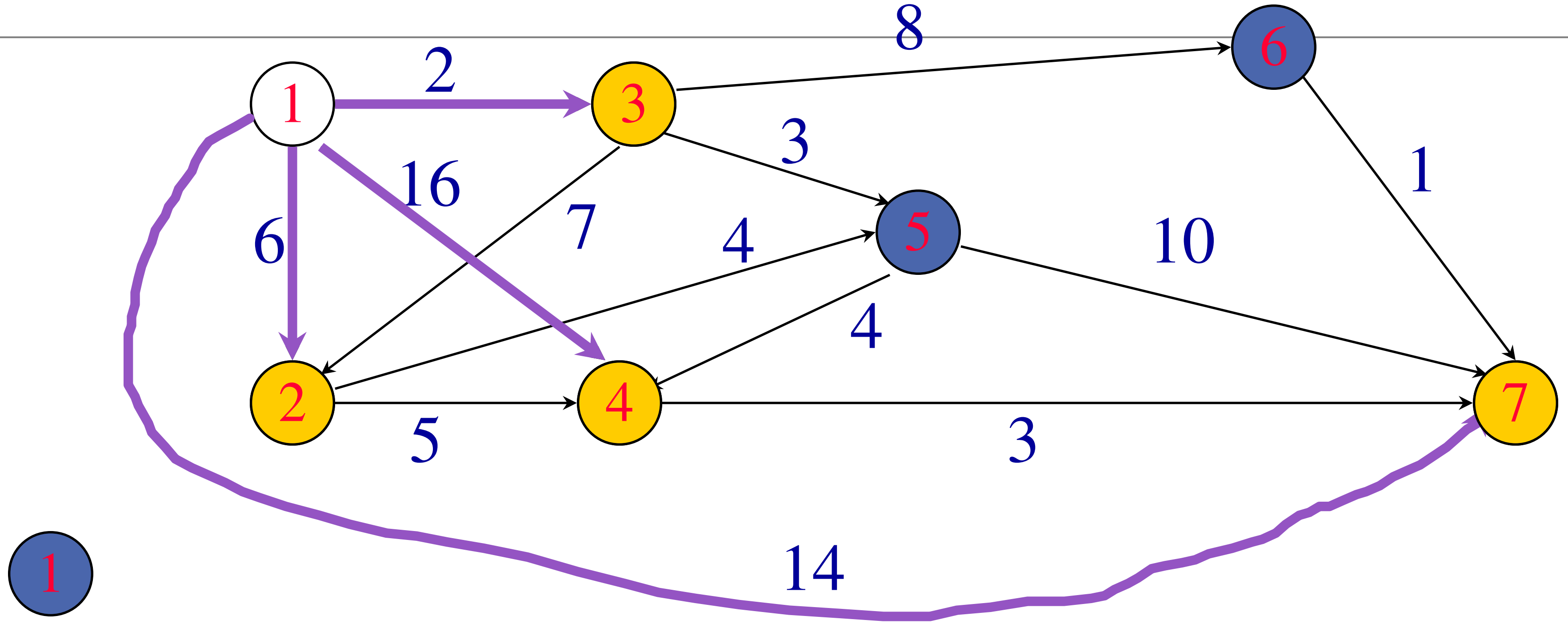
Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	
				9	

S: { A, C, E, B, D }

Dijkstra Algorithm

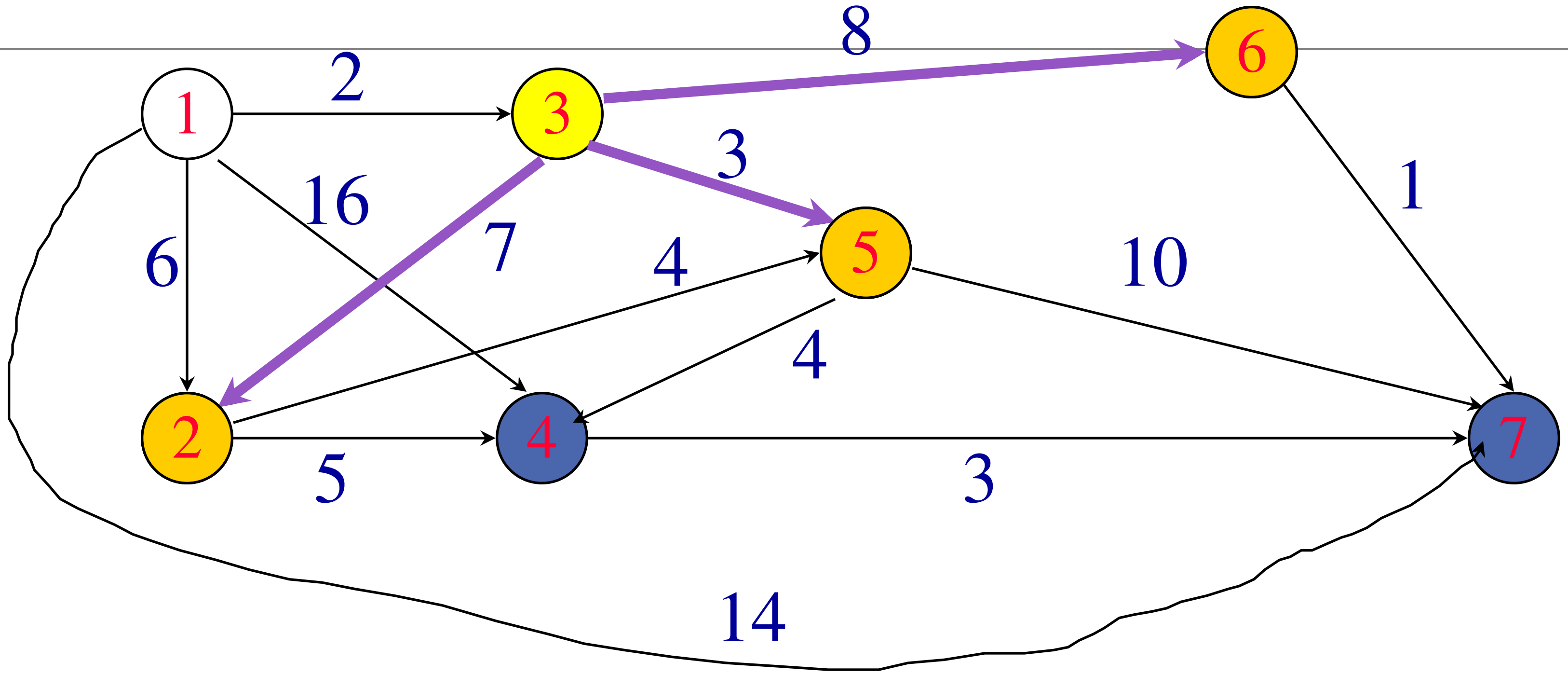
```
Algorithm shortestPath(v)
{ //v is source vertex
      for (i=0;i<n; i++)
      {
            s[i] = 0;
            dist[i]=cost[v][i];
      }
      //put v into s
       s[v] = 1;
      dist[v] =0;
      for(j=2;j<n;  j++)
      {
            //choose u from among those vertices not in s
               such that dist[u] is minimum;
            s[u] = 1;
            for each(w adjacent to u with s[w] = 0)
            {
               if(dist[w] > dist[u]+cost[u][w] )
                        dist[w] = dist[u]+cost[u][w];
            }
      }
}
```

