Lab 3.1: Putting opamps to 'gainful' use [Negative Feedback]

Note: This assignment contains only design and simulation questions.
Hardware demo questions are in Lab 3.2 **[60]**

All question material is in blue. Please put your answers in black color font.
Note on symbols for Gain:
In earlier labs we used the symbol **G** to denote the open-loop gain of an opamp. For the LM741 **G~10⁶**(effectively infinite as approximated in most equations).
As discussed in class, the opamp working in open-loop mode is not very useful. Our objective is to design and build an opamp circuit with a finite gain – we will call this gain G_f **i.e. the gain with feedback in place.** G_f **is a finite number whose value is specified by design**

Part A: Simple Negative Feedback

A.1) Simple negative feedback to set finite voltage gain G_f

A.1.1 non-inverting negative feedback

In the pre-lab session, the following scheme of negative feedback was discussed to arrange a circuit with gain $G_f = \frac{1}{x}$:

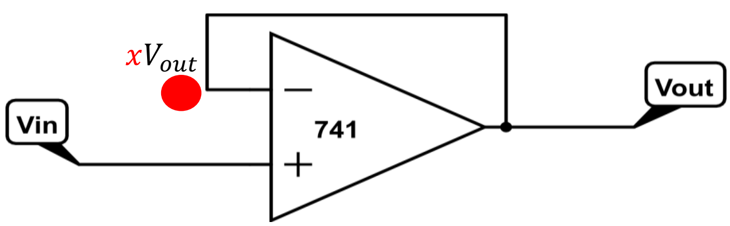


Fig 1: Basic scheme for opamp negative feedback, preserving the sign of V_{in}

A.1.1.2) LTSpice simulation for $G_f = +10$

Arrange a resistor divider to feedback a fraction xV_{out} of the output voltage V_{out} back to the V_- input of the opamp. Be careful to specify the nodes where the resistors in the resistor divider are connected

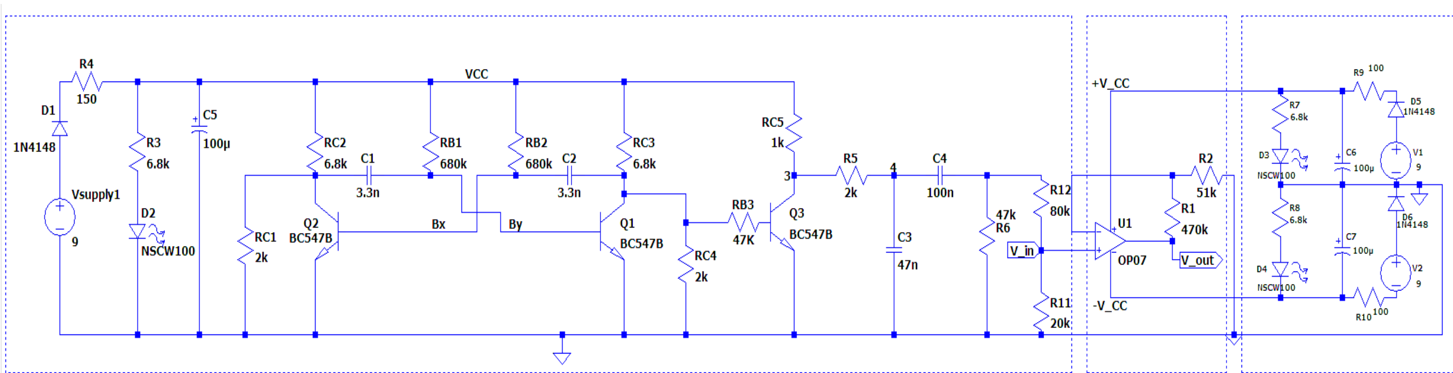
Draw your circuit diagram in LTSpice.
Include the FG simulation used earlier in PH231 to get an accurate idea of the V_{in} waveform you will be experimenting with later.
You may use the standard opamp available in LTSpice OP07 as a stand-in replacement for LM741

Run the simulation in LTSpice and put below a plot of your simulated $V_{in} \rightarrow V_{out}$ signals.
Now obviously you need to include the FG simulation for this step.Remember to put large value electrolytic (100μF/47μF/22μF/10μF) power supply bypass capacitors for both FG and opamp power rails. Note that you will be running the opamp with ≈ ±8V V_{CC} rail voltage as setup in Lab 2.
From the simulation determine a suitable amplitude of V_{in} such that V_{out} does not hit the opamp

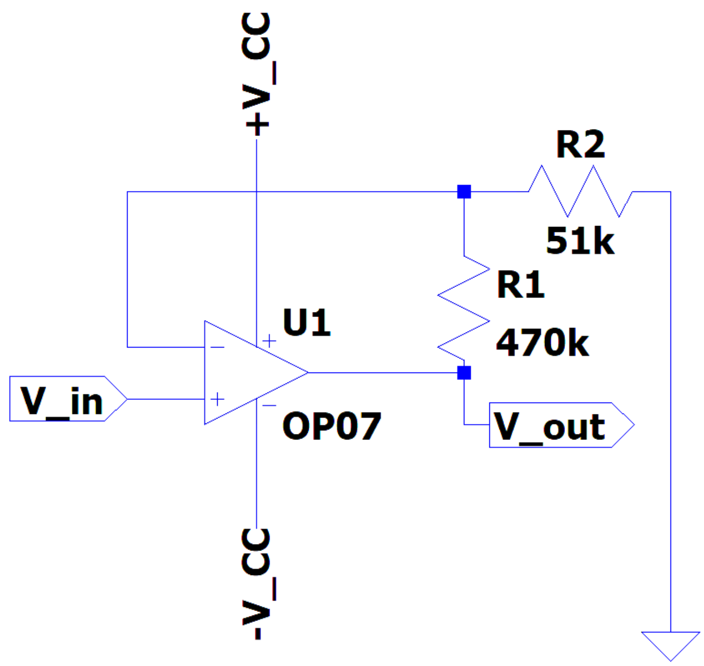
saturation voltage limits when you test the circuit experimentally.
Objective of your simulation: Verify the transfer function $V_{in} \rightarrow V_{out}$
PLOTS EXPECTED:

[*] LTSpice diagram, including the FG used earlier from PH231 [5]

Complete Circuit:



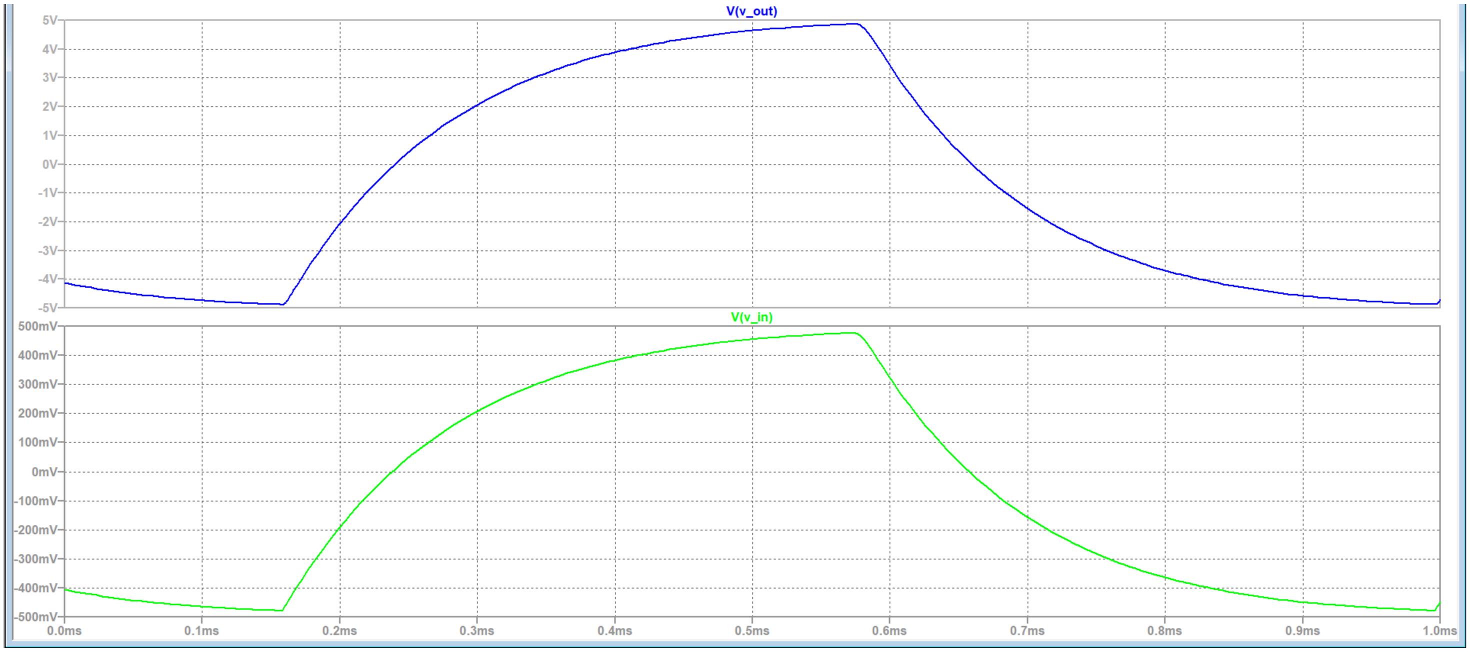
Non-Inverting Amplifier block:



