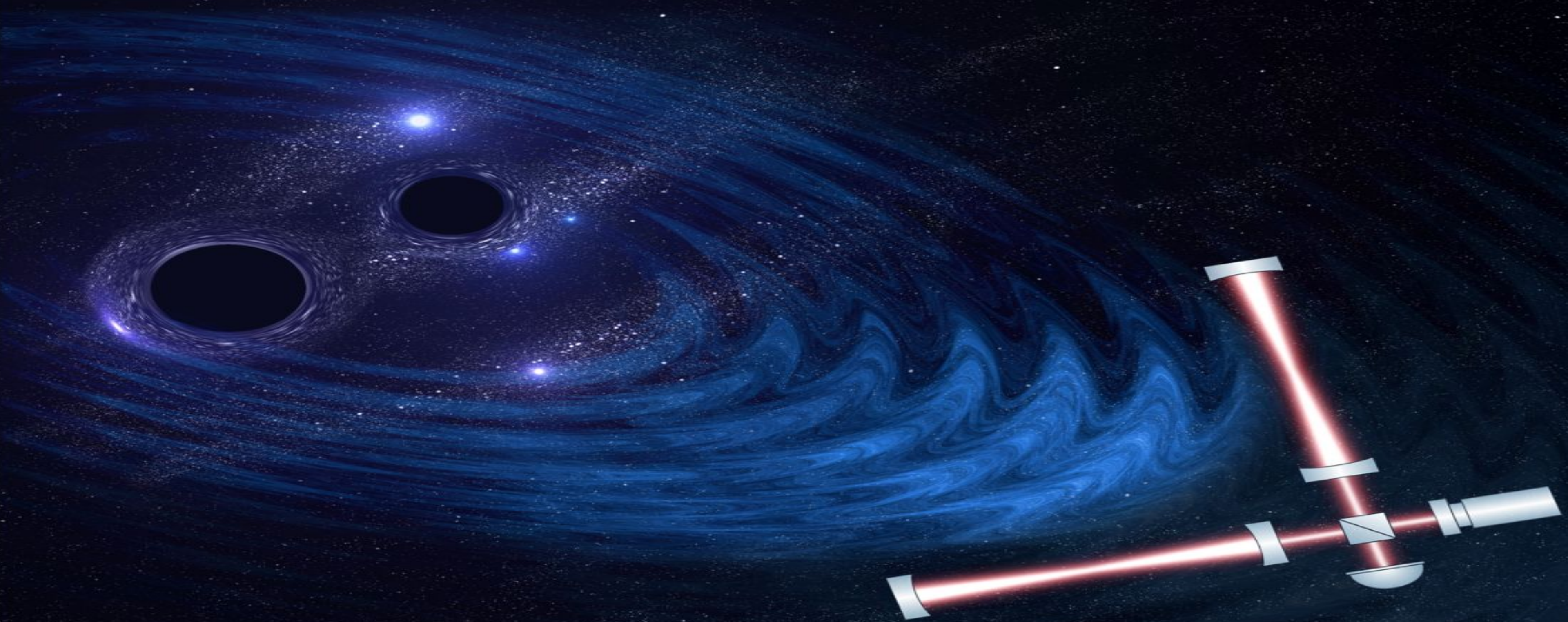

Event Rates and Waveform models of EMRI

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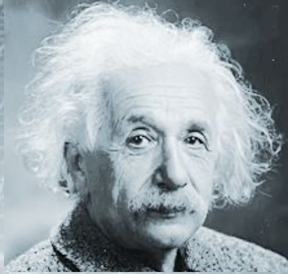
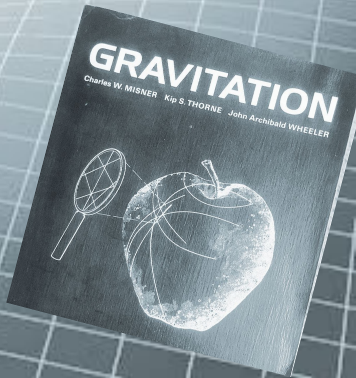
Introduction

Gravity as an Insight to Geometry



Space-time tells matter how to move; matter tells space-time how to curve.