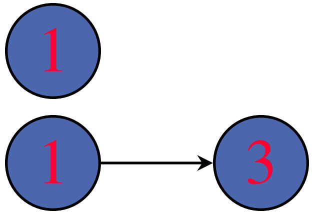
Dijkstra Algorithm



	[1]	[2]	[3]	[4]	[5]	[6]	[7]
d	0	6	2	16	-	-	14
p	-	1	1	1	-	-	1

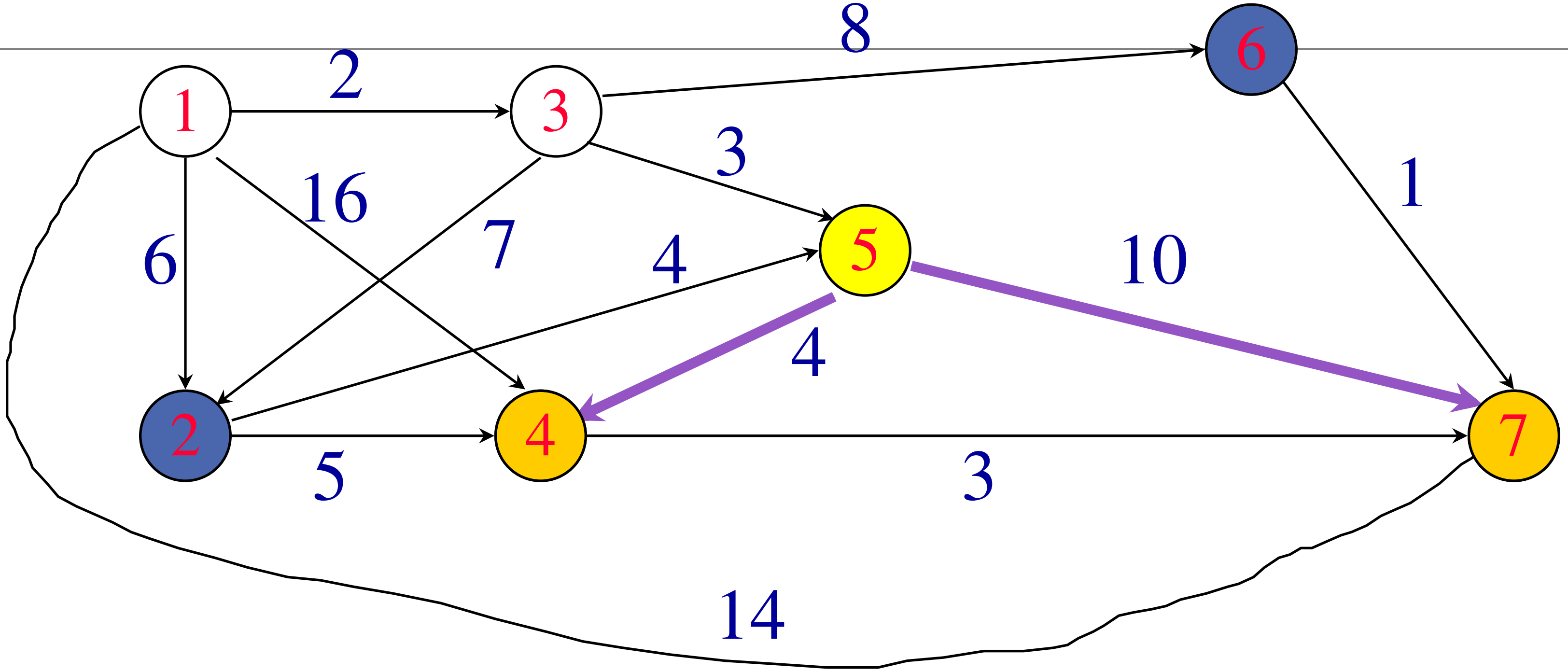
Dijkstra Algorithm

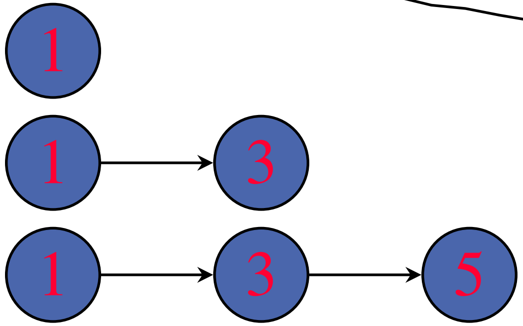




	[1]	[2]	[3]	[4]	[5]	[6]	[7]
d	0	6	2	16	5	10	14
p	-	1	1	1	3	3	1

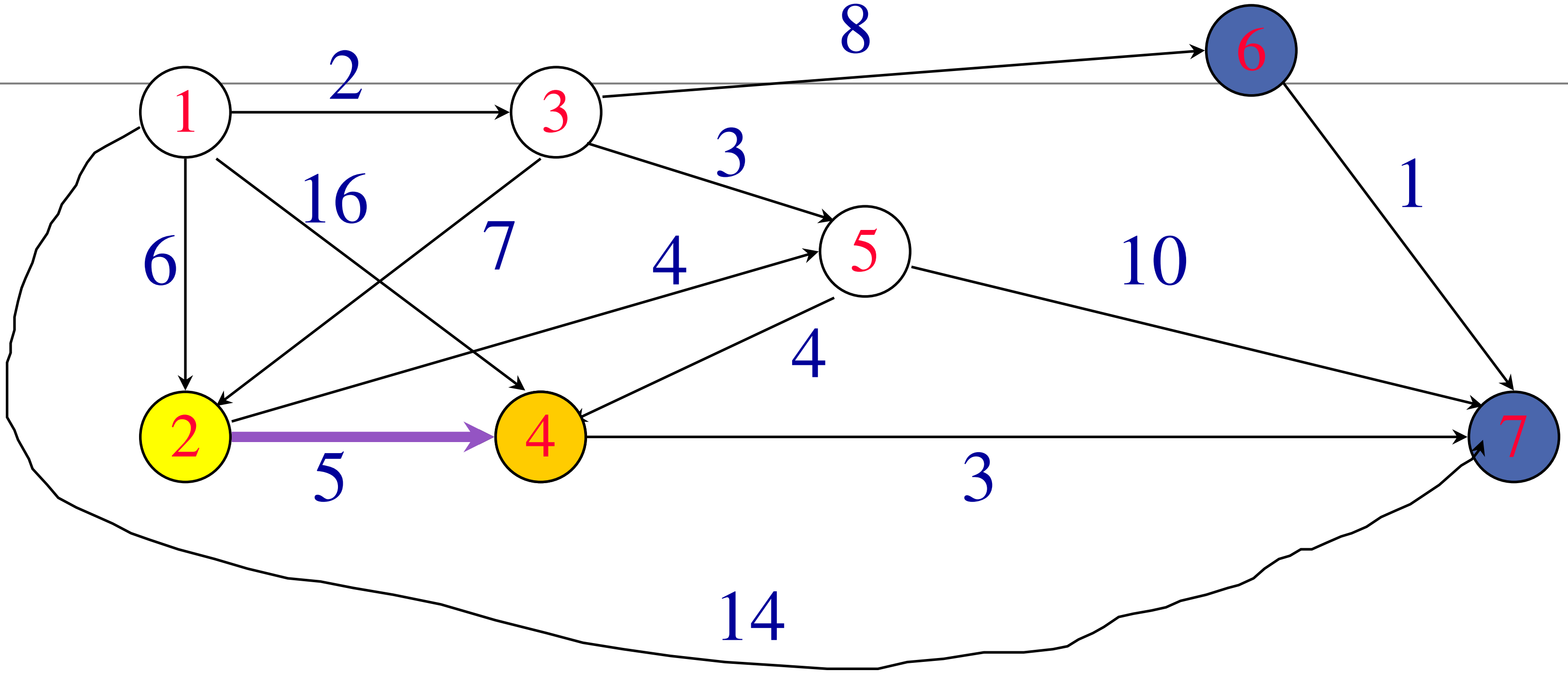
