Unit II - Regular Expressions.

* Regular Expression - RE are short notations that can denote complex and infinite regular languages.

- RE over Σ include letters, φ [empty set] and ε [empty string of length zero]

- Operators in RE-

*	:	Closure
.	:	Concatenation
+	:	Union

↓ Precedence.

* - zero or more occurrences

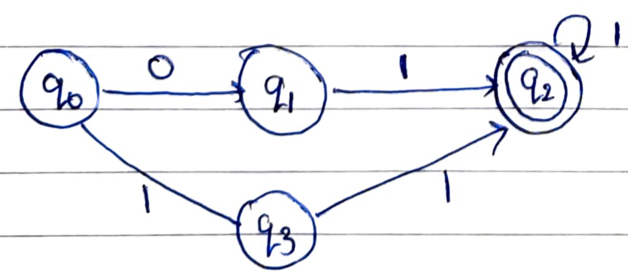
. - serial connection

\+ - parallel connection

a^+ - one or more occurrences

L(r) ↓ Lang representatⁿ

eg- $01^* + 1$



Set of all strings over {0,1} consisting of 1 or 0 followed by zero or more no. of 1's

Eg - 01, 011, 0111111, ...

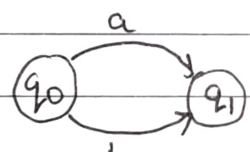
__/__/__

eg - $(01)^* + 1$

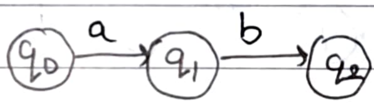
set of all strings over $\{0,1\}$ consisting of 1 or 0 or more no. of 01.

eg, 01, 0101, 01, 01011 ...

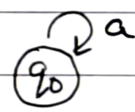
- $a+b \rightarrow$ parallel (either a or b)
- $a \cdot b \rightarrow$ series (followed by b)
- $a^* \rightarrow$ closure



1]



2]



3]

* Identities of Regular Expression.

1] $R \cup \phi = R$ [adding empty language]

2] $\phi . R = R \cdot \phi = \phi$

3] $\epsilon \cdot R = R \cdot \epsilon = R$ [joining empty string to any string will not change]