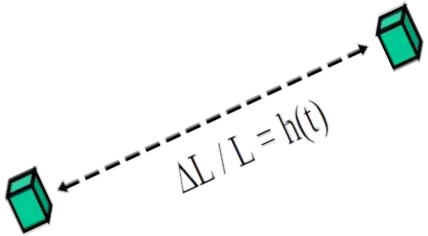
—— John Archibald Wheeler



Introduction

Wave Generation and Propagation

- **Gravitational waves** are the ripples in the fabric of the cosmos
  - Produced by rapidly rotating massive astronomical objects
  - Propagating with speed of light out to infinity from the source
  - The form of periodic perturbations in the space–time.
- The effect of gravitational radiation, is measured by strain amplitude h given by



$$h = \frac{\Delta L}{L}$$

ΔL –change in the length between two freely falling proof masses as the GW passes by
L – the measure of displacement between test masses in the absence of gravitational perturbations

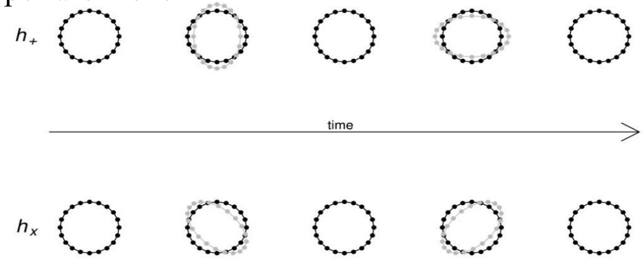
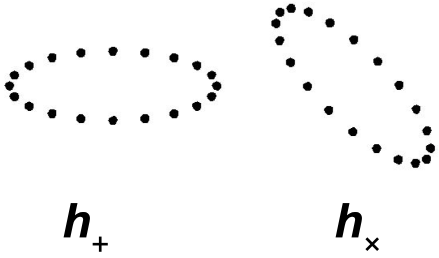
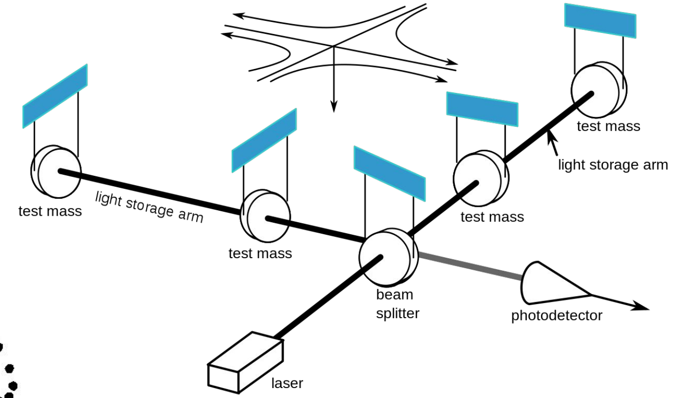


Figure: Ali et al. Bayesian Inference on Gravitational Waves. Pakistan Journal of Statistics and Operation Research, XI(4):645-665, 2015.