[1] Time domain at a fixed frequency: Simulate with the FG at fixed f = 1kHz [5]

To check the phase difference, you must display both V_{in} and V_{out} with individual scaling on Y-axis: First plot just V_{in} using the voltage probe tool. Then, select the Output window and choose "Add Plot Pane" option from the "Plot Settings" drop-down control. Next, plot V_{out} as the second voltage using the voltage probe tool.

$$V_{in|max} = 480mV, V_{out|max} = 4.8V \ll V_{sat} \simeq 8V$$

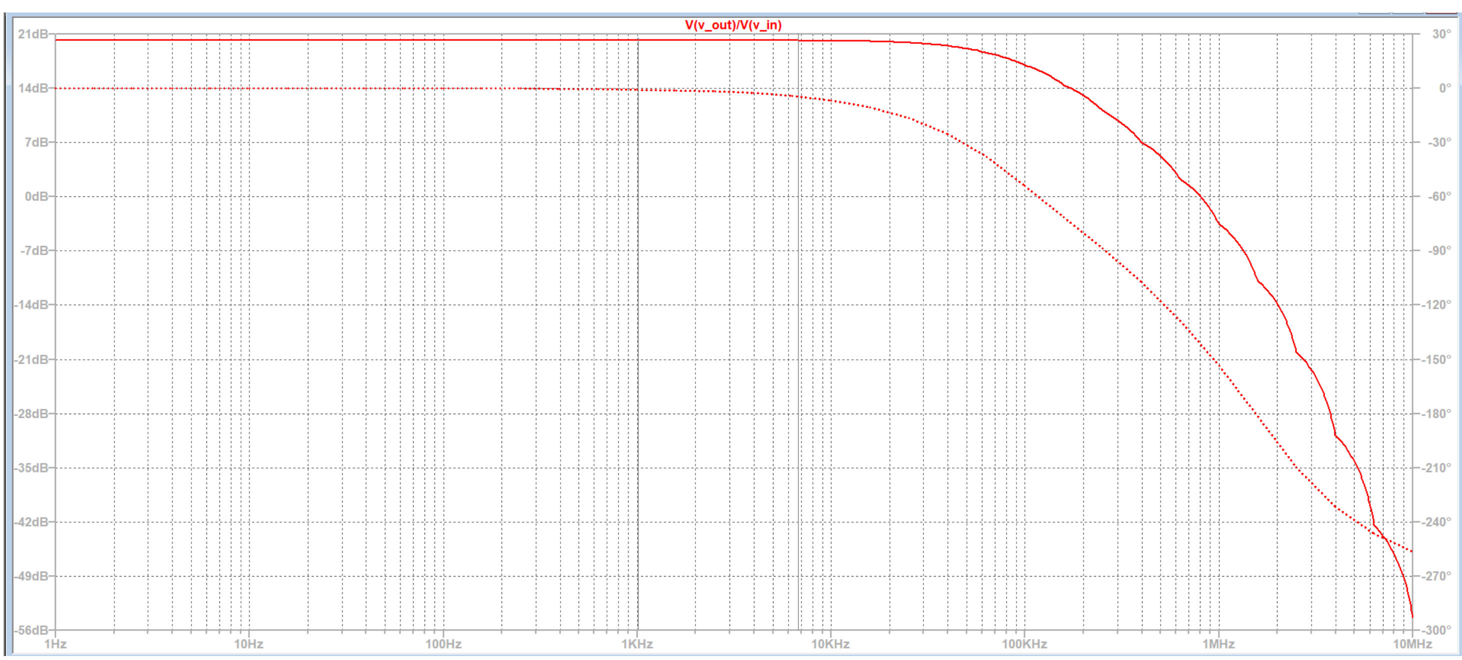


Since the frequency is well within the plateau region ($\simeq 0°$) of the Bode plot, the phase offset between input and output is indeed close to $0°$.

[2] **Frequency response** **[5]**

The frequency analysis function of LTSpice uses a specific AC voltage source.
What is the time domain waveform of this source? [look up LTSpice documentation!]
Obviously, your PH231 designed FG has a fixed frequency, so it can't be used for the frequency sweep. In the simulation, use a separate V_{in} signal source as required for frequency analysis. Plot the frequency dependent magnitude and phase response of your circuit (i.e., Bode plot) up to 10MHz

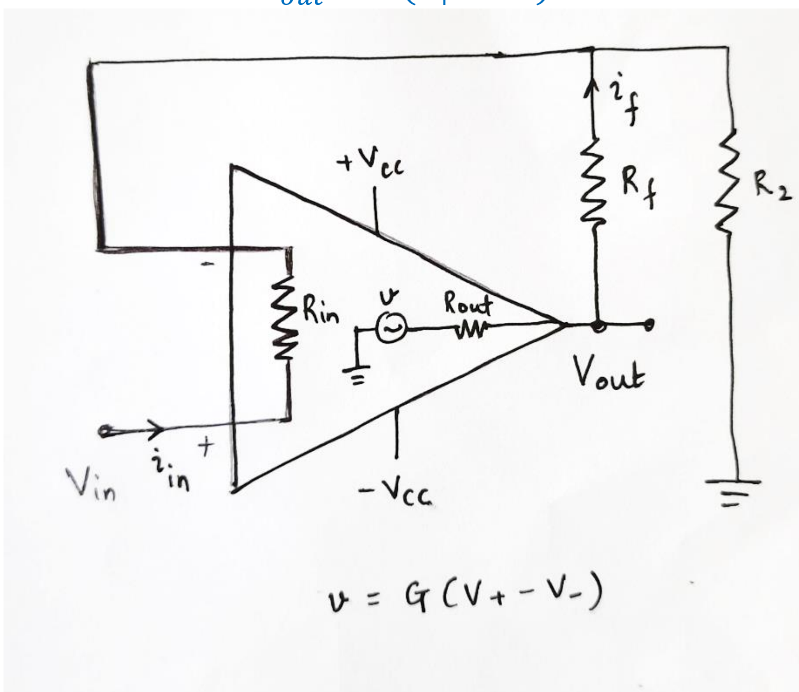
For frequency analysis LTSpice uses a sinusoidal signal of variable frequency. The Bode plot is given below: (dotted is the phase shift, solid curve is the gain amplitude)



A.1.1.2) Design math **[2]**

Work out the design equations to calculate values of the resistor divider for $G_f = +10$ in your circuit design. Justify the choice of particular resistor values you use. Write all steps of the design equation here: Use 'Insert equation' in MS-Word or Libre-Office to format your equations correctly including subscripts. A prototype equation is provided below, which you may copy-paste and reuse. One equation per line looks good!

$$V_{out} = G(V_+ - V_-)$$



$$V_+ = V_{in}$$

$$V_- = V_+ - i_{in}R_{in} = V_+ - i_{in}R_{in}$$

$$V_- = (i_f + i_{in})R_2 \ (\because current\ through\ R_2\ is\ i_{in} + i_f)$$

$$i_f = \frac{V_{out} - V_-}{R_f}\ and\ i_{in} = (V_{in} - V_-)/R_{in}$$

Thus, we have two equations: $V_{out} = G(V_{in} - V_-)\ and\ V_- = \frac{R_2(V_{out} - V_-)}{R_f} + \frac{R_2(V_{in} - V_-)}{R_{in}}$

But here we know that $R_{in} \gg R_2 \Rightarrow V_- \simeq V_{out}\left(\frac{R_2}{R_2 + R_f}\right)$

$$\therefore V_{out} = G\left(V_{in} - V_{out}\left(\frac{R_2}{R_2 + R_f}\right)\right)$$

$$\Rightarrow \frac{\boldsymbol{V_{out}}}{\boldsymbol{V_{in}}} = \boldsymbol{G_f} = \frac{\boldsymbol{G}}{\boldsymbol{1} + \boldsymbol{G}\left(\frac{\boldsymbol{R_2}}{\boldsymbol{R_2} + \boldsymbol{R_f}}\right)} \simeq \boldsymbol{1} + \frac{\boldsymbol{R_f}}{\boldsymbol{R_2}}$$

$For\ G_f = 10, R_f = 9R_2$

Values of resistors chosen for the circuit: $R_f = 470k\Omega\ and\ R_2 = 51k\Omega$

The analysis is done without invoking the approximation that $i_{in} \simeq 0$ beforehand in order to make the impedance analysis easier in further questions.

