# Sequence & Series

## Lecture-9

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Type-4

# How to Identify Type - 4

In the given series if difference b/w 2 consecutive terms forms an A.P or G.P then the problem belongs to Type-4 which is also known as **method of difference**

e.g. i) 1, 3, 6, 10, 15, 21 → Type-4 M.O.D.

Differences: 2, 3, 4, 5, 6

Differences are in A.P.

9      81

ii> 1, 4, 13, 40, 121. → Type-4

3      27

M.O.D.

Differences are in G.P.

# Steps to solve any problem of M.O.D.

1. Just write $T_r$

2. Rewrite the series by shifting one term each.

3. Subtract to find $T_r = ?$

4. $S_n = \sum_{r=1}^{n} T_r$ "Type-1"

Q.1 Find sum of series upto n terms:

$$S = 6 + 13 + 22 + 33 + 46 + \text{------}$$

Soln

7 11

$$S = 6 + 13 + 22 + 33 + 46 + \text{------}$$

9 13

To find: $S_n = ?$ $\rightsquigarrow$ M.O.D.

Differences form an A.P.

$$S = 6 + 13 + 22 + 33 + 46 + \cdots + T_r$$

$$S = \quad 6 + 13 + 22 + 33 + \cdots + T_r$$

$$- \quad - \quad - \quad - \quad - \quad - \quad -$$

$$0 = 6 + 7 + 9 + 11 + 13 + \cdots - T_r$$

Sum of A.P of "r-1" terms.

$$T_r = 6 + 7 + 9 + 11 + 13 + \cdots$$

$$T_r = 6 + \frac{r-1}{2}\left[2.7 + (r-1-1).2\right]$$

$$T_r = 6 + \frac{r-1}{2}\left[2.7 + (r-1-1).2\right]$$

$$T_r = 6 + \frac{r-1}{2}\left[14 + 2r - 4\right]$$

$$T_r = 6 + \frac{r-1}{2}\left[2r + 10\right]$$

$$T_r = 6 + (r-1)(r+5)$$

$$T_r = 6 + r^2 + 4r - 5$$

$$T_r = r^2 + 4r + 1$$

$$S_n = \sum_{r=1}^{n} r^2 + 4r + 1$$

$$= \sum_{r=1}^{n} r^2 + 4 \sum_{r=1}^{n} r + \sum_{r=1}^{n} 1$$

$$= \frac{n(n+1)(2n+1)}{6} + 4 \cdot \frac{n(n+1)}{2} + n$$

Q. 2 Find sum of series upto n terms :

$$S = 5 + 7 + 13 + 31 + 85 + \dots$$

Sol$^{\underline{n}}$

6      54

$$S = 5 + 7 + 13 + 31 + 85 + \dots$$

2      18

M.O.D.

To find : $S_n = ?$

Differences are in G.P.

$$S = 5 + 7 + 13 + 31 + 85 + \_\_\_\_ \_\_\_\_ + T_r$$

$$S = \quad 5 + 7 + 13 + 31 + \_\_\_\_ \_\_ + T_r$$

$$- \quad - \quad - \quad - \quad - \quad -$$

$$0 = 5 + 2 + 6 + 18 + 54 + \_\_ \_\_ - T_r$$

$$T_r = 5 + 2 + 6 + 18 + 54 + \_\_\_\_\_\_$$

↳ Sum of G.P of

r − 1 terms.

$$T_r = 5 + \frac{2\left[3^{r-1} - 1\right]}{3 - 1}.$$

$$T_r = 5 + \frac{\cancel{2}\left[3^{r-1} - 1\right]}{\cancel{3-1}}.$$

$$T_r = 5 + 3^{r-1} - 1$$

$$T_r = 3^{r-1} + 4$$

$$S_n = \sum_{r=1}^{n} 3^{r-1} + 4$$

$$S_n = \sum_{r=1}^{n} 3^{r-1} + 4$$

$$S_n = \sum_{r=1}^{n} 3^{r-1} + 4 \sum_{r=1}^{n} 1$$

$$S_n = 1 + 3 + 9 + \_\_\_ + 3^{n-1} + 4n$$

Sum of G.P of "n terms".

$$S_n = 1 + 3 + 9 + \_\_\_ + 3^{n-1} + 4n$$

$$S_n = \frac{1\left(3^n - 1\right)}{3 - 1} + 4n$$

$$S_n = \frac{3^n - 1}{2} + 4n$$ Ans.

Type-5

# Application of most general formula of $T_n$

$$\boxed{T_n = S_n - S_{n-1}}$$

## TJN

Jab bhi kisi sawaal mai $S_n$ diya jaaye to hamesha yehi formula lagana chahe kuch bhi pucha ho.

Q.3 If sum of n terms of an A.P. is given by $S_n = a + bn + cn^2$. Then find its common difference.

