Gravitational-wave Data Analysis: Parameter Estimation

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What can be measured?

1. GW detection leads to measurements of many properties related to the compact binaries with varying amounts of accuracies.

2. Intrinsic parameters:
   a. Masses (chirp mass and mass ratio)
   b. Spin and precession
   c. Tidal deformity

3. Extrinsic parameters:
   a. Right ascension, declination and inclination angle
   b. Luminosity distance
   c. Polarisation angle and phase

How do we measure the properties?

1. Parameter estimation: matching waveforms with data to obtain distributions for parameter values.

2. Since the data has noise, we do not predict the exact values for the parameters. Instead, we obtain distributions.

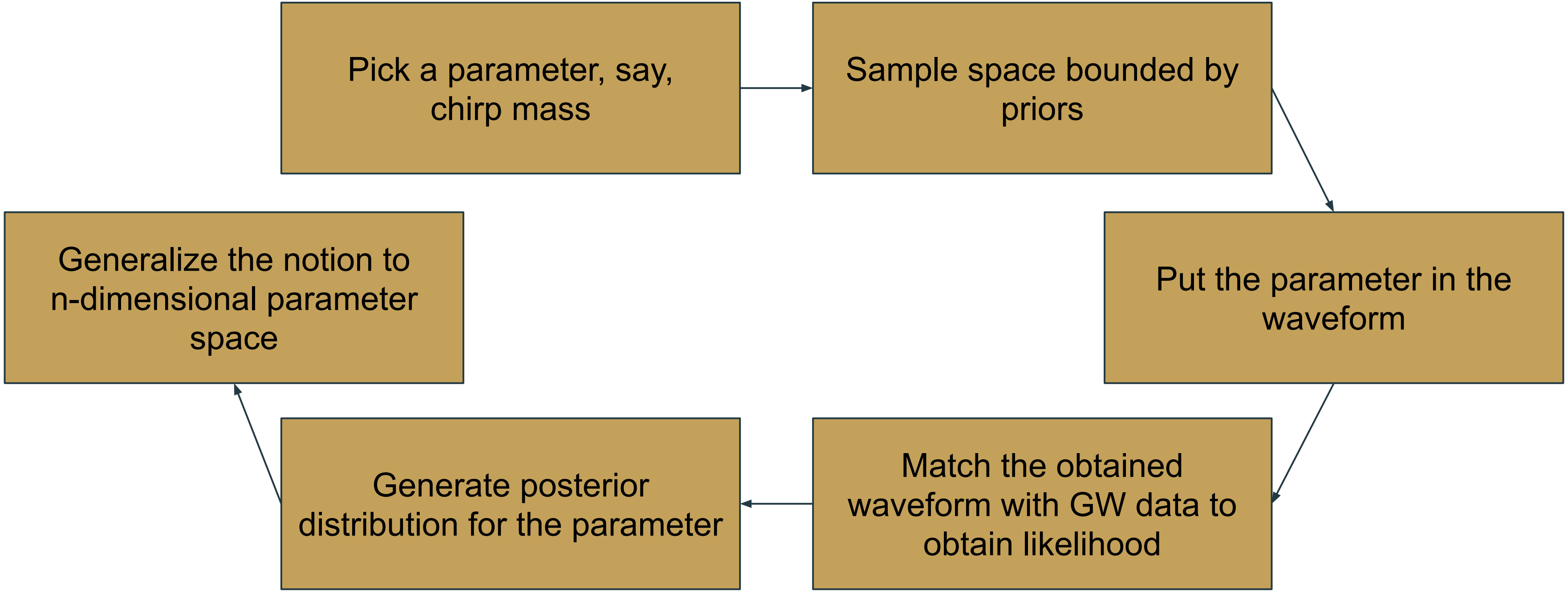
3. Base of the techniques used is the Bayes!