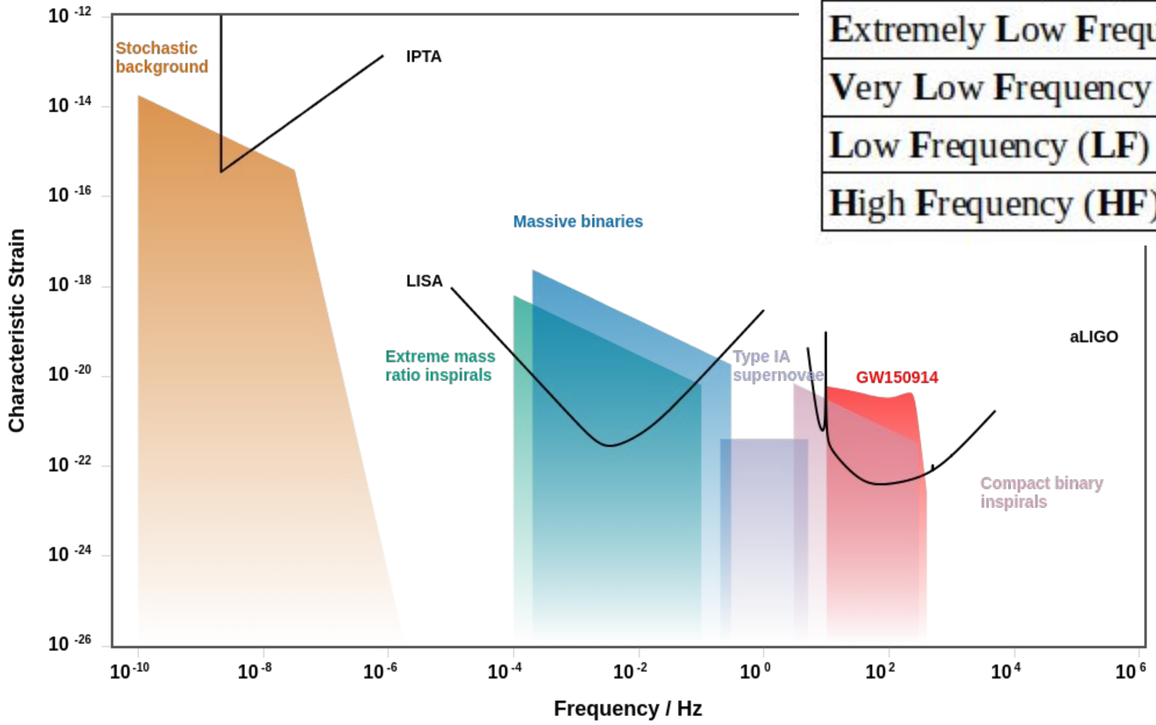




Credit: Caltech/ MIT/ LIGO Lab

Introduction

Gravitational Wave Window



Frequency Band	Frequency (Hz)
Extremely Low Frequency (ELF)	$10^{-18} - 10^{-15}$
Very Low Frequency (VLF)	$10^{-9} - 10^{-6}$
Low Frequency (LF)	$10^{-6} - 1$
High Frequency (HF)	$1 - 10^{4}$

http://gwplotter.com/

Introduction

Parameterization of Relativistic Binaries

- Binary systems are parameterized by the field strengths and mass ratio.
- Define a dimensionless parameter to parameterize determine gravitational field strength given as

$$\epsilon = \frac{GM}{rc^2} = \frac{r_g}{r}$$

where gravitational radius r_g is given as $r_g = \frac{GM}{c^2}$

  - *M – corresponds to the mass of the gravitating object*
  - *r – typically represents the distance to the source*

- As potential continually decreases → the field weakens up to the flat space–time metric in the limit.
- ε << 1 effectively drop to the scale of Newtonian field limit.
- Within close range of horizon radii of BHs potential approaches unity.

Dimitrios Psaltis. Probes and Tests of Strong-Field Gravity with Observations in the Electromagnetic Spectrum. Living Reviews in Relativity, 11(9), 2008.