A.2) Negative feedback, inverting the sign of V_{in}

By a rearrangement of node connections in Fig 1 it is possible to apply a negative sign to the gain: $V_{out} = -G_f V_{in}$.

This is called an 'inverting' configuration since V_{out} is out of phase with respect to V_{in} by 180°

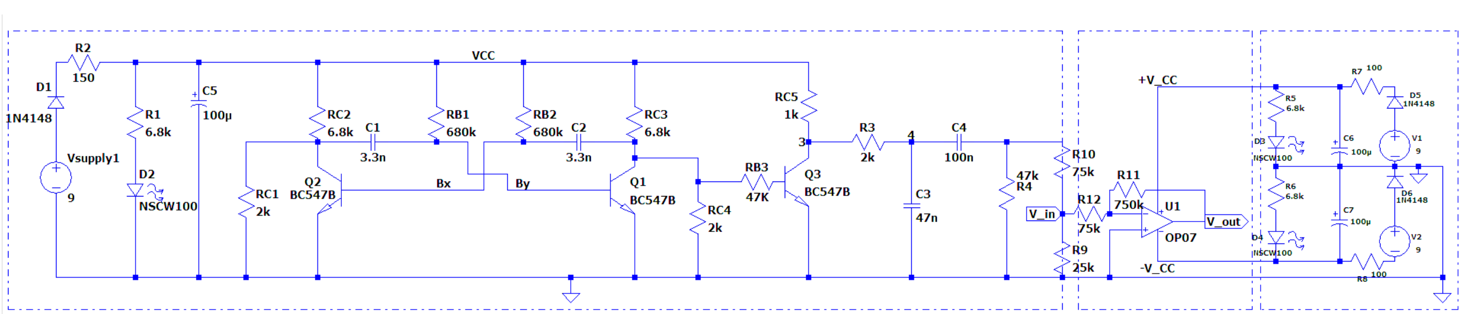
Keep in mind that for negative feedback, by definition a fraction of V_{out} must be applied to the V_- terminal of the opamp. However, you are free to cleverly connect V_{in} to either V_+ or V_-

Work out your circuit design and component values for implementing the experimental relation $\boldsymbol{V_{out} = -10 \times V_{in}}$: the absolute value of the G_f is the same, but now there is a negative sign!

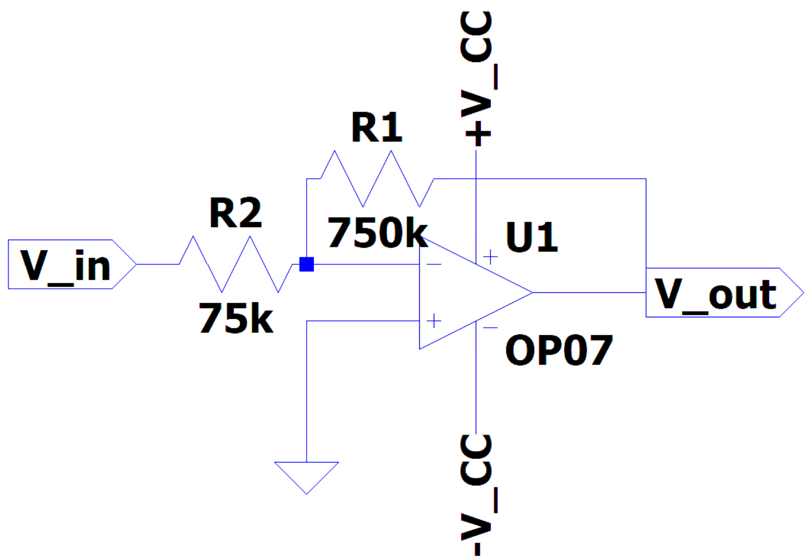
A.2.1) [10]

Circuit design: **3 marks**

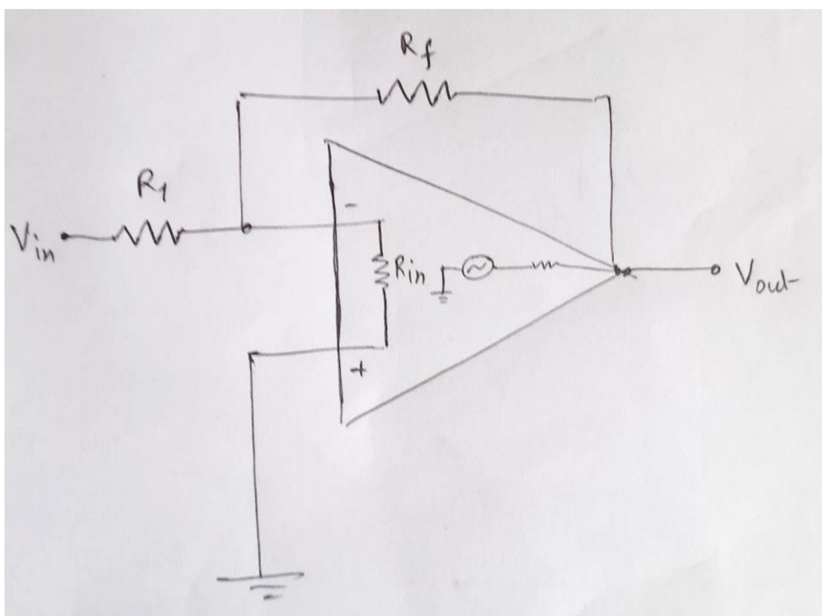
Complete Circuit:



Inverting Amplidier block:



Equations: **2 marks**



$Let\ i_1 \equiv current\ through\ R_1, i_{in} \equiv current\ through\ R_{in}$

$\Rightarrow current\ through\ R_f \equiv i_1 - i_{in}$

$V_+ = 0V\ (GND)$

$\Rightarrow V_{out} = G(V_+ - V_-) = -GV_-$

$$V_- = V_{in} - i_1 R_1$$

$$Also, V_- = i_{in} R_{in}$$

$$\Rightarrow V_{out} = V_- - (i_1 - i_{in}) R_f$$

$$\Rightarrow V_{out} = V_- - \frac{(V_{in} - V_-) R_f}{R_1} + \frac{V_- R_f}{R_{in}}$$