Bayesian Analysis

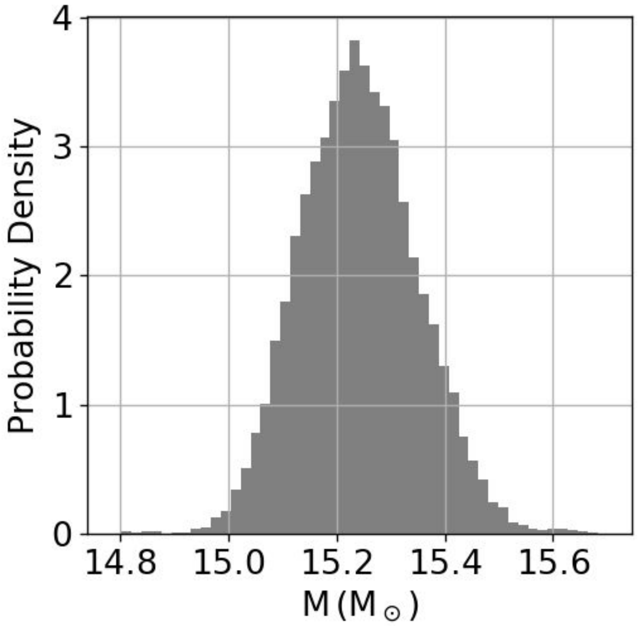
$$P(\mathcal{E} \mid d, H) = \frac{P(\mathcal{E} \mid H)P(d \mid \mathcal{E}, H)}{P(d \mid H)}$$

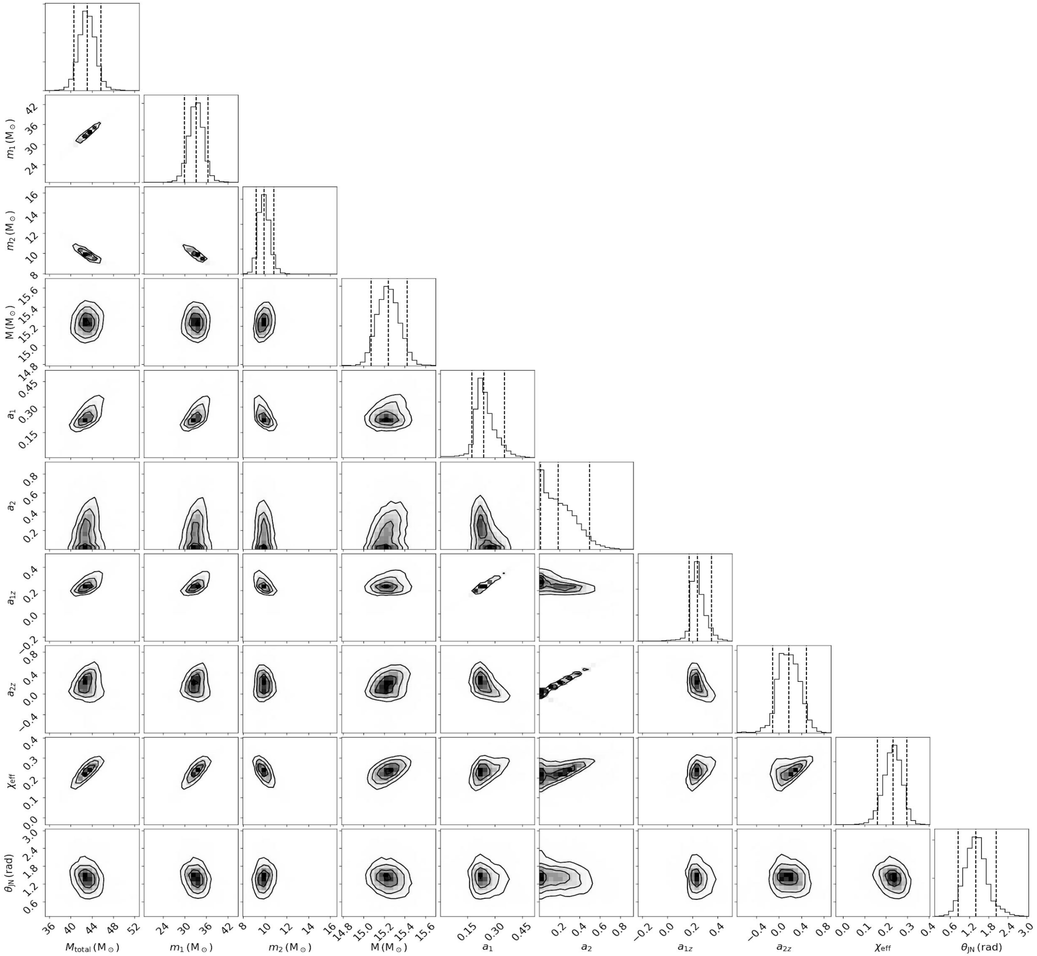
- $P(\mathcal{E} \mid H)$ is called the *prior*. It is the probability distribution of the parameters based on our hypothesis.
- $P(d \mid \mathcal{E}, H)$ is called the *likelihood*. It is the probability that the data contains the signal described by the parameters and our hypothesis.
- $P(d \mid H)$ is the normalization constant and is also referred to as the *evidence*.
- $P(\mathcal{E} \mid d, H)$ is called the *posterior*. It gives the probability distribution of parameters given the data and the hypothesis.