Introduction

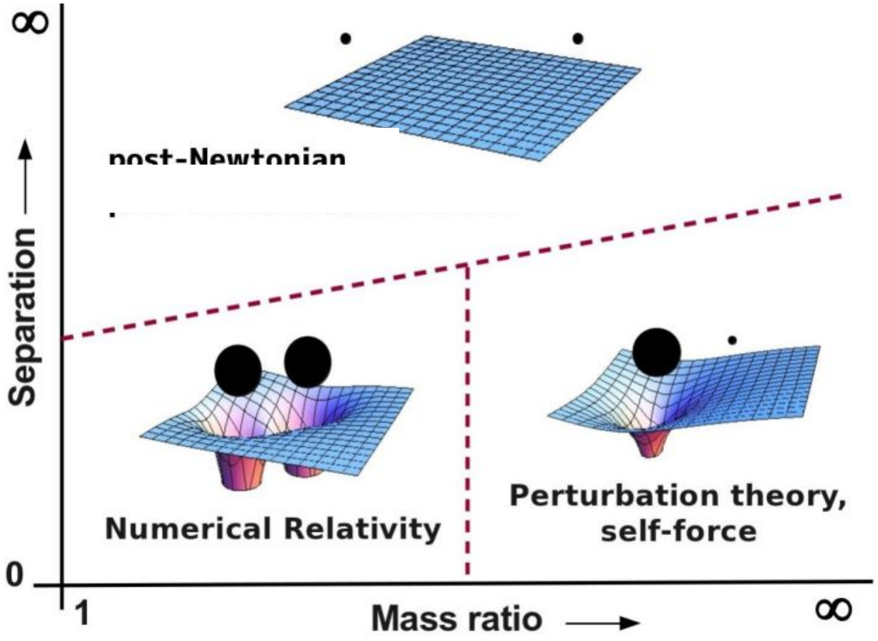
Methods of Analytic Approximation

- Analytic approximation methods in GR ranges with
  - The mass ratio of the binaries
  - Separation between them.

- **Post–Newtonian (PN)** approximation well qualifies the primary dynamical waveform evolution of inspiraling binaries perturbative expansions in the successive powers of ~$(v/c)^2$, where v is the orbital velocity.

- Strong field dynamics of comparable binaries are then unfolded using the tool of **Numerical Relativity (NR)** where discretized space–time metric is traced using iterative numerical methods.

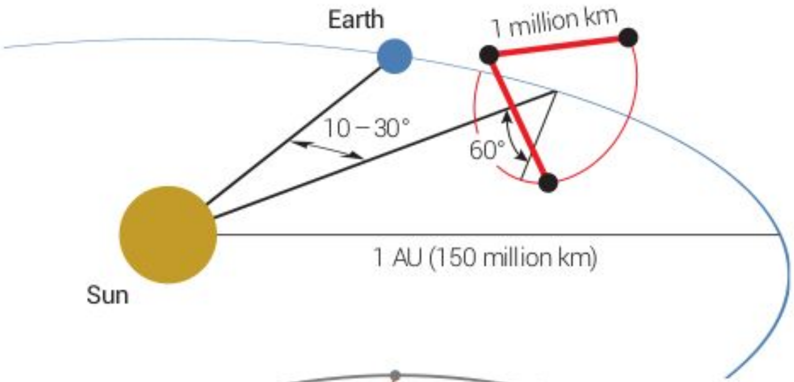
- **Gravitational Self–Force (GSF)** where compact object is presumed as a point particle moving along geodesics about a large MBH.



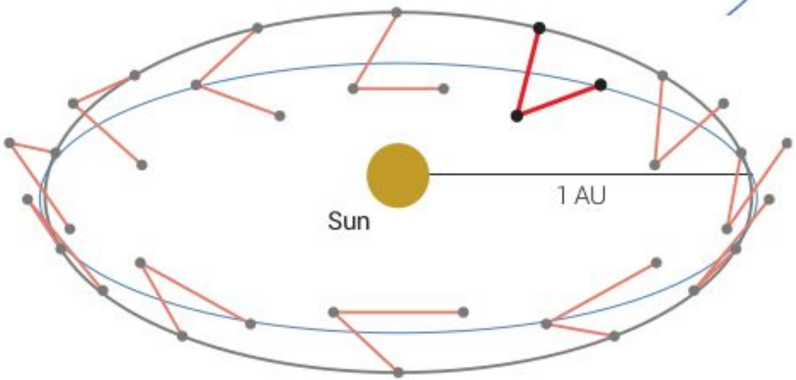
Luc Blanchet. Analytic Approximations in GR and Gravitational Waves. International Journal of Modern Physics D, 28(6):1930011, 2019.
Figure: Deyan P. Mihaylov and Jonathan R. Gair. Transition of EMRIs through resonance: corrections to higher order in the on-resonance flux modification. Journal of Mathematical Physics 58, 112501, 2017.