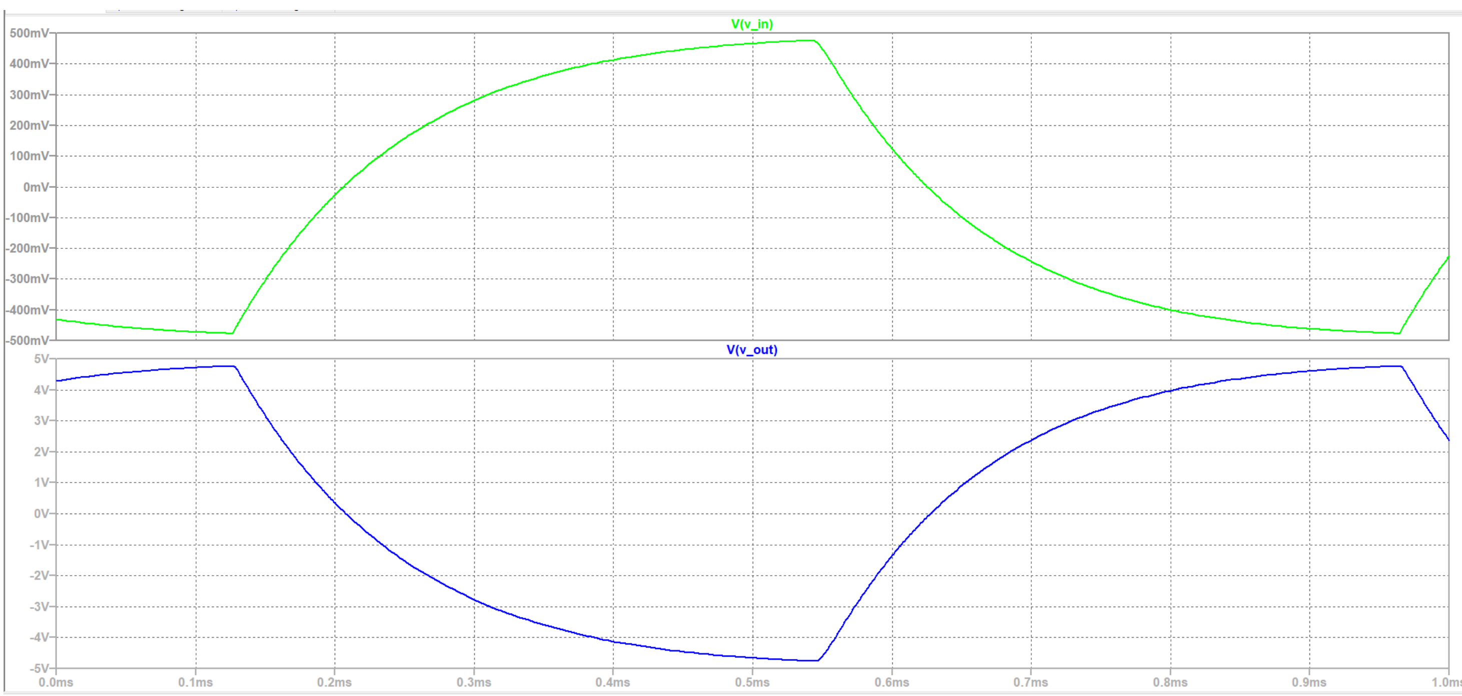
$$Here, R_{in} \gg R_f \text{ also we know that } V_{out} = -GV_-$$

$$\Rightarrow V_{out} = -\frac{V_{out}}{G} - \frac{\left(V_{in} - \frac{V_{out}}{G}\right) R_f}{R_1} + \frac{V_{out} R_f}{G R_{in}} \simeq -\frac{R_f}{R_1} V_{in}$$

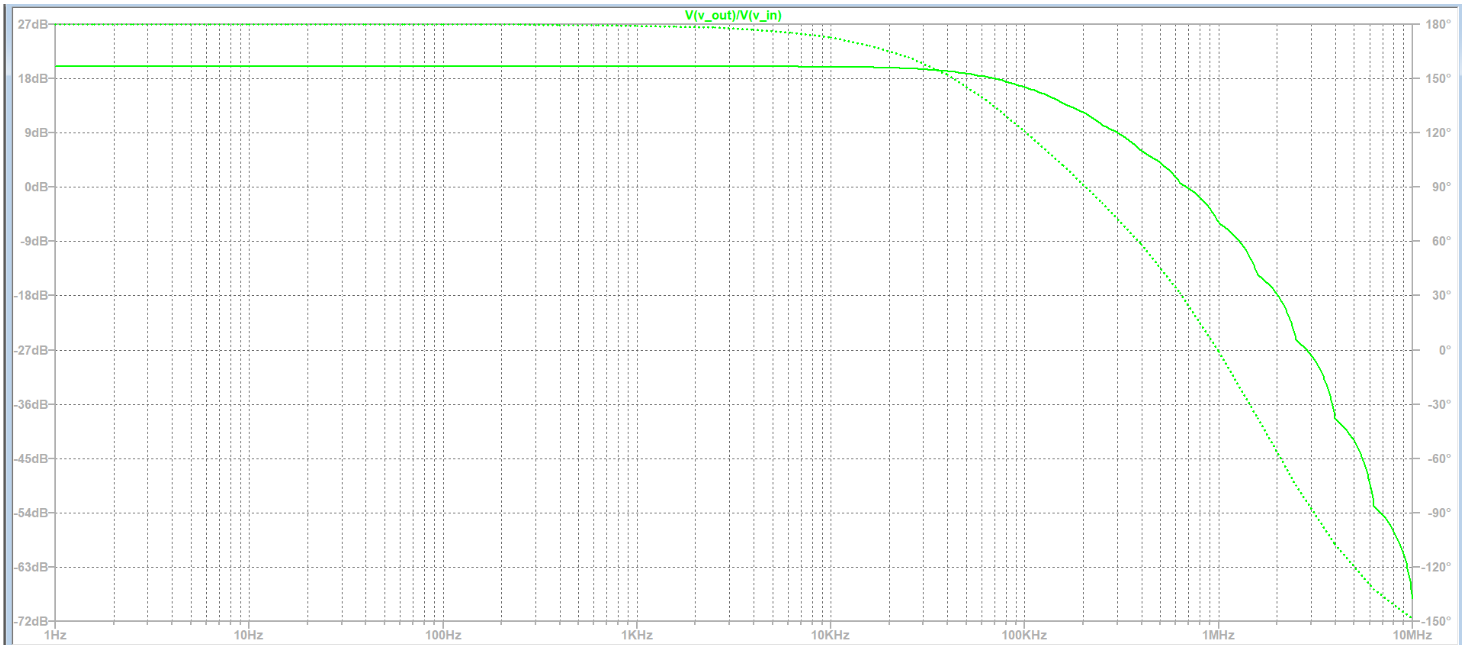
$$\therefore G_f = -\frac{R_f}{R_1}$$

LTSpice simulation: **5 marks (time domain with FG, and frequency sweep as in A.1)**

Time domain analysis for FG: ($V_{in} \simeq \pm 460mV$ and $V_{out} \simeq \mp 4.6V$)



Frequency domain, Bode plot:



A.2.2) [Thought experiment] With only a single channel measurement on the DSO, when you build the circuit, how will you confirm that the amplifier is inverting the sign of V_{in}? [5]

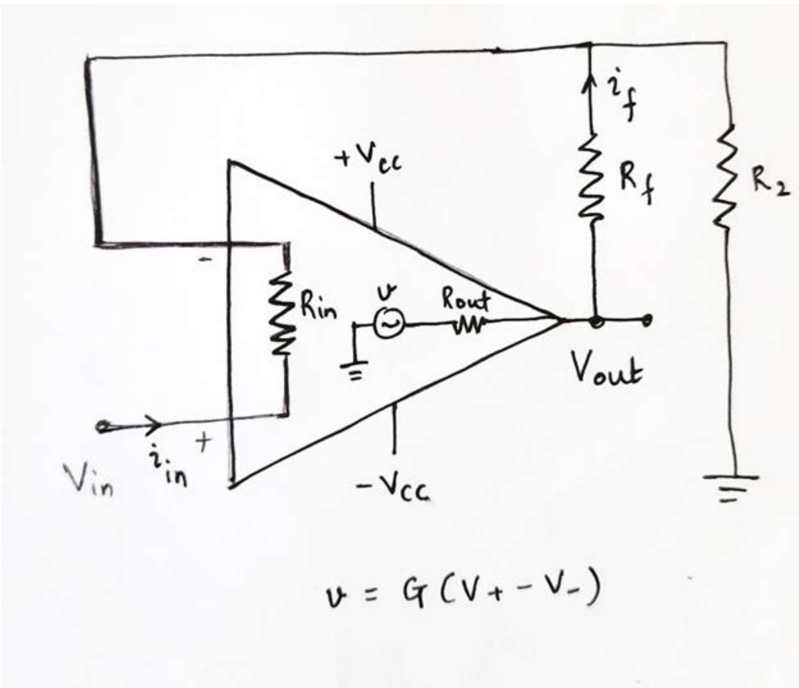
The trick: The DSO can measure voltage difference between any two nodes at a time. Thus, given that all voltages are well defined with respect to an absolute ground we can easily estimate whether two AC voltages are in phase ($0°$) or out of phase ($180°$). (Here we assume that we have V_{out} exactly in/out of phase not in-between)
Procedure:

1) Measure V_{in} with respect to the absolute ground- we get an AC signal of amplitude V, say.
2) Measure V_{out} with respect to the absolute ground- we get an AC signal of amplitude $G_f V$, say. (where G_f is the absolute value of amplification)
3) Now measure V_{out} *with respect to* V_{in} i.e., connect the red probe to V_{out} and the black to V_{in}. When the Amplification is out of phase the DSO output must show AC output of amplitude $(G_f - 1)V$ which verifies what we want.

A.3) Input impedances?