“Sampling over parameters”- the idea



Examples of posteriors





LALSuite

LALSimulation

Waveform Generation

Contains families of waveforms, like Taylor, IMR, SEOBNR, etc. for time-domain and frequency-domain

LALInference

Parameter Estimation

Allows for sampling over the parameters to be estimated to generate posteriors

Some important steps

1. Acquiring data
    a. GW event data (GraceDB)
    b. Injections- Using available waveforms
2. config.ini file- Configuration file that specifies the following:
    a. Data being used (GW or injection)
    b. Noise settings
        i. PSD (Power Spectral Density) data for events
        ii. Zero noise setting can be used for injections
    c. Sampler settings
        i. MCMC
        ii. Nested
        iii. Dynesty (in case of bilby)
    d. Prior bounds for parameters
    e. Version of LALSuite being used
    f. Output directory

Bilby!

1. Why work with bilby instead of LALInference?
    a. New and fast
    b. Easy to install and execute
    c. Python is easier than C
2. Bilby uses LALSimulation to generate the waveforms and GraceDB to obtain the event data.
3. We don’t have a config.ini file here. Instead we have a python script that contains everything.

Sample scripts are available for you to perform parameter estimation by yourself.

Additional Resources

1. https://arxiv.org/abs/0903.0338: Sathyaprakash B.S. and Schutz B.F.; *Physics, Astrophysics and Cosmology with Gravitational Waves*- A thorough review of the theoretical aspects of gravitational waves, detectors and the science behind gravitational-wave data analysis.

2. https://www.gw-openscience.org/tutorials/: Tutorials on GW data analysis by LIGO.

3. https://lscsoft.docs.ligo.org/bilby/: Documentation for bilby.

4. https://lscsoft.docs.ligo.org/bilby/examples.html: Examples to help you use bilby.

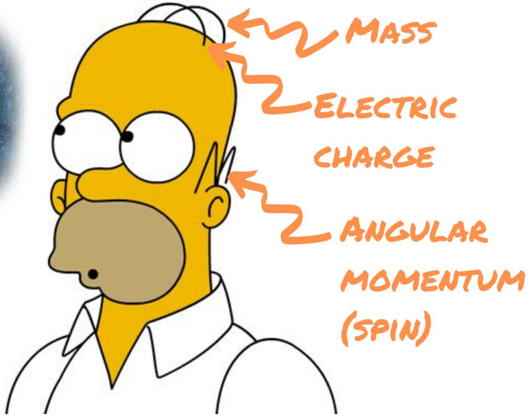
My Research

Testing the no-hair theorem

Black hole no-hair theorem

1. General relativity predicts that a black hole can be completely characterised by their mass, charge and spin.

2. So, binary black holes can also be completely characterised by a set of intrinsic parameters (like chirp mass, mass ratio and spins).