Dijkstra Algorithm

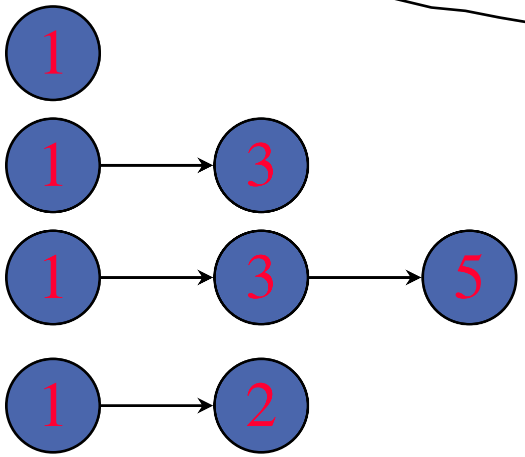




	[1]	[2]	[3]	[4]	[5]	[6]	[7]
d	0	6	2	9	5	10	14
p	-	1	1	5	3	3	1

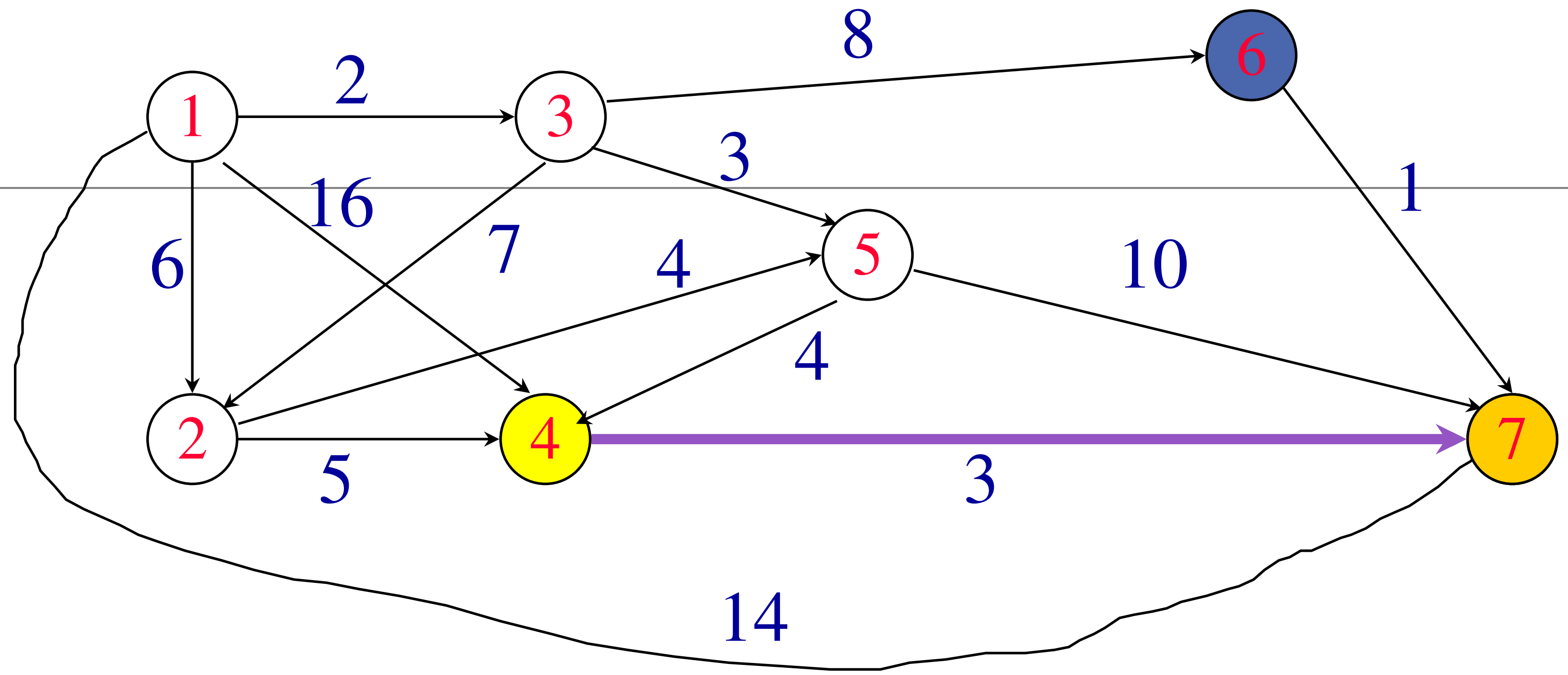
Dijkstra Algorithm

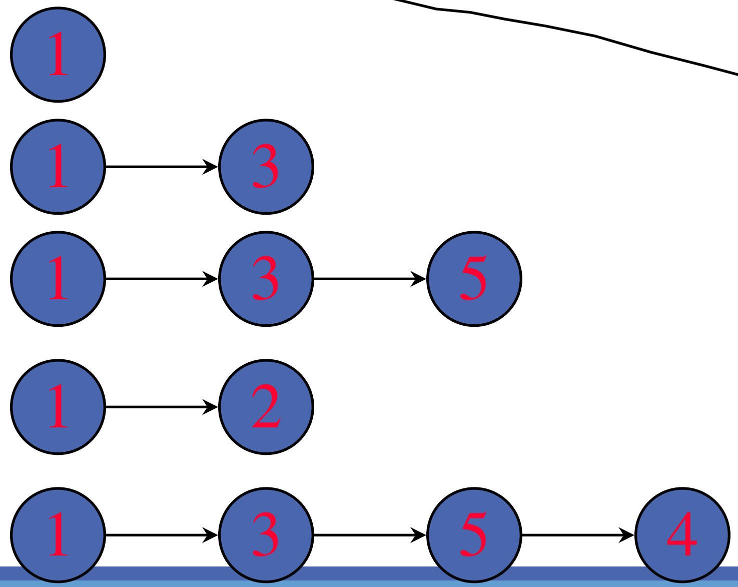




	[1]	[2]	[3]	[4]	[5]	[6]	[7]
d	0	6	2	9	5	10	14
p	-	1	1	5	3	3	1

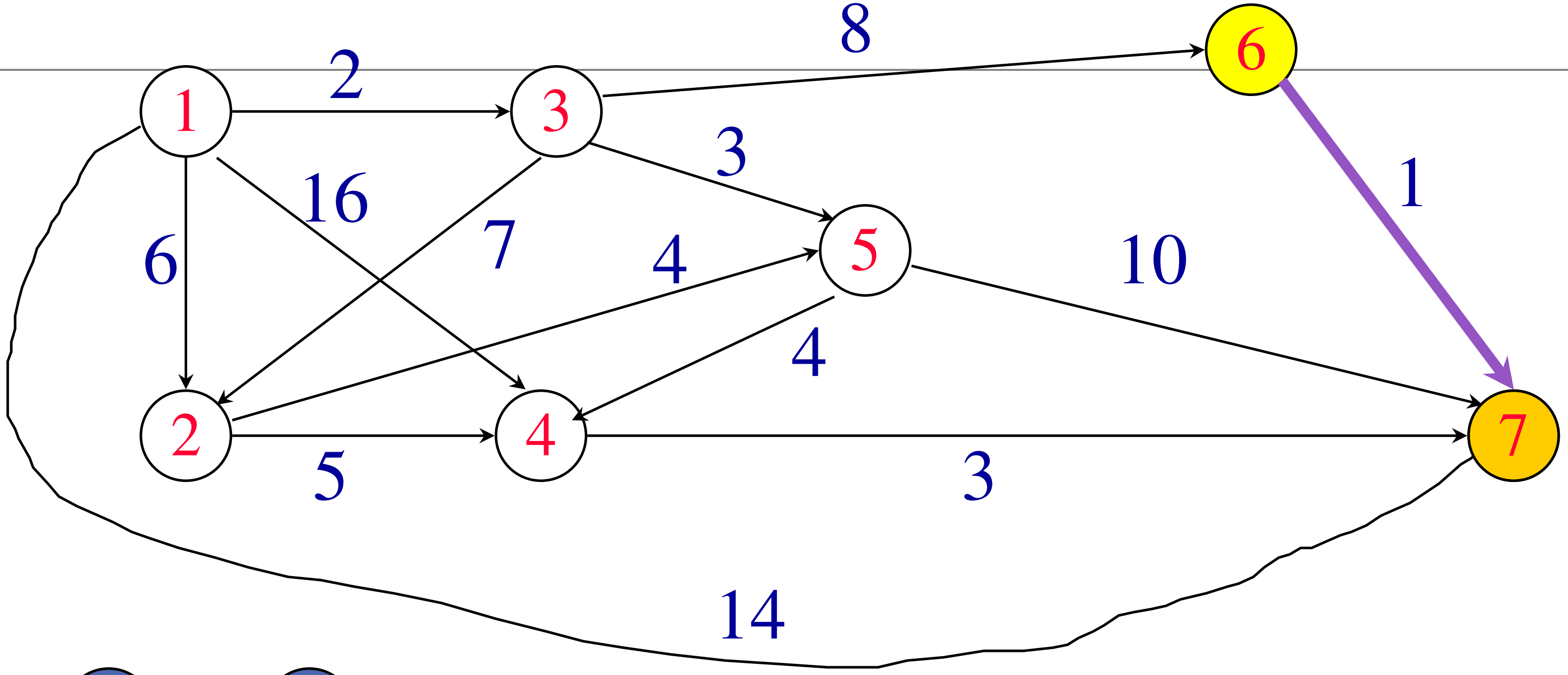
Dijkstra Algorithm

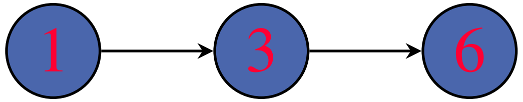




	[1]	[2]	[3]	[4]	[5]	[6]	[7]
d	0	6	2	9	5	10	12
p	-	1	1	5	3	3	4

Dijkstra Algorithm





	[1]	[2]	[3]	[4]	[5]	[6]	[7]
d	0	6	2	9	5	10	11
p	-	1	1	5	3	3	6

Dijkstra Algorithm

Path	Length
1	0
1 → 3	2
1 → 3 → 5	5
1 → 2	6
1 → 3 → 5 → 4	9
1 → 3 → 6	10
1 → 3 → 6 → 7	11

