20BCE204

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Assignment 1

1. Find correlation coefficient and equations of regression lines.

X	Y	XY	X^2	Y^2
1	2	2	1	4
2	5	10	4	25
3	3	9	9	9
4	8	32	16	64
5	7	35	25	49
15	25	88	55	151

$\bar{x} = \frac{15}{5} = 3$

$\bar{y} = \frac{25}{5} = 5$

Regression line y on x:

$$b_{yx} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$b_{yx} = \frac{5(88) - 15(25)}{5(55) - (15)^2} = \frac{440 - 375}{275 - 225}$$

$$b_{yx} = \frac{65}{50} = 1.3$$

Regression line x on y

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - 5 = 1.3(x - 3)$$

$$y = 1.3x + 1.1$$

$$b_{xy} = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum y^2 - (\sum y)^2}$$

$$= \frac{5(88) - 15(25)}{5(151) - (25)^2} = \frac{440-375}{775-625} = \frac{65}{130}$$

$$= 0.5$$

Regression line x On y

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$x - 3 = 0.5(y - 5)$$

$$x = 0.5y + 0.5$$

Correlation coefficient $r^2 = 1.3 \times 0.5$

$$= 0.65$$

$$r = 0.806$$

2. Paper 1 = X Paper 2 = Y

X	Y	XY	X^2	Y^2
80	81	6480	6400	6561
45	56	2520	2025	3136
55	50	2750	3025	2500
56	48	2688	3136	2304
58	60	3480	3364	3600
60	62	3720	3600	3844
65	64	4160	4225	4096
68	65	4420	4624	4225
70	70	4900	4900	4900
75	74	5550	5625	5476
85	90	7650	7225	8100
717	720	48318	48149	48742

Regression line y on x :

$$b_{yx} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

$$= \frac{11 \cdot 48318 - 717(720)}{11(48149) - (717)^2}$$

$$= \frac{15258}{15550}$$

$$b_{yx} = 0.9812$$

$\bar{x} = \frac{717}{11} = 65.18$ $\qquad$ $\bar{y} = \frac{720}{11} = 65.45$

~~Regression line~~

$$\cancel{x - \bar{x} = b_{xy}(y - \bar{y})}$$

$$\cancel{x - 65.18 = 0.98(y - 65.45)}$$

$$\cancel{x = 0.859y + 8.9547}$$

Regression line y on x

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$y - 65.45 = 0.98(x - 65.18)$$

$$y = 0.9812x - 1.49$$

$$b_{xy} = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum y^2 - (\sum y)^2}$$

$$b_{xy} = \frac{11(48318) - 717(720)}{11(48742) - (720)^2} = \frac{15258}{17762} = 0.86$$

Regression line x on y

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$x - 65.18 = 0.859(y - 65.45)$$

$$x = 0.859y + 8.9547$$

Coefficient of correlation $r = \sqrt{0.98 \times 0.86}$

$$r = 0.9180$$

3. Two regression coefficients are 0.8 and 0.2

$\therefore$ coefficient of correlation

$$r^2 = 0.8 \times 0.2$$

$$r = 0.4$$

4. $\sigma_x^2 = 9 \qquad \sigma_x = 3$

$$8x - 10y + 66 = 0$$

$$40x - 18y - 214 = 0$$

a) Mean of x and y

$$40x - 18y - 214 = 0$$

$$\underline{40x - 50y + 330 = 0}$$

$\therefore y = 17$ & $x = \cancel{8}\ 13$

$\therefore$ Hence $\bar{x} = 13 \qquad \bar{y} = 17$

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c) Coefficient of correlation

$$x = \frac{18}{40}y + \frac{214}{40} \qquad y = \frac{8}{10}x + \frac{66}{10}$$

$$x = 0.45y + 5.35 \qquad y = 0.8x + 6.6$$

$$b_{xy} = 0.45 \qquad b_{yx} = 0.8$$

$$\therefore r^2 = 0.45 \times 0.8$$

$$r = 0.6$$

b) Standard deviation of y

$$b_{xy} = r\frac{\sigma_x}{\sigma_y}$$

$$0.45 = 0.6 \times \frac{3}{\sigma_y}$$

$$\sigma_y = 4$$

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5. $20x - 9y - 107 = 0$ $\sigma_x^2 = 9$
$4x - 5y + 33 = 0$ $\sigma_x = 3$

a) Mean values of x and y

$$\begin{array}{r} 20x - 9y - 107 = 0 \\ 20x - 25y + 165 = 0 \\ - \quad + \quad - \\ \hline 16y = 272 \end{array}$$