1) What is the input impedance of the straightforward "non-inverting" configuration of A.1, when $V_{out} = +G_f V_{in}$? **[4]**

Explain your answer with calculation steps & logical reasoning of the flow of current into/out of various nodes (you can check this in your LTSpice simulation)



The flow of current is as indicated in the diagram:

i_{in} is fed into the Op-Amp due to V_{in} whereas i_f flows through R_f.

The current through R_2 can simply be found by Kirchhoff's junction law.

Note: The variables used here and their relations have properly been defined and derived in A.1.1.2

$$By\ definition\ Z_{in} = \frac{V_{in}}{i_{in}}$$

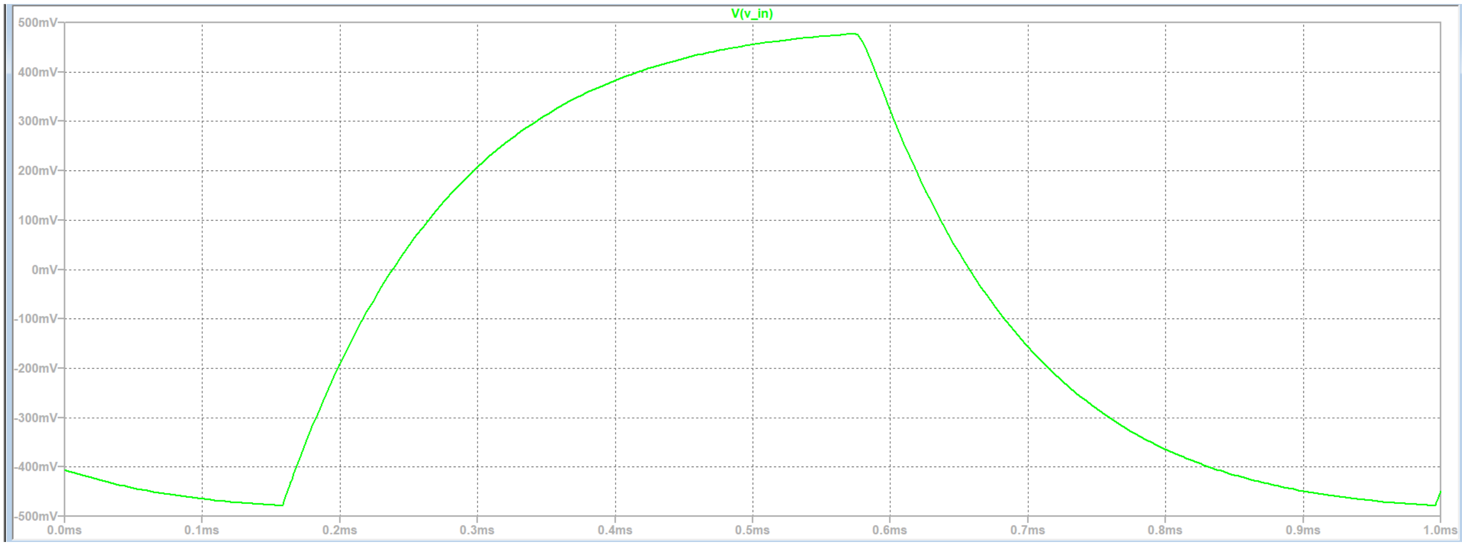
$$i_{in} = \frac{V_{in} - V_-}{R_{in}} = \frac{V_{out}}{GR_{in}} = \frac{V_{in}\left(1 + \frac{R_f}{R_2}\right)}{GR_{in}} \quad \{derived\ in\ A.1.1.2\}$$

$$\Rightarrow Z_{in} = \frac{V_{in}R_{in}}{V_{in} - V_-} = \frac{GR_{in}}{1 + \frac{R_f}{R_2}}$$

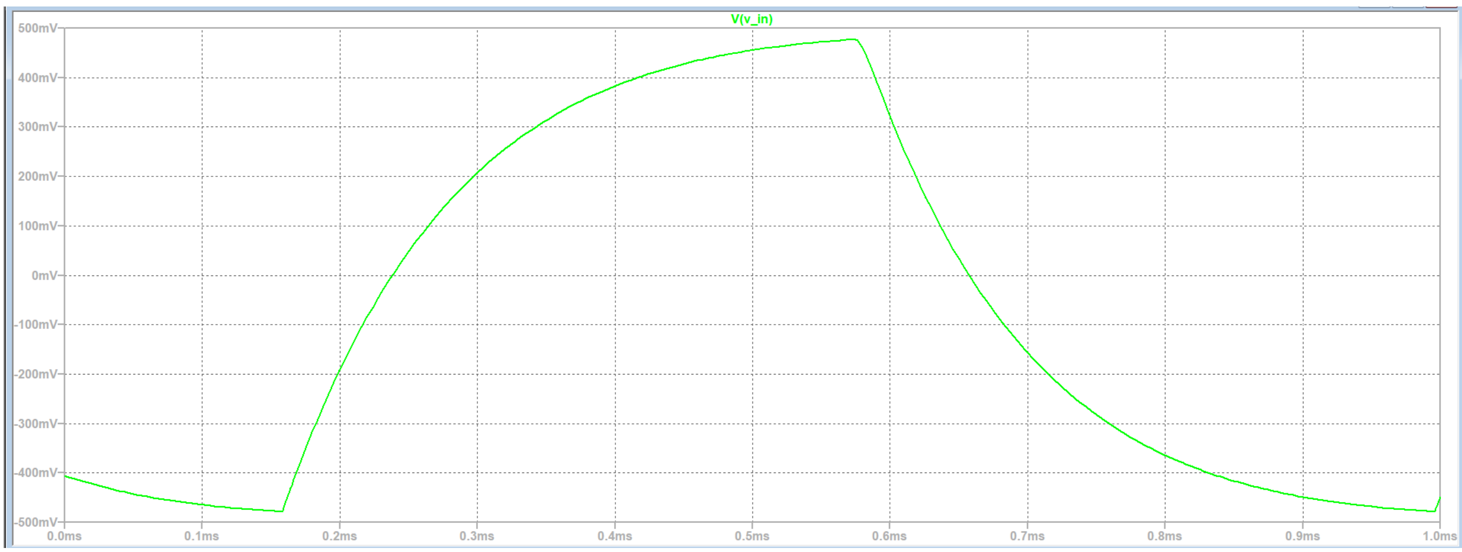
Thus, as expected the input impedance is huge, effectively infinite.

This is evident from the fact that the FG output is unaffected after connection of Op-Amp:

Without Op-Amp: ($V_{in} = \pm 480mV$)

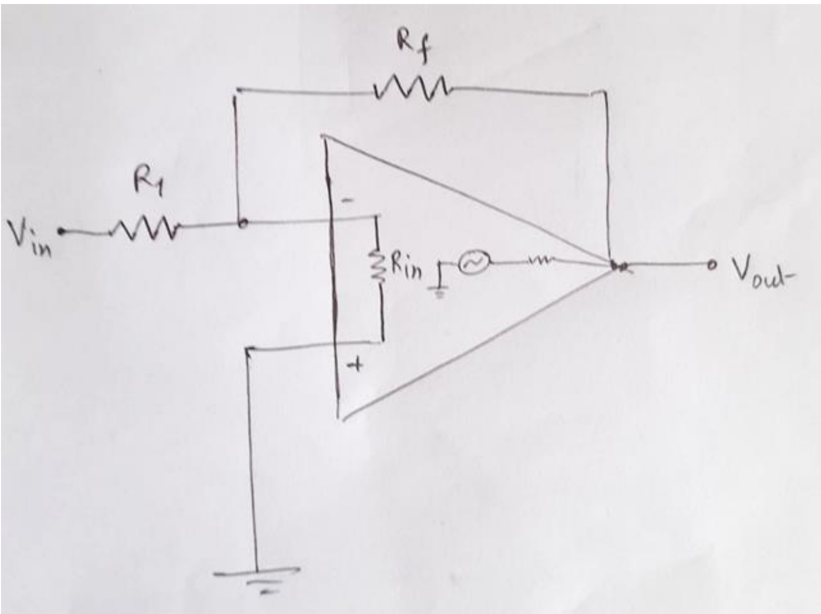


With Op-Amp: ($V_{in} = \pm 480mV$)



2) What is the input impedance of the "inverting" configuration of A.2, when $V_{out} = -G_f V_{in}$? **[4]**

Explain your answer with calculation steps & logical reasoning of the flow of current into/out of various nodes (you can check this in your LTSpice simulation)



The current i_1 flows from V_{in} into the node V_- through R_1

A fraction of this current flows into the Op-Amp, call it i_{in}

The current through the feedback branch found by simply applying Kirchhoff's junction rule.