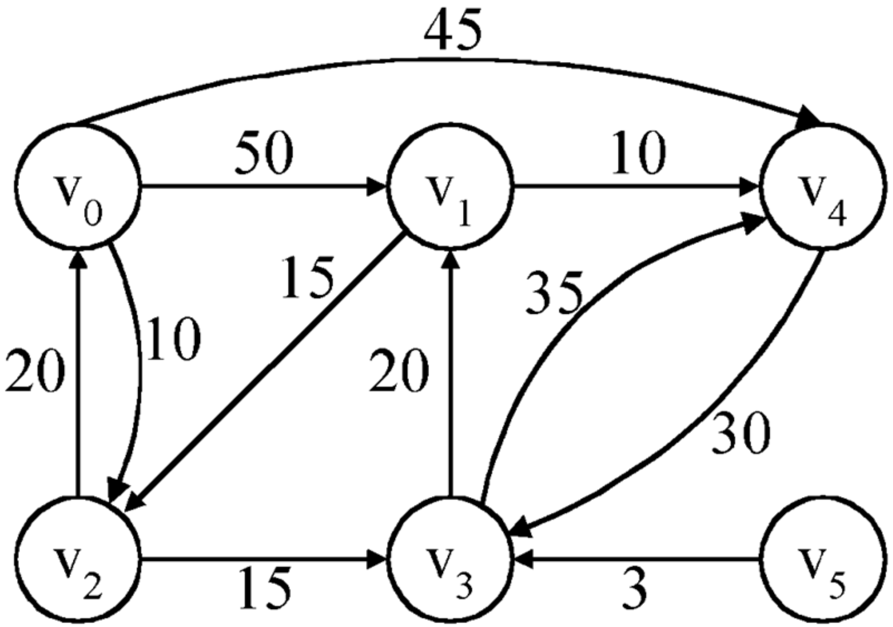
[1]	[2]	[3]	[4]	[5]	[6]	[7]
0	6	2	9	5	10	11
-	1	1	5	3	3	6

Analysis of Dijkstra Algorithm

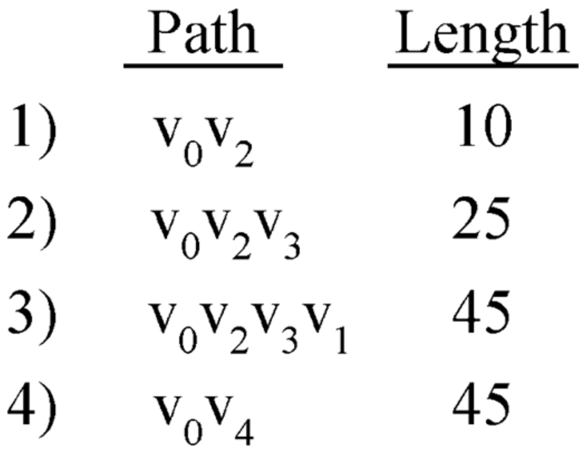
- O(n) to select next destination vertex.
- O(out-degree) to update d() and p() values when adjacency lists are used.
- O(n) to update d() and p() values when adjacency matrix is used.
- Selection and update done once for each vertex to which a shortest path is found.
- Total time is $O(n^2 + e) = O(n^2)$.

Dijkstra Algorithm

shortest paths from v_0(Single source) to all destinations



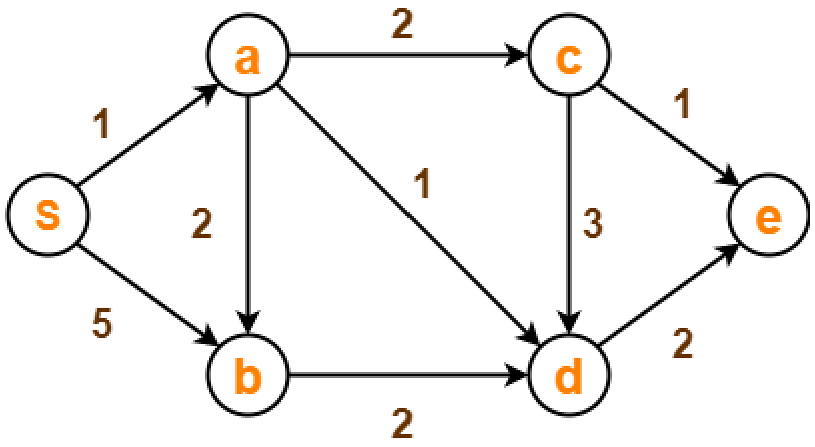
(a)

