Maxwell Boltzmann statistics for IDEAL GASES : you can track electrons

- The particles are identical in terms of physical properties but distinguishable in terms of position, path, or trajectory. It will be demonstrated later in this chapter that this assumption is equivalent to the statement that the particle size is small compared with the average distance between particles.
- The equilibrium distribution is the most probable way of distributing the particles among various allowed energy states subject to the constraints of a fixed number of particles and fixed total energy.
- There is no theoretical limit on the number of particles in a given energy state, but the density of particles is sufficiently low and the temperature sufficiently high that no more than one particle is likely to be in a given state at the same time.

**Assumptions of the Maxwell–Boltzmann distribution**

- Postulate: Every $\mu$-state is as Likely as any other
- Obs: we are more likely to find the energy uniformly distributed among all particles rather than being conc. to a few.

for $N \equiv$ No. of particles ~ Large

$$\boxed{f_{MB} = A e^{-E_i/k_B T}}$$

~ $f_{MB} \equiv$ Probab of finding particle with energy $E_i$

If Energy state $E_i$ is $g_i$ fold degenerate then $\boxed{n_i = g_i f_{MB}(E_i)}$

for a Continuum of Energy values (allowed energy states are very closely spaced)

$g_i \rightarrow g(E)$ ~ Density of states (Deg.)

$f_{MB} \rightarrow A e^{-E/k_B T}$

$$\Rightarrow \boxed{n(E) = g(E) f_{MB}(E)}$$

$n(E) = \dfrac{1}{V}\dfrac{dN}{dE} \equiv$ No of particles per unit Vol per unit energy

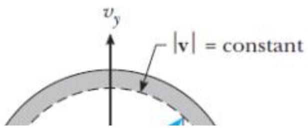
$\Downarrow$

$$\frac{dN}{V} = n(E)\,dE \Rightarrow \frac{N}{V} = \int_0^\infty n(E)\,dE$$

Now consider a Non interacting Ideal Gas

$$E(v) = \frac{M}{2}v^2 \equiv KE \Rightarrow n(E)\,dE = \underline{\underline{g(E)}}\, A e^{-mv^2/2k_B T}\, dE$$

Density of state (Degenerate)




No. of states with velo btwn $v$ and $v+dv$

$\propto$ Vol of Shell

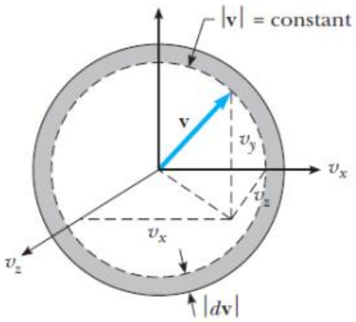



**Figure 10.5** Velocity space. The number of states with speeds between $v$ and $v + dv$ is proportional to the volume of a spherical shell with radius $v$ and thickness $dv$.

$\propto$ Vol of Shell

$\Rightarrow f(v)\,dv = C \cdot 4\pi v^2 dv$

since E is an explicit func of velo

$g(E)\,dE = C \cdot 4\pi v^2 dv \Rightarrow n(E)\,dE = A \cdot 4\pi v^2 e^{-mv^2/2k_BT} dv$

$\Rightarrow n(v)\,dv = \frac{N}{V}\left(\frac{m}{2\pi k_B T}\right)^{3/2} \times 4\pi v^2 e^{-mv^2/2k_BT} dv$

A was found using $\frac{N}{V} = \int_0^\infty n(v)\,dv$

$$\langle v \rangle = \bar{v} = \text{mean/avg} = \frac{\int_0^\infty v\,(n(v))\,dv}{N/V} = \sqrt{\frac{8k_BT}{\pi m}}$$

$$\langle v^2 \rangle = \overline{v^2} = \text{RMS} = \frac{\int_0^\infty v^2\, n(v)\,dv}{N/V} = \sqrt{\frac{3k_BT}{m}}$$

It is useful to develop a criterion to determine when the classical distribution is valid. We may say that the **Maxwell–Boltzmann distribution is valid when the average distance between particles, $d$, is large compared with the quantum uncertainty in particle position, $\Delta x$, or**

$$\Delta x \ll d \tag{10.15}$$

Look at See 10.2 Serway

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Quantum distributions:

Ex. 2 $e^-$ WF overlap → Can't comment which part is which → We can NO MORE TRACK these $e^-$

Consider an Ex: 2 particles in a 1D $\infty$ well (NON Interacting)

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2}\psi(x_1, x_2) - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2}\psi(x_1, x_2) = E\,\psi(x_1, x_2)$$

Since $V(x_1, x_2) = 0 \rightsquigarrow \psi(x_1, x_2) = \phi(x_1)\cdot\phi(x_2)$ – Seperable

Now if $E = E_{n_1,n_2} = (n_1^2 + n_2^2)E_0$

Then what is the state of the system?