4] $\epsilon^* = \epsilon$ and $\phi^* = \epsilon$

5] $R + R = R$

6] $R^* R^* = R^*$

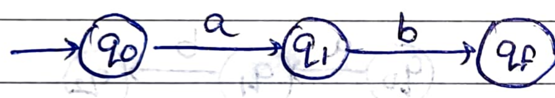
7) $RR^* = R^*R$

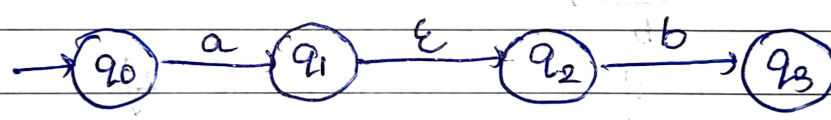
8] $(R^*) = R^*$

9] $\epsilon + RR^* = R^*$
$= \epsilon + R^*R$

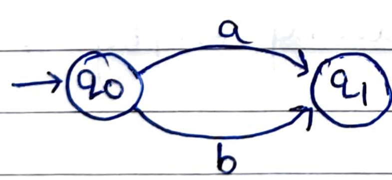
* Equivalence of FA & RE

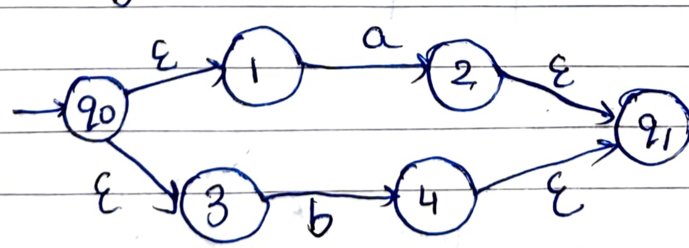
1] $a \cdot b$ $\Sigma = \{a, b\}$

NFA 

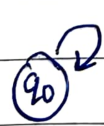
NFA - ϵ 

2] $a+b$

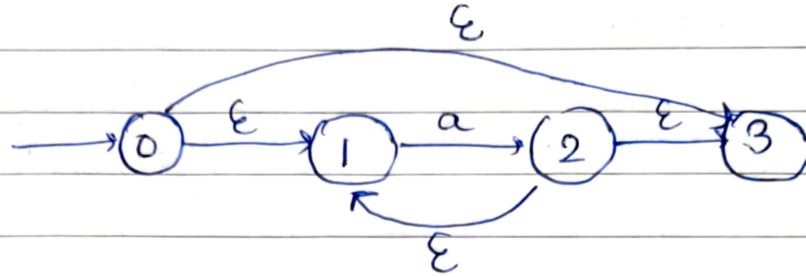
NFA 

NFA-ϵ 

3] a^*

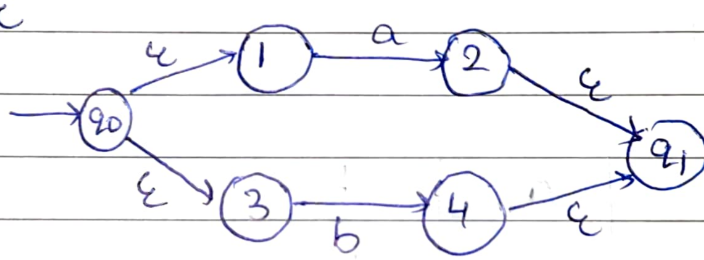
NFA 

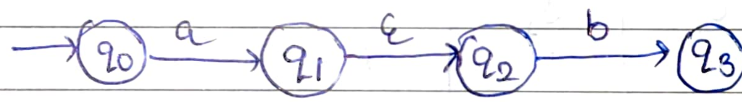
NFA - ε



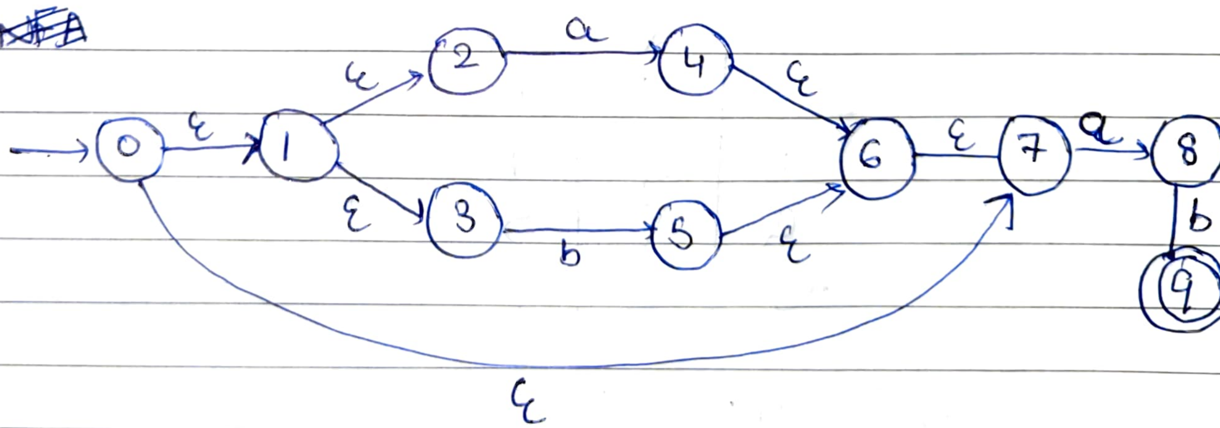
Q] $(a+b)^* ab$

NFA - ε

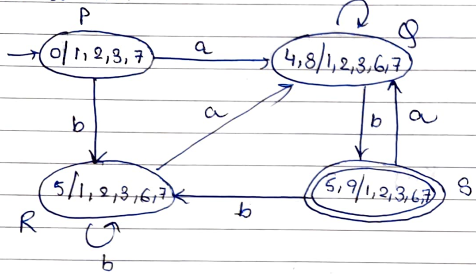




~~NFA~~



RE to NFA



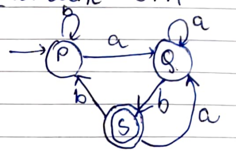
TT

	a	b
P	Q	R
Q	Q	S
R	Q	R
*S	Q	R

↓

	a	b
P	Q	P
Q	Q	S
*S	Q	P

Equivalent DFA



* DFA to RE : Arden's Theorem.

Statement – Let P & Q be two regular expression. If P does not contain null string (ε) then $R = Q + RP$ has a unique solution that is $R = QP^*$

Proof: $R = Q + (Q+RP)P$ [After putting the value $R = Q + RP$]

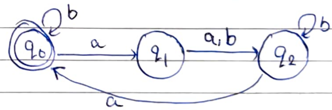
$= Q + QP + RPP$

$= Q + QP + QP^2 + RP^3$

$= Q(\varepsilon + P + P^2) + RP^3$

$= QP^*$ [As P^* represents $(\varepsilon + P + P^2 + P^3 + \cdots)$]

Eg-



$q_0 = q_0 b + q_2 a + \varepsilon$

$q_1 = q_0 a$

$q_2 = q_1 a + q_1 b + q_2 b$

Substitution → $q_2 = q_0 a^2 + q_0 ab + q_2 b$

$= q_0(a^2 + ab) + q_2 b$

$q_0 = q_0(a^2 + ab) \cdot b^*$

Substituting for q_2 in q_0

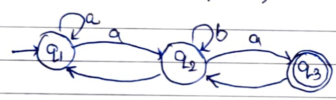
$q_0 = q_0 b + q_2 a + \varepsilon$

$q_0 = q_0 b + [q_0 (a^2 + ab)\ b^*]\ a + \varepsilon$

$= q_0 (b + a(a+b)\ b^* a) + \varepsilon$

$q_0 = \varepsilon (b + a(a+b) b^* a)^*$

Eg-



R=Q+RP

$q_1 = q_2 b + q_1 a$

$q_2 = q_3 b + q_2 b + q_1 a$

$q_3 = q_2 a$.

~~q2 in q1~~ q_3 in q_2 $(q_3 = q_2 \cdot a)$

$q_2 = q_1 a + q_2 b + q_3 a$

$= q_1 a + q_2 b + q_2 a^2$

$= q_1 a + q_2 (b + a^2)$

$q_2 = q_1 a (b + a^2)^*$

Using q_2 in q_1 $(q_2 = q_1 a (b + aa)^*)$

$q_1 = q_1 a + q_2 b + \varepsilon$

$= q_1 a + [q_1 \cdot a (b + a^2)^*] + \varepsilon$

$= \varepsilon + q_1 [a + a(b + a^2)^* \cdot b]$

$q_1 = \varepsilon [a + a(b + aa)^* \cdot b]$

$q_1 = [a + a(b + a^2)^* b]^*$

$q_2 = [a + a(b + a^2)^* \cdot b]^* \cdot a(b + a^2)^*$

$q_3 = [a + a[b + a^2]^* b]^* \cdot a(b + a^2)^* a$