Bound State : $E < V_0$

Scattering State : $E > V_0$ } for finite Box

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi(x) = E\psi(x)$$

Steps :

1) Solve TISE Independently in all regions

2) Apply Boundary Conditions
- Differentiability → for finite potential discont.
- Continuity → Must always hold

$$(\mathrm{I})\quad \psi_{\mathrm{I}}(x) = Ae^{-\alpha x} + Be^{+\alpha x} \qquad k = \sqrt{\frac{2mE}{\hbar^2}}$$

$$(\mathrm{II})\quad \psi_{\mathrm{II}}(x) = C\sin(kx) + D\cos(kx)$$

$$(\mathrm{III})\quad \psi_{\mathrm{III}}(x) = Ee^{-\alpha x} + Fe^{\alpha x} \qquad \alpha = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$$

Boundary Condition :

$\psi(x \to \pm\infty) = 0 \Rightarrow A = 0$ and $F = 0$
- $\psi_{\mathrm{I}} = Be^{\alpha x}$
- $\psi_{\mathrm{III}} = Ee^{-\alpha x}$

} Don't blow up

$$\psi(\underbrace{0^+}_{\text{RHL}}) = \psi(\underbrace{0^-}_{\text{LHS}}) \Rightarrow B = D \Rightarrow \psi_{\mathrm{II}} = C\sin(kx) + B\cos(kx)$$

$$\psi'(0^+) = \psi'(0^-) \Rightarrow B\alpha = Ck$$

$$\psi(L^-) = \psi(L^+) \Rightarrow Ee^{-\alpha L} = \left(\frac{B\alpha}{k}\right)\sin(kL) + B\cos(kL)$$

$$\psi'(L^-) = \psi'(L^+) \Rightarrow -\alpha Ee^{-\alpha L} = (B\alpha)\cos(kL) - Bk\sin(kL)$$

} Divide

$$-\alpha = \frac{\alpha\cos(kL) - k\sin(kL)}{\frac{\alpha}{k}\sin(kL) + \cos(kL)} \Rightarrow +\frac{\alpha}{k} = \frac{\sin(kL) - \frac{\alpha}{k}\cos(kL)}{\frac{\alpha}{k}\sin(kL) + \cos(kL)}$$

let $\alpha/k = \tan\theta$ $\qquad \tan(\theta - \phi) = \sim$

$$\tan\theta = \frac{\tan(kL) - \tan\theta}{\quad} \Rightarrow \tan\theta = \tan(kL-\theta)$$

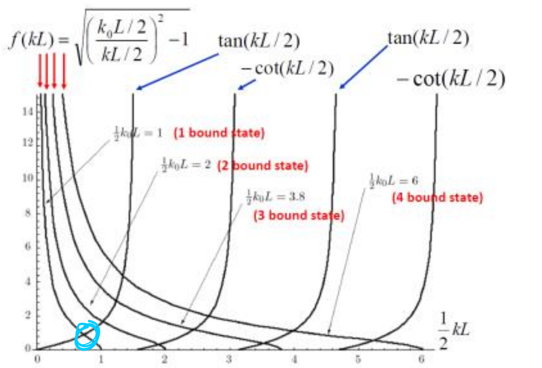
$$\theta = (kL - \theta) \pm n\pi$$

$$1 + \tan\theta \tan(kL)$$

$$2\theta = kL \pm n\pi \Rightarrow \theta = \frac{kL}{2} \pm \frac{n\pi}{2} = \tan^{-1}\left(\frac{\alpha}{k}\right)$$

$$\Rightarrow \frac{\alpha}{k} = \tan\left(\frac{kL}{2}\right) ; \quad \frac{\alpha}{k} = -\cot\left(\frac{kL}{2}\right)$$

$$\frac{\alpha}{k} = \sqrt{\frac{(k_0L/2)^2}{(kL/2)^2} - 1}$$

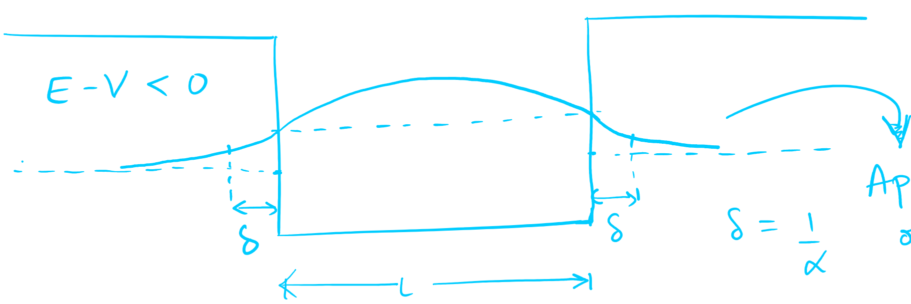


$k_0 = \sqrt{2mV_0/\hbar^2}$ As V_0 increases, it admits more and more bound states

⇝ Imp: for $E < V_0$ we have

At least one B.S.

G.S.:



$\delta = \frac{1}{\alpha}$

$$\frac{\hat{p}^2}{2m}$$

Apparent violation of KE > 0 Law?

NOTE: (E - V) in QM is NOT the "usual" kinetic energy

NO

Ehrenfest Theorem: EXPECTATION VALUES OBEY CLASSICAL LAWS

$\therefore \langle E - V \rangle \equiv$ "Usual" K.E $\Rightarrow \langle E - V \rangle > 0$ for Every W.F.