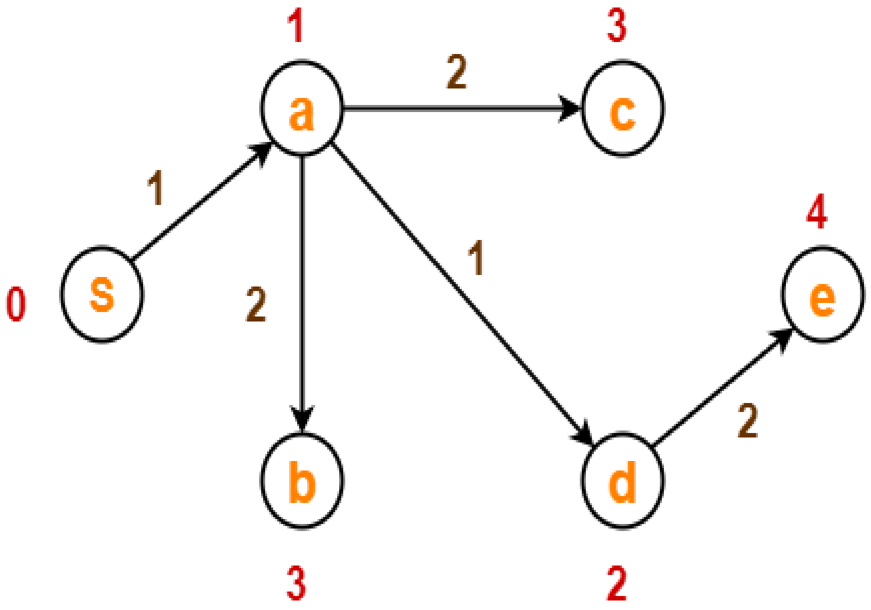


	Path	Length
1)	v_0v_2	10
2)	$v_0v_2v_3$	25
3)	$v_0v_2v_3v_1$	45
4)	v_0v_4	45

(b)

Practice Problem





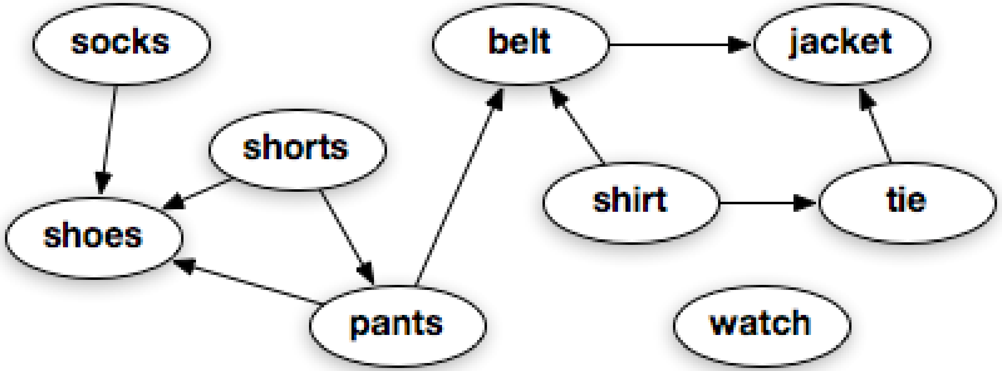
Shortest Path Tree

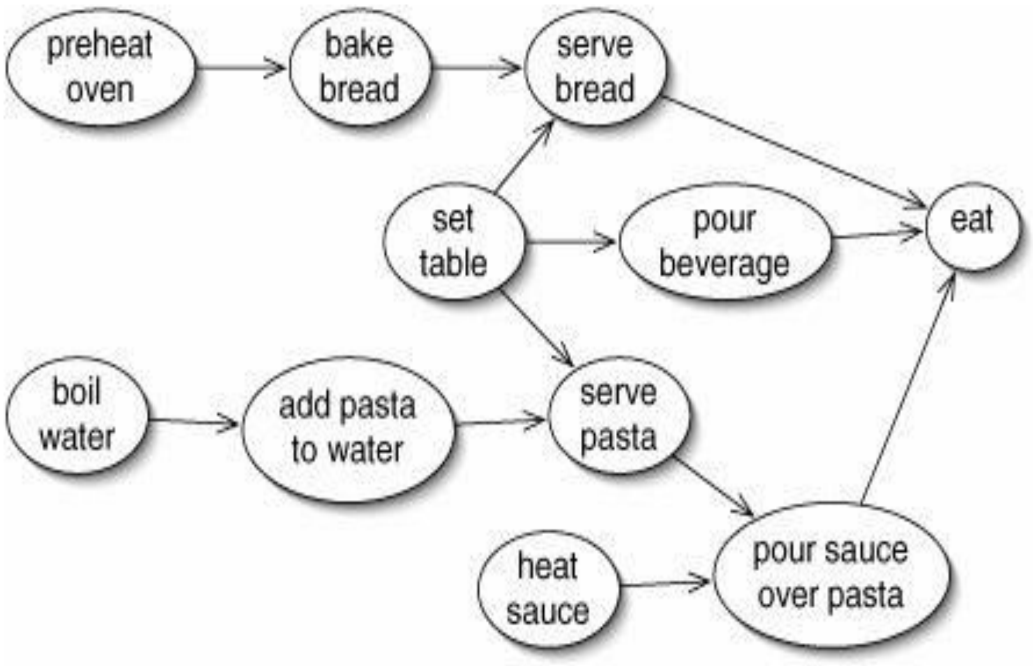
Topological sort

- We have a set of tasks and a set of dependencies (precedence constraints) of form *"task A must be done before task B"*
- Topological sort: An ordering of the tasks that conforms with the given dependencies
- Goal: Find a topological sort of the tasks or decide that there is no such ordering

Topological sort

- Applications
  - □ Assembly lines in industries
  - □ Courses arrangement in schools
  - □ Life related applications: Dressing order





Activity on vertex (AOV) network

- **Activity on vertex (AOV) network**
  - □ An activity on vertex(AOV)network, is a digraph G in which the vertices represent tasks or activities and the edges represent precedence relations between tasks.
- **Predecessor :**
  - □ Vertex i in an AOV network G is a predecessor of vertex j iff there is a directed path from vertex i to vertex j
  - □ Vertex i is an immediate predecessor of vertex j iff <i, j> is an edge in G
- **Successor :**
  - □ If i is a predecessor of j, then j is a successor of i.
  - □ If i is an immediate predecessor of j, then j is an immediate successor of i

Activity on vertex (AOV) network

- Example 1:

□ Prerequisites define precedence relations between courses. The relationship defined may be more clearly represented using a directed graph in which the vertices represent courses & the directed edges represent prerequisites.

