Fundamentals of Astrodynamics

A bit of history…

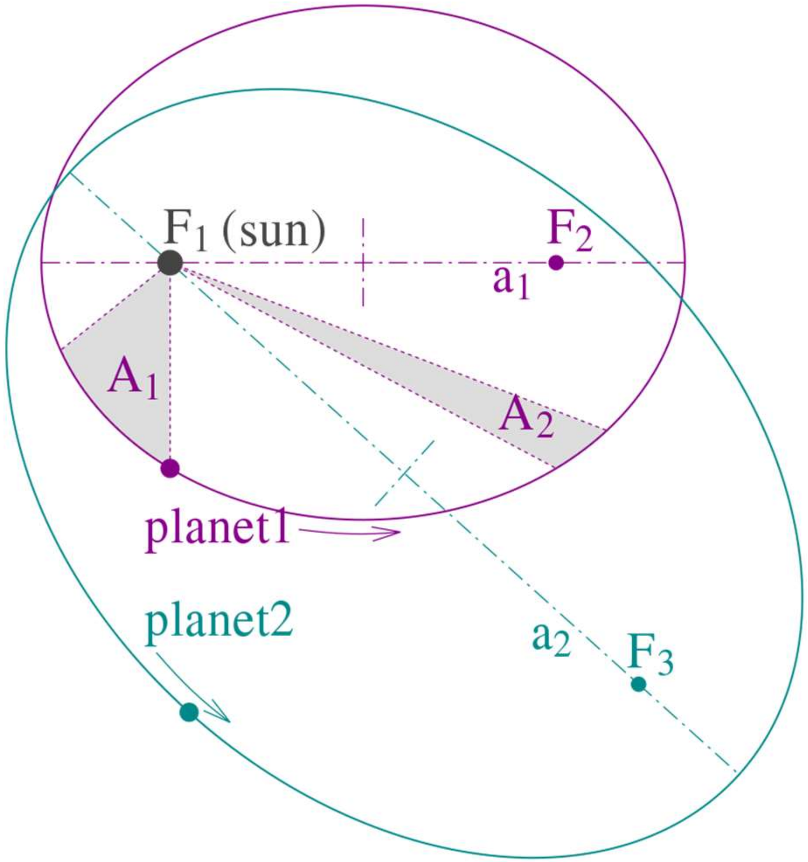
Tycho Brahe and Johannes Kepler, Statue in Prague, Czech Republic

Issac Newton and Edmund Halley

Rules of thumb

- Newton’s laws of motion
- Newton’s law of gravitation

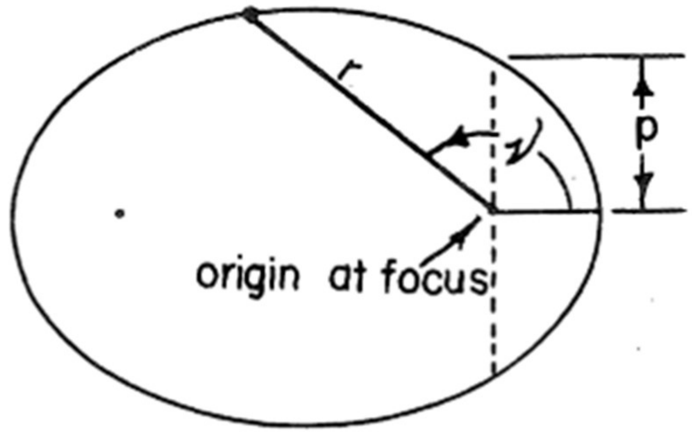
- Kepler’s laws of planetary motion
  - <u>First Law</u> - The orbit of each planet is an ellipse, with the Sun at a focus
  - <u>Second Law</u> - The line joining the planet to the Sun sweeps out equal areas in equal times.
  - <u>Third Law</u> - The square of the period of a planet is proportional to the cube of its mean distance from the Sun.



Mathematical description of orbits

Equations of motion: $\ddot{\boldsymbol{r}} + \frac{\mu}{r^3}\boldsymbol{r} = 0$

Analytical solution: $r = \frac{h^2/\mu}{1+(B/\mu)\cos(\nu)}$



Source: [1]

General equation of a conic section:

$$r = \frac{p}{1 + e\cos(\nu)}$$

All orbits in a two-body system are conic sections!

Geometry of an orbit

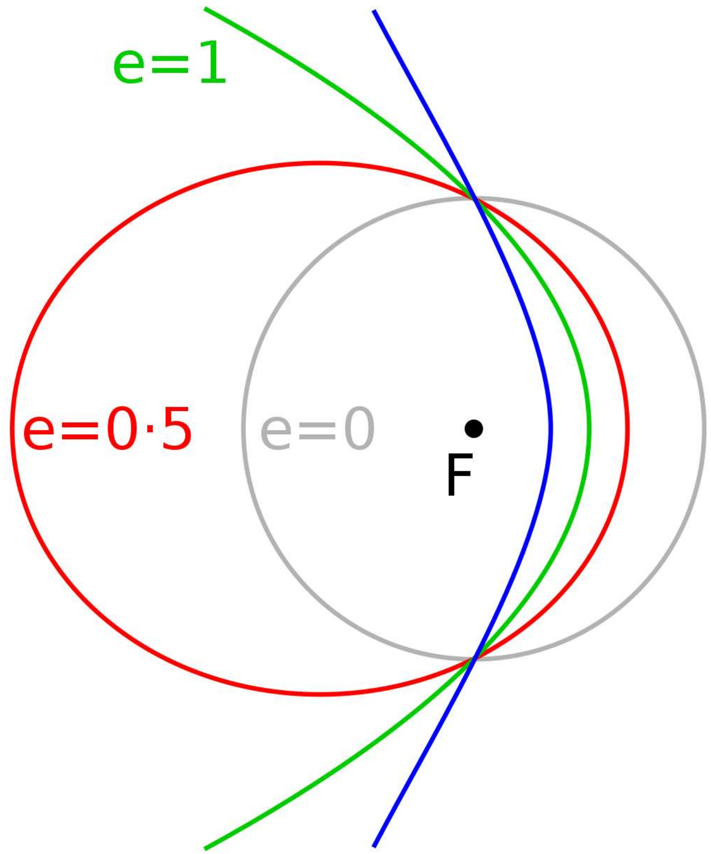
Relationship between Energy and geometry of orbit:

$$\mathcal{E} = -\frac{\mu}{2a}$$

$$e = \sqrt{1 + \frac{2\mathcal{E}h^2}{\mu^2}}$$

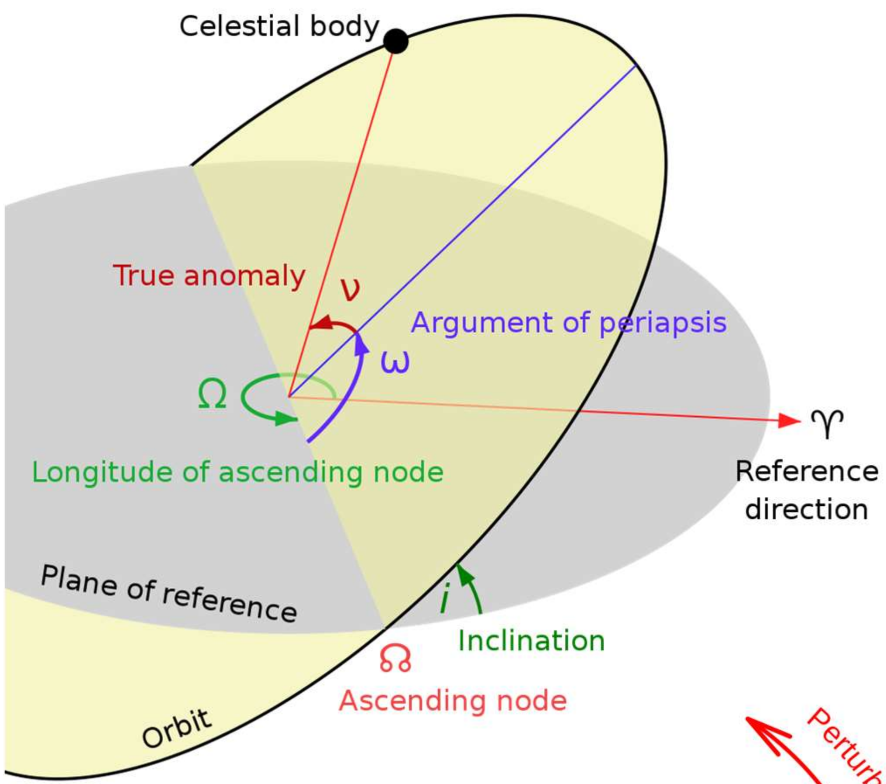
where $\mathcal{E} = E/\mu$ is specific orbital energy

- Elliptical ($0 \leq e < 1$, $\mathcal{E} < 0$)
- Parabolic ($e = 1$, $\mathcal{E} = 0$)
- Hyperbolic ($e > 1$, $\mathcal{E} > 0$)

