4. *Calculate the group and phase velocities for the wave packet corresponding to a relativistic particle.

$$V_p = \frac{E}{p} \quad , \quad V_g = \frac{dE}{dp}$$

$$E = \sqrt{p^2c^2 + (m_0c^2)^2}$$

$$\Rightarrow V_p = \sqrt{c^2 + \left(\frac{m_0 c}{p}\right)^2 \cdot c^2} = c\sqrt{1 + \left(\frac{m_0 c}{p}\right)^2}$$

$$\Rightarrow V_g = \frac{1}{\sqrt{p^2c^2 + (m_0c^2)^2}} \cdot \frac{1}{\cancel{2}} \times \cancel{2}pc^2 = c \cdot \left[\frac{pc}{\sqrt{p^2c^2 + (m_0c^2)^2}}\right] = \frac{c}{\left[\sqrt{1 + \left(\frac{m_0 c}{p}\right)^2}\right]}$$

Clearly: $\boxed{V_g \cdot V_p = c^2}$

For a photon $V_g = V_p = c$ (in vacuum)

For a photon in medium of refractive index μ

The ratio btwn c and V_p is defined to be μ

$$\frac{c}{V_p} = \mu \Rightarrow \frac{c}{(\omega/k)} = \mu \Rightarrow \frac{ck}{\mu} = \omega \qquad \boxed{V_p = \frac{c}{\mu}}$$

$$\frac{d\omega}{dk} = \boxed{\frac{c}{\mu} = V_g}$$ $\longrightarrow$ This is true ~~is~~ if μ is same $\forall\, k$ i.e. $\frac{d\mu}{dk} = 0$

$$\therefore \boxed{V_g \cdot V_p = (c/\mu)^2}$$