$y = 16$

$\therefore x = 11.75$

Mean of $x = 11.75$ and $y = 16$

b) standard deviation of y

$x = \frac{9}{20} y + \frac{107}{20}$ $y = 0.8x + 6.6$

$x = 0.45 y + 107/20$

$\therefore b_{xy} = 0.45$ $b_{yx} = 0.8$

$r^2 = 0.45 \times 0.8$

$r = 0.6$

std. dev of y $b_{xy} = r \frac{\sigma_x}{\sigma_y}$

$\sigma_y = 4$

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6. $3x + 2y = 26$
$6x + y = 31$

a) Mean Values

$\therefore 6x + y = 31$
$6x + 4y = 52$
$- \quad - \quad -$

$-3y = -21$
$y = 7$
$\therefore x = 4$

$\therefore$ Mean of $x = 4$
Mean of $y = 7$

b) $3x + 2y = 26$ $\qquad$ $6x + y = 31$

$y = \frac{-3}{2}x + \frac{26}{2}$ $\qquad$ $x = \frac{-1}{6}y + \frac{31}{6}$

$b_{yx} = -1.5$ $\qquad$ $b_{xy} = -0.1667$

$r = \sqrt{-1.5 \times (-0.1667}$

$r = \sqrt{0.25}$

$r = -0.5$ $\qquad$ as b_{xy} & b_{yx} are -ve

7. Pearson's coefficient of correlation

X	Y	dx	dy	dx^2	dy^2	dx.dy
160	292	-5	33	25	1089	-165
164	280	-3	21	9	441	-63
172	260	1	1	1	1	1
182	234	6	-25	36	625	-150
166	266	-2	7	4	49	-14
170	254	0	-5	0	25	0
178	230	4	-29	16	841	-116
		1	3	91	3071	507

$\bar{x} = \frac{1192}{7}$ $\bar{y} = \frac{1816}{7}$

$x = 170.28$ $= 259.42$

Assumed mean $A = 170$ $B = 259$

$dx = \frac{x - 170}{2}$ $dy = \frac{y - 259}{1}$

$$b_{yx} = \frac{n \sum dx\, dy - (\sum dx)(\sum dy)}{n \sum dx^2 - (\sum dx)^2}$$

$$= \frac{7(-507) - 1(3)}{7(91) - (1)^2}$$

$$= \frac{-3552}{636}$$

$$= -5.58$$

Regression line y on x

$y - \bar{y} = b_{yx}(x - \bar{x})$

$y - 259.42 = -5.58(x - 170.28)$

$y = -5.58x + 1210.45$

$$b_{xy} = \frac{n\sum dxdy - (\sum dx)(\sum dy)}{n\sum dy^2 - (\sum dy)^2}$$

$$= \frac{7(-507) - 1(3)}{7(3071) - (3)^2}$$

$$= \frac{-3552}{21488}$$

$$b_{xy} = -0.1653$$

Regression line x on y

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$x - 170.28 = -0.1653(y - 259.42)$$

$$x = -0.165y + 213.169$$

8. $PV^{\gamma} = c$

$$V = \left(\frac{c}{P}\right)^{\frac{1}{\gamma}} = c^{\frac{1}{\gamma}} P^{-\frac{1}{\gamma}}$$

$$\therefore \log V = \frac{1}{\gamma}\log c - \frac{1}{\gamma}\log P$$

$$Y = A + BX$$

$$\therefore Y = \log V \quad X = \log P \quad A = \frac{1}{\gamma}\log c$$

$$B = -\frac{1}{\gamma}$$

P	V	X	Y	XY	X^2
0.5	1.6	-0.301	0.2041	-0.061	0.09062
1	1	0	0	0	0
1.5	0.75	0.1760	-0.1249	-0.0219	0.03101
2	0.62	0.3010	-0.2076	-0.0624	0.09062
2.5	0.52	0.3979	-0.2839	-0.1128	0.1583
3	0.46	0.4771	-0.3372	-0.1608	0.2276
		1.051	-0.7495	-0.255	0.59825

$$\Sigma y = nA + \Sigma x B$$

$$\Sigma xy = \Sigma x A + \Sigma x^2 B$$

$$\therefore -0.7495 = 6A + 1.051B$$

$$-0.255 = 1.051A + 0.5925B$$

$$\therefore A = 0.09124$$

$$B = -0.592241$$

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$\gamma = \frac{-1}{B} = \frac{1}{0.592241}$

$\gamma = 1.688$

$\log c = \gamma A$

$= 1.688(0.09124)$

$\log c = 0.1540$

$c = 1.4256$

$\therefore \quad PV^{1.688} = 1.4256$

Pressure = 3.5

$V^{1.688} = \frac{1.4256}{3.5}$

$V^{1.688} = 0.4073$

$V = (0.4073)^{\frac{1}{1.688}}$

$V = 0.5873$