Note: The variables used here and their relations have properly been defined and derived in A.2.1

$$Z_{in} = \frac{V_{in}}{i_1}$$

$$i_1 = \frac{V_{in} - V_-}{R_1} \; and \; V_{out} = -GV_- = -\frac{R_f}{R_1} V_{in}$$

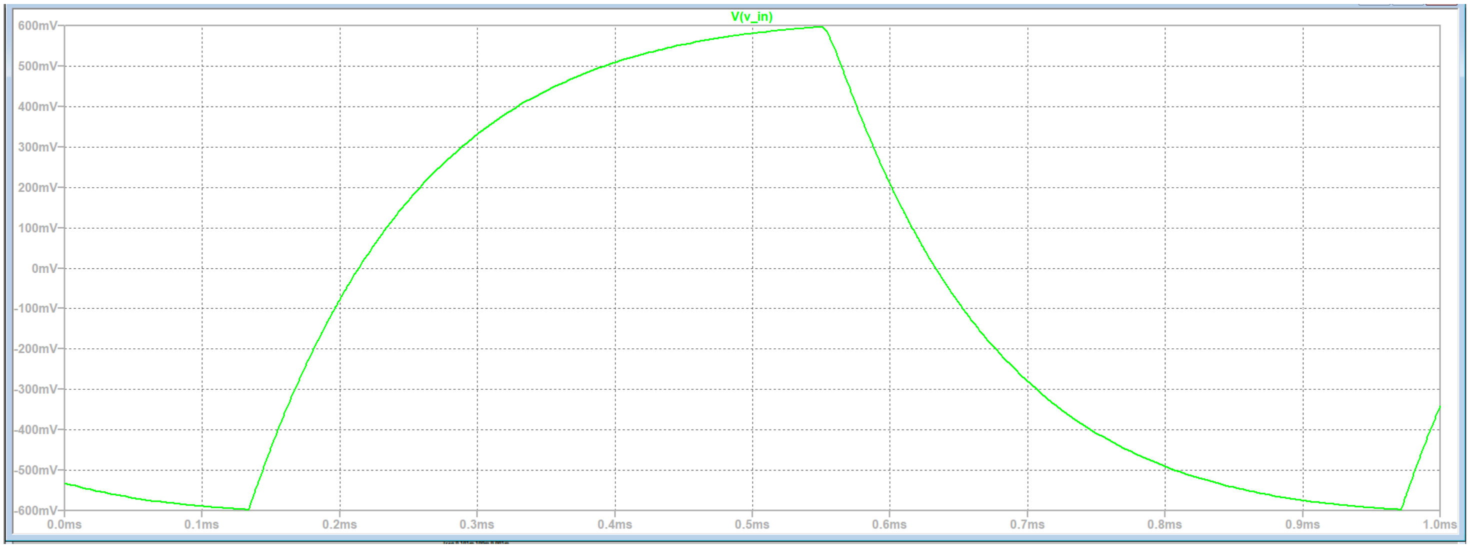
$$\Rightarrow V_- = \frac{R_f}{GR_1} V_{in} \;\Rightarrow Z_{in} = \frac{V_{in} R_1}{V_{in} - \frac{R_f}{GR_1} V_{in}}$$

$$\Rightarrow Z_{in} \simeq R_1 \quad \left\{ \frac{GR_f}{R_1} \ll 1 \right\}$$

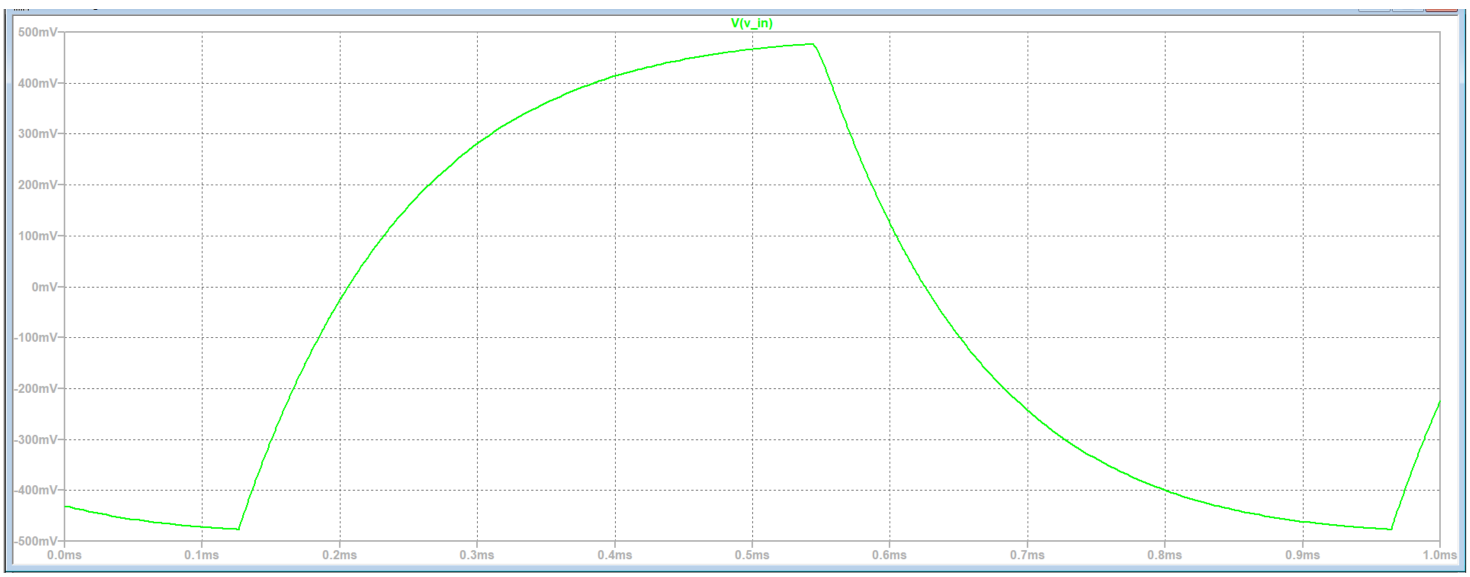
As observed the impedance is quite small compared to the non-inverting case.

This is evident from the fact that the FG output shows considerable change on connecting the Op-Amp circuit:

Without Op-Amp: ($V_{in} = \pm 600mV$)



With Op-Amp: ($V_{in} = \pm 460mV$)



Part B) "Complex" negative feedback

For Part B be sure to start from the <u>INVERTING</u> amplifier configuration of A.3

In part A, for the fraction xV_{out} sent as feedback to the V_{-} input of the opamp, we used x implicitly as a real number, formed by a resistor divider.

Generalize the idea – if a reactive component like a capacitor is used in the feedback loop, the $V_{in} \rightarrow V_{out}$ relation becomes a function of the signal frequency.

B.1) Integrator / low pass filter

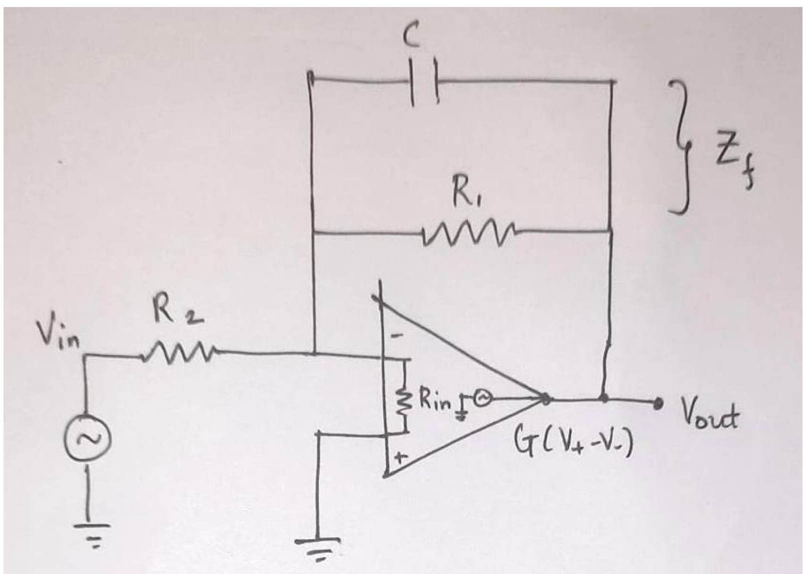
Redesign the circuit of Part A.3 such that the circuit works as a low-pass filter in the frequency domain, and an integrator in the time domain.