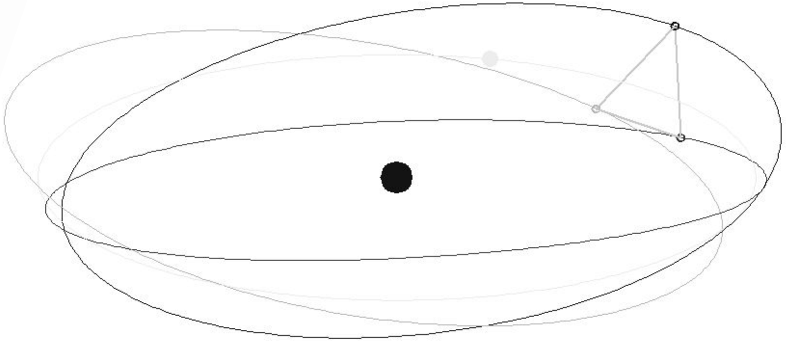
Inspiralling sources in the Range of LISA

LISA — Detector



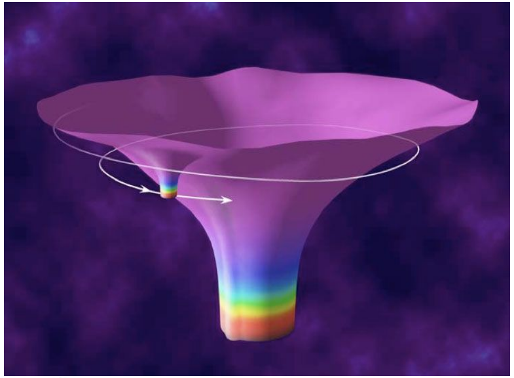

THE GRAVITATIONAL UNIVERSE
A science theme addressed by the eLISA mission observing the entire Universe

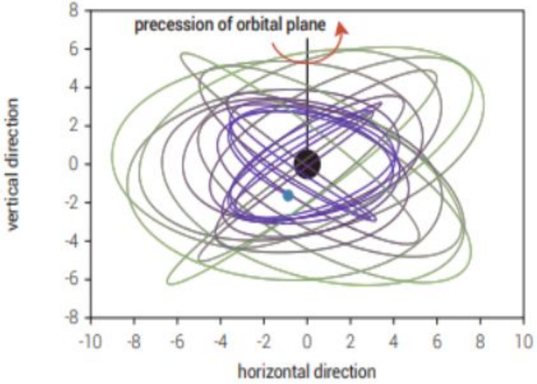




Inspiralling sources in the Range of LISA Detector and Detection Methodology

Extreme Mass Ratio Inspirals (EMRIs)