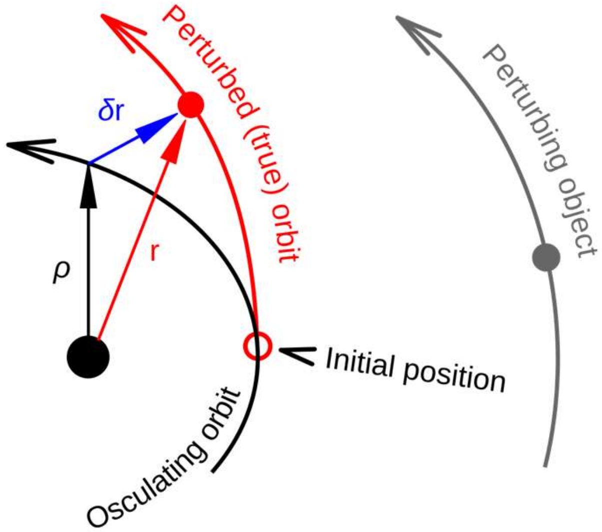


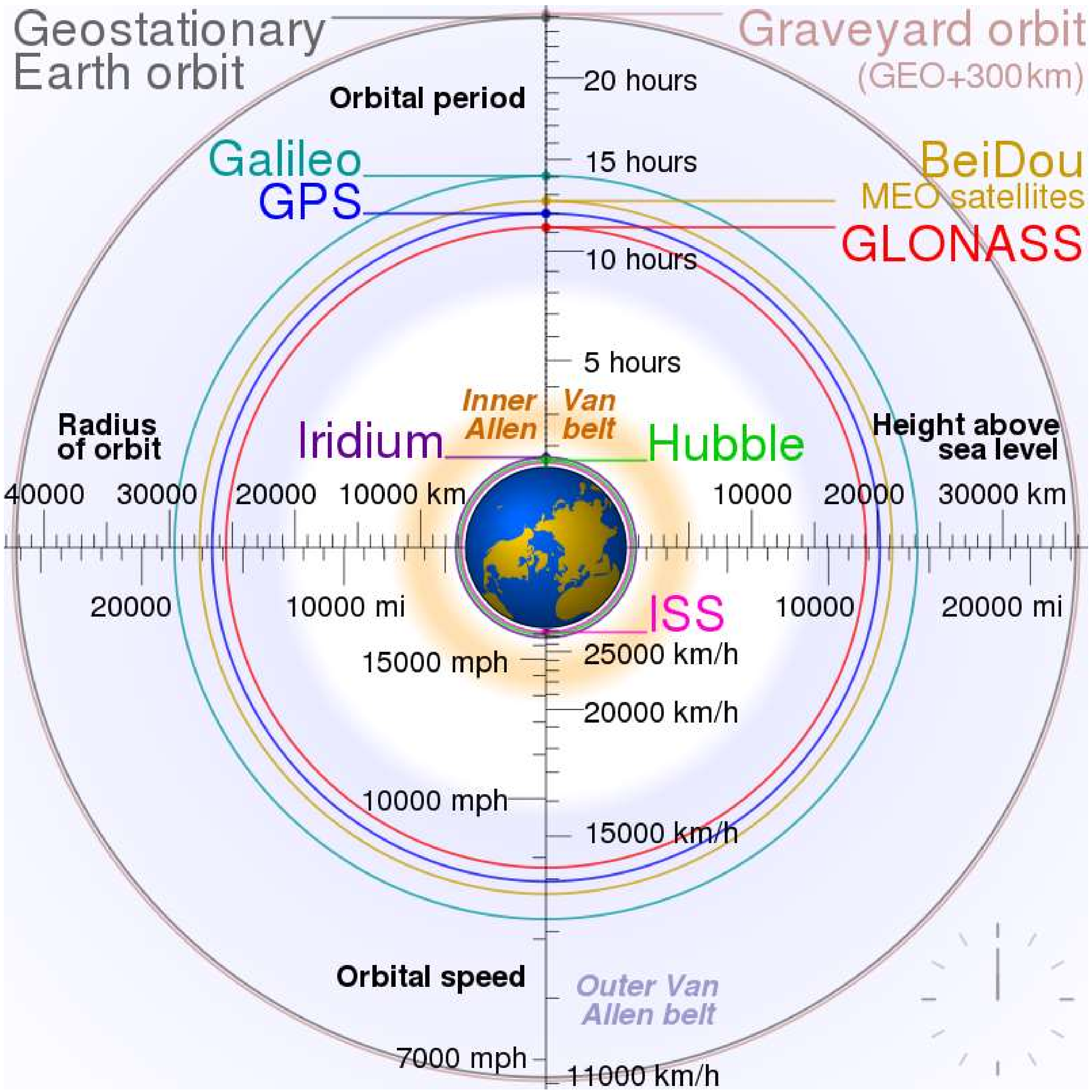
Orbital elements

- Semi-major axis (a)
- Eccentricity (e)
- Inclination (i)
- Longitude of the ascending node (Ω)
- Argument of periapses (ω)
- True anomaly (ν)



- Osculating elements
- Mean elements





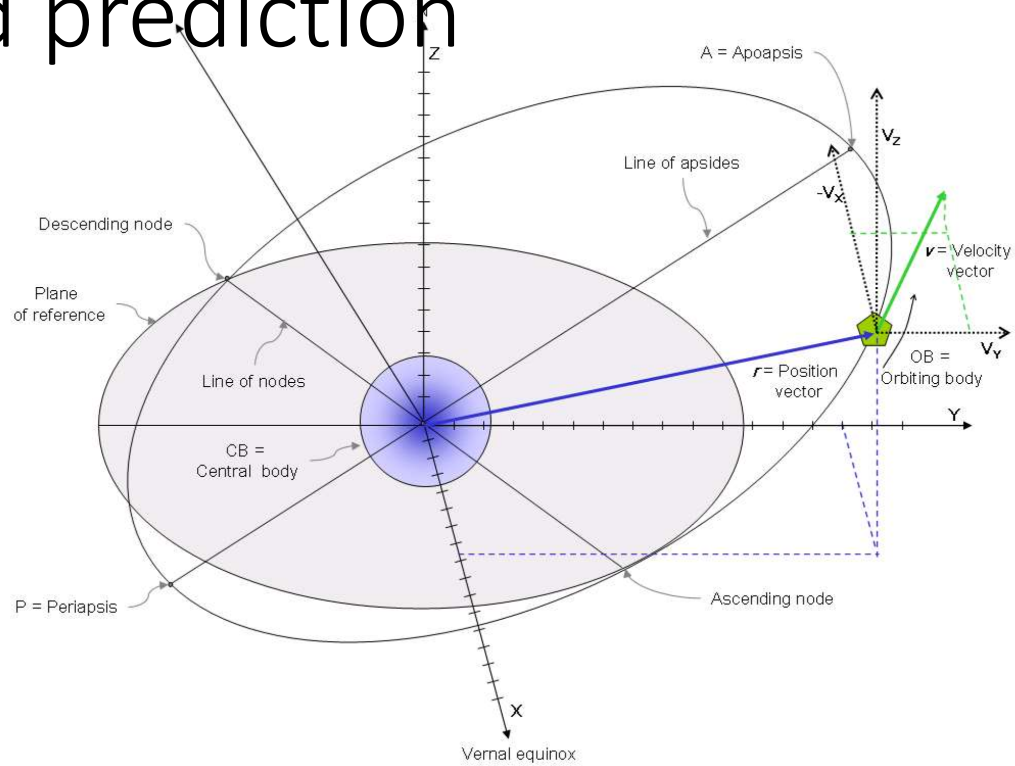
Geostationary Earth orbit
Graveyard orbit
(GEO+300km)
Orbital period
20 hours
15 hours
10 hours
5 hours
Galileo
GPS
BeiDou
MEO satellites
GLONASS
Inner Van Allen belt
Radius of orbit
Iridium
Hubble
Height above sea level
40000
30000
20000
10000 km
10000
20000
30000 km
20000
10000 mi
10000
20000 mi
ISS
15000 mph
25000 km/h
20000 km/h
10000 mph
15000 km/h
Orbital speed
Outer Van Allen belt
7000 mph
11000 km/h