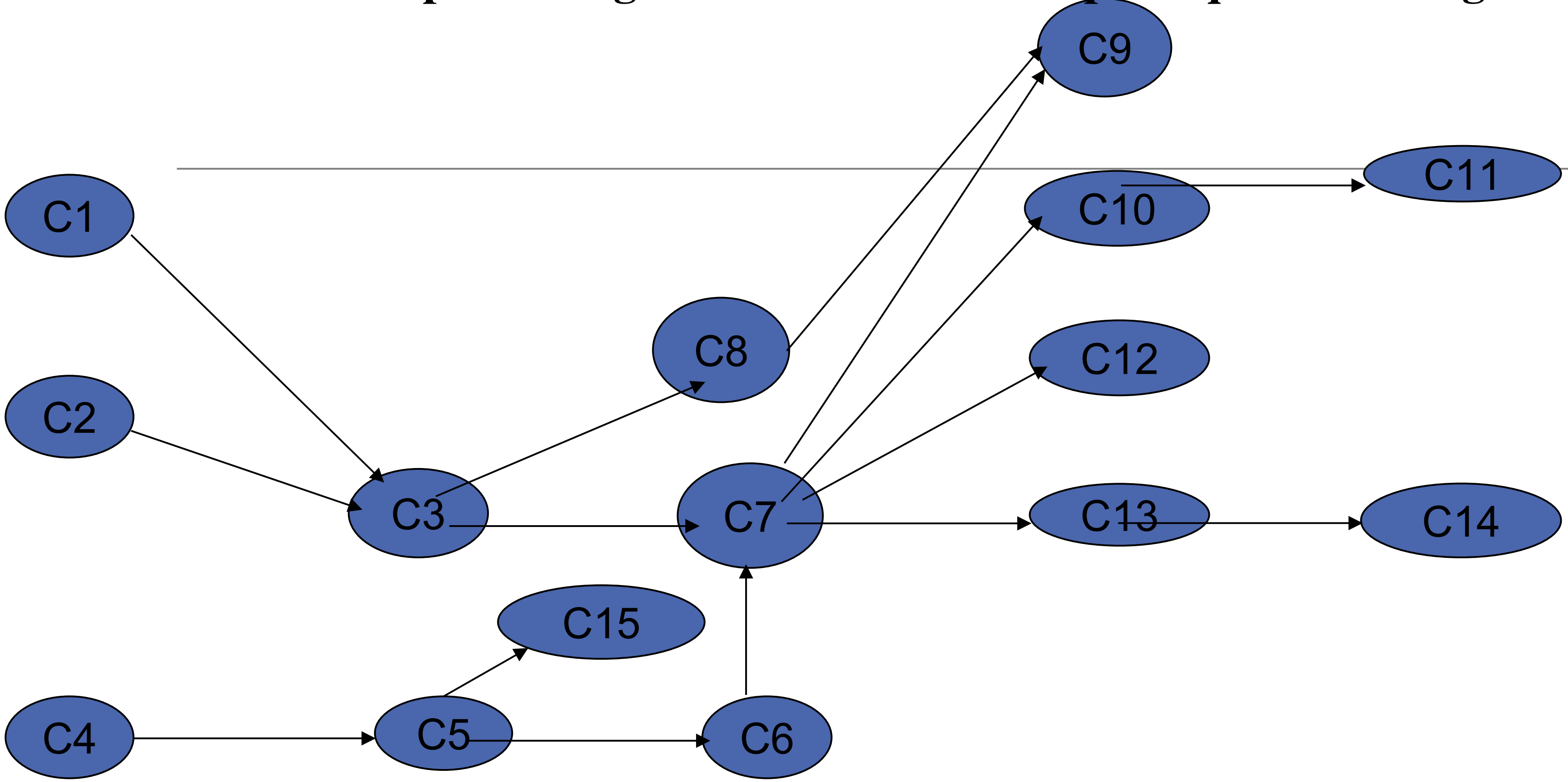
  - Each edge <i,j> implies that course i is a perquisite of course j

Activity on vertex (AOV) network

Course number	Course name	Prerequisites
C1	Programming I	None
C2	Discrete Mathematics	None
C3	Data Structures	C1, C2
C4	Calculus I	None
C5	Calculus II	C4
C6	Linear Algebra	C5
C7	Analysis of Algorithms	C3, C6
C8	Assembly Language	C3
C9	Operating Systems	C7, C8
C10	Programming Languages	C7
C11	Compiler Design	C10
C12	Artificial Intelligence	C7
C13	Computational Theory	C7
C14	Parallel Algorithms	C13
C15	Numerical Analysis	C5

Activity on vertex (AOV) network

AOV n/w representing courses as vertices & prerequisites as edges



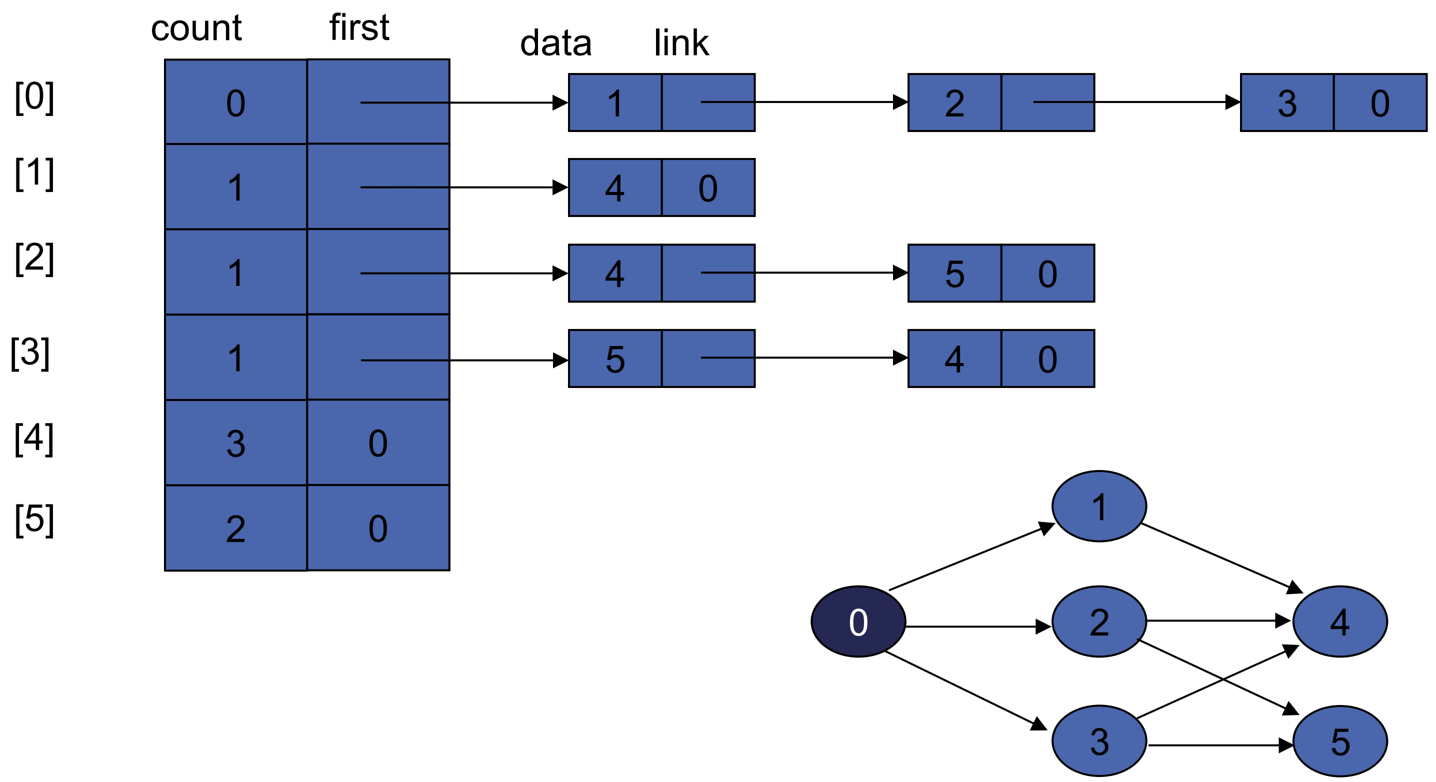
Course number	Prerequisites
C1	None
C2	None
C3	C1, C2
C4	None
C5	C4
C6	C5
C7	C3, C6
C8	C3
C9	C7, C8
C10	C7
C11	C10
C12	C7
C13	C7
C14	C13
C15	C5