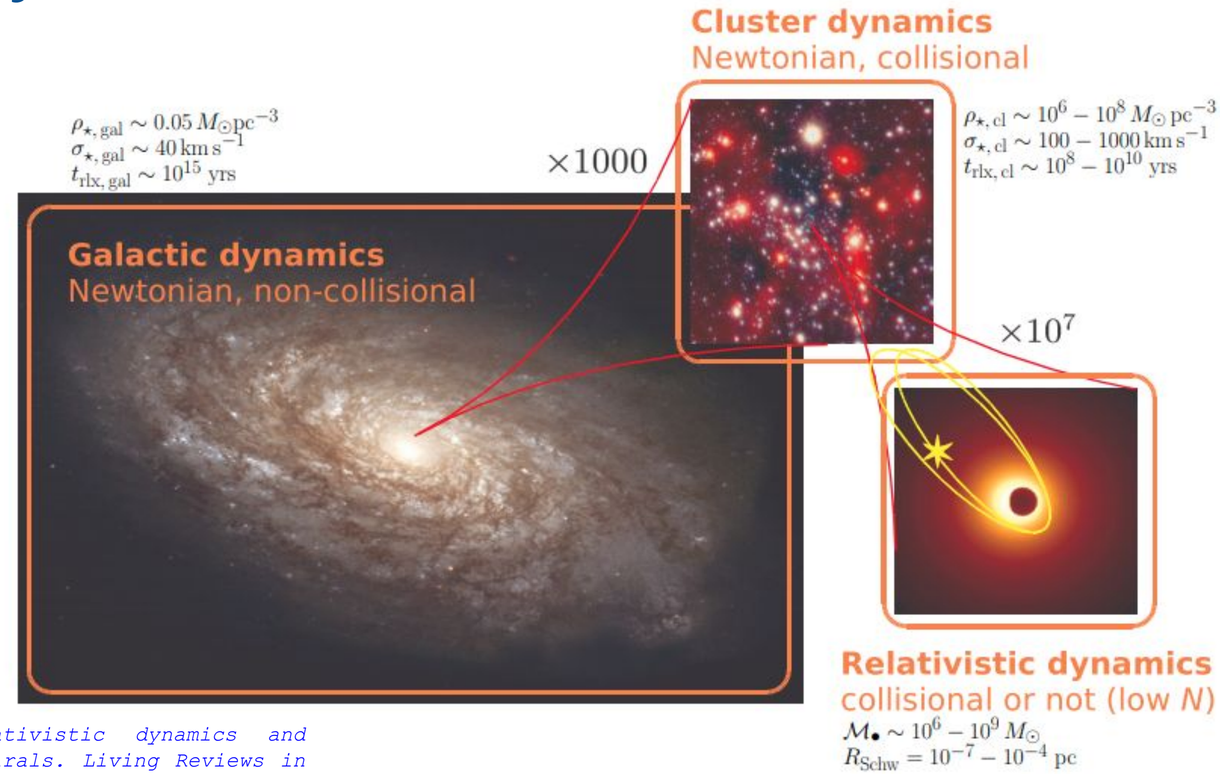




- Galaxies that hosts a MBH ~ $10^4 – 10^7 M_\odot$ in their galactic center
- **EMRIs**: A stellar remnant **compact objects (COs)** can either be
  - **Stellar mass black holes (BHs)**
  - **Neutron stars (NSs)**
  - **White dwarfs (WDs)** with diminishing mass ratio **$q \ll 1$** and prolonged cycles ~ $10^4 – 10^5$

Event Rates and Astrophysical Dynamics of EMRIs in the Galactic Centers

Stellar Dynamics and EMRIs



Amaro-Seoane. P., Relativistic dynamics and extreme mass ratio inspirals. Living Reviews in Relativity, 21: 4, 2018.

Event Rates and Astrophysical Dynamics of EMRIs in the Galactic Centers

Event Rates

- We are in the quest of computing the astrophysical probability of EMRI events per MBH of GC using LISA sensitivity
- **Astrophysical EMRI Model for Galactic Center:**
  - **MBH population**
    - The scaling relation of mass function that is independent of red–shift factor given as

$$\frac{dn}{d(lnM)} = n_0\Big(\frac{M}{3\times 10^6 M_\odot}\Big)^\beta$$

J. R. Gair et al. LISA extreme-mass-ratio inspiral events as probes of the black hole mass function. Physical Review D, 81:104014, 2010.

    - For range MBH falling in LISA sensitivity band n_0 = 0.002 M pc $^{-3}$ and β = 0.3.

  - **Intrinsic Rates of stellar remnants around MBH**
    - Set the range of CO for stellar mass BHs μ
    - The intrinsic rates of CO's population in GC is given by power law

$$\mathcal{R}(M) = \mathcal{R}_0\Big(\frac{M}{10^6 M_\odot}\Big)^\alpha$$

Clovis Hopman. Extreme mass ratio inspiral rates: dependence on the massive black hole mass. Classical and Quantum Gravity, 26(9), 2009.

    - The scaling factor α = {−0.15, −0.25, −0.25} with event rates R_0= {400, 7, 20}Gyr $^{-1}$ for BHs, NSs and WDs respectively.

Event Rates and Astrophysical Dynamics of EMRIs in the Galactic Centers

Event Rates

- The details of calculations of **event rates** for **detectable EMRIs** using mission life-time of LISA t_{life} **= *2 years***, we will require the **number density of comoving MBHs *dn/d(lnM)*** and **intrinsic rate of probable EMRIs per MBH** $\mathcal{R}$.
- MBH's **spin** remains **highly uncertain**, hence, the integrating probability of spin distribution ***p(a) da*** is normalized to **1** with uniform range of spins ranging from 0 to 1, considering the prograde spin orbits (aligned).
- Retrograde (anti-aligned) spin ranges between −1 to 0.
- The number of EMRI events falling in LISA frequency band is given by

$$N_{EMRI} = t_{life} \int_{M=M_{min}}^{M_{max}} \mathcal{R} \, \frac{dn}{d(lnM)} \, d \, lnM$$

where $M_{min} = 10^4 \, M_\odot$ and $M_{max} = 10^7 \, M_\odot$.