Achieving and Maintaining Orbit

Credits: NASA

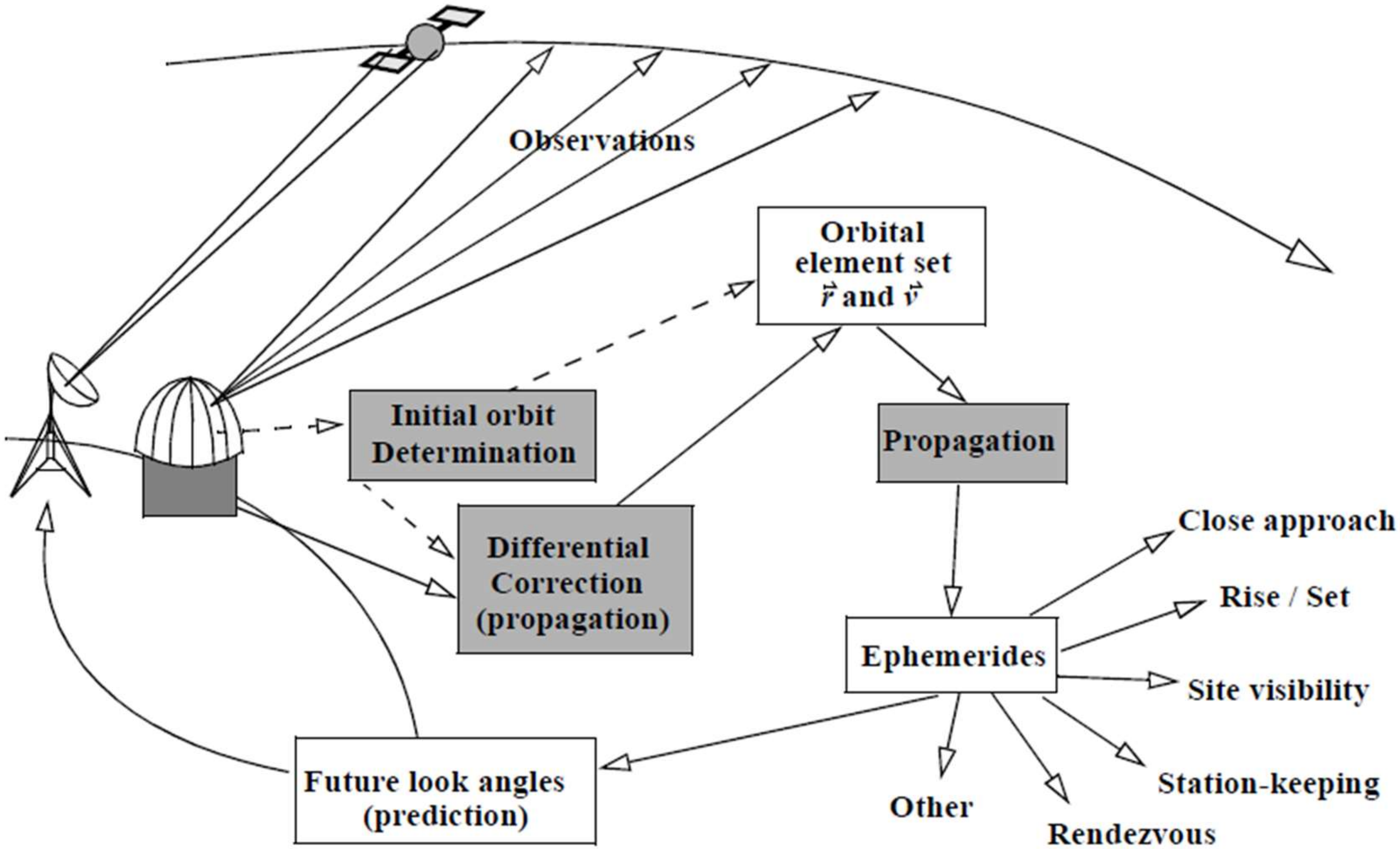
Orbit determination and prediction

The Raisting Satellite Earth Station, Germany



Orbital state vectors of a satellite in orbit around earth

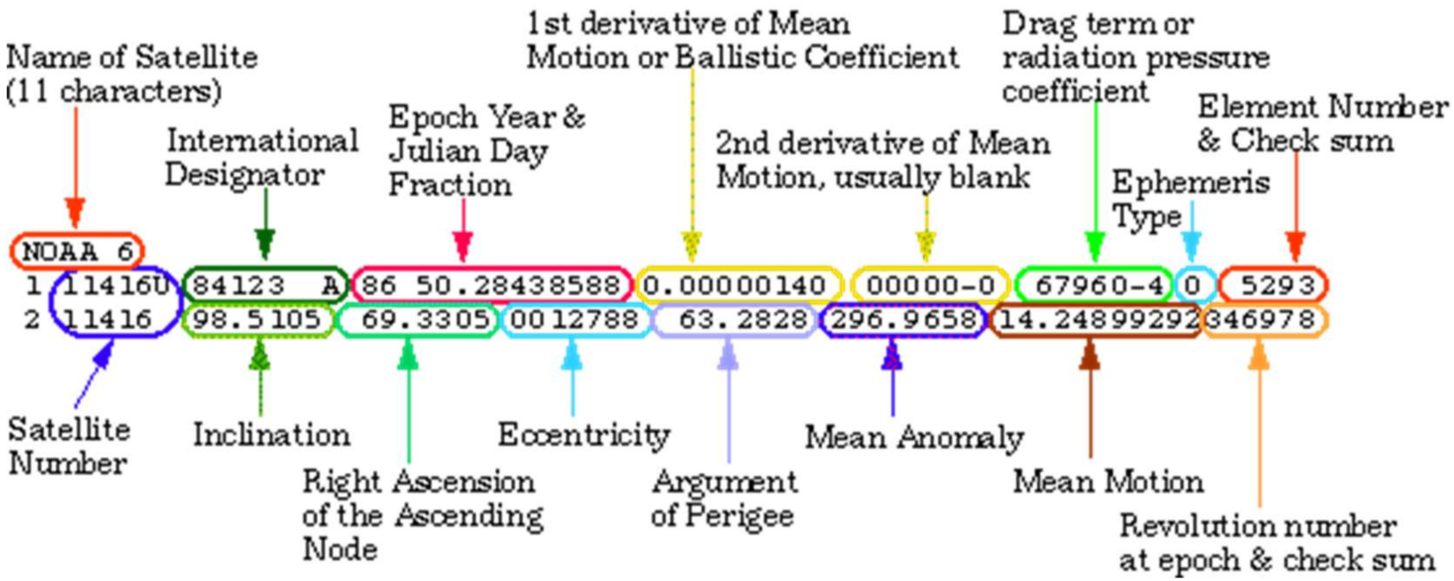
Orbit determination and prediction



Credits: David A Vallado

Orbit determination and prediction

TLEs along with Simplified General Perturbations models (SGP, SGP4, SDP4, SGP8 and SDP8) are used for orbit prediction