Credits: Matt Groening and Fox Broadcasting

The test we implement

Allow inconsistencies in higher order modes (H.O.M) by introducing deviation parameters

Dominant mode

Subdominant modes

$$h(t;\mathbf{n},\lambda,\Delta\lambda) = \sum_{m=\pm 2} Y_{2m}^{-2}(\mathbf{n})\, h_{2m}(t,\lambda) + \sum_{\text{H.O.M}} Y_{\ell m}^{-2}(\mathbf{n})\, h_{\ell m}(t,\lambda+\Delta\lambda)$$

We introduce deviations in chirp mass and mass ratio. If the deviation is obtained to be 0, this will verify the claims of general relativity.

LALSuite

LALSimulation

Waveform Generation

Include deviation parameters in the respective waveform models

LALInference

Parameter Estimation

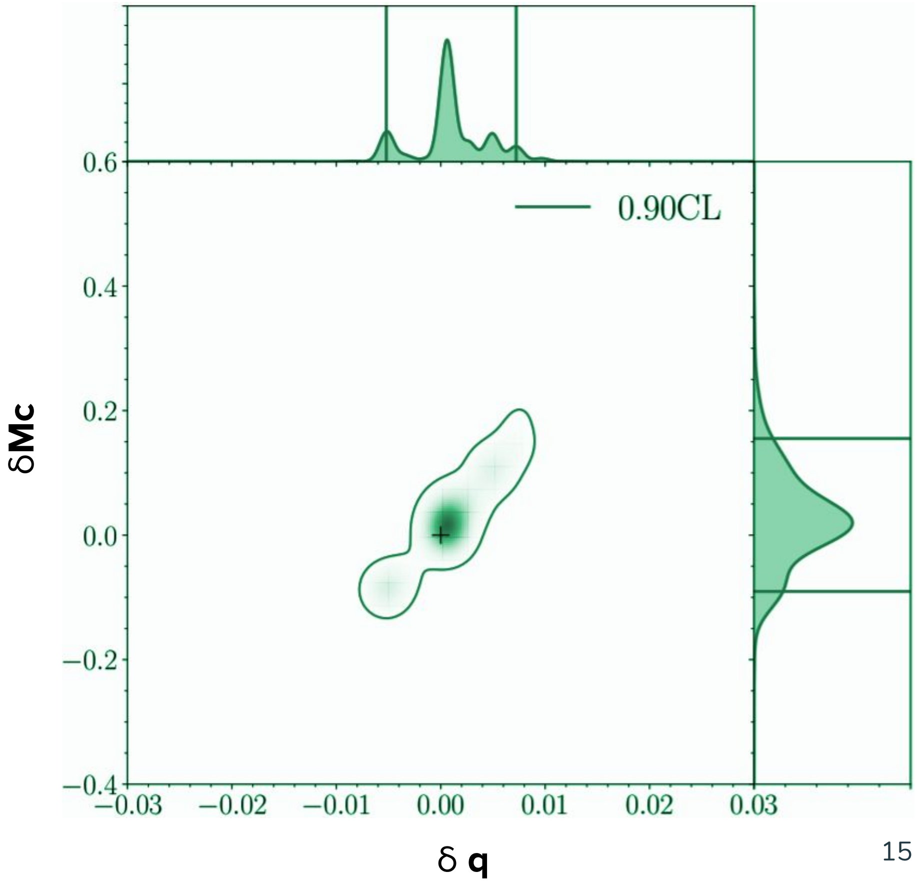
Sample over the introduced deviation variables during parameter estimation

PARAMETER ESTIMATION

System Configuration

- For injection:
    - Mc = 9.43
    - q = 1/9 = 0.11
- Waveform: IMRPhenomHM (including Mc_{HM} and q_{HM})

We inject a GR waveform and retrieve using modified GR version of our model, i.e. allowing Mc_{HM} and q_{HM} to deviate from Mc_{22} and q_{22}.



Questions?