$\phi_{n_1}(x_1)\cdot\phi_{n_2}(x_2)$ or $\phi_{n_2}(x_1)\cdot\phi_{n_1}(x_2)$

Ans : Superposition of Both :

For 2x Fermions Pauli exclusion $\Rightarrow n_1 \neq n_2$ (Same Quantum State NOT ALLOWED)

$$\Rightarrow \psi_{fermions}(x_1, x_2) = \frac{\phi_{n_1}(x_1)\,\phi_{n_2}(x_2) - \phi_{n_2}(x_1)\cdot\phi_{n_1}(x_2)}{\sqrt{2}}$$

$$\psi_f(x_1, x_2) = -\psi_f(x_2, x_1)$$

For 2x Bosons

$$\Rightarrow \psi_{Bosons}(x_1, x_2) = \frac{\phi_{n_1}(x_1)\,\phi_{n_2}(x_2) + \phi_{n_2}(x_1)\cdot\phi_{n_1}(x_2)}{\sqrt{2}}$$

Can be Generalized further (in slides)

for Bosons $\psi_{b,n_1=n_2} = \sqrt{2}\,\phi_n(x_1)\,\phi_n(x_2) \Rightarrow |\psi_{b,n_1=n_2}|^2 = 2\times|\psi_{MB}|^2$

$$\boxed{f_{BE}(E) = \frac{1}{Be^{E/k_BT}-1} \quad \& \quad f_{FD} = \frac{1}{e^{(E-E_F)/k_BT}+1}}$$

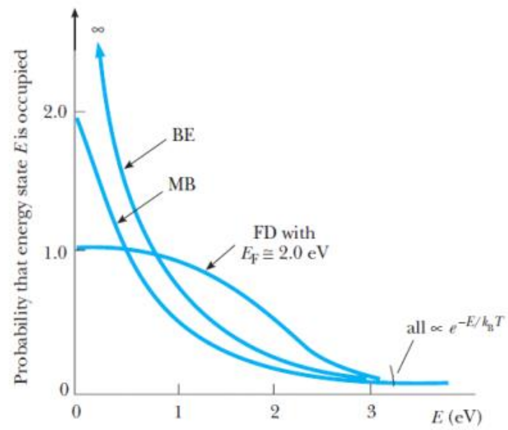
Bose-Einstein

Fermi Dirac

$B=1$ for photons & phonons

**This expression shows the meaning of the Fermi energy: The probability of finding an electron with an energy equal to the Fermi energy is exactly 1/2 at any temperature.**

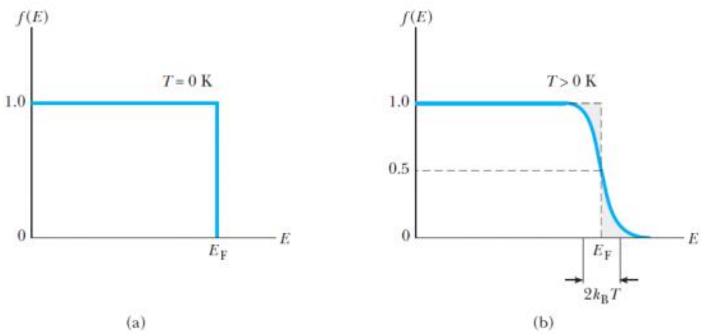
A plot comparing the Maxwell–Boltzmann, Bose–Einstein, and Fermi–Dirac distributions as functions of energy at a common temperature of 5000 K is shown in Figure 10.8. Note that for large $E$, all occupation probabilities decrease to zero as $e^{-E/k_BT}$. For small values of $E$, the FD probability saturates at 1 as required by the exclusion principle, the MB probability constantly increases but remains finite, and the BE probability tends to infinity. This very high probability for bosons to have low energies means that at low temperatures most of the particles drop into the ground state. When this happens, a new phase of matter with different physical properties can occur. This change in phase for a system of bosons is called a **Bose–Einstein condensation (BEC),** and it occurs in liquid helium



$$g(k)\ dk = \frac{k^2\ dk}{2\pi^2} \tag{3.44}$$

To apply this expression to electrons in a metal, we must multiply it by a factor of 2 to account for the two allowed spin states of an electron with a given momentum or energy:

$$g(k)\ dk = \frac{k^2\ dk}{\pi^2} \tag{10.36}$$




(a) (b)