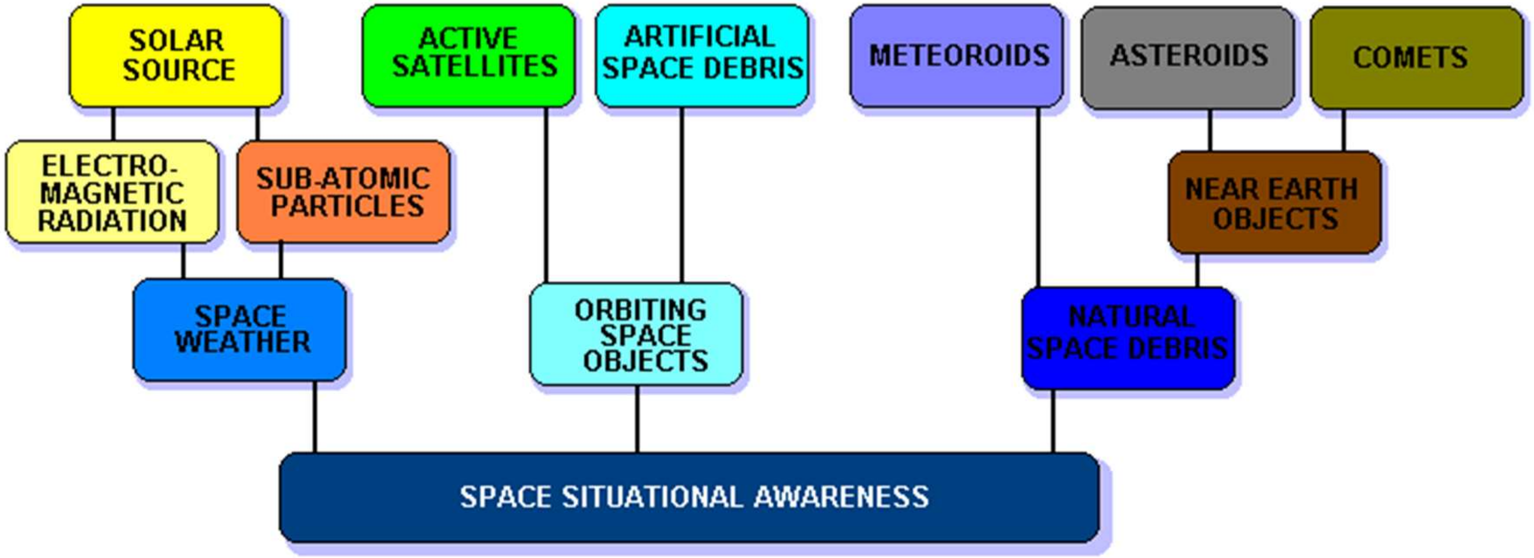


Credits: NASA

Intermission

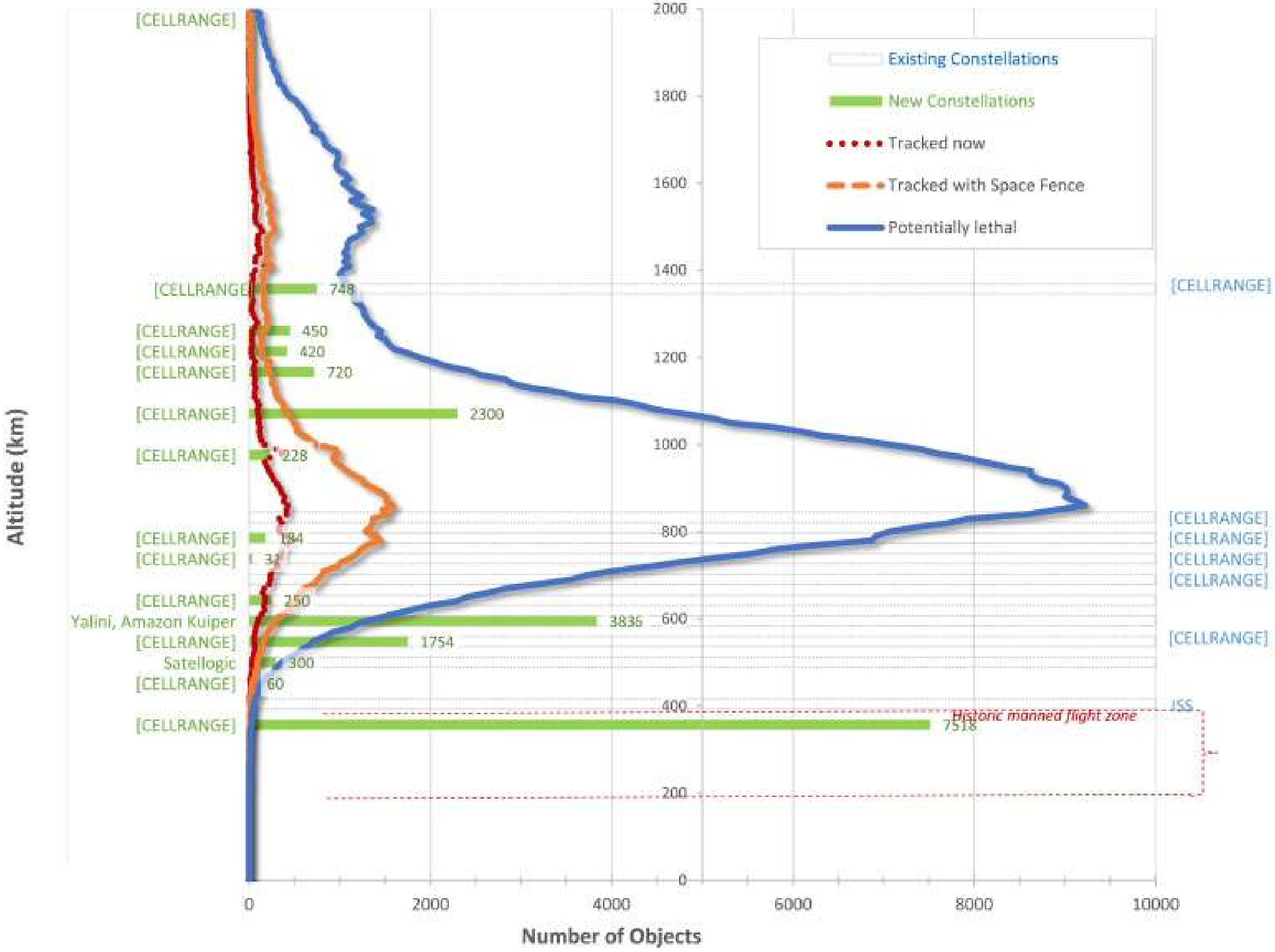
Space Situational Awareness

Components of SSA



Source: [3]

Why is SSA important?