The following parts are expected for the solution:

B.1.1) Circuit design and equation calculation steps **[5]**

Design your low pass filter with a corner frequency $f_{3dB} = 160Hz$

Choose R, C values such that the absolute magnitude of the gain in the pass band is 10



Notice that the circuit skeleton is essentially the same as that of an inverting amplifier with just one addition, R_f is replaced by a parallel combination of the reactance X_C of the capacitor and the resistor R_1.

$$Z_f = X_C || R_1 = \frac{R_1}{1 + j\omega R_1 C}$$

Where $\omega = 2\pi f$ and $j = \sqrt{-1}$

The analysis of the circuit remains the same except for the change mentioned above.

$$Let\ i_2 \equiv current\ through\ R_2, i_{in} \equiv current\ through\ R_{in}$$

$$\Rightarrow current\ through\ Z_f \equiv i_2 - i_{in}$$

$$V_+ = 0V\ (GND) \Rightarrow V_{out} = G(V_+ - V_-) = -GV_-$$

$$V_- = V_{in} - i_2 R_2$$

$$Also, V_- = i_{in} R_{in}$$

$$\Rightarrow V_{out} = V_- - (i_2 - i_{in})Z_f \Rightarrow V_{out} = V_- - \frac{(V_{in} - V_-)Z_f}{R_2} + \frac{V_- Z_f}{R_{in}}$$

$$Here, R_{in} \gg |Z_f|\ also\ we\ know\ that\ V_{out} = -GV_-$$

$$\Rightarrow V_{out} = -\frac{V_{out}}{G} - \frac{\left(V_{in} - \frac{V_{out}}{G}\right) Z_f}{R_2} + \frac{V_{out} Z_f}{G R_{in}} \simeq -\frac{Z_f}{R_2} V_{in}$$

$$\therefore G_f = -\frac{Z_f}{R_2} = -\left(\frac{R_1}{R_2}\right)\left(\frac{1}{1 + j\omega R_1 C}\right)$$

In the pass band for $f \simeq 0, |G_f| = \left|\frac{R_1}{R_2}\right| = 10$

For $R_1 = 750k\Omega \; and \; R_2 = 75k\Omega;$

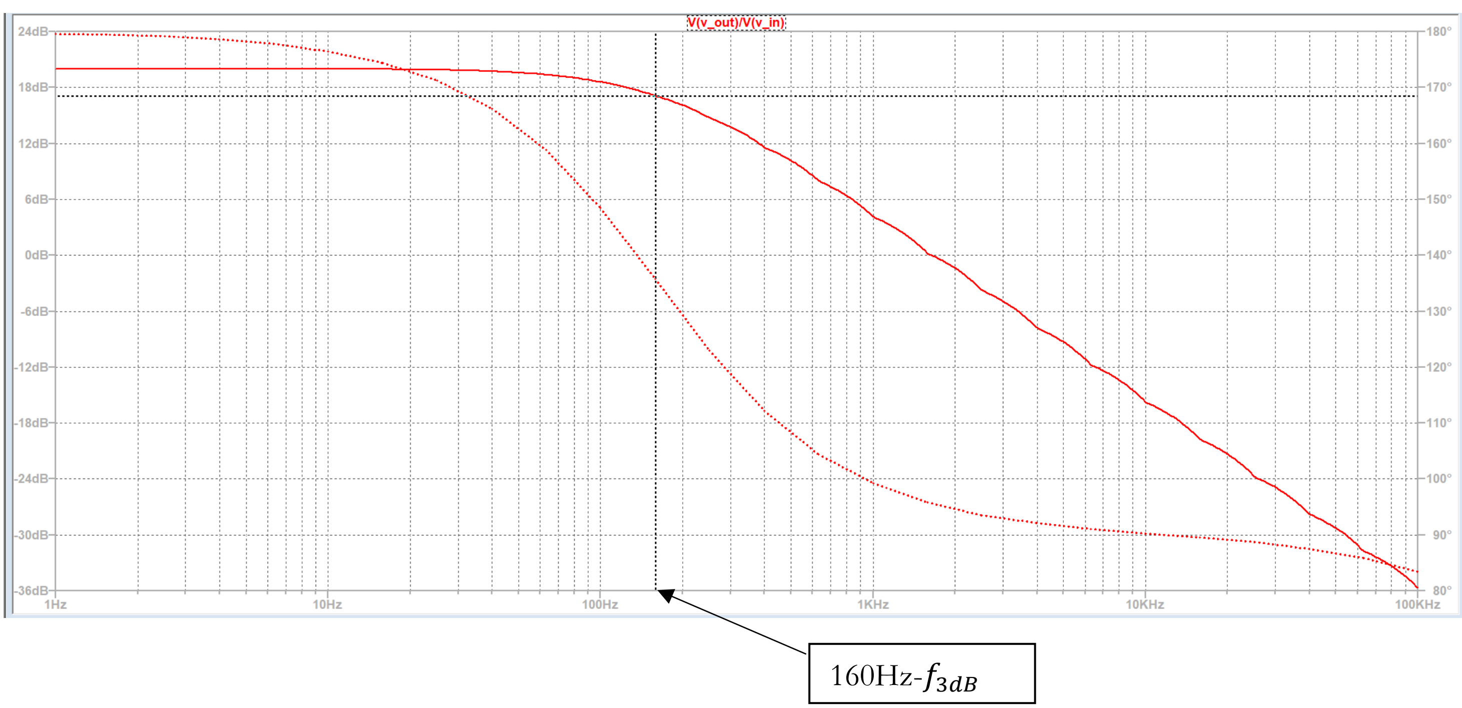
$$f_{3dB} = \frac{1}{2\pi f C R_1} = 160Hz\,(given)$$

$$\Rightarrow C = 1.32nF$$

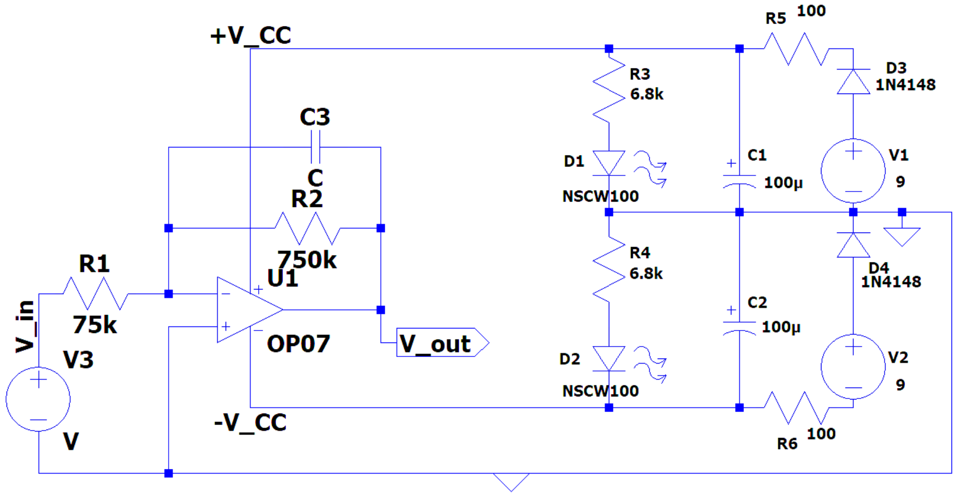
B.1.2) LTSpice simulation of the circuit frequency response **[5]**

With LTSpice simulation you can check the frequency response of your circuit design and verify that it does indeed work as a low pass filter. Use an ideal voltage source as V_{in} to your circuit, instead of the fixed frequency FG we are using in practice.
Discover the tools provided in LTSpice to sweep through a range of frequencies. Perform the required signal response analysis of your circuit design and show the Bode plots below, verifying the low pass frequency as designed in (B.1.1) above

Clearly, the circuit is an active low pass filter with corner frequency 160Hz.