~ Have a quick look @ section 10.5 for Appl$^n$ of FD statistics

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$Q(N_i) \rightarrow$ No of ways that we can get a particular E config

Classical (MB) :

$$\boxed{Q_{MB}(N_i) = \frac{N!}{\prod_i (N_i)!} \prod_i (g_i)^{N_i}}$$

$N \equiv$ Total ; $N_i \equiv$ particles with $E_i$ ; $g_i \equiv$ Deg of $E_i$

$\Rightarrow \sum_i N_i = N$ & $E = \sum_i E_i N_i$

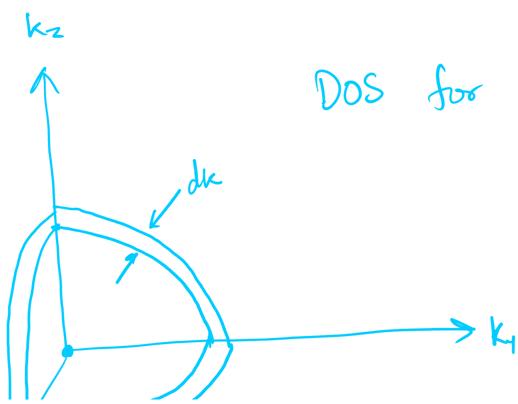
Fermions :

$$\boxed{Q_F(N_i) = \prod_i \left( \frac{g_i!}{N_i!\,(N_i - g_i)!} \right)}$$

Bosons :

$$\boxed{Q_B(N_i) = \prod_i \left( \frac{(N_i + g_i - 1)!}{N_i!\ (g_i - 1)!} \right)}$$

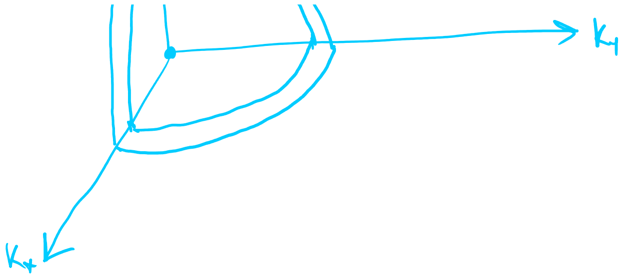
} Indistinguishable.




DOS for hv

$$L_x = \frac{n\lambda}{2} = \frac{n\pi}{k_x}$$

$$L_y = \frac{n\pi}{k_y} \ ; \ L_z = \frac{n\pi}{k_z}$$




$$k^2 = k_x^2 + k_y^2 + k_z^2 \equiv$$ Particles with Same wave no $\Downarrow$ same En

We essentially want to calculate no of units of k in a volume element

$$E = \frac{hc}{\lambda} = \hbar k c = h\nu$$

No of states $\propto \dfrac{4\pi k^2 dk}{8}$

$$g(k)\,dk = \frac{1}{V}\left(\frac{4\pi k^2\,dk}{8V_0}\right) = \frac{1}{\cancel{L_xL_yL_z}} \times \frac{4\pi k^2\,dk}{8(\pi)^3}\,\cancel{L_xL_yL_z} = \frac{4k^2dk}{8\pi^2}$$

$$g(k)\,dk = g(\nu)\,d\nu = \left(\frac{8\pi\nu^2\,d\nu}{2c^3}\right)$$

$$\left\{\begin{array}{l} \dfrac{kc}{2\pi} = \nu \\ k = \left(\dfrac{2\pi}{c}\nu\right) \end{array}\right\}$$

for 2 $\perp$ polarizations of photons: $\boxed{g(\nu)\,d\nu = \dfrac{8\pi\nu^2\,d\nu}{c^3}}$

$\Rightarrow$ Using BE: $\quad n(\nu)\,d\nu = g(\nu)\,f_{BE}(\nu)\,d\nu$

$$\Rightarrow u(\nu)\,d\nu = g(\nu) \times h\nu \times f_{BE}(\nu)\,d\nu$$

$$\Rightarrow \boxed{u(\nu)\,d\nu = \left(\frac{8\pi\nu^2}{c^3}\right) \times \frac{h\nu}{e^{h\nu/k_BT}+1}\,d\nu}$$