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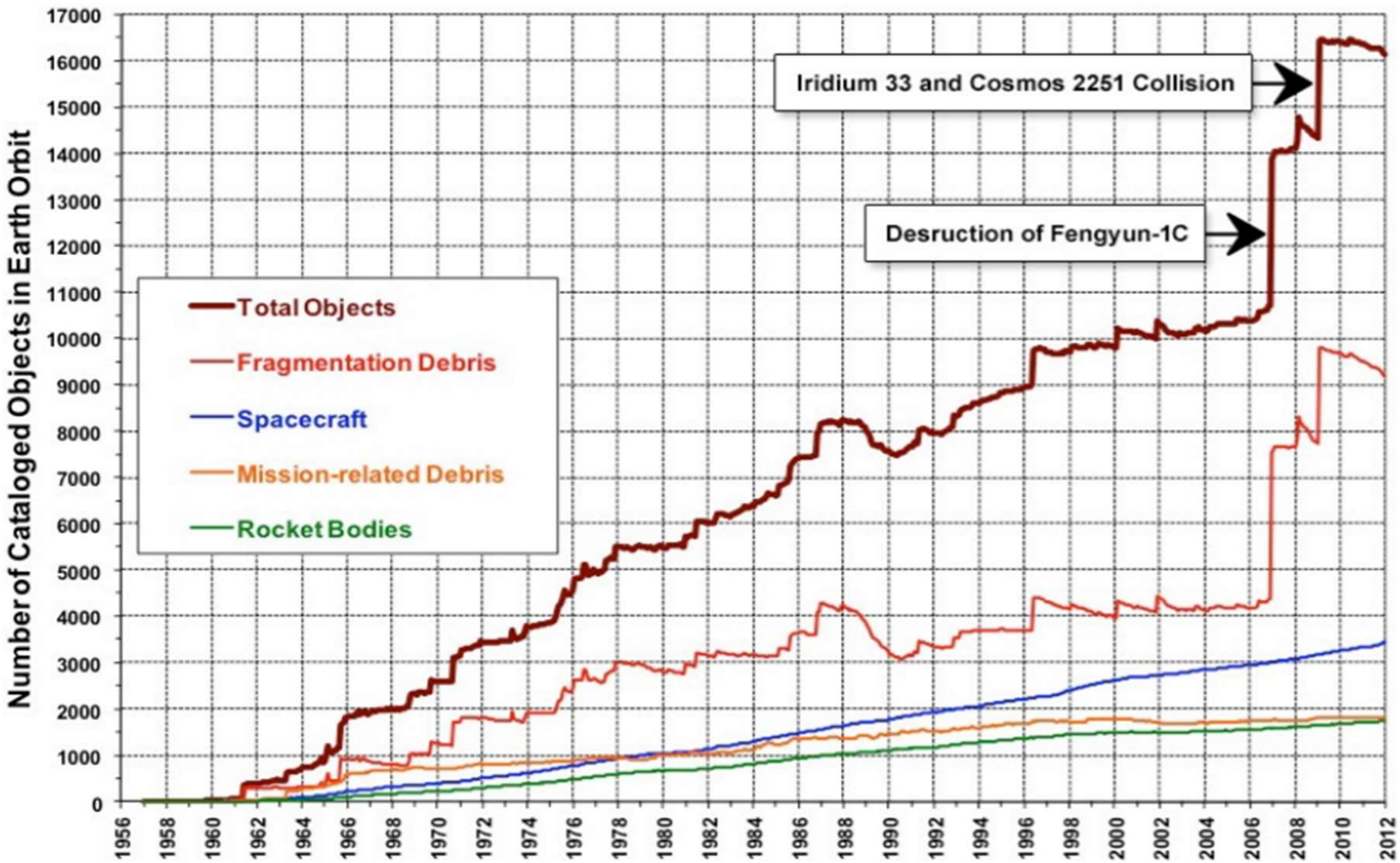
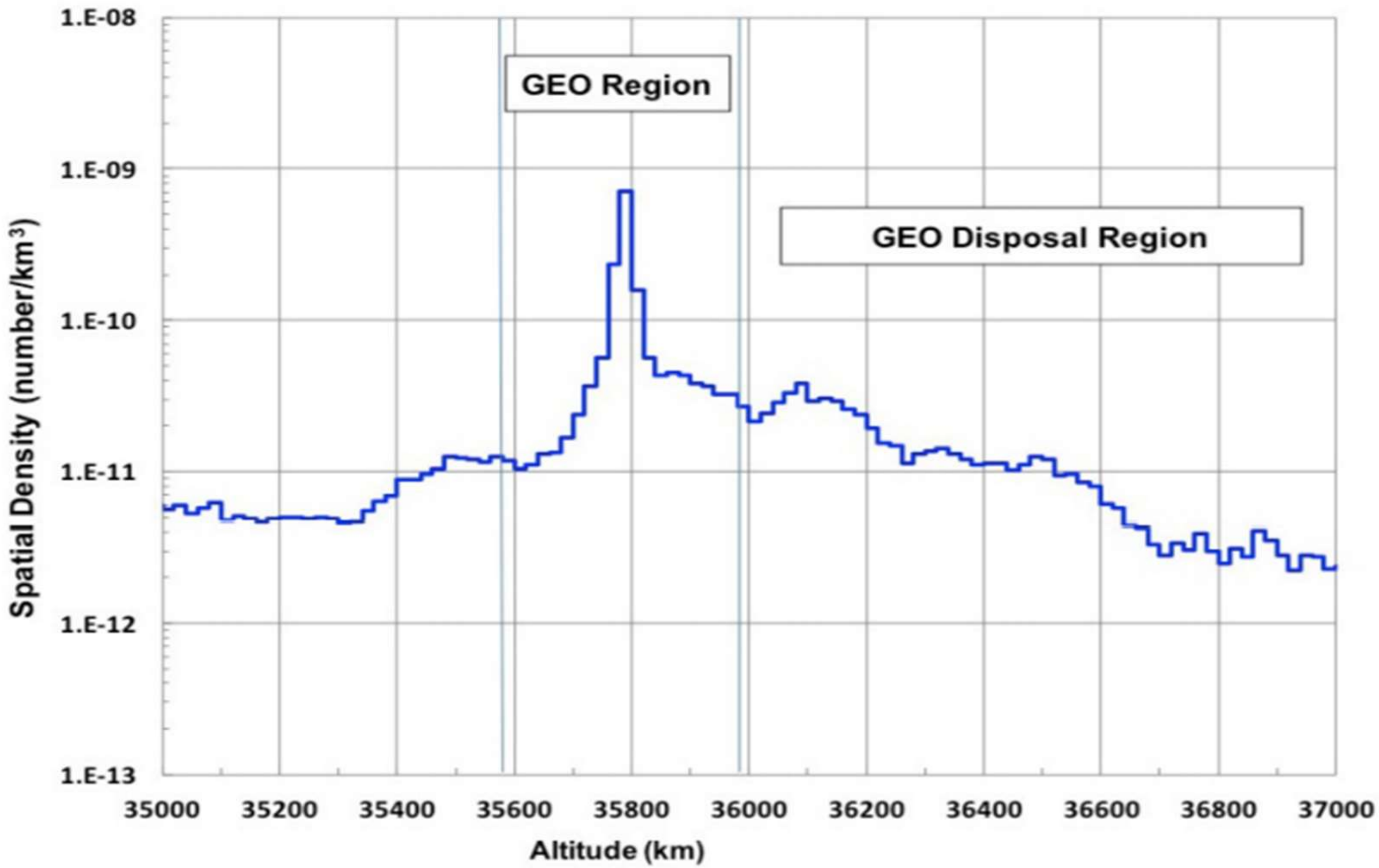
Number of Cataloged Objects in LEO