Complete circuit:

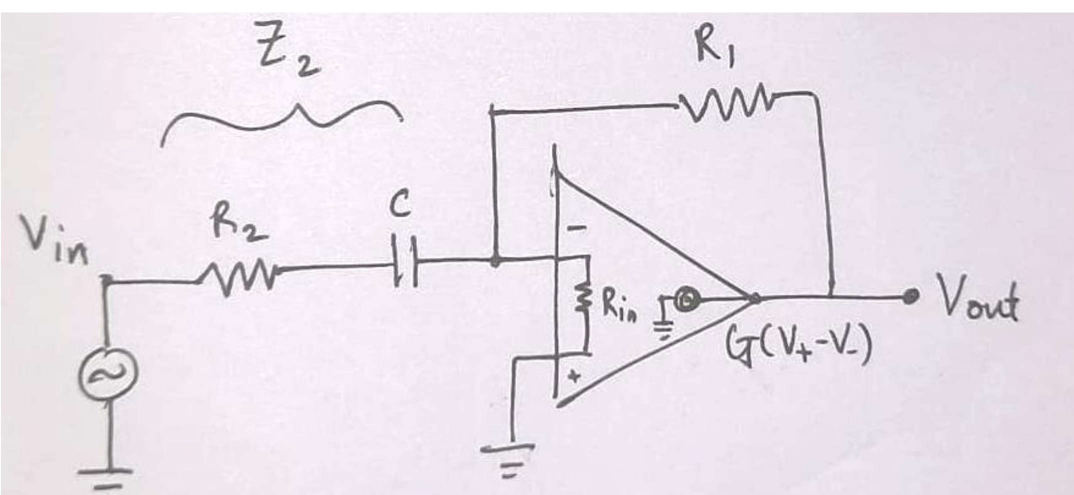


B.2) Differentiator / high pass filter

As in B.1, change the inverting negative feedback design such that the circuit works as a high-pass filter in the frequency domain, and a differentiator in the time domain.
The following parts are expected for the solution:

B.2.1) Circuit design and equation calculation steps [5]

Design your low pass filter with a corner frequency $f_{3dB} = 16kHz$
As before, keep the absolute magnitude of the gain as close to 10 as possible in the pass band, with the component values available in your kit.



Notice that the circuit skeleton is essentially the same as that of an inverting amplifier with just one addition, resistor at the input is replaced by a series combination of the reactance X_C of the capacitor and the resistor R_2.

$$Z_2 = X_C + R_2 = R_2 + \frac{1}{j\omega C}$$

Where $\omega = 2\pi f$ and $j = \sqrt{-1}$

Proceeding in the exact same fashion as the previous question we get:

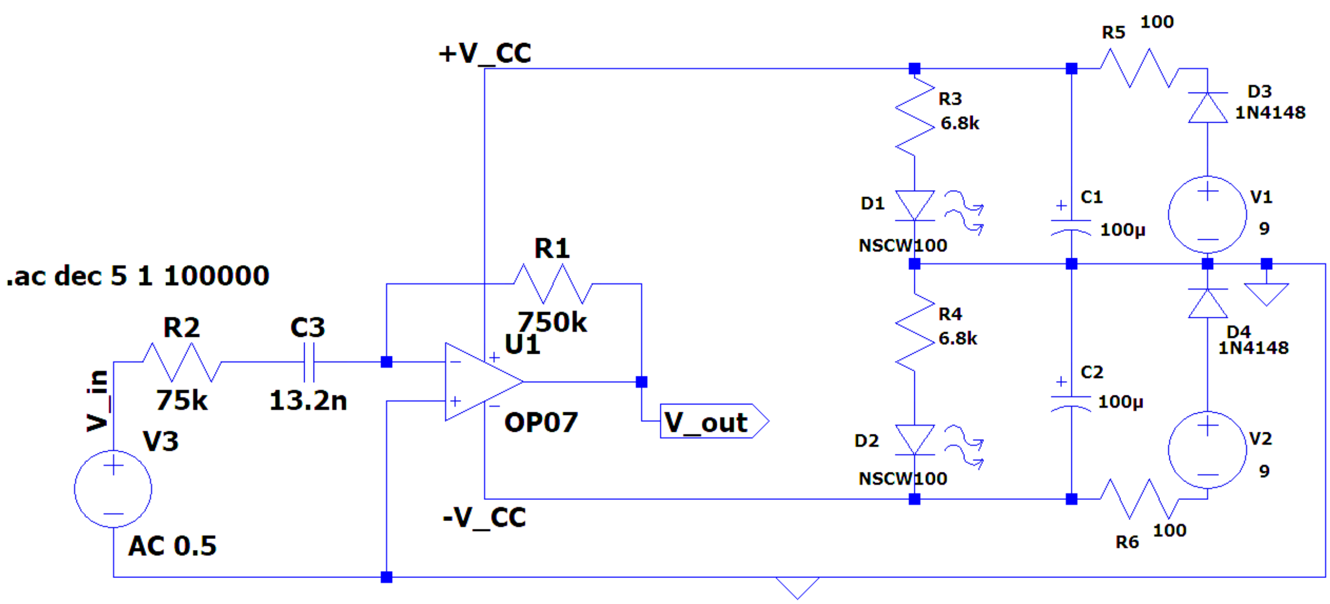
$$\therefore G_f = -\frac{R_1}{Z_2} = \frac{R_1}{X_C + R_2} = \frac{R_1 j\omega C}{R_2 j\omega C + 1}, |G_f| \simeq 10\ for\ large\ \omega$$

We see that this filter acts like a high pass filter up to reasonably high frequencies.
We have $R_2 = 75k\Omega\ and\ R_1 = 750k\Omega$
With $f_{3dB} = \frac{1}{2\pi C R_2} = 16kHz \Rightarrow C = 132pF$
But one might notice that the circuit really acts like a band-pass filter i.e., at extremely high frequencies the gain magnitude again starts to drop!

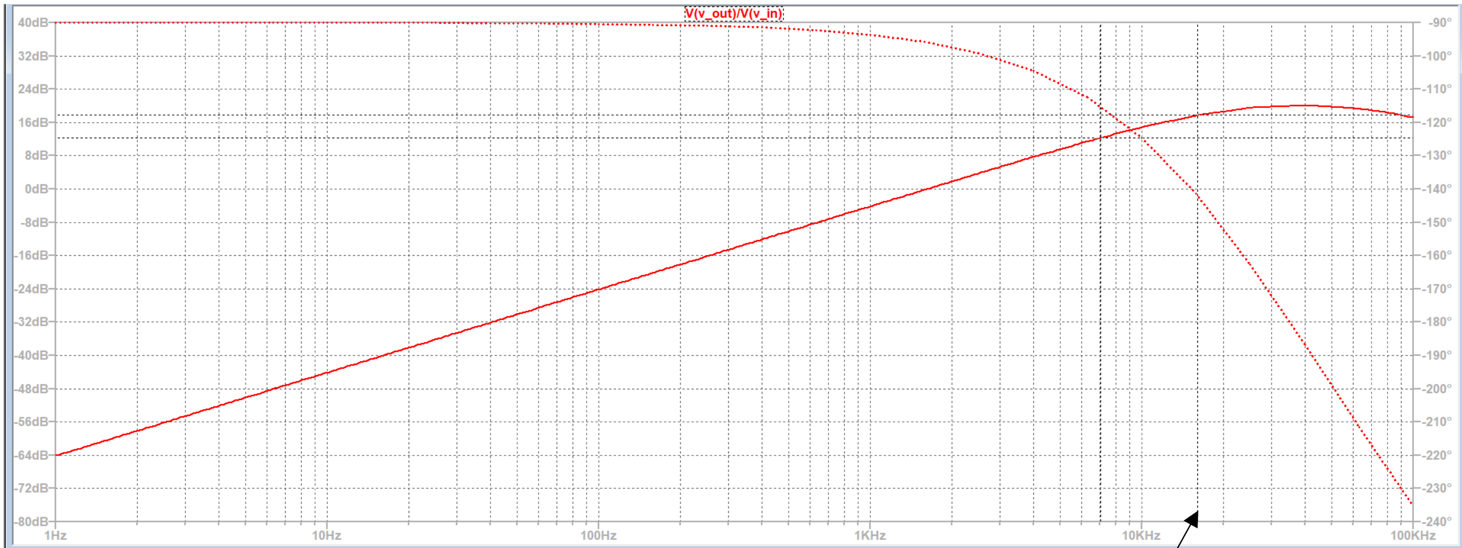
B.2.2) LTSpice simulation of the circuit frequency response [5]

As with the Integrator, run an ac signal analysis simulation to obtain a Bode plot of response $V_{in} \rightarrow V_{out}$ as a function of frequency proving that your design works as a high pass filter.

Circuit diagram:



Frequency response, Bode plot: High pass filter



$f_{3dB} = 16kHz$