- This gives the lower end of the N_{EMRI} within the specified threshold for most optimistic cases for ensemble of galaxies.

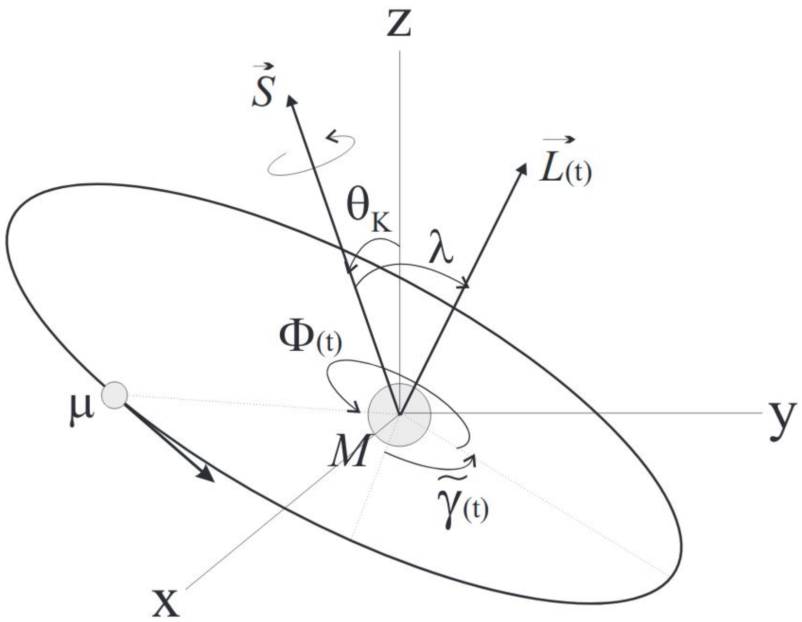
J. R. Gair. Probing black holes at low redshift using LISA EMRI observations. Classical and Quantum Gravity, 29:094034, 2009. URL arXiv:0811.0188.

Waveform modeling of EMRIs

Waveform Models:

17 physical parameter fully describes the **CO–MBH system** that is further reduced to 14 parameters by ignoring the spin of CO:

Parameters	*Symbol*	*Unit*
Initial azimuthal orbital frequency	ν_0	Hertz
Mass of CO	μ	$M_\odot$
Mass of MBH	M	$M_\odot$
Spin of MBH	$\tilde{a}$	M^2
Initial eccentricity	e_0	1
Orbital inclination angle	ι	Radian
Ecliptic latitude	θ_s	Radian
Ecliptic longitude	ϕ_s	Radian
Polar spin angle of MBH	θ_k	Radian
Azimuthal spin angle of MBH	ϕ_k	Radian
Distance to the source	D_L	Parsec
Initial direction of pericenter	$\tilde{\gamma}_0$	Radian
Initial azimuthal orbital phase angle	Φ_0	Radian
Initial azimuthal angle of orbital angular momentum	α_0	Radian



Leor Barack and Curt Cutler. LISA Capture Sources: Approximate Waveforms, Signal-to-Noise Ratios, and Parameter Estimation Accuracy. Physical Review D, 69:082005, 2004. URL arXiv:gr-qc/0310125.

Future Implications and Science Return of EMRIs

Astrophysics

- The key objectives of **LISA mission** is to scrutinize the **MBHs**, inhabited at the galactic centers.
- Investigate the underlying understanding of the **origin** and **growth of MBHs.**
- MBHs undergoes episodes of **mergers** and **accretions** that alters theirs **masses** and **spins.**
- Spins of these MBHs being deterministic to the merger histories, **imprinted onto the GWs**, will inform about the **coherent or chaotic evolution of MBH spin**, consequently, the evolution of the galaxies.
- EMRIs will give an insight to the galactic dynamics –the the regions that are unseen by EM observations which includes
    - Population of MBHs
    - Mass spectrum of MBHs
    - Dynamical processes leading to the formation of EMRIs
    - Formation histories of MBHs and their co-evolution of host galaxies.
- Galactic EMRIs in MW will provide unprecedented measurements on spin parameter of MBH.
- Detection counts will inform us about the stellar population around MBH and dynamical processes in extreme environs.