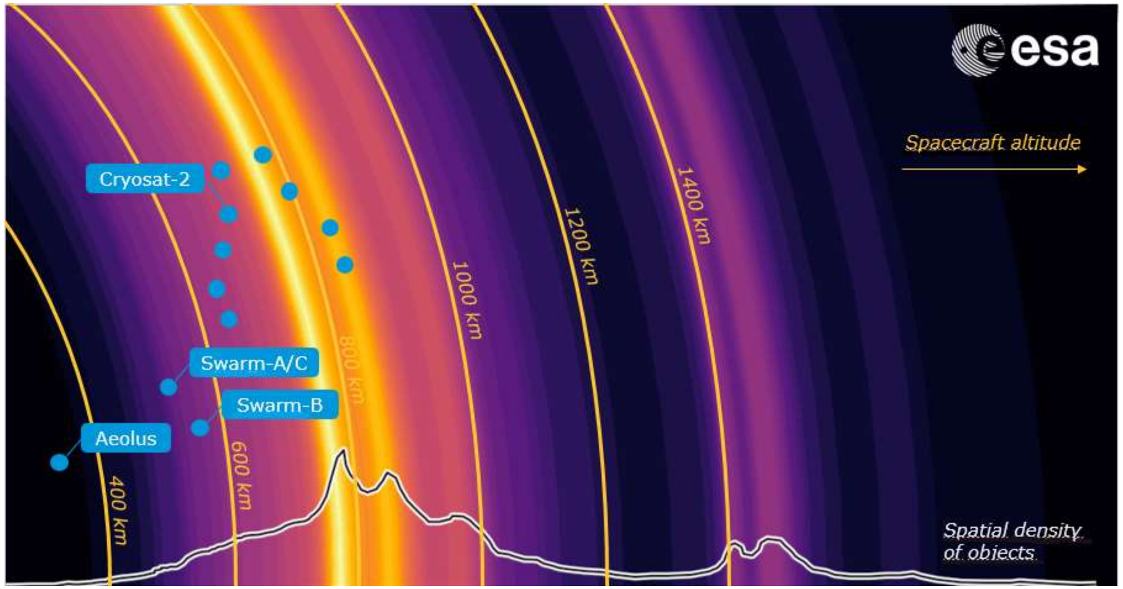
Source: NASA

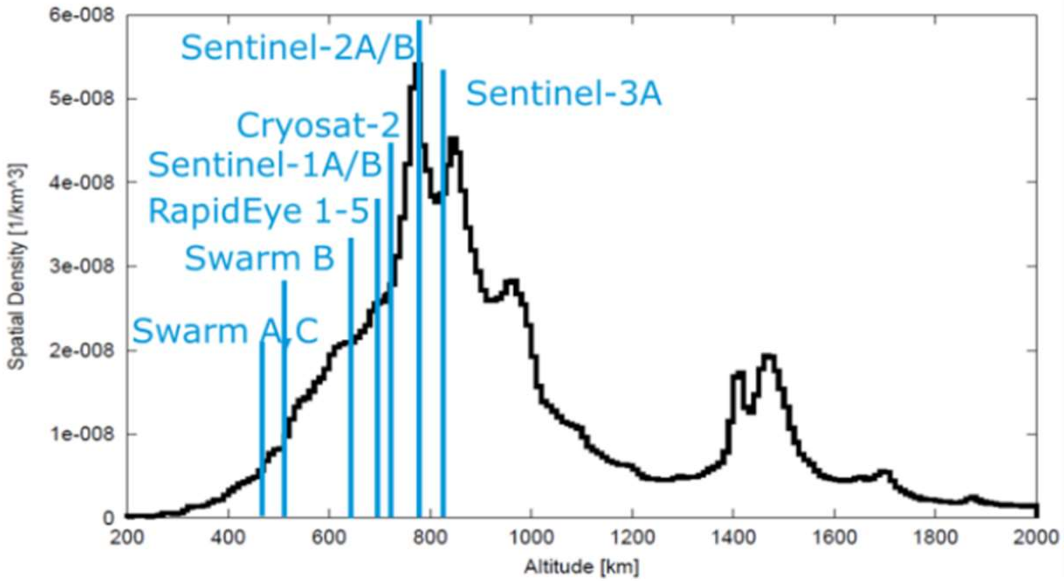
Number of Cataloged Objects in GEO



Source: NASA

The threat of orbital space debris

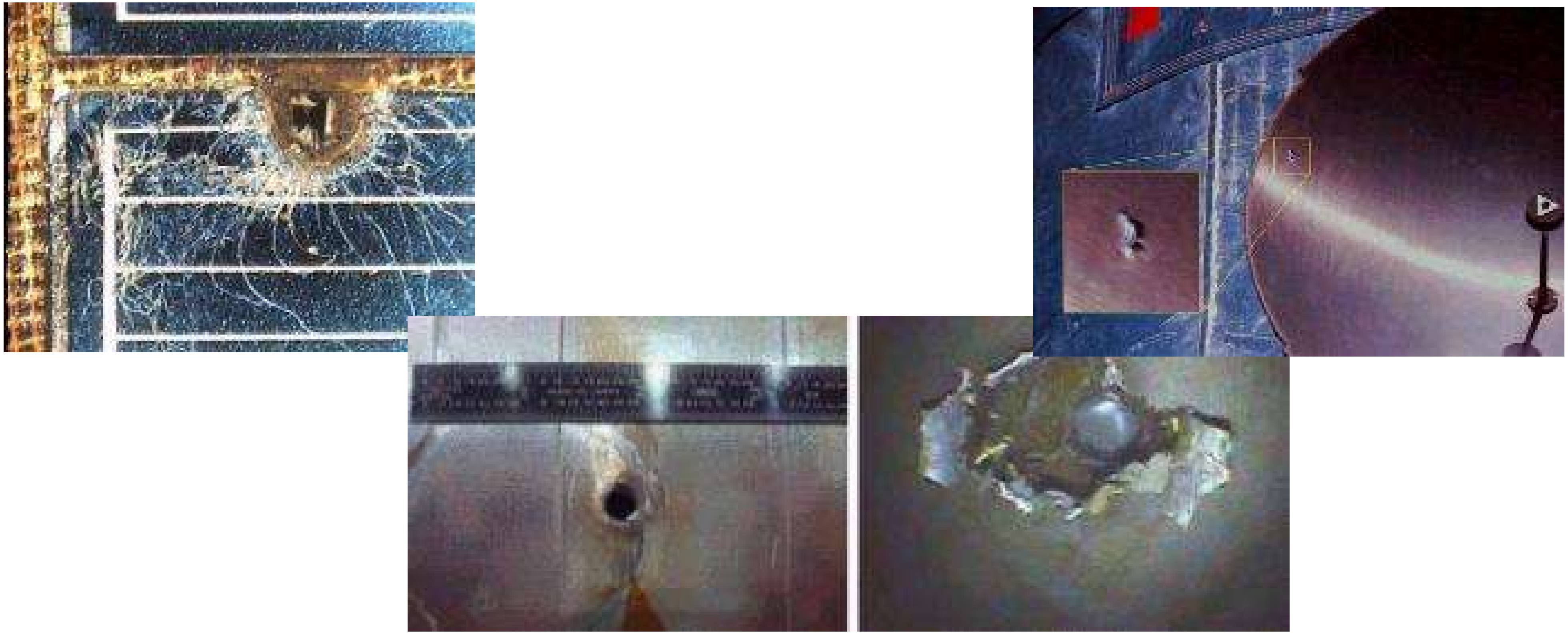




Spatial density of objects > 10 cm

Credits: ESA

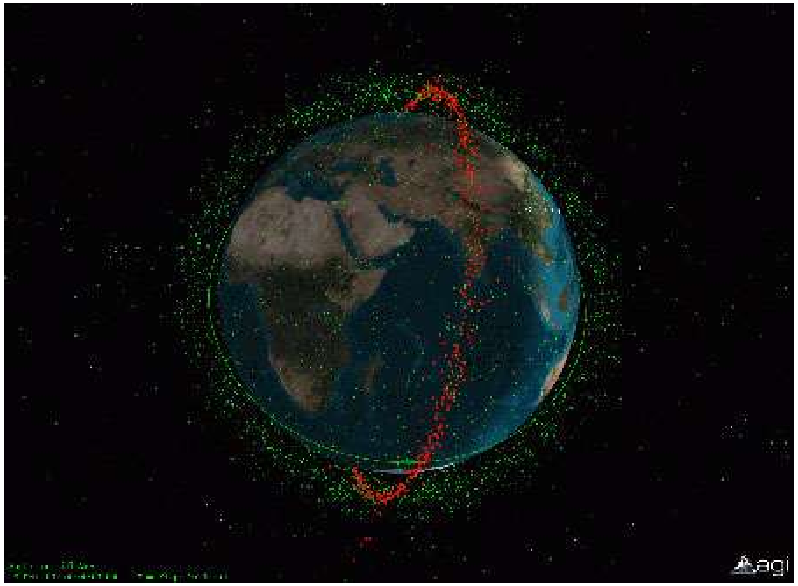
Debris damage



Credits: https://www.spaceacademy.net.au/watch/debris/gsd/gsd.htm

Worst collisions in history

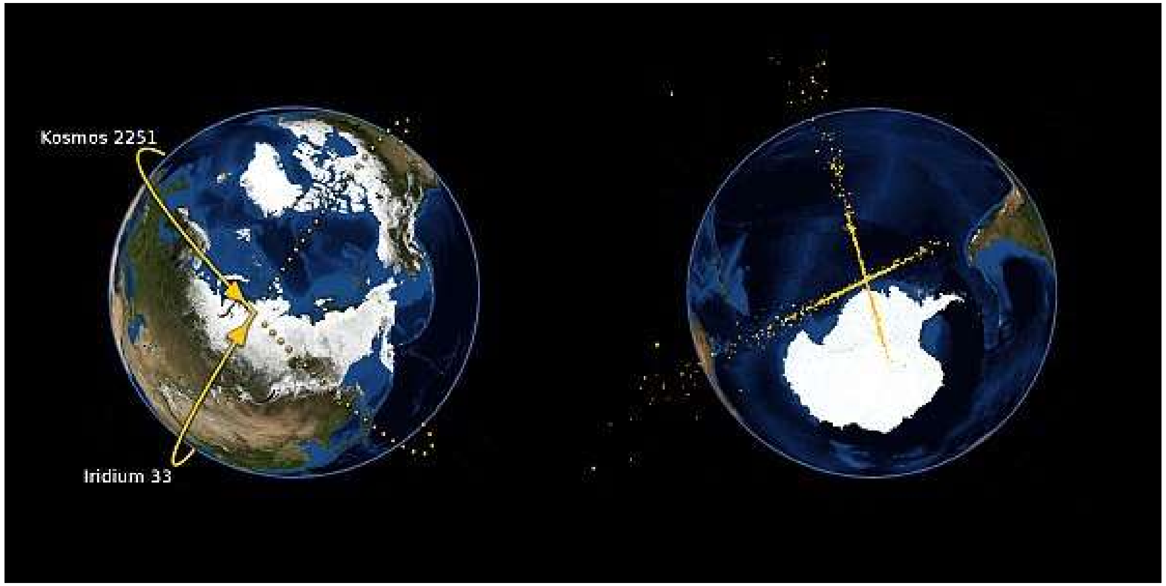
2007 Chinese anti-satellite missile test



View of ISS Orbit (green band) and LEO satellites (green dots) along with debris ring (red) from Chinese ASAT Test

Credit: Dr. Thomas Kelso at CSSI (Center for Space Standards and Innovation)

2009 Iridium – Cosmos collision



Point of collision between Iridium 33 and Kosmos 2251 alongside the debris field after 50 minutes

Collision Avoidance

Identify conjunctions

- by propagating objects’ orbits into the future

Generate CDMs

- containing information regarding the conjunction

Propose maneuvers

- By analyzing CDMs that show high risk of collision

Sustainability of Space

Debris Mitigation - Minimize creation of future debris

- Prevent in-orbit explosions and collisions
- Post-mission disposal

Space Traffic Management - Minimize effect of debris on spacecraft

- Collision avoidance
- Spacecraft shielding

Debris Removal - Remove existing debris from orbit

- Dedicated space mission

Further Reading

1. *Fundamentals of Astrodynamics* by Roger R. Bate, Donald D. Mueller and Jerry E. White
2. *Fundamentals of Astrodynamics and Applications* by David A. Vallado
3. *An Overview Of Space Situational Awareness.* https://www.spaceacademy.net.au/intell/ssa.htm
4. *SSA Programme Overview.* https://www.esa.int/Safety_Security/SSA_Programme_overview
5. *Space traffic management in the new space era.* Muelhaupt et. al. The Journal of Space Safety Engineering (2019)