- **Possible Topological orders:**
- c1, c2, c4, c5, c3, c6, c8, c7, c10, c13, c12, c14, c15, c11, c9
- c4, c5, c2, c1, c6, c3, c8, c15, c7, c9, c10, c11, c12, c13, c14

Topological sort

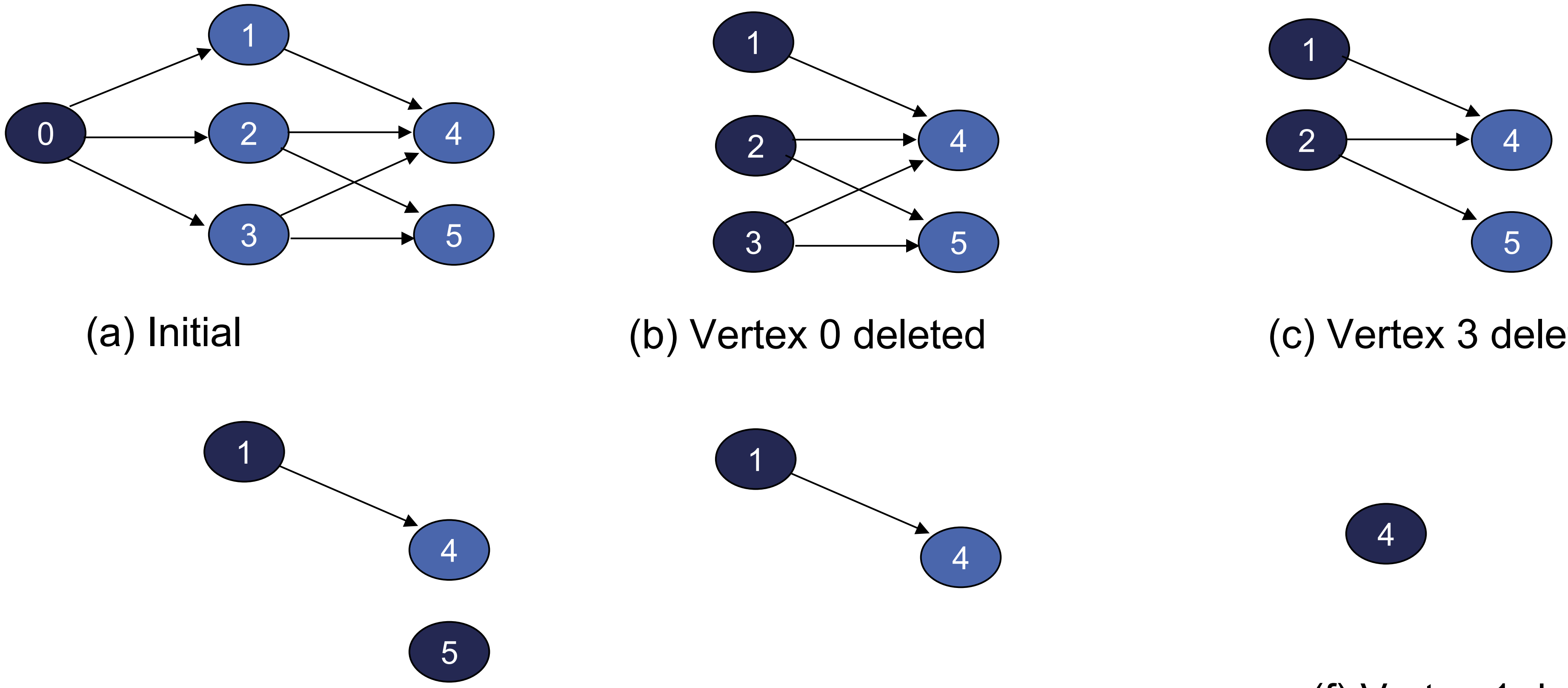
```
void topological_sort()
{
for (i = 0; i <n; i++) {
     if every vertex has a predecessor {
         fprintf(stderr, “Network has a cycle. \n “ );
         exit(1);
      }
      pick a vertex v that has no predecessors;
      output v;
      delete v and all edges leading out of v    from the network;
}
```

Internal representation used by topological sorting algorithm



	count	first
[0]	0	
[1]	1	
[2]	1	
[3]	1	
[4]	3	0
[5]	2	0

An AOV network



(a) Initial

(b) Vertex 0 deleted

(c) Vertex 3 deleted

(d) Vertex 2 deleted

(e) Vertex 5 deleted

(f) Vertex 1 deleted