Christopher P. L. Berry et al. The unique potential of extreme mass-ratio inspirals for gravitational-wave astronomy. White paper submitted to Astro2020 (2020 Decadal Survey on Astronomy and Astrophysics).

Future Implications and Science Return of EMRIs

Fundamental Physics

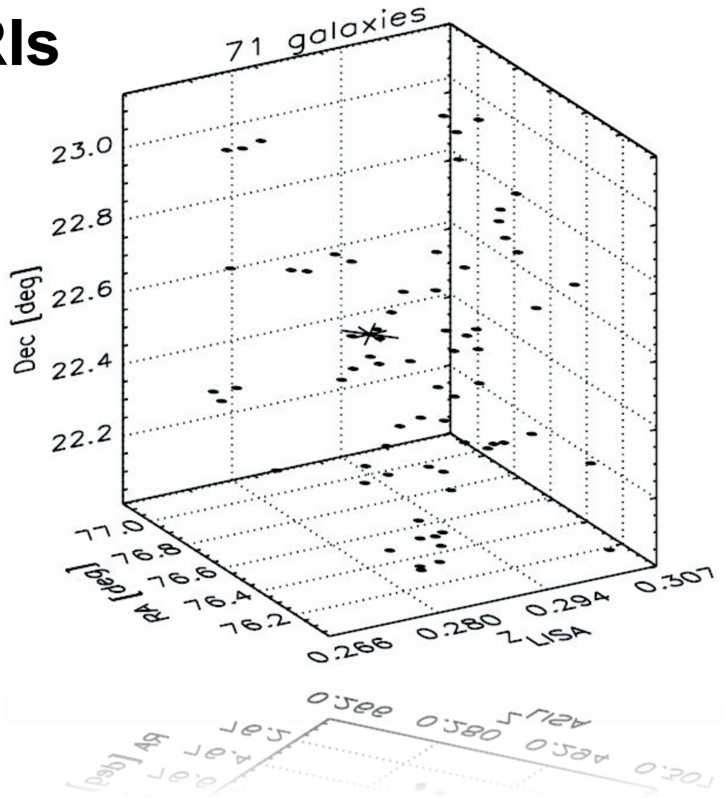
- EMRI measurements leads to **testify GR** in strong gravity regime.
- **GR** might not be the **ultimate theory of gravity.**
- Testing GR have strong implications p**robing new physics.**
- In GR, stationary and axisymmetric solutions of Kerr BHs can explicitly described in terms of its infinite number multipole moments.
    - Mass
    - Spin
- Deviations from the expected waveform signals will be evidence of new physics.
- If the gravitating massive object is indeed a BH, emission of GWs will be ceased as it reaches the LSO
    - Boson stars and Gravastars will continue the emission spiralling towards the center.
    - The additional oscillatory modes in characteristic signal amplitude
- Studies of GWs be dependent on matched filtering of the signals based on available waveform models.
    - These models are sensitive to the minimal changes in the orbital evolution of CO.
    - These additional effects will be observed by mismatching of signal with theoretical models.

Christopher P. L. Berry et al. The unique potential of extreme mass-ratio inspirals for gravitational-wave astronomy. White paper submitted to Astro2020 (2020 Decadal Survey on Astronomy and Astrophysics).

Future Implications and Science Return of EMRIs

Cosmography

- Using available EM data from galactic catalog that makes use of cluster of the galaxies correlating to the cosmic web, at particular z, to spot the MBH hosting EMRI.
- EMRI signals will allow us to measure luminosity distance in the local universe.
- Distance of the source can be deduced from the amplitude of the signal, z in turn can be measured.
- The data can be used to compute cosmological parameters like Hubble constant H_0.



L. MacLeod and Craig J. Hogan. Precision of Hubble constant derived using black hole binary absolute distances and statistical redshift information. Physical Review D, 77:043512, 2008.

Thank You