Sol$\underline{^n}$ A.P.

$$S_n = a + bn + cn^2$$

$$T_n = S_n - S_{n-1}$$

$$T_n = (a + bn + cn^2) - \left[a + b(n-1) + c(n-1)^2\right]$$

$$T_n = (a + bn + cn^2) - \left[a + b(n-1) + c(n-1)^2\right]$$

$$T_n = \cancel{a} + \cancel{bn} + \cancel{cn^2} - \cancel{a} - \cancel{bn} + b - \cancel{cn^2} + 2cn - c$$

$$T_n = 2cn + b - c$$

To find : $d = ?$

$$d = T_2 - T_1$$

$$= (4c + b - c) - (2c + b - c)$$

$$d = 2c$$ Ans.

Q.4 If for the given series sum of n terms is $n(n+1)(n+2)$. Then find $\sum_{n=1}^{\infty} \frac{1}{T_n}$ where $T_n$ is the $n^{th}$ term of given series.

Imp for J. M ****

J. Adv **

Sol$^{\underline{n}}$ Given Series :

$$S_n = n(n+1)(n+2)$$

$$T_n = S_n - S_{n-1}$$

$$S_n = n(n+1)(n+2)$$

$$T_n = S_n - \boxed{S_{n-1}}$$

$$T_n = n(n+1)(n+2) - (n-1)(n-1+1)(n-1+2)$$

$$T_n = n(n+1)(n+2) - (n-1)\,n\,(n+1)$$

$$T_n = n(n+1)\left[\cancel{n} + 2 - \cancel{n} + 1\right]$$

$$T_n = 3n(n+1)$$

To find : $\displaystyle\sum_{n=1}^{\infty} \frac{1}{T_n}$

$$\Rightarrow \sum_{n=1}^{\infty} \frac{1}{3n(n+1)}$$

$$T_r = \frac{1}{3r(r+1)}$$

Type - 3

$S_\infty = ?$

$$T_r = \frac{1}{3r(r+1)}\left[(r+1) - r\right]$$

$$T_r = \frac{1}{3}\left[\frac{1}{r} - \frac{1}{r+1}\right]$$

$$T_1 = \frac{1}{3}\left[\frac{1}{1} - \cancel{\frac{1}{2}}\right]$$

$$T_2 = \frac{1}{3}\left[\cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}\right]$$

$$\vdots$$

$$T_n = \frac{1}{3}\left[\cancel{\frac{1}{n}} - \frac{1}{n+1}\right]$$

$$S_n = \frac{1}{3}\left[1 - \frac{1}{n+1}\right]$$

$$S_\infty = \frac{1}{3}\left[1 - \cancel{\frac{1}{\infty}}^{\,0}\right]$$

$$S_\infty = 1/3$$

Q.4

If $\sum_{r=1}^{n} I(r) = (3^n - 1)$, then $\sum_{r=1}^{n} \frac{1}{I(r)}$ is equal to:

**(A)** $2\left(1-\left(\frac{1}{3}\right)^n\right)$ **(B)** $\left(1-\left(\frac{1}{3}\right)^n\right)$ **(C)** $\frac{3}{4}\left(1-\left(\frac{1}{3}\right)^n\right)$ **(D)** $\frac{4}{3}\left(1-\left(\frac{1}{3}\right)^n\right)$

Sol$\underline{^n}$

Given

general term

$$\sum_{r=1}^{n} I(r) = S_n = 3^n - 1$$

$$T_n = S_n - S_{n-1}$$

$$= (3^n - \cancel{1}) - (3^{n-1} - \cancel{1})$$

$$T_n = 3^n - 3^{n-1}$$

$$T_n = 3^{n-1}[3 - 1]$$

$$T_n = 2 \cdot 3^{n-1}$$

general term.

$$I(r) = 2 \cdot 3^{r-1}$$

To find : $\displaystyle\sum_{r=1}^{n} \frac{1}{I(r)}$

$$\frac{1}{2} \sum_{r=1}^{n} \frac{1}{3^{r-1}}$$

$$\Rightarrow \frac{1}{2}\left[\underbrace{\frac{1}{1} + \frac{1}{3} + \frac{1}{3^2} + \text{——} + \frac{1}{3^{n-1}}}_{\text{Sum of G. P of n terms.}}\right]$$

Q . 5

If $\sum_{r=1}^{n} I(r) = n(2n^2 + 9n + 13)$, then $\sum_{r=1}^{n} \sqrt{I(r)}$ is equal to:

**(A)** $\frac{\sqrt{3}}{2}\left(n^2 + 3n\right)$ **(B)** $\frac{3}{2}\left(n^2 + 3n\right)$ **(C)** $\sqrt{\frac{3}{2}}\left(n^2 + 3n\right)$ **(D)** $\frac{3}{\sqrt{2}}\left(n^2 + 3n\right)$

# Home Assignment

Q.No. 46-49,51,56-58,60,62,80,82,